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Abstract

We analyze an oligopoly where public and private firms compete in quantity and R&D. Using general functions, we show that an output subsidy and an R&D tax can achieve the first-best allocation. Moreover, the degree of privatization does not influence the optimal output subsidy but does influence the optimal R&D tax.

JEL Classification: H42; L13; L32;

Keywords: R&D subsidy; Output subsidy; Mixed oligopoly; Partial privatization

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1 Introduction

The relationship between subsidization and privatization has been discussed intensively in the existing literature on mixed oligopolies. [9] showed that a uniform output subsidy yields the first-best outcome from the viewpoint of welfare in mixed and private oligopolies. Subsequent studies proved that his result is quite robust in various economic circumstances; [8] considered the order of firms' moves; [3] generalized firms' objective functions; and [6] extended the model of [9] to output regulation. These results are known as *privatization neutrality theorem* (PNT), which claims that the first-best allocation should be achieved under an identical, uniform subsidy to whatever extent a public firm is privatized.

The existing studies on subsidized mixed oligopolies have focused mainly on the effect of output subsidies on production allocation. In particular, they have assumed that public and private firms have a given identical production technology. However, R&D efforts could work to improve firms' technologies, thereby affecting production allocation. Inevitably, social benefits depend not only on the production allocation but also on an allocation of firms' R&D investments. As such, adjusting the allocations of production and R&D is required to achieve the first-best outcome.

As a key to the adjustment of both allocations of production and R&D, we consider a policy mix of output and R&D subsidies. Recently, some existing works have analyzed an impact of subsidies in mixed oligopolies with R&D activities. [2] showed that the socially optimal R&D subsidy increases total R&D and production, but it does not lead to an efficient distribution of production costs. [4] showed that an R&D subsidy gives rise to higher (res. lower) welfare than an output subsidy when the extent of R&D spillovers is high (res. low). However, these studies focused only on a situation in which a single subsidy policy is employed. Instead, considering both output and R&D subsidies, we examine how they affect the allocations of production and R&D investments. In particular, we analyze whether the PNT holds in the presence of R&D activities.

2 Model

Consider an industry with $(n+1)$ firms producing a homogeneous good and engaging in cost-reducing R&D investments. We define the set of firms by $N = \{0, 1, 2, \cdots, n\}$. Firm 0 is a public firm and firm $j \in N \setminus \{0\}$ is a private firm. Let $P(Q)$ be the inverse demand function, where $Q = \sum_{i=0}^{n} q_i$ is the total market output and q_i is the output of firm $i \in N$. Let $C(q_i, x_i)$ and $\Gamma(x_i)$ be the costs of production and R&D of firm $i \in N$, respectively, where x_i is the amount of R&D. We use a bold character to represent a vector, such as $\boldsymbol{q} = (q_0, q_1, \cdots, q_n)$ and $\boldsymbol{x} = (x_0, x_1, \cdots, x_n)$. Throughout the paper, the following is assumed:

Assumption 1. *A finite number $\bar{Q} > 0$ exists such that $P(Q) > 0$ if $Q < \bar{Q}$ and $P(Q) = 0$ otherwise. Moreover, $P(Q)$ is twice continuously differentiable with $P'(Q) < 0$ for $Q < \bar{Q}$ and*

$$\varepsilon(Q) \equiv \frac{P''(Q)Q}{P'(Q)} > -1.$$

Assumption 2. *$C(q_i, x_i)$ satisfies (a) $(\partial/\partial q_i)C(q_i, x_i) > 0$ and $(\partial^2/\partial q_i^2)C(q_i, x_i) > 0$, and (b) $(\partial/\partial x_i)C(q_i, x_i) \leq 0$, $(\partial^2/\partial x_i^2)C(q_i, x_i) \leq 0$, and $(\partial^2/\partial x_i \partial q_i)C(q_i, x_i) < 0$.*

Assumption 3. *$\Gamma(x_i)$ satisfies $\Gamma'(x_i) > 0$ and $\Gamma''(x_i) > 0$.*

The government provides all the firms with two types of subsidies: an output subsidy and an R&D subsidy. Let $\boldsymbol{s} = (s_q, s_x)$ denote a pair of output and R&D subsidy rates. The profit of firm $i \in N$ is then given by

$$\Pi_i(\boldsymbol{q}, x_i, \boldsymbol{s}) \equiv \pi_i(\boldsymbol{q}, x_i, s_q) - \Gamma(x_i) + s_x x_i, \quad \text{where} \quad \pi_i(\boldsymbol{q}, x_i, s_q) \equiv P(Q)q_i - C(q_i, x_i) + s_q q_i.$$

Welfare is

$$W(\boldsymbol{a}) \equiv w(\boldsymbol{a}) - \sum_{i=0}^{n} \Gamma(x_i), \quad \text{where} \quad w(\boldsymbol{a}) \equiv \int_0^Q P(z)dz - \sum_{i=0}^{n} C(q_i, x_i) \quad \text{and} \quad \boldsymbol{a} = (\boldsymbol{q}, \boldsymbol{x}) \in \mathbb{R}_+^{2(n+1)}.$$

Under Assumptions 1–3, the first-best allocation $\boldsymbol{a}^f = (\boldsymbol{q}^f, \boldsymbol{x}^f)$ must satisfy the marginal-cost pricing principle $(\partial/\partial q_i)W(\boldsymbol{a}^f) = P(Q^f) - (\partial/\partial q_i)C(q^f, x^f) = 0$ and the cost-minimization condition $(\partial/\partial x_i)W(\boldsymbol{a}^f) = -(\partial/\partial x_i)C(q^f, x^f) - \Gamma'(x^f) = 0$ for all $i \in N$, where $Q^f = (n+1)q^f$.

The government can sell its stocks of firm 0 to private investors. Let $\theta \in [0, 1]$ denote the private investors' shareholdings in firm 0 (henceforth, the *degree of privatization*). We follow [5] by assuming that each private firm maximizes its profit, whereas firm 0 maximizes a convex combination of its profit and welfare, $V(\boldsymbol{a}, \boldsymbol{s}, \theta) = (1-\theta)W(\boldsymbol{a}) + \theta\Pi_0(\boldsymbol{q}, x_0, \boldsymbol{s})$, that is,

$$V(\boldsymbol{a}, \boldsymbol{s}, \theta) \equiv v(\boldsymbol{a}, s_q, \theta) - \Gamma(x_0) - (1-\theta)\sum_{j=1}^{n}\Gamma(x_j) + \theta s_x x_0,$$

$$\text{where} \quad v(\boldsymbol{a}, s_q, \theta) \equiv (1-\theta)w(\boldsymbol{a}) + \theta\pi_0(\boldsymbol{q}, x_0, s_q).$$

We consider the following three-stage game. In the first stage, the government sets $\boldsymbol{s} = (s_q, s_x)$ for a given θ. Observing the choice made by the government, all the firms simultaneously and independently choose their R&D investments in the second stage and their outputs in the third stage. We solve this game by backward induction. As easily confirmed, in the third stage of the game, Assumptions 1 and 2 warrant the second-order conditions, the strategic substitutability, and the stability of the Cournot-Nash equilibrium.

For the result presented in the next section, we define some functions. First, let the output vector of the third-stage equilibrium be $\boldsymbol{q}^*(\boldsymbol{x}, s_q, \theta)$, which is characterized by the equation system $(\partial/\partial q_0)v(\boldsymbol{q}^*(\boldsymbol{x}, s_q, \theta), \boldsymbol{x}, s_q, \theta) = 0$ and $(\partial/\partial q_j)\pi_j(\boldsymbol{q}^*(\boldsymbol{x}, s_q, \theta), x_j, s_q) = 0$ for any $j \in N \setminus \{0\}$, with its Jacobian matrix Ω negative definite. Second, we denote the reduced forms of firms' objective functions by $\tilde{V}(\boldsymbol{x}, \boldsymbol{s}, \theta) \equiv V(\boldsymbol{q}^*(\boldsymbol{x}, s_q, \theta), \boldsymbol{x}, \boldsymbol{s}, \theta)$ and $\tilde{\Pi}_j(\boldsymbol{x}, \boldsymbol{s}, \theta) \equiv \Pi_j(\boldsymbol{q}^*(\boldsymbol{x}, s_q, \theta), x_j, \boldsymbol{s})$ for $j \in N \setminus \{0\}$. Finally, we denote the allocation and subsidy profile in the subgame perfect Nash equilibrium by $\boldsymbol{a}^{**}(\theta) = (\boldsymbol{q}^{**}(\theta), \boldsymbol{x}^{**}(\theta))$ and $\boldsymbol{s}^{**}(\theta) = (s_q^{**}(\theta), s_x^{**}(\theta))$, respectively.

3 Main theorem

We say that the PNT holds if and only if $\boldsymbol{a}^{**}(\theta) = \boldsymbol{a}^f$ and $\boldsymbol{s}^{**\prime}(\theta) = \boldsymbol{0}$ (i.e., $(s_q^{**\prime}(\theta), s_x^{**\prime}(\theta)) = (0,0)$). The existing studies have shown that this theorem holds if R&D activities are not taken into account. This can be expressed in our model as follows:

Proposition 1. *$\boldsymbol{q}^*(\boldsymbol{x}^f, s_q^e, \theta) = \boldsymbol{q}^f$, where $s_q^e \equiv -P'(Q^f)q^f > 0$.*

Proof: q^f is the best response of firm 0 when the other firms choose q^f. Indeed, it follows from the definition of $\boldsymbol{a}^f$ that

$$\frac{\partial}{\partial q_0} v(\boldsymbol{a}^f, s_q^e, \theta) = P(Q^f) - \frac{\partial}{\partial q_0} C(q^f, x^f) + \theta P'(Q^f)q^f + \theta s_q^e = \theta \left[P'(Q^f)q^f + s_q^e \right] = 0.$$

By the same procedure, we can easily show that $q_j = q^f$ is the best response of firm $j \in N \setminus \{0\}$. **Q.E.D.**

We finally examine whether the PNT holds if R&D activities are introduced. As indicated by the following theorem, it never holds in the sense that the optimal R&D subsidy depends on θ even if the first-best allocation is achieved.

Theorem 1. *Suppose that $s_q = s_q^e > 0$ and $s_x = s_x^e(\theta) \equiv n s_q^e (\partial/\partial x_0) q_1^*(\boldsymbol{x}^f, s_q^e, \theta) < 0$. There holds $\boldsymbol{a}^{**}(\theta) = \boldsymbol{a}^f$ if and only if either (i) $\theta = 1$ or (ii) $\varepsilon(Q^f) = \Psi$ holds, where*

$$\Psi \equiv -\frac{(n-1)(n+1)P'(Q^f)}{nP'(Q^f) - (\partial^2/\partial q_0^2)C(q^f, x^f)} \leq 0, \quad \text{with equality if and only if } n = 1.$$

Proof: First, we show that $s_x^e(\theta) < 0$. Appendix shows that

$$\frac{\partial}{\partial x_0} q_1^*(\boldsymbol{x}^f, s_q^e, \theta) = -\frac{1}{\det \Omega} \left[P'(Q^f) + P''(Q^f)q^f \right] \left[P'(Q^f) - \frac{\partial^2}{\partial q_1^2} C(q^f, x^f) \right]^{n-1} \frac{\partial^2}{\partial x_0 \partial q_0} C(q^f, x^f),$$

where $\det \Omega$ is the determinant of Ω. Since sign $\det \Omega = \text{sign } (-1)^{n+1}$ holds because of its negative definiteness, we obtain $(\partial/\partial x_0) q_1^*(\boldsymbol{x}^f, s_q^e, \theta) < 0$ and thus, $s_x^e(\theta)$ is negative.

We next show that $\boldsymbol{x} = \boldsymbol{x}^f$ can be the Nash equilibrium in the second stage under $\boldsymbol{s}^e(\theta) = (s_q^e, s_x^e(\theta))$. First, we prove that $x_0 = x^f$ is firm 0's best response to the R&D investments of the other firms $\boldsymbol{x}_{-0} = (x^f, \cdots, x^f) \in \mathbb{R}^n_+$. By symmetry among private firms and the definition of $\boldsymbol{a}^f$, we obtain

$$\frac{\partial}{\partial x_0}\tilde{V}(\boldsymbol{x}^f, \boldsymbol{s}^e(\theta), \theta) = \theta\left[nP'(Q^f)q^f \frac{\partial}{\partial x_0} q_1^*(\boldsymbol{x}^f, s_q^e, \theta) + s_x^e(\theta)\right] = 0.$$

Coupled with this, Proposition 1 suggests that firm 0 chooses x^f as the best response to $\boldsymbol{x}_{-0}$. Similarly, for firm $j \in N \setminus \{0\}$, we obtain

$$\begin{aligned}\frac{\partial}{\partial x_j}\tilde{\Pi}_j(\boldsymbol{x}^f, \boldsymbol{s}^e(\theta), \theta) &= P'(Q^f)q^f\left[\frac{\partial}{\partial x_2}q_0^*(\boldsymbol{x}^f, s_q^e, \theta) - n\left(\frac{\partial}{\partial x_0}q_1^*(\boldsymbol{x}^f, s_q^e, \theta)\right) + (n-1)\left(\frac{\partial}{\partial x_2}q_1^*(\boldsymbol{x}^f, s_q^e, \theta)\right)\right] \\ &= \frac{(1-\theta)\Phi P'(Q^f)q^f}{(n+1)(\det\Omega)}\left[\varepsilon(Q^f) + \frac{(n-1)(n+1)P'(Q^f)}{nP'(Q^f) - (\partial^2/\partial q_0^2)C(q^f, x^f)}\right]\frac{\partial^2}{\partial x_0 \partial q_0}C(q^f, x^f),\end{aligned}$$

where

$$\Phi = P'(Q^f)\left[nP'(Q^f) - \frac{\partial^2}{\partial q_0^2}C(q^f, x^f)\right]\left[P'(Q^f) - \frac{\partial^2}{\partial q_0^2}C(q^f, x^f)\right]^{n-2} \neq 0.$$

Thus, firm j chooses $x_j = x^f$ as the best response if and only if either (i) $\theta = 1$ or (ii) $\varepsilon(Q^f) = \Psi$. **Q.E.D.**

Finally, we make several remarks on Theorem 1.

Remark 1. As stated by Theorem 1, output and R&D subsidies can yield the first-best allocation even if firms' strategic choices of R&D are taken into account. In particular, the first-best outcome is obtained if demand is linear (i.e., $\varepsilon(Q) = 0$) and $n = 1$. However, if demand is strictly convex (i.e., $\varepsilon(Q) > 0$) and firm 0 is not fully privatized (i.e., $\theta \in [0, 1)$), the subsidies do not remove the distortions enough to achieve the first-best allocation.

Remark 2. Using a mixed duopoly with linear demand and quadratic costs, [10] showed that if the government provides both production and R&D subsidies to firms, the optimal R&D subsidy is negative irrespective of whether a public firm is fully privatized or fully nationalized. Theorem 1 extends [10] by (i) generalizing demand and costs, (ii) introducing partial privatization, and (iii) allowing for more than one private firm. The negativity of the optimal R&D subsidy is explained as follows. The production subsidy plays a role to reduce the distortion due to firms' underproduction. On the other hand, it encourages private firms to overinvest because the greater investments lead to the higher market shares. Thus, the government attempts to remedy the overinvestments by imposing a R&D tax.

Remark 3. Some existing studies have presented the failure of the PNT by showing that subsidies cannot achieve the first-best allocation ([1], [7]). By contrast, Theorem 1 suggests that while the first-best allocation is achievable, the degree of privatization does influence the optimal R&D subsidy. Indeed, the optimal R&D subsidy rate increases with the degree of privatization. We briefly explain the intuition, relegating the proof to Appendix. An increase in θ makes firm 0 produce less for a given R&D profile, which enlarges each private firm's output because of strategic substitution. Accordingly, private firms lose their incentives to conduct R&D investments and thus, the government can raise the R&D subsidy rate.

Appendix

The derivation of derivatives

The optimality conditions in the third stage are given by $(\partial/\partial q_0)v(\boldsymbol{q}^*(\boldsymbol{x}, s_q, \theta), \boldsymbol{x}, s_q, \theta) = 0$ and $(\partial/\partial q_j)\pi_j(\boldsymbol{q}^*(\boldsymbol{x}, s_q, \theta), x_j, s_q) = 0$ for $j \in N \setminus 0$. We differentiate this equation system with respect to x_0 to obtain

$$\Omega \begin{pmatrix} (\partial/\partial x_0) q_0^*(\boldsymbol{x}, s_q, \theta) \\ (\partial/\partial x_0) q_1^*(\boldsymbol{x}, s_q, \theta) \\ \vdots \\ (\partial/\partial x_0) q_n^*(\boldsymbol{x}, s_q, \theta) \end{pmatrix} = \begin{pmatrix} (\partial^2/\partial x_0 \partial q_0) C(q_0^*(\boldsymbol{x}, s_q, \theta), x_0) \\ 0 \\ \vdots \\ 0 \end{pmatrix}. \tag{1}$$

Recall that $q_i^*(\boldsymbol{x}^f, s_q^e, \theta) = q^f$ for any $i \in N$ by Proposition 1. This implies that

$$(\partial^2/\partial q_i^2) C(q_i^*(\boldsymbol{x}^f, s_q^e, \theta), x^f) = (\partial^2/\partial q_j^2) C(q_j^*(\boldsymbol{x}^f, s_q^e, \theta), x^f),$$
$$(\partial^2/\partial x_i \partial q_i) C(q_i^*(\boldsymbol{x}^f, s_q^e, \theta), x^f) = (\partial^2/\partial x_j \partial q_j) C(q_j^*(\boldsymbol{x}^f, s_q^e, \theta), x^f),$$

for $i, j \in N$ and $i \neq j$. We then use F and G to represent $(\partial^2/\partial q_i^2) C(q^f, x^f)$ and $(\partial^2/\partial x_i \partial q_i) C(q^f, x^f)$, respectively. Setting $\boldsymbol{x} = \boldsymbol{x}^f$ and $s = s_q^e$ and solving the equation system (1), we obtain

$$\begin{aligned}
&(\det \Omega) \frac{\partial}{\partial x_0} q_1^*(\boldsymbol{x}^f, s_q^e, \theta) \\
&= \begin{vmatrix} (1+\theta)P'(Q^f) + \theta P''(Q^f)q^f - F & G & P'(Q^f) + \theta P''(Q^f)q^f & \cdots & P'(Q^f) + \theta P''(Q^f)q^f \\ P'(Q^f) + P''(Q^f)q^f & 0 & P'(Q^f) + P''(Q^f)q^f & \cdots & P'(Q^f) + P''(Q^f)q^f \\ P'(Q^f) + P''(Q^f)q^f & 0 & 2P'(Q^f) + P''(Q^f)q^f - F & \cdots & P'(Q^f) + P''(Q^f)q^f \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ P'(Q^f) + P''(Q^f)q^f & 0 & P'(Q^f) + P''(Q^f)q^f & \cdots & 2P'(Q^f) + P''(Q^f)q^f - F \end{vmatrix} \\
&= -\left[P'(Q^f) + P''(Q^f)q^f\right] G \begin{vmatrix} 1 & 1 & 1 & \cdots & 1 \\ 0 & P'(Q^f) - F & 0 & \cdots & 0 \\ 0 & 0 & P'(Q^f) - F & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & P'(Q^f) - F \end{vmatrix} \\
&= -\left[P'(Q^f) + P''(Q^f)q^f\right] \left[P'(Q^f) - F\right]^{n-1} G
\end{aligned}$$

Likewise, we find that

$$\frac{\partial}{\partial x_2} q_0^*(\boldsymbol{x}^f, s_q^e, \theta) = -\frac{1}{\det\Omega}\left[P'(Q^f) + \theta P''(Q^f)q^f\right]\left[P'(Q^f) - F\right]^{n-1} G,$$
$$\frac{\partial}{\partial x_2} q_1^*(\boldsymbol{x}^f, s_q^e, \theta) = -\frac{1}{\det\Omega}\left[P'(Q^f) + P''(Q^f)q^f\right]\left[\theta P'(Q^f) - F\right]\left[P'(Q^f) - F\right]^{n-2} G.$$

Proof of $s_x^{e\prime}(\theta) > 0$

Let us use $H(\theta)$ to represent $\det\Omega$ under $\boldsymbol{x} = \boldsymbol{x}^f$ and $s_q = s_q^e$. Straightforward computation shows that

$$H'(\theta) = \begin{vmatrix} P'(Q^f) + P''(Q^f)q^f & P''(Q^f)q^f & P''(Q^f)q^f & \cdots & P''(Q^f)q^f \\ P'(Q^f) + P''(Q^f)q^f & 2P'(Q^f) + P''(Q^f)q^f - F & P'(Q^f) + P''(Q^f)q^f & \cdots & P'(Q^f) + P''(Q^f)q^f \\ P'(Q^f) + P''(Q^f)q^f & P'(Q^f) + P''(Q^f)q^f & 2P'(Q^f) + P''(Q^f)q^f - F & \cdots & P'(Q^f) + P''(Q^f)q^f \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ P'(Q^f) + P''(Q^f)q^f & P'(Q^f) + P''(Q^f)q^f & P'(Q^f) + P''(Q^f)q^f & \cdots & 2P'(Q^f) + P''(Q^f)q^f - F \end{vmatrix}$$
$$= \left[P'(Q^f) + P''(Q^f)q^f\right]\left[(n+1)P'(Q^f) - F\right]\left[P'(Q^f) - F\right]^{n-1},$$

which implies that $\text{sign}H'(\theta) = \text{sign}(-1)^{n+1}$ because of Assumptions 1 and 2. Thus, it follows from $G < 0$ that

$$s^{e\prime}(\theta) = \frac{nGs_q^e H'(\theta)}{(\det\Omega)^2}\left[P'(Q^f) - F\right]^{n-1}\left[P'(Q^f) + P''(Q^f)q^f\right] > 0.$$

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