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Subsidization Policy on the Social Enterprise for the Underprivileged*

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We formulate an oligopoly model in which social enterprise for the underprivileged competes with private enterprises under government subsidization, and examine the market role of private leadership. We show that Stackelberg private leadership is better from the viewpoint of total social welfare, while Cournot followership is better when the social provisions for the underprivileged are emphasized. We also find that both cost inefficiency and the number of private enterprises affect the profitability and welfare consequences. We then investigate the rationing policy on the production of social enterprise and show that output rationing is superior to market share rationing not only for the social concerns of the underprivileged but also for total social welfare, even though it is less attractive than subsidy policy. Finally, we find that there is a strategic over-incentive to pursue social activities under government subsidization.

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I. Introduction

Since the 1990s, the emergence of social enterprises (SEs) has received increasing attention to qualify the entrepreneurial initiatives in different industries in order to

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tackle challenging problems in the market economy. In general, SEs can be regarded as specific non-profit organizations (NPOs) that provide general-interest services and manage production activities that benefit the entire society. SEs' initiatives include those supplying social services and generating social value as well as those supporting social welfare, health, and education by creating jobs for disadvantaged workers, combating poverty and undernourishment, and promoting ethical financing such as micro credit and fair trade. In these areas, for-profit private enterprises (PEs) are generally ineffective because of the low profitability and public nature of the services provided.[1]

SEs explicitly aim to generate social value through the private use and management of human and financial resources that are partially generated by market trade and are partially supported by government. These initiatives are not confined to pursue profit maximization, but rather to use market mechanisms innovatively to underwrite the provision of goods and services that have a social impact. Thus, their governance and the implications of their performance depend heavily upon market competition and government regulation.

Over the past decade, SEs have made amazing breakthroughs in Western Europe, East Asia, Latin America, and the former Soviet Union. In East Asian countries, in particular, SE initiatives have started to emerge in response to structural changes, including the dislocation of the manufacturing industry and deindustrialization in the process of accelerated globalization, high unemployment, and growing social inequality problems. The most striking feature of SEs in East Asian countries continues to be the government's strong involvement and control. For example, South Korea's parliament passed a law in 2006 for the promotion of SEs dedicated to provide social services or work integration for hard-to-place job seekers.

Several theoretical studies have recently examined the efficiency of market competition for SEs and PEs as well as the performance of government regulations such as organizational ownership and subsidies. For example, Goering (2012, 2014), Lambertini and Tampieri (2015), Kopel (2015), Brand and Grothe (2015), Liu *et al.* (2015) and Hirose *et al.* (2017) analyzed the optimal strategies of corporate social responsible firms, while Kopel and Marini (2014) and Marini *et al.* (2015) examined consumer cooperatives, which compete with profit-maximizing private firms. They show that firms' profits and social welfare can be improved from the normative perspective of society when firms consider consumer welfare and environmental pollution to be social concerns.

In their analyses, SEs are assumed to both pursue profit and address social concerns such as consumer surplus and/or environmental damage. This approach follows the literature on canonical mixed market models in which public firm

[1] For more detailed discussions, see Borzaga and Defourny (2001), Borzaga and Tortia (2009), Defourny (2010), and Aldashev *et al.* (2015).

competes with private firms and behaves strategically to maximize either social welfare (De Fraja and Delbono; 1989) or a weighted objective function between welfare and its profit (Matsumura; 1998). However, if we consider SEs to be special NPOs, the profit in the objective function of SEs may be restrictive for predicting the behavior of real-world SEs and the resulting market outcomes. For example, SEs for the underprivileged generates social values by creating jobs for disadvantaged workers and the elderly, as well as producing goods and services with social goals (e.g. combating poverty and undernourishment). Thus, the formulation in which profit itself is included in the objective function should be re-evaluated.

In this study, we take an alternative approach in Crémer *et al*. (1989), Estrin and de Meza (1995), and Bennett and La Manna (2012) by confining the objective function of SE to social concerns. In particular, we assume that SEs produce their output under a break-even constraint. This formulation is sufficiently general to cover a wide range of managerial delegations such as employment level, and is informationally less demanding to enforce from the viewpoint of government. Then, by adopting this approach to address on the objective function of SE for the underprivileged, we re-examine the behavior of SE and the resulting market outcomes.

This research investigates the subsidy policy on SE for the underprivileged and examines how market competition can affect the role of SE not only for social welfare, but also for social concerns. In the contemporary market economy where non-profit-maximizing SE competes with profit-maximizing PEs, we examine two competition modes; Cournot competition where PEs take private followership and Stackelberg competition where PEs take private leadership. We then compare the equilibrium results between these two competition modes and show that there is a trade-off between Cournot private followership and Stackelberg price leadership. In particular, private leadership is better from the viewpoint of total social welfare, whereas private followership is better when social concerns are emphasized. We also find that both cost inefficiency of SE and the number of PEs affect the profitability and welfare consequences, which is contrast to the literature on private or/and mixed markets.

We also consider the extent to which rationing policy influences an SE's production and compare the results between output rationing and market share rationing. We show that output rationing is superior to market share rationing not only for the social concerns of the underprivileged but also for total social welfare. This finding implies that even though rationing policy is less attractive than subsidy policy, output rationing is the better alternative.

Finally, we examine the strategic incentives on the social activities of SE and show that there is an over-incentive to pursue social activities because these neglect the profitability of enterprises, which is part of total social welfare. Therefore, it is important to improve an SE's profitability for supporting the sustainability of SEs.

The remainder of this paper is organized as follows. In section 2, we provide the basic model. In section 3 characterize the socially optimal first-best outcome where the benevolent government can allocate to maximize total social welfare. In section 4, we analyze two cases of quantity competition, Cournot followership and Stackelberg private leadership, and compare the equilibrium and welfare consequences. In section 5, we examine the effectiveness of output rationing and market share rationing, and investigate the strategic incentives on the social activities of SE. We finally provide policy implications in section 6.

II. The Model

We consider an oligopoly market in which $n+1$ firms produce homogeneous products, where one SE (social enterprise for the underprivileged) and $n(\geq 1)$ PEs (private enterprises for the general) compete with a perfectly substitutable output in the marketplace. We denote q_0 as the output of the SE, firm 0, and q_i as the output of the PEs, firm $i(=1,\ldots,n)$. Market price P is given by a linear inverse demand function:

$$P = A - Q \text{, where } Q = q_0 + \textstyle\sum_{i=1}^{n} q_i \text{, } i = 1,2 \ldots n. \tag{1}$$

Consumer surplus is denoted by $CS = \frac{1}{2}Q^2$. We assume that the SE employs a large number of underprivileged and thus has diseconomies of scale with the quadratic cost function, $c(q_0) = cq_0^2$ where $c > 0$. We assume that the production cost of PE is constant and normalized to zero.[2] Thus, the higher cost of the SE requires external subsidization to ensure its long-term sustainability under market competition. We assume that the government can provide a certain subsidy to the SE to support the co-existence of both types of enterprises in the equilibrium. Denoting the output subsidy by the rate of s, the profits of the SE and PEs are respectively as follows:

$$\pi_0 = (A - q_0 - \textstyle\sum_{i=1}^{n} q_i)q_0 - cq_0^2 + sq_0, \tag{2}$$

$$\pi_i = (A - q_0 - \textstyle\sum_{i=1}^{n} q_i)q_i, \ i = 1, 2 \ldots n \tag{3}$$

While the PE maximizes its profit, the SE aims to create social value such as the well-being of cooperatives and large employment of disadvantaged and/or aged

[2] As far as PEs have constant marginal costs, the assumption on the quadratic cost function of the SE ensures interior solutions in the market equilibrium. If we assume a constant marginal cost for the SE, corner solutions appear in the equilibrium, depending on the relative size of marginal social value, b, and marginal cost, c.

workers despite incurring high costs. Specifically, we confine our focus to the case that the SE maximizes both economic market value, which is defined as consumer surplus in the market trade, and intrinsic social value, which is created from its social activities.[3] For example, job creation or wage expenditure on the underprivileged should be counted as social concerns, as these can be used to combat poverty and undernourishment. We assume that social value is proportional to the SE's output level. In sum, the objective of the SE is to maximize the following function under non-negative profit:[4]

$$G = CS + bq_0, \ \ s.t \ \ \pi_0 \geq 0 \tag{4}$$

where $b(>0)$ is interpreted as the marginal social value of the economic activities of the SE. Note that the activities of the SE are constrained by the economic constraint of non-negative profit, which supports its survival under subsidy policy. To analyze the interior solutions in the equilibrium, we assume that $0 < b < 2Ac$. This assumption implies that the marginal benefit over the marginal cost of social activities, $b/2c$, should be lower than the market size, A. It implies that the economic value of market activities outweighs the social value of social activities in the market equilibrium.

Finally, the benevolent government maximizes total social welfare, which is defined as the sum of economic welfare and social value, where economic welfare contains consumer surplus (CS) and producer surplus ($\pi_0 + \sum_{i=1}^{n} \pi_i$) minus the subsidy expenditures of the government (sq_0), as follows:

$$W = CS + \pi_0 + \sum_{i=1}^{n} \pi_i - sq_0 + bq_0 \tag{5}$$

Note that there is a trade-off between total output, which contributes to economic value, and the output of the SE, which contributes to social value for the underprivileged.

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[3] In microeconomics, social goods (or merit goods) are defined as private goods that have excludability and rivalry. Further, they have not only marketability by nature but also social value as an externality caused mostly by public concern about income inequality or justice. This can be provided by government or designated SEs under government subsidization. In our model, social value is not incorporated into the economic market value of consumer surplus because we assume that products are perfectly substitutable, where consumers do not differentiate between goods but the government cares for the externality.

[4] This formulation with a break-even constraint is sufficiently general to cover a wide range of managerial delegation such as employment level and social activities. See, for example, Cremer et al. (1989), Estrin and de Meza (1995), and Bennett and La Manna (2012). Compared with their models, however, we explicitly consider the social value created by the social activities of the SE to be a positive externality of market activities.

III. The First-best Outcome

From the normative perspective, we identify the conditions under which (i) SE exists in the market economy at the first-best optimum and (ii) it cannot be sustainable without government subsidization. To show this, when there is no subsidy, the total social welfare in (5) can be rewritten as follows:

$$W = \frac{1}{2}(q_0 + \Sigma_{i=1}^{n} q_i)^2 + bq_0 + [(A - q_0 - \Sigma_{i=1}^{n} q_i)q_0 - cq_0^2] \\ + n[(A - q_0 - \Sigma_{i=1}^{n} q_i)q_i]$$

(i) At the first-best, where the government can directly allocate the output levels at the maximized social welfare,[5] the socially optimal output level can be found from the first-order conditions with respect to q_0 and q_i:

$$\frac{\partial W}{\partial q_0} = A + b - q_0 - nq_i - 2cq_0 = 0\,, \tag{6}$$

$$\frac{\partial W}{\partial q_i} = (q_0 + nq_i) - q_0 + n(A - q_0 - nq_i - q_i) = 0, \quad i = 1,2\ldots n \tag{7}$$

These two equations simply state that the marginal benefits to the society of the SE and the PEs should be equal to their marginal costs. Combining these equations yields the first-best outcome:

$$q_0 = \frac{b}{2c}\,, \quad q_i = \frac{2Ac - b}{2cn}\,, \text{ and } \ Q = A \tag{8}$$

Note that interior solutions exist only when $0 < b < 2Ac$. Further, the optimal output of the SE is independent of the number of the PEs, while that of the PEs decreases as their number increases. We also have $q_0 \gtrless q_i$ if $b \gtrless \frac{2Ac}{1+n}$.

(ii) The profit of PEs is zero, while that of the SE is negative at the first-best outcome, that is,

$$\pi_0 = -\frac{b^2}{4c} < 0 \quad \text{and} \quad \pi_i = 0 \tag{9}$$

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[5] We assume that the benevolent government, as a social planner, has complete information and that it directly controls the first-best efficient allocations of the SE and PEs. Note that all of second-order conditions are satisfied in this study.

Total social welfare at the first-best is as follows where the superscript SO stands for the social optimum:

$$W^{SO} = \frac{b^2 + 2A^2c}{4c} \tag{10}$$

It represents that the socially beneficial SE cannot be sustainable at the first-best even though it should produce a positive output at the maximized total social welfare. Thus, the most efficient way in which to achieve the first-best and support the economic sustainability of the SE is to provide a lump-sum subsidy.[6] However, the government needs full information to calculate the exact lump-sum subsidy, which incurs additional administrative costs and leads to moral hazard problems of the SE. Furthermore, the government should allocate the output levels of the SE and PEs to achieve the first-best, which is impossible in a market economy where economic agents can make decisions independently. Therefore, an appropriate subsidy policy under market competition should be devised, not only to support the economic sustainability of the SE but also to enhance total social welfare.

IV. Subsidization and Market Competition

We focus on the survival of the SE at the market equilibrium and analyze the optimal output subsidy policy that supports its sustainability. We assume that the government can subsidize the SE under the financial requirement that its economic profit should be non-negative in (4).[7,8] Then, we analyze two modes of market competition between the SE and PEs under subsidization and compare the results and welfare consequences. The first case is Cournot competition in which both the SE and the PEs are followers and the other is Stackelberg competition in which the PEs lead and the SE follows.[9]

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[6] In a hypothetical world, social planner can achieve the first-best outcomes in (8) if she has full information and perfect authority without transaction costs. For example, if the lump-sum subsidy is determined at $b^2/4c$, the profit of the SE in (9) is zero and this is sustainable at the first-best outcome.

[7] Alternatively, the government can impose an output tax on the PEs, instead of an output subsidy on the SE, and then provide the necessary lump-sum subsidy to the SE. However, we can prove that this policy is inferior to subsidy policy for the SE because it indirectly supports its sustainability.

[8] In Appendix A, we also examine the results with the same rate of output subsidy between Cournot and Stackelberg competition, and show that most of the results in propositions still hold when the subsidy rate is not so high, which includes a zero subsidy.

[9] Note that the market role of the SE does not affect the market equilibrium when the PEs follow under market competition because the reactions of the PEs are given when they are followers. However, the SE does not behave strategically by choosing the optimal decision, which is simply

4.1. Private Followership: Cournot Competition

In Cournot competition, the timing of the game is as follows: In the first stage, the government sets the output subsidy to maximize total social welfare. In the second stage, the SE and PEs choose their outputs simultaneously. We solve the subgame-perfect Nash equilibrium by backward induction.

In the second stage, the profit-maximization condition of the PEs in (3) provides the following reaction function:

$$q_i = \frac{A - q_0 - \sum_{j \neq i, j=1} q_j}{2} \tag{11}$$

For the case of the SE, we know that G in (4) is increasing in q_0. This implies that the zero-profit condition of the SE is binding at the optimum. That is, the SE do not behave strategically by choosing the optimal decision, which is simply determined by the zero-profit condition. Thus, the zero-profit condition of the SE in (2) provides the following reaction function:

$$q_0 = \frac{A + s - \sum_{i=1}^{n} q_i}{1+c} \tag{12}$$

Note that products are strategic substitutes and the reaction function of the SE is less sensitive, i.e., $\frac{\partial q_0}{\partial q_i} = -\frac{1}{1+c} > -1$. Thus, we have the following equilibrium outcomes from the symmetric output of the PEs:

$$q_0 = \frac{A + (1+n)s}{1+(1+n)c}, \quad q_i = \frac{Ac - s}{1+(1+n)c}, \text{ and } Q = \frac{A(1+cn)+s}{1+(1+n)c} \tag{13}$$

The marginal social value does not directly affect equilibrium output. As the subsidy increases, the output of the SE increases while that of the PEs decreases. Thus, the equilibrium output of the SE is greater than that of the PEs if the subsidy is large. That is, $q_0 \gtrless q_i$ if $s \gtrless \frac{A(c-1)}{2+n}$. As the number of PEs approaches infinity, $q_0 > q_i$ with a positive subsidy. However, when $s = 0$, $q_0 \gtrless q_i$ if $c \gtrless 1$.

Total social welfare is as follows:

$$W = \frac{2Ab(1+c+cn) + A^2(1+cn(2+c(2+n))) - 2Ac(1+2n)s + s(2b(1+n)(1+c+cn) - (1+2c(1+n)^2)s)}{2(1+c+cn)^2}. \tag{14}$$

determined from the zero-profit condition, as shown in (12). Therefore, Cournot competition where the SE and PEs choose their output simultaneously provides the same results as Stackelberg competition where the SE leads and the PEs follow sequentially.

In the first stage, the differentiation of W with respect to s yields the following optimal subsidy where the superscript F stands for private followership:

$$s^F = \frac{b(1+n)(1+c+cn) - Ac(1+2n)}{1+2c(1+n)^2} \gtrless 0 \quad \text{if} \quad b \gtrless \frac{Ac(1+2n)}{(1+n)(1+c+cn)} \tag{15}$$

Since $b < 2Ac$, we have $s^F < Ac$. Thus, the optimal subsidy can be positive or negative, depending on the marginal social value, marginal production cost of the SE, and number of PEs. If the marginal social value is sufficiently large, it is socially beneficial to provide a positive subsidy to encourage the production of the SE. However, if the marginal production cost of SE is sufficiently large, it is socially beneficial to impose an output tax, instead of a subsidy, to discourage the production of the SE.

We have the following comparative statics:

$$\frac{\partial s^F}{\partial b} = \frac{(1+n)(1+c+cn)}{1+2c(1+n)^2} > 0, \quad \frac{\partial s^F}{\partial c} = -\frac{(1+2n)(A+b(1+n)^2)}{(1+2c(1+n)^2)^2} < 0, \text{ and}$$

$$\frac{\partial s^F}{\partial n} = \frac{(2Ac-b)(2cn(1+n)-1)}{(1+2c(1+n)^2)^2} \gtrless 0 \quad \text{if} \quad n \gtrless \frac{1}{c+\sqrt{c(2+c)}}$$

It represents that the optimal subsidy rate increases as the marginal social value increases or the marginal production cost decreases. As expected, an increased marginal social value or decreased marginal production cost raises the subsidy rate for encouraging the production of the SE. However, the optimal subsidy depends on the number of PEs. As the number of PEs increases, this induces tough competition and thus reduces the output of the SE but increases total output. In return, the government, which is concerned about the trade-off between total output (economic value) and the output of SE (social value), increases (decreases) the subsidy rate when the number of PEs is sufficiently large (small). In particular, when $c > 1/4$, the optimal subsidy always increases as the number of PEs increases.

Substituting s^F provides the following equilibrium output:

$$q_0^F = \frac{A+b(1+n)^2}{1+2c(1+n)^2}, \quad q_i^F = \frac{(2Ac-b)(1+n)}{1+2c(1+n)^2}, \text{ and}$$

$$Q^F = \frac{b(1+n)+A(1+2cn(1+n))}{1+2c(1+n)^2}. \tag{16}$$

Then, $q_0^F \gtrless q_i^F$ if $b \gtrless \frac{A(2c(1+n)-1)}{(1+n)(2+n)}$. As a sufficient condition, $q_0^F > q_i^F$ if

$0 < c < \frac{1}{2(1+n)}$. This implies that when the marginal production cost of the SE is very small, its output is always greater than the output of the PEs under private followership.

The market price is as follows:

$$p^F = \frac{(2Ac-b)(1+n)}{1+2c(1+n)^2} \tag{17}$$

The profit of the PEs and total social welfare are respectively as follows:

$$\pi_i^F = \frac{(2Ac-b)^2(1+n)^2}{(1+2c(1+n)^2)^2} \tag{18}$$

$$W^F = \frac{2Ab+b^2(1+n)^2+A^2(1+2cn(2+n))}{2+4c(1+n)^2} \tag{19}$$

Thus, total social welfare with subsidization under private followership is lower than that of the first-best in (10), i.e., $W^F < W^{SO}$. However, the SE can earn zero profit under subsidization (i.e., it can be economically sustainable.)

4.2. Private Leadership: Stackelberg Competition

In Stackelberg competition, profit-maximizing PEs play market leaders and thus move first with the SE following sequentially. The timing of the game is as follows: In the first stage, the government sets the subsidy to maximize total social welfare. In the second stage, the PEs simultaneously chooses their output. In the third stage, the SE chooses its output by observing the outputs of the PEs. We solve the subgame perfect Nash equilibrium by backward induction.

As explained before, the SE does not choose its output strategically and thus the reaction function is the same as that in (12) in the third stage. Then, in the second stage, the profit function of the PEs becomes:

$$\pi_i = (A - q_0 - \Sigma_{i=1}^n q_i)q_i = \left(A - \left(\frac{A+s-\Sigma_{i=1}^n q_i}{1+c}\right) - \Sigma_{i=1}^n q_i\right)q_i \tag{20}$$

The first-order condition of the PEs is as follows:

$$\frac{\partial \pi_i}{\partial q_i} = \left(A - \left(\frac{A+s-\Sigma_{i=1}^n q_i}{1+c}\right) - \Sigma_{i=1}^n q_i\right) + q_i\left(\frac{1}{1+c} - 1\right) = 0 \tag{21}$$

Solving these equations provides the following equilibrium outcomes:

$$q_0 = \frac{Ac + (c+n+cn)s}{c(1+c)(1+n)}, \quad q_i = \frac{Ac - s}{c(1+n)}, \text{ and } \quad Q = \frac{Ac(1+n+c)+(1+c)sn}{c(1+c)(1+n)} \tag{22}$$

Again, the marginal social value does not directly affect the equilibrium outputs. As the subsidy increases, the output of the SE increases while that of the PEs decreases. Thus, the equilibrium output of the SE is greater than that of the PEs if the subsidy is large. That is, $q_0 \gtrless q_i$ if $s \gtrless \frac{Ac^2}{1+n+c(2+n)}$. As the number of PEs approaches infinity, $q_0 > q_i$ with a positive subsidy. Further, when $s = 0$, $q_i > q_0$ for all $c > 0$.

Total social welfare is as follows:

$$W = \frac{\begin{array}{c} A^2c[1+(1+c)^2 n(2+n)] + 2Ac[b(1+c)(1+n) - (c+2(1+c)n)s] \\ -[c(1+2c)+4c(1+c)n+2(1+c)^2 n^2]s^2 + 2sb(1+c)(1+n)(c+n+cn) \end{array}}{2c(1+c)^2(1+n)^2} \tag{23}$$

In the first stage, the differentiation of W with respect to s yields the following optimal subsidy where the superscript L stands for private leadership:

$$s^L = \frac{b(1+c)(1+n)(c+n+cn) - Ac(c+2(1+c)n)}{2n^2 + 2c^2(1+n)^2 + c(1+2n)^2} \gtrless 0 \quad \text{if}$$
$$b \gtrless \frac{Ac(c+2n+2cn)}{(1+c)(1+n)(c+n+cn)} \tag{24}$$

Since $b < 2Ac$, we have $s^L < Ac$. Thus, the optimal output subsidy can also be positive or negative, depending on the size of the marginal social value, marginal production cost of the SE, and number of PEs. If the marginal social value is sufficiently large, it is socially beneficial to provide a positive subsidy. However, if the marginal production cost of the SE is sufficiently large, it is socially beneficial to impose an output tax, instead of a subsidy.

We have the following comparative statics:

$$\frac{\partial s^L}{\partial b} = \frac{(1+c)(1+n)(c+n+cn)}{2n^2 + 2c^2(1+n)^2 + c(1+2n)^2} > 0,$$
$$\frac{\partial s^L}{\partial c} = -\frac{b(1+n)(c^2+n+c(4+3c)n+2(1+c)^2n^2) + A(c^2+2c^2n+4c(1+c)n^2+4(1+c)^2n^3)}{(2n^2+2c^2(1+n)^2+c(1+2n)^2)^2} < 0 \quad \text{and}$$

$$\frac{\partial s^L}{\partial n}=\frac{(1+c)(2Ac-b)(c(2n-1+2cn)+2(1+c)^2n^2)}{(2n^2+2c^2(1+n)^2+c(1+2n)^2)^2}>0$$

Similar to private followership, the optimal subsidy rate increases as the marginal social value increases or as the marginal production cost decreases. However, contrary to private followership, the optimal subsidy increases as the number of PEs rises. Under private leadership, PEs are the first-movers and thus they increase their outputs and total output, compared with under private followership. This also reduces the output of the SE and thus the optimal subsidy under private leadership should be higher than that under private followership, as discussed later. Thus, when tough competition occurs under private leadership, the government increases the subsidy rate to encourage the production of the SE.

Substituting s^L provides the equilibrium output levels:

$$q_0^L=\frac{Ac^2+b(c+n+cn)^2}{c(2n^2+2c^2(1+n)^2+c(1+2n)^2)},\quad q_i^L=\frac{(1+c)(2Ac-b)(c+n+cn)}{c(2n^2+2c^2(1+n)^2+c(1+2n)^2)}\quad \text{and}$$

$$Q^L=\frac{(A+b)c+(1+c)(b+2Ac)n+2A(1+c)^2n^2}{2n^2+2c^2(1+n)^2+c(1+2n)^2} \tag{25}$$

Thus, $q_0^L \gtrless q_i^L$ if $b \gtrless \frac{Ac(c(1+2c)+2(1+c)^2n)}{(c+n+cn)(1+n+c(2+n))}$. Note that when the marginal production cost of the SE is sufficiently small, its output is greater than that of the PEs under private leadership.

The market price is as follows:

$$p^L=\frac{(2Ac-b)(c+n+cn)}{2n^2+2c^2(1+n)^2+c(1+2n)^2} \tag{26}$$

The profit of the PEs and total social welfare are respectively as follows:

$$\pi_i^L=\frac{(1+c)(2Ac-b)^2(c+n+cn)^2}{c(2n^2+2c^2(1+n)^2+c(1+2n)^2)^2} \tag{27}$$

$$W^L=\frac{2Abc^2+b^2(c+n+cn)^2+A^2c(c+4c(1+c)n+2(1+c)^2n^2)}{2c(2n^2+2c^2(1+n)^2+c(1+2n)^2)} \tag{28}$$

Hence, total social welfare under private leadership is lower than that of the first-best in (10), i.e., $W^L<W^{SO}$. However, the SE can also earn zero profit under subsidization. (i.e., it is economically sustainable.)

4.3. Comparisons

We now compare the outcomes under Cournot private followership and Stackelberg private leadership.

Proposition 1: *The output of the SE under private followership is higher than that under private leadership, while both the output of the PEs and total market outputs under private followership are lower than those under private leadership.*

Proof: Comparing the output levels in (16) and (25) yields the followings:

$$q_0^F - q_0^L = \frac{(2Ac-b)n(n+2c(1+n))}{c(1+2c(1+n)^2)(2n^2+2c^2(1+n)^2+c(1+2n)^2)} > 0,$$

$$q_i^F - q_i^L = -\frac{(2Ac-b)(n+c(1+2c(1+n)^2+2n(2+n)))}{c(1+2c(1+n)^2)(2n^2+2c^2(1+n)^2+c(1+2n)^2)} < 0 \quad \text{and}$$

$$Q^F - Q^L = -\frac{(2Ac-b)n(+2n(1+n)+2c(1+n)^2-1)}{(1+2c(1+n)^2)(2n^2+2c^2(1+n)^2+c(1+2n)^2)} < 0. \quad \text{Q.E.D.}$$

Proposition 1 resembles the well-known results in the IO literature on the comparisons between Cournot competition and Stackelberg competition. As Gal-Or (1985), Dowrick (1986), and Vives (1999) showed in symmetric duopolies with homogeneous products, the first-mover has the advantage under stability conditions. Thus, under Stackelberg competition, the first-mover advantage increases the output of the PEs and total output compared to Cournot competition. However, it reduces the output of the SE because outputs are strategic substitutes.

Proposition 1 supports that this holds true in a homogeneous product market with different objective functions between enterprises. In our model, the SE expands its output more under private followership to increase social value, while the PEs care only for their profits and expand their output more under private leadership. As shown in (12), the SE is less sensitive to changing output and thus, when the PEs are the first movers and increase their output, total market output under private leadership is also greater than that under private followership.

Proposition 2: *The optimal subsidy rate under private followership is lower than that under private leadership.*

Proof: Comparing the optimal subsidies in (15) and (24) yields the followings:

$$s^F - s^L = -\frac{(2Ac-b)n(1+n)(2n-1+2cn)}{(1+2c(1+n)^2)(2n^2+2c^2(1+n)^2+c(1+2n)^2)} < 0. \quad \text{Q.E.D.}$$

Proposition 2 represents that the optimal subsidy rate depends crucially on the role of market leadership. In particular, the subsidy program for the SE under private leadership (private followership) should be high-powered (low-powered), and its rate should be higher when market competition is more competitive and thus the sustainability of SE is very weak. This is because private leadership increases the output of the PEs but decreases the output of the SE, as shown in Proposition 1. However, as the number of PEs approaches infinity, the difference between the subsidy rates becomes zero.

Proposition 3: *The profit of the PEs under private followership is lower than that under private leadership when* $n=1$*, but the result is reversed when* $n \geq 2$ *and* $c \geq 1$.

Proof: Comparing the profits of the PEs in (18) and (27) yields the following: $\pi_i^F - \pi_i^L = \frac{(2Ac-b)^2 K(n,c)}{H(n,c)}$ where $H(n,c) = (1+2c(1+n)^2)^2 c(2n^2 + 2c^2(1+n)^2 + c(1+2n)^2)^2$ and $K(n,c) = c(2n^2 + 2c^2(1+n)^2 + c(1+2n)^2)^2(1+n)^2 - (1+c)(c+n+cn)^2(1+2c(1+n)^2)^2$. Since $2Ac - b > 0$ and $H(n,c) > 0$, the comparison depends on the sign of $K(n,c)$. Fig. 1 in Appendix B shows that $K(1,c) < 0$ for all $c > 0$, and $K(n,c) > 0$ when both $n \geq 2$ and $c \geq 1$. Q.E.D.

Proposition 3 accompanies Proposition 1. Consider the duopoly case where $n=1$. Under private leadership, the first-mover advantage always appears and thus the PEs increase output to earn higher profits even though the market price decreases because of the expansion of total market output. It is also well-known that the first-mover under Stackelberg competition has a higher output effect compared with the price effect, which increases profits.[10]

Proposition 3 also states that when the number of PEs is more than two, one sufficient conditions to support the first-mover advantage is that $c \geq 1$. When the SE is a follower and has a higher cost, the first-mover advantage disappears as the number of firms increases. (see also the figure in Appendix B.) Thus, both the number of PEs and cost inefficiency of the SE affect the welfare consequences under private leadership.

This finding also resembles the result in the IO literature on Stackelberg competition. Daughety (1990) and Ino and Matsumura (2012) investigated a Stackelberg model in which m leaders and $n-m$ followers compete in a homogeneous goods market with identical cost functions and found an inverse U-shaped relationship between m and economic welfare. Contrary to their results,

[10] Ono (1978) considered a profit-maximizing duopoly in a homogeneous product market with cost asymmetry and showed that the firm with the lower (higher) cost prefers the role of the leader (follower) if the cost difference is sufficiently large. Van Damme and Hurkens (2004) and Amir and Stepanova (2006) showed that this holds true in a differentiated product market.

Proposition 3 shows that cost differences and payoffs asymmetry change the previous results. In particular, as shown in the figure in Appendix B, as the number of the PEs increases, competition between private leaders intensifies and thus the first-mover advantage disappears. This is because the degree of competition induces that the output effect can be dominated by the price effect and thus profits fall.

Proposition 3 indicates that we can endogenize the timing of the game in choosing the market role, in which PEs face the observable delay game, provided by Hamilton and Slutsky (1990).[11] That is, when choosing the role of market leadership, the PEs choose whether to be leader or follower depending on the number of the PEs and cost inefficiency of the SE endogenously. This is because SE is always a follower since it does not choose its role strategically but meet the zero-profit condition, in which the output of the PEs is given. This finding implies that total social welfare also depends on the choice of private leadership in the endogenous game.[12]

Proposition 4: *Total social welfare under private followership is lower than that under private leadership*.

Proof: Comparing the social welfare in (19) and (28) yields the following:

$$W^L - W^F = \frac{(2Ac-b)^2 n(n+2c(1+n))}{2c(1+2c(1+n)^2)(2n^2+2c^2(1+n)^2+c(1+2n)^2)} > 0 . \quad \text{Q.E.D.}$$

Propositions 1 and 4 show that not only total output but also total social welfare under private leadership are greater than those under private followership. This finding supports the previous result in the literature on mixed markets where a profit-maximizing private firm competes against a welfare-maximizing public firm. (see the literature described in footnote 11.) They examined the role of both public and private firms with asymmetric payoffs and showed that either private or public leadership can be the equilibrium but private leadership is socially desirable from the welfare viewpoint. However, we also show that private leadership can be worse when the social impacts of social activities are taken into policy consideration.

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[11] Hamilton and Slutsky (1990) examined an observable delay game and showed that if two firms have the same costs, the Stackelberg equilibrium appear as an equilibrium of endogenous timing game. Amir and Stepanova (2006) demonstrated that if two firms have different costs, the firm with the lower cost becomes the leader. Regarding the endogenous timing game in the literature on mixed markets, see, for example, see Pal (1998), Matsumura (2003), Lu (2006), Bárcena-Ruiz and Garzón (2010), Tomaru and Kiyono (2010), Tomaru and Saito (2010), Ino and Matsumura (2010), Matsumura and Ogawa (2014) and Marcella (2014) among others.

[12] Tomaru and Saito (2010) and Marcella (2014) showed that with subsidization, either Stackelberg leadership competition or Cournot equilibrium can be the equilibrium of the endogenous timing game.

V. Policy Discussions

5.1. Rationing Policy for the Social Enterprise

In 2006, South Korea's parliament passed a law for the promotion of SEs dedicated to provide social services or work integration for hard-to-place jobseekers. In the legislative process, however, there are still debates about market share rationing policy of SEs to support its sustainability, such as the extent to which the government should provide a rationing on the market share of SEs, which ranges from 5% to 7% of total market share. Then, it is needed to examine how this rationing policy is effective compared with subsidy policy.

In the following, we compare output-based rationing with market share-based rationing, and analyze the welfare consequences. Output-based rationing policy is *ex-ante* rationing on the first-best outcome in which the government pre-commits to impose the output level of the SE with that in the first-best level before market competition. On the contrary, in market share rationing, the government expects the market equilibrium outcomes and pre-commits to require the optimal market share of the SE, which is *ex-post* rationing after market competition.

The timing of the game is the same as in the previous section, but the government decides its rationing policy in the first stage. The first case is to set the optimal output rationing for the SE. The equilibrium output of the SE under output rationing is $q_0^A = b/2c$, where the superscript A stands for output rationing.[13] Then, from the reaction function of the PEs in (11), we have the following equilibrium:

$$q_0^A = \frac{b}{2c}, \quad q_i^A = \frac{2Ac-b}{2c(1+n)}, \text{ and } \quad Q^A = \frac{b+2Acn}{2c(1+n)} \tag{29}$$

In the output rationing equilibrium, compared with the non-rationing equilibrium, the output of the SE is higher and thus the output of the PEs is lower, but total market output is higher.

The profits of the SE and PEs are as follows:

$$\pi_0^A = \frac{b(2Ac-b(1+c+cn))}{4c^2(1+n)} \quad \text{and} \quad \pi_i^A = \frac{(b-2Ac)^2}{4c^2(1+n)^2} \tag{30}$$

[13] Note that the first-best output of the SE is $q_0 = \frac{b}{2c}$, and total market output is $Q = A$. Then, the SE's market share is $\frac{q_0^{SO}}{Q} = \frac{b}{2Ac} < 1$. In fact, output rationing requires the minimum amount of output of the SE, $q_0^A \geq \frac{b}{2c}$. However, the effective constraint is binding in this inequality. Otherwise, it is an unnecessary regulation.

Note that the PEs earn positive profits, while the SE might face losses, i.e., $\pi_0^A \gtreqless 0$ if $2Ac \gtreqless b(1+c+cn)$. In particular, if $b < 2Ac < b(1+c+cn)$, the SE earns a negative profit. Thus it is still necessary to provide a lump-sum subsidy to support the non-negative profit of the SE.

Total social welfare under optimal rationing is as follows:

$$W^A = \frac{4Abc + 4A^2c^2n(2+n) + b^2(-1+2c(1+n)^2)}{8c^2(1+n)^2} \tag{31}$$

Proposition 5: *Total social welfare under output subsidy is higher than that under the optimal output rationing policy.*

Proof: Comparing total welfare between Cournot followership and output rationing yields the followings: $W^F - W^A = \frac{(b-2Ac)^2}{8c^2(1+n)^2(1+2c(1+n)^2)} > 0$. Then, Proposition 4 ensures that $W^L > W^F > W^A$. Q.E.D.

Proposition 5 simply states that direct rationing based on the first-best output level is inferior to an indirect subsidy policy. Furthermore, output rationing policy might require the subsidization of SEs.

The second case is the optimal market share rationing for the SE. The equilibrium market share of the SE under market share rationing is $q_0^{SO}/Q = b/2AC < 1$. Again, from the reaction function of the PEs in (11), we have the following equilibrium:[14]

$$q_0^m = \frac{bnA}{2cA(1+n)-b}, \quad q_i^m = \frac{A(2Ac-b)}{2cA(1+n)-b}, \text{ and } Q^m = \frac{2A^2cn}{2cA(1+n)-b} \tag{32}$$

In the market share rationing equilibrium, compared with the non-rationing equilibrium, the SE's output is higher and so the PE's output is lower, but total market output is higher. Note that the optimal market share of the SE is achieved in the equilibrium, that is, $\frac{q_0^m}{Q^m} = \frac{b}{2Ac}$.

The profits of the SE and the PEs are as follows:

$$\pi_0^m = \frac{A^2bn(2Ac - b(1+cn))}{(b-2Ac(1+n))^2} \text{ and } \pi_i^m = \frac{A^2(b-2Ac)^2}{(b-2Ac(1+n))^2} \tag{33}$$

[14] By using the relations that $Q = q_0 + nq_i = \frac{An+q_0}{1+n}$ and $\frac{(1+n)q_0}{An+q_0} = \frac{b}{2Ac}$, we have the following results under market share rationing.

Note that the PEs earn positive profits, while the SE might also face losses, i.e., $\pi_0^m \gtreqless 0$ if $2Ac \gtreqless b(1+cn)$. In particular, if $b < 2Ac < b(1+cn)$, the SE earns a negative profit. Thus, it is still necessary to provide a lump-sum subsidy to support the non-negative profit of the SE. However, the possibility of this subsidization is lower than that in the output rationing case.

Total social welfare under optimal rationing is as follows:

$$W^m = \frac{A(b^2 + 2A^2c)n(-b + Ac(2+n))}{(b - 2Ac(1+n))^2} \tag{34}$$

Proposition 6: *Total social welfare under output subsidy is higher than that under the optimal market share rationing policy.*

Proof: Comparing total social welfare between private followership and market share rationing yields the following:

$$W^F - W^m = \frac{(b-2Ac)^2(A+b+bn)^2}{2(b-2Ac(1+n))^2(1+2c(1+n)^2)} > 0. \quad \text{Q.E.D.}$$

Thus, we have $W^L > W^F > W^m$ from Proposition 4. Proposition 6 also represents that direct rationing based on the first-best market share level is inferior to indirect subsidy policy. Furthermore, market share rationing policy might also require the subsidization of SEs.

Proposition 7: *Under market share rationing policy, both the output of the SE and total market outputs are lower than those under output rationing policy. Total social welfare is also lower than that under output rationing policy.*

Proof: Comparing the results under output rationing and in market share rationing yields the following:

$$q_0^A - q_0^m = \frac{b(2Ac-b)}{2c(2Ac+2Acn-b)} > 0, \quad Q^A - Q^m = \frac{b(2Ac-b)}{2c(1+n)(2Ac(1+n)-b)} > 0, \text{ and}$$

$$W^A - W^m = \frac{b(b-2Ac)^2(4Ac(1+n) - b(1-2c(1+n)^2))}{8c^2(1+n)^2(b-2Ac(1+n))^2} > 0. \quad \text{Q.E.D.}$$

Proposition 7 represents that output rationing policy is superior to market share rationing policy not only for social concerns but also for total social welfare, even though it is less attractive policy than output subsidy policy. This is because direct rationing on the output level increases not only the output of the SE but also total market outputs.

5.2. Strategic Incentives on the Social Activities

In the previous analysis, we showed that private leadership is the superior policy, but it requires more subsidies to the SE, which might increase both the financial burden of government and the financial dependency of SE. This fact implies that if the subsidy program heavily supports the survival of the SE, it has a strategic incentive to put more effort into social activities. That is, if a high-powered incentive subsidy is available, the SE would increase the social value created from its increased output of the SE, but might reduce total social welfare deprived from their decreased output of the PEs. On the contrary, if the subsidy program weakly supports the survival of the SE, it has a strategic incentive to put less effort into social activities. Thus, if a low-powered incentive subsidy exists, the SE decreases not only social value but also total social welfare.

We now examine the strategic incentives on the social activities of the SE under a subsidy program and its welfare consequences. We endogenizes social value as a function of social activities. Let the social activities of the SE be a_0 and the marginal social value is a function of a_0, i.e., $b(a_0)$, where $b'(a_0)>0$ and $b''(a_0)\leq 0$. We also assume that the cost of social activities is $d(a_0)$, where $d'(a_0)>0$ and $d''(a_0)\geq 0$. Then, the objective function of the SE in (4) is transformed as follows:

$$G^s(a_0)=\mathrm{CS}+b(a_0)q_0-d(a_0)=\frac{1}{2}Q^2+b(a_0)q_0-d(a_0)\,. \tag{35}$$

The SE decides its strategic social activities to maximize its objective function under the zero-profit constraint. We assume that the SE decides its social activities before the government decides its subsidy rate. Then, the optimal output subsidy under private followership in (15) or private leadership in (22) is a function of a_0. The first-order condition of a_0 yields the following optimal social activities of the SE:

$$\frac{dG^s}{d\mathrm{a}_0}=b'\left[\frac{\partial q_0}{\partial b}(Q+b)+Q\sum_{i=1}^{n}\frac{\partial q_i}{\partial b}+q_0\right]-d'=0 \tag{36}$$

Let the strategic choice of the optimal social activities of the SE in (36) be denoted by a_0^*.

On the other hand, total social welfare in (5) can be rewritten as follows:

$$W=\mathrm{G}^S+\pi_0+\sum_{i=1}^{n}\pi_i-sq_0 \tag{37}$$

The derivative of (37) yields the following:

$$\frac{\partial W}{\partial a_0}=\frac{\partial G^S}{\partial a_0}+b'\left[\frac{\partial q_0}{\partial b}\left(P'q_0+P-2cq_0+\Sigma_{i=1}^n\frac{\partial \pi_i}{\partial q_0}\right)+\Sigma_{i=1}^n\frac{\partial q_i}{\partial b}\left(P'q_0+\frac{\partial \pi_i}{\partial q_i}+\Sigma_{i\neq j}^n\frac{\partial \pi_j}{\partial q_i}\right)\right] \quad (38)$$

In the market equilibrium, we can evaluate the level of strategic social activities of the SE at the optimal choice in (36):

$$\left.\frac{\partial W}{\partial a_0}\right|_{a_0^*}=\frac{\partial G^S}{\partial a_0}+b'\left[\frac{\partial q_0}{\partial b}(-q_0+(A-Q-2cq_0)-\Sigma_{i=1}^n q_i)+\Sigma_{i=1}^n\frac{\partial q_i}{\partial b}\left(-q_0+\frac{\partial \pi_i}{\partial q_i}-\Sigma_{i\neq j}^n q_j\right)\right]$$

$$=b'\left[\frac{\partial q_0}{\partial b}(A-2Q-2cq_0)+\Sigma_{i=1}^n\frac{\partial q_i}{\partial b}(q_i-Q)\right] \quad (39)$$

Note that $\frac{\partial G^S}{\partial a_0}=0$ and $\frac{\partial \pi_i}{\partial q_i}=0$ in the market equilibrium.

Proposition 8: *When subsidy policy supports the sustainability of the SE, its social activities are always higher than those at the social optimum.*

Proof: First, in the case of private followership where $a_0^*=a_0^F$, we have in (39)

$$\left.\frac{dW^F}{da_0}\right|_{a_0^F}=(q_i^F-Q^F-2cq_0^F)-2cnq_0^F<0 \quad \text{since} \quad (q_i^F-Q^F-2cq_0^F)<0.$$

Second, in the case of private leadership where $a_0^*=a_0^L$, we have in (39)

$$\left.\frac{dW^L}{da_0}\right|_{a_0^L}=c\left(q_i^L-Q^L-\frac{q_i^L}{1+c}-2cq_0^L\right)-n(1+c)\left(\frac{q_i^L}{1+c}+2cq_0^L\right)<0 \quad \text{since}$$

$$\left(q_i^L-Q^L-\frac{q_i^L}{1+c}-2cq_0^L\right)<0.$$

Owing to the convexity of total social welfare function, we can state that a_0^* is higher than the social optimum, which satisfies $\frac{\partial W}{\partial a_0}=0$ in (38). Q.E.D.

Proposition 8 implies that the SE has a strategic incentive to over-invest in social activities because it aims to promote consumer surplus and social value but neglects profitability, which is part of total social welfare. This over-incentive of the SE provides a higher supply of production but reduces rivals' production, which worsens the economic incentives of the SE to be profitable in the market. Therefore, it is important to improve the SE's profitability under subsidy policy to support its sustainability.

Finally, we compare the relative social activities of the two competition modes where the subsidy rate under private leadership is higher than that under private followership. Since $b'(a_0) > 0$ and $d'(a_0) > 0$, the first-order condition in (36) yields the following:

$$\left[\frac{\partial q_0}{\partial b}(Q + b(a_0)) + Q\sum_{i=1}^{n}\frac{\partial q_i}{\partial b} + q_0\right] = \frac{d'(a_0)}{b'(a_0)} > 0 \tag{40}$$

Further, since $b''(a_0) \le 0$ and $d''(a_0) \ge 0$, (40) implies that

$$a_0^F \gtreqless a_0^L \quad \text{iff}$$

$$\frac{\partial q_0^F}{\partial b}(Q^F + b(a_0^F)) + Q^F \textstyle\sum_{i=1}^{n} \dfrac{\partial q_i^F}{\partial b} + q_0^F \gtreqless \dfrac{\partial q_0^L}{\partial b}(Q^L + b(a_0^L)) + Q^L \textstyle\sum_{i=1}^{n} \dfrac{\partial q_i^L}{\partial b} + q_0^L .$$

Because of analytical complexity, we impose specific assumptions; the marginal social value is linear, $b(a_0) = ba_0$, and the cost of social activities is quadratic, $d(a_0) = a_0^2$. Then, by using the equilibrium results under private followership in (16) and private leadership in (25), we show that there exists a threshold which supports that $a_0^F \gtrless a_0^L$ holds, depending on the parameters such as n, c, and b. For example, if we let $n = c = 1$, we can show that $a_0^F \gtrless a_0^L$ if $b \gtrless 1.38$. This relation implies that social activities under private followership can be higher (lower) than those under private leadership as the marginal benefit of social activities increases (decreases).

VI. Conclusion

The recent emergence of the SEs requires the economic analysis of the relationship between market performance and government policy. In this study, we investigated market equilibrium under subsidy policy on the SE for the underprivileged and found a trade-off between private followership and private leadership. In particular, we showed that private leadership is better for total social welfare, whereas private followership is better when social concerns for the underprivileged are emphasized. We also found that the number of PEs and cost inefficiency of SE both affect profitability and welfare consequences, in contrast to the previous literature. We then showed that both output rationing and market share rationing are inferior to subsidy policy. However, if the government has to fulfill a rationing policy for some political and/or informational reasons, output rationing is the better alternative. Finally, we showed that SEs are over-incentivized to pursue social activities and thus, it is important to improve SE's profitability under subsidy policy. This finding also implies that the government should investigate the moral hazard problem in which a for-profit pseudo-SE wastes or abuses the resources through its over-investment in the social activities to obtain a higher subsidy from the government. This calls for incentive mechanism and continuous monitoring system accompanied by certification with an appropriate subsidy policy. Therefore, in a subsidization regime, the profitability and accountability are important policy principles for the sustainability of SE in a market economy.

There are some future research topics on market competition with the SEs and the PEs. First, our results depend on the number of SEs and specific functional forms such as linear demand and quadratic cost functions. Thus, there is a need to examine the robustness of the results in the context of general functions. Second, the practical and innovative objective function of SEs, including those generating social value with differentiated products and services and those promoting ethical financing such as micro-credit and fair trade is a promising topic for future research. Finally, different organizational structure between managers and employees in the SEs should be analyzed to understand their profit-sharing scheme.

Appendix A

We examine the comparisons between private followership and private leadership under the same output subsidy rate. We can show that the subsidy rate is critical to determine the equilibrium outputs. In the below, we summarize the propositions with the assumption that $0 \le s < Ac$, in which both optimal subsidy rates, s^F in (15) and s^L in (24), are supportable from the government. Note that this case also includes a zero subsidy case.

Proposition A1: *When government imposes the same subsidy rate, Proposition 1 holds.*

Proof: Comparing the output levels yields the followings:

$$q_0^F - q_0^L = \frac{n(Ac-s)}{(c+c^2)(1+c+n+2cn+cn^2)} \gtrless 0 \text{, Thus } q_0^F \gtrless q_0^L \text{ iff } s \lessgtr Ac.$$

$$q_i^F - q_i^L = \frac{-Ac+s}{c(1+n)(1+c+cn)} \lessgtr 0 \text{, Thus } q_i^F \lessgtr q_i^L \text{ iff } s \lessgtr Ac.$$

$$Q^F - Q^L = \frac{n(-Ac+s)}{(1+c)(1+n)(1+c+cn)} \lessgtr 0 \text{, Thus } Q^F \lessgtr Q^L \text{ iff } s \lessgtr Ac. \quad \text{Q.E.D.}$$

Proposition A2: *Irrespective of the subsidy rate, Proposition 3 holds.*

Proof: Comparing the profits of the PEs yields the followings:

$$\pi_i^F - \pi_i^L = \frac{cn^2-(1+c)}{c(1+c)(1+n)^2(1+c+cn)^2}(Ac-s)^2 \gtrless 0 \text{ iff } n^2 \gtrless 1+\frac{1}{c}. \quad \text{Q.E.D.}$$

Proposition A3: *When government imposes the same subsidy rate, Proposition 4 holds if* $\bar{s} < s < Ac$.

Proof: Comparing social welfare yields the followings:

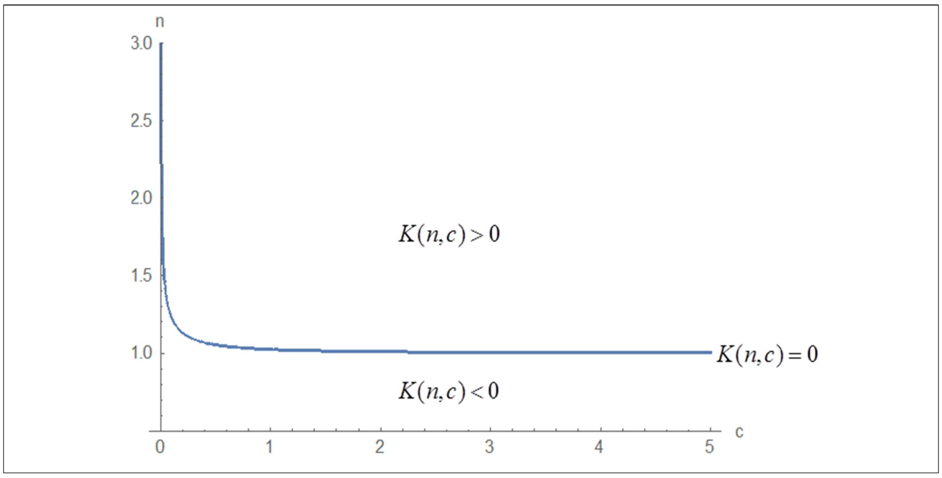
$$W^F - W^L = \frac{n(Ac-s)(2b(1+c)(1+n)(1+c+cn) - Ac(2+c)(2+n+2c(1+n)) - (2n+c(2+2c+7n+6cn+4(1+c)n^2))s)}{2c(1+c)^2(1+n)^2(1+c+cn)^2}$$

Let $\bar{s} = \dfrac{2b(1+c)(1+n)(1+c+cn) - Ac(2+c)(2+n+2c(1+n))}{2n+c(2+2c(1+n)(1+2n)+n(7+4n))}$. Since $\bar{s} < Ac$, we have the followings: $W^F \gtrless W^L$ iff $s \lessgtr \bar{s}$. Q.E.D.

Appendix B

We can draw the figure of $K(n,c)=0$ where
$K(n,c)=c(2n^2+2c^2(1+n)^2+c(1+2n)^2)^2(1+n)^2-(1+c)(c+n+cn)^2(1+2c(1+n)^2)^2$.

[Figure 1] The value of $K(n,c)$



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