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Chun-Hui Miao*

Abstract

In many markets, consumers obtain price quotes before making purchases. This paper considers a fixed-sample size model of consumer search for price quotes when sellers must spend resources to learn the true cost of providing goods/services. It is found that (1) even with ex ante identical consumers and sellers, there is price dispersion in the equilibrium; (2) the expected equilibrium price can decrease with the search cost of consumers; (3) consumers may engage in excessive search that is detrimental to their own welfare; (4) a decline in the search cost can leave consumers worse off, due to their lack of commitment. (JEL D40, L00)

Keywords: Price dispersion, Precontract cost, Search cost, Sealed-bid Auction.

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In order to make an informed purchase decision, consumers search. In order to earn their business, sellers provide relevant information such as prices. The standard economic models of consumer search assume that price search is costly, but price setting is costless.¹ The best example of sellers in these models is perhaps a supermarket, which sets prices *before* consumer search takes place. Consumers are assumed to know the distribution of prices (presumably from repeated purchases), but do not know the prices charged by particular stores until they search.²

However, in many markets, even a simple price quote may involve nontrivial costs for the seller. For example, a repair shop must diagnose the problem before giving a cost estimate; a mortgage lender must evaluate a borrower's creditworthiness before issuing a rate quote; an insurance agent must assess an applicant's risk characteristics before determining the premium; a travel agent may have to compare different itineraries and consult with multiple airlines before offering an airfare deal. Unlike a supermarket, sellers in these markets make costly efforts to learn the production costs based on the needs of the individual consumers, and therefore can only set prices *after* consumer search takes place. The process of consumer search works much like a first-price sealed-bid procurement auction: each seller submits a bid without knowing its competitors' bids. A consumer can canvass a number of sellers, but cannot impose an alternative mechanism within which the sellers compete, nor is the purchase made frequently such that the consumer and the seller have a long-term relationship, the lack of which precludes the parties from contracting on the seller's effort in preparing the price quote.

This paper incorporates the above features into a theory of consumer search, which nests a model of sealed-bid common value auction, to study the impact of precontract costs including consumer search cost and price setting cost. The latter cost is due to uncertainty in the production cost. To facilitate comparison with relevant papers, I use an example adapted from Pesendorfer and Wolinsky (2003) to illustrate. Consider a homeowner who finds that her air conditioning does not work properly. She asks contractors for repair cost estimates. A contractor may send a skilled worker at a high cost or an unskilled worker at a low cost. The unskilled worker recommends a

¹It is a long tradition that began with Stigler's seminal paper (1961). More recently, a class of search models with an "information clearinghouse" assume nearly the opposite, that is, zero (marginal) search cost but positive (fixed) advertising cost (Baye and Morgan 2001).

²This assumption is relaxed in Rothschild (1974), Benabou and Gertner (1993) and Dana (1994), where consumers learn about the distribution of prices as they search, and in Daughety (1992), where consumers search without price precommitment by sellers.

generic repair (e.g., replacement of an expensive part), whereas the skilled worker might be able to diagnose the actual problem and find a less expensive solution.³ The homeowner cannot distinguish the skilled from the unskilled worker. By assuming that the repair outcome is not contractible (but price search is costless),⁴ Pesendorfer and Wolinsky (2003) examine the market inefficiencies when a consumer must rely on second opinions to pick the right contractor.

In contrast, this paper assumes that the success of the repair can be verified, so the homeowner only needs to select the contractor who offers the lowest price (along with the same warranty), but the contractors' problem is complicated by the need to diagnose before competitive bidding: if he expects the homeowner to do comparison shopping and canvass a large number of contractors, then he may be less inclined to send the skilled worker, a sunk cost that cannot be recovered in a bidding war. In the equilibrium, some contractors send skilled workers before submitting informed bids, but their expected profits net of the diagnosis cost must be equal to those of contractors that send unskilled workers to submit blind quotes. Ultimately the consumer bears the diagnosis cost. While this observation suggests an equivalence between consumer search and contractor diagnosis — both are simultaneously beneficial (discovery of a less expensive solution) and wasteful (duplicative efforts) — our analysis of the contractor's problem reveals an important difference: the only way to save on the total search cost is to search less, but paradoxically a consumer can potentially save on the total diagnosis cost by searching more. The latter is true if the contractors' willingness to incur the diagnosis cost drops sharply when they face more competitors. Consequently, price setting cost and search cost can have very different, even opposite, impacts on consumer search behavior. For example, if the price setting cost is zero then the optimal number of searches is two, but it may be infinity when the search cost vanishes. More generally, while the optimal number of searches always *decreases* with the search cost, it can sometimes *increase* with the price setting cost. Furthermore, price dispersion persists even with ex ante identical consumers and sellers and even if consumer search is costless.

The introduction of price setting cost also affects the timing of the game, which raises a commitment issue that has not been previously investigated. If sellers set prices before consumer search

³This is where my example differs from the model of Pesendorfer and Wolinsky (2003), which assumes that the costs of performing any potential services are identical, but the outcomes can be different. In addition, their model assumes that prices are set before the diagnosis, not after.

⁴Their model contains a search cost, but it is not paid until a contract offer is accepted.

takes place, as in all models with posted prices, then sellers are always the Stackelberg leader; but if prices are set upon consumer request, then the game between the consumer and sellers can either be sequential in nature, or simultaneous, depending on whether the consumer can commit. If a consumer can act as a Stackelberg leader and precommit to the number of searches before sellers submit their bids, then the number of searches will be optimal, but this is not always the case. As argued by Wolinsky (2005), an individual consumer may not have significant commitment power. In the current paper, the lack of commitment is due to a credibility problem: holding constant the sellers' pricing strategies, a searcher always benefits from sampling a larger size, and therefore she cannot credibly convince sellers that her actual number of searches will be the announced number. This is especially true if the number of price quotes is private information. In this setting, I find that the expected equilibrium price can decrease with search cost. Moreover, consumers may engage in excessive search that is detrimental to their own welfare, and therefore a decline in the search cost can leave consumers worse off.

The literature on consumer search originates from Stigler (1961), which discusses the optimal search strategy of a consumer faced with an exogenously specified price distribution. While his paper shows that price dispersion is consistent with costly search, it does not really explain why sellers charge different prices. Numerous papers have since been written and provided a variety of explanations, but none of them has deviated from Stigler's assumption that consumers take the price distribution as given. In these models, regardless of whether consumers engage in simultaneous search or sequential search, equilibrium is derived by equating the cost of obtaining an additional price quote to the expected price reduction from the additional search.⁵ Underlying it all is the presumption that, with price dispersion, more searches lead to discovery of better prices. Consequently, despite many differences, these papers have the following results in common: *a)* expected price increases with search cost; *b)* consumer welfare decreases with search cost; *c)* the degree of price dispersion among homogeneous sellers decreases with search cost. This paper shows that none of these results is necessarily true in markets where prices are set after consumer search. This is because, contrary to Stigler's assumption, price distributions will be affected by consumers' search intensity in these markets.

⁵In the class of "information clearinghouse" models exemplified by Baye and Morgan (2001), the number of consumer searches is exogenous, but sellers' participation decisions are endogenous.

More broadly, this paper contributes to our understanding of transaction costs. Coase (1960) defines transaction cost as follows: "In order to carry out a market transaction it is necessary to discover who it is that one wishes to deal with, to inform people that one wishes to deal and on what terms, to conduct negotiations leading up to a bargain, to draw up the contract, to undertake the inspection needed to make sure that the terms of the contract are being observed, and so on." More succinctly, Dahlman (1979) classifies transaction costs into three categories based on the stages of a contract: search and information costs (precontract), bargaining and decision costs (contract), policing and enforcement costs (postcontract). While the impact of consumer search cost on market outcome has spawned a huge amount of research efforts, its interaction with sellers' precontract cost has so far received scant attention. A notable exception is French and McKormick (1984), whose informal analysis of the service market anticipates many of the themes explored in this paper. Their paper was also the first to point out the analogue between the usual process for purchasing services in the consumer goods market and a sealed-bid auction.⁶ After showing that the winner's expected profit equals the sum of his competitors' sunk costs of bid preparation under a free-entry condition, they argue that consumers indirectly pay for sellers' precontract costs. The focus of their paper, however, is on sellers' marketing strategies, such as how likely sellers are to charge for their estimates or advertise, whereas the focus of this paper is on the problem faced by the consumer side.⁷

To the best of my knowledge, Pesendorfer and Wolinsky (2003) and Wolinsky (2005) are the only other papers that consider consumer search in the presence of precontract costs. Both study the markets for procurement contracts, but focus on different information problems. Wolinsky (2005)'s main finding that consumers engage in an inefficiently excessive search is the same as mine. The underlying mechanism is also similar: the impossibility of contracting on the sellers' effort distorts consumers' search incentive. However, there is an important difference: in Wolinsky's model sellers' efforts improve the quality of service, but in the current model efforts are made to extract information rents. As a result, the two models have opposite predictions about the welfare properties of efforts. In addition, in his model, while search intensity is excessive from a

⁶ Other papers that have emphasized the close connection between auction and search include McAfee and McMillan (1988) and Baye, Morgan and Scholten (2006). The latter also provides an extensive review of the search literature.

⁷ Due to the lack of formal game-theoretic analysis, the connection between assumptions and results is somewhat opaque in their paper. For example, it is not clear whether the predicted pattern is the result of collective behavior among sellers or the noncooperative outcome.

social planner's point of view, it is optimal for the consumer, but in this model excessive search is detrimental to the consumer herself as well as social welfare. The papers also use different modeling approaches: he uses a sequential search model in the context of a market for a differentiated product, whereas I use a fixed-sample size model with homogeneous products. The two approaches are complementary.

The remainder of this paper is organized as follows. Section 1 introduces the model and discusses assumptions. Section 2 presents some preliminary results. Section 3 characterizes the full equilibrium for both when the consumer is able to commit to the optimal number of searches and when she is not. Section 4 concludes.

1 The Model

A consumer is willing to pay v for a good or service (henceforth, the product), which can be provided by any one of the $N \geq 2$ sellers. In order to find the best deal, the consumer visits sellers to collect price quotes. The cost of effort for each visit is s , i.e., the search cost. Sellers face the same production cost, but it can take two values: a low cost of c_l with a probability of q or a high cost of c_h with a probability of $1 - q$. At a cost of t , a seller can acquire information to learn the actual production cost. Sellers are risk neutral. The values of all above variables are assumed to be common knowledge.

The game is played in the following order:

1. The consumer requests prices quotes from n sellers;⁸
2. Each seller, upon request, chooses whether to incur t to acquire information about the production cost: if a seller acquires information, it quotes a price from the distribution of either $F_l(p)$ or $F_h(p)$, depending on whether the cost is c_l or c_h ; if a seller does not acquire information, it quotes a price from the distribution of $F_b(p)$.
3. After receiving all price quotes, the consumer buys from the seller that offers the lowest price.

⁸Since each seller quotes one price, n represents the number of price quotes as well as the number of consumers searches or the number of sellers visited by the consumer, or simply the search intensity as in Janssen and Moraga-Gonzalez (2004).

A complete specification of the game also requires us to specify whether n belongs to the sellers' information set when they compete at stage 2. If n is publicly observable, then the consumer can act as a Stackelberg leader and commit to the optimal number of searches; but if n is non-contractible private information, then the consumer and sellers will be playing a simultaneous move game at the first two stages. Both possibilities will be considered.

It is not difficult to see that $F_h(p)$ is a degenerate distribution, where $F_h(p) = 0$ for $p < c_h$ and $F_h(p) = 1$ for $p \geq c_h$. Therefore, the equilibrium will be characterized by the consumer's choice of n and sellers' strategies in a triplet $\{\alpha, F_l(p), F_b(p)\}$, where α is the probability of a seller choosing to assess the production cost, $F_l(p)$ is the cumulative distribution function of price quote when a seller learns that the production cost is low, and $F_b(p)$ is the distribution function when a seller submits a "blind" quote. Sellers use pure strategies in pricing if and only if price distributions are degenerate. The solution concept is a Bayesian Nash equilibrium. The consumer and sellers form beliefs about each other's strategies. In the equilibrium, their beliefs are correct.

For ease of exposition, I impose the following restrictions on parameter values. First, I assume that $v >> s$ so that it is never optimal for the consumer to search just one seller, in which case she minimizes the total search costs but has to pay a monopoly price for the product; in other words, $n \geq 2$. Second, I assume that the pool of sellers is so large that the consumer's number of searches is never limited by the number of sellers, i.e., $N \rightarrow \infty$. Last, I assume that the cost of information acquisition does not exceed the private value of the information, i.e., $t < q(1 - q)(c_h - c_l)$. These restrictions cut down the number of cases we have to consider and allow us to focus on only the nontrivial cases, but they do not change or weaken any of the qualitative results.

1.1 Discussion of Assumptions

In order to keep the model tractable and have a stark contrast with the existing literature, this paper has introduced a very simple model that focuses on only the most important aspects of the markets and assumes away some of the admittedly more realistic features. Here I discuss the key assumptions and justify the use of some simplifying assumptions.

Homogenous Sellers At the last stage of the game, the consumer choice is based on price only. This is a common assumption in the consumer search literature. At the same time, it is a

natural assumption given the paper's focus on the transaction costs at the precontract stage. After all, without having to incur additional transaction costs at later stages, sellers can offer complete contracts that cover all aspects of the product, including design, quality, warranty, price, etc. This means that we can collapse all these variables into a single one — "price" — and assume all other aspects of the product to be uniform across sellers. Essentially, sellers can be viewed as competing in utility space.

At the bidding stage, the game is modeled as a common value procurement auction since sellers are assumed to be *ex ante* identical. This may not be a realistic assumption: sellers' costs are likely correlated, but not necessarily the same. Nonetheless, I make this assumption to ensure that equilibrium price dispersion cannot be attributed to different cost realizations across sellers.

The assumption of homogenous sellers is also consistent with empirical research in related markets. For example, in her study of the auto insurance market, Honka (2014) assumes that the only search dimension is the premium charged by each provider. Moreover, she reports that over 93% of consumers kept their coverage choice the same during the last shopping occasion and were searching only for the lowest premium. Similarly, in their study of the Canadian mortgage market, Allen, Clark and Houde (2014) find that contracts are homogeneous, and for a given consumer costs are mostly common across lenders due to loan securitization and a government insurance program.

Uncertainty in Production Cost The production cost is assumed to be a binary random variable for tractability. Admittedly, this is a crude assumption, but it is in line with the home repair example. For other markets, it is also relevant: a mortgage broker or an insurance agent's precontract effort involves identifying the variety of discounts available to an applicant based on her risk characteristics; a travel agent or a car dealer's effort involves finding out the existence of airline promotions or manufacturer incentives. The same assumption is also used by Pesendorfer and Wolinsky (2003). As for the source of the cost uncertainty, it can be internal, (i.e., consumer specific), or external, (e.g., due to input cost variations).

Fixed-sample Size Search There are two basic types of models used in the search literature: fixed-sample size search models assume that consumers sample a fixed number of sellers and choose

to buy the lowest priced alternative,⁹ whereas sequential search models assume that consumers visit sellers one-by-one and do not stop searching until their reservation price is met.

There are several reasons why I assume fixed-sample size search. First, existing empirical evidence suggests that fixed-sample size search provides a more accurate description of observed consumer search behavior (De los Santos, Hortacsu, and Wildenbeest 2012, Honka and Chintagunta 2017); second, fixed-sample size search is optimal when there is a fixed-cost component to search. Recall the example of a homeowner in need of repair cost estimates: if she has to take a day off from work for the visits of the contractors, then it will be more efficient to schedule multiple appointments beforehand so that all visits take place during the same day than to follow a prolonged process from sequential searches. Third, but particularly relevant to this model, when price setting involves costly assessment, a price quote often comes after a delay so the consumer can engage in additional searches while waiting for the outcome from a previous search.¹⁰

It should also be noted that there is a subtle difference between the commitment problem behind the common criticism against the fixed-sample size approach of modeling consumer search and the commitment problem highlighted in this paper. The former is about a consumer's inability to commit to the fixed-sample size *strategy* itself when the expected marginal benefit of an extra search exceeds the cost, but the latter is the consumer's inability to commit to the optimal *number* of searches. In economic environments where fixed-sample size search is more advantageous than sequential search, the issue of commitment to strategy does not bite, but the issue of commitment to the number of searches may remain; it goes away only if either the number of searches or the sellers' assessment effort becomes publicly observable and contractible, as shown in the analysis below.

⁹ MacMinn (1980) is the first to show that equilibrium price dispersion can arise when consumers engage in fixed-sample size search, but his result relies on cost heterogeneity among sellers. Burdett and Judd (1983) provides a model of equilibrium price dispersion with ex ante identical consumers and sellers. In the equilibrium price distribution, all sellers charge positive markups. A fraction of consumers visit one store and purchase, while the remaining fraction of consumers search two stores and buy from whichever offers the lower price. Janssen and Moraga-Gonzalez (2004) develop an oligopolistic version of Burdett and Judd (1983) where some consumers search costlessly. They show that the equilibrium expected price may be constant, increasing or non-monotonic in the number of sellers, depending on the equilibrium consumers' search intensity and the existing number of sellers. In particular, they find that duopoly yields identical expected price and price dispersion but higher welfare than an infinite number of sellers.

¹⁰ In the same vein, Morgan and Manning (1985) and Janssen and Moraga-Gonzalez (2004) argue that fixed-sample size search is more appealing when a consumer needs to gather price information quickly.

Estimation Fee A problem highlighted in this paper is that sellers cannot be compensated directly for their precontract efforts. One may wonder whether charging consumers an estimation fee upfront will solve the problem. In fact, a large part of French and McCormick's discussion centers around the use of an estimation fee to recoup the costs of sellers' efforts. There are two reasons why the current model does not include an estimation fee. First, as emphasized by Pesendorfer and Wolinsky (2003), the diagnosis and estimation of the repair cost is a type of "credence" service — the sellers' assessment effort and outcome are unobservable and thus incontractible. Paying an estimation fee does not guarantee that a seller will put forth the effort after pocketing the payment. Second, French and McCormick's discussion is based on the assumption that a repair shop has some captive consumers — due to prohibitively high search costs — who will choose to pay the estimation fee even if other repair shops can provide costless estimates.¹¹ In the current model, all consumers have the same search cost, and therefore all sellers will end up charging zero estimation fee (or a fee equal to the assessment cost of an unskilled worker) even if they are free to charge a fee. The same result, based on the standard Bertrand style argument, is obtained in Wolinsky (2005).

2 Preliminary Results

2.1 A Benchmark Result

A useful point of departure is to consider what happens if $t = 0$, i.e., sellers can costlessly assess their production costs. This is not only the assumption of a frictionless market, but also the working assumption of almost all consumer search models. In this case, based on the standard Bertrand style argument, we can see that the prices will be set equal to the (realized) production cost as long as there are at least two sellers. A consumer cannot do better by visiting more sellers because she cannot get a better offer, nor can she do better by visiting just one seller, who will charge a monopoly price. Therefore, it is sufficient for the consumer to visit just two sellers to obtain competitive price quotes while economizing on the search costs. The consumer earns a surplus of $v - c_E - 2s$, where $c_E = qc_l + (1 - q)c_h$ is the expected production cost. This serves as a natural benchmark for the current analysis.

¹¹One of their remarks - shops in low-income neighborhoods are more likely to charge for estimates because of potential customers' lower search costs - appears to contradict this assumption.

Zero assessment cost, however, is not a necessary condition for the above benchmark outcome. The same outcome can be obtained even if the assessment cost is positive. It is not difficult to see why: acquiring information about the production cost allows sellers to earn information rents, but it is wasteful from the consumer's point of view. If she and sellers can contract on the latter's assessment efforts, then the consumer can prevent sellers from earning information rents by simply requiring sellers not to engage in the discovery of production costs.¹² The sellers will again compete *a la Bertrand*, with each of them quoting a price of c_E and earning zero profits. The consumer earns the same amount of surplus as in the benchmark. Albert straightforward, this result demonstrates that the assessment cost, in itself, does not necessarily cause welfare loss for the consumer. Rather, any loss of efficiency is due to the inability to contract on sellers' assessment efforts. Of course, if the cost of effort is so high that it exceeds the private value of information, then neither seller will make an effort even if they are not contractually precluded from doing so. In the model, the private value of the information on the production cost equals $q(1 - q)(c_h - c_l)$. The assumption that $t < q(1 - q)(c_h - c_l)$ allows us to focus on the nontrivial cases.

In both cases, consumer surplus is already maximized when the consumer obtains two competing price quotes. There is no further gain from requesting additional price quotes.¹³ Therefore, it does not matter whether the consumer can commit. Accordingly, I summarize the above results in the following proposition:

Proposition 1 *Regardless of whether the number of price quotes is publicly observable or non-contractible private information, consumer surplus will be maximized in the unique equilibrium if either (i) $t = 0$; or (ii) $t \geq q(1 - q)(c_h - c_l)$, or (iii) the consumer and sellers can contract on the assessment effort. In all three cases, the optimal number of searches is two.*

Interestingly, Proposition 1 also shows that the optimal number of searches is the same for extreme values of t : This suggests that the optimal number of searches may not be monotonic in t , an observation that will be confirmed later in the analysis.

¹²This requirement may not work if the consumer has private information, which can potentially lead to an adverse selection problem. Here I assume away consumer private information on the ground that sellers have more expertise than consumers in the relevant markets.

¹³There will be savings from requesting one fewer price quote so the consumer might randomize between searching once and twice, giving rise to the same result as in Burdett and Judd (1983), but this possibility can be ruled out by the assumption that $n \geq 2$.

2.2 The Subgame Equilibrium at the "Bidding" Stage

In the (stage 2) subgame, n sellers receive request for price quotes. When deciding what price to quote, these sellers face a problem similar to a sealed-bid common value auction: each seller chooses whether (or not) to incur a cost to assess the production cost before submitting a bid without knowing his competitors' bids. This is not a trivial problem, because a pure strategy equilibrium does not exist for any small but positive assessment cost, i.e., $t \in (0, q(1-q)(c_h - c_l))$.

Lemma 1 *If $t \in (0, q(1-q)(c_h - c_l))$, no pure strategy equilibrium exists at the "bidding" stage in which all sellers make an effort to learn the true cost.*

Proof. If all sellers learn the true cost, since this is a common value auction, they will all quote the same price. As a result, net of the assessment cost t , their profits are negative. If a seller deviates by not incurring the assessment cost and quoting c_h , his profit will be zero, so the deviation is profitable. ■

Lemma 2 *If $t \in (0, q(1-q)(c_h - c_l))$, no pure strategy equilibrium exists at the "bidding" stage in which no seller makes an effort to learn the true cost.*

Proof. If no sellers learns the true cost, then the price must be c_E and sellers' profits will be zero. If a seller learns the true cost, he can charge $c_E - \varepsilon$ if the production cost is revealed to be c_l . This happens with a probability of q , so his expected profit is $q(c_E - c_l) = q(1-q)(c_h - c_l)$. It is a profitable deviation if $t < q(1-q)(c_h - c_l)$. ■

Having ruled out the existence of a symmetric pure strategy equilibrium for the relevant range of parameter values, I now examine symmetric mixed-strategy equilibrium in the subgame at the bidding stage. The mixed-strategy equilibrium involves two randomizations for sellers: first, sellers randomize between submitting an informed quote and submitting a blind quote; second, sellers randomize their price quotes. It is tedious but not difficult to verify that, in a symmetric equilibrium, the supports of the two price distributions $F_l(p)$ and $F_b(p)$ do not overlap. Given this observation, if seller i chooses not to make the assessment effort, then his expected profit is

$$\pi_i = \int_{\underline{p}_b}^{\bar{p}_b} dF_b(p) \left(q(p - c_l)(1 - \alpha)^{n-1} \left(1 - \tilde{F}_l(p)\right)^{n-1} + (1 - q)(p - c_h) \sum_{k=0}^{n-1} \binom{n-1}{k} \alpha^k (1 - \alpha)^{n-1-k} \left(1 - \tilde{F}_b(p)\right)^{n-1-k} \right),$$

where $\tilde{F}_l(p)$ and $\tilde{F}_b(p)$ are the corresponding price distributions for the $n - 1$ other sellers. Because of symmetry, $\tilde{F}_l(p) \equiv F_l(p)$ and $\tilde{F}_b(p) \equiv F_b(p)$. In a mixed strategy equilibrium, seller i must be indifferent among all choices of p on the support of $[\underline{p}_l, \bar{p}_l]$, i.e.,

(1)

$$q(p - c_l)(1 - \alpha)^{n-1}(1 - F_l(p))^{n-1} + (1 - q)(p - c_h) \sum_{k=0}^{n-1} \binom{n-1}{k} \alpha^k (1 - \alpha)^{n-1-k} (1 - F_b(p))^{n-1-k} = \Delta_1,$$

where Δ_1 is a constant. If seller i chooses to learn the cost, his expected profit can be written as

$\pi_i = q \int_{\frac{\underline{p}_l}{p}}^{\bar{p}_l} (p - c_l) \sum_{k=0}^{n-1} \binom{n-1}{k} \alpha^k (1 - \alpha)^{n-1-k} (1 - \tilde{F}_l(p))^k dF_l(p) - t$. Applying again the indifference principle, we have

$$(2) \quad (p - c_l) \sum_{k=0}^{n-1} \binom{n-1}{k} \alpha^k (1 - \alpha)^{n-1-k} (1 - F_l(p))^k = \Delta_2.$$

Since competing sellers are identical, they must all earn zero expected profits in the equilibrium.

Hence, we must have $\Delta_1 = 0$ and $\Delta_2 = t/q$. Thus, from (1), we can get $q(p - c_l)(1 - \alpha)^{n-1}(1 - F_b(p))^{n-1} + (1 - q)(p - c_h)(\alpha + (1 - \alpha)(1 - F_b(p)))^{n-1} = 0$, i.e., $F_b(p) = 1 - \frac{\alpha}{1-\alpha} \left(\left(\frac{q(p-c_l)}{(1-q)(c_h-p)} \right)^{\frac{1}{n-1}} - 1 \right)^{-1}$;

from (2), we can get $(\alpha - \alpha F_l(p) + (1 - \alpha))^{n-1} = \frac{t}{q(p-c_l)}$, i.e., $F_l(p) = \frac{1 - \left(\frac{t}{q(p-c_l)} \right)^{\frac{1}{n-1}}}{\alpha}$. Since $F_b(\underline{p}_b) = 0$, we must have $q(\underline{p}_b - c_l)(1 - \alpha)^{n-1} + (1 - q)(\underline{p}_b - c_h) = 0$. Solving, we obtain $\underline{p}_b = \frac{(1-q)c_h + q(1-\alpha)^{n-1}c_l}{(1-q)+q(1-\alpha)^{n-1}}$. Similarly, we can obtain $\bar{p}_b = c_h$, $\underline{p}_l = c_l + t/q$ and $\bar{p}_l = c_l + \frac{t}{q(1-\alpha)^{n-1}}$. In addition, we have $\bar{p}_l = \underline{p}_b$, so $c_l + \frac{t}{q(1-\alpha)^{n-1}} = \frac{(1-q)c_h + q(1-\alpha)^{n-1}c_l}{(1-q)+q(1-\alpha)^{n-1}}$. From this, we can solve for the

assessment probability: $\alpha = 1 - \left(\frac{t(1-q)}{q((c_h - c_l)(1-q) - t)} \right)^{1/(n-1)}$. The expected price is

$$\begin{aligned}
E(p) &= \alpha^n (1 - q) c_h + \alpha^n q \int pd(1 - (1 - F_l(p))^n) + (1 - \alpha)^n \int pd(1 - (1 - F_b(p))^n) \\
&\quad + \sum_{k=1}^{n-1} \binom{n-1}{k} \alpha^k (1 - \alpha)^{n-k} \left((1 - q) \int pd(1 - (1 - F_b(p))^{n-k}) + q \int pd(1 - (1 - F_l(p))^k) \right) \\
&= \alpha^n (1 - q) c_h + \int_0^1 \left(\begin{array}{l} \alpha^n n q F_l^{-1}(F) (1 - F)^{n-1} \\ \quad + (1 - \alpha)^n n F_b^{-1}(F) (1 - F)^{n-1} \\ \quad + \sum_{k=1}^{n-1} \binom{n-1}{k} \alpha^k (1 - \alpha)^{n-k} \left(\begin{array}{l} (1 - q) (n - k) F_b^{-1}(F) (1 - F)^{n-k-1} \\ \quad + q k F_l^{-1}(F) (1 - F)^{k-1} \end{array} \right) \end{array} \right) dF \\
(3) \quad &= \alpha^n (1 - q) c_h + n \int_0^1 \left(\begin{array}{l} q \alpha (1 - \alpha F)^{n-1} F_l^{-1}(F) \\ \quad + (1 - \alpha)^n (1 - F)^{n-1} \left(q + (1 - q) \left(\frac{\alpha}{(1-\alpha)(1-F)} + 1 \right)^{n-1} \right) F_b^{-1}(F) \end{array} \right) dF \\
&= \alpha^n (1 - q) c_h + n \int_0^1 \left(\begin{array}{l} \alpha t + q \alpha (1 - \alpha F)^{n-1} c_l \\ \quad + (1 - \alpha)^n (1 - F)^{n-1} \left(q c_l + (1 - q) c_h \left(\frac{\alpha}{(1-\alpha)(1-F)} + 1 \right)^{n-1} \right) \end{array} \right) dF \\
&= q c_l + (1 - q) c_h + n \alpha t = c_E + n \alpha t
\end{aligned}$$

Summarizing, we have the following results:

Lemma 3 *If the consumer searches $n \geq 2$ sellers, each seller chooses to assess the production cost with a probability of $\alpha = 1 - \left(\frac{t(1-q)}{q((c_h - c_l)(1-q) - t)} \right)^{1/(n-1)}$. If a seller learns that the production cost is c_h , he quotes a price of c_h ; otherwise he quotes a price according to the distribution of $F_l(p) = \frac{1 - \left(\frac{t}{q(p - c_l)} \right)^{1/(n-1)}}{\alpha}$, with $\underline{p}_l = c_l + t/q$ and $\bar{p}_l = \frac{(1-q)c_h + q(1-\alpha)^{n-1}c_l}{(1-q) + q(1-\alpha)^{n-1}}$. If a seller does not make the assessment effort, he quotes a price according to the price distribution of $F_b(p) = 1 - \frac{\alpha}{1-\alpha} \left(\left(\frac{q(p - c_l)}{(1-q)(c_h - p)} \right)^{\frac{1}{n-1}} - 1 \right)^{-1}$, with $\underline{p}_b = \bar{p}_l$ and $\bar{p}_b = c_h$. The expected price is $c_E + n \alpha t$.*

Figure 1 illustrates how the assessment cost affects the price distributions. The red solid curve depicts the price distribution when $t = 0.2(c_h - c_l)$ and the black dashed curve when $t = 0.1(c_h - c_l)$. For each level of assessment cost, there are two segments of price distributions, corresponding to informed bids and blind bids.

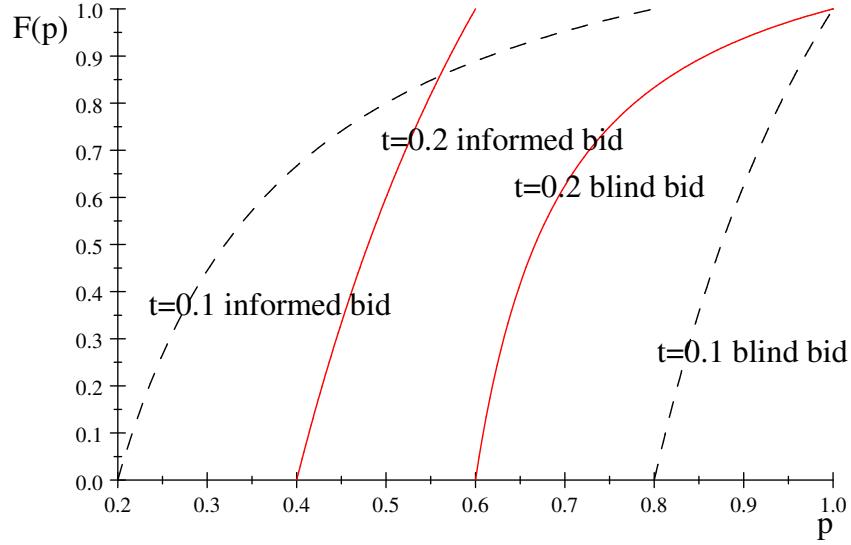


Figure 1: The red solid curve depicts the price distribution when $t = 0.2 (c_h - c_l)$ and the black dashed curve when $t = 0.1 (c_h - c_l)$. For both, $q = 1/2$.

From the graph, we can see that sellers are more likely to set low blind bids when t is large. Intuitively, a larger t makes sellers less likely to do assessments and this gives uninformed sellers a greater chance to win. In response to that, uninformed sellers bid more aggressively. The effects of a larger t on the bidding behavior of a seller informed of a low cost, however, are more complicated: on one hand, having fewer competing bids raises the lower bound of informed bids; on the other hand, more aggressive bidding by uninformed sellers lowers the upper bound of informed bids. Therefore, a larger t decreases the degree of dispersion in informed bids.

3 Equilibrium Properties

Consumer surplus is maximized in the benchmark case, but it requires the consumer and sellers to be able to contract on the latter's assessment effort. In reality, sellers' assessment effort is not verifiable so the benchmark outcome cannot be obtained. The second best outcome is for the consumer to commit to the number of searches before sellers offer competing bids. This is possible only if the number of searches is publicly observable. Accordingly, I analyze two cases, depending on whether n is publicly observable or private information.

3.1 The Number of Price Quotes is Publicly Observable

The consumer surplus can be written as $v - c_E - \min_{n \geq 2} (\alpha t + s) n$. Relative to the benchmark case, it is lower by $\min_{n \geq 2} (\alpha t + s) n - 2s$. The term $(\alpha t + s) n$ captures the overall impact of precontract costs, including consumer search cost and price setting cost, on consumer welfare. It does not contribute to sellers' profit margin and is merely a waste caused by market frictions, but for the lack of better names I shall call it the expected markup and denote it by $\psi(n, s, t)$. Lemma 4 summarizes how the expected markup (essentially the negative of consumer surplus) varies with the number of searches for different combinations of parameter values. For ease of exposition, I ignore the integer constraint on n in this section.

Lemma 4 *For given values of s and t , let $\psi(n) = \psi(n, s, t) = (\alpha t + s) n$ and $\gamma = -\ln \frac{t(1-q)}{q((c_h - c_l)(1-q) - t)}$,*

- (i) $\psi(n)$ monotonically increases with n on $(1, \infty)$ if $1 + s/t \geq e^{\gamma-2} (4/\gamma - 1)$; otherwise,
- (ii.a) if $s = 0$, then $\psi(n)$ is unimodal on $(1, \infty)$;
- (ii.b) If $s > 0$, then $\psi(n)$ has two critical points on $(1, \infty)$. Denote them by $\{n_1, n_2\}$, where $n_1 < n_2$, $\psi(n)$ is maximized at n_1 and minimized at n_2 . If $1 + s/t < e^{-\gamma} (2\gamma + 1)$ and $\gamma < 1.256$, then $n_1 < 2 < n_2$; if $1 + s/t \in (e^{-\gamma} (2\gamma + 1), e^{\gamma-2} (4/\gamma - 1))$ and $\gamma < 1$, then $n_1 < n_2 \leq 2$; if $1 + s/t \in (e^{-\gamma} (2\gamma + 1), e^{\gamma-2} (4/\gamma - 1))$ and $\gamma > 1$, then $2 < n_1 < n_2$.
- (iii) if $n \rightarrow \infty$, then $\psi(n) \rightarrow ns + \gamma t$.

Proof. Let $z = \gamma/(n-1)$, we have $n = \gamma/z + 1$, $\alpha = 1 - \exp(-z)$, $\frac{\partial \gamma}{\partial t} < 0$, and $\frac{\partial}{\partial n} z = -\frac{\gamma}{(n-1)^2} = -\frac{z^2}{\gamma}$. Thus, $\psi(n)$ can be rewritten as $((1 - \exp(-\gamma/(n-1)))t + s)n = ((1 - \exp(-z(n)))t + s)n = t\left(\frac{(1+s/t)\exp z - 1}{z}\right)(\gamma + z)\exp(-z)$. Hence, $\psi'(n) = tz\exp(-z)\left(\frac{(1+s/t)\exp z - 1}{z} - \frac{\gamma+z}{\gamma}\right)$. Let

$$(4) \quad \phi(z, \gamma) = \gamma \frac{(1 + s/t) \exp z - 1}{z(z + \gamma)}.$$

Using L'Hopital's rule (Estrada and Pavlovic 2017), we can verify that $\phi(z, \gamma)$ is convex in z and increases with γ , $\lim_{z \rightarrow 0} \phi(z, \gamma) = \gamma \frac{(1+s/t)\exp z}{2z+\gamma}|_{z=0} = 1+s/t \geq 1$ and $\lim_{z \rightarrow 0} \phi_z(z, \gamma) = -\frac{1+s/t}{2} < 0$.

- (i) Since $\phi_z(z, \gamma) = \gamma \frac{(\gamma+2z)+e^z(-2z-\gamma+z\gamma+z^2)(1+s/t)}{z^2(z+\gamma)^2}$, $\phi(z, \gamma)$ is minimized when

$$(5) \quad (1 + s/t) \exp z = \frac{\gamma + 2z}{\gamma + 2z - z\gamma - z^2}.$$

Since $\phi(z, \gamma)$ is convex in z , (5) has a unique solution. Denote it by z^* . Hence, $\min_z \phi(z, \gamma) = \gamma \frac{(1+s/t) \exp z^* - 1}{z^*(z^* + \gamma)} = \frac{\gamma}{\gamma + 2z^* - z^*\gamma - z^{*2}} = 1$ if $z^* = 2 - \gamma$. Since z^* solves (5), $\min_z \phi(z, \gamma) = 1$ if and only if (5) holds when $z = 2 - \gamma$, i.e., $(1+s/t) \exp(2 - \gamma) = 4/\gamma - 1$. Denote its solution by γ^* . Thus, $\min_z \phi(z, \gamma^*) = 1$. Since $\phi_\gamma(z, \gamma) > 0$, we must have $\min_z \phi(z, \gamma) \geq 1$ for all $\gamma \geq \gamma^*$ by the Envelope Theorem. Therefore, $\psi'(n) \geq 0$ if $\gamma \geq \gamma^*$, i.e., $1 + s/t \geq e^{\gamma-2} (4/\gamma - 1)$.

(ii) Now suppose $\gamma < \gamma^*$. If n is a critical point, then we must have $\phi(z, \gamma) = 1$. There are two critical points if and only if $\min_z \phi(z, \gamma) < 1$.

(ii.a) If $s = 0$, then $\gamma^* = 2$. Since $\phi(z, \gamma)$ is convex in z , $\lim_{z \rightarrow 0} \phi(z, \gamma) = 1$ and $\lim_{z \rightarrow 0} \phi_z(z, \gamma) < 0$. There is a unique solution of z for $\phi(z, \gamma) = 1$ on $(0, \infty)$. Denote it by \hat{z} . $\psi'(n) > 0$ if and only if $z > \hat{z}$. Thus $\hat{n} = \gamma/\hat{z} + 1$ must be the unique critical point on $(1, \infty)$ and $\psi'(n) \geq 0$ if $n \leq \hat{n}$.

(ii.b) If $s > 0$, then $\gamma^* < 2$, since $e^{\gamma-2} (4/\gamma - 1)$ decreases with γ . For $\gamma < \gamma^*$, $e^{\gamma-2} (4/\gamma - 1) > 1 + s/t$, there are two solutions of z for $\phi(z, \gamma) = 1$ on $(0, \infty)$. Denote them by z_1 and z_2 , where $z_1 > z_2$. Thus, $\psi'(n) < 0$ if and only if $z \in (z_2, z_1)$. Hence, $n_i = \gamma/z_i + 1$, $i = 1, 2$, must be the only two critical points on $(1, \infty)$, with $\psi'(n) < 0$ if and only if $n \in (n_1, n_2)$. As a result, n_1 maximizes $\psi(n)$ and n_2 minimizes $\psi(n)$. If $s/t < e^{-\gamma} (2\gamma + 1) - 1$, then $\phi(z, \gamma)|_{z=\gamma} < 1$. Note that $e^{-\gamma} (2\gamma + 1) > 1$ if and only if $\gamma < 1.256$. Since $\phi(z, \gamma)$ is convex in z and $\phi(z_1, \gamma) = \phi(z_2, \gamma) = 1$, we must have $z_2 < \gamma < z_1$, i.e., $n_1 < 2 < n_2$. If $s/t > e^{-\gamma} (2\gamma + 1) - 1$, then $\phi(z, \gamma)|_{z=\gamma} > 1$. It is easy to verify that $e^{-\gamma} (2\gamma + 1) < e^{\gamma-2} (4/\gamma - 1)$ if $\gamma \neq 1$ and $e^{-\gamma} (2\gamma + 1) = e^{\gamma-2} (4/\gamma - 1) = 3/e$ if $\gamma = 1$. Since $\phi_z(z, \gamma)|_{z=\gamma} = 0$ when $\gamma = 1$ and $\phi_{z\gamma}(z, \gamma) > 0$, we must have $\phi_z(z, \gamma)|_{z=\gamma} \geq 0$ when $\gamma \geq 1$. Thus, if $\gamma < 1$, we have $\phi_z(z, \gamma)|_{z=\gamma} < 0$, so $\gamma < z_2 < z_1$, i.e., $n_1 < n_2 < 2$; otherwise, $z_2 < z_1 < \gamma$, i.e., $2 < n_1 < n_2$. Last, if $\gamma = 1$, then we cannot have $e^{-\gamma} (2\gamma + 1) < 1 + s/t < e^{\gamma-2} (4/\gamma - 1)$.

(iii) $\lim_{n \rightarrow \infty} \alpha n = \lim_{z \rightarrow 0} (1 - e^{-z}) \frac{\gamma + z}{z} = \lim_{z \rightarrow 0} e^{-z} \frac{(\gamma + z)^2}{\gamma} = \gamma$, so $\lim_{n \rightarrow \infty} (\alpha t + s) n = \gamma t + sn$. ■

According to Lemma 4, there are three possible patterns of how the expected markup varies with the number of searches. Figure 2 illustrates these possibilities. When the assessment cost t is small (the blue dashed curve at the bottom), the expected markup monotonically increases with n ; when t is large but s is zero (the black dotted curve in the middle), the expected markup first increases with n , then decreases; when t is large and s is positive (the red solid curve at the top), the expected markup increases between 0 and n_1 , then decreases between n_1 and n_2 , and

then increases again for $n > n_2$, where n_1 is a local maxima and n_2 is a local minima as defined in Lemma 4.

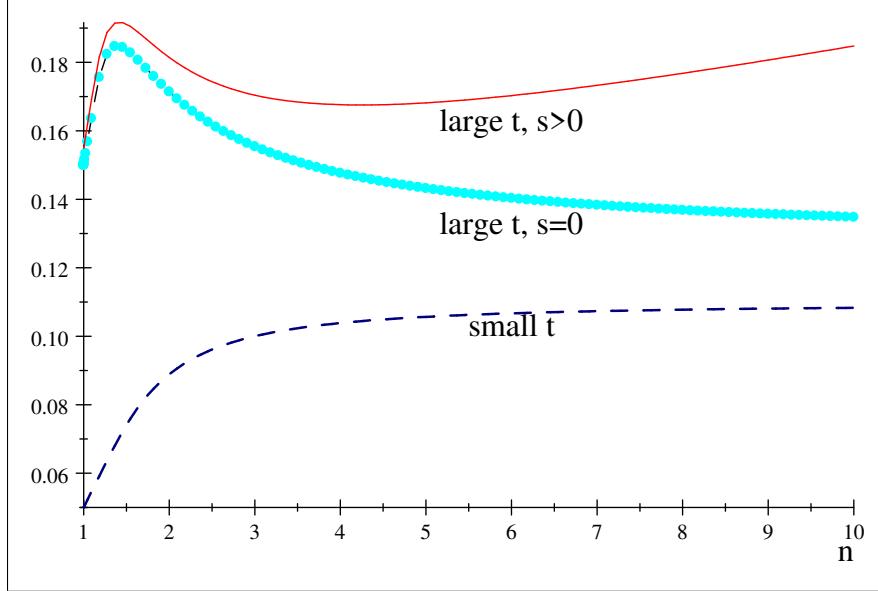


Figure 2 ($q = 1/2$): The expected markup as a function of n . Blue dashed $(t = 0.05 (c_h - c_l), s = 0)$, Black dotted $(t = 0.15 (c_h - c_l), s = 0)$, Red solid $(t = 0.15 (c_h - c_l), s = 0.005 (c_h - c_l))$.

Hence, there are at most three candidates for the optimal number of price quotes: 2, n_2 , or infinity, with the last candidate, infinity, being optimal only if $s = 0$. Therefore, the optimal number of price quotes can simply be determined by comparing three numbers, $\psi(2)$, $\psi(n_2)$ and $\psi(\infty)$. The only remaining difficulty is to determine $\psi(n_2)$, as n_2 does not have an analytical solution, but we can rely on studying the monotonicity and oscillation of the expected markup as a function of n to accomplish the task.

3.1.1 The Optimal Number of Searches

Suppose that n^o is the optimal number of price quotes, then we must have $n^o = \arg \min_{n \geq 2} \psi(n, s, t)$. Proposition 2 summarizes the choices of n^o for different parameter values of s and t .

Proposition 2 Suppose that the number of price quotes is publicly observable. If $s = 0$, then $n^o > 2$ if and only if $\frac{t}{c_h - c_l} > \frac{q(1-q)}{4.92(1-q)+q}$; if $s > 0$, then $n^o > 2$ if and only if $\frac{t}{c_h - c_l} > \frac{q(1-q)}{(1-q)e+q}$ and $s/t < e^{-\gamma} (2\gamma + 1) - 1$, or $\frac{t}{c_h - c_l} < \frac{q(1-q)}{(1-q)e+q}$ and $s/t < e^{\gamma-2} (4/\gamma - 1) - 1$ and $2(1 + s/t - e^{-\gamma}) > \frac{(z^*+\gamma)^2}{\gamma} \exp(-z^*)$, where z^* is the larger root for $\gamma \frac{(1+s/t)\exp z-1}{z(z+\gamma)} = 1$ and $\gamma = -\ln \frac{t(1-q)}{q((c_h - c_l)(1-q)-t)}$.

Proof. If $s = 0$, by Lemma 4 (ii.a), we only need to compare $\psi(2)$ and $\psi(\infty)$. By Lemma 4 (iii), $\lim_{n \rightarrow \infty} \psi(n) = t\gamma$, whereas $\psi(2) = 2t \left(1 - \left(\frac{t(1-q)}{q((c_h - c_l)(1-q) - t)}\right)\right)$. Since $1 - \exp(-\gamma) < \gamma/2$ for $\gamma > 1.594$, we have $\psi(2) < \lim_{n \rightarrow \infty} \psi(n)$ if and only if $\frac{t(1-q)}{q((c_h - c_l)(1-q) - t)} < \exp(-1.594)$, i.e., $t < \frac{q(1-q)}{4.92(1-q)+q} (c_h - c_l)$. The reason why $s = 0$ has to be considered as a special case is due its different asymptotic behavior: $\lim_{n \rightarrow \infty} \psi'(n) = s$ if $s > 0$, but $\psi'(n) < 0$ and $\lim_{n \rightarrow \infty} \psi'(n) = 0$ if $s = 0$.

If $s > 0$, by Lemma 4 (ii.b), there are four possibilities:

(i) $\psi(n)$ is increasing on $(1, \infty)$ when $\gamma > \gamma^*$, i.e., $\frac{t}{c_h - c_l} \leq \frac{q(1-q)}{(1-q)e^{\gamma^*} + q}$. Hence, $n^o = 2$.

(ii) $n_1 < 2 < n_2$ when $1 + s/t < e^{-\gamma}(2\gamma + 1)$ and $\gamma < 1.256$. Since $\psi(n)$ is decreasing on the interval of $[2, n_2]$, $\psi(n_2) < \psi(2)$. Therefore, $n^o = n_2 > 2$.

(iii) $n_1 < n_2 < 2$ when $1 + s/t \in (e^{-\gamma}(2\gamma + 1), e^{\gamma-2}(4/\gamma - 1))$ and $\gamma < 1$. Since $\psi(n)$ is increasing on the interval of (n_2, ∞) , $\psi(n_2) < \psi(2) < \lim_{n \rightarrow \infty} \psi(n)$. In addition, $n^o \geq 2$, so we must have $n^o = 2$.

(iv) $2 < n_1 < n_2$ when $1 + s/t \in (e^{-\gamma}(2\gamma + 1), e^{\gamma-2}(4/\gamma - 1))$ and $\gamma > 1$. Since $\psi(n)$ increases on $[2, n_1]$ and then decreases on $[n_1, n_2]$, $n^o = \arg \min_{n \in \{2, n_2\}} \psi(n)$. Since $\psi(2) = \left(\frac{(1+s/y)\exp z-1}{z}\right)(\gamma + z)\exp(-z)|_{z=\gamma} = 2(1 + s/t - e^{-\gamma})$ and $\psi(n_2) = \left(\frac{(1+s)\exp z-1}{z}\right)(\gamma + z)\exp(-z)$ where $\gamma \frac{(1+s/t)\exp z-1}{z(z+\gamma)} = 1$, $n^o > 2$ if $2(1 + s/t - e^{-\gamma}) > \frac{(z+\gamma)^2}{\gamma} \exp(-z)$.

Therefore, in order for $n^o > 2$, if $\gamma < 1$, i.e., $\frac{t}{c_h - c_l} > \frac{q(1-q)}{(1-q)e^{\gamma^*} + q}$, then we must have $s/t < e^{-\gamma}(2\gamma + 1) - 1$; if $\gamma > 1$, i.e., $\frac{t}{c_h - c_l} < \frac{q(1-q)}{(1-q)e^{\gamma^*} + q}$, then we must have $s/t < e^{\gamma-2}(4/\gamma - 1) - 1$ and $2(1 + s/t - e^{-\gamma}) > \frac{(z^*+\gamma)^2}{\gamma} \exp(-z^*)$. ■

Proposition 2 shows that, frequently, the optimal number of searches is two.¹⁴ We can see this result more clearly from Figure 3, where $q = 1/2$. The optimal number of searches is two in all regions but the lower-right corner. It is true even when the search cost goes to zero. This result is reminiscent of the model of Fullerton and McAfee (1999), which finds it optimal to include only two contestants in a research tournament. In their model, restricting entry to two competitors decreases the coordination problem of competing contestants and minimizes the duplication of fixed costs. Similarly, in the current model, limiting the number of bidders creates better incentives at the assessment stage and reduces duplication in efforts. One should be careful, however, about applying this result in practice because the model incorporates two features that lower the benefit

¹⁴Honka (2014) finds that consumers get on average 2.96 quotes with the majority of consumers collecting two or three quotes when purchasing auto insurance policies.

from additional searches. First, sellers face the same production costs, so additional search does not offer opportunities to discover more efficient sellers; second, only one product meets the need of the consumer, so additional searches does not offer opportunities to discover new solutions. If those opportunities are present in a more realistic environment, then a consumer may want to search more than two sellers.

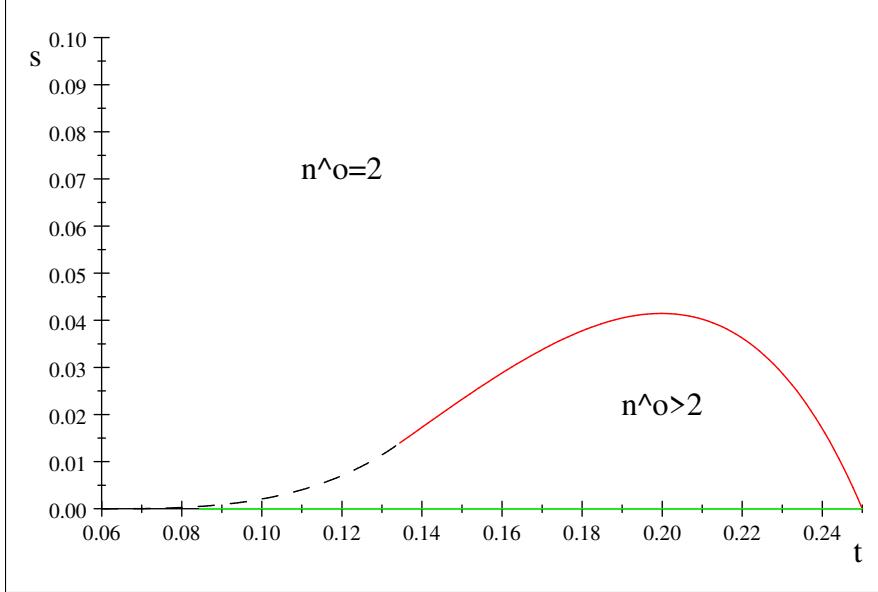


Figure 3 ($q = 1/2$): the solid curve represents $s/t = e^{-\gamma} (2\gamma + 1) - 1$ and the dashed one represents $e^{\gamma-2} (4/\gamma - 1) - 1$. The green segment ($t > 0.0845$) on the horizontal axis represents the values of t such that $n^o = 2$.

While the equilibrium outcome shares some similarities with existing models of consumer search, e.g., there is price dispersion even with ex ante identical and rational consumers and sellers, there are important differences. In all existing search models with homogeneous sellers, the limiting result when search cost approaches zero is the Bertrand competition outcome, in which there is no price dispersion,¹⁵ but price dispersion persists in this model even when search cost vanishes. At first glance, this result is not surprising: after all, sellers' assessment cost can be seen as a part of the overall search cost that is indirectly paid by the consumer, so zero direct search cost does not imply zero overall search cost. However, it will be wrong to extend this line of reasoning to conclude that the introduction of price setting cost adds nothing but splitting the difference. As shown in Proposition 2, the consumer is more likely to search a large number of sellers when s is

¹⁵Sellers in Baye and Morgan (2001) have identical costs, but each of them is also a local monopoly.

small and when t is large. The first effect is quite obvious, but the second one is not. When t rises, one might think that the consumer should search less since she has to indirectly pay for sellers' assessment cost, but this intuition is incomplete because it ignores the effect of an increase in n on sellers' propensity to assess. As shown in the following comparative statics analysis, the effects of search cost and assessment cost are quite different.

3.1.2 Comparative Statics

In this section, I examine the impact of search cost and assessment cost on the equilibrium outcome. Proposition 3 summarizes the impact of s and t on the optimal number of searches.

Proposition 3 *Suppose that the number of price quotes is publicly observable, the optimal number of searches n^o*

- (i) *decreases with s ;*
- (ii) *is non-monotonic in t if $s > 0$, but increases with t if $s = 0$.*

Proof. (i) Suppose that n^o changes to $n^{o'}$ and n_2 changes to n'_2 when s increases to $s' > s$. To prove that n^o is (weakly) decreasing in s , we need to show $n^{o'} \leq n^o$.

If $s = 0$, by Lemma 4 (ii.a), $n^o = \arg \min_{n \in \{2, \infty\}} \psi(n, 0, t)$ and $n^{o'} = \arg \min_{n \in \{2, n'_2\}} \psi(n, s', t)$. We only need to prove that $n^{o'}$ cannot be $n'_2 > 2$ when $n^o = 2$, i.e., $\psi(n'_2, s', t) > \psi(2, s', t)$ when $\psi(n'_2, 0, t) > \psi(2, 0, t)$, but this is true because $\psi(n'_2, s', t) = \psi(n'_2, 0, t) + n'_2 s' > \psi(2, 0, t) + 2s' = \psi(2, s', t)$.

If $s > 0$, by Lemma 4 (ii.b), $n^o = \arg \min_{n \in \{2, n_2\}} \psi(n, s, t)$ and $n^{o'} = \arg \min_{n \in \{2, n'_2\}} \psi(n, s', t)$.

By the Envelope theorem,

$$(6) \quad \text{sign} \frac{dn_2}{ds} = \text{sign} - \frac{\partial^2}{\partial n \partial s} \psi(n, s, t) = (-),$$

since $\frac{\partial^2}{\partial n \partial s} \psi(n, s, t) = 1$. Hence, $n'_2 < n_2$. Suppose that $n^o = n_2$, then $n^{o'}$ will be greater than n^o only if $n'_2 > n^o$, but $n'_2 < n_2 = n^o$ by (6). Suppose that $n^o = 2$, then $n^{o'}$ will be greater than n^o only if $n'_2 > 2$, i.e., $\psi(2, s, t) < \psi(n_2, s, t)$ and $\psi(2, s', t) > \psi(n'_2, s', t)$, but $\psi(2, s', t) = \psi(2, s, t) + 2(s' - s) < \psi(n_2, s, t) + 2(s' - s) \leq \psi(n_2, s, t) + n'_2(s' - s) < \psi(n'_2, s, t) + n_2(s' - s) = \psi(n'_2, s', t)$, contradiction.

(ii) is immediate from Proposition 2. ■

According to Proposition 3, search cost and assessment cost can sometimes have opposite effects on the optimal number of searches. To gain some intuition behind this surprising result, consider the limiting cases where $s = 0$. If $t = 0$, then the consumer only needs to visit two sellers to obtain the competitive price according to Proposition 1. In contrast, if $t \rightarrow \infty$, then no seller will ever make an effort to discover the true cost, but it will be best for the consumer to visit as many sellers as possible. Hence, the consumer ends up visiting more sellers when the sellers' effort becomes more costly. As such, search cost and assessment cost can have very different impacts on consumers' search behavior, even though they both appear to contribute to the overall precontract cost. This means that it is not only the total cost, but also the composition of the cost, that matters to consumer search. A similar result exists for two-sided markets, but the underlying mechanism is much different. In two-sided markets the composition of the cost matters because the cost imposed on one side cannot be fully internalized by the other side of the market, whereas in this model it matters despite consumers' full internalization of sellers' costs.

Let $\varphi(s, t) = \min_{n \geq 2} (\alpha t + s) n$ denote the minimized expected markup as a function of s and t . Proposition 4 examines the welfare impact of the two costs.

Proposition 4 Suppose that the number of price quotes is publicly observable, $\varphi(s, t)$

- (i) monotonically increases with s ;
- (ii) is unimodal in t .

Proof. (i) By Lemma 4, $\varphi(s, t) = \min \{\psi(2), \psi(n_2), \psi(\infty)\}$. By the Envelope Theorem, $\frac{d}{ds}\psi(n_2) = n_2 > 0$. At the same time, $\frac{d}{ds}\psi(2) = 2 > 0$ and $\frac{d}{ds}\psi(\infty) = n > 0$. Therefore, $\frac{\partial}{\partial s}\varphi(s, t) > 0$.

(ii) First, $\frac{d}{dt}\psi(2) = \frac{\partial}{\partial t} \left(1 - \frac{t(1-q)}{q((1-q)-t)} \right) t = \frac{t^2 - 2t(1-q) + q(1-q)^2}{q(1-q-t)^2}$, $\frac{d^2}{dt^2}\psi(2) = \frac{\partial^2}{\partial t^2} \left(1 - \frac{t(1-q)}{q((1-q)-t)} \right) t = -\frac{2}{q(1-q-t)^3} < 0$, so $\psi(2)$ increases with t if and only if $t < 1-q-\sqrt{(1-q)^3}$. Second, by the Envelope Theorem, $\frac{d}{dt}\psi(n_2) = \frac{\partial}{\partial t} (1 - \exp(-\gamma/(n_2-1))) tn_2 = -n_2 \left(e^{-\frac{\gamma}{n_2-1}} - 1 \right) \frac{\partial \gamma}{\partial t} < 0$, i.e., $\psi(n_2)$ decreases with t . Third, by Lemma 4 (iii), $\frac{d^2}{dt^2}\psi(\infty) = \frac{\partial^2}{\partial t^2} \left(-t \ln \frac{t(1-q)}{q((1-q)-t)} \right) = -\frac{1}{t} \frac{(1-q)^2}{(1-q-t)^2} < 0$. If $s > 0$, by Lemma 4 (ii.b), $\varphi(s, t) = \min \{\psi(2), \psi(n_2)\}$, Since $\psi(2)$ and $\psi(n_2)$ are both quasi-concave, $\varphi(s, t)$ must be quasi-concave in t . If $s = 0$, by Lemma 4 (ii.a), $\varphi(s, t) = \min \{\psi(2), \psi(\infty)\}$. Since $\psi(2)$ and $\psi(\infty)$ are both concave in t , $\varphi(s, t)$ must be concave in t . Last, since $\varphi(s, 0) = \varphi(s, \bar{t})$, where $\bar{t} = q(1-q)(c_h - c_l)$, $\varphi(s, t)$ must be unimodal in t . ■

Perhaps not surprisingly, the consumer benefits from a lower search cost, though it turns out to be true only if the consumer has the ability to commit, otherwise the opposite result may be obtained, as shown in the next section. More interestingly, the consumer can benefit from a further increase in the assessment cost when it is already large. This directly follows from our earlier observation that a high assessment cost can discourage sellers from engaging in wasteful and duplicative assessment efforts, which are indirectly paid by the consumer. Another way to understand why the expected total expense is not monotonic in the assessment cost is to recall Proposition 1: when $t = 0$, by definition the cost of assessment effort will be zero; when $t \geq q(1 - q)(c_h - c_l)$, there will be no wasteful expenditure on assessment because the cost exceeds the value of the information. Between the two extremes, however, there will be positive amounts of (wasteful) assessment efforts.

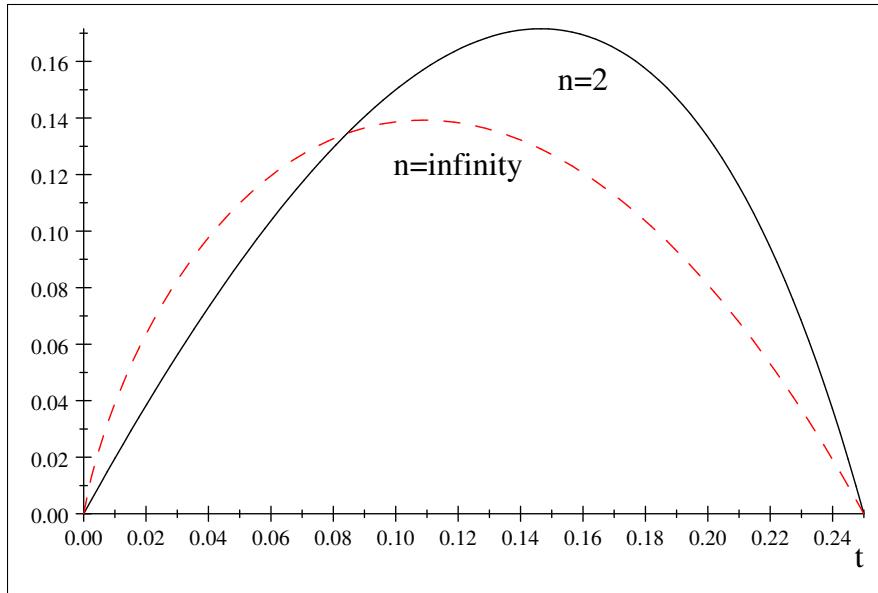


Figure 4: The black solid curve depicts $\psi(2)$ and the red dashed curve is $\psi(\infty)$, with both as functions of t . The lower envelope is the expected markup.

Figure 4 illustrates in more detail how the expected markup varies with t . When t is small, the cost of information is small relative to its value, so sellers are more likely to make the assessment effort. At the same time, given that sellers have the same production cost, the gain from having additional competition is relatively small, so the consumer is better off by visiting just two sellers. This minimizes search costs as well as sellers' information rents. When t is large, α decreases with n at a faster rate so the consumer will be better off by increasing n . To see this effect more clearly,

I plot $\alpha(n)/\alpha(2)$ in Figure 5. The graph illustrates three results: first, the propensity to assess α decreases with n across all ranges of t ; second, sellers are less likely to make the assessment effort when t is large; these two results are obvious. Less obvious is the third result, namely, α decreases at a faster rate when t is large. This means that increasing the number of searches can potentially reduce the wasteful and duplicative assessment efforts when t is large. This is true if the rate of decrease in α is greater than the rate of increase in n .

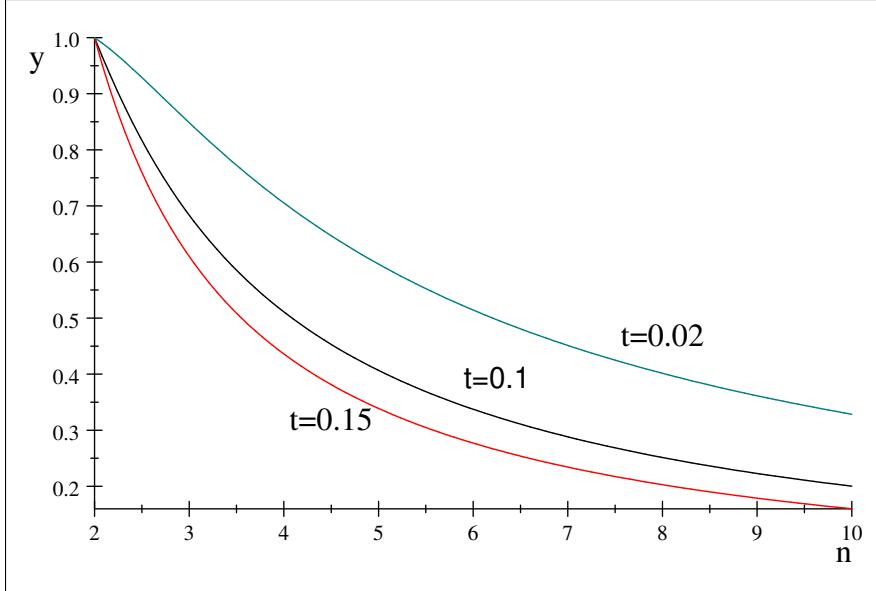


Figure 5: $y = \alpha(n)/\alpha(2)$, when $t = 0.02, 0.1, 0.15$

Up to this point, we have focused on the consumer's total expense, next I consider the expected price. The two are obviously correlated, but the correlation is not perfect: even if an additional search leads to a lower price, the total expense could increase because of the extra search cost. Hence, we cannot simply use the results above to predict the price effects. In order to have empirically testable predictions on prices and compare with the noncommitment case discussed later, we need the following results:

Proposition 5 Suppose that the number of price quotes n is publicly observable, the expected price

- (i) increases with n if $\frac{t}{c_h - c_l} < \frac{q(1-q)}{(1-q)e^2 + q}$;
- (ii.a) decreases with n if $\frac{t}{c_h - c_l} \geq \frac{q(1-q)}{(1-q)e^2 + q}$ and $n_2 \leq 2$; (ii.b) increases with n on the interval of $[2, n_2]$ and then decreases with n on (n_2, ∞) if $\frac{t}{c_h - c_l} \geq \frac{q(1-q)}{(1-q)e^2 + q}$ and $n_2 > 2$;
- (iii) increases with s .

Proof. (i) and (ii) are immediate from the proof of Lemma 4 (i) and (ii.a).

(iii) By the Envelope theorem, $\frac{d}{ds}E(p|n=n_2) = \frac{d}{ds}\psi(n_2) - \frac{d}{ds}n_2s = n_2 - n_2 - s\frac{dn_2}{ds} \geq 0$.

At the same time, $E(p|n=2)$ is a constant with respect to s . By Proposition 3, n^o decreases with s . This means that, when s increases to s' , we cannot have $n^o = 2$ and $n^{o'} = n_2 > 2$. Hence, the only possibility for a price decreases after an increase in s is to have $\psi(n_2) < \psi(2)$ but $E(p|n=n_2) > E(p|n=2)$. If this is true, then we must have $\psi(n_2) - 2s < \psi(2) - 2s = E(p|n=2) < E(p|n=n_2) = \psi(n_2) - n_2s$, contradiction. ■

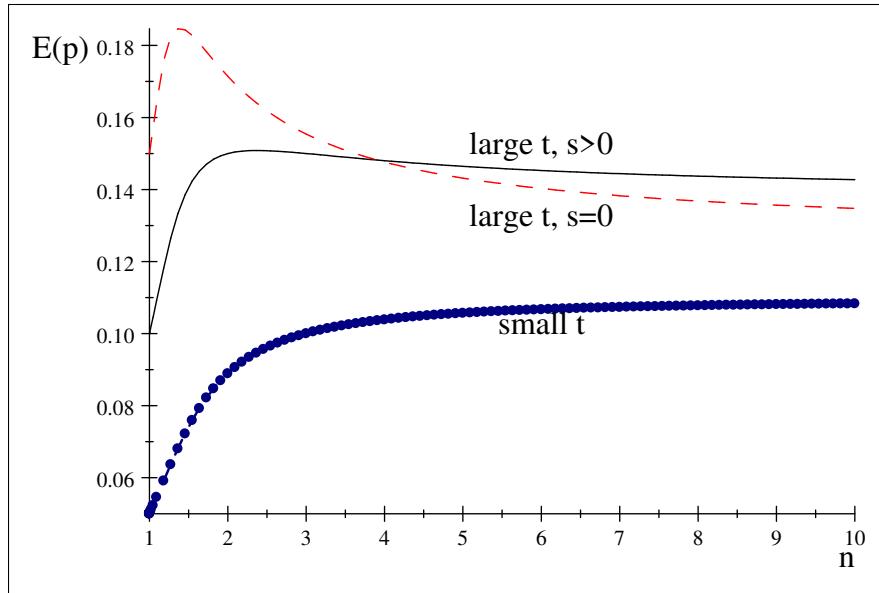


Figure 6: The expected price as a function of n . Blue dotted ($t = 0.05(c_h - c_l), s = 0$), Black solid ($t = 0.15(c_h - c_l), s = 0$), Red dashed ($t = 0.15(c_h - c_l), s = 0.005(c_h - c_l)$). In all cases,

$$q = 1/2.$$

In classic models of consumer search, prices are either set before consumer search or at the same time. This implies that consumers' search intensity cannot alter price distributions, so the expected price has to decrease with the number of searches. However, if prices are set after consumer search, then the effect of more searches on the final price is not obvious: on one hand, competition lowers price; on the other hand, competition causes sellers more likely to submit blind quotes that are inflated to avoid winner's curse. Indeed, as illustrated in Figure 6, it is possible that more searches result in a high price. To understand why, recall that the consumer indirectly pays for the assessment cost. When the consumer searches more sellers, each seller will have a smaller chance of being the winning bidder and so will in equilibrium seek a higher expected profit in the event

of winning, which is translated into a higher price. To researchers of auctions, this result is not surprising. It is by now a well-known result that the expected revenue may be decreasing in the number of potential bidders when there are participation costs (Harstad 1990). Proposition 5 shows that the same can happen in a search model.¹⁶

Unlike other models of costly consumer search, the current model does not predict that the expected price approaches the marginal cost when $s \rightarrow 0$, but it so far still maintains the standard result that the expected price increases with the search cost, according to Proposition 5 (iii). This result, however, will be overturned once we relax the assumption that the consumer can commit to the optimal number of searches, as shown below.

3.1.3 Economic Significance of Assessment Costs

While the above model generates some interesting results, they will not matter much if the assessment cost only has a small impact on consumer welfare. In order to evaluate its economic significance, I compare the expected total expense under costly effort to the benchmark case, in which the total expense is just the expected production cost plus the search costs, $c_E + 2s$. To focus on the impact of assessment cost, I further assume $s = 0$, in which case n^o is either 2 or ∞ . Another measure that can be used is the expected price plus the assessment costs, but it generates the same qualitative result.¹⁷

¹⁶Satterthwaite (1979) shows that an increased number of sellers may perversely cause the equilibrium price to rise in a monopolistically competitive industry that sells a reputation good.

¹⁷Consumer surplus is a less appropriate measure, because it depends on v , an arbitrary parameter in the model.

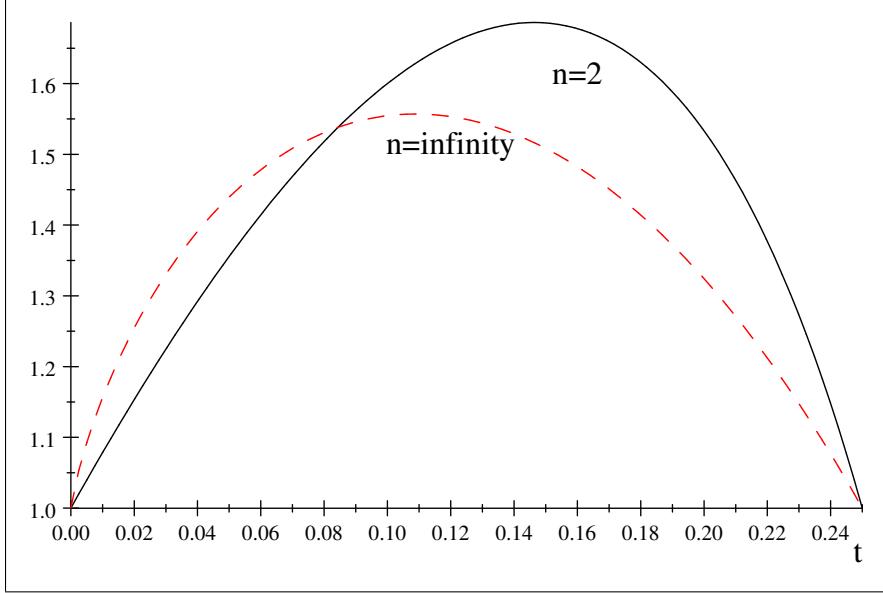


Figure 7: $\varphi(s, t) / c_E$ as a function of $t / (c_h - c_l)$ when $q = 1/2$. The solid curve has $n = 2$ and the dashed $n \rightarrow \infty$.

Figure 7 plots $\varphi(s, t) / c_E$ when $q = 1/2$, with the solid curve corresponding to the case of $n = 2$ and the dashed $n \rightarrow \infty$, so the lower envelope is the expected cost when n is optimally chosen. As we can see from the graph, the existence of an incontractible assessment cost can potentially increase the consumer's total expense by more than one half. This clearly demonstrates that it is not a negligible cost and should be taken seriously not only for its theoretical interests, but also for its practical importance. As such, it offers an alternative explanation for why car buyers obtain significantly more of the surplus available under customer rebate than under dealer discount, a finding that is counter to the simple invariance of incidence analogy (Busse, Silva-Risso and Zettelmeyer 2006). Busse et al. test several hypotheses and find evidence consistent with the asymmetric information hypothesis, that is, car buyers are disadvantaged in negotiations because they are less informed than dealers about the availability of dealer discounts. In contrast, the parties are symmetrically informed about the availability of customer rebates, which are always publicized to potential customers, often in prime-time television advertisements. Note that their explanation is based on the assumption that the information about dealer discounts is readily accessible to dealers. However, these discounts are often in the form of conditional discounts, depending on geographical location and/or specific equipment package, or “trim level”. This means that there may be a

higher assessment costs for dealer discounts than for customer rebate.¹⁸ Thus, the pass-through from dealer discounts can be lower as a result.

3.2 The Number of Price Quotes is Non-contractible Private Information

As mentioned earlier, if n is non-contractible private information, then the consumer faces a credibility problem: sellers' bidding strategy depends on their conjecture of the number of competitors; but holding their belief and the corresponding bidding strategy constant, the consumer always benefits from having a larger sample size. To see this, consider the lowest price from obtaining n price quotes, taking the price distribution as given. Denote it by p_{\min} . Since the overall CDF for a seller's price distribution is $F(p) = (1 - \alpha)F_b(p) + \alpha(qF_l(p) + (1 - q)\mathbf{1}(p \geq c_h))$, where $\mathbf{1}(\cdot)$ is the indicator function, $F_l(p)$ and $F_b(p)$ are given in Lemma 3, the CDF of p_{\min} is $\Pr(p_{\min} < p) = 1 - \prod_{i=1}^n \Pr(p_i > p) = 1 - (1 - F(p))^n$. Assuming that these expectations are finite, we have $E(p_{\min}) = \int_0^\infty (1 - (1 - F(p))^n) dF(p) = \int_0^\infty (1 - F(p))^n dn$.

$$(7) \quad \frac{d}{dn}E(p_{\min}) = \int_0^\infty (1 - F(p))^n \ln(1 - F(p)) dp < 0.$$

$$(8) \quad \frac{d^2}{dn^2}E(p_{\min}) = \int_0^\infty (1 - F(p))^n (\ln(1 - F(p)))^2 dp > 0.$$

Equation (7) and (8) show that, given the sellers' bidding strategies, a consumer obtains lower prices as she searches more sellers, but the incremental gain from further price reductions becomes smaller and smaller. This is essentially the original insight of Stigler (1961). Under the assumption that the price distributions are completely exogenous, he argues that increased search will yield positive but diminishing returns as measured by the expected reduction in the minimum asking price whatever the precise distribution of prices.¹⁹ Figure 8 illustrates (7) and (8) for the case

¹⁸ According to a web site specialized on automobile markets: "Even if you are the only customer in the dealership, there is still no guarantee you'll be able to get a deal offer in a flash. If you're taking out a loan, the sales manager might have to run your credit to get your credit score. He'll call the finance department to get your interest rate, and then look up specials and incentives on your car to make sure you're getting the right program offer for the right car. Sometimes it just takes a while to get all the information together." Matt Jones, "Behind the Scenes at A Car Dealership", April 29th, 2016, <https://www.edmunds.com/car-buying/behind-the-scenes-at-a-car-dealership.html>.

¹⁹ His article contains a sketch of the proof, attributed to Robert Solow.

where sellers believe they only face one competitor, but the consumer engages in $n = 1, 2$ or 3 searches (The mathematical expressions are derived in Appendix A).

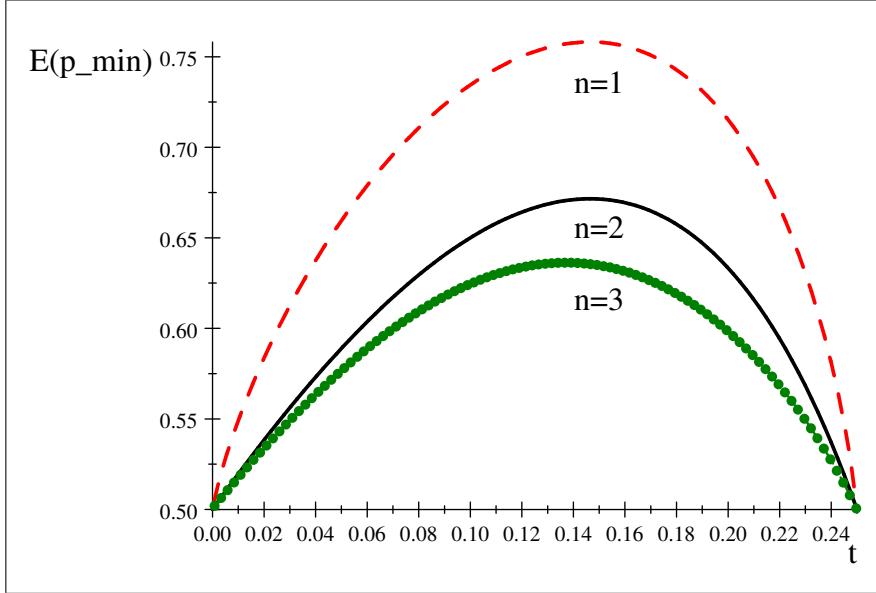


Figure 8: $E(p_{\min})$ as a function of $t/(c_h - c_l)$. The dashed line on the top represents $E(p_{\min})$ when $n = 1$, the solid one in the middle $E(p_{\min})$ when $n = 2$, the dotted one at the bottom $E(p_{\min})$ when $n = 3$. In all case, $q = 1/2$.

In this section, I model the consumer's problem by studying a simultaneous move version of the game. More specifically, I modify the original game by assuming that stage 1 consumer search and stage 2 seller bidding take place at the same time. In other words, n is not observed by the sellers when they choose their bids and the consumer cannot precommit to the optimal number of searches. Instead, sellers must form beliefs about the number of other sellers visited by the consumer. In the Nash equilibrium, their beliefs must be correct. At the same time, given sellers' beliefs and pricing strategies, the consumer has no incentive to search more or fewer sellers. Formally, let $\Psi(n, m)$ denote the expected markup, where n represents the actual number of price quotes and m a seller's belief about the number of price quotes including his own. Let the corresponding lowest price quote p_{\min} be denoted by $E(p_{\min}|n, m)$. The equilibrium conditions are: (1) $m = n$; (2) $n = \arg \min_k \Psi(k, n)$, where $\Psi(k, n) = E(p_{\min}|k, n) + ks$; and (3) $\Psi(n, n) = \psi(n) = (t\alpha + s)n$. I further assume that $s < E(p_{\min}|1, 2) - E(p_{\min}|2, 2)$ so that the consumer will not use mixed strategies in the equilibrium.²⁰

²⁰If $s > E(p_{\min}|1, 2) - E(p_{\min}|2, 2)$, then again the consumer might randomize between searching once and twice, as in footnote 13. This will make the equilibrium exceedingly difficult to solve.

Suppose that n^{PI} is the equilibrium number of price quotes when n is private information, then we must have $\Psi(n^{PI}, n^{PI}) \leq \Psi(n, n^{PI})$ for any n , i.e.,

$$(9) \quad E(p_{\min}|n^{PI}, n^{PI}) + n^{PI}s \leq E(p_{\min}|n, n^{PI}) + ns.$$

By (7) and (8), we know that $E(p_{\min}|n, m) - E(p_{\min}|n+1, m)$ is positive and strictly decreases with n . Condition (9) can thus be rewritten as

$$(10) \quad E(p_{\min}|n^{PI}, n^{PI}) - E(p_{\min}|n^{PI} + 1, n^{PI}) \leq s < E(p_{\min}|n^{PI} - 1, n^{PI}) - E(p_{\min}|n^{PI}, n^{PI}),$$

which is the necessary and sufficient condition for the existence of a search equilibrium. From (10), we can easily see that $n^{PI} \rightarrow \infty$ when $s \rightarrow 0$, regardless of the optimal number of price quotes. This observation gives us the basic intuition why the equilibrium number of price quotes may deviate from the optimum.

The full characterization of the equilibrium is challenging and the uniqueness of the equilibrium is not guaranteed. Fortunately, they are not required for us to see the main qualitative results. In the following analysis, I will demonstrate by way of examples that the conventional wisdom on the market impact of search cost does not extend to the current setting.

3.2.1 Search Cost and Price

Proposition 6 Suppose that the number of price quotes n is private information, the expected equilibrium price may decrease with s if $\frac{t}{c_h - c_l} < \frac{q(1-q)}{4.92(1-q)+q}$.

Proof. Suppose that initially $s \geq E(p_{\min}|2, 2) - E(p_{\min}|3, 2)$, then we must have $n^{PI} = 2$ according to (10) and the condition that $s < E(p_{\min}|1, 2) - E(p_{\min}|2, 2)$. If s decreases to 0, then $n^{PI} > 2$, but $\Psi(n, n) = \psi(n) > \Psi(2, 2) = \psi(2)$ for $n > 2$ if $\frac{t}{c_h - c_l} < \frac{q(1-q)}{4.92(1-q)+q}$ by Proposition 2. ■

Proposition 6 considers the price impact of search costs. In contrast to Proposition 5 (iii), the expected price is no longer monotonically increasing in the search cost if the consumer is unable to precommit. This is because, relative to the commitment case, the search cost has a more direct impact on the number of searches. While a consumer with commitment must take into account the impact of additional searches on the price distribution, a consumer without commitment takes the

price distribution as given. Consequently, a lower search cost is more likely to tempt a consumer without the commitment power to engage in additional searches, but this can cause sellers to quote higher prices in order to compensate for their smaller chances of winning the bidding war.

3.2.2 Excessive Search

Not only can a decrease in the search cost result in higher prices, as shown in Proposition 6, but it can also lead to excessive searches.

Proposition 7 *Suppose that the number of price quotes n is private information, $n^{PI} > n^o$ as long as $n^o = 2$ and $s < E(p_{\min}|2, 2) - E(p_{\min}|3, 2)$.*

Proof. Obvious. ■

In the noncommitment case, when a consumer decides whether to have one more price quote, it is driven by the expected price decrease, holding sellers' belief and strategy constant. In the commitment case, the marginal benefit of searching one more firm is smaller. Therefore, given the same search cost, the consumer has a stronger incentive to search more sellers if she is unable to commit, but this incentive to search is too strong to maximize consumer surplus. In Figure 9, the dashed line represents $E(p_{\min}|1, 2) - E(p_{\min}|2, 2)$ and the dotted one $E(p_{\min}|2, 2) - E(p_{\min}|3, 2)$. Recall from Figure 3 that n^o is two in the area above the solid line. Between the dashed line and the dotted line, $E(p_{\min}|2, 2) - E(p_{\min}|3, 2) < s < E(p_{\min}|1, 2) - E(p_{\min}|2, 2)$, hence $n^{PI} = 2$. However, between the dotted line and the solid curve, $s < E(p_{\min}|2, 2) - E(p_{\min}|3, 2)$, hence $n^{PI} > 2$. Clearly, there is a big range of parameter values for which consumer search may be excessive.²¹

²¹The discontinuity is due to the integer constraint on the number of price quotes.

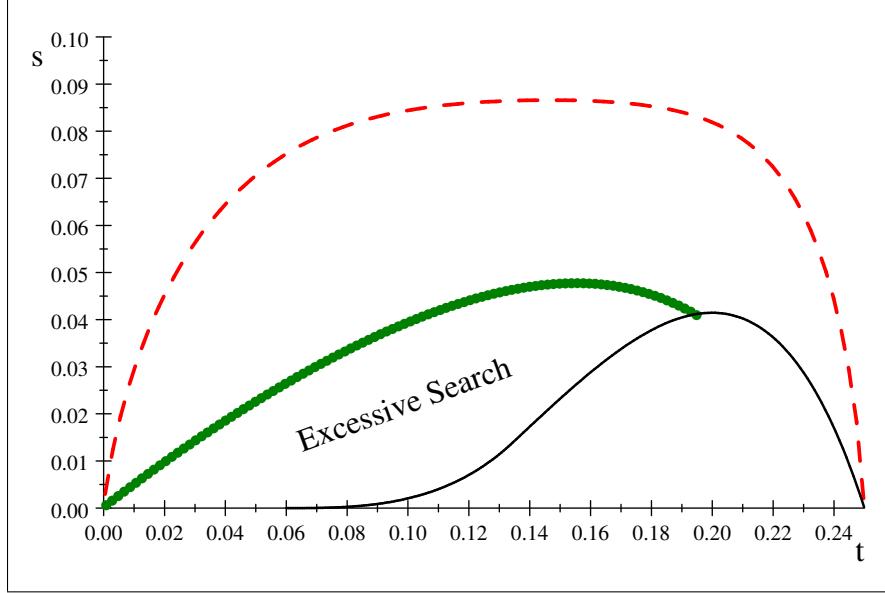


Figure 9: the dashed line represents $E(p_{\min}|1, 2) - E(p_{\min}|2, 2)$, the dotted one $E(p_{\min}|2, 2) - E(p_{\min}|3, 2)$. In the area above the solid line but below the dotted one, $n^o = 2$ but $n^{PI} > 2$.

It is worth noting that the current model assumes that the consumer cannot engage in multiple rounds of searches. This assumption ensures that the consumer cannot first infer the production cost from her previous round of search and then use the information to incite a bidding war. Assuming otherwise will only serve to exacerbate her commitment problem.

Corollary 1 Suppose that the number of price quotes n is private information, (i) consumers can be worse off when s decreases; (ii) both consumer surplus and social welfare can be increased if n becomes publicly observable.

With the advance of the Internet, a lot of research has been devoted to understanding the impact of search cost on consumer welfare. It is generally accepted that consumers benefit from lower search costs (Bakos 1997, Brown and Goolsbee 2002, Lin and Wildenbeest 2015). Even in Wolinsky (2005), which otherwise finds that consumer search in the market for procurement contracts can be excessive, the effect of a lower search cost on welfare remains positive. Corollary 1, however, shows that a lower search cost, contrary to consensus, can sometimes leave consumers worse off. This finding has two important implications: first, when there are multiple sources of transaction costs, police makers should be careful in prescribing cost reduction as the panacea for

market imperfections; second, it explains the use of intermediaries as a commitment mechanism in such markets.²² One such example is the online mortgage referral agent LendingTree.com, which matches potential borrowers with various loan programs. A consumer who applies to LendingTree receives four different offers and each one quotes a price including interest rates and up-front fees.²³

4 Conclusion

When consumers search, they incur costs. In order to provide consumers the information they search for, sellers may also incur costs. This paper departs from the extant literature by assuming that sellers must make an effort to learn the true cost of providing the goods/services before they bid against other sellers, but the consumer is unable to verify or contract on the seller's effort in preparing their bids, i.e., the price quotes. Despite its simplicity, the current model is a faithful snapshot of procurement markets. It allows us to derive a number of new results that do not exist in the search models of posted-price markets. These results caution against studying the impact of consumer search cost without taking into account sellers' costly effort in providing the information sought by consumers. In particular, it shows that a decline of search cost does not necessarily benefit consumers, providing yet another argument why the advance of the Internet or even the elimination of consumer search cost may not lead to the "law of one price."

These results also have important empirical implications. First, they suggest that the choice of a small sample size when consumers search is not necessarily due to high search costs. Recent empirical studies have documented surprisingly few searches conducted by consumers when shopping for financial products.²⁴ The lack of consumer search has been attributed to high search costs and non-price preferences, but it is also consistent with the existence of price setting costs. Indeed, as shown in this paper, the optimal number of searches is frequently two. Empirical studies that

²²It should be noted that the type of intermediation suggested by this paper is different from market-making as in Gehrig (1993), or match-making as in Spulber (1996) or information gatekeeping as in Baye and Morgan (2001), but rather the referral types of services. Recent papers have also emphasized the role of intermediaries in reducing search costs (Allen, Clark and Houde 2014) and generating search-externalities (Armstrong 2015, Salz 2017).

²³Other examples include Better Business Bureau in the service market, Travelsuperlink.com in the travel market, Insure.com and Quickquote.com in the insurance market. Salz (2017) gives a detailed account of intermediation in the New York City's trade waste market (see also <https://www.nytimes.com/2014/10/26/business/dividing-and-conquering-the-trash.html>).

²⁴Honka (2014) documents evidence from the US auto insurance market, Allen, Clark and Houde (2014) and Alexandrov and Koulayev (2017) from the Canadian and US mortgage markets, respectively, and Stango and Zinman (2015) from the credit card market.

do not take into account sellers' price setting costs may overestimate consumer search costs or the impact of other factors. Second, they pose a challenge to empirical studies that attempt to recover consumer search costs from the observed price distributions. In posted price markets, the standard estimation strategy relies on a monotonicity assumption, i.e., prices are monotonically increasing functions of the search costs (Hortacsu and Syverson 2004, Hong and Shum 2006), but the breakdown of monotonicity cautions against the direct extension of this approach to non-posted price markets.

For policy makers, this paper offers a cautionary message: regulations that aim to eliminate market frictions but focus only on consumer search cost may end up worsening consumers' commitment problem. A standard exercise, when making policy recommendations, is the use of counterfactual analyses, but one must be careful in interpreting the results obtained from any of those analyses that presume consumers would gain from reductions of search costs.

This paper is only the first step in trying to understand the impact of sellers' precontract cost on consumer search behavior. For further exploration, it can be extended in a number of directions. First, if the source of uncertainty in product costs is consumer specific, then a consumer may have some private information. This is especially true for insurance markets. How consumer private information affects their search behavior remains an open question. Second, the current model does not consider the possibility that custom solutions exist for different realizations of the production costs. This effectively rules out second-degree price discrimination by a single seller or vertical differentiation among multiple sellers. Third, the production cost is assumed to be a binary variable that can be learned with perfect precision. Last, sellers are assumed to compete in a common value auction.²⁵ Extending the model by relaxing some of these assumptions can be the basis of fruitful future work.

²⁵In MacMinn (1980) and Spulber (1995), sellers' price setting is equivalent to bidding in a private value auction. Price dispersion arises from cost heterogeneity of sellers.

References

- ALEXANDROV, A., AND S. KOULAYEV (2017): “No shopping in the us mortgage market: Direct and strategic effects of providing more information,” Consumer Financial Protection Bureau Office of Research Working Paper No. 2017-01.
- ALLEN, J., R. CLARK, AND J.-F. HOUDE (2014): “The effect of mergers in search markets: Evidence from the Canadian mortgage industry,” *The American Economic Review*, 104(10), 3365–3396.
- ARMSTRONG, M. (2015): “Search and ripoff externalities,” *Review of Industrial Organization*, 47(3), 273–302.
- BAKOS, J. Y. (1997): “Reducing buyer search costs: Implications for electronic marketplaces,” *Management science*, 43(12), 1676–1692.
- BAYE, M. R., AND J. MORGAN (2001): “Information gatekeepers on the internet and the competitiveness of homogeneous product markets,” *American Economic Review*, pp. 454–474.
- BAYE, M. R., J. MORGAN, P. SCHOLTEN, ET AL. (2006): “Information, search, and price dispersion,” in *Handbook on economics and information systems*, ed. by T. Hendershott, vol. 1, pp. 323–77. Elsevier.
- BENABOU, R., AND R. GERTNER (1993): “Search with learning from prices: does increased inflationary uncertainty lead to higher markups?,” *The Review of Economic Studies*, 60(1), 69–93.
- BROWN, J. R., AND A. GOOLSBEY (2002): “Does the Internet make markets more competitive? Evidence from the life insurance industry,” *Journal of political economy*, 110(3), 481–507.
- BURDETT, K., AND K. L. JUDD (1983): “Equilibrium price dispersion,” *Econometrica: Journal of the Econometric Society*, 51(4), 955–969.
- BUSSE, M., J. SILVA-RISSO, AND F. ZETTELMEYER (2006): “\$1,000 cash back: The pass-through of auto manufacturer promotions,” *The American economic review*, 96(4), 1253–1270.
- COASE, R. H. (1960): “The problem of social cost,” *The journal of Law and Economics*, 56(4), 837–877.

- DAHLMAN, C. J. (1979): “The problem of externality,” *The journal of law and economics*, 22(1), 141–162.
- DANA, JR., J. D. (1994): “Learning in an equilibrium search model,” *International Economic Review*, pp. 745–771.
- DAUGHEY, A. F. (1992): “A model of search and shopping by homogeneous customers without price precommitment by firms,” *Journal of Economics & Management Strategy*, 1(3), 455–473.
- ESTRADA, R., AND M. PAVLOVIC (2017): “L’Hôpital’s monotone rule, Gromov’s theorem, and operations that preserve the monotonicity of quotients,” *Publications de l’Institut Mathématique*, 101(115), 11–24.
- FRENCH, K. R., AND R. E. MCCORMICK (1984): “Sealed Bids, Sunk Costs, and the Process of Competition,” *The Journal of Business*, 57(4), 417.
- FULLERTON, R. L., AND R. P. McAFFEE (1999): “Auctioning Entry into Tournaments,” *Journal of Political Economy*, 107(3), 573–605.
- GEHRIG, T. (1993): “Intermediation in search markets,” *Journal of Economics & Management Strategy*, 2(1), 97–120.
- HARSTAD, R. M. (1990): “Alternative Common-Value Auction Procedures: Revenue Comparisons with Free Entry,” *Journal of Political Economy*, 98(2), 421–429.
- HONG, H., AND M. SHUM (2006): “Using price distributions to estimate search costs,” *The RAND Journal of Economics*, 37(2), 257–275.
- HONKA, E. (2014): “Quantifying search and switching costs in the US auto insurance industry,” *The RAND Journal of Economics*, 45(4), 847–884.
- HONKA, E., AND P. CHINTAGUNTA (2017): “Simultaneous or sequential? search strategies in the us auto insurance industry,” *Marketing Science*, 36(1), 21–42.
- HORTAÇSU, A., AND C. SYVERSON (2004): “Product differentiation, search costs, and competition in the mutual fund industry: A case study of S&P 500 index funds,” *The Quarterly Journal of Economics*, 119(2), 403–456.

- JANSSEN, M. C. W., AND J. L. MORAGA-GONZÁLEZ (2004): “Strategic Pricing, Consumer Search and the Number of Firms,” *Review of Economic Studies*, 71(4), 1089–1118.
- LIN, H., M. R. WILDENBEEST, ET AL. (2015): “Search and prices in the medigap insurance market,” Discussion paper, Indiana University, Bloomington.
- LOS SANTOS, B. D., A. HORTAÇSU, AND M. R. WILDENBEEST (2012): “Testing Models of Consumer Search Using Data on Web Browsing and Purchasing Behavior,” *American Economic Review*, 102(6), 2955–2980.
- MACMINN, R. D. (1980): “Search and market equilibrium,” *Journal of Political Economy*, 88(2), 308–327.
- MCAFEE, R. P., AND J. McMILLAN (1988): “Search mechanisms,” *Journal of Economic Theory*, 44(1), 99–123.
- MORGAN, P., AND R. MANNING (1985): “Optimal search,” *Econometrica: Journal of the Econometric Society*, pp. 923–944.
- PESENDORFER, W., AND A. WOLINSKY (2003): “Second Opinions and Price Competition: Inefficiency in the Market for Expert Advice,” *Review of Economic Studies*, 70(2), 417–437.
- ROTHSCHILD, M. (1974): “Searching for the Lowest Price When the Distribution of Prices Is Unknown,” *Journal of Political Economy*, 82(4), 689–711.
- SATTERTHWAITE, M. A. (1979): “Consumer Information, Equilibrium Industry Price, and the Number of Sellers,” *The Bell Journal of Economics*, 10(2), 483–502.
- SPULBER, D. F. (1995): “Bertrand competition when rivals’ costs are unknown,” *The Journal of Industrial Economics*, pp. 1–11.
- (1996): “Market making by price-setting firms,” *The Review of Economic Studies*, 63(4), 559–580.
- STANGO, V., AND J. ZINMAN (2015): “Borrowing high versus borrowing higher: price dispersion and shopping behavior in the US credit card market,” *The Review of Financial Studies*, 29(4), 979–1006.

STIGLER, G. J. (1961): "The Economics of Information," *Journal of Political Economy*, 69(3), 213–225.

WOLINSKY, A. (2005): "Procurement via Sequential Search," *Journal of Political Economy*, 113(4), 785–810.

Appendix

A The Expected Price When the Number of Price Quotes is Private Information

Let $E(p_{\min}|n, m)$ denote the lowest price quoted by n sellers when each of them believes that the consumer obtains m price quotes. Also let $F_{l,m}(p)$ denote the corresponding price distribution of informed bids, $F_{b,m}(p)$ that of blind bids, and α the probability of a seller choosing to assess the production cost. From (3), we obtain that

$$\begin{aligned}
 & E(p_{\min}|n, m) \\
 &= \alpha^n (1-q) c_h + \alpha^n q \int pd(1 - (1 - F_{l,m}(p))^n) + (1-\alpha)^n \int pd(1 - (1 - F_{b,m}(p))^n) \\
 &+ \sum_{k=1}^{n-1} \binom{n-1}{k} \alpha^k (1-\alpha)^{n-k} \left((1-q) \int pd(1 - (1 - F_{b,m}(p))^{n-k}) + q \int pd(1 - (1 - F_{l,m}(p))^k) \right) \\
 &= \alpha^n (1-q) c_h + n \int_0^1 \left(\begin{array}{c} q\alpha (1-\alpha F)^{n-1} F_{l,m}^{-1}(F) \\ + (1-\alpha)^n (1-F)^{n-1} \left(q + (1-q) \left(\frac{\alpha}{(1-\alpha)(1-F)} + 1 \right)^{n-1} \right) F_{b,m}^{-1}(F) \end{array} \right) dF \\
 &= \alpha^n (1-q) c_h + n \int_0^1 \left(\begin{array}{c} q\alpha (1-\alpha F)^{n-1} \left(c_l + \frac{t/q}{(1-\alpha F)^{m-1}} \right) + (1-\alpha)^n (1-F)^{n-1} \\ * \left(q + (1-q) \left(\frac{\alpha}{(1-\alpha)(1-F)} + 1 \right)^{n-1} \right) \frac{qc_l + (1-q)c_h \left(\frac{\alpha}{(1-\alpha)(1-F)} + 1 \right)^{m-1}}{q + (1-q) \left(\frac{\alpha}{(1-\alpha)(1-F)} + 1 \right)^{m-1}} \end{array} \right) dF \\
 (11) \quad &= \alpha^n (1-q) c_h + qc_l (1 - (1 - \alpha)^n) + n \int_0^1 \alpha t (1 - \alpha F)^{n-m} dF \\
 &+ n \int_0^{1-\alpha} G^{n-1} \left(q + (1-q) \left(\frac{\alpha}{G} + 1 \right)^{n-1} \right) \left(\frac{qc_l + (1-q)c_h (\frac{\alpha}{G} + 1)^{m-1}}{q + (1-q) (\frac{\alpha}{G} + 1)^{m-1}} \right) dG.
 \end{aligned}$$

From (11), we can get $E(p_{\min}|1, 2) = c_E + \alpha q (1-q) (c_h - c_l) \ln \frac{1-\alpha q}{\alpha(1-q)} - t \ln (1-\alpha)$, $E(p_{\min}|2, 2) = c_E + 2at$ and $E(p_{\min}|3, 2) = c_E + 3\alpha t (1-\alpha/2) + 3\alpha q (1-q) (c_h - c_l) \cdot \left(\alpha^2 q (1-q) \ln \frac{1-\alpha q}{\alpha(1-q)} - \frac{1}{2} (1-\alpha) (1-\alpha + 2q\alpha) \right)$. It is not difficult to verify that (7) and (8) hold.