The Choice of Technology and Equilibrium Wage Rigidity

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Abstract

In this general equilibrium model, firms engage in oligopolistic competition and choose increasing returns technologies to maximize profits. Capital and labor are the two factors of production. The existence of efficiency wages leads to unemployment. The model can explain some interesting observations of the labor market. First, even though there is neither long-term labor contract nor costs of wage adjustment, wage rigidity is an equilibrium phenomenon: an increase in the exogenous job separation rate, the size of the population, the cost of exerting effort, and the probability that shirking is detected will not change the equilibrium wage rate. Second, the equilibrium wage rate increases with the level of capital stock. Third, a higher level of capital stock does not necessarily reduce the unemployment rate. That is, there is no monotonic relationship between capital accumulation and the unemployment rate.

Keywords: Unemployment, efficiency wages, wage rigidity, the choice of technology, oligopolistic competition

JEL Classification Numbers: E24, J64, L13

1. Introduction

There are some interesting observations of the labor market. First, industries with higher capital intensities are associated with higher wage rates, as discussed in Dickens and Katz (1987) and Abowd, Kramarz, and Margolis (1999). Second, there is no long-term relationship between capital accumulation and the unemployment rate, as discussed in Romer (2006, chap. 9). Third, there is likely existence of wage rigidity, as discussed in Gottschalk (2005), Dickens et al. (2007), and Schmitt-Grohe and Uribe (2013). As stated in Romer (2006, chap. 9), there is no study explaining the above three features of the labor market in a unified formal model.

In this paper, we provide explanations to the above three observations on the labor market in a simple general equilibrium model. We show that wage rigidity is an equilibrium phenomenon even though there is neither long-term labor contract nor costs of wage adjustment. Following Shapiro and Stiglitz (1984), a worker chooses whether to exert effort or shirk. If a worker is detected shirking, this worker will be fired. A worker also faces an exogenous rate of separation from his job. To deter shirking, the wage rate is higher than the market clearing wage rate and unemployment results. Firms engage in Cournot competition. In addition to choosing output, a firm chooses from a continuum of technologies to maximize its profit. For each technology, capital is the fixed cost and labor is the marginal cost of production. A more advanced technology has a higher fixed cost but a lower marginal cost of production. The existence of fixed costs leads to increasing returns in production.

First, we show that an increase in capital leads firms to choose more advanced technologies and the equilibrium wage rate increases. This result is consistent with empirical evidence that industries with higher capital-labor ratios pay higher wage rates.

Second, we show that an increase in capital does not necessarily reduce the unemployment rate. The reason is as follows. When the amount of capital increases, there are two effects affecting the unemployment rate working in opposite directions. On the one hand, a more advanced technology uses a lower amount of labor to produce each unit of output. This tends to decrease the demand for labor. On the other hand, an increase in capital increases total output. This latter effect tends to increase the demand for labor. Overall an increase in capital does not necessarily increase the total demand for labor and the impact on the unemployment rate is ambiguous. This result is consistent with the observation that there is no long run relationship between capital accumulation and the unemployment rate in macroeconomics. This result is also consistent with the criticism of the Lewis model that capital accumulation may not be the same as job creation in economic development (Todaro and Smith, 2012, p. 118).

Third, while the wage rate is not exogenously given, wage rigidity is an equilibrium phenomenon in this model: the equilibrium wage rate changes neither with an increase in the exogenous job separation rate, the size of the population, the cost of exerting effort, nor with the probability that shirking is detected. To understand this kind of wage rigidity, in this model the equilibrium technology depends on the amount of capital only. The reason is as follows. From the equilibrium condition determining a firm's optimal choice of technology, a firm's technology is affected by the wage rate, the interest rate, and its level of output. However, the wage rate, the interest rate, and the level of output are endogenously determined in this model. The impact from an increase in the interest rate is cancelled out by that from an increase in output. The remaining variable affecting the equilibrium technology of a firm is the wage rate, which is affected by the amount of capital. Thus, the equilibrium technology is affected by the amount of capital only. Since the equilibrium technology changes neither with the size of the population, the cost of effort, nor the probability that shirking is detected, the equilibrium wage rate does not change with any of these factors. Thus, in a business cycle if some factors such as the cost of effort change, the wage rate may not change.

Why unemployment exists is a complicated issue. There are many interesting and valuable models of unemployment, such as the search models and the efficiency wage models. Wage stickiness can be generated in search models such as in Hall (2005) and Rudanko (2011). This paper is directly related to the literature on unemployment as a result of the existence of efficiency wages as studied in the stimulating paper of Shapiro and Stiglitz (1984). The Shapiro-Stiglitz model has been extended and generalized in various directions. For example, the out of steady state dynamics is explored in Kimball (1994) and productivity shock is studied in Fella (2000). The Shapiro-Stiglitz model has also been used as a vehicle to address the hotly debated issue of the impact of international trade on unemployment: in Matusz (1996), firms engage in monopolistic competition; in Hoon (2001), out of steady state dynamics is an essential part of his analysis; in Davis and Harrigan (2011), firm heterogeneity is introduced through differences in monitoring intensities of firms. The choice of technology by firms is not addressed in the above models.

In terms of the choice of technologies, this paper is related to Zhou (2004, 2009, 2013). Zhou (2004) studies the mutual dependence between the division of labor and the extent of the market in a general equilibrium model. Zhou (2009) addresses the impact of population growth on an economy's possibility of achieving industrialization. One difference between this paper and Zhou (2004, 2009) is that there is no unemployment in Zhou (2004, 2009) while in this model there is unemployment in equilibrium. Zhou (2013) explores the choice of technology in a two-sector model of economic development in which urban unemployment exists. One difference between this paper and Zhou (2013) is that the wage rate is exogenously given in Zhou (2013) while in this model the wage rate is endogenously determined.

The plan of the paper is as follows. Section 2 specifies the model by studying a consumer's behavior, a firm's behavior, and market clearing conditions. Section 3 establishes the existence of a unique steady state and conducts comparative statics to explore the properties of the steady state. Section 4 concludes. By specifying marginal and fixed costs, the Appendix provides closed form solutions to equilibrium variables.

2. Model specification

In this model, time is continuous. If there is no confusion, we do not index variables with time periods. Capital and labor are the two factors of production. Capital does not depreciate and the amount of exogenously given capital is K. Capital is owned equally by all individuals. The interest rate is r, which will be determined endogenously. Individuals live forever. The size of the population is L and does not change over time.

In the following, first we study a consumer's utility optimization, then we study a firm's profit maximization, finally we establish market clearing conditions, such as markets for capital and the consumption good.

2.1. Consumer behavior

Individuals are risk neutral. Each individual is endowed with one unit of labor. The wage rate is w and the unemployment rate is u. For an individual, the per capita income from ownership of capital is η. An individual's total expected income is I, which is the sum of income from owning capital and expected wage income:

$$I = (1-u)w + \eta . \tag{1}$$

This individual's level of consumption is c. The cost of effort for a worker if this worker does not shirk is e. The price of the consumption good is p. A consumer's utility function is specified as (whether the time preference is equal to interest rate does not affect main results)

$$\int_0^\infty u(t)e^{-rt}dt .$$

$$u(t) = \frac{I}{p} - s\,e . \tag{2}$$

In the above specification, $s \in \{0,1\}$. If a worker shirks, then $s = 0$ and no cost of effort is incurred by this worker; if a worker does not shirk, then $s = 1$ and the cost of effort is incurred by the worker. A consumer's budget constraint states that

$$p\,c = I .$$

If a worker shirks, the probability that shirking is detected is q. Once detected shirking, this worker will be fired. In addition to being fired, the exogenous job separation rate is b. The expected lifetime utility of an employed shirker is V_E^S and the expected lifetime utility of an unemployed individual is V_u. At any point in time, an individual is either employed or unemployed. When an individual is employed, this individual's income is the sum of the wage income and the revenue from owning capital. Similar to Shapiro and Stiglitz (1984), for a shirker, the asset equation is given by

$$rV_E^S = U(w+\eta) + (b+q)(V_u - V_E^S).$$

The above equation states that a shirker enjoys instant utility of $U(w+\eta)$, the possibility of job separation at each moment is $b+q$ and thus a change of asset value of $V_u - V_E^S$. Rearrangement of this equation leads to

$$V_E^S = \frac{U(w+\eta) + (b+q)V_u}{r+b+q}. \tag{3}$$

The expected lifetime utility of an employed nonshirker is V_E^N. For a nonshirker, the possibility of job separation at each moment is b, which is smaller than the possibility of job separation for a shirker $b+q$. However, a nonshirker incurs a disutility from working. The asset equation for a nonshirker is given by

$$rV_E^N = U(w+\eta) - e + b(V_u - V_E^N).$$

Rearrangement of this equation leads to

$$V_E^N = \frac{U(w+\eta) - e + bV_u}{r+b}. \tag{4}$$

A worker will choose not to shirk if the value from nonshirking is not smaller than that from shirking: $V_E^N \geq V_E^S$. From equations (3) and (4), the no-shirking condition is

$$U(w+\eta) \geq rV_u + \frac{(r+b+q)e}{q}. \tag{5}$$

The level of unemployment benefit is $\overline{w}$. When an individual becomes unemployed, this individual still gets income from owning capital. The instant rate for an unemployed individual to find a job is a. The asset equation for an unemployed individual is

$$rV_u = U(\eta + \overline{w}) + a(V_E - V_u). \tag{6}$$

From equations (4) and (6), we have

$$rV_u = \frac{a[u(w+\eta) - e] + (r+b)u(\eta)}{a+b+r}. \tag{7}$$

2.2. Firm behavior

The consumption good is produced by m identical firms.[1] Firms engage in Cournot competition.[2] To produce the consumption good, similar to Zhou (2004, 2009, 2013), we assume that a firm may choose from a continuum of technologies indexed by a positive number n. A higher value of n indicates a more advanced technology. For each technology, capital is specified as the fixed cost and labor is specified as the marginal cost of production. Different technologies have different levels of fixed and marginal costs. The level of fixed cost in terms of the amount of capital used associated with technology n is $f(n)$ and the level of marginal cost in terms of the amount of labor used is $\beta(n)$. To capture the substitution between capital and labor in production, we assume that a more advanced technology has a higher fixed cost but a lower marginal cost of production: $f'(n) > 0$ and $\beta'(n) < 0$.

The motivation behind the above assumption on technologies is as follows. In reality, there are various examples that technologies exhibit increasing returns and firms choose technologies with different levels of marginal and fixed costs. First, Porter (1990, p. 97) discusses choices of printers. There are three types of printers and the most expensive one (with a price tag of dozens of millions dollars) is the fastest and is suitable for large volume printing such as printing newspapers. The costs of purchasing printers are fixed costs and the existence of fixed costs leads to increasing returns. An increase in fixed costs leads to a decrease in marginal costs of printing. Second, Prendergast (1990) discusses the tradeoff between fixed and marginal costs in three industries: nuts and bolts, iron founding, and machine tools. In those industries, technologies with higher fixed costs have lower marginal costs of production. Prendergast shows that a firm's choice of technology is affected by its level of output: a higher level of output is associated with the usage of technologies with higher fixed costs of production. Third, Levinson (2006) discusses the adoption of containers and the choice of technology in the transportation sector. The adoption of containers was one of the most important innovations in the transportation sector in the twentieth century. Containers were introduced into the transportation sector in the 1950s. At that time, the

[1] For simplicity, the number of firms is a real number rather than restricted to be an integer number.

[2] Oligopolistic competition is an important type of market structure for a modern society. Chandler (1990) discusses the rise of oligopoly in the United States. Modern production is associated with the extensive use of machines, which are fixed costs of production. With the existence of significant fixed costs, a firm with a higher level of output has a cost advantage over a firm with a lower level of output. Increasing returns in production, distribution, and management led to the formation of large firms in many important industries, such as the steel industry. With antitrust laws preventing monopolies from happening, those industries began dominated by small numbers of firms engaging in oligopolistic competition.

loading and unloading of cargos were handled by longshoremen and were labor intensive. With high wage rates, marginal costs were high. Because specially designed cranes, containerships, and container ports had to be built, the adoption of containers led to sharp rises in fixed costs of production. However, marginal costs of loading and unloading decreased sharply.

For a firm with output level x, its revenue is px. This firm's costs of capital are $f(n)r$, costs of labor are $\beta(n)xw$, thus its profit is $px - f(n)r - \beta(n)xw$. A firm chooses output and technology optimally to maximize its profit. First, a firm's optimal choice of output leads to $p + x\frac{\partial p}{\partial x} - \beta w = 0$. With the specification of the utility function in (2), the absolute value of a consumer's elasticity of demand for the consumption good is one. Plugging this result into a firm's optimal choice of output leads to the following familiar equation stating that marginal revenue equals marginal cost:

$$p\left(1 - \frac{1}{m}\right) = \beta w. \tag{8}$$

Second, the first order condition for a firm's optimal choice of technology requires that

$$-f'(n)r - \beta'(n)xw = 0. \tag{9}$$

The second order condition for the optimal choice of technology requires that $-f''(n)r - \beta''(n)xw \leq 0$. We assume that $f'' \geq 0$ and $\beta'' \geq 0$. That is, when more advanced technologies are adopted, fixed costs increase at a nondecreasing rate and marginal costs decrease at a nonincreasing rate. The restriction $f'' \geq 0$ and $\beta'' \geq 0$ is sufficient for the second order condition to be satisfied.

The choice of technology can lead to firm heterogeneity among countries. Suppose two countries differ in their endowments of capital. Even though firms may have access to the same set of technologies, firms in the country with a higher endowment of capital will choose more advanced technologies and workers will have higher wage rates (shown later in Proposition 1 for different industries in a given country). This can lead to correlations among the choice of technology, capital stock, and general macroeconomic conditions.[3] For example, if a country experiences capital inflow, we may expect that firms will choose more advanced technologies and the wage rate will increase.

[3] I thank a referee for this suggestion that the choice of technology may lead to firm heterogeneity.

2.3. Market clearing conditions

Similar to the argument in Shapiro and Stiglitz (1984), a firm will set the unemployment benefit to zero. Plugging equation (7) into (5), for a worker not to shirk, the wage rate needs to satisfy the following condition:

$$u(w+\eta) \geq \frac{a[u(w+\eta)-e]+(r+b)u(\eta)}{a+b+r} + \frac{(r+b+q)e}{q}.$$

In equilibrium, the above relationship will hold with equality. With the utility function specified in equation (2), from the above expression, the wage rate is defined by

$$u(w) = e + \frac{e}{q}\left(\frac{bL}{L-m\beta x} + r\right). \tag{10}$$

For the market for labor, each firm demands βx units of labor and total demand for labor from m firms is $m\beta x$. In equilibrium, all the employed individuals do not shirk. Each individual supplies one unit of labor and total effective supply of labor in this economy is $(1-u)L$. Equilibrium in the labor market requires that

$$m\beta x = (1-u)L. \tag{11}$$

For the market for capital, each firm demands f units of capital and total demand for capital from m firms is mf. Total supply of capital is K. The clearance of the market for capital requires that

$$mf = K. \tag{12}$$

For the market for the consumption good, total demand from all individuals is LI/p (since preferences are homothetic, the distribution of income will not affect the total demand for the consumption good). Total supply of the consumption good from all firms is mx. The clearance of the market for the consumption good requires that

$$\frac{LI}{p} = mx. \tag{13}$$

Since the number of firms is a real number rather than restricted to be an integer number, firms will enter until the level of profit is zero.[4] Zero profit for a firm requires that

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[4] For examples of Cournot competition with zero profits for firms, see Mankiw and Whinston (1986), Sections 3.7 and 4.5 of Brander (1995), and Zhang (2007).

$$p\,x - f\,r - \beta\,x\,w = 0\,. \tag{14}$$

In equilibrium, the total amount of revenue received by individuals as owners of capital $\eta\,L$ should be equal to the return to capital $r\,K$:

$$r\,K = \eta\,L\,. \tag{15}$$

In a steady state, equations (1) and (8)-(15) form a system of 9 equations defining a system of 9 variables p, r, x, m, I, n, w, η, and u as functions of exogenous parameters. A steady state is a tuple (p, r, x, m, I, n, w, η, u) satisfying equations (1) and (8)-(15).[5] For the rest of the paper, the consumption good is used as the numeraire:

$$p \equiv 1\,.$$

With the price of the consumption good normalized to one, equation (10) simplifies to

$$w = e + \frac{e}{q}\left(\frac{b\,L}{L - m\,\beta\,x} + r\right). \tag{16}$$

3. Comparative statics

From equations (1) and (8)-(15), we can derive the following system of three equations defining three endogenous variables u, r, and n as functions of exogenous parameters:[6]

$$V_1 \equiv \frac{1}{\beta}\left(1 - \frac{f}{K}\right) - e - \frac{e}{q}\left(\frac{b}{u} + r\right) = 0\,, \tag{17}$$

$$V_2 \equiv -\beta\,f' - \beta'(K - f) = 0\,, \tag{18}$$

$$V_3 \equiv (1 - u)\,f\,L - \beta\,r\,K^2 = 0\,. \tag{19}$$

Partial differentiation of the system of equations (17)-(19) with respect to u, r, n, K, b, L, e, and q leads to[7]

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[5] For this system of equations, when equations (1), (8)-(12), (14), and (15) are satisfied, equation (13) is automatically satisfied. That is, one equation is redundant. With Walras's law in mind, this redundancy is not surprising.

[6] Equations (17)-(19) are derived as follows. First, equation (17) is derived by plugging the value of m from equation (12) into equation (8), then plugging the resulting value of w into equation (16). Second, equation (18) is derived by plugging the value of x from equation (14) into equation (9). Third, equation (19) is derived by plugging the value of m from equation (12), the value of x from equation (14), and the value of $1 - \beta\,w$ from equations (8) and (12) into equation (11).

[7] Equation (18) is used to show that $\partial V_1 / \partial n = 0$.

$$\begin{pmatrix} \frac{\partial V_1}{\partial u} & \frac{\partial V_1}{\partial r} & 0 \\ 0 & 0 & \frac{\partial V_2}{\partial n} \\ \frac{\partial V_3}{\partial u} & \frac{\partial V_3}{\partial r} & \frac{\partial V_3}{\partial n} \end{pmatrix} \begin{pmatrix} du \\ dr \\ dn \end{pmatrix} = -\begin{pmatrix} \frac{\partial V_1}{\partial K} \\ \frac{\partial V_2}{\partial K} \\ \frac{\partial V_3}{\partial K} \end{pmatrix} dK - \begin{pmatrix} \frac{\partial V_1}{\partial b} \\ 0 \\ 0 \end{pmatrix} db$$

$$- \begin{pmatrix} 0 \\ 0 \\ \frac{\partial V_3}{\partial L} \end{pmatrix} dL - \begin{pmatrix} \frac{\partial V_1}{\partial e} \\ 0 \\ 0 \end{pmatrix} de - \begin{pmatrix} \frac{\partial V_1}{\partial q} \\ 0 \\ 0 \end{pmatrix} dq \,. \tag{20}$$

Let Δ denote the determinant of the coefficient matrix of endogenous variables of the system (20):

$$\Delta \equiv \frac{\partial V_2}{\partial n}\left(\frac{\partial V_1}{\partial r}\frac{\partial V_3}{\partial u} - \frac{\partial V_1}{\partial u}\frac{\partial V_3}{\partial r} \right).$$

From equation (18), $\frac{\partial V_2}{\partial n} = -\beta f'' - \beta'' K$. With $f'' \geq 0$ and $\beta'' \geq 0$, $\frac{\partial V_2}{\partial n} < 0$. Since $\frac{\partial V_1}{\partial r} < 0$, $\frac{\partial V_1}{\partial u} > 0$, $\frac{\partial V_3}{\partial u} < 0$, and $\frac{\partial V_3}{\partial r} < 0$, we have $\Delta < 0$. With Δ nonsingular, a unique equilibrium exists.[8]

In macroeconomics, there is no monotonic relationship between capital accumulation and the unemployment rate. In the Lewis model in economic development, it is assumed that capital accumulation leads to an increase in the level of employment in the manufacturing sector. The Lewis model has been criticized because capital accumulation may lead to the adoption of technologies saving labor while employment in the manufacturing sector may not increase. The following proposition studies whether an increase in the amount of capital leads to a decrease in the unemployment rate.

[8] Turnovsky (1977, chap. 2) discusses conditions for the existence of a unique local equilibrium and a unique global equilibrium. He demonstrates that conditions for the existence of a unique global equilibrium are very strict. Thus we focus on the existence of a unique local equilibrium.

Proposition 1: An increase in the amount of capital leads firms to choose more advanced technologies and the wage rate increases. The impact on the interest rate and the unemployment rate is ambiguous.[9]

Proof: Plugging the value of m from equation (12) into equation (8) yields

$$w = \frac{1}{\beta}\left(1 - \frac{f}{K}\right). \tag{21}$$

An application of envelope theorem on equation (21) leads to

$$\frac{dw}{dK} = \frac{\partial w}{\partial K} > 0 .$$

An application of Cramer's rule on the system (20) reveals

$$\frac{dn}{dK} = \frac{\partial V_2}{\partial K}\left(\frac{\partial V_1}{\partial u}\frac{\partial V_3}{\partial r} - \frac{\partial V_1}{\partial r}\frac{\partial V_3}{\partial u}\right) / \Delta ,$$

$$\frac{dr}{dK} = \left(\frac{\partial V_1}{\partial u}\frac{\partial V_2}{\partial n}\frac{\partial V_3}{\partial K} - \frac{\partial V_1}{\partial u}\frac{\partial V_2}{\partial K}\frac{\partial V_3}{\partial n} - \frac{\partial V_1}{\partial K}\frac{\partial V_2}{\partial n}\frac{\partial V_3}{\partial u}\right) / \Delta ,$$

$$\frac{du}{dK} = \left(\frac{\partial V_1}{\partial K}\frac{\partial V_2}{\partial n}\frac{\partial V_3}{\partial r} + \frac{\partial V_1}{\partial r}\frac{\partial V_2}{\partial K}\frac{\partial V_3}{\partial n} - \frac{\partial V_1}{\partial r}\frac{\partial V_2}{\partial n}\frac{\partial V_3}{\partial K}\right) / \Delta .$$

Since $\frac{\partial V_2}{\partial K} > 0$ and $\frac{\partial V_1}{\partial u}\frac{\partial V_3}{\partial r} - \frac{\partial V_1}{\partial r}\frac{\partial V_3}{\partial u} < 0$, $\frac{dn}{dK} > 0$. Since the sign of $\frac{\partial V_1}{\partial u}\frac{\partial V_2}{\partial n}\frac{\partial V_3}{\partial K} - \frac{\partial V_1}{\partial u}\frac{\partial V_2}{\partial K}\frac{\partial V_3}{\partial n} - \frac{\partial V_1}{\partial K}\frac{\partial V_2}{\partial n}\frac{\partial V_3}{\partial u}$ is undetermined, the sign of $\frac{dr}{dK}$ undetermined. Since the sign of $\frac{\partial V_1}{\partial K}\frac{\partial V_2}{\partial n}\frac{\partial V_3}{\partial r} + \frac{\partial V_1}{\partial r}\frac{\partial V_2}{\partial K}\frac{\partial V_3}{\partial n} - \frac{\partial V_1}{\partial r}\frac{\partial V_2}{\partial n}\frac{\partial V_3}{\partial K}$ is undetermined, the sign of $\frac{du}{dK}$ is undetermined.

From Proposition 1, the wage rate increases with the amount of capital. Since the price of the consumption good is normalized to one, an increase in the wage rate is an increase in both nominal and real wage rates. To understand Proposition 1, when the amount of capital increases, the number of firms may be either constant or increase. First, if the number of firms is constant, each firm uses more capital and the productivity of labor increases. As a result, the wage rate

[9] In a model with constant returns to scale in the production of the consumption good, an increase in the amount of capital can decrease the interest rate. With increasing returns in this model, the interest rate may not decrease when the amount of capital increases.

increases. Second, if the equilibrium number of firms increases, an increase in the number of firms increases the degree of competition in the labor market and the wage rate increases.

From Proposition 1, a higher amount of capital leads firms to choose more advanced technologies. However, the impact on the unemployment rate is ambiguous.[10] The intuition behind this result is as follows. When the amount of capital increases, there are two effects. First, since a firm chooses a more advanced technology, the amount of labor required to produce each unit of output decreases. This tends to decrease the demand for labor. Second, an increase in the amount of capital increases one factor of production and total output in this economy increases. An increase in output increases the demand for labor. Since the two effects work in opposite directions, overall the total impact on the demand for labor and thus on the unemployment rate is ambiguous.

In Shapiro and Stiglitz (1984), the impact of an increase in the amount of capital on the unemployment rate is not studied explicitly. However, if we view that an increase in the amount of capital increases the marginal product of a worker, an increase in the amount of capital will decrease the unemployment rate in their model. The reason for this difference between their result and the result here is that the choice of technology is not incorporated in their model. With the presence of the second effect (an increase in a factor of production) only, an increase in the amount of capital always decreases unemployment rate in their model.

The following proposition studies the impact of an increase in the exogenous job separation rate.

Proposition 2: An increase in the exogenous job separation rate increases the unemployment rate and decreases the interest rate, but changes neither the equilibrium technology nor the wage rate.

Proof: An application of Cramer's rule on the system (20) leads to

$$\frac{dn}{db} = 0,$$

$$\frac{du}{db} = \frac{\partial V_1}{\partial b}\frac{\partial V_2}{\partial n}\frac{\partial V_3}{\partial r} / \Delta > 0,$$

[10] In this model, when the amount of capital increases, the impact on the interest rate is ambiguous.

$$\frac{dr}{db} = -\frac{\partial V_1}{\partial b}\frac{\partial V_2}{\partial n}\frac{\partial V_3}{\partial u} / \Delta < 0 .$$

From equation (21), since $\frac{dn}{db} = 0$, $\frac{dw}{db} = 0$.

To understand the result that an increase in the exogenous job separation rate does not affect the equilibrium technology in this model, from equation (18), the equilibrium technology depends on the amount of capital only. The intuition behind this observation is as follows. Equation (9) is the condition for a firm's optimal choice of technology. In this equation, a firm's technology is affected by the wage rate, the interest rate, and its level of output. However, the wage rate, the interest rate, and the level of output are endogenously determined in the model. When there is an increase in the interest rate, the level of output of a firm also increases, and the impact from an increase in the interest rate is cancelled out by that from an increase in output. Thus the remaining variable affecting the equilibrium technology is the wage rate. From equation (21), the wage rate is determined by the amount of capital. As a result, in equilibrium, the equilibrium technology is affected by the amount of capital only.

This result is different from that in Shapiro and Stiglitz (1984) in which an increase in the exogenous job separation rate (later the cost of exerting effort) increases the equilibrium wage rate. This difference can be understood as follows. The adjustment mechanisms for the nonshirking condition to hold are different in the two models. In their model, the interest rate is exogenously given. When the exogenous job separation rate (or the cost of exerting effort) increases, both the wage rate and the unemployment rate increase so that the nonshirking condition is satisfied. In this model, since the interest rate is endogenously determined, the adjustment of the interest rate provides an additional channel for the nonshirking condition to be satisfied. When the exogenous job separation rate (or the cost of exerting effort) increases, the interest rate decreases and the unemployment rate increases so that the nonshirking condition is satisfied.

In Proposition 2, the interest rate decreases when the exogenous job separation rate increases. How to understand this result? From equations (8), (12), and (14), a firm's output can be expressed as $x = \frac{f\,r}{1-\beta w} = m\,f\,r = K\,r$. With $x = K\,r$, since the amount of capital is fixed, the interest rate is directly related to the level of output of a firm. Intuitively, when the interest rate increases, a firm needs to produce a higher level of output to break even. An alternative interpretation is that a firm can afford a higher level of interest rate payment when its output increases. With the interest rate increases with the level of output, factors leading to an increase in the level of output will lead to an increase in the interest rate. When the exogenous job separation rate increases, through the nonshirking condition, the wage rate increases. Other things equal, an increase in the wage rate increases a firm's marginal cost and output decreases. Thus, the interest rate decreases. This explanation can also be used to explain results shown later. First, an increase in population increases the interest rate because an increase in population increases the output of a firm. Second, an increase in the cost of exerting effort decreases the interest rate because an increase in the cost of exerting effort decreases the level of output through the nonshirking condition. Third, an increase in the probability that shirking is detected increases the interest rate because an increase in the probability that shirking is detected decreases the wage rate through the nonshirking condition and output increases.

When there is an increase in the exogenous job separation rate, since the amount of capital and the level of technology do not change, the equilibrium wage rate does not change. Since the equilibrium technology does not change, from equation (12), the equilibrium number of firms does not change. However, since the equilibrium interest rate decreases, each firm produces a lower level of output.

An increase in population increases the supply of labor. The following proposition studies whether this will decrease the wage rate.

Proposition 3: An increase in population increases the unemployment rate and the interest rate, but changes neither the equilibrium technology nor the equilibrium wage rate.

Proof: An application of Cramer's rule on the system (20) leads to

$$\frac{dn}{dL} = 0,$$

$$\frac{du}{dL} = -\frac{\partial V_1}{\partial r}\frac{\partial V_2}{\partial n}\frac{\partial V_3}{\partial L} / \Delta > 0,$$

$$\frac{dr}{dL} = \frac{\partial V_1}{\partial u}\frac{\partial V_2}{\partial n}\frac{\partial V_3}{\partial L} / \Delta > 0.$$

From equation (21), since $\frac{dn}{dL} = 0$, $\frac{dw}{dL} = 0$.

As discussed in the paragraph after Proposition 2, even though the wage rate is endogenously determined in this model and is subject to change, in equilibrium the wage rate does not decrease when the size of the population increases.

The impact of an increase in population on other variables can also be studied. First, since the unemployment rate increases and the equilibrium wage rate does not change, the expected labor income of a worker decreases.[11] Since both the expected labor income and the return from capital decrease, an individual is worse off when there is an increase in population. Second, when there is an increase in population, since the amount of capital and the equilibrium technology do not change, the equilibrium number of firms does not change. Third, since the equilibrium interest rate increases, from equation (9), each firm produces a higher level of output. Finally, from equation (11), since the number of firms and technology do not change and the level of output increases, the number of individuals employed increases with the size of the population. The unemployment rate increases because the rate of the increase in the number of individuals employed is smaller than the rate of the increase in population.

If the cost of exerting effort increases, the marginal cost of supplying labor is higher. The following proposition studies whether an increase in the cost of exerting effort increases the equilibrium wage rate.

Proposition 4: An increase in the cost of exerting effort increases the unemployment rate and decreases the interest rate, but changes neither the equilibrium technology nor the equilibrium wage rate.

Proof: An application of Cramer's rule on the system (20) leads to

$$\frac{dn}{de} = 0,$$

$$\frac{du}{de} = \frac{\partial V_1}{\partial e}\frac{\partial V_2}{\partial n}\frac{\partial V_3}{\partial r} / \Delta > 0,$$

$$\frac{dr}{de} = -\frac{\partial V_1}{\partial e}\frac{\partial V_2}{\partial n}\frac{\partial V_3}{\partial u} / \Delta < 0.$$

[11] The result that an increase in population leads to an increase in the unemployment rate is similar to that in Shapiro and Stiglitz (1984). In their model, an increase in population also increases the unemployment rate.

From equation (21), since $\frac{dn}{de} = 0$, $\frac{dw}{de} = 0$.

As discussed in the paragraph after Proposition 2, since the equilibrium technology does not change, the equilibrium wage rate does not change even though the cost of exerting effort increases. Since both the expected labor income and the return from capital decrease and the cost of exerting effort increases, an individual is worse off when there is an increase in the cost of effort.

The probability that shirking is detected is related to the level of efficiency in the labor market. The following proposition studies the impact of an increase in the probability that shirking is detected.

Proposition 5: An increase in the probability that shirking is detected leads to a decrease in the unemployment rate and an increase in the interest rate, but changes neither the equilibrium technology nor the equilibrium wage rate.

Proof: An application of Cramer's rule on the system (20) leads to

$$\frac{dn}{dq} = 0,$$

$$\frac{du}{dq} = \frac{\partial V_1}{\partial q}\frac{\partial V_2}{\partial n}\frac{\partial V_3}{\partial r} / \Delta < 0,$$

$$\frac{dr}{dq} = -\frac{\partial V_1}{\partial q}\frac{\partial V_2}{\partial n}\frac{\partial V_3}{\partial u} / \Delta > 0.$$

From equation (21), since $\frac{dn}{dq} = 0$, $\frac{dw}{dq} = 0$.

The equilibrium wage rate does not change when there is an increase in the probability that shirking is detected. Since both the expected labor income and the return from capital increase, an individual is better off rather than worse off when there is an increase in the probability that shirking is detected.

The impact of an increase in the probability that shirking is detected on other variables can also be studied. First, the equilibrium number of firms does not change because the amount of capital and the equilibrium technology do not change. Second, since the interest rate increases, from equation (9), each firm produces a higher level of output.

4. Conclusion

In this paper, we have shown that by incorporating the choice of technology into the Shapiro-Stiglitz model, even though there is neither long-term labor contract nor costs of wage adjustment, we are able to explain some interesting observations of the labor market in a unified model. First, an increase in capital leads firms to choose more advanced technologies and the equilibrium wage rate increases. However, the impact on the unemployment rate is ambiguous. Second, an increase in the level of exogenous job separation rate, the size of the population, or the cost of exerting effort leads to an increase in the unemployment rate and a decrease in the interest rate, but changes neither the equilibrium level of technology nor the wage rate. An individual is worse off when there is an increase in population. Third, an increase in the probability that shirking is detected increases the employment rate and the interest rate, but neither changes the equilibrium technology nor the wage rate. An individual is better off rather than worse off when there is an increase in the probability that shirking is detected.

Appendix: A closed form solution to equations (17)-(19)

In this Appendix, by specifying functional forms for the marginal and fixed costs of production, we can solve equations (17)-(19) explicitly. We specify

$$f(n) = n, \tag{A1}$$

$$\beta(n) = \frac{1}{n}. \tag{A2}$$

Plugging (A1) and (A2) into equation (18) yields the level of technology

$$n = \frac{K}{2}. \tag{A3}$$

Plugging (A1), (A2), and (A3) into equation (19) yields the interest rate

$$r = \frac{1-u}{4} L. \tag{A4}$$

Plugging (A1), (A2), (A3), and (A4) into equation (17) yields the following equation defining the unemployment rate implicitly

$$\Phi \equiv Lu^2 - \left(L - \frac{Kq}{e} - 4q \right) u - 4b = 0 . \tag{A5}$$

From (A5), $\frac{du}{dL} = -\frac{\partial \Phi / \partial L}{\partial \Phi / \partial u} = \frac{u(1-u)}{2Lu - L - 4q + \frac{Kq}{e}}$. If $2Lu - L - 4q + \frac{Kq}{e} > 0$, or $u > \frac{L - \frac{Kq}{e} + 4q}{2L}$, then $\frac{du}{dL} > 0$.

Equation (A5) has two roots. One root is $u = \frac{L - \frac{Kq}{e} + 4q - \sqrt{\left(L - \frac{Kq}{e} + 4q \right)^2 + 16Lb}}{2L}$. This root indicates a negative unemployment rate and is thus discarded. The other root is kept:

$$u = \frac{L - \frac{Kq}{e} + 4q + \sqrt{\left(L - \frac{Kq}{e} + 4q \right)^2 + 16Lb}}{2L} . \tag{A6}$$

From (A6), it is clear that $u > \frac{L - \frac{Kq}{e} + 4q}{2L}$. Thus $\frac{du}{dL} > 0$.

Plugging (A6) into (A4) yields

$$r = \frac{L}{8} + \frac{Kq}{8e} - \frac{q}{2} - \frac{1}{8}\sqrt{\left(L - \frac{Kq}{e} + 4q \right)^2 + 16Lb} . \tag{A7}$$

From (A7), $\frac{dr}{dL} > 0$ if $q\left(\frac{K}{e} - 4 \right) > 4b$. The validity of this inequality can be checked by (A6) when we require that the unemployment rate should not be larger than one.

Equations (A3), (A6), and (A7) provide a set of solutions for the system of equations (17)-(19). With the values of u, r, and n determined, the values of other variables can also be determined. For example, plugging equations (A1) and (A3) into equation (12) yields the equilibrium number of firms

$$m = 2 .$$

Plugging (A1), (A2), and (A3) into equation (21) yields the equilibrium wage rate

$$w = \frac{K}{4} . \tag{A8}$$

Plugging (A1), (A2), (A3), (A7), and (A8) into equation (14) yields the equilibrium level of output of a firm

$$x = K\left(\frac{L}{8} + \frac{Kq}{8e} - \frac{q}{2} - \frac{1}{8}\sqrt{\left(L - \frac{Kq}{e} + 4q\right)^2 + 16Lb}\right).$$

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