A glance at Solow's growth theory

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Abstract

This paper examines the growth theory of Robert Solow[1], which has been a point of reference of economic growth since the 1950s. First, the article analyzes the path-breaking model of growth contained in Solow's article "A Contribution to the Theory of Economic Growth" published in *The Quarterly Journal of Economics* (1956). Second, it looks at the contribution of Solow to growth accounting and to the new method of studying capital formation in economic growth through the vintage approach. Therefore, the work analyzes the article "Technical Change and the Aggregate Production Function" published in *The Review of Economics and Statistics* (1957). In the latter publication, Solow, through the aggregate production function, tries to measure growth and provide an explanation of the nature of technical progress. The article also examines Solow's 1960 essay "Investment and Technical Progress" based on the hypothesis of embodied technological progress and the vintage approach.

Keyword: Aggregate Production Function, Capital Accumulation, Solow's Models of Growth, Technological Change.
Jel Classification: B220, E10, E230, O10, O33.

1. INTRODUCTION

The dynamics of modern industrial economies are increasingly characterized by innovation and rapid transformations in technological knowledge that tend to change their structure systems (Schilirò, 2006, 2012a, 2012b). Schumpeter (1934) provided and highlighted a theory of innovation as the main factor influencing long-term growth. However, neoclassical growth models developed in the 1950s took a different direction. They did not provide a theory of innovation or technological change. Instead, they recognized the major role of technological change in growth.

The initial formulation of the neoclassical theory of growth is due to the contributions of Swann (1956) and Solow (1956, 1957). These models, which were based on capital accumulation, were inspired by Ramsey's pioneering essay (1928) on the role of saving to achieve the utmost utility.

The neoclassical growth theory intends to explain the continuing rise in per capita income. This has characterized many market economies over the last two centuries. In the neoclassical growth model,

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[1] Robert Solow was awarded the Nobel Prize for Economics in 1987 for his contributions to the theory and measurement of economic growth.

the production takes place in conditions of competition, whereas capital accumulation is the engine of output growth.

2. SOLOW'S MODEL OF GROWTH

It is known that the theory of growth used Solow's 1956 growth model, contained in the article "A Contribution to the Theory of Economic Growth", as its point of reference. The article contains a mathematical model in the form of a differential equation to describe how increased capital stock generates greater per capita production. Solow begins with the proposition that society saves a given constant proportion of its incomes. The population and the supply of labor grow at a constant rate. Capital intensity (or capital per employee) can be regulated. Capital intensity is determined by the prices of production factor. Due to diminishing returns, additional capital increases (or increasing capital intensity) make ever smaller contributions to production. In this model, in the long term and under the condition of absence of technological progress, a steady-state growth path is reached when output, capital, and labor grow at the same rate. Therefore, output per worker and capital per worker are constant[2], and the economy approaches a condition of identical growth rates for capital, labor and total production. An increase in the proportion of saved incomes cannot lead to a permanent increase in the rate of growth. In fact, in the absence of technological progress, the rate of growth will remain the same (irrespective of the share of savings), and will be purely dependent on an increased supply of labor.

Solow's theoretical model of growth had an enormous impact on economic analysis. According to Acemoglou (2009), this model shaped the way we approach both economic growth and the entire field of macroeconomics.

2.1. THE MODEL

Solow (1956) criticizes the Keynesian Harrod-Domar long term growth model[3] for the crucial assumption that production takes place under conditions of fixed proportions. Consequently, these conditions cause potential dysfunctional aspects of economic growth (for example, increased unemployment or prolonged inflation). Thus, Solow (1956, p.66) proposed a model of long-run growth "which accepts all the Harrod-Domar assumptions except that of fixed proportion" in production. It considers labor-capital substitution, that is, the change in production technique as a response to changes in relative prices of labor and capital. This implies that there is the neoclassical aggregate production function at the center of Solow's growth model.

The main hypotheses of such a model include:

- Markets in balance due to a market clearing mechanism
- Presence of declining marginal returns
- Convergence of economies with the same initial conditions
- Nil long-term growth rate, or case corresponding to an exogenous increase rate of technical progress

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[2] Per capita production and real wage no longer increase.
[3] The Harrod–Domar model is a Keynesian model of economic growth. The model was developed independently by Roy F. Harrod (1939) and Evsey Domar (1946).

This growth model is characterized by the presence of exogenous variables, including savings rate, growth rate of population and growth rate of technical progress[4]. The factors of production, capital K, and labor L, change as a result of investment and population growth respectively. Moreover, markets are in perfect competition (Aghion, & Howitt, 2003; Helpman, 2004; Schilirò, 1986).

The model can be represented as follows.

First, there is an aggregate production function to determine the technological possibilities. Y represents the output (net output after depreciation of capital); K and L represent capital and labor inputs in "physical" units. Then

$$Y = F(K, L) \qquad (1)$$

Equation (1) represents the aggregate production function, which is assumed to satisfy a series of technical conditions:
- Increasing in both arguments
- Displaying decreasing marginal returns to each factor
- Displaying constant returns to scale[5]

Cobb-Douglas is a production function satisfying these properties. It assumes that F exhibits constant returns to scale in K and L (i.e., it is linearly homogeneous [homogeneous of degree 1] in these two variables).

The Solow model is also characterized by a law of motion for the stock of capital.
The stock of capital K(t) takes the form of an accumulation of the composite commodity. Net investment I(t) is the rate of increase of this capital stock dK/dt. Threfore, we have the basic identity at every instant of time.

$$dK/dt \equiv \dot{K} = I(t) \qquad (2)$$

The third fundamental equation of the Solow model is the savings/investment function. This is assumed to be of a Keynesian nature. Savings and investment (in a closed economy) are a constant fraction s of total income Y (t)[6]:

$$S(t) = I(t) = s\,Y(t) \qquad (3)$$

Inserting equation (1) in equation (3) gives us:

$$\dot{K} = s\,F(K, L) \qquad (4)$$

Solow closes his model following Harrod. As a result of exogenous population growth, the labor force increases at a constant relative rate *n*. Since Solow assumes the absence of technological change, *n* corresponds to Harrod's natural rate of growth.

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[4] The reason this model is called "exogenous" growth model is the saving rate is taken to be exogenously given.
[5] Constant returns to scale implies that by multiplying each input by factor z, output changes by a multiple of that same factor: zQ = f (zK, zL). Thus, the production function is homogeneous of first degree (Solow, 1956, p.67).
[6] The model can be easily extended to include a household's problem with a dynamic consumption/saving decision (called the Ramsey-Cass-Koopmans model, Aghion & Howitt, 2003, pp.17-22).

$$L(t) = L_0 e^{nt} \qquad\qquad (5)$$

The complete set of three equations consists of equation (4), equation (5) and $\dfrac{\partial F(K,L)}{\partial L} = w$. The last equation is the marginal productivity equation that determines the real wage rate.

In equation (4), L represents total employment. In equation (5), L represents the available supply of labor. Solow assumes that full employment is perpetually maintained by identifying the two variables.

Inserting equation (5) in equation (4) gives us:

$$\dot{K} = s\, F\,(K, L_0 e^{nt}) \qquad\qquad (6)$$

This equation determines the time path of capital accumulation that must be followed if all available labor is to be employed.

The equation (6) is a differential equation in the single variable K(t). Its solution gives the only time profile of the capital stock of the economy to fully employ available labor.

The time path of capital stock and labor force is determined by equation (5) and equation (6), it is possible to compute from the production function equation (1) the corresponding time path of real output. An assumption of full employment of the available stock of capital is also present in this model.

Solow (1956, p. 68) explains the process of capital accumulation by stating that:

"at any moment of time the available labor supply is given by (5) and the available stock of capital is also a datum. Since the real return to factors will adjust to bring about full employment of labor and capital we can use the production function (1) to find the current rate of output. Then the propensity to save tells us how much of net output will be saved and invested. Hence, we know the net accumulation of capital during the current period. Added to the already accumulated stock this gives the capital available for the next period, and the whole process can be repeated".

Thus, Solow shows possible growth patterns by analyzing if there is always a capital accumulation path consistent with any rate of growth of the labor force. He starts from the differential equation (6).

First Solow introduces a new variable, $r = K/L$, the ratio of capital to labor (capital intensity).

We have $K = r\,L = r\,L_0 e^{nt}$.

Differentiating with respect to time we get:

$$\dot{K} = L_0 e^{nt} + n\,\dot{r}\,L_0 e^{nt}. \qquad\qquad (7)$$

Substituting equation (7) in equation (6), and, because of constant returns to scale, we can divide both variables in F by $L = L_0 e^{nt}$. We obtain, $(\dot{r} + n\,r)\,L_0 e^{nt} = s\,L_0 e^{nt}\,F(K/L_0 e^{nt}, 1)$.

By dividing out the common factor $L_0 e^{nt}$ we get

$$\dot{r} = s\,F(r,1) - n\,r. \tag{8}$$

Equation (8) represents the rate of change of the capital-labor ratio as the difference of two terms. One representing the increment of capital and one the increment of labor. In particular, the function $F(r, 1)$ in equation (8) is the total product curve as varying amounts r of capital are employed with one unit of labor. Alternatively, it gives output per worker as a function of capital per worker.

When $\dot{r} = 0$, the capital-labor ratio is a constant, and the capital stock must be expanding at the same rate as the labor force, n. That is, the warranted rate of growth. This rate of growth is warranted by the appropriate real rate of return to capital and equals the natural rate.
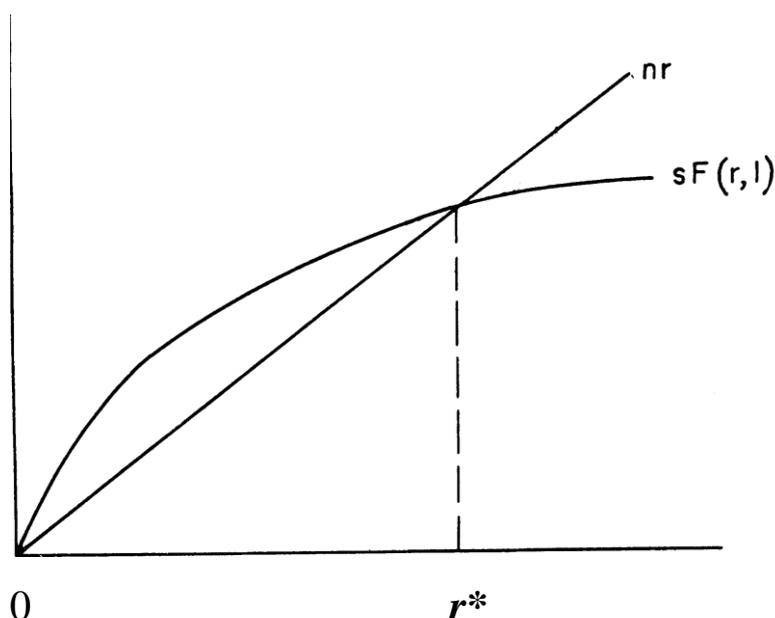

Figure 1.

In Figure 1 the straight line at 45 degrees passing through the origin with slope n represents the function nr. The curve is the function $sF(r,1)$. This curve passes through the origin and is convex upward. Therefore, there is no output unless both inputs are positive. There is diminishing marginal productivity of capital, as would be the case, for example, with the Cobb-Douglas function.

At the point of intersection $nr = sF(r,1)$ and $\dot{r} = 0$

Solow (1956, p. 70) explained that: *"If the capital-labor ratio r* should ever be established, it will be maintained, and capital and labor will grow thenceforward in proportion. By constant returns to scale, real output (Y) will also grow at the same relative rate n, and output per head of labor force will be constant."*

Moreover, according to Solow (1956), the equilibrium value r^* is stable. Whatever the initial value of the capital-labor ratio, the system will develop toward a state of balanced growth at the natural rate.

If the initial capital stock is below the equilibrium ratio, capital and output will grow at a faster pace than the labor force until the equilibrium ratio is approached. If the initial ratio is above the equilibrium value, capital and output will grow more slowly than the labor force. The growth of output is always intermediate between those of labor and capital.

Solow's basic conclusion (1956, 73) is that:

"when production takes place under the usual neoclassical conditions of variable proportions and constant returns to scale, no simple opposition between natural and warranted rates of growth is possible. There may not be - in fact in the case of the Cobb-Douglas function there never can be - any knife-edge. The system can adjust to any given rate of growth of the labor force, and eventually approach a state of steady proportional expansion"[7].

In Solow's model, a steady-state growth path is reached when output, capital and labor grow at the same rate. Output per worker and capital per worker are both constant. The model shows that a sustained rise in capital investment (accumulation of capital) − and therefore an increase in the proportion of incomes which is saved[8] − increases the growth rate only temporarily, because the ratio of capital to labor goes up (increasing capital intensity). In fact, the marginal product of additional units of capital declines due to diminishing returns. Thus, an economy returns to a long-term growth path with total production growing at the same rate as labor (under the condition that there is no technological progress). This involves a situation in which per capita production and real wage no longer increase. (Nobelprize.org, 1987).

In this model an implication of diminishing returns is that the equilibrium rate of growth is proportional to the saving (investment) rate. In addition, it is independent of the saving (investment) rate (Solow, 1988).

Since, the model of Solow maintains that growth is due to capital accumulation, then, according to the model, growth will be very strong when countries first begin to accumulate capital, and will slow down as the process of accumulation continues, because of diminishing returns of capital.

So, the model defines the conditions for the tendency of different nations to approach an equilibrium (steady-state) level of the capital stock.

In addition, the Solow growth model suggests that countries will tend to converge in output per capita and in standard of living[9]. In fact, the model implies, because of a higher marginal rate of return on invested capital in faster-growing countries, that income levels in poor economies will grow relatively faster than developed countries and eventually converge or catch up to the economies of the latter through capital accumulation, assuming that all countries have the same access (due to spillover effects) to technologies. Several years later, in commenting on his article, Solow (1988) states that «a developing economy that succeeds in permanently increasing its saving (investment) rate will have a higher level of output than if it had not done so, and must therefore grow faster for a while. But it will not achieve a permanently higher rate of growth of output. More precisely: the permanent rate of growth of output per unit of labor input is independent of the saving (investment) rate and depends entirely on the rate of technological progress in the broadest sense» (1988, p.308).

However, these propositions have been criticized, since with just a few exceptions, that is not happening. Moreover, the extent of catch up in living standards is questioned. Finally, some literature

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[7] Solow (1956, pp.76-77) in the example of the Cobb-Douglas production function shows that the asymptotic behavior of the system is balanced growth at the natural rate. Thus, in the long-run equilibrium growth, the natural rate equals "the" warranted rate.

[8] It is assumed that savings correspond to an equivalent amount of (planned) capital investment.

[9] Aghion, Howitt (2003, p.17) state that the model exhibits 'conditional convergence', in the sense that convergence is conditional on the determinants of the countries' steady state levels of output per person.

has proved the existence of the middle-income trap when growing economies find it hard to sustain growth and rising per capita incomes beyond a certain level.

Summing up:
Growth in output per capita (*y*) occurs only as an economy moves to the steady state.
– In steady state, there is no growth in *y*.
– The aggregate output of the economy (Y) grows at rate of *n*.
An important implication of the model is that countries with low *r* should grow more quickly than countries with *r* closer to steady state. Another implication is that it predicts high growth in poor countries. Also, in this model differences in income levels between countries are attributed to different rates of savings.

Furthermore, Solow aims to set out the price-wage-interest behavior appropriate to the growth paths sketched earlier (Solow, 1956, p.79). Solow examines the four prices involved in the system: (1) the selling price of a unit of real output (and since real output serves also as capital this is the transfer price of a unit of capital stock) p(t); (2) the money wage rate w(t); (3) the money rental per unit of time of a unit of capital stock q(t); (4) the rate of interest i(t). He takes p(t), the price of real output, as given, since "in the real system we are working with there is nothing to determine the absolute price level" (ibid., p.79). According to Solow, in general, a stable growth path exists, thus the fall in the real wage or real rental is needed to get to it. In particular, if there is an initial shortage of labor (compared with the equilibrium ratio) the real wage will have to fall (Solow, 1956, p.83). Thus, factor price flexibility is a fundamental condition to get on a stable growth path.

In the last part of the article, Solow discusses possible extensions of his model (1956, pp. 85-91). In particular he presents a model that takes into account neutral technical change. In this special case of neutral technical change. Shifts in the production function are defined as neutral if they leave marginal rates of substitution untouched but simply increase or decrease the output attainable from given inputs. In that case the production function [1] takes the special form

$$Y = A(t)f(K, L) \qquad\qquad [1a]$$

where the A (t) measures the cumulated effect of shifts over time and it represents the technical change, that is, an expression for any kind of shift in the production function.
Solow explains that "the way in which the (now ever-changing) equilibrium capital-labor ratio is affected can be seen on a diagram like Figure 1 by "blowing up" the function sF(r,1)" (Solow, 1956, p.85).

By adopting the Cobb-Douglas function in this case, we take $A(t) = e^{gt}$, then the basic differential equation becomes

$$\dot{K} = s\, e^{gt}\, K^a\, (L_o\, e^{nt})^{1-a} = s\, K^a\, L_o^{1-a}\, e^{(n(1-a)+g)t} \qquad\qquad [9]$$

whose solution is

$$K(t) = [\ K_o{}^b - \frac{bs}{nb+g}\ L_o{}^b + \frac{bs}{nb*g}\ L_o{}^b e^{(nb*g)t}\ ]^{1/b} \qquad [10]$$

where $b = 1 - a$.

In this new model with neutral technological progress, in the long run the capital stock increases at the relative rate $n + g/b$ (compared with n in the case of no techno-logical change). The eventual rate of increase of real output is $n + ag/b$, which is than n[10].

In conclusion, Solow's Model is a valuable contribution to modeling economic growth. It does not explain every factor that influences growth. What it illuminates (i. e, accumulation of capital) is not the most important aspect of growth. Critics point out that treating human capital as exogenous renders us unable to analyze an important factor of production and the policies that may influence that factor. Another criticism is that the Solow Model lacks micro-foundations. Households make their savings and consumption decisions mechanically, s is not the result of a utility maximization problem. As a result, equilibrium may not be efficient.

3. SOLOW'S GROWTH ACCOUNTING

In 'Technical Change and the Aggregate Production Function' (1957), Solow attempted to quantify the effect of individual factors on the pace of growth. Thus, he carried out an empirical analysis of the long-term growth of the U.S. economy, and based his model on time series figures for total production, factors of production and the cost shares of these factors in total production. According to Solow, the key to economic growth in the period 1909-1949 was technological progress, rather than the production factors of labor and capital.

3.1. The Theoretical Model

Solow defines the following aggregate production function in closed economy:

$$Q = F\ (K, L; t). \qquad [11]$$

Q is the aggregate output, K and L the factors of production capital and labor, and the variable t for time appears in F to allow for technical change. Solow uses «the phrase "technical change" as a short-hand expression for *any kind of shift* in the production function. Thus slowdowns, speed-ups, improvements in the education of the labor force, and all sorts of things will appear as "technical change."» (Solow, 1957, p.352). Thus, Solow adopts a broad notion of technology.
In particular, Solow starts assuming *neutral technical change*. The technical change is *neutral* when shifts in the production function leave marginal rates of substitution untouched and simply increase or decrease the output attainable from given inputs.
Thus, the aggregate production function for the composite final output takes the special form

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[10] Solow (1956, p.85) also considers and explains the special case with $a > \frac{1}{2}$. Then the rate of increase of real output may even be faster than $n + g$.

$$Q = A\,(t)\,f\,(\text{K, L})' \qquad\qquad\qquad \text{[11a]}$$

where the multiplicative factor $A\,(t)$ measures the cumulated effect of shifts over time, and it represents the technological progress or, as it is usually named, the total factor productivity (TFP) growth. TFP refers to all inputs that affect the aggregate output (Q) except labor and capital[11].

An important assumption in this model is that A is exogenous. The model is thus making no effort to explain why A equals a particular value. Thus, the model is assuming that neither policy nor the choices of agents in the model affect its value.

Differentiate [11a] totally with respect to time and divide by Q, we obtain

$$\dot{Q}/Q = \dot{A}/A + A\,\frac{\partial f}{\partial K}\frac{\dot{K}}{Q} + A\,\frac{\partial f}{\partial L}\frac{\dot{L}}{Q}$$

since $w_k = \dfrac{\partial f}{\partial K}\dfrac{K}{Q}$ represents the relative share of capital, and $w_L = \dfrac{\partial f}{\partial L}\dfrac{K}{Q}$ is the relative share of labor,

then we can write the above equation as follows:

$$\dot{Q}/Q = \dot{A}/A + w_k\frac{\dot{K}}{K} + w_L\frac{\dot{L}}{L} \qquad\qquad\qquad \text{[12]}$$

In this model it is assumed that factors are paid their marginal products, w_k and w_L. Since all factor inputs are classified either as K or L, then w_k plus w_L add up to one. This is equivalent to assuming the hypotheses of Euler's theorem. Hence, F is homogeneous of degree one.

Then let $Q/L = q$, $K/L = k$, $w_L, = 1 - w_k$; note that $\dot{q}/q = \dot{Q}/Q - \dot{L}/L$ etc., and [12] becomes

$$\dot{Q}/Q = \dot{A}/A + w_k\frac{\dot{k}}{k} \qquad\qquad\qquad \text{[12a]}$$

If technological change is constant in time then $A(t) = e^{at}$.

The case of neutral shifts and constant returns to scale can be represented graphically as in Figure 2.

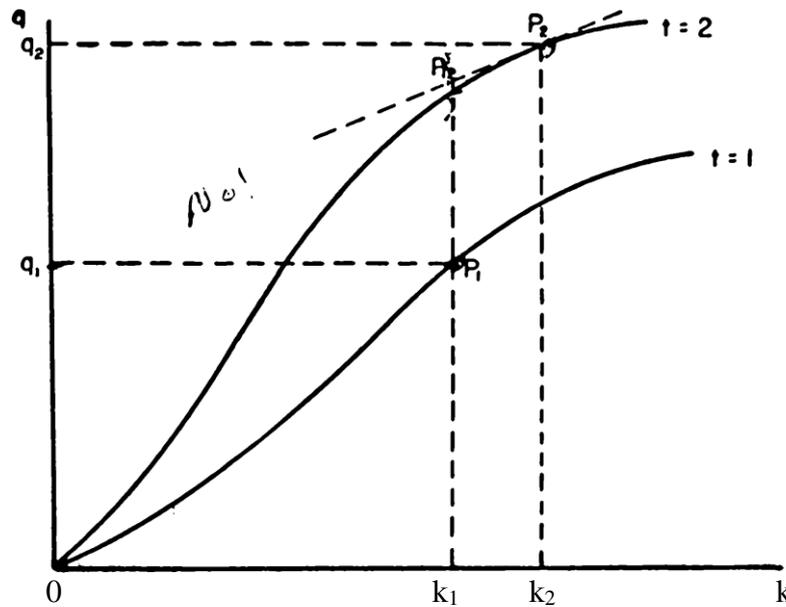[11] TFP and technology are sometimes used interchangeably.

Fig.2

In Figure 2 the production function is shifting in time, "so that if we observe points in the (q, k) plane, their movements are compounded out of movements along the curve and shifts of the curve" (Solow, 1957, p. 313). The problem then is distinguishing between the two kinds of changes.

3.2. The Empirical Results in Solow's Growth Model

In order to isolate (empirically) shifts of the aggregate production function from movements along it, Solow uses the following time series: output per unit of labor, capital per unit of labor, and the share of capital. Moreover, as a measure of aggregate output Solow adopts the GNP. Thus, the q is a time series of real private non-farm GNP per man hour[12].

After having carried out his empirical study, Solow comes to the following general conclusion: "over the 40 year period output per man hour approximately doubled. At the same time, according to Chart 2, the cumulative upward shift in the production function was about 80 per cent. It is possible to argue that about one-eighth of the total increase is traceable to increased capital per man hour, and the remaining seven-eighths to technical change" (1957, p.316).

Yet, Solow adds the clarification that much, perhaps nearly all, innovation (and therefore technical progress) must be embodied in new plant and equipment to be realized at all.

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[12] In [12] or [12a] it is possible to replace the time-derivatives by year-to-year changes and calculate the following expression: A q/q - w_k A k/k. The result is an estimate of Δ F/F or Δ A/A, depending on whether these relative shifts appear to be neutral or not.

In addition, by assuming constant returns to scale and representing the production function in the following simplified form:

$$q = A(t) f(k,1), \qquad\qquad [13]$$

Solow shows that from [13] by plotting q(t) /A (t) against k (t) he gets (mild) diminishing returns and that the Cobb-Douglas functional form performs better than other parametric forms of production function (Solow, 1957, pp.318-319).

In short, in this contribution Solow suggested a method to separate shifts of the aggregate production function from movements along it. The method rests on the assumption that factors are paid their marginal products, that is, on the assumption of competitive factor markets.

The conclusions highlighted by Solow about his empirical analysis to America data from 1909 to 1949 are the following: i) Technical change during that period was neutral on average (i.e., the distribution of GNP between wage earnings and capital yield was not affected by technical change). ii) The upward shift in the production function was at a rate of about 1 per cent per year for the first half of the period, and 2 per cent per year for the last half. iii) Gross output per man hour doubled over the interval, with 87.5 per cent of the increase attributable to technical change and the remaining 12.5 per cent to increased use of capital. iv) The aggregate production function, corrected for technical change, gives a distinct impression of diminishing returns, but the curvature is not violent (Solow, 1957, p.320).

Therefore, although the outcome of Solow's growth model seems steady state growth because of diminishing returns, however Solow aims to show that economic history suggests accelerating progress.
This seminal paper on "growth accounting" had a great impact on the future empirical literature on growth, many studies were undertaken in other countries. Later discussion was mainly on how to measure the contributions of production factors to total production.

3.3. Capital Formation and the Vintage Approach

In 'Investment and technical progress' (1960), Solow gave on the topic concerning how to measure the contributions of inputs to total production an interesting contribution. In this essay, he presents a new method of studying capital formation in economic growth, based on the vintage approach and the hypothesis of embodied technological progress[13].
In his previous contribution on growth theory, Solow worked with disembodied technological progress, which is not tied to the replacement of old equipment with new. Such technological progress has the character of an exogenous quantity, not related to the introduction of production factors into the production process.

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[13] This hypothesis means that technical progress is built into machines and other capital goods, and that it must be taken into account in making empirical measurements of the role played by capital.

Instead, embodied technological progress reckons on the fact that older production equipment is gradually replaced with new and improved equipment. By making the hypothesis of embodied technological progress, Solow disaggregated capital according to its age structure and therefore also according to its technical level. Thus, the vintage approach assumes that new investments are characterized by the most modern technologies, and that the resulting capital does not change in qualitative terms over its remaining life.

In such an understanding of technological progress, capital as a share of economic growth is substantially higher at the expense of disembodied technological progress. Solow's new analytical framework based on this vintage approach permitted empirical calculations to be made, and the empirical results gave the formation of capital a major role in explaining the increase in production per employee.

Conclusion

A major proposition that it is possible to draw from Solow's growth model is that accumulation alone cannot yield lasting progress. The most important source of wealth for a country (an industry, a firm) is technological progress, while investments take less importance. However, in Solow technological advance becomes the exogenous force driving growth. Actually, technological progress that is what really matter for growth has been little illuminated by Solow' growth articles model. The "Solow residual" represents just the rate of technological change that explains the difference between real income growth and growth explicable by growth in labor and capital. In his 1957 article Solow tried to measure technological change, due to its relevance on growth, but without explaining it. However, the contributions of Solow to growth theory have broadened the technological framework of growth theory. Since then, as a consequence, economists and policy makers have given priority to technical progress and how to go about accelerating it.

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