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Abstract

We propose a model for optimizing structured portfolios with liquidity-adjusted Value-at-Risk (*LVaR*) constraints, whereby linear correlations between assets are replaced by the multivariate nonlinear dependence structure based on Dynamic Conditional Correlation *t*-copula modeling. Our portfolio optimization algorithm minimizes the *LVaR* function under adverse market circumstances and multiple operational and financial constraints. When we consider a diversified portfolio of international stock and commodity market indices under multiple realistic portfolio optimization scenarios, the obtained results consistently show the superiority of our approach relative to other competing portfolio strategies including the minimum-variance, risk-parity and equally weighted portfolio allocations.

Keywords: Dynamic copulas, LVaR, dependence structure, portfolio optimization algorithm.

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1. Introduction

Trading risks in financial markets are usually associated with potential losses arising not only from security price changes and interdependence among different asset classes (e.g., equities, currencies, interest rates, and commodities), but also from their negative tail co-movements in bearish market conditions. Over the last three decades, the measurement and forecasting of financial risk have greatly evolved from modest indicators of market risk and linear correlation to multifaceted measures of risk and interdependence based on more sophisticated time-dependent and market context-based modeling techniques. The latter include, among others, scenario-analysis, contemporary stress-testing procedures, Value-at-Risk (*VaR*), and dynamic conditional correlation (*DCC*) copulas for dependence estimation.

VaR techniques have recently become important and useful tools for monitoring and forecasting market and liquidity risk, following the recommendations of the Bank for International Settlements (BIS) and the Basel Committee on capital adequacy and banking regulations.[1] The main advantage of the *VaR* models for risk management decision-making is their focus on downside-risk (i.e. the impact of bad outcomes) and their straightforward interpretation in monetary terms. Despite their simple implementation, traditional *VaR* models do not adequately take nonlinear dependence between assets within a portfolio into account and become inefficient under illiquid market scenarios, particularly in times of financial turbulence.

Since the 2008-2009 global financial crisis, Liquidity-adjusted Value-at-Risk (*LVaR*) techniques recognized the grown prominence of asset liquidity risk assessment as an essential element of risk management processes (Ruozi and Ferrari, 2013). Market downturns and financial crises particularly require an adequate modeling of liquidity risk taking into account multivariate dependence patterns in financial assets as well as the evaluation of their impact on the performance and optimal design of structured trading portfolios, subject to financially meaningful operational constraints under adverse and stress market circumstances.[2]

The assessment and forecasting of liquidity risk typically depend on many interlinked factors, such as the dependence between asset prices and their time-variations, sector-specific market frictions, financial and market information availability from and across market sectors, stock market confidence, financial trading regulations in stress markets, sudden market shocks resulting in market downturns and contractions in capital inflow and outflow, and cap-

[1] See, Bank for International Settlements (2009, 2013).

[2] The concept of liquidity risk in financial markets and institutions could refer to either, the added transaction costs related to trading large quantities of a certain financial security or, the ability to trade this financial security without triggering significant changes in its market prices (see, Roch and Soner, 2013 for further details).

ital reserve levels of financial and trading institutions. In spite of several works on liquidity risk (Berkowitz, 2000; Bangia et al., 2002; Angelidis and Benos, 2006; Al Janabi, 2013, 2014; Weiß and Supper, 2013), accurate estimations of market liquidity risk and its application to the problem of portfolio optimization remain as challenging tasks for financial entities.

This paper investigates the above-mentioned issue by developing and implementing robust modeling techniques to assess liquidity risk under illiquid market scenarios, while taking into account multivariate asset dependence. We also attempt to examine the impacts of changes in estimated liquidity risk on the optimal portfolio allocation. For these purposes, our modeling approach combines *LVaR* algorithms for liquidity risk measurement, Dynamic Conditional Correlation (*DCC*) *t*-copula models for dependence structure estimation and nonlinear optimization algorithms. Note that copulas, particularly *DCC* copulas, are nowadays considered to be appropriate tools for modeling the conditional dependence structure of financial assets since they offer the possibility to accurately account for nonlinear co-movements and changing patterns of dependence across various market conditions (e.g., Rodriguez, 2007; Ye et al., 2012; Laih, 2014), which are neglected by linear correlation-based models. We empirically show the usefulness of our approach by modeling a diversified portfolio consisting of several international stock market indices and two global commodity market indices (namely, gold and oil). Our ultimate goal is thus to scrutinize whether the realistic copula-LVaR-based optimization algorithms are capable of producing improved optimal multi-asset allocation under adverse market scenarios, while taking into account operational and financial boundary constraints, largely evidenced by illiquidity shocks during the 2008-2009 global financial crisis.

Our modeling framework belongs to the portfolio optimization and risk measurement literature pioneered by the seminal works of Markowitz (1952) and (Morgan, 1996), and is broadly linked to the studies by Rockafellar and Uryasev (2002), Garcia et al. (2007), and Ho et al. (2008), where portfolio optimization with respect to variance, *VaR* and *CVaR* is conducted under normal distributions. It also connects to the studies by Campbell et al. (2001), and Alexander and Baptista (2004, 2008) who consider downside risk and *VaR* contraints, instead of the mean-variance framework.[3] Our study is more closely related to the literature that focuses on liquidity *VaR* and portfolio optimization. On this specific line of research, Jarrow and Subramanian (1997) consider the optimal liquidation of portfolios and provide a market

[3] Campbell et al. (2001), and Alexander and Baptista (2004) improve the optimal portfolio selection by maximizing expected return subject to a downside-risk constraint and to a VaR constraint, respectively. Alexander and Baptista (2004) also analyze the effect of the VaR constraint on portfolio selection.

liquidity impact model. Bangia et al. (2002) propose an exogenous liquidity *VaR* adjusted model that better accounts for tradable assets' risk exposure. More recently, Al Janabi (2013, 2014) tackles the issue of adverse market price impacts on liquidity risk and coherent portfolio optimization using a parametric liquidity-adjusted *VaR* methodology.[4] On the subject of dependence estimation using copulas, our paper is related to the recent studies by Low et al. (2013) and Weiß and Supper (2013). The former forecasts portfolio returns with both symmetric and asymmetric copula models, subject to no short-sales constraints and the minimization of *CVaR*. The latter uses vine copula models to examine the issue of liquidity-adjusted intraday *VaR* forecasting for a portfolio of five NASDAQ listed stocks, and finds evidence of their suitability in predicting intraday liquidity-adjusted portfolio performance. Other studies have also combined copula models with portfolio optimization (see, Bekiros et al., 2015; and references therein).

Overall, this paper contributes to the liquidity risk and portfolio optimization literature on several fronts. First, it develops and implements a portfolio optimization modeling framework that combines *LVaR* and *DCC t*-copula algorithms for liquidity risk assessment and multivariate dependence structure estimation in order to improve the asset allocation under illiquid market scenarios. This specific type of modeling is new in the literature and permits portfolio managers to designate the required liquidity horizons (close-out periods) and to determine robust asset allocation according to realistic market conditions. Second, our proposed approach, which consists of replacing the variance risk measure by the *LVaR* algorithms and the linear correlation between assets by their multivariate nonlinear dependence structure based on the *DCC t*-copula, is a thorough enhancement of the traditional Markowitz (1952) mean-variance portfolio optimization given the relevance of these factors in asset pricing and allocation (Liu, 2006; Heinen and Valdesogo, 2008; Cornett et al., 2011). Third, our study is among few studies (Angelidis and Benos, 2006; Al Janabi, 2013, 2014; Weiß and Supper, 2013) that examine liquidity risk, based on the use of daily data of stock market indices from developed and emerging markets, along with two major commodities: gold and oil. This research design is advantageous in that country indices better capture the effects of liquidity on asset prices and market drivers, due to aggregation. Lastly, our copula-LVaR-based portfolio optimization considers crisis market situations whereby illiquidity is a critical factor.

[4] The issues of liquidity risk management, asset pricing, and portfolio selection have also been addressed in, Berkowitz (2000), Madhavan et al. (1997), Hisata and Yamai (2000), Le Saout (2002), Takahashi and Alexander (2002), Amihud et al. (2005), Cochrane (2005), Angelidis and Benos (2006), and Meucci (2009).

Under multiple realistic scenarios, our empirical results consistently show the superiority of our copula-LVaR-based optimization approach relative to the minimum variance Markowitz optimal portfolio. The observed risk allocation superiority of our algorithm on a 10-day holding period, relative to the standard *VaR* approach that employs linear correlations, stems from the flexibility of the *DDC t*-copula in capturing more accurately the negatively-skewed behavior of the marginal distributions and the negative tail asymmetric dependence of the stock and commodity asset returns. The *LVaR* algorithm implemented maintains its risk estimation edge over the standard *VaR*. The obtained optimal *LVaR* frontier under adverse conditions supports the findings for realistic assumptions and constraints for realistic and structured portfolios. An out-of-sample analysis also confirms the superior performance of our approach over other frameworks considering the mean-VaR efficient, risk-parity and equally weighted portfolios.

The remainder of the paper is structured as follows. Section 2 introduces the integrated framework for *LVaR* measurement and portfolio optimization algorithm with respect to *LVaR* using pair vine copulas. Section 3 shows how our framework can be applied to a portfolio of international stock market indices and global commodity markets. Section 4 concludes the paper.

2. Models

2.1 Parametric LVaR model under adverse market perspectives

The calculation of the parametric *VaR* entails the extraction of the volatility from each risk factor (financial asset) based on a pre-defined historical observation period. Moreover, this type of *VaR* estimate can also be obtained by fitting a GARCH-class model while considering adverse market condition assumptions. The potential risk effect of each asset in the trading portfolio can then be determined and aggregated by taking into consideration the correlation parameters among different risk factors, to provide the overall portfolio *VaR* for a given confidence level. Accordingly, the absolute *VaR* in monetary terms for a single trading position can be defined as follows:

$$\mathrm{VaR}_i = \left| (\mu_i - \alpha * \sigma_i)(\mathrm{Asset}_i * \mathrm{Fx}_i) \right| \tag{1}$$

where μ_i denotes the expected average return of asset i, α is the confidence level of risk assessment and σ_i is the conditional volatility of the return. The term Asset_i indicates the current mark-to-market monetary amount of asset i, while Fx_i represents the foreign exchange unit applicable to asset i.[5] For the particular case when the expected average return of the asset μ_i is small or close to zero, Eq. (1) can be reduced to[6]:

$$VaR_i = |\, \alpha * \sigma_i * \mathrm{Asset}_i * \mathrm{Fx}_i \,| \qquad (2)$$

For multi-asset portfolios, the *VaR* can be expressed as in Eq. (3):

$$VaR_P = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} VaR_i \, VaR_j \rho_{i,j}} \;=\; \sqrt{VaR^T [\rho][VaR]} \qquad (3)$$

Eq. (3) is a general formula for *VaR* estimation regardless of the size of the portfolio and $[\rho_{i,j}]$ denotes the correlation parameters among different assets. The matrix form of the second term in Eq. (3) simplifies the programming process and the inclusion of short selling transactions in the risk evaluation process (Al Janabi, 2012, 2013).

While liquidity risk is an important factor in portfolio management, risk models have not yet dealt with it adequately. Illiquid trading positions considerably increase the risk of loss, while sending negative signals to traders who realize the need for a higher expected return under those stress market conditions. As such, the notion of asset liquidity during the unwinding period is notably important to accurately estimate *VaR*; and recent financial market upheavals have confirmed these observations.

If returns are independent and multivariate elliptical distributed, then the liquidity adjusted *VaR* (*LVaR*) for any liquidation horizon (*t*) can be estimated as follows:

$$LVaR(t - day) = VaR(1 - day)\sqrt{t} \qquad (4)$$

Eq. (4) has been recommended by *J.P. Morgan* in their prior *RiskMetrics*™ technique (1994). However, this approach does not reflect real-world trading circumstances since it implies, indirectly, that the unwinding of assets happens when the close-out period ends.

In what follows, we discuss a *LVaR* algorithm that can be implemented for the assessment of investable portfolios. This practical framework incorporates and evaluates *LVaR* for illiquid assets under multiple horizons and can be applied to multi-asset portfolios.[7]

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5 In dealing with internationally diversified and structured portfolios that consist of stock indices from different countries, the effects of foreign exchange rates can be excluded when all indices are expressed in the same currency, which is US dollar in our case.

6 Eq. (2) is built on a simplifying assumption frequently applied in financial markets to estimate the *VaR* of a particular asset (Al Janabi, 2014).

7 The *LVaR* mathematical approach presented herein is partially drawn from Al Janabi (2012, 2013).

As in Al Janabi (2012, 2013), let t denote the unwinding period (i.e., the liquidation horizon or close-out period), whereas σ_{adj}^2 and σ_{adj} indicate respectively the estimated variance and standard deviation of any particular asset within the trading portfolio. As a result, if one assumes that the trading assets can be liquidated linearly across t-days unwinding period, then the estimated variance of any particular asset within the trading portfolio σ_i^2, can be stated as the sum of the variances, for $i = 1,2 \ldots t$ days. The following equation expresses this relationship:

$$\sigma_{adj}^2 = (\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \cdots + \sigma_{t-2}^2 + \sigma_{t-1}^2 + \sigma_t^2) \tag{5}$$

The square root-t approach of Eq. (4) is a special case of Eq. (5) since for this special situation the following equality holds: $\sigma_{adj}^2 = (\sigma_1^2 + \sigma_1^2 + \sigma_1^2 + \cdots + \sigma_1^2) = t\sigma_1^2$.

The main assumption for daily linear liquidation of assets is that the estimated variance of the first trading day for any specific asset falls linearly as a function of t and, as such, it enables one to derive the following analytical expression:

$$\sigma_{adj}^2 = \left\{(\tfrac{t}{t})^2\sigma_1^2 + (\tfrac{t-1}{t})^2\sigma_1^2 + (\tfrac{t-2}{t})^2\sigma_1^2 + (\tfrac{t-3}{t})^2\sigma_1^2 + \cdots + (\tfrac{3}{t})^2\sigma_1^2 + (\tfrac{2}{t})^2\sigma_1^2 + (\tfrac{1}{t})^2\sigma_1^2\right\} \tag{6}$$

For the case of t-days unwinding horizon, the estimated variance of any specific asset, which is shaped and influenced by the time or number of days factor to close-out the trading position, is given by Eq. (7):

$$\sigma_{adj}^2 = \sigma_1^2\left\{(\tfrac{t}{t})^2 + (\tfrac{t-1}{t})^2 + (\tfrac{t-2}{t})^2 + (\tfrac{t-3}{t})^2 + \cdots + (\tfrac{3}{t})^2 + (\tfrac{2}{t})^2 + (\tfrac{1}{t})^2\right\} \tag{7}$$

Using finite series mathematical shortcuts, we obtain the following relationship:

$$(t)^2 + (t-1)^2 + (t-2)^2 + (t-3)^2 + \cdots + (3)^2 + (2)^2 + (1)^2 = \frac{t(t+1)(2t+1)}{6} \tag{8}$$

Then, by substituting Eq. (8) into Eq. (7), we obtain the following equality:

$$\sigma_{adj}^2 = \sigma_1^2\left[\frac{1}{t^2}\{(t)^2 + (t-1)^2 + (t-2)^2 + (t-3)^2 + \cdots + (3)^2 + (2)^2 + (1)^2\}\right]$$

Or $\sigma_{adj}^2 = \sigma_1^2\left(\frac{(2t+1)(t+1)}{6t}\right)$ (9)

From Eq. (9), the liquidity risk parameter can be formulated in terms of volatility as:

$$\sigma_{adj} = \sigma_1\left\{\sqrt{\frac{1}{t^2}[(t)^2 + (t-1)^2 + (t-2)^2 + (t-3)^2 + \cdots + (3)^2 + (2)^2 + (1)^2]}\right\}$$

Or $\sigma_{adj} = \sigma_1 \left\{ \sqrt{\frac{(2t+1)(t+1)}{6t}} \right\}$ (10)

A distinctive feature of Eq. (10) is that it is a function of time t and not the square root-t method that we have discussed earlier. Using Eq. (10), the *LVaR* for any time horizon and under illiquid market conditions can be estimated as follows[8]:

$$LVaR_{adj} = VaR\sqrt{\frac{(2t+1)(t+1)}{6t}} \quad (11)$$

Eq. (11) indicates that $LVaR_{adj} > VaR$ and when the number of days to unwind assets is equal one, the following equality $LVaR_{adj} = VaR$ holds. Furthermore, an equation for estimating the liquidation horizon can be defined as follows:

$$t = Total\ Market\ Value\ of\ Asset_i\ /\ Average\ Daily\ Volume\ of\ Asset_i \quad (12)$$

With the objective of assessing *LVaR* for the entire trading portfolio under illiquid and adverse market conditions (i.e. $LVaR_{P_{adj}}$), we can implement, in line with Al Janabi (2012, 2013), the following model, which is an extension of Eq. (3):

$$LVaR_{P_{adj}} = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} LVaR_{i_{adj}}\, LVaR_{j_{adj}}\rho_{i,j}} = \sqrt{[LVaR_{adj}]^T[\rho][LVaR_{adj}]} \quad (13)$$

Once having stated the model to estimate the *LVaR* for a trading portfolio under illiquid market conditions, we present the portfolio optimization model that integrates *LVaR* and asset dependence structure. This model minimizes *LVaR* subject to multiple meaningful operational and financial constraints under adverse and realistic market circumstances, which effectively improves the traditional Markowitz (1952) mean-variance method where the variance risk is used. The model also allows us to maximize the portfolio's expected return while controlling for large risk exposures. The analytical portfolio optimization model is as follows:

$$Min{:}\ LVaR_{P_{adj}} = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} LVaR_{i_{adj}}\, LVaR_{j_{adj}}\rho_{i,j}} = \sqrt{[LVaR_{adj}]^T[\rho][LVaR_{adj}]} \quad (14)$$

The objective function in Eq. (14) can be minimized subject to several meaningful portfolio management constraints. For our purpose, the minimization process is modelled by defining the following operational and financial boundary limits:

$$\sum_{i=1}^{n} R_i x_i = R_P\ ;\ l_i \le x_i \le u_i \quad i = 1,2,\dots,n \quad (15)$$

[8] Eq. (11) can be applied to estimate the *LVaR* for any particular time horizon if the total *LVaR* does not exceed the total trading volume of the portfolio.

$\sum_{i=1}^{n} x_i = 1.0$; $l_i \leq x_i \leq u_i \quad i = 1,2, \dots , n$ (16)

$\sum_{i=1}^{n} V_i = V_P \quad i = 1,2, \dots , n$ (17)

$[LHF] \geq 1.0$;$\forall_i$ i=1,2,…,n (18)

where each element of the vector in Eq. (18) can be expressed as:

$$LHF_i = \left[\sqrt{\frac{(2t_i+1)(t_i+1)}{6t_i}} \right] \geq 1.0 \text{ ;} \forall_i \quad \text{i=1,2,…,n} \tag{19}$$

In Eqs. (15)-(19) above, R_P and V_P indicate respectively the expected average return and total trading volume of the portfolio, while x_i is the allocation (weight) for every trading asset. The values l_i and μ_i, for $i = 1, 2, \dots, n$ in Eqs. (15)-(16) represent the lower and upper limits of the portfolio asset allocation. If we select $l_i = 0$, $i = 1, 2, \dots, n$, then we end up with the case where no short selling operations are permitted. Finally, $[LHF]$ denotes an $(n \times 1)$ vector of the particular unwinding periods (i.e. the liquidity close-out horizons) of each asset for all $i = 1, 2, \dots, n$.

2.2 LVaR algorithm based on time-varying t-copula

2.2.1 Marginal model

Eq. (2) shows that the conditional volatility is a crucial input factor for adequate Value-at-Risk estimates. Following previous studies (e.g., Grégoire et al., 2008; Aloui et al., 2011), we also employ a GARCH (1,1) approach to model the dynamics of financial returns and in order to capture some of their stylized characteristics such as volatility clustering and time-varying heteroscedastic volatility (Engle, 1982; Bollerslev, 1986). The GARCH model estimates will then be used to specify the marginal univariate distributions which are required for the estimation of the time-varying t-copula parameters.

As in Bollerslev (1986), let r_t denote the daily log return of the asset under consideration at time *t*. The conditional mean equation takes the following form:

$$r_t = \mu_t + \varepsilon_t \sigma_t \tag{20}$$

where ε_t is an independent and identically distributed (*iid)* (0,1) random variable and σ_t is the time-dependent standard deviation. Since we deal with daily log returns, we set $\mu_t = 0$ and model the conditional volatility by specifying a GARCH(1,1) process as follows:

$$\sigma_t^2 = \omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{21}$$

where ω as a constant and the scalars $\alpha + \beta \leq 1$. The standardized returns are then achieved by

$$\frac{r_t}{\sigma_t} = \varepsilon_t \tag{22}$$

Given the fat-tailed behavior of the return distributions, the marginal model is estimated by assuming that the standardized returns follow a Student-*t* distribution.

2.2.2 DCC t-copula

In order to adequately capture the potential of nonlinear dependence between the assets under consideration, we apply a DCC t-copula approach to GARCH filtered returns. The copula approach, in its general form, derives from the theorem of Sklar (1959). The time-varying DCC t-copula we fit is suitable because it leads to positive-semidefinite correlation matrices (Walker, 2003) that enable the estimation of the assets' dependence structure.

The setup of the *DCC t*-copula is as follows:

$$C_{DCC}^{t}(u_1, \dots, u_n) = t_{\rho_t,v}(t_v^{-1}(u_1) \dots, t_v^{-1}(u_n)) \tag{23}$$

where $t_{\rho_t,v}$ is the multivariate *t*-distribution with correlation ρ and v degrees of freedom. The parameter t_v^{-1} represents the inverse of the univariate *t*-distribution, and *u* represents the returns transformed by their individual cumulative distribution function (cdf). As part of the multivariate *t*-distribution, the degrees of freedom v capture joint extreme observations and as $v \longrightarrow \infty$ the *t*-copula approximates the Gaussian copula. Moreover, given the GARCH filtered returns, the dynamic conditional correlation (*DCC*) process is modelled along the lines of Engle (2002):

$$\rho_t = diag\left\{Q_t^{-1/2}\right\} Q_t \, diag\left\{Q_t^{-1/2}\right\} \tag{24}$$

where $Q_t = \Omega + \delta\varepsilon_{t-1}\varepsilon_{t-1}' + \gamma Q_{t-1}$

Analogous to a GARCH process, the dependence paths are described by the persistence parameter γ and by news impact parameter δ, whereas ε_{t-1} describes the one-period lagged value of the GARCH filtered returns. Following Patton (2006), we estimate these parameters via maximum likelihood. Also, since we attempt to optimize large portfolios, we adopt a two-step maximum likelihood method for the estimation of all parameters as indicated in Joe (1996). In the first step, all parameters related to *n* individual univariate margins, based on *t*-periods, are estimated by:

$$\theta_1 = ArgMax_{\theta_1} \sum_{t=1}^{T} \sum_{j=1}^{n} lnf_j\ (r_{jt;\theta_1}) \tag{25}$$

Then, based on θ_1 the copula parameters can be estimated in the second step as follows:

$$\theta_2 = ArgMax_{\theta_2(\widehat{\theta_1})} \sum_{t=1}^{T} \text{lnc}(F_1(r_{1t}), F_2(r_{2t}), \dots, F_n(r_{nt})) \tag{26}$$

This estimation method is referred to as the Inference for the Margins (IFM), $\theta_{IFM} = (\theta_1, \theta_2)'$. Overall, the time-varying DCC t-copula improves traditional portfolio optimization models by accounting for nonlinearities in the dependence between portfolio's assets and dynamic changes of the dependence structure. Additionally, we are able to achieve VaR figures, based on the calibrated DCC t-copula by simulating *N* observations characterized by the DCC t-copula dependence structure (Palaro and Hotta, 2006). Following Berger (2013), we simulate 10,000 observations for each day and the $LVaR$ is determined by the empirical quantile.

3. Data and Empirical Application

3.1 Data and Stochastic Properties

The dataset we select to implement our modeling framework consists of log return series for 12 national equity market indices (MSCI equity indices) and two commodity price series spanning from January 1, 2004 to January 31, 2014. The equity market indices considered include a group of developed markets (USA, Japan, UK, Italy, France, Germany and Canada) and a group of emerging markets (Brazil, Russia, India, China and South Africa), while the commodity indices are the Brent crude oil and the gold bullion. All these indices, called assets in our research, are denominated in US dollars.

The selection of developed and emerging market indices enables us to establish a comparison in terms of liquidity risk, as well as to identify their market and liquidity risk profile. The selected sample data also allow for a comparison of $LVaR$ estimates in diverse market conditions. The oil and gold commodity indices are included in the copula-LVaR-based portfolio optimization because they have historically been observed to influence market liquidity during crisis and non-crisis periods, and display patterns of time-varying interdependence. At the empirical level, daily log returns of the assets are employed to conduct the $LVaR$ portfolio optimization and the time-varying copula is implemented to estimate the asset return dependence structure. The data have been downloaded from DataStream International.

Table 1: Stochastic properties of return series

	Mean	Std Dev	Max	Min	Skew	Kurt	JB	Qstat	LM
USA	0.00	0.01	0.11	-0.10	-0.35	14.58	29.50	85085.45	2609.57
Japan	0.00	0.01	0.11	-0.10	-0.21	8.37	214.91	86723.58	2602.83
UK	0.00	0.01	0.12	-0.10	-0.12	12.75	30.16	87592.43	2607.01
Italy	0.00	0.02	0.12	-0.11	-0.04	8.84	201.03	91666.50	2615.91
France	0.00	0.02	0.12	-0.12	0.00	9.83	233.50	86508.36	2606.22
Germany	0.00	0.02	0.12	-0.10	-0.06	9.07	133.11	84822.96	2608.40
Canada	0.00	0.02	0.10	-0.14	-0.80	13.20	182.24	86040.37	2610.04
Brazil	0.00	0.02	0.17	-0.18	-0.36	11.39	111.38	87857.38	2612.39
Russia	0.00	0.03	0.24	-0.26	-0.55	18.41	524.51	88223.04	2613.11
India	0.00	0.02	0.19	-0.12	-0.01	11.18	68.44	87208.51	2612.85
China	0.00	0.02	0.14	-0.13	-0.02	9.71	52.93	87703.43	2612.05
South Africa	0.00	0.02	0.12	-0.14	-0.32	7.55	214.13	85856.76	2607.64
Crude oil	0.00	0.02	0.14	-0.11	0.07	6.44	151.73	86712.22	2613.40
Gold	0.00	0.01	0.07	-0.10	-0.56	7.92	203.66	92687.96	2618.56

Notes: the table presents the stochastic properties of the log return series we consider over the period from January 1, 2004 to January 30, 2014. The Jarque-Bera test indicates the absence of normality in the return distribution. The fitted Q-Stat and LM (20) statistic reveal the presence of serial correlation and heteroscedasticity for squared returns.

Table 1 presents the stochastic properties of the return series under consideration. The Jarque-Bera test shows departure from normality for all return series distributions. The fitted LM (20) statistics indicate evidence of heteroscedasticity in the squared returns. Emerging markets display larger minimum and maximum returns as compared to developed markets.

3.2 Estimation results

Table 2 presents the GARCH parameters and degrees of freedom for the fitted t-distribution. While all assets display fat tails, the gold commodity index has the largest number of degrees of freedom. Moreover, all GARCH parameters are significant and all return series are characterized by a strong degree of persistence (β varies around 0.9 for all assets). Russia and India display the largest α values, indicating that both markets' conditional volatility reacts more sharply to market shocks.

Table 2: Estimation results of the GARCH (1,1) model with t-distribution

	ω	α	β	dof	LL
USA	0.00	0.09	0.90	5.80	8594.55
	(3.47)	(7.50)	(77.8)	(7.84)	
Japan	0.00	0.09	0.88	11.36	7788.41
	(3.63)	(6.22)	(43.3)	(5.34)	
UK	0.00	0.08	0.91	8.79	8038.47
	(2.99)	(7.20)	(77.2)	(6.20)	
Italy	0.00	0.08	0.92	9.20	7476.11
	(2.51)	(7.77)	(95.6)	(5.97)	
France	0.00	0.07	0.92	8.11	7582.05
	(3.09)	(7.42)	(91.5)	(6.54)	
Germany	0.00	0.08	0.92	8.11	7601.34
	(3.12)	(7.63)	(92.2)	(6.36)	
Canada	0.00	0.08	0.92	11.02	7916.29
	(2.75)	(7.40)	(85.5)	(5.12)	
Brazil	0.00	0.08	0.91	7.84	6708.30
	(3.22)	(7.02)	(67.5)	(6.93)	
Russia	0.00	0.10	0.89	5.18	6682.87
	(3.32)	(7.13)	(67.6)	(9.70)	
India	0.00	0.11	0.87	6.46	7215.60
	(4.22)	(7.98)	(58.3)	(8.05)	
China	0.00	0.08	0.92	6.61	7302.56
	(2.94)	(7.14)	(81.9)	(7.39)	
South Africa	0.00	0.09	0.89	10.27	6993.19
	(3.52)	(6.93)	(54.7)	(5.47)	
Crude oil	0.00	0.05	0.95	8.46	6838.46
	(1.43)	(7.00)	(130.7)	(6.46)	
Gold	0.00	0.04	0.95	4.39	8112.79
	(2.92)	(6.59)	(140.8)	(10.76)	

Notes: the table presents GARCH (1,1) parameters and the respective *t*-values for the return series of 14 indices from January 1, 2004-January 30, 2014. The abbreviations *dof* and *LL* stand for degrees of freedom and Log Likelihood. The numbers between parentheses represent the *t*-values.

Table 3: DCC t-copula coefficients of portfolio's asset returns

	USA	Japan	UK	Italy	France	Germany	Canada	Brazil	Russia	India	China	S. Africa	Crude oil	Gold
USA	1.00	0.04	0.52	0.50	0.54	0.54	0.65	0.61	0.36	0.21	0.16	0.36	0.17	0.06
Japan	0.04	1.00	0.22	0.19	0.22	0.22	0.18	0.15	0.20	0.25	0.44	0.27	0.12	0.19
UK	0.52	0.22	1.00	0.82	0.88	0.86	0.63	0.59	0.57	0.40	0.38	0.67	0.35	0.29
Italy	0.50	0.19	0.82	1.00	0.91	0.89	0.59	0.56	0.53	0.39	0.32	0.63	0.30	0.26
France	0.54	0.22	0.88	0.91	1.00	0.95	0.63	0.60	0.58	0.41	0.36	0.68	0.32	0.26
Germany	0.54	0.22	0.86	0.89	0.95	1.00	0.61	0.59	0.57	0.41	0.37	0.66	0.30	0.26
Canada	0.65	0.18	0.63	0.59	0.63	0.61	1.00	0.66	0.51	0.32	0.31	0.56	0.40	0.34
Brazil	0.61	0.15	0.59	0.56	0.60	0.59	0.66	1.00	0.52	0.35	0.34	0.56	0.30	0.23
Russia	0.36	0.20	0.57	0.53	0.58	0.57	0.51	0.52	1.00	0.39	0.39	0.59	0.40	0.25
India	0.21	0.25	0.40	0.39	0.41	0.41	0.32	0.35	0.39	1.00	0.50	0.43	0.19	0.17
China	0.16	0.44	0.38	0.32	0.36	0.37	0.31	0.34	0.39	0.50	1.00	0.44	0.18	0.16
S. Africa	0.36	0.27	0.67	0.63	0.68	0.66	0.56	0.56	0.59	0.43	0.44	1.00	0.33	0.38
Crude oil	0.17	0.12	0.35	0.30	0.32	0.30	0.40	0.30	0.40	0.19	0.18	0.33	1.00	0.31
Gold	0.06	0.19	0.29	0.26	0.26	0.26	0.34	0.23	0.25	0.17	0.16	0.38	0.31	1.00

Notes: the table displays the DCC *t*-copula coefficients between the country and commodity indices. USA, Japan, UK, Italy, France, Germany and Canada represent the group of developed markets. Brazil, Russia, India, China and South Africa are the group of emerging markets.

Table 3 displays the DCC *t*-copula coefficients for each pair of assets in our representative portfolio. The strongest dependence occurs between the European Union countries (e.g., UK, Italy, France and Germany). The oil index return displays strong dependence with equity index return of Russia, Canada and the UK, while the gold index return exhibits grater dependence on crude oil index return and equity index return of South Africa and Canada. The strongest return dependence among developed markets occurs between Germany and France.

3.3 Analysis of Optimal Portfolios and Efficient Frontiers

Analogous to the graphical analysis of Dang and Forsyth (2016), we assess optimal portfolio allocations subject to realistic budget constraints under multiple illiquid market scenarios, in order to show the flexibility and adequacy of the proposed copula-LVaR portfolio optimization approach. We also use the regulatory parameterisation of daily *VaR* estimates as the benchmark in our analysis. The relevant *VaR* parameterisation is defined as a 99% confidence interval and with 10 days (t=10) holding period.[9] In what follows, we discuss four different portfolio optimization scenarios to stress on the flexibility of copula-LVaR approach and its superiority over the classical mean-variance approach. We particularly focus on the outcomes resulting from various restrictions placed on the portfolio optimization algorithm, and the impact of each restriction on the efficient frontier that determines the optimal portfolio allocations with respect to risk and return.

The first scenario assesses portfolio allocations in the absence of short selling. As such it presents a widely accepted optimization setup (e.g., Kwan and Yuan, 1993; Arreola-Hernandez et al., 2015) that leads us to emphasize the strength of the copula-LVaR-based optimization, relative to the classical Markowitz portfolio optimization approach. The second scenario deals with budget restrictions that are of practical relevance (e.g., Al Janabi, 2013; Ji and Lejeune, 2015;). In this regard, we introduce budget restrictions for particular asset classes in order to illustrate the performance of the copula-LVaR approach. The third scenario shows the impact of short selling on the portfolio allocation. We specifically allow for short selling to ensure realistic hedging scenarios as in Jacobs et al. (2005) and Al Janabi (2012, 2014). The fourth scenario shows the flexibility of the introduced approach and deals with individual liquidation periods for each asset class. In this scenario, we apply different liquidation periods to each asset class and account for liquid and non-liquid markets.

[9] Notice that our results do not depend on the choice of holding period and are valid for all liquidation scenarios.

Similar to Maillet et al. (2015), we compare the copula-LVaR-based efficient portfolio allocations for each scenario against the mean-VaR efficient frontier of the Markowitz approach. t-distributed returns and an adjusted liquidation period are assumed for both types of optimization in order to highlight the differences between approaches.[10] A multivariate CCC-GARCH model as in Engle (2009) is also used to show the robustness of the proposed copula-LVaR approach and to assess alternative trading strategies as a point of reference (Weiß, 2013; Berger and Missong, 2013). Finally, we provide sufficient out-of-sample analysis and assessment of the copula-LVaR-based portfolio allocation performance.

For the purpose of stressing on the flexibility of the time-varying DCC t-copula approach implemented, the degrees of freedom (*dof*) for each marginal return distribution are modelled individually. We assess the dependence structure of the underlying assets by focussing on the mean-VaR efficient portfolio allocations. The tails of each return series, indicated by different *dof*, are given in Table 2, whereas the joint tail dependence, measured and captured using the DCC t-copula, is given in Table 3. As the Markowitz mean-variance efficient portfolio allocations do not allow for an individual assessment of each return series, we assume 8 degrees of freedom across all assets. This choice is justified by the average of the degrees of freedom for the individual assets under consideration (see Table 2).

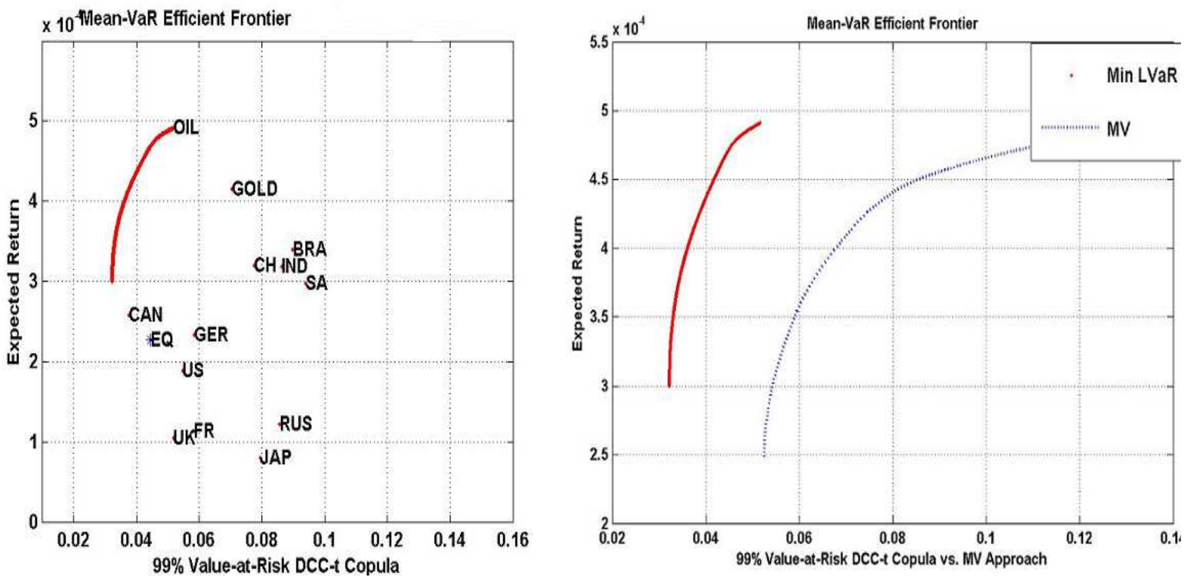

Figure 1: Mean-*VaR* efficient frontiers
On the left is displayed the efficient frontier of *LVaR* for the assessed portfolio against the individual assets and the equal weighted portfolio (EQ). On the right is displayed the efficient frontier produced by the proposed copula-LVaR-based optimization, and is compared with the Markowitz mean-variance approach.

[10] As the liquidation period $\sqrt{10}$ is by definition larger than the liquidation period based on t = 10, we omit this step since it simply describes a linear transformation of the results. The results for that step are available upon request.

Figure 1 displays the expected return and 99% confidence level *VaR* efficient frontiers for the mean-variance and mean-*LVaR* optimized portfolios. The plot on the left hand side shows that investments in the oil and gold commodity indices, as well as in the Canadian index are attractive in terms of risk and return, under the specific time horizon selected. It is therefore not surprising that those assets are visibly crucial in the shaping of the risk-return portfolio allocation efficient frontier. The plot on the right hand side displays both, the mean-VaR efficient frontier of the Markowitz portfolio algorithm and the mean-VaR efficient frontier produced by implemented time varying copula algorithm. The efficient frontier generated by the *LVaR* model indicates the presence of portfolio allocations having lower *VaR* for a given level of expected returns and confidence level. Overall, and as indicated by the plot on the right hand side, the flexible time varying DCC t-copula copula applied leads to portfolio allocations that categorically outperform the classical Markowitz approach. This result is doubtlessly due to the liquidity-adjusted *VaR* component of our approach, which is capable of capturing the market and liquidity risk of the assets, and the integration of the dependence structure of the assets into the parent optimization algorithm.

Table 4: Portfolio weights for each scenario

	Scenario 1		Scenario 2		Scenario 3		Scenario 4	
	LVaR	MV	LVaR	MV	LVaR	MV	LVaR	MV
USA	0.12	0.37	0.10	0.20	0.20	0.30	0.10	0.37
Japan	0.08	0.24	0.20	0.17	0.04	0.24	0.10	0.24
UK	0.00	0.00	0.00	0.12	0.05	0.20	0.00	0.00
Italy	0.02	0.00	0.00	0.00	0.27	0.05	0.05	0.00
France	0.00	0.00	0.00	0.00	-0.24	-0.22	0.00	0.00
Germany	0.00	0.00	0.00	0.00	-0.11	-0.04	0.00	0.00
Canada	0.43	0.00	0.20	0.00	0.30	-0.04	0.53	0.00
Brazil	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00
Russia	0.00	0.00	0.02	0.00	0.05	-0.03	0.00	0.00
India	0.00	0.03	0.09	0.15	0.10	0.15	0.00	0.03
China	0.01	0.00	0.18	0.12	0.17	0.09	0.00	0.00
South Africa	0.00	0.00	0.00	0.04	-0.03	0.09	0.00	0.00
Brent crude oi	0.23	0.03	0.12	0.00	0.12	-0.02	0.15	0.03
Gold Bullion	0.11	0.33	0.08	0.20	0.08	0.22	0.06	0.33

Notes: The table displays the portfolio weights resulting from the mean-VaR efficient portfolio allocation for each scenario. LVaR = DCC-copula Liquidity adjusted VaR, MV=Mean-Variance efficient approach.

Table 4 presents the weights corresponding to the global mean-VaR efficient portfolio allocation for each of the four scenarios considered. Interestingly, assets preferred by the DCC-copula algorithm (e.g., Canada and crude oil) are ignored by the classical Markowitz portfolio allocation approach. The US and Japanese assets (i.e., country equity market indices) are highlighted as important in the Markowitz portfolio composition (see Scenario 1). The incorporation of time varying tail dependence and the individual assessment of marginal return distributions is observed to heavily impact the portfolio allocations.

Next, we expand the setup of the first scenario and restrict the portfolio weights to realistic boundaries and budget constraints, and assess their impact on the relevant portfolio allocations. The budget constraints delimit the portfolio composition to 50% investment in developed markets (i.e., USA, Japan, UK, Italy, Germany, France, Canada), 30% in emerging markets (i.e., Brazil, Russia, India, China, South Africa) and 20% in commodities (i.e., crude oil, gold). Additionally, we restrict the maximum weight on every individual asset to a maximum of 20%. Figure 2 shows the efficient frontiers produced by the copula-LVaR-based portfolio optimization and the mean-variance Markowitz portfolio optimization. Under this optimization scenario the efficient frontier of the copula-LVaR-based approach also leads to portfolio allocations that have more efficient VaR-return ratios, thus categorically outperforming the Markowitz optimization. The achievement of lower *VaR* values for given levels of expected returns (as in Scenario 1) stems from the effect of the selected budget constraints. Although both optimization approaches are influenced by the realistic constraints, the traditional Markowitz portfolio risk-return ratio remains higher and any investment in that specific type of constrained portfolio is riskier. By contrast, an investment strategy based on the proposed copula-LVaR-based portfolio optimization should be seen as a promising alternative.

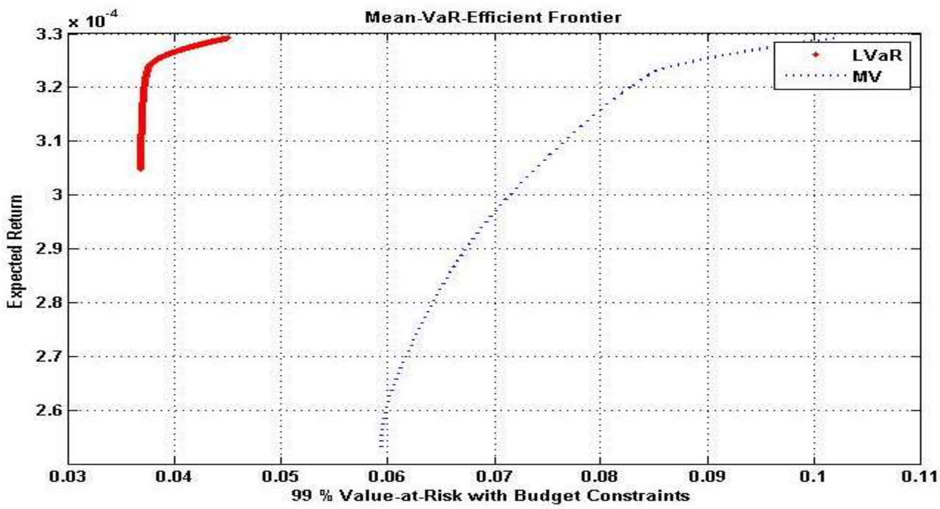

Figure 2: Mean-*VaR* efficient frontiers with budget constraints and weight allocation restrictions
This figure shows the expected portfolio return (vertical axis) and the *VaR* of the portfolios for a 99% confidence level (the horizontal axis).

As to the third scenario, we allow for short selling up to –20 % for each individual asset. Table 4 shows that both approaches lead to portfolio allocations that put negative weights in the French market index. French stocks could thus be used for hedging in market downturns and portfolio risk diversification. As opposed to the Markowitz model, most likely due to tail dependence, larger negative weights are given to German market index, another indication of the Markowitz's model tendency to underestimate risk and overestimate return.

Figure 3 displays the impact of short selling on the constrained portfolio problem. In Scenario 3, contrary to Scenario 1 and Scenario 2, higher expected returns are attained from the inclusion of short selling in the portfolio optimization process. The tail dependence and time-varying dependence effects lead to higher return portfolio allocations, for a given level of *VaR*. Once more, the proposed copula-LVaR-based optimization outperforms the traditional Markowitz minimum variance portfolio in terms of risk-return trade-off. With respect to Scenario 4, the applied time-varying elliptical *t*-copula does not only allow for individual return distributions, but also for the consideration of individual holding periods for different asset classes, by assessing different liquidation periods we take a more realistic setup into account. Specifically, we set $t = 5$ for developed markets, $t = 10$ for emerging markets and $t = 8$ for commodities. The selected liquidation horizons are used to show the flexibility of our modeling approach, which cannot be specified in the conventional mean-variance method.

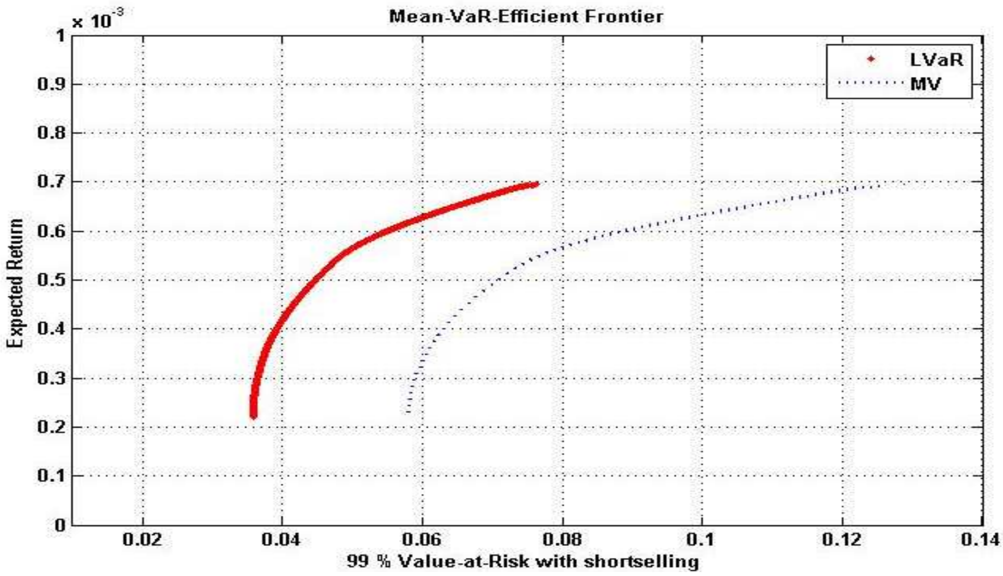

Figure 3: Mean-*VaR* efficient frontiers for a portfolio without short selling constraints
This figure shows the expected portfolio return (vertical axis) and the *VaR* of the portfolios for a 99% confidence level (horizontal axis).

Figure 4 describes the efficient frontiers of both approaches, where different liquidation periods for different asset classes are taken into account. In comparison to Scenario 1, in Scenario 4 different liquidation periods lead to slightly lower risk figures. The portfolio efficient frontiers displayed in Figure 4 are in line with the findings from Scenarios 1, 2 and 3 and confirm the superiority of the copula-LVaR-based optimization over the minimum variance portfolio in terms of risk-adjusted return when realistic constraints, such as individual liquidation periods for different markets, are taken into account.

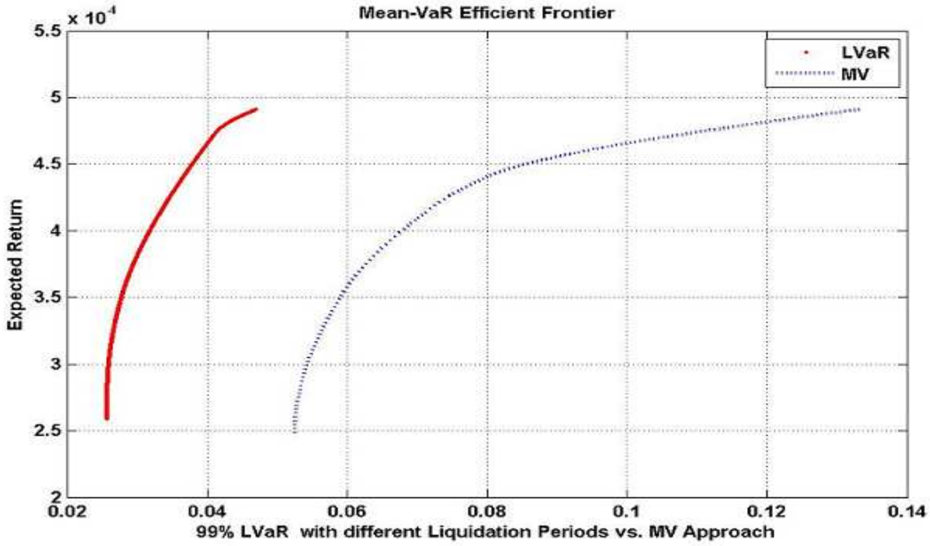

Figure 4: Mean-*VaR* efficient frontiers for a portfolio with different liquidation periods
This figure shows the expected portfolio return (vertical axis) and the *VaR* of the portfolios for a 99% confidence level (horizontal axis).

3.4 Out-of-Sample Analysis of Optimal Portfolios

In order to test for the robustness of the performance of the introduced time-varying copula optimization algorithm within a portfolio management setup, additionally to the graphical analysis of efficient frontiers, we conduct several out-of-sample tests. Specifically, we track the daily performance of portfolio allocations based on the competing algorithms (i.e., the traditional Markowitz, a multivariate CCC-GARCH and the time-varying copula-LVaR approach) for each of the introduced scenarios. We specifically do so by focusing on the optimal mean-VaR efficient portfolio allocation and by assessing the daily performance of each portfolio algorithm for the period of January 1st 2004 – January 31st 2014.

The daily portfolio performance for each of the scenarios considered is analyzed in terms of daily average returns, average 99% *VaR* forecasts and risk adjusted returns. Moreover, we draw on De Miguel et al. (2009) and apply a 1/N strategy as a benchmark for the assessed portfolio allocations and assess risk parity portfolio allocations to underline the flexibility of our analysis.[11] Since the equally weighted risk contribution (ERC) strategy represents a middle way between the extreme mean-VaR efficient portfolio allocation and the 1/N strategy (Maillard et al., 2009), we indicate the full range of potential portfolio allocations. In the remainder of this analysis, we assume a 100,000 USD investment and present the performance of each strategy based on competing methodologies for each of the introduced market scenarios.

Table 5: Summary of optimal portfolios' return and risk across various scenarios

	Exp. Return	Min	Max	99% VaR	VaR Ratio
Scenario 1: No Short Sales					
Mean-VaR DCC-Copula	29.96	-8594.67	6964.29	-3223.37	**0.0093**
Mean-VaR CCC-GARCH	29.56	-9342.31	6837.59	-3385.95	0.0087
Mean-VaR	24.90	-5203.05	4585.40	-4524.76	0.0055
1/N	22.72	-8859.17	8113.98	-6779.47	0.0034
Scenario 2: Budget Restrictions					
Mean-VaR DCC-Copula	30.48	-8520.09	7648.50	-3676.81	**0.0083**
Mean-VaR CCC-GARCH	31.12	-8865.39	7768.81	-3832.65	0.0081
Mean-VaR	25.32	-5700.69	6072.20	-5119.29	0.0049
1/N	23.71	-8174.26	7674.25	-6632.15	0.0036
Scenario 3: Short Sales					
Mean-VaR DCC-Copula	21.97	-7539.02	7719.30	-3573.53	**0.0061**
Mean-VaR CCC-GARCH	21.09	-7807.28	7733.54	-3674.75	0.0057
Mean-VaR	23.04	-5676.49	4994.71	-5004.52	0.0046
1/N	22.72	-8859.17	8113.98	-6779.47	0.0034
Scenario 4: Individual Holding Periods					
Mean-VaR DCC-Copula	25.88	-9670.32	6998.83	-2565.04	**0.0101**
Mean-VaR CCC-GARCH	25.48	-9767.39	7001.21	-2769.57	0.0092
Mean-VaR	24.90	-5203.05	4585.40	-4524.76	0.0055
1/N	22.72	-8859.17	8113.98	-5048.89	0.0045
Scenario 5: Risk Parity					
ERC DCC-Copula	24.15	-7459.03	7299,64	-6422,63	**0.0038**
ERC CCC-GARCH	23.80	-7458.29	7249,60	-6430,20	0.0037
ERC	23.75	-7316.37	7001,43	-6492.66	0.0036

Notes: the table provides the returns of the optimal portfolios as well as their 99% confidence level *VaR* values for each of the four scenarios considered, from January 1st 2004 - January 31st 2014. All figures are presented in US dollars. The abbreviations *Exp. Return*, *Min*, *Max, VaR 99%* and *VaR Ratio* stand for daily average return, minimum return, maximum return, 99% *VaR* for the presented strategy with a liquidity adjusted holding period, and *VaR* adjusted return. The highest ratios are indicated in bold. ERC refers to equally weighted risk contribution strategy.

Table 5 summarizes the portfolio outcomes for each scenario under consideration with an investment budget of US$100,000. As indicated by the efficient frontier analysis, op-

[11] The risk parity allocation implies that each asset describes the same marginal risk contribution to the assessed portfolio allocation (Maillard et al., 2009).

timal portfolios based on the copula-LVaR-based portfolio optimization approach are characterized by higher performance relative to the competing optimization approach. Although mean-VaR efficient portfolios based on the applied CCC-GARCH approach outperform the parsimonious Markowitz approach based on historical covariance matrices, the applied copula algorithm leads to higher risk adjusted performance for the assessed sample for all scenarios. As both approaches CCC-GARCH and copula approach take individual GARCH volatilities into account, the superior performance of the introduced copula approach stems from the ability to take time varying dependence as well as tail dependence into account. The fourth scenario leads to the highest risk adjusted return, indicating the flexibility of the introduced approach. The results in Scenario 5 from the combined risk parity and copula-LVaR approach also indicate the robustness of our approach, as the applied ERC strategy presents a middle way between 1/N and MV. Nevertheless, the highest risk-adjusted returns for the assessed period are produced by the flexibility portfolio optimization approach that combines the DCC *t*-copula, *LVaR* and GARCH processes for the marginals.

4. Conclusion

The main objective of this paper is to introduce a flexible copula-LVaR-based optimization approach that is able to deal with realistic constraints such as budget, liquidity and maximum trading limit thresholds, as well as individual holding periods and short selling that are commonly found in structured portfolio management. The introduced approach overcomes the shortcomings of the standard *VaR* measures of market risk and proposes an integrated and comprehensive framework to accurately measure the risk of loss on a multi-asset portfolio with exposure to liquidity risk and time-varying dependence between assets. By considering the time-varying copula-based dependence structure and various market conditions, the proposed approach offers a flexible and effective way to accurately capture the dependence risk stemming from the co-movement of portfolios' financial assets in the left tail of the return distributions.

A thorough comparison between the introduced time-varying *t*-copula-LVaR portfolio optimization and traditional Markowitz portfolio optimization approach indicates the adequacy of the proposed approach. Although the Markowitz approach provides a straightforward way to identify the optimal portfolio weights for each asset, it has several shortcomings and is not able to take different liquidation periods into account. The obtained empirical results consistently show, under multiple illiquid, adverse and realistic market portfolio optimization scenarios, the superiority of our approach relative to the traditional mean-VaR Markowitz approach and other competing portfolio strategies.

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