The Relation between Absences and Grades: A Statistical Analysis

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Abstract
The paper investigates the relation between absences and grades by employing statistical modelling and using data from a private hospitality school. The obtained parameters estimates verify the assumption that there exists an inverse relationship between absences and grades. It is shown that one unit increase in normalized absences leads to 0.814 units decrease in the average class grade. Further, a dynamic interaction between absences and grades is examined by means of a VAR model. No evidence that the links between absences and grades are propagated over time is found. The system has no memory: each term and / or course defines its own dynamics which is not spread over other terms and / or other courses.

Keywords: Students' performance, tardiness, absences, education, GARCH models, VAR models.

1. Introduction

The relation between absences and students performance is one of the most discussed topics in Education at all levels. Intuition and common sense as well as academic research suggest an inverse relation between absences and students' performance as this reflected on their grades. In this context, inverse relation is understood as, as long as absences increase (decrease), school performance decrease (increase). Studies from several researchers such as, for instance, Mizell (1987), Ligon and Jackson (1988), Cuellar (1992), Escourt (1986), Ediger (1987), all cited by Weade in her Master's thesis (2004), have shown that tardiness (late arrival in the classroom) and/or absences are known factors which contribute to failure, dropout and lower academic performance. In the same thesis, Weade (op.cited) has shown that unexcused absences and GPA are negatively correlated, as it is evidenced by a negative Pearson correlation coefficient equal to -0.519. Similar results have also been observed by Silvestri (2003), Callahan (1993), Hammen and Keeland (1994). In general, all the relevant literature shows an inverse relation between absences and performance which is also independent of the subject of study (LeBlanc III, 2005).

The vast majority of researchers employ statistical tools such as descriptive statistics, the correlation coefficient and/or classical multivariate regression analysis. The present paper attempts to investigate the relation between class attendance and students performance by employing relatively advanced statistical modelling. In particular, the main focus on the paper is the quantification of the response of grades to the students' absences using relevant statistical data obtained from the records of a private hospitality school[2]. The advanced modelling techniques in the present paper refer to the use of GARCH models (Engle, 1982) as a potential tool for measuring volatility clustering, possibly existing in educational time series data, and, also, to the use of a Vector Autoregression Model (Lütkepohl and Krätzig, 2004) as a device of measuring dynamic interaction between grades and absences over time.

The paper is organized as follows: In Section 1, a descriptive analysis of the variables of consideration is presented. In Section 2, a series of models is employed in order to arrive at a statistically admissible and reliable model. The significance of relevant diagnostic and misspecification tests, as well as estimates of parameters of interest, are also presented here. In Section 3, the obtained results are discussed and future research paths are suggested. Diagrams, graphs and other important tools of analysis are presented in the Appendix under the generic term Figure.

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2. Statistical Analysis
2.1 Descriptive Statistics

Data and Variables

In this paper, the data refer to the period winter 2013 - fall 2014 (8 three-month terms) and concern the courses of Microeconomics, Macroeconomics, Mathematics, Calculus and Statistics. There are 35 observations in total, from which the first 18 refer to 2013 and the remaining 17 refer to 2014. The data have been obtained from the schools' management system, based on the class record book and the examination papers. The variables under consideration are: the number of students, absences and grades.

Number of Students

The analysis of the data shows that no change in the mean number of students over the two-year period is observed. However, the volatility increases significantly. It is observed that unconditional volatility, measured by the unconditional variance, in 2014 is almost double of the volatility in 2013: variance (2014) = 59 whereas variance (2013) = 29.15. Conditional variance, modelled by a GARCH (0,1) process, is statistically significant at 10% significance level. However, the effect of volatility clustering, measured by the GARCH model, is rather marginal and, therefore, the normality hypothesis of the number of students cannot be rejected at 5% significance level. See Figures 1a - 1d.

Absences

The total number of absences depend on the number of students enrolled in a class. Therefore, a measure independent of the number of students is needed. This leads to the introduction of the concept of normalized absences. It is defined as: Number of Absences / (Number of Students x Number of Contact Hours). In the school the contact hours are 40. Throughout the paper, several diagnostics and misspecification tests are involved in the various models proposed. Indicatively, for the detection of the order of autocorrelation the Hannan - Quinn criterion (1979) is used, for the unit roots test the Dickey-Fuller test (1979) with the MacKinnon, Haug Michelis (1999) critical values is employed while for normality the Jarque-Berra test is used. Several other tests employed in the paper, such as the Chow test, the Ramsey misspecification test, the Breusch-Pagan-Godfrey heteroskedasticity test, the Lagrange multiplier serial correlation test or, where appropriate, the Durbin-Watson test for first order autocorrelation, can be found in introductory econometrics texts.

From the descriptive analysis, it is observed that the distribution of normalized absences is positively asymmetric. This is attributed to exceptional number of absences in some courses, as, for example, in Mathematics in winter 2013. It also turns out from the analysis that there is no change in the average number of normalized absences over the two-year period under consideration. This finding is supported by relevant diagnostic tests, that is, no ARIMA and/or GARCH processes are detected, no autocorrelation, heteroskedasticity and lack of normality are present and no structural break takes place over the two-year period under consideration. Therefore, we may safely assume that normalized absences follow a white noise process with mean 6.3 and standard deviation 3.56. The mean value of 6.3 suggests that, on average, 6.3% of the taught hours are missing due to absences. The standard deviation, in combination to the fact that the theoretical distribution is normal, implies that the probability of normalized absences, being between 2.74% and 9.86%, is approximately equal to 68%. Details of the analysis are presented in Figures 2a - 2k.

Grades

The statistical analysis of grades shows that there is a statistically significant, at 5% significance level, decrease of the average classroom grade for 2014 in comparison to 2013. This is estimated by means of a dummy variable (DUM) in the intercept which takes the value 0 for 2013 and 1 for 2014. Therefore, we may suggest that a structural break, present in the grades over time, takes place: the average class grade in 2013 equals 85.37 but the average grade in 2014 equals 80.48, that is a difference of 4.89 grade points. Based on the relevant diagnostic and misspecification tests, no autocorrelation, heteroskedasticity, lack of normality at 5% significance level or other instabilities exist. Hence, the model may be safely considered statistically admissible. Given these results, the theoretical probabilities of the average class grades are also estimated. They are as follows: probability of grade A=14.95%, probability of grade B=52.24%, probability of grade C=30.12%, probability of grade D=2.66%, probability of grade < D=0.03%. More details are presented in Figures 3a-3k.

2.2 Statistical Modelling of the Relation between Absences and Grades

2.2.1 Response of Grades to Absences: the Correlation Coefficient

In the following, where absences are mentioned, they are understood as normalized absences. A first measure of the relation between grades and absences can be obtained by the correlation coefficient which is a measure of linear association between these two variables. With the given data, the correlation coefficient equals -0.45, a moderate inverse relation between absences and grades. This is expected, since absences partly but significantly affect class performance, as intuition, common sense and existing research have shown.

2.2.2 Response of Grades to Absences: Searching for a Suitable Statistical Model

The models below attempt to quantify the relationship between grades and absences. Model 1, estimated by maximum likelihood, is an exponential GARCH (1,1) model which shows that there is no GARCH process in the data. Therefore, a model without GARCH process is estimated by OLS. This is the Model 2a which shows that a deterministic trend is statistically insignificant. Because of the insignificance of the trend, the deterministic trend is removed and the next Model 2b is estimated. This model does not show autocorrelation, heteroskedasticity, AR-GARCH effects, or lack of normality.

This is a better model than Model 2a but it displays instability in the beginning of 2014 as the Chow test shows. Therefore, an introduction of a dummy variable, with values 0 for 2013 and 1 for 2014, is added to the model. This is a new model, the Model 2c, which, based on all diagnostic tests, is statistically admissible. The estimates are: $y = e^{4.51-0.01x} \text{ for } 2013$ and $y = e^{4.45-0.01x} \text{ for } 2014$. The fit of the model to the data, as it is measured by the coefficient of multiple determination R square, is 32%, suggesting that absences explain 32% of the grades, whereas the remaining 68% is not captured by the model. To make the interpretation of this model easier, a linear model, with a dummy variable introduced as above, is estimated as an alternative to Model 2c. This is an almost statistically equally accepted model and it is the final model on which the interpretation of the relation between grades and absences is based.

The estimated models are:
$y = 90.33 - 0.814x \text{ for } 2013$ and $y = 85.81 - 0.814x \text{ for } 2014$. The interpretation of the estimates is as follows:

Interpretation of the linear model for 2013:
If normalized absences increase (decrease) by 1 unit, then the grade will decrease (increase) on average by 0.814 units, provided that all implicitly considered variables included in the intercept remain constant. If there were no absences (x=0), then the average grade would be 90.33.

Interpretation of the linear model for 2014:
If normalized absences increase (decrease) by 1 unit, then the grade will decrease (increase) on average by 0.814 units, provided that all implicitly considered variables included in the intercept remain constant. If there were no absences (x=0), then the average grade would be 85.81.

These estimates and their interpretation suggest that, although the response of grades to absences did not change at all over the two years, other factors, not captured by this model and included collectively in the intercept of the model, affect negatively the class performance. That is, if there were no absences, the average class grade would be 90.33 for 2013 and 85.81 for 2014. All models, their diagnostics and a deterministic simulation are displayed in Figures 4a-4p.

2.2.3 Response of Grades to Absences: Dynamic Interaction

There is a theoretical possibility that the effects of grades and absences are propagated over time due to conscious or subconscious memory effects: the students may remember the relation between grades and absences from their own experience and, also, lectures may remember the same relation from their own experience too. For this purpose, a dynamic model has been built and estimated in order to investigate to which extent grades and absences interact over time. The model employs an impulse mechanism (the error term) and a propagation mechanism (a time lag structure). The precise structure of these two mechanisms has been found from the estimates and the diagnostics of a Vector Autoregression Model (VAR). The estimates, on the basis of several lag selection criteria (see, for example, Akaike, 1974 or Hannan - Quinn, 1979), suggest a model with one time lag and white noise residuals. The model, without the dummy variable, has the functional form:

$\mathbf{y}_t = \mathbf{A}\mathbf{y}_{t-1} + \mathbf{u}_t = \begin{pmatrix} y_t \\ x_t \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} y_{t-1} \\ x_{t-1} \end{pmatrix} + \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix}$, where $\mathbf{y}$ is the vector of endogenous variables of grades and absences, $\mathbf{u}$ is the vector of error terms and $\mathbf{A}$ is a matrix of parameters to be estimated. The experimentation with this model, by means of impulse-response functions, shows that grades and absences do not dynamically interact over time. Put it differently, grades and absences do not "remember" each other over time. A possible interpretation might be that the lecturer is not biased against students at any current term and at any current course because of students' absences at any course at the previous term. Also, students do not "remember" the effect of their previous absences on their current grades. The system has no memory: absences and grades are not dynamically linked. Each term and / or course defines its own dynamics which is not spread over other terms and / or other courses. See Figures 4q-4u.

3. Conclusion and Suggestions for Further Research

The present paper is an attempt to quantify the relation between the relation of absences and average class performance by means of statistical modelling. The models employed have been thoroughly tested for statistical pitfalls by means of appropriate diagnostic and misspecification tests. In this context, the obtained estimates may be considered quite reliable. The finally chosen models (Model 2c and the linear model) establish that the relation between grades and absences is statistically very significant and verify the common intuition. The findings are also consistent with the existing literature in that absences affect negatively the grades: one unit increase in normalized absences leads to 0.814 units decrease in the average class grade. Although absences do play a role in class performance, they contribute only by 32% to the explanation of the class grades. It must, however, be noted that, given that the models are statistically well-behaved, the addition of other explanatory variables does not alter the quantitative relation between grades and absences. That is, it is expected that, again, one unit increase in normalized absences leads to 0.814 units decrease in the average class grade. This stability is an important property of the established statistical adequacy of the employed models. Further, the evidence of no interaction between grades and absences over time may imply that the lecturer is not biased against students at any current term and at any current course because of students' absences at any course at a previous term. The findings of the present paper strongly suggest the formulation of attendance policies which take into consideration the relation between grades and academic performance more effectively.

The above exposed analysis may also be enriched with some additional elements. For example, it is reasonable to assume that the background of the students before their admission to the school is a very important explanatory factor of their performance in the school. Therefore, data referring to the students' background and introduced in an appropriate statistical model, may significantly enhance the explanatory power of the analysis.

The present models employed average class grades and average class absences. Another possible research avenue would be the exploration of the relation between absences and grades at the level of individual courses. As a last point, another interesting research question would be the effect of individual student absences on the individual student grades.

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APPENDIX

Figure 1a: No of Students.

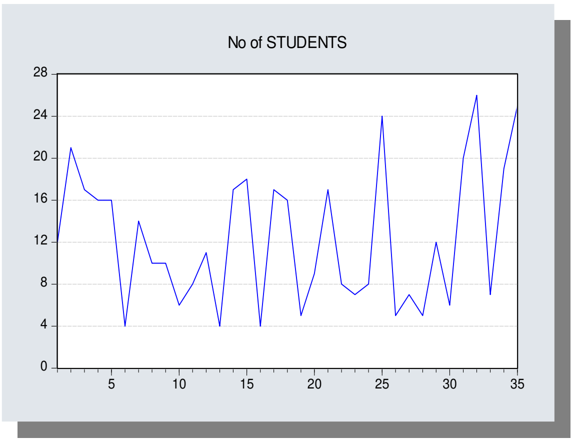

Figure 1b: Conditional Variance of No of Students.

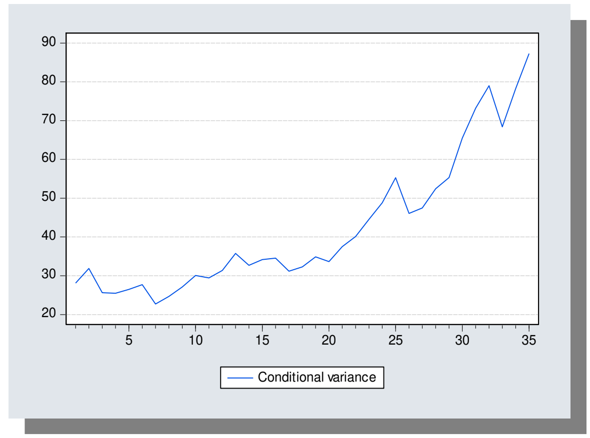

Figure 1c: Histogram and Descriptive Measures

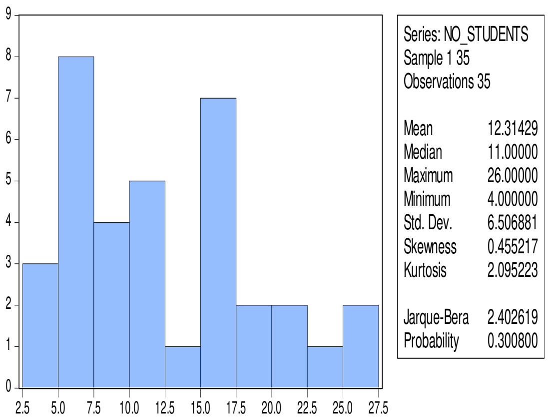

Figure 1d: Histogram and the Corresponding Theoretical Normal Distribution.

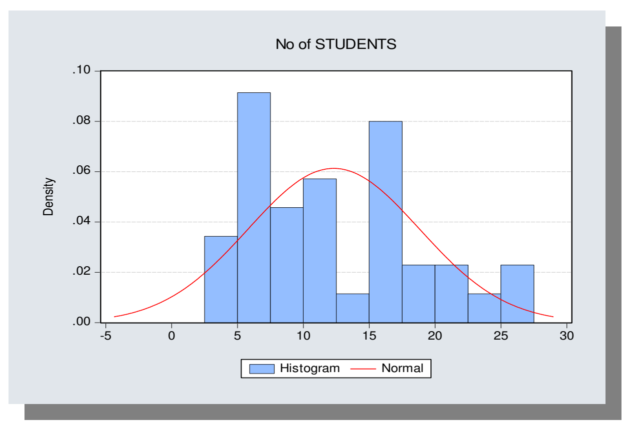

Figure 2a: Absences for all Taught Subjects.

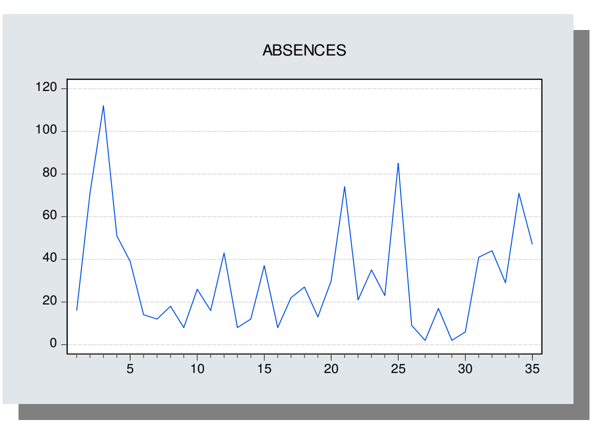

Figure 2b: Histogram of Absences and Descriptive Measures.

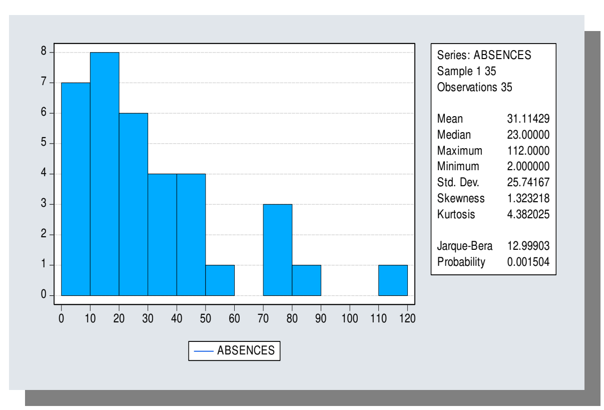

Figure 2c: Histogram and Kernel: Unusually many absences: Mathematics, Winter 2013.

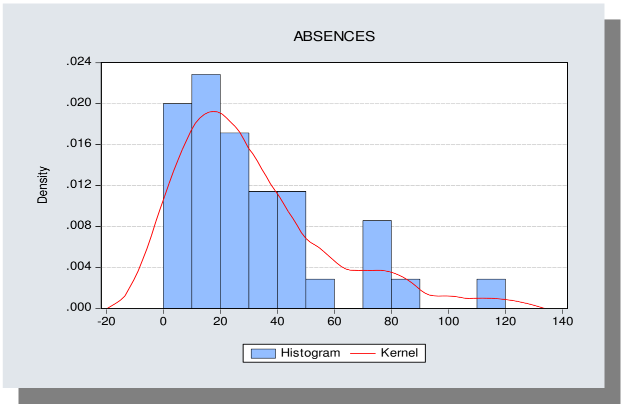

Figure 2d: Box-Plot of Absences: Positively Asymmetric Distribution.

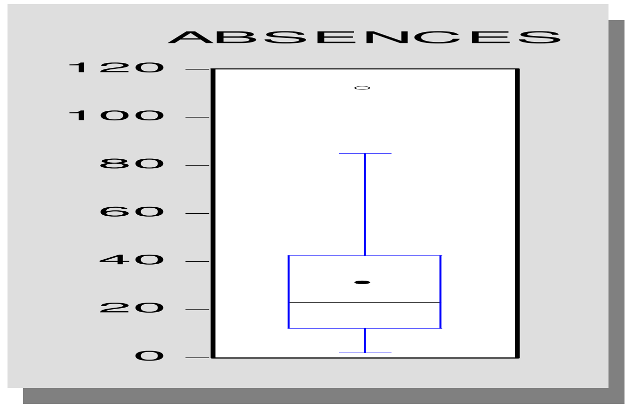

Figure 2e: Absence per Student for all Taught Subjects.

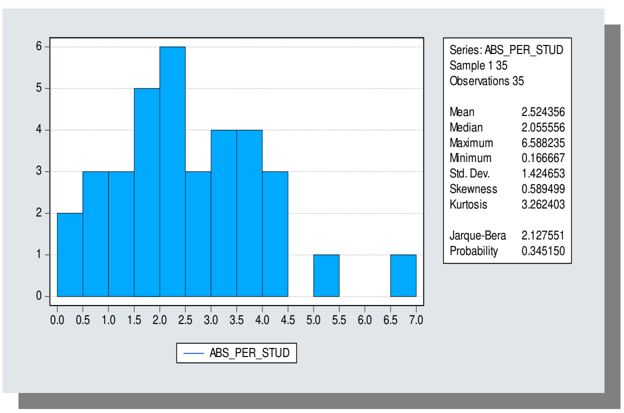

Figure 2f: Absence for all Taught Subjects.

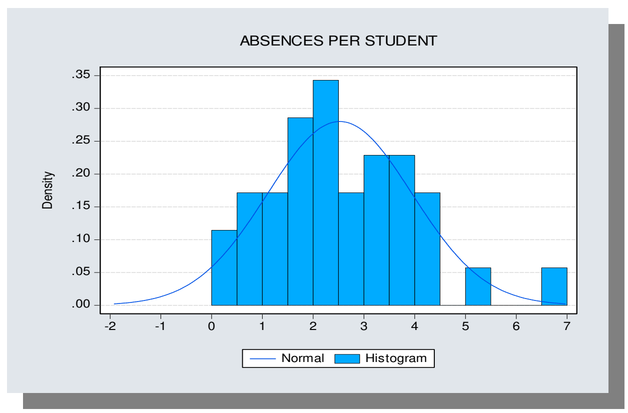

Figure 2g: Normalized Absences for all Taught by Subjects.

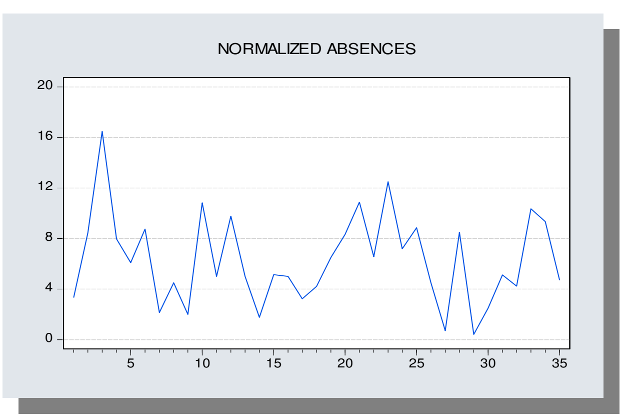

Figure 2h: Stability of the Estimated Constant By means of the Cumulative Sum of Squares.

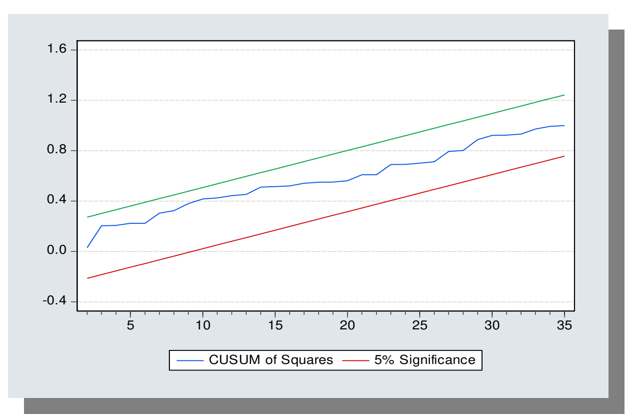

Figure 2i: Histogram of Normalized Absences and Descriptive Measures.

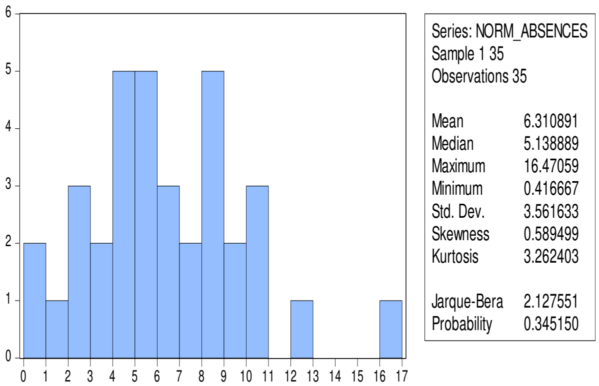

Figure 2j: Histogram of Normalized Absences and the Theoretical Normal Distribution.

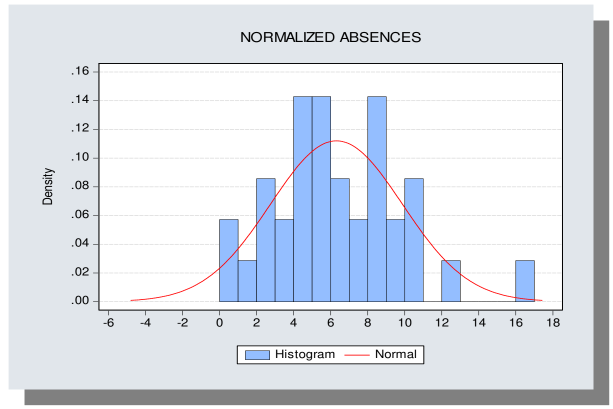

Figure 2k: No change over time. No ARIMA and/or GARCH processes are detected. No autocorrelation, heteroskedasticity and lack of normality are present. No structural break. The process is white noise.

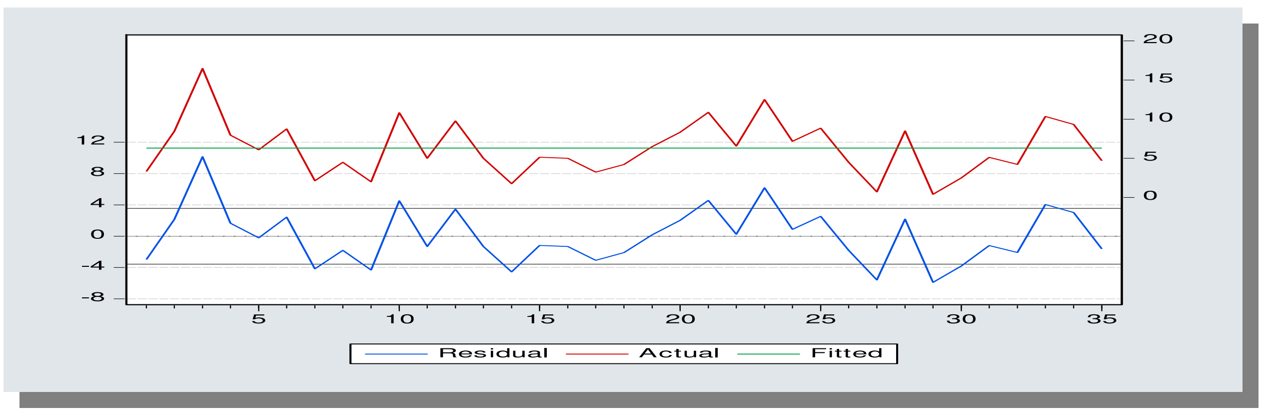

Figure 3a: Grades over Time.

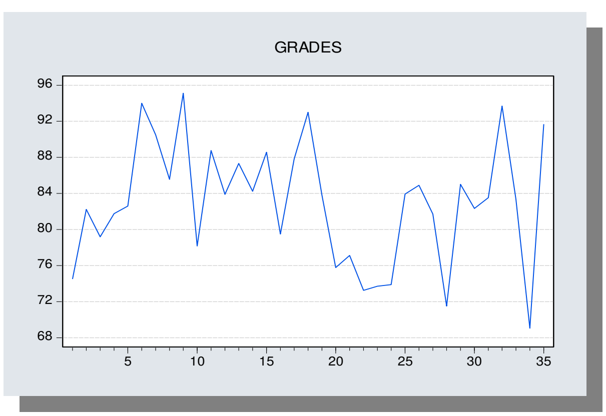

Figure 3b: Structural Break in the Intercept.

Dependent Variable: GRADE
Method: Least Squares

Sample: 1 35
Included observations: 35

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	85.37167	1.498169	56.98402	0.0000
DUM	$\boxed{-4.889902}$	2.149662	-2.274730	0.0295

R-squared	0.135546	Mean dependent var	82.99657
Adjusted R-squared	0.109351	S.D. dependent var	6.735092
S.E. of regression	6.356191	Akaike info criterion	6.592181
Sum squared resid	1333.238	Schwarz criterion	6.681058
Log likelihood	-113.3632	Hannan-Quinn criter.	6.622861
F-statistic	5.174398	Durbin-Watson stat	1.805356
Prob(F-statistic)	0.029549		

Figure 3c: Serial Correlation and Heteroskedasticity Tests.

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	0.005903	Prob. F(2,31) 0.9941
Obs*R-squared	0.013325	Prob. Chi-Square(2) 0.9934

Heteroskedasticity Test: Breusch-Pagan-Godfrey

F-statistic	0.898368	Prob. F(1,33) 0.3501
Obs*R-square	0.927563	Prob. Chi-Square(1) 0.3355
Scaled explained SS	0.535751	Prob. Chi-Square(1) 0.4642

Figure 3d: Stability of the Intercept after the Introduction of the Dummy Variable.

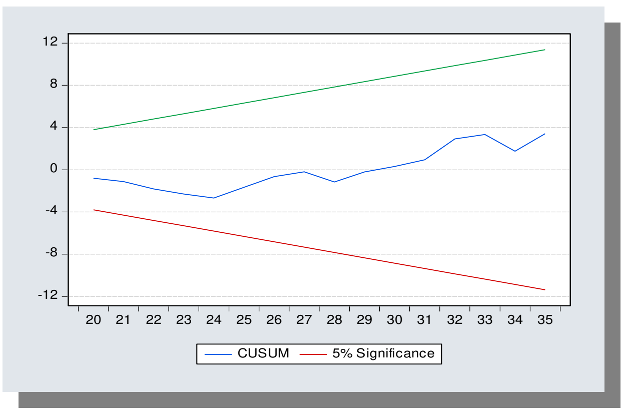

Figure 3e: Histogram of Grades.

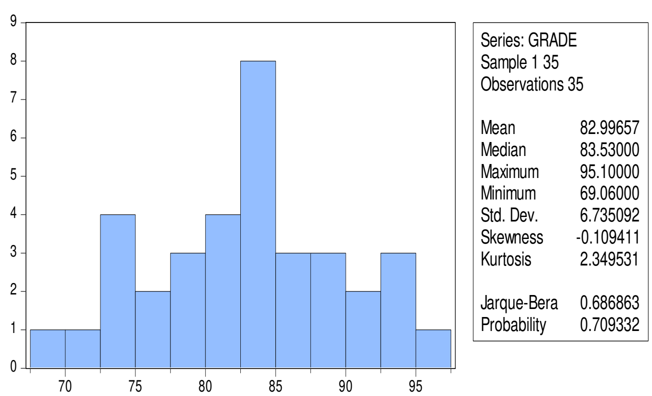

Figure 3f: Histogram and the Theoretical Normal Distribution of Grades.

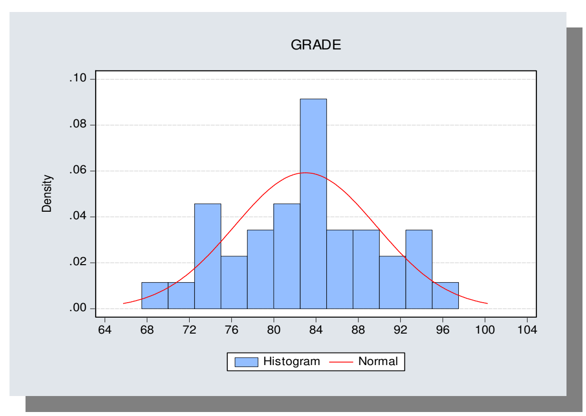

Figure 3g: Probability of Grade A = 14.95%.

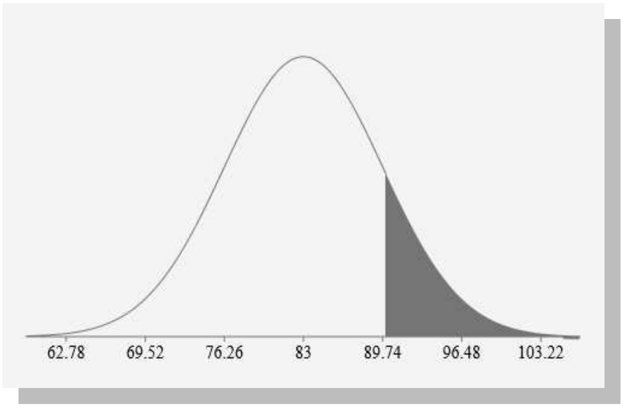

Figure 3h: Probability of Grade B = 52.24%.

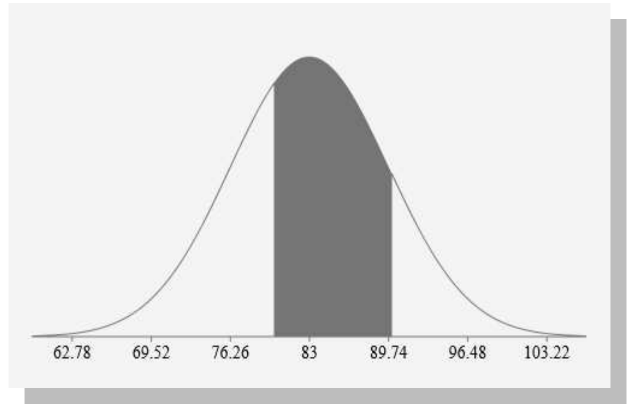

Figure 3i: Probability of Grade C = 30.12%.

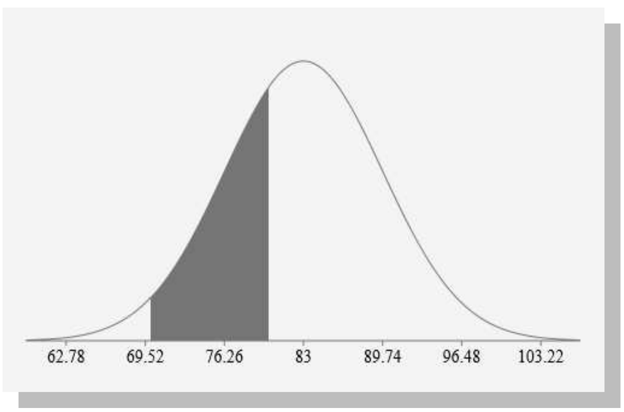

Figure 3j: Probability of Grade D = 2.66%.

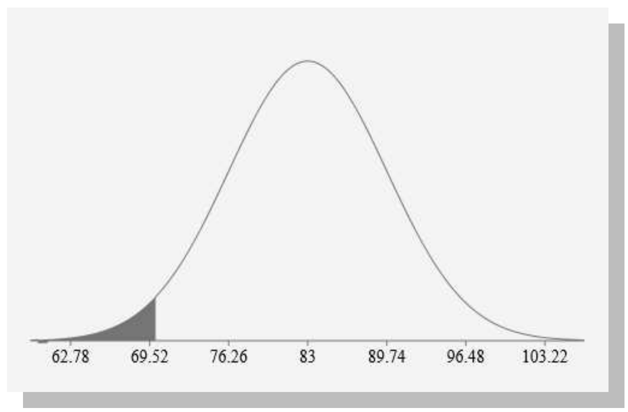

Figure 3k: Probability of Grade < D = 0.03%.

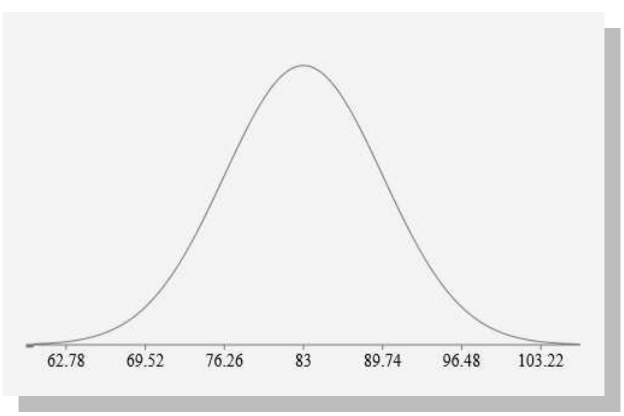

Figure 4a: Correlation Coefficient between Grades and Absences = -0.45.

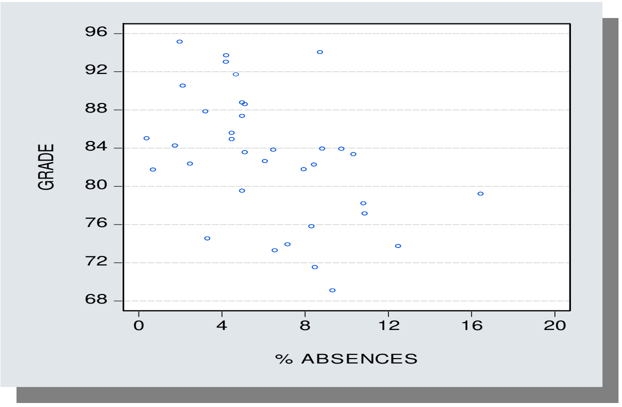

Figure 4b: Model 1: Exponential GARCH model. Estimation of Parameters and Diagnostics.

Dependent Variable: LOG(GRADE)
Method: ML - ARCH (Marquardt) - Normal distribution

Sample: 1 35
Included observations: 35
Failure to improve Likelihood after 12 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(4) + C(5)*RESID(-1)^2 + C(6)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	4.524684	0.034712	130.3490	0.0000
NORM_ABSENCES	-0.012831	0.004023	-3.189690	0.0014
@TREND	-0.001525	0.000855	-1.784624	0.0743
Variance Equation				
C	0.003917	0.003410	1.148841	0.2506
RESID(-1)^2	-0.198709	0.198259	-1.002268	0.3162
GARCH(-1)	0.349822	0.515423	0.678709	0.4973

R-squared	0.238789	Mean dependent var	4.415557
Adjusted R-squared	0.191214	S.D. dependent var	0.082011
S.E. of regression	0.073755	Akaike info criterion	-2.241896
Sum squared resid	0.174073	Schwarz criterion	-1.975265
Log likelihood	45.23317	Hannan-Quinn criter.	-2.149855
Durbin-Watson stat	1.796515		

Figure 4c: Model 2a: Exponential OLS Model with Trend. Estimation of Parameters and Diagnostics.

Dependent Variable: LOG(GRADE)
Method: Least Squares

Sample: 1 35
Included observations: 35

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	4.512705	0.034716	129.9889	0.0000
NORM_ABSENCES	-0.010957	0.003556	-3.081589	0.0042
@TREND	-0.001647	0.001236	-1.332585	0.1921

R-squared	0.246677	Mean dependent var	4.415557
Adjusted R-squared	0.199594	S.D. dependent var	0.082011
S.E. of regression	0.073372	Akaike info criterion	-2.304738
Sum squared resid	0.172269	Schwarz criterion	-2.171422
Log likelihood	43.33291	Hannan-Quinn criter.	-2.258717
F-statistic	5.239217	Durbin-Watson stat	$\boxed{1.772797}$
Prob(F-statistic)	0.010757		

Figure 4d: Model 2b: Exponential OLS Model without Trend. Estimation of Parameters and Diagnostics.

Dependent Variable: LOG(GRADE)
Method: Least Squares

Sample: 1 35
Included observations: 35

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	4.481332	0.025812	173.6174	0.0000
NORM_ABSENCES	$\boxed{-0.010422}$	0.003574	-2.915950	0.0063

R-squared	0.204872	Mean dependent var	4.415557
Adjusted R-squared	0.180777	S.D. dependent var	0.082011
S.E. of regression	0.074229	Akaike info criterion	-2.307873
Sum squared resid	0.181829	Schwarz criterion	-2.218996
Log likelihood	42.38777	Hannan-Quinn criter.	-2.277192
F-statistic	8.502763	Durbin-Watson stat	1.668732
Prob(F-statistic)	0.006330		

Figure 4e: Model 2b: Estimates of the Parameters.

$$y = e^{4.48-0.01x}$$

$$e = \lim_{n\to\infty}(1+\frac{1}{n})^n = 2.7182818...$$

x=absences, y=grades.

Figure 4f: Model 2b is free of autocorrelaiton, heteroskedasticity, lack of normality but it shows structural instability in the beginning of 2014.

Chow breakpoint test in observation 19 (beginning of 2014) shows structural break:

F-statistic	3.235015
Prob. F(2,31)	0.0530

Figure 4g: A Dummy Variable is introduced in Model 2b. This leads to Model 2c.

Dependent Variable: LOG(GRADE)
Method: Least Squares

Sample: 1 35
Included observations: 35

Variable	Coefficient	Std. Error	t-Statistic	Prob.
NORM_ABSENCES	-0.009914	0.003354	-2.955904	0.0058
DUM	-0.055889	0.023558	-2.372432	0.0239
C	4.505272	0.026194	171.9980	0.0000

R-squared	0.323807	Mean dependent var	4.415557
Adjusted R-squared	0.281545	S.D. dependent var	0.082011
S.E. of regression	0.069514	Akaike info criterion	-2.412754
Sum squared resid	0.154631	Schwarz criterion	-2.279438
Log likelihood	45.22319	Hannan-Quinn criter.	-2.366733
F-statistic	7.661877	Durbin-Watson stat	$\boxed{1.918981}$
Prob(F-statistic)	0.001910		

Figure 4h: Estimates from Model 2c.

$$y = e^{4.51-0.01x} \text{ for } 2013$$
$$y = e^{4.45-0.01x} \text{ for } 2014$$

x=absences, y=grades.

Figure 4i: Diagnostics of Model 2c.

Sample: 1 35
Included observations: 35

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.087	-0.087	0.2876	0.592
		2	-0.112	-0.120	0.7772	0.678
		3	0.047	0.026	0.8650	0.834
		4	-0.054	-0.062	0.9864	0.912
		5	-0.275	-0.284	4.2405	0.515
		6	0.258	0.210	7.2138	0.302
		7	0.040	0.020	7.2889	0.399
		8	-0.106	-0.056	7.8252	0.451
		9	-0.117	-0.184	8.5026	0.484
		10	-0.081	-0.189	8.8423	0.547
		11	-0.148	-0.081	10.024	0.528
		12	0.138	0.067	11.092	0.521
		13	-0.026	-0.127	11.131	0.600
		14	0.110	0.065	11.879	0.616
		15	-0.011	-0.038	11.886	0.688
		16	-0.144	-0.155	13.304	0.650

Figure 4j: The Residuals are Distributed Normally.

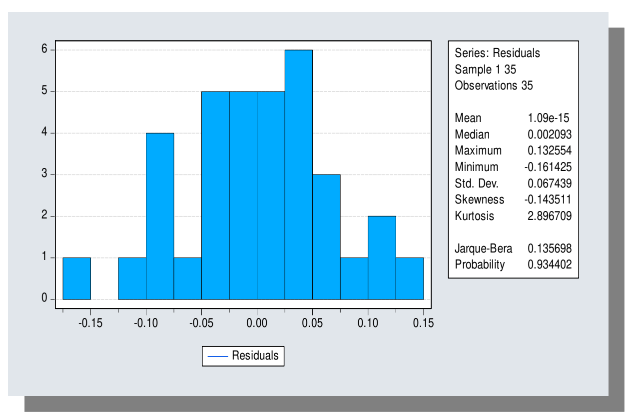

Figure 4k: No misspecification, autocorrelation and heteroskedasticity exist. The model is well-behaved.

Ramsey RESET Misspecification Test
Specification: LOG(GRADE) NORM_ABSENCES DUM C
Omitted Variables: Squares of fitted values

	Value	df	Probability
t-statistic	0.793527	31	0.4335
F-statistic	0.629684	(1, 31)	0.4335
Likelihood ratio	0.703810		0.4015

Breusch-Godfrey Serial Correlation LM Test

F-statistic	0.368527	Prob. F(2,30)	0.6948
Obs*R-squared	0.839277	Prob. Chi-Square(2)	0.6573

Heteroskedasticity Test: Breusch-Pagan-Godfrey

F-statistic	0.250909	Prob. F(2,32)	0.7796
Obs*R-squared	0.540390	Prob. Chi-Square(2)	0.7632
Scaled expl. SS	0.428390	Prob. Chi-Square(2)	0.8072

Figure 4l: Structural Stability of the Parameters of the Model 2c: the Model is Stable.

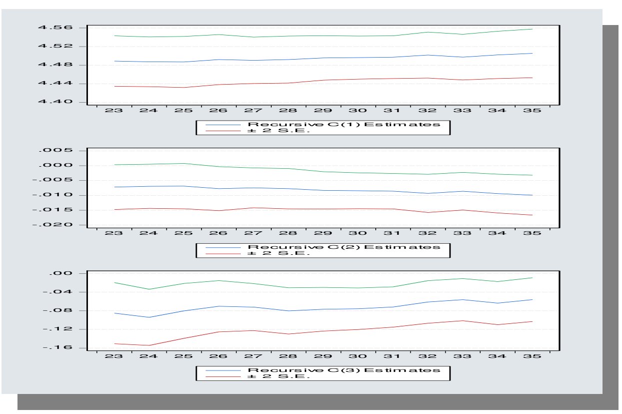

Figure 4m: Structural Stability of the Residuals of the Model 2c: the Model is Stable.

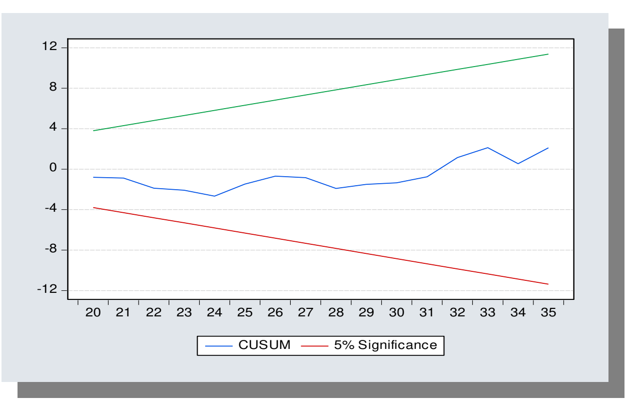

Figure 4n: Actual, Fitted Grades and Residuals: Normalized absences explain 32% of the total variation of the grades in the present model 2c. The remaining 68% of the variation cannot be explained by this model.

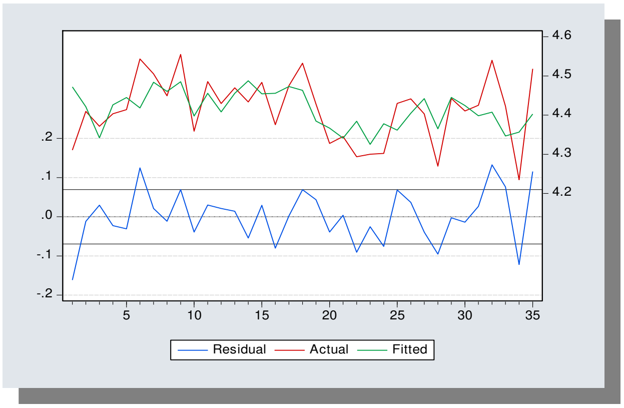

Figure 4o: Static Forecast with 95% Confidence Interval.

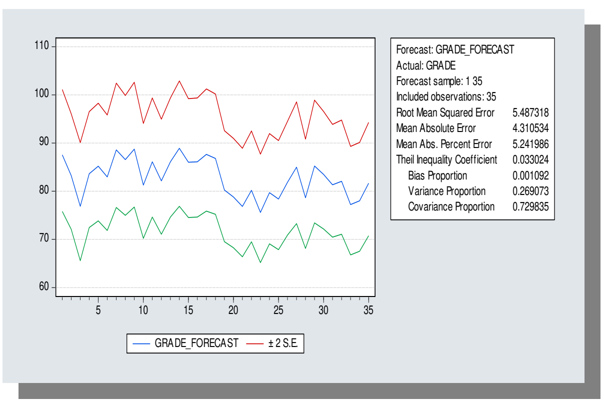

Figure 4p: Deterministic Simulation of the Relation between Absences and Absences and Grades.

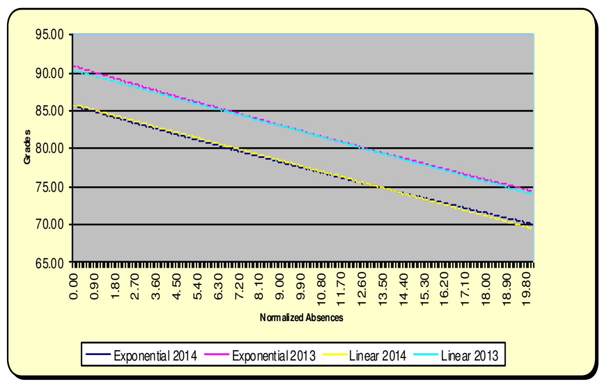

Figure 4q: Lag Selection Criteria: 1 lag is selected.

1 time lag has been selected on the basis of all lag order selection criteria.

VAR Lag Order Selection Criteria
Endogenous variables: GRADE NORM_ABSENCES DUM
Exogenous variables: C

Sample: 1 35
Included observations: 31

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-196.4939	NA	78.02342	12.87057	13.00935	12.91581
$\boxed{1}$	-162.4226	59.35002*	15.54561*	11.25307*	11.80816*	11.43402*
2	-160.6624	2.725507	25.28670	11.72015	12.69156	12.03681
3	-157.6285	4.110317	38.98205	12.10507	13.49280	12.55743
4	-151.5525	7.056062	51.69714	12.29371	14.09776	12.88178

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

Figure 4r: Stability of the VAR Process: All roots are inside the unit circle.

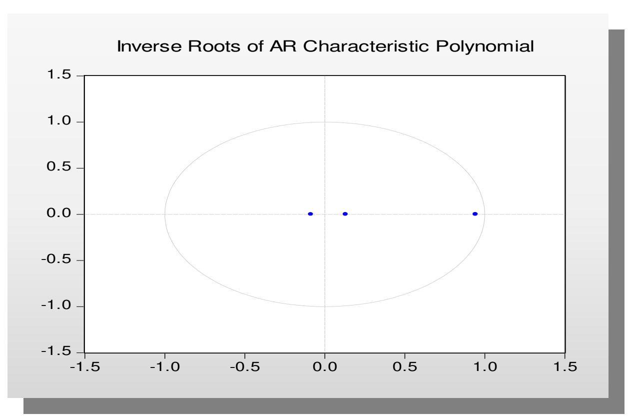

Figure 4s: Roots of Characteristic Polynomial.

Root	Modulus
0.942682	0.942682
0.131343	0.131343
-0.085906	0.085906

No root lies outside the unit circle.
VAR satisfies the stability condition

Figure 4t: Statistical Significance of the Lagged Parameters.
None is significant.

Vector Autoregression Estimates

Sample (adjusted): 2 35
Included observations: 34 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	GRADE	NORM_ABS...	DUM
GRADE(-1)	-0.108040	-0.009185	0.006527
	(0.20288)	(0.12083)	(0.00561)
	[-0.53253]	[-0.07602]	[1.16282]
NORM_ABSENCES(-1)	-0.298841	0.115579	0.000624
	(0.34053)	(0.20281)	(0.00942)
	[-0.87757]	[0.56989]	[0.06623]
DUM(-1)	-6.049470	0.158636	0.980580
	(2.39333)	(1.42538)	(0.06622)
	[-2.52764]	[0.11129]	[14.8077]
C	96.93274	6.348920	-0.505507
	(18.4235)	(10.9724)	(0.50976)
	[5.26137]	[0.57863]	[-0.99166]
R-squared	0.202890	0.016885	0.894609
Adj. R-squared	0.123179	-0.081426	0.884070
Sum sq. resids	1170.137	415.0429	0.895821
S.E. equation	6.245364	3.719511	0.172802
F-statistic	2.545325	0.171751	84.88507
Log likelihood	-108.3987	-90.77827	13.57448
Akaike AIC	6.611687	5.575193	-0.563204
Schwarz SC	6.791259	5.754764	-0.383633
Mean dependent	83.24647	6.398466	0.500000
S.D. dependent	6.669639	3.576740	0.507519
Determinant resid covariance (dof adj.)		11.49551	
Determinant resid covariance		7.896872	
Log likelihood		-179.8617	
Akaike information criterion		11.28598	
Schwarz criterion		11.82470	

Figure 4u: Impulse - Response Functions: Very Fast Monotonic Convergence to Zero.