Inflation and Growth: A Non-Monotonic Relationship in an Innovation-Driven Economy

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Abstract

This paper investigates the effects of monetary policy on long-run economic growth via different cash-in-advance constraints on R&D in a Schumpeterian growth model with vertical and horizontal innovation. The relationship between inflation and growth is contingent on the relative extents of CIA constraints and diminishing returns to two types of innovation. The model can generate a mixed (monotonic or non-monotonic) relationship between inflation and growth, given that the relative strength of monetary effects on growth between different CIA constraints and that of R&D-labor-reallocation effects between different diminishing returns vary with the nominal interest rate. In the empirically relevant case where horizontal R&D suffers from greater diminishing returns than vertical R&D, inflation and growth can exhibit an inverted-U relationship when the CIA constraint on horizontal R&D is sufficiently larger than that on vertical R&D. Finally, the model is calibrated to the US economy, and we find that the growth-maximizing rate of inflation is around 2.8%, which is closely consistent with recent empirical estimates.

JEL classification: O30; O40; E41.
Keywords: Inflation; Endogenous growth; CIA constraint on R&D

1 Introduction

The relationship between inflation and growth has long been debated among monetary economists. Is inflation negatively related to long-run economic growth conclusively? Furthermore, do they maintain a steadily monotonic relationship regardless of the inflation level? Earlier studies indeed find a negative relationship between steady inflation and output/growth across

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countries (such as Fischer (1983) and Cooley and Hansen (1989)), whereas later works by Bullard and Keating (1995), Bruno and Easterly (1998), and Ahmed and Rogers (2000) seemingly find no robust or even a positive correlation in low-inflation industrialized economies.

Recent empirical works challenge most previous studies that document only monotonic relationships between inflation and growth. They suggest a non-monotonic relationship whereby the real growth effect of inflation could be either positive or negative, depending on the status quo inflation rate. This series of studies can be traced back to Sarel (1996), who identifies a structural break in the function that relates growth rates to inflation, showing that when inflation is low, specifically eight percent annually, there is no significant negative effect (or even a slightly positive effect) on economic growth. When inflation is high, however, there exists a robust, statistically significant negative effect on growth. Several studies (Ghosh and Phillips (1998); Khan and Senhadji (2001); Burdekin et al. (2004); Eggoh and Khan (2014)) demonstrate successively that there is a nonlinear correlation, but the specific threshold remains inconclusive, varying from 1% to 15-18%.[1] In this study, our model is calibrated to the aggregate data of the US economy to provide a quantitative analysis. We find that the growth-maximizing inflation rate is within the range for industrialized economies, i.e., 1-8%. Furthermore, we show that the fraction of the CIA constraint on consumption is crucial in determining the inflation threshold.

In the present study, we reconcile the theories and recent empirical evidence on inflation and growth in the context of an innovation-driven growth model characterized by two modes of innovation. More precisely, various CIA constraints on R&D are incorporated, which sheds light on how monetary policy can generate a non-monotonic relationship between inflation and growth through these constraints. In particular, our analysis builds on the growth model developed by Howitt (1999) and Segerstrom (2000), featuring both horizontal and vertical innovation.[2] Vertical innovation serves to improve the quality of existing products whereas horizontal innovation aims at expanding product varieties, both of which are conducted by forward-looking entrepreneurs. Monetary policy, which is exercised through nominal interest rate targeting, affects the long-run growth rate by affecting the two types of innovation through the relative extents of different CIA constraints and diminishing returns to two types of R&D.

Imposing (CIA) constraints on R&D is consistent with the following empirical findings. First, monetary evidence (e.g., Hall (1992) and Himmelberg and Petersen (1994)) report a strong R&D-cash flow sensitivity for firms. Hall and Lerner (2010) reports that more than 50 percent of R&D spending is accounted for by the wages and salaries of R&D personnel. Hiring scientists and engineers usually involves a very high adjustment cost.[3] R&D-intensive firms are required to hold cash in order to smooth their R&D spending over time. Brown and Petersen (2011) provides direct evidence that US firms relied heavily on cash reserves to smooth R&D spending during the 1998-2002 boom. The above evidence suggests that, relative to the traditional physical

[1] As is documented in López-Villavicencio and Mignon (2011), the reasons for the controversies include the frequency of data, the considered framework and the methodologies applied, the countries under study, and the existence of high-inflation observations.

[2] This class of R&D-driven growth models with two-dimensional innovations developed by Smulders and Van de Klundert (1995), Peretto (1996, 1998), Dinopoulos and Thompson (1998), Young (1998) and Howitt (1999) have received strong empirical support in more recent years, such as Laincz and Peretto (2006), Ha and Howitt (2007), Madsen (2008), and Ang and Madsen (2011).

[3] Because their skills are highly specific and unique, their absence may make the whole R&D process fail and dramatically decrease the firm's profit. See Hall and Lerner (2010).

investment, R&D activities exhibit a stronger investment-cash flow sensitivity.

In addition, several important empirical findings concerning firm characteristics motivate us to capture these insights through an endogenous growth model with two modes of innovation. First, larger firms induce a relatively greater amount of investment in process and incremental (vertical) R&D, while smaller firms are usually involved in more radical (horizontal) product innovation (e.g., Cohen and Klepper (1996); Akcigit (2009); Janiak and Monteiro (2011)). Second, the requirements of cash holdings exhibit distinct patterns in relation to these two modes of innovation. Existing empirical evidence shows that there is a stronger impact from cash holdings on R&D in smaller firms, which are more likely to confront binding liquidity and financing constraints (see Brown and Petersen (2009), Brown and Petersen (2011), Brown et al. (2012) and Caggese (2015)). Together with the fact that radical and original R&D are more adequately represented by horizontal innovation (see Acemoglu et al. (2014)), it is reasonable to consider that horizontal R&D is subject to a more severe CIA constraint than vertical R&D. Accordingly, vertical innovation gains a cost advantage relative to horizontal innovation. Finally, empirical evidence in management (e.g., McDermott and O'Connor (2002)) shows that radical innovations rely on less standardized capital, are often involved in new facilities and equipments, and face higher technological uncertainty compared to incremental innovation.[4] These features are consistent with the findings in Audretsch et al. (2006) and are captured by Howitt (1999) in that radical innovation is prone to suffering from greater diminishing returns than incremental innovation.

By taking into consideration various CIA constraints, monetary policy in this study can generate different impacts of inflation on economic growth subject to the relative extents of the CIA constraints and the different diminishing returns to two innovations. To be specific, with a change in the nominal interest rate, different CIA constraints imply the existence of a force that transmits different inflation costs, which distort the incentives and the use of economic resources in different sectors; at the same time, different extents of diminishing returns to R&D imply another force that triggers a reallocation of resources between the two types of R&D activities. Both forces jointly determine the long-run relationship between inflation and growth.

We first investigate the cases subject to each single type of CIA constraint and the results are as follows. In the presence of a CIA constraint on consumption only, increasing the nominal interest rate increases (decreases) the economic growth rate if horizontal R&D exhibits greater (smaller) diminishing returns than vertical R&D. In this case, the degree of relative diminishing returns to R&D plays a crucial role in determining the allocation of R&D resources, whereby with along a rise in the nominal interest rate, a lager (smaller) diminishing returns to horizontal R&D allow more R&D resources to be allocated to horizontal (vertical) innovation than to vertical (horizontal) innovation, thereby increasing the growth of variety (quality) at the expense of the growth of quality (variety) and thus leading to a decrease (an increase) in the long-run economic growth. By contrast, in the presence of a CIA constraint on vertical (horizontal) R&D only, increasing the nominal interest rate always decreases (increases) economic growth regardless of the relative diminishing returns to both types of R&D. The reason is that R&D resources will always be shifted away from the CIA-constrained sector to the non-constrained one regardless of which R&D sector exhibits higher diminishing returns. The diminishing returns to R&D in these cases only govern "the amount", but not "the direction" of, the shifting R&D resources.

[4] Klepper and Simons (2005) document that firms engaging in incremental innovative activities are more likely to gain success in their R&D projects.

More interestingly, incorporating all the CIA constraints into the model yields a diverse relationship between inflation and growth. In particular, by focusing on the empirically relevant scenario where horizontal R&D exhibits greater diminishing returns, we find that increasing the nominal interest rate may induce a non-monotonic (inverted-U) relationship between inflation and growth, provided that the CIA constraint on horizontal R&D is sufficiently stronger than that on vertical R&D. Specifically, with a sufficiently large CIA constraint on horizontal relative to that on vertical R&D, increasing the nominal interest rate from a low level yields a strong positive growth effect from the CIA constraint on horizontal R&D, which dominates the negative growth effects from the CIA constraints on consumption and vertical R&D; thus, a positive relationship between inflation and growth is generated. Nevertheless, as the nominal interest rate increases and then exceeds a threshold, the positive growth effect is dampened and becomes overwhelmed, leading to a negative relationship between inflation and growth. Overall, a non-monotonic relationship (inverted-U shape) is formed in these circumstances.

By applying US aggregate data, our quantitative analysis in the benchmark case generates an inverted-U relationship between inflation and growth, showing that the threshold value of the inflation rate is around 2.8%, which is closely in line with the recent empirical estimates in Ghosh and Phillips (1998) (i.e., 2.5%), López-Villavicencio and Mignon (2011)(i.e., 2.7%), and Eggoh and Khan (2014)(i.e., 3.4%). Welfare, however, is monotonically decreasing in inflation, implying that the Friedman rule is optimal. Interestingly, when the relative extent of the CIA constraint on horizontal to vertical R&D decreases, the inflation-growth relationship becomes negative, which conforms to our analytical finding. Welfare, instead, becomes inverted-U shaped in inflation, implying the sub-optimality of the Friedman rule. Finally, sensitivity analysis is performed with alternative calibrated values for several key parameters, and it shows that our quantitative results are robust.

The literature pertaining to the analysis of monetary policy and growth is too large and diverse for a detailed review. Nevertheless, the closely related works are those that use endogenous growth models with R&D to analyze the effects of monetary policy on long-run growth. The pioneering work is that of Marquis and Reffett (1994) who explore the effects of monetary policy on growth via a CIA constraint on consumption in the framework of Romer (1990). Subsequent studies (e.g., Chu and Lai (2013), Chu and Cozzi (2014), and Oikawa and Ueda (2015)) analyze monetary policy in a Schumpeterian model with a quality ladder. The present study differs from the above works by considering a scale-invariant Schumpetarian growth model that features two dimensions of innovation. Another strand of the literature such as Huang et al. (2015) and Chu and Ji (2016) also analyzes the growth and welfare effects of monetary policy in a scale-invariant fully endogenous growth model based on Peretto (1998) that features both horizontal and vertical innovation. Nonetheless, their models only predict a monotonic linkage between inflation and long-run growth, whereas our model can yield a non-monotonic relationship between them, depending on the status quo inflation. Finally, to characterize a non-linear relationship between inflation and growth, Arawatari et al. (2017) use a variety expansion model with heterogeneous R&D abilities. They find a cutoff inflation level around which a negative nonlinear relationship between inflation and growth is found to exist but they do not generate a non-monotonic relationship between them. Wang and Xie (2013), however, find evidence of a non-monotonic relationship between inflation and growth (i.e., a growth-maximizing inflation rate) based on a neoclassical model that features labor friction. One notable exception is Chu et al. (2017), who also find evidence of an inverted-U relationship between inflation and growth in a canonical Schumpeterian growth model featuring random quality improvements. Our results complement their work in several respects. First, their framework only considers vertical innovation, whereas our model considers vertical innovation in addition to horizontal innovation, and these two types of innovation are shown to play very different roles in explaining the impact of monetary policy on economic growth. Second, the model in Chu et al. (2017) removes scale effects by normalizing the size of the population, whereas our model is made to be scale invariant by taking into account the product proliferation. Furthermore, when the model setting is generalized to allow for elastic labor supply, the result of an inverted-U relationship between inflation and growth does not hold in Chu et al. (2017), whereas our model is still robust to produce this relationship between inflation and growth.

The remainder of this study proceeds as follows. The basic model is spelled out in section 2. Section 3 analyzes the effects of monetary policy in different cases for CIA constraints. The numerical analysis is displayed in section 4, and the final section concludes.

2 Model

We consider a monetary variant of Howitt (1999) and Segerstrom (2000) that features two dimensional innovations. The model is extended to examine the effects of monetary policy by allowing for an elastic labor supply and various CIA constraints on consumption and R&D investments. The economy consists of households, firms (including the incumbents for intermediate goods production and entrants for two types of R&D (i.e., vertical and horizontal R&D)), and a government that is solely represented by the monetary authority.

2.1 The Household

Consider a closed economy that admits a household and is populated by a mass of individuals L_t with the population size growing at an exponential rate g_L. Each individual supplies labor elastically and faces a life-time utility function given by

$$U = \int_0^\infty e^{-\rho t}[\ln c_t + \theta \ln(1 - l_t)]dt, \tag{1}$$

where ρ is the discount rate, c_t is the consumption of final goods per capita at time t, l_t is the supply of labor per person at time t, and θ determines the preference for leisure relative to consumption.

An individual maximizes (1) subject to the budget constraint and a CIA constraint, which are respectively given by:

$$\dot{a}_t + \dot{m}_t = (r_t - g_L)a_t + w_t l_t + i_t b_t + \zeta_t - (\pi_t + g_L)m_t - c_t, \tag{2}$$

and

$$\xi_c c_t + b_t \leq m_t, \tag{3}$$

where a_t refers to the real assets owned by each person, and r_t is the real interest rate. Each individual supplies labor l_t to earn a real wage rate w_t, and loans out an amount b_t of money to the entrepreneurs, with a return rate i_t (i.e., the nominal interest rate). Each individual receives a lump-sum transfer ζ_t from the government. Moreover, m_t represents the real money balances held by the individual, and π_t is the inflation rate.

The CIA constraint in (3) states that the holding of real money balances m_t by each household is used not only to finance the R&D investments, but also to partly purchase consumption c_t, where $\xi_c \in [0, 1]$ represents the share of consumption required to be purchased by cash/money.

Denote η_t, ω_t as the multipliers associated with the budget constraint in (2) and the CIA constraint in (3), respectively. The utility in (1) is maximized subject to (2) and (3) from which the first-order conditions for c_t, l_t, a_t, m_t, and b_t can be derived. After some manipulations, the first-order conditions can be reduced to the following optimality conditions. The standard Euler equation governs the growth of consumption given by

$$\frac{\dot{c}_t}{c_t} = r_t - \rho - g_L. \tag{4}$$

The optimal condition determines the consumption-leisure tradeoff such that

$$w_t(1 - l_t) = \theta c_t(1 + \xi_c i_t), \tag{5}$$

and the no-arbitrage condition between all assets and money implies the Fisher equation given by

$$i_t = r_t + \pi_t. \tag{6}$$

2.2 Final Goods

Final goods are produced by a mass of identical perfectly competitive firms that employ labor and a continuum of intermediate inputs according to the same constant returns to scale production technology. The production function of a typical firm k at time t is:

$$Y_{kt} = L_{ykt}^{1-\alpha} \int_0^{N_t} A_{it} x_{ikt}^{\alpha} di, \tag{7}$$

where L_{ykt} is the amount of labor employed by final-good firm k. N_t is the number of input varieties (or industries). A_{it} is the productivity level attached to the latest version of intermediate product i. x_{ikt} is the i-th type of intermediate inputs employed by firm k, and $\alpha \in (0, 1)$ is the elasticity of demand for intermediate products.

Firm k faces the following profit-maximization problem

$$\max_{L_{ykt}, x_{ikt}} \pi_{kt} = p_{yt} Y_{kt} - w_t L_{ykt} - p_{it} x_{ikt}$$

subject to (7) in which the final-good price p_{yt} is set as the numeraire (i.e., $p_{yt} \equiv 1$). Firm k chooses the amount of labor L_{ykt} and intermediate input x_{ikt} to maximize its profit, taking as given the wage rate w_t and the price of intermediate input p_{it}. The first-order condition with respect to x_{ikt} leads to the inverse demand for x_{ikt}:

$$p_{it} = \alpha A_{it}(L_{ykt}/x_{ikt})^{1-\alpha}.$$

Since all firms face the same price p_{it}, the input ratios must be identical across firms (i.e., $L_{ykt}/x_{ikt} = L_{yt}/x_{it}$), where $L_{yt} = \int L_{ykt}dk$ and $x_{it} = \int x_{ikt}dk$. Therefore, the above expression can be reduced to

$$p_{it} = \alpha A_{it}(L_{yt}/x_{it})^{1-\alpha}. \tag{8}$$

Similarly, the inverse demand for L_{ykt} is given by

$$w_t = (1-\alpha)\int_0^{N_t} A_{it}\left(\frac{x_{it}}{L_{yt}}\right)^{\alpha} di. \tag{9}$$

2.3 Incumbents

There is a continuum of industries N_t producing differentiated intermediate goods. Each industry is occupied by an industry leader that holds a patent on the latest innovation and monopolizes the production of one differentiated intermediate good i. The monopolistic leader dominates the market temporarily until its displacement by the next innovation.

The production technology across all incumbent firms is assumed to be identical, in which each incumbent requires α^2 units of final goods to produce one unit of intermediate good as in Acemoglu et al. (2012). Accordingly, firm i faces the following profit-maximization problem:

$$\max_{x_{it}} \pi_{it} = p_{it}x_{it} - \alpha^2 x_{it}.$$

Then the solution yields the optimal price $p_{it} = \alpha$, and thus the quantity of intermediate product i is given by

$$x_{it} = L_{yt}A_{it}^{\frac{1}{1-\alpha}}. \tag{10}$$

Substituting these results into π_{it} yields the equilibrium profit:

$$\pi_{it} = \alpha(1-\alpha)x_{it} = \alpha(1-\alpha)L_{yt}A_{it}^{\frac{1}{1-\alpha}}. \tag{11}$$

The industry leader i possesses this profit flow in each period until the arrival of next innovation.

2.4 Entrants

Following Howitt (1999) and Segerstrom (2000), a new firm (an entrant) can enter the market by either engaging in a vertical or a horizontal innovation. An entrant that engages in vertical innovation targets an existing industrial product line and devotes resources to improving the quality of that product. The product with the improved quality allows the innovator to replace the incumbent that introduced the original product and then become the industry leader until the next innovation in this industry occurs.

An entrant that engages in horizontal innovation devotes resources to create an entirely new industry. She then becomes a new industry leader with an exclusive patent right to produce a differentiated good until the arrival of the next vertical innovation targeted at this industry.

2.4.1 Vertical R&D

First, consider that the entrant j engages in vertical R&D by targeting an existing industry i to improve its product quality at time t with a successful rate of innovation ϕ_{ijt} that follows a Poisson process, which is given by

$$\phi_{ijt} = \frac{\lambda(L_{v,ijt})^{\delta}(K_{ijt})^{1-\delta}}{A_t}; \quad 0 < \delta < 1, \tag{12}$$

where λ is a positive R&D productivity parameter, $L_{v,ijt}$ is the level of firm j's R&D employment, K_{ijt} is the stock of the firm-specific knowledge possessed by firm j, and δ measures the degree of diminishing returns to vertical R&D expenditures. A_t is the leading-edge productivity parameter at time t defined as $A_t \equiv \max\{A_{it}; i \in [0, N_t]\}$, and is also interpreted as the force of increasing research complexity. The evolution of A_t will be discussed in detail in a later subsection.

To capture the monetary effect of the CIA constraint on vertical R&D, we assume that a fraction ξ_v of vertical R&D spending is constrained by cash/money. This cash constraint forces the R&D firm to borrow an amount $\xi_v w_t L_{v,ijt}$ of money at the nominal interest rate i from the household for financing the R&D expenditure. Accordingly, the profit-maximization problem for each potential entrant is

$$\begin{aligned} \max_{L_{v,ijt}} \quad & \phi_{ijt}\Pi_{vt} - w_t L_{v,ijt}(1-\xi_v) - w_t L_{v,ijt}\xi_v(1+i_t) \\ &= \phi_{ijt}\Pi_{vt} - w_t L_{v,ijt}(1+\xi_v i_t), \end{aligned}$$

where $\Pi_{vt} \equiv \int_t^{\infty} e^{-\int_t^{\tau}(r+\phi_s)ds}\hat{\pi}_{t\tau}d\tau$ is the expected present value of the innovative firm's profit flows before the replacement of the next successful innovation, and $\hat{\pi}_{t\tau}$ is the monopoly profit flow at time τ from a firm whose technology is of vintage t. As assumed in Howitt (1999) and Segerstrom (2000), each innovation at time t produces a new generation of products in that industry, which embodies the leading-edge productivity parameter A_t. This results in a continuous flow of the same monopoly profit $\hat{\pi}_{t\tau}$ across industries after time t and is given by $\hat{\pi}_{t\tau} = \alpha(1-\alpha)L_{y\tau}A_t^{1/(1-\alpha)}$. Moreover, r is the instantaneous interest rate, and ϕ_s is the rate of creative destruction, namely, the instantaneous flow probability of being displaced by an innovation. Along with the same instantaneous discount rate $r+\phi_s$ applying the same amount of profit flow $\hat{\pi}_{t\tau}$ earned by each industry leader, it is easy to deduce that the expected reward for vertical innovation Π_{vt} does not vary across industries.

At time t, a potential entrant j that targets the vertical R&D in industry i solves the above profit-maximization problem, yielding the first-order condition such that

$$\frac{\lambda\delta\Pi_{vt}}{A_t}\left(\frac{L_{v,ijt}}{K_{ijt}}\right)^{\delta-1} = w_t(1+\xi_v i_t), \tag{13}$$

which reveals that the marginal expected benefit of an extra unit of vertical R&D equals its marginal cost. It is clear from (13) that the marginal cost is positively correlated with the parameter ξ_v, capturing the adverse effect of the nominal interest rate i on the firm's R&D decision $L_{v,ijt}$ through increasing the marginal cost of vertical innovation.

Following Segerstrom (2000), K_{ijt} is considered to be the same and infinitesimally small for all j.[5] Given this assumption, (13) implies that $L_{v,ijt}/K_{ijt} = L_{v,it}/K_{it}$ for all j, where $L_{v,it} = \int L_{v,ijt}dj$ and $K_{it} = \int K_{ijt}dj$. In addition, we assume that $K_{it} \equiv \int K_{ijt}dj = L_t/N_t$ for all i, which is in line with Romer (1990), Segerstrom (2000), and Ha and Howitt (2007).[6] Thus, (13) can be re-expressed as

$$\frac{\lambda\delta\Pi_{vt}}{A_t}(l_{vt})^{\delta-1} = w_t(1+\xi_v i_t), \tag{14}$$

where $l_{vt} \equiv L_{vt}/L_t$ ($L_{vt} \equiv \int L_{v,it}di$) is the fraction of total labor employment that is allocated to vertical R&D. We further assume that the returns on conducting vertical R&D are identical across firm j and across time (see Segerstrom (2000)). This assumption together with the fact that $l_{vt} \equiv L_{vt}/L_t$ and $K_{it} = L_t/N_t$ indicates that the Poisson arrival rate of vertical innovation in each industry becomes

$$\phi_t = \int \phi_{ijt}dj = \frac{\lambda(L_{vt}/N_t)^{\delta}(L_t/N_t)^{1-\delta}}{A_t} = \lambda l_{vt}^{\delta}\iota_t, \tag{15}$$

where $\iota_t \equiv L_t/(A_t N_t)$. The expression (15) shows that the arrival rate of vertical innovations is increasing in per industry vertical R&D expenditure L_{vt}/N_t and the knowledge spillover L_t/N_t but decreasing in the R&D difficulty term A_t.

2.4.2 Horizontal R&D

An entrant q that engages in horizontal innovation devotes resources to create a new variety (and thus an entirely new industry). She faces the following rate of discovering new innovations, denoted as $\dot{N}_{qt}$:

$$\dot{N}_{qt} = \frac{\lambda(L_{hqt})^{\gamma}(\mathcal{K}_{qt})^{1-\gamma}}{A_t}; \quad 0 < \gamma < 1, \tag{16}$$

where L_{hqt} is the level of firm q's R&D employment, $\mathcal{K}_{qt}$ is the firm-specific knowledge possessed by firm q that is useful for horizontal innovation, and the exponent γ measures the degree of diminishing returns to horizontal R&D expenditures. A_t reflects the fact of increasing research complexity.

As in Howitt (1999) and Segerstrom (2000), we assume that each horizontal innovation at time t results in a new intermediate good variety whose productivity parameter is drawn randomly from an invariant long-run distribution of the existing productivity parameters A_{it} across industries i. This assumption ensures that the process of expanding variety will not affect the convergence of the distribution of existing parameters A_{it} to an invariant distribution in the long run. See the detailed discussion in the next subsection.

Next, to capture the monetary effect of the CIA constraint on horizontal R&D, we assume that a fraction ξ_h of horizontal R&D expenditure is constrained by cash/money. This cash constraint

[5] An infinitesimally small K_{ijt} implies that the optimal amount of firms' R&D resources $L_{v,ijt}$ is also infinitesimally small, governed by (13). Hence, the likelihood of any one firm winning a vertical R&D race can be negligible, given that the vertical R&D races are perfectly competitive.

[6] We mainly follow Ha and Howitt (2007) to capture the insight that the total amount of firm-specific knowledge in each industry equals per industry labor, which grows over time in equilibrium.

forces the innovative firm to borrow an amount $\xi_h w_t L_{hqt}$ of money at the nominal interest rate i from the household for financing the R&D expenditure. In addition, throughout the rest of this study, the assumption that $\xi_h > \xi_v$ is imposed to capture the empirical evidence that the investment on radical innovations is more constrained by cash/money than that on incremental innovations (e.g., Akcigit (2009) and Caggese (2015)). Accordingly, the profit-maximization problem for horizontal R&D firm q is

$$\max_{L_{hqt}} \pi_{hqt} = \dot{N}_{qt}\Pi_{ht} - w_t L_{hqt}(1+\xi_h i_t),$$

and

$$\Pi_{ht} = \Gamma^{-1}\Pi_{vt}, \tag{17}$$

where $\Gamma \equiv 1 + [\sigma/(1-\alpha)]$ and Π_{ht} is the expected value of a successful horizontal innovation. (17) reveals the relationship between Π_{ht} and Π_{vt} from the aforementioned assumption regarding the random draw of the productivity parameters, and the derivation of (17) will be provided in the next subsection.

Then, the first-order condition for horizontal R&D firms profit maximization is given by

$$\frac{\lambda\gamma\Pi_{ht}}{A_t}\left(\frac{L_{hqt}}{\mathcal{K}_{qt}}\right)^{\gamma-1} = w_t(1+\xi_h i_t). \tag{18}$$

This equation clearly shows that the marginal cost is positively related to the CIA parameter ξ_h, capturing the negative effect of the nominal interest rate i_t on the firm's R&D decision L_{hqt} through increasing the marginal cost of horizontal innovation $w_t(1+\xi_h i_t)$.

Moreover, (18) states that Π_{ht} only scales Π_{vt} with a constant factor, implying that Π_{ht} is also identical across all entrants q. Together with the same marginal cost faced by each entrant, the above first-order condition implies that $L_{hqt}/\mathcal{K}_{qt} = L_{ht}/\mathcal{K}_t$ for all q, where $L_{ht} \equiv \int L_{hqt}dq$ and $\mathcal{K}_t \equiv \int \mathcal{K}_{qt}dq$. Furthermore, a similar assumption is made such that $\mathcal{K}_{qt} = L_t/N_t$ for all q as in the previous subsection. Substituting $\mathcal{K}_{qt} = L_t/N_t$ and $L_{hqt}/\mathcal{K}_{qt} = L_{ht}/\mathcal{K}_t$ into (18) yields:

$$\frac{\gamma\lambda\Pi_{ht}}{A_t}(l_{ht})^{\gamma-1} = w_t(1+\xi_h i_t), \tag{19}$$

where $l_{ht} \equiv L_{ht}/L_t$ is the fraction of labor allocated to horizontal R&D. The growth rate of the measure of industries is the summation of the discovery rates for all the individual firms that engage in horizontal R&D, such that

$$g_{Nt} \equiv \frac{\dot{N}_t}{N_t} = \int \frac{\dot{N}_{qt}}{N_t}dq = \frac{\lambda(L_{ht}/N_t)^{\gamma}(L_t/N_t)^{1-\gamma}}{A_t} = \lambda l_{ht}^{\gamma}\iota_t. \tag{20}$$

2.4.3 Spillovers

As in Caballero and Jaffe (1993), Howitt (1999), and Segerstrom (2000), the leading-edge productivity parameter A_t grows over time as a result of knowledge spillovers produced by vertical innovation. The growth rate of A_t is proposed to take the following standard form

$$g_{At} \equiv \frac{\dot{A}_t}{A_t} = \left(\frac{\sigma}{N_t}\right)(\phi_t N_t) = \sigma\phi_t = \sigma\lambda l_{vt}^{\delta} l_t, \tag{21}$$

where $\sigma > 0$ measures the R&D spillover effect and $\phi_t = \int \phi_{ijt} dj$ is the Poisson arrival rate of vertical innovation in each industry $i \in [0, N_t]$ (namely, a summation of all potential vertical entrants).

As shown in (21), g_{A_t} can essentially be decomposed as a product of two factors σ/N_t and $\phi_t N_t$, where $\phi_t N_t$ is the aggregate flow of vertical innovation in this economy. (21) states that the growth of knowledge spillovers is assumed to be proportional to the aggregate flow of vertical innovation $\phi_t N_t$. The factor of proportionality σ/N_t measures the marginal effect of each vertical innovation on the stock of public knowledge. The divisor N_t captures the fact that each vertical innovation has a smaller impact on the aggregate economy as the number of specialized products expands with the development of the economy.

Because the distribution of productivity parameters among new products at any time is identical to the distribution across existing products at that time, one can show that the distribution of relative productivity parameters, which is defined as $z_{it} \equiv A_{it}/A_t$, will converge monotonically to the invariant distribution $Pr\{z_{it} \le z\} \equiv F(z) = z^{1/\sigma}$, wherein $0 < z \le 1$. It follows that in the long run:

$$E\left[\left(\frac{A_{it}}{A_t}\right)^{1/(1-\alpha)}\right] = \Gamma^{-1}, \tag{22}$$

where $\Gamma \equiv 1 + [\sigma/(1-\alpha)]$.[7]

Recall that the productivity parameter of each new innovative variety is drawn randomly from the above distribution. This implies that the realized monopoly profit flow for each horizontal R&D firm at date τ and its realized present value at time t are $\pi_{i\tau} = \alpha(1-\alpha)L_{y\tau}A_{it}^{1/(1-\alpha)}$ and $\Pi_{ht} = \int_t^{\infty} e^{-\int_t^{\tau}(r+\phi_s)ds}\pi_{i\tau}d\tau$, respectively. Along with the fact that a successful vertical innovation gains the profit flow $\hat{\pi}_{t\tau} = \alpha(1-\alpha)L_{y\tau}A_t^{1/(1-\alpha)}$ with the leading-edge productivity parameter A_t, it is easy to deduce that $\Pi_{ht} = (A_{it}/A_t)^{1/(1-\alpha)}\,\Pi_{vt}$. Taking expectations on both sides of this equation yields (17).

2.5 Monetary Authority

The monetary authority implements its monetary policy by targeting a long-run nominal interest rate i_t. Denote the nominal money supply by M_t, and thus the growth rate of nominal money supply is $\dot{M}_t/M_t = \mu_t$. Recall that m_t is real money balances per capita and is given by $m_t = M_t/(L_t p_{yt})$, so the growth rate of real money balance per capita is $g_{mt} \equiv \dot{m}_t/m_t = \mu_t - \pi_t - g_L$. Substituting this expression and the Euler equation (4) into the Fisher equation (6),

[7] See Howitt (1999) and Segerstrom (2000) for the detailed proof.

along with the fact that $g_{mt} = g_{ct}$ in the steady state,[8] we obtain

$$i_t = r_t + \pi_t = (\rho + g_{ct} + g_L) + (\mu_t - g_{mt} - g_L) = \rho + \mu_t. \tag{23}$$

This equation illustrates the existence of a one-by-one monotonic relationship between the nominal interest rate i_t and the growth rate of nominal money supply μ_t, which indicates an isomorphic choice of monetary instruments between i_t and μ_t. Specifically, an exogenous increase in i_t corresponds to an endogenous increase in μ_t.

Upon increasing the nominal interest rate i_t, the government earns the seigniorage revenue through an inflation tax. To balance the budget, it is assumed that the government returns the revenues as a lump-sum transfer to the household. Therefore, the government's budget constraint (in terms of per capita level) is given by $\dot{M}_t/(L_t p_{yt}) = \dot{m}_t + (\pi_t + g_L)m_t = \zeta_t$.

2.6 Characterization of Equilibrium

The equilibrium in this economy consists of a time path of prices $\{w_t, r_t, i_t, p_{it}, p_{yt}\}_{t=0}^{\infty}$, and a time path of allocations $\{c_t, m_t, l_t, Y_{kt}, Y_t, x_{it}, x_t, L_{ykt}, L_{v,ijt}, L_{hqt}\}_{t=0}^{\infty}$, where $Y_t = \int Y_{kt} dk$ and $x_t = \int_0^{N_t} x_{it} di$. Moreover, at each instant of time,

- individuals maximize utility taking $\{i_t, r_t, w_t\}$ as given;
- the competitive final-goods firms produce $\{y_{kt}\}$ to maximize profits taking $\{p_{yt}\}$ as given;
- the monopolistic intermediate-goods firms produce $\{x_{it}\}$ and choose $\{Y_t, p_{it}\}$ to maximize profits taking $\{p_{yt}\}$ as given;
- the labor market clears such that $L_{yt} + L_{vt} + L_{ht} = l_t L_t$;
- the final-goods market clears such that $Y_t = C_t + x_t$;
- the asset market clears such that the value of monopolistic firms adds up to the value of households' assets: $\Pi_{vt} + \Pi_{ht} = a_t L_t$; and
- the amount of money borrowed by the two types of innovation entrants is given by $b_t L_t = \xi_v w_t L_{vt} + \xi_h w_t L_{ht}$.

Using (22), we obtain $\int_0^{N_t} A_{it}^{\frac{1}{1-\alpha}} di = A_t^{\frac{1}{1-\alpha}} N_t \int_0^1 z^{\frac{1}{1-\alpha}} F'(z) dz = A_t^{\frac{1}{1-\alpha}} N_t \Gamma^{-1}$. Substituting this equation, (10), and $Y_t = \int Y_{kt} dk$ into (7) yields the equilibrium final-goods production function:

$$Y_t = \frac{L_{yt} A_t^{\frac{1}{1-\alpha}} N_t}{\Gamma}. \tag{24}$$

Accordingly, the per-capita consumption and the production-labor shares of output are, respectively,

$$c_t = \frac{(1-\alpha^2) l_{yt} A_t^{\frac{1}{1-\alpha}} N_t}{\Gamma}, \tag{25}$$

and

$$w_t = (1-\alpha)\frac{Y_t}{L_{yt}} = \frac{(1-\alpha) A_t^{\frac{1}{1-\alpha}} N_t}{\Gamma}. \tag{26}$$

[8] According to the CIA constraint (3), it can be shown that on the balanced growth path, m_t and c_t grow at the same rate.

2.7 Balanced-Growth Properties

In this section, we follow Segerstrom (2000) to focus on the analysis of the balanced-growth equilibrium properties of the model. In the balanced-growth equilibrium, the fraction of labor supplied to each sector must be constant over time (i.e., $l_{vt} = l_v, l_{ht} = l_h, l_{yt} = l_y$ for all t). Since both g_{At} and g_{Nt} must be constant in a balanced-growth equilibrium, (12) implies that the arrival rate of vertical innovations must be constant as well (i.e., $\phi_t = \phi$ for all t). Furthermore, according to (21) and (20), ι_t must be constant in the balanced-growth equilibrium (i.e., $\iota_t = \iota$ for all t). Thus, the quality and variety growth rates can, respectively, be written as

$$g_A = \sigma \lambda l_v^{\delta} \iota, \tag{27}$$

and

$$g_N = \lambda l_h^{\gamma} \iota. \tag{28}$$

2.7.1 Economic Growth

Denote by g the growth rate of consumption per capita c_t on the balanced-growth path (and economic growth rate thereafter). Differentiating the per-capita consumption share of output (25) with respect to time yields

$$g = g_N + \frac{1}{1-\alpha} g_A. \tag{29}$$

This equation, called the *iso-growth* condition, demonstrates that on the balanced-growth path (BGP), the growth rate of the measure of industries g_N and the growth rate of productivity of industries g_A jointly determine the overall rate of economic growth g.

2.7.2 Population-Growth Condition

Moreover, differentiating $\iota_t = \frac{L_t}{A_t N_t} = \iota$ with respect to time t yields the population-growth condition such that

$$g_L = g_A + g_N. \tag{30}$$

This equation states that to guarantee a BGP, the growth rate of the leading-edge productivity parameter A_t and that of the measure of variety N_t are required to grow in a manner in the sense that these growth rates are constrained by the population-growth rate g_L. The intuition behind this constraint is as follows. As the economy grows with higher levels of A_t and N_t, research becomes more complex, and thus the productivity of researchers ι_t falls in response. To maintain a constant innovation rate in g_N and g_A over time as stipulated in (20) and (21), more workers are needed to focus on R&D activities. The population-growth rate g_L determines the rates at which labor resources can be devoted into both horizontal and vertical R&D activities and therefore determines the overall growth rate of the economy.

Additionally, examining both equations (29) and (30) yields the following result:

Lemma 1. *In the steady-state equilibrium, the economic growth rate is increasing in the vertical R&D growth rate.*

The intuition underlying this lemma is straightforward. The population-growth condition (30) implies that there is an equal tradeoff between g_A and g_N (i.e., an increase in g_A comes at the cost of an identical amount of reduction in g_N to maintain a constant population-growth rate). However, the iso-growth condition (29) reveals that the economic growth rate stems from a larger contribution of g_A than g_N (i.e., $1/(1-\alpha) > 1$). Therefore, an increase in g_A at the sacrifice of g_N comes with a higher economic growth rate. This theoretical attribute is also available in Howitt (1999) in that the economic growth rate is eventually supported by the growth from creative destruction (vertical innovation) rather than variety expansion (horizontal innovation) in the steady-state equilibrium. In addition, this implication is consistent with the empirical finding in Garcia-Macia et al. (2016), who decompose the aggregate total factor productivity growth for the US within the periods 1976-1986 and 2003-2013 and find that the contribution of growth from creative destruction is overwhelmingly larger than that from new varieties.

3 Growth Effects of Monetary Policy

In this section, we analyze the growth effects of monetary policy (in terms of nominal interest rate targeting) on growth with various CIA constraints. To fully comprehend the underlying mechanism, we first proceed in our analysis with different scenarios, each of which is subject to one distinct type of CIA constraint. After picking up the intuition behind each scenario, we impose all types of CIA constraints simultaneously and then provide a complete analysis.[9]

3.1 CIA constraint on Consumption

First, we analyze the case in which only a CIA constraint on consumption is present, and the following proposition is obtained:

Proposition 1. *In the presence of a CIA constraint on consumption only (i.e., $\xi_c > 0$, $\xi_v = \xi_h = 0$), a higher nominal interest rate decreases (increases) the economic growth rate if $\gamma < \delta$ ($\gamma > \delta$).*

Fig.1 illustrates the effects of a permanent increase in the nominal interest rate i on the economic growth rate when the model only features a CIA constraint on consumption. Using both the iso-growth condition in (29) and the population-growth condition in (30), we can derive two downward sloping lines with a slope of $-1/(1-\alpha)$ and of -1, respectively, in the (g_A, g_N) space. Thus, the slope of each iso-growth line exceeds the slope of the population-growth condition (in absolute value terms).

To better understand the intuition underlying Proposition 1, we, first, analyze the instant effect of raising i starting from the initial balanced-growth equilibrium. When only consumption is subject to the CIA constraint, increasing the nominal interest rate i raises the cost for consumption purchases relative to leisure. As a result, individuals enjoy more leisure by reducing their labor supply, and thus the equilibrium labor for both R&D activities l_h and l_v declines. More importantly, l_h decreases by a smaller (larger) amount than l_v if horizontal R&D exhibits greater (smaller) diminishing returns than vertical R&D (i.e., $\gamma < (>) \delta$). In Fig.1, to reflect the case where $\gamma < \delta$, a higher i leads the economy to jump from the initial steady state A to B' with a

[9] The detailed technical proofs of the propositions in this section are available from the authors upon request.

smaller reduction in g_N than in g_A. By contrast, to reflect the case where $\gamma > \delta$, a higher i shifts the economy from A to C', with a larger reduction in g_N than in g_A.

Next, we follow Segerstrom (2000) to provide an intuitive explanation about how the economy adjusts after its instant shift off the balanced-growth path. The corresponding decreases in g_A and g_N indicate that the research complexity grows at a slower rate than usual. It follows that the research productivity ι_t rises gradually over time, which drives up g_A and g_N again as indicated in (27) and (28), until they are back to the balanced-growth equilibrium. That is, the population-growth condition is satisfied again. Therefore, there are two cases to be considered.

When $\gamma < \delta$, raising i initially drives the economy to jump from A to B' (i.e., a larger decrease in g_A than in g_N). Then the research productivity ι_t rises over time, driving up g_A and g_N gradually by a similar magnitude, which induces the economy to move from point B' to the new balanced-growth path B. It is clear that the long-run effect of raising the nominal interest rate boosts the horizontal innovation rate g_N at the expense of reducing g_A. Then the economic growth rate will decrease in response as shown in Lemma 1.

When $\gamma > \delta$, raising i initially drives the economy to jump from A to C' (i.e., a smaller decrease in g_A than in g_N). This force subsequently induces the economy to move from point C' to the new balanced-growth path C. In this case, the long-run effect of raising the nominal interest rate boosts the vertical innovation rate g_A at the expense of reducing g_N. As a result, the economic growth rate will increase in response as implied by Lemma 1.

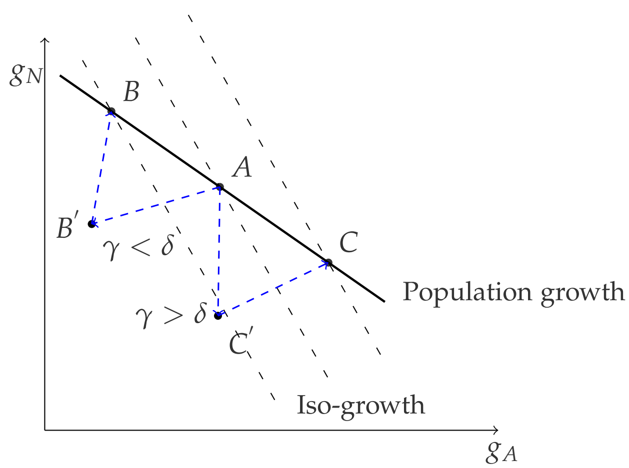

Fig. 1. Adjustment process: CIA constraint on consumption.

3.2 CIA constraint on Vertical R&D

In this subsection, we analyze the case in which only a CIA constraint on vertical R&D is present, and the following result is obtained:

Proposition 2. *In the presence of a CIA constraint on vertical R&D only (i.e., $\xi_v > 0$, $\xi_c = \xi_h = 0$), a higher nominal interest rate decreases the economic growth rate under both $\gamma < \delta$ and $\gamma > \delta$, but by a larger amount under $\gamma < \delta$.*

Fig.2 illustrates the effects of a permanent increase in the nominal interest rate i on growth when the model only features a CIA constraint on vertical R&D. Similar to Subsection 3.1, the analysis starts off by exploring the instant effect of raising i from the initial balanced-growth equilibrium.

When only vertical R&D is subject to the CIA constraint, an increase in i raises the cost for vertical R&D. The labor force will be reallocated from vertical R&D l_v to production l_y, horizontal R&D l_h, and leisure. Under $\gamma < \delta$, greater diminishing returns to horizontal R&D will reallocate less labor force to l_h, allowing only for a smaller increase in g_N; in addition, a high level of δ causes a decrease in l_v to transmit a larger reduction in g_A as shown in (27). In Fig.2, the economy, therefore, moves from A to $B^{'}$ in this case. By contrast, under $\gamma > \delta$, smaller diminishing returns to horizontal R&D will reallocate more labor force to l_h, leading to a higher g_N; in addition, a low level of δ also causes a decrease in l_v to transmit a smaller reduction in g_A. In Fig.2, the economy will move from A to $C^{'}$ if the gap between γ and δ is small (i.e., γ is slightly larger than δ), and thus the magnitudes of the changes in g_N and g_A are close. Otherwise, the economy will move from A to $C^{''}$ if the gap between γ and δ is large (i.e., γ is much larger than δ), and thus the size of the increase in g_N is much more significant than that of the decrease in g_A.

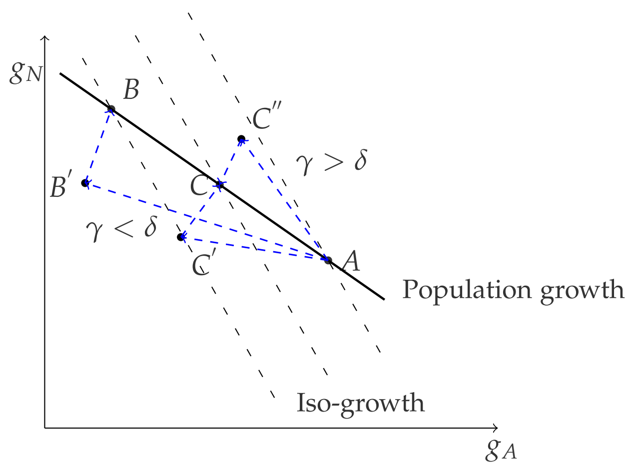

Fig. 2. Adjustment process: CIA constraint on vertical R&D.

Next, we turn to intuitively explain the adjustment process. There are three scenarios to be considered. First, when $\gamma < \delta$, since the magnitude of the decrease in g_A is much larger than that of the increase in g_N as shown in the movement from point A to point $B^{'}$, the growth of research complexity is driven down to a lower rate than usual. It follows immediately that the research productivity ι_t rises over time. Hence, g_A and g_N grow gradually in a similar manor, inducing the economy to move from point $B^{'}$ to the new balanced-growth path B. Second, when $\gamma > \delta$ and the gap between γ and δ is small, the close magnitudes of the changes in g_A and g_N may still drive down the growth of research complexity to a lower rate than usual. It then follows that ι_t rises over time. Hence, g_A and g_N grow gradually in a similar manner, inducing the economy to move from point $C^{'}$ to the new balanced-growth path C. Third, when $\gamma > \delta$ and the gap between γ and δ is large, the magnitude of the increase in g_N is greater than that of the decrease in g_A. In this case, the growth of research complexity is driven up to a higher rate than usual. It follows that the research productivity ι_t will fall over time. Hence, g_A and g_N are lowered gradually in a similar manner, inducing the economy to move from point C'' to the new balanced-growth path C.

To sum up, the long-run growth effect of raising i increases g_N at the expense of reducing g_A regardless of the comparison between γ and δ. Nevertheless, the reduction in g_A turns out to be more significant under $\gamma < \delta$ than under $\gamma > \delta$. Consequently, according to Lemma 1, the economic growth rate is decreasing in i more considerably under $\gamma < \delta$ than under $\gamma > \delta$.

3.3 CIA constraint on Horizontal R&D

In this subsection, we analyze the case in which only a CIA constraint on horizontal R&D is present, and the following result is obtained:

Proposition 3. *In the presence of a CIA constraint on horizontal R&D only (i.e., $\xi_h > 0$, $\xi_v = \xi_c = 0$), a higher nominal interest rate increases the economic growth rate under both $\gamma < \delta$ and $\gamma > \delta$, but by a larger amount under $\gamma < \delta$.*

Fig.3 illustrates the growth effects of a permanent increase in the nominal interest rate i when the model only features a CIA constraint on horizontal R&D. Again, the analysis starts off by studying the instant effect of raising i from the initial balanced-growth equilibrium.

When horizontal R&D is subject to the CIA constraint, the instant effects of raising i are just the opposite to those in Subsection 3.2. An increase in i raises the cost for horizontal R&D, reallocating workers from horizontal R&D l_h to production l_y, vertical R&D l_v, and leisure. On the one hand, when $\gamma < \delta$, namely, the diminishing returns to vertical R&D are small, more labor force will be reallocated to l_v leading to a larger rise in g_A. In Fig.3, if the gap between γ and δ is small, then the economy will move from point A to B', since the magnitude of the increase in g_A is not significant compared to the magnitude of the decrease in g_N, as shown in (27) and (28). By contrast, if the gap between γ and δ is large, the economy would move from point A to B'', since the magnitude of the increase in g_A becomes larger than the magnitude of the decrease in g_N. On the other hand, when $\gamma > \delta$, namely, the diminishing returns to vertical R&D are large, less labor force will be reallocated to l_v leading to a smaller rise in g_A. Therefore, the economy will move from point A to C', given that the size of the decrease in g_N is significantly larger than that of the increase in g_A.

Now, we turn to intuitively explain the adjustment process. Under $\gamma < \delta$, if the gap between γ and δ is small, the increase in g_A is not significant enough to dominate the decrease in g_N. As a result, the research productivity ι_t grows over time, driving up both g_A and g_N, and therefore the economy moves from point B' to the new balanced-growth path B, as displayed in Fig.3. However, if the gap between γ and δ is large, the increase in g_A is, instead, more likely to dominate the decrease in g_N, which drives up the growth of research complexity to a higher rate than usual. As a result, the research productivity falls over time and g_A and g_N are reduced, so the economy moves from point B'' to B.

Under $\gamma > \delta$, since the magnitude of the decrease in g_N is much larger than that of the increase in g_A as shown in the movement from point A to point C', the growth of research complexity is driven down to a lower rate than usual. It follows immediately that the research productivity ι_t rises over time. Hence, g_A and g_N grow gradually in a similar manner, inducing the economy to move from point C' to the new balanced-growth path C.

In sum, the long-run growth effect of raising i increases g_A at the expense of reducing g_N regardless of the comparison between γ and δ. Nevertheless, the increase in g_A turns out to be more significant under $\gamma < \delta$ than under $\gamma > \delta$. Consequently, according to Lemma 1, the economic growth rate is increasing in i more considerably under $\gamma < \delta$ than under $\gamma > \delta$.

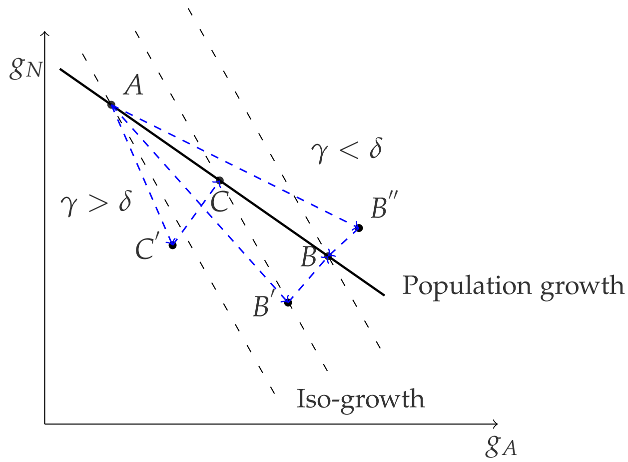

Fig. 3. Adjustment process: CIA constraint on horizontal R&D.

3.4 CIA constraint on Consumption and R&D

After building up the intuition underlying each scenario in which only one type of the CIA constraints is present, we are now in a position to analyze a more general case by incorporating all types of CIA constraints into the model. To highlight the interesting non-monotonic relationship between inflation and growth, our analysis focuses on the empirically relevant case where $\gamma < \delta$.[10] Accordingly, we obtain the following result:

Proposition 4. *Suppose that $\gamma < \delta$ holds. Then (a) for a sufficiently large gap between ξ_h and ξ_v, the economic growth rate g has a non-monotonic (i.e., an inverted-U) relationship with the nominal interest rate i, and there exists a threshold value i^* below (above) which g is increasing (decreasing) in i; (b) for an insufficiently large gap between ξ_h and ξ_v, g is monotonically decreasing in i.*

To intuitively explain the results of Proposition 4, we need to combine the results obtained in Propositions 1-3. Recall that from Propositions 1 and 2, in the case where $\gamma < \delta$, the CIA constraints on both consumption and vertical R&D yield a negative relationship between the nominal interest rate i and the economic growth rate g, and only Proposition 3 (i.e., the CIA constraint on horizontal R&D) can generate a positive relationship between them. It is obvious that an inverted-U shape requires a positive relationship between the nominal interest rate and economic growth at the relatively low levels of i. This implies that the growth effect of i from

[10]In the case where $\gamma > \delta$, the economic growth rate is monotonically increasing in the inflation rate when all CIA constraints are present. The detailed analysis for this case is available from the authors upon request.

the CIA constraint on horizontal R&D has to be relatively strong to dominate the other two effects from the CIA constraints on consumption and vertical R&D. At the initial increase in i, the distortions of the CIA constraints are mild, which implies that the above three effects are all weak. Therefore, to ensure a strong positive effect of the constraint on horizontal R&D at the initial increase in i, there must be a sufficiently large extent of the constraint on horizontal R&D relative to vertical R&D (i.e., a sufficiently large gap in $\xi_h > \xi_v$), so that raising i yields a strong reallocation effect from l_h to l_v to generate a high level of g_A to enhance g.

Nevertheless, as i continues to rise, the greater diminishing returns to horizontal R&D relative to vertical R&D (i.e., $\gamma < \delta$) come into play and exert a counter-force that weakens the reallocation effect from l_h to l_v. This counter-force increases non-linearly in i which, henceforth, results in the negative growth effects from the constraints on vertical R&D and consumption becoming stronger than the positive effects from the constraint on horizontal R&D. This implies that there will be a threshold rate of nominal interest i^* across which the two negative growth effects play the dominant role, so that g becomes monotonically decreasing in i.

Finally, if the gap in $\xi_h > \xi_v$ is not sufficiently large, the reallocation effect of i from the constraint on horizontal R&D is weak at the initial increase in i, so it will be dominated by the two negative effects as i rises. Accordingly, it is straightforward to explain that g becomes monotonically decreasing in i for all levels of i.

4 Quantitative Analysis

In this section, our model is calibrated to quantify the growth effects of monetary policy. We show that by using an empirically plausible range of parameter values, there exists an inverted-U relationship between the nominal interest rate and economic growth in the calibrated economy, which is consistent with the existing empirical findings. In addition, we evaluate the effects of monetary policy on (steady-state) social welfare.

4.1 Calibration

To make the quantitative analysis more realistic, our model is calibrated to match the aggregate data of the US economy. Our model features the following set of parameters $\{\rho, \alpha, g_L, \xi_c, \xi_v, \xi_h, \gamma, \delta\}$ and the policy variable i. The discount rate is set to a standard value $\rho = 0.02$ as in Grossmann et al. (2013). We follow Jones and Williams (2000) to set the capital share to a standard value such that $\alpha = 0.36$, which is an estimate of the US economy during 1951-2000. According to the Conference Board Total Economy Database, g_L is thereafter set to 1.2% to correspond to the population growth rate in the US during this period.

As for the degree of various types of CIA constraints $\{\xi_c, \xi_v, \xi_h\}$, the degree of the CIA constraint on consumption ξ_c is set to 0.29, which lies in a reasonable range of M1-consumption ratios (see, for example, Dotsey and Sarte (2000) and Chu et al. (2010)). We focus on the case where $\xi_h > \xi_v$ to capture the empirical findings that firms which tend to engage in radical innovation are more constrained by cash/money than their old and large counterparts that conduct incremental innovation (e.g., Akcigit (2009) and Caggese (2015)). Additionally, following Chu et al. (2015), the strengths of CIA constraints on horizontal and vertical innovative activities are set to $\xi_h = 0.6$ and $\xi_v = 0.4$, respectively, as the benchmark values, which are considered to feature a sufficiently large gap between ξ_h and ξ_v. Our analysis will also consider the case with $\xi_h = 0.5$ and $\xi_v = 0.4$, and the case where $\xi_h = 0.45$ and $\xi_v = 0.4$, respectively, which are referred to as the cases of an insufficiently large gap in the CIA constraints between the two R&D activities.

Next, as for the values of the two R&D diminishing returns γ and δ, we choose the empirically plausible case of $\gamma < \delta$ in the benchmark.[11] Specifically, we set $\delta = 0.8$ and $\gamma = 0.6$,[12] respectively, both of which lie in the range of the estimated elasticity of innovative outputs with respect to R&D expenditures documented in Anselin et al. (1997) and Acs et al. (2002).[13].

Moreover, we follow Jones and Williams (2000) to set the equilibrium rate of economic growth in the benchmark as $g = 1.25\%$. Then the market-level nominal interest rate i is calibrated by targeting at $\pi = 2.5\%$, which is in line with the average inflation rate in the US economy. Given the above calibrated parameter values, we calibrate σ and θ simultaneously to match the equilibrium growth rate and the standard time of employment $l = 0.33$ by using the labor-leisure choice (5), the first-order conditions for the vertical and horizontal R&D (13) and (18), the iso-growth condition (29), the population growth condition (30), and the labor-market-clearing condition. The detailed calibration procedure is relegated to the online Appendix. The parameter values are summarized in Table 1.

Table 1: Calibration

Targets						r	g	π
						0.0445	0.0125	0.025
Parameters	ξ_h	ξ_v	ξ_c	γ	δ	θ	σ	i
	0.6	0.4	0.29	0.6	0.8	1.5051	0.2789	0.0695
	0.5	0.4	0.29	0.6	0.8	1.5054	0.2812	0.0695
	0.45	0.4	0.29	0.6	0.8	1.5056	0.2824	0.0695

4.2 Growth and Welfare Implications of Monetary Policy

Fig.4 displays the quantitative results under the benchmark parameter values, wherein we find that the rate of economic growth is an inverted-U function of the inflation rate given that $\xi_h - \xi_v = 0.2$. This result supports the implication of Proposition 4; when the difference between ξ_v and ξ_h is relatively large, at low levels of i the positive growth effect of i through the CIA constraint on horizontal R&D strictly dominates the negative growth effects through the CIA constraints on vertical R&D and consumption. Nevertheless, as i rises, this domination becomes increasingly weaker and finally the negative effects overwhelm the positive one. In addition, the threshold value of i is roughly 2.8%, which is in line with the empirical estimates of Ghosh and Phillips (1998) (i.e., 2.5%), López-Villavicencio and Mignon (2011) (i.e., 2.7%), and Eggoh and Khan (2014) (i.e., 3.4%).

[11] We examine $\gamma > \delta$ in the sensitivity check to complete the quantitative illustrations.

[12] We also perform a sensitivity analysis by choosing alternative values to show that our basic results are robust to this change.

[13] Anselin et al. (1997) and Acs et al. (2002) estimate this elasticity by using the number of patents as a proxy for new knowledge, resulting in a range of [0.54, 0.85].

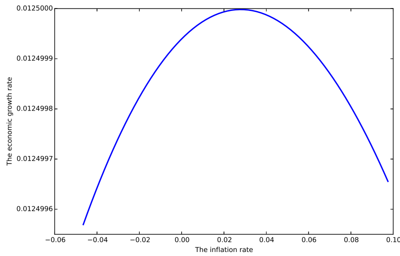

Fig. 4. Inflation and economic growth($\xi_h - \xi_v = 0.2$)

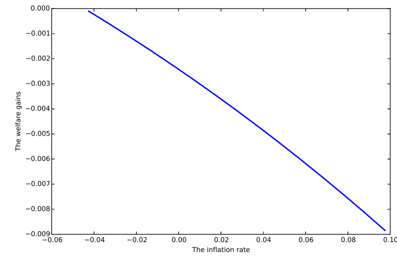

Fig. 5. Inflation and social welfare($\xi_h - \xi_v = 0.2$)

To explore the welfare effects of monetary policy, we impose balanced growth on (1) to derive the steady-state welfare function

$$U = \frac{1}{\rho}\left[\ln c_0 + \frac{g}{\rho} + \theta \ln(1-l)\right] \tag{31}$$

where the exogenous terms have been dropped and $c_0 = (1-\alpha^2)l_y/\Gamma$ is the steady-state level of consumption along the BGP. Fig.5 accordingly depicts the welfare effect of the inflation rate. It is shown that the level of welfare is monotonically decreasing in the inflation rate. Specifically, a 10 percentage points increase in the inflation rate, from -4.45% (corresponding to the zero nominal interest rate) to 5.55%, leads approximately to a welfare loss of 0.587%. The intuition can be explained as follows. There are two positive welfare effects of a higher rate of inflation (or raising the nominal interest rate). The first effect stems from the growth effect for an inflation rate that is below the threshold value, as previously mentioned. The second effect comes from the increase in leisure, which leads to a higher utility level. However, given our benchmark parameter values, these two positive welfare effects are completely dominated by the negative welfare effect from the decrease in the households' initial income level. This mainly arises from the CIA constraint on consumption, which reduces labor employment in the final-goods sector and hence the level of c_0. Furthermore, as the inflation rate increases to a permanently higher rate that is above the threshold, the positive growth effect becomes negative, resulting in the overall welfare effect of inflation always be negative. Therefore, this model predicts a monotonically decreasing relationship between welfare and inflation in the benchmark case.

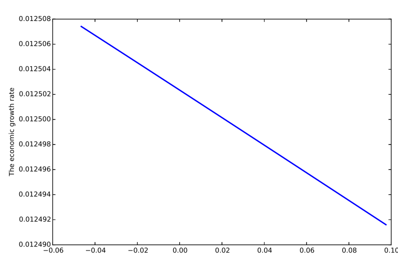

Fig. 6. Inflation and economic growth($\xi_h - \xi_v = 0.1$)

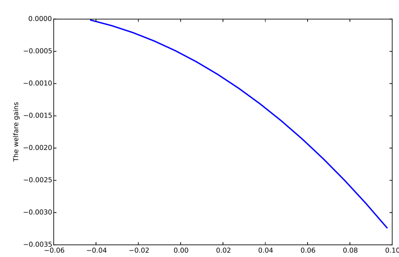

Fig. 7. Inflation and social welfare($\xi_h - \xi_v = 0.1$)

Nevertheless, some existing empirical studies (e.g.,Vaona (2012) and Barro (2013)) also find a evidence of long-run negative effect of inflation on economic growth. In fact, given that the majority of the current calibrated values of parameters are preserved, our model is also sufficiently flexible to generate a negative relationship between inflation and economic growth. Fig.6 illustrates this scenario accordingly. We recalibrate the values of the parameters such that the gap between ξ_v and ξ_h from 0.2 (i.e., $\xi_v = 0.4$, $\xi_h = 0.6$) is shrunk to 0.1 (i.e., $\xi_v = 0.4$, $\xi_h = 0.5$). It is found that the inflation-growth relationship becomes strictly negative, which is still consistent with the predictions of the analytical part. The welfare is also decreasing in the rate of inflation, as shown in Fig.7. There is approximately a 0.180% welfare loss when the inflation rate is increased by 10 percentage points (from -4.45% to 5.55%), and a larger welfare loss (i.e., 0.592%) is thereafter attained when the inflation rate continues to increase from 5.55% to 15.55%.

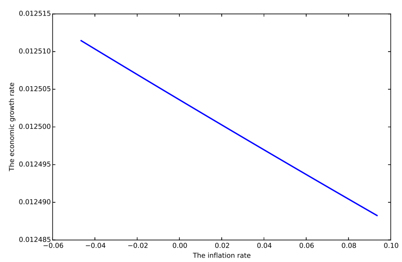

Fig. 8. Inflation and economic growth($\xi_h - \xi_v = 0.05$)

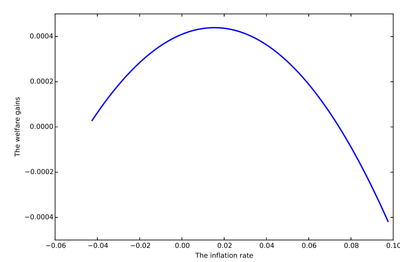

Fig. 9. Inflation and social welfare($\xi_h - \xi_v = 0.05$)

Moreover, if the gap between ξ_v and ξ_h is shrunk to an even smaller value of 0.05 (i.e., $\xi_v = 0.4$, $\xi_h = 0.45$), Fig.8 shows that the monotonically decreasing relationship between the inflation rate and economic growth rate continues to hold. Interestingly, a higher inflation rate generates an inverted-U shaped effect on welfare in this case (see Fig.9). As the inflation rate becomes higher, the positive welfare effect from the increase in leisure will initially dominate the negative welfare effects from a lower economic growth rate and a lower level of consumption, but this domination is reversed as the inflation rate increases. This in turn implies that the Friedman rule, which is optimal in the aforementioned cases, becomes suboptimal. In this case, our model predicts a welfare-maximizing inflation rate of 1.7%.

4.3 Sensitivity

In this subsection, we undertake sensitivity checks to test the robustness of our numerical results in terms of quantitative magnitudes. Specifically, this sensitivity exercise is conducted by varying several key parameters. The parameter values that will be altered are summarized in Table 2.

First, we perform a sensitivity analysis by examining the analytical results in three special scenarios, namely, $\xi_c > 0(\xi_v = \xi_h = 0)$, $\xi_v > 0(\xi_c = \xi_h = 0)$, and $\xi_h > 0(\xi_c = \xi_v = 0)$, respectively. Fig.10, Fig.12, and Fig.14 illustrate that the inflation rate and economic growth rate are negatively correlated in the cases of the CIA constraint on only consumption and on only vertical R&D, but positively correlated in the case of the CIA constraint on only horizontal R&D. These quantitative results are in line with our analytical implications in Propositions 1-3. In addition, the corresponding effects of a higher inflation rate on social welfare are depicted in

Table 2: Sensitivity

Targets						r	g	π
						0.0445	0.0125	0.025
Parameters	ξ_h	ξ_v	ξ_c	θ	σ	γ	δ	i
	0	0	0.29	1.6152	0.2862	0.6	0.8	0.0695
	0	0.4	0	1.5374	0.2937	0.6	0.8	0.0695
	0.6	0	0	1.6449	0.2713	0.6	0.8	0.0695
	0.6	0.4	0.26	1.5082	0.2788	0.6	0.8	0.0695
	0.6	0.4	0.29	0.0439	0.0211	0.8	0.6	0.0695
	0.6	0.4	0.29	1.4811	0.3511	0.5	0.75	0.0695
	0.7	0.4	0.29	1.4807	0.3489	0.5	0.75	0.0695

Note: The parameters θ and σ are re-calibrated in each case to maintain the standard moments of the equilibrium economic growth rate and time of employment as shown in the benchmark case.

Fig.11, Fig.13, and Fig.15, respectively. That is, the inflation-welfare relationship is inverted-U-shaped under $\xi_c > 0(\xi_v = \xi_h = 0)$, positive under $\xi_v > 0(\xi_c = \xi_h = 0)$, and negative under $\xi_h > 0(\xi_c = \xi_v = 0)$, respectively.

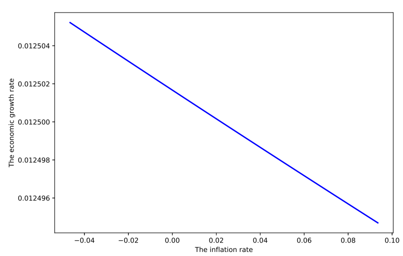

Fig. 10. Inflation and economic growth ($\xi_c > 0$).

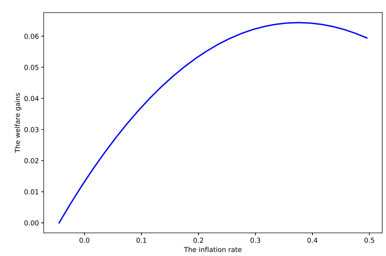

Fig. 11. Inflation and social welfare ($\xi_c > 0$).

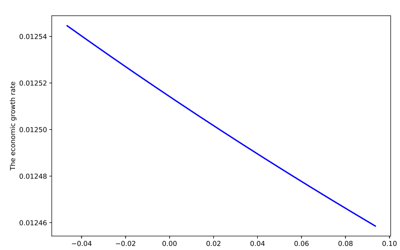

Fig. 12. Inflation and economic growth ($\xi_v > 0$).

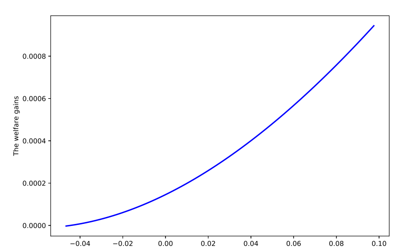

Fig. 13. Inflation and social welfare ($\xi_v > 0$).

Second, a sensitivity analysis is performed by changing the value of parameter ξ_c from 0.29 in the benchmark to 0.26. By comparing Fig.4 and Fig.16, a lower degree of ξ_c shifts the inflation-growth curve to the right, implying a higher threshold value of inflation i^*. A smaller ξ_c weakens

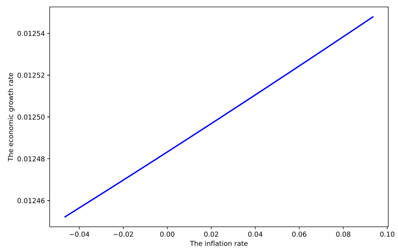

Fig. 14. Inflation and economic growth ($\xi_h > 0$).

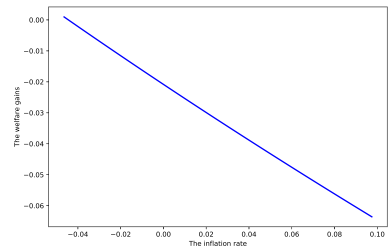

Fig. 15. Inflation and social welfare ($\xi_h > 0$).

the negative inflation-growth effect arising from the CIA constraint on consumption, as shown in Proposition 1. Therefore, for a given level of ξ_v and of ξ_h, which, respectively, determines the negative inflation-growth effect and the positive one, a larger increase in the inflation rate (and also the nominal interest rate) is required to make the negative inflation-growth effect sufficiently strong to dominate the positive inflation-growth effect from the CIA constraint on horizontal innovation. As a result, the threshold value i^* increases to around 8%, which is close to the empirical estimates in Sarel (1996), Burdekin et al. (2004), and Yilmazkuday (2013) (i.e., 8%). As for social welfare, a 10 percentage points increase in the inflation rate slightly enlarges the welfare loss under a smaller ξ_c (i.e., 0.773%) as compared to the benchmark case (i.e., 0.587%).

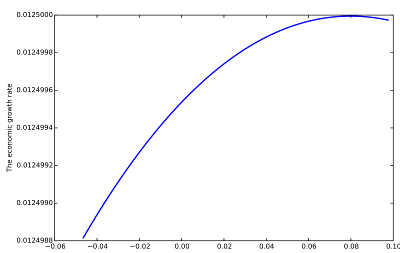

Fig. 16. Inflation and economic growth ($\xi_c = 0.26$)

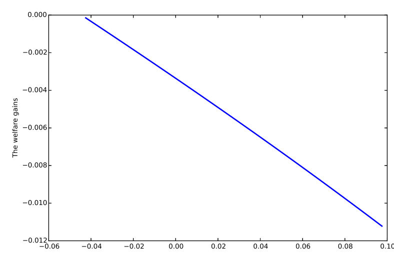

Fig. 17. Inflation and social welfare ($\xi_c = 0.26$)

Third, to capture the impacts of diminishing returns in the two types of innovation, we consider an alternative case in which $\gamma > \delta$ (i.e., $\gamma = 0.8$ and $\delta = 0.6$), although it is less empirically realistic. Our model predicts that under $\gamma > \delta$, the economic growth rate is monotonically increasing in the inflation rate as shown in footnote 9. The quantitative result displayed in Fig.18 is consistent with the prediction of the model; an increase in the nominal interest rate (and then an increase in the inflation rate) monotonically raises the economic growth rate. As illustrated in Proposition 1, $\gamma > \delta$ leads to a positive inflation-growth effect in the presence of the CIA constraint on consumption, and, as a result, together with another positive effect from the CIA constraint on horizontal R&D, an increase in the inflation rate always raises the economic growth rate. In addition, the inflation rate is monotonically increasing in social welfare (see Fig.19). In particular, raising the inflation rate by 10 percentage points from -4.45% to 5.55% yields a welfare gain of 3.889%. In this case, the positive welfare effect, stemming from the positive growth effect under a higher inflation rate, reinforces another positive welfare effect from a higher degree of leisure to strictly dominate the negative welfare effect due to a lower initial income level. Therefore, a higher inflation rate results in welfare gains.

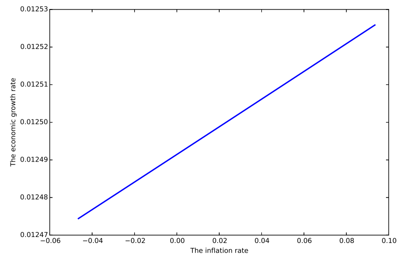

Fig. 18. Inflation and economic growth ($\gamma > \delta$).

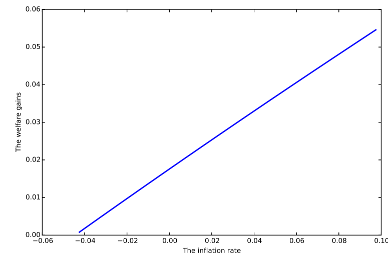

Fig. 19. Inflation and social welfare ($\gamma > \delta$).

Fourth, we perform a sensitivity analysis by altering the values for γ and δ to $\gamma = 0.5$ and $\delta = 0.75$, respectively, while other parameters remain unchanged. In this case, the growth rate is monotonically decreasing in the inflation rate (see Fig.20). Moreover, the welfare level is an inverted-U function of the inflation rate (see Fig.21), in which the welfare-maximizing rate of inflation is roughly 2%, implying that the Friedman rule is suboptimal.

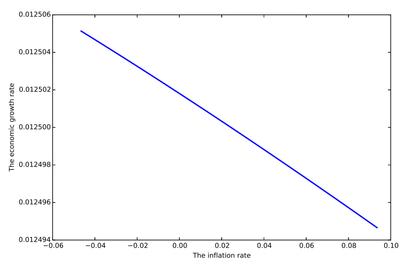

Fig. 20. Inflation and economic growth.

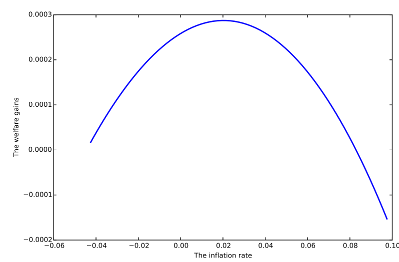

Fig. 21. Inflation and social welfare.

Finally, we consider a case in which the gap between ξ_v and ξ_h is enlarged from 0.2 to 0.3 by singly raising $\xi_h = 0.7$, while other parameters remain the same as in the previous case (where $\gamma = 0.5$ and $\delta = 0.75$). The purpose of this experiment is to show that the inverted-U relationship between inflation and economic growth rate is robust to the changes in the two parameters that govern the extent of diminishing returns. Fig.22 shows that the inverted-U relationship between inflation and economic growth still holds, which is similar to Fig.4. A comparison of Fig.20 and Fig.22 shows that a larger ξ_h strengthens the positive inflation-growth effect arising from the CIA constraint on horizontal R&D, which in turn makes it possible for the positive inflation-growth effect to initially dominate the negative inflation-growth effects. As explained in the benchmark case, the negative inflation-growth effects from the CIA constraints on consumption and vertical R&D eventually dominate the positive effect as the inflation rate becomes higher. Thus, the inverted-U pattern for inflation and economic growth is generated, and the threshold value in this case is approximately 1.8%, which is close to the empirical estimates in Khan and Senhadji (2001) (i.e., 1-3%) and Omay and Kan (2010) (i.e., 2%). In addition, social welfare is also monotonically decreasing in the inflation rate as shown in Fig.23. For example, a 10 percentage points increase in the inflation rate, from -4.45% to 5.55%, leads to a welfare loss of roughly 0.215%, which is smaller as compared to the benchmark case (i.e., 0.587%).

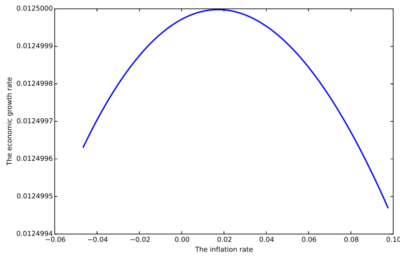

Fig. 22. Inflation and economic growth.

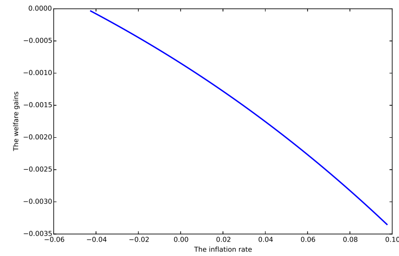

Fig. 23. Inflation and social welfare.

5 Conclusion

In this study, we explore the growth and welfare effects of monetary policy in an endogenous growth model with both vertical and horizontal innovation by incorporating cash-in-advance constraints on consumption and two R&D sectors. The novel contribution of this work, in contrast to most of the previous studies, is that our model is flexible enough to generate a mixed (i.e., monotonically decreasing or an inverted-U) relationship between inflation and economic growth, depending on the relative extents of CIA constraints and of diminishing returns to the two types of innovation. In particular, in a more empirically supportive case where horizontal R&D suffers a greater diminishing returns than vertical R&D, our model can generate an inverted-U relationship between inflation and growth when a sufficiently larger extent of a CIA constraint on horizontal R&D than on vertical R&D is met. This result holds in our general model setting with elastic labor supply and without a scale effect, which differs from Chu et al. (2017). Moreover, our model is calibrated by applying aggregate data for the US economy. We find that the threshold value of the inflation rate is around 2.8%, which is consistent with recent empirical estimates.

A.1 Proofs of Proposition 1, 2, and 3.

To analytically prove these propositions, first, we follow Segerstrom (2000) to establish the mutual R&D condition. This condition is derived from both vertical and horizontal R&D no-arbitrage conditions (14) and (19) for vertical R&D and horizontal R&D. As for the expected profit of each successful vertical innovator, substituting (11) into (14) yields

$$\Pi_{vt} = \int_t^{\infty} e^{-\int_t^{\tau}(r+\phi_s)ds}\hat{\pi}_{t\tau}d\tau = \frac{\alpha(1-\alpha)L_{yt}A_t^{\frac{1}{1-\alpha}}}{\rho + g_L + \left(\frac{1}{1-\alpha} - 1 + \frac{1}{\sigma}\right)g_A}. \tag{A.1.1}$$

Thus, the two R&D conditions are, respectively,

$$\frac{\delta\Gamma\alpha\lambda l_y\iota}{\rho + g_L + \left(\frac{1}{1-\alpha} - 1 + \frac{1}{\sigma}\right)g_A} l_v^{\delta-1} = 1 + \xi_v i, \tag{A.1.2}$$

and

$$\frac{\gamma\alpha\lambda l_y \iota}{\rho + g_L + \left(\frac{1}{1-\alpha} - 1 + \frac{1}{\sigma}\right) g_A} l_h^{\gamma-1} = 1 + \xi_h i. \tag{A.1.3}$$

Combining (A.1.3) and (A.1.2) yields the mutual R&D condition such that

$$\frac{\delta \Gamma l_v^{\delta-1}}{1+\xi_v i} = \frac{\gamma l_h^{\gamma-1}}{1+\xi_h i}. \tag{A.1.4}$$

Also, using (27) and (28), (A.1.4) can be re-expressed as a relationship with two growth rates such that

$$g_N = \sigma \Omega^{\frac{\gamma}{\gamma-1}} l_v^{\frac{\gamma-\delta}{1-\gamma}} g_A, \tag{A.1.5}$$

where $\Omega = \frac{1+\xi_h i}{1+\xi_v i}\Psi$, and $\Psi = \frac{\delta\Gamma}{\gamma}$. Plugging (24), (26) and $c_t = C_t/L_t$ into the individuals' consumption-leisure condition (5)yields the relationship between leisure and the production labor

$$l = 1 - \theta(1+\alpha)(1+\xi_c i) l_y. \tag{A.1.6}$$

Using (A.1.4), (A.1.6), and the labor-market-clearing condition $l_y + l_v + l_h = l$ yields

$$l_y = \frac{1 - l_v - \Omega^{\frac{1}{\gamma-1}} l_v^{\frac{1-\delta}{1-\gamma}}}{\Upsilon}, \tag{A.1.7}$$

where $\Upsilon = 1 + \theta(1+\alpha)(1+\xi_c i)$. Substituting (A.1.7) into (A.1.3) yields the general R&D condition:

$$g_A \left\{ \frac{1-l_v}{(1+\xi_v i) l_v} - \frac{\Omega^{\frac{1}{\gamma-1}} l_v^{\frac{\gamma-\delta}{1-\gamma}}}{1+\xi_v i} - \frac{\Upsilon\left[1+\sigma\left(\frac{1}{1-\alpha}-1\right)\right]}{\Gamma\delta\alpha} \right\} = \frac{\sigma\Upsilon(\rho+g_L)}{\Gamma\delta\alpha}. \tag{A.1.8}$$

In addition, substituting (A.1.6) into the population-growth condition (30) yields

$$g_L = \left(1 + \sigma\Omega^{\frac{\gamma}{\gamma-1}} l_v^{\frac{\gamma-\delta}{1-\gamma}}\right) g_A. \tag{A.1.9}$$

Consequently, (A.1.8) and (A.1.9) represent a system of two equations with two unknowns (l_v and g_A), which can be solved for a balanced-growth equilibrium.

Lemma 2. *The model has a unique balanced-growth equilibrium. In the equilibrium with a CIA constraint on consumption only, a permanent increase in the nominal interest rate i (a) decreases the fraction of labor allocated to vertical R&D l_v and increases the long-run product-quality growth rate g_A if $\gamma > \delta$, and (b) decreases l_v and g_A if $\gamma < \delta$.*

Proof. Imposing $\xi_v = \xi_h = 0$ reduces (A.1.5), (A.1.8) and (A.1.9) to

$$g_N = \sigma \Psi^{\frac{\gamma}{\gamma-1}} l_v^{\frac{\gamma-\delta}{1-\gamma}} g_A, \tag{A.1.10}$$

$$g_A \left\{ \frac{1-l_v}{l_v} - \Psi^{\frac{1}{\gamma-1}} l_v^{\frac{\gamma-\delta}{1-\gamma}} - \frac{\Upsilon\left[1+\sigma\left(\frac{1}{1-\alpha}-1\right)\right]}{\Gamma\delta\alpha} \right\} = \frac{\sigma\Upsilon(\rho+g_L)}{\Gamma\delta\alpha}, \tag{A.1.11}$$

and

$$g_L = \left(1 + \sigma \Psi^{\frac{\gamma}{\gamma-1}} l_v^{\frac{\gamma-\delta}{1-\gamma}}\right) g_A, \tag{A.1.12}$$

respectively. The last two equations are graphed in Fig.24 assuming that $\gamma > \delta$. The curve for the R&D condition (A.1.11) is unambiguously upward sloping and goes through the origin, whereas the curve for the population-growth condition (A.1.12) is unambiguously downward sloping and has a strictly positive vertical intercept. As illustrated in Fig.24, there is a unique intersection of these two curves at point A, which pins down the balanced-growth equilibrium values of l_v and g_A. With these values determined, (A.1.10) pins down g_N, (27) pins down ι, and thereby (28) pins down l_h. Thus, the model has a unique balanced-growth equilibrium when $\gamma > \delta$.

The effect of permanently increasing the nominal interest rate i is illustrated in Fig.24 by the movement from point A to point B. An increase in i unambiguously causes the curve for the R&D condition (A.1.11) to shift up, whereas it has no effect on the curve for the population-growth condition (A.1.12). Thus, a higher nominal interest rate decreases l_v and increases g_A if $\gamma > \delta$.

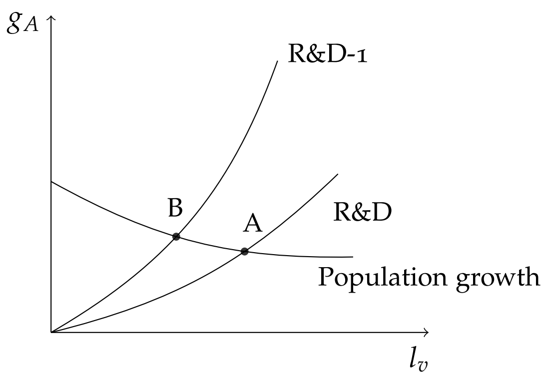

Fig. 24. The effect of a higher nominal interest rate for $\gamma > \delta$.

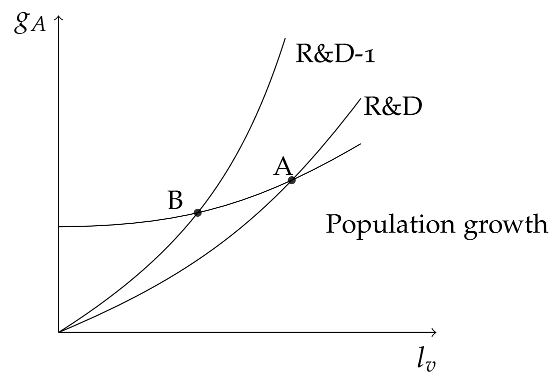

Fig. 25. The effect of a higher nominal interest rate for $\gamma < \delta$.

Equations (A.1.11) and (A.1.12) are graphed in Fig.25 assuming that $\gamma < \delta$. For $\gamma < \delta$, the slope of the curve for the population-growth condition turns to be positive because a higher l_v is correlated with a higher g_A, whereas the positiveness of the slope of the curve for the R&D condition remains unchanged. Again, there is a unique intersection of these two curves at point A, which pins down the balanced-growth equilibrium values of l_v and g_A in addition to other variables. Thus, the model also has a unique balanced-growth equilibrium if $\gamma < \delta$.

The effect of permanently increasing the nominal interest rate i is illustrated in Fig.25 by the movement from point A to point B. An increase in i unambiguously shifts the curve for the R&D condition (A.1.11) upward, whereas it has no effect on the curve for the population-growth condition (A.1.12). Thus, a higher nominal interest rate decreases l_v and decreases g_A if $\gamma < \delta$.

Proof of Proposition 1. Based on the above results, we now proceed to the analysis of the overall effects of monetary policies on g_A and g_N. In the (g_A, g_N) space, the slope of each iso-growth line(i.e.,$1/(1-\alpha)$) exceeds the slope of the population-growth condition(i.e.,1) (in absolute value). The effects of a higher nominal interest rate are illustrated in Fig.26 accordingly. The mutual R&D condition (given by (A.1.13)) is an upward-sloping line that goes through the

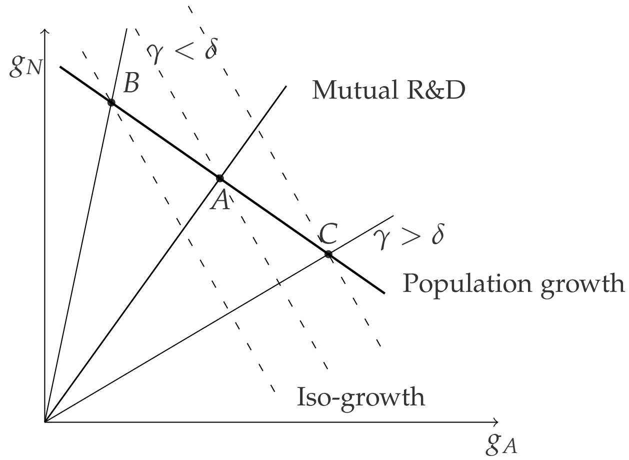

Fig. 26. The growth effect of a higher i with CIA constraint on consumption.

origin in the (g_A, g_N) space, when l_v is fixed at the initial equilibrium value. An increase in i shifts down the mutual R&D condition to a new intersection C if $\gamma > \delta$, leading to an increase in g_A as shown in Lemma 2. In contrast, an identical increase in i shifts up the mutual R&D condition to another new intersection B if $\gamma < \delta$, leading to an decrease in g_A. Combining (29) with (30) to express the aggregate economic growth rate exclusively as the vertical innovation growth rate such that $g = g_L + [1/(1-\alpha) - 1]g_A$. It then shows that an increase in i, which leads to an decrease in g_A when $\gamma < \delta$, decreases the long-run growth rate g(i.e., the movement from A to B); while an identical increase in i, which results in an increase in g_A when $\gamma > \delta$, increases the long-run growth rate g(i.e., the movement from A to C).

Lemma 3. *The model has a unique balanced-growth equilibrium. In the equilibrium with a CIA constraint on vertical R&D only, a permanent increase in i (a) decreases l_v and g_A if $\gamma > \delta$, and (b) yields an identical decreasing and larger effect on l_v and g_A if $\gamma < \delta$.*

Proof. We make use of $\xi_c = \xi_h = 0$ to reduce (A.1.5), (A.1.8) and (A.1.9) to

$$g_N = \sigma \Psi^{\frac{\gamma}{\gamma-1}} (1+\xi_v i)^{\frac{\gamma}{1-\gamma}} l_v^{\frac{\gamma-\delta}{1-\gamma}} g_A, \tag{A.1.13}$$

$$g_A \left[\underbrace{\frac{1-l_v}{l_v(1+\xi_v i)} - \Psi^{\frac{1}{\gamma-1}} (1+\xi_v i)^{\frac{\gamma}{1-\gamma}} l_v^{\frac{\gamma-\delta}{1-\gamma}}}_{(-)} - \frac{(1+\theta+\theta\alpha)(\Gamma-\sigma)}{\Gamma\delta\alpha} \right] = \frac{\sigma(1+\theta+\theta\alpha)(\rho+g_L)}{\Gamma\delta\alpha}, \tag{A.1.14}$$

and

$$g_L = \left[1 + \sigma \Psi^{\frac{\gamma}{\gamma-1}} (1+\xi_v i)^{\frac{\gamma}{1-\gamma}} l_v^{\frac{\gamma-\delta}{1-\gamma}} \right] g_A, \tag{A.1.15}$$

respectively. Equations (A.1.14) and (A.1.15) are graphed in Fig.27 given $\gamma > \delta$. There is a unique intersection of these two curves at point A, which pins down the balanced-growth equilibrium values of all endogenous variables as in the previous case (in which only CIA constraint on consumption is present). Thus, the model has a unique balanced-growth equilibrium when $\gamma > \delta$. The effect of permanently increasing the nominal interest rate i is illustrated in Fig.27 by the movement from point A to point B. An increase in i unambiguously causes the curve for the R&D condition (A.1.14) (the negative sign means that the value of those terms overall decreases as i increases) to shift upward, and unambiguously causes the curve for the population-growth condition (A.1.15) to shift downward. Hence, a higher rate of nominal interest surely decreases l_v.

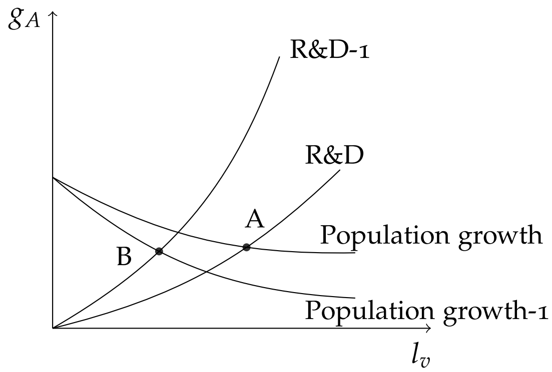

Fig. 27. The effect of a higher nominal interest rate for $\gamma > \delta$.

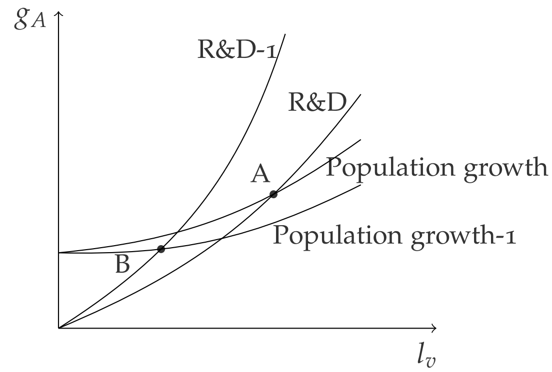

Fig. 28. The effect of a higher nominal interest rate for $\gamma < \delta$.

As to determine the effect on g_A, first, suppose that for some $\gamma > \delta$, an increase in i increases (or has no effect on) g_A. According to (A.1.15), this implies that $(1+\xi_v i)^{\gamma/(1-\gamma)} l_v^{(\gamma-\delta)/(1-\gamma)}$ must decrease (or remain unchanged) when i increases. That is, $[(1+\xi_v i)l_v]^{-1} l_v^{\delta/\gamma}$ must increase (or remain unchanged). Given that l_v decreases as i increases, $[(1+\xi_v i)l_v]^{-1}$ must increase in response. Therefore, (A.1.14) implies that $(1-l_v)/[(1+\xi_v i)l_v] - \Psi^{1/(\gamma-1)}(1+\xi_v i)^{\frac{\gamma}{1-\gamma}} l_v^{(\gamma-\delta)/(1-\gamma)}$ must increase, and thereby g_A will decrease. This yields a contradiction. As a result, g_A must always decrease in response to an increase in i when $\gamma > \delta$.

Equation (A.1.14) and (A.1.15) for $\gamma < \delta$ are graphed in Fig.28. There is still a unique intersection of these two curves at point A, so the model has a unique balanced-growth equilibrium when $\gamma < \delta$. The effect of permanently increasing i is illustrated in Fig.28 by the movement from point A to point B. An increase in i unambiguously causes the curve for the R&D condition (A.1.14) to shift upward, while it unambiguously shifts the curve for the population-growth condition (A.1.15) downward. Hence, a higher i unambiguously decreases l_v. A similar proof applies for the change in g_A. The only difference in $\gamma < \delta$ from $\gamma > \delta$ is that a higher i amplifies the negative effect on g_A in the former case, leading to a larger magnitude of the reduction in g_A.

Proof of Proposition 2. The effects of a higher rate of nominal interest on the aggregate rate of economic growth g are displayed in Fig.29. An increase in i shifts up the line for the mutual R&D condition (given by (A.1.13)), which eventually decreases the vertical R&D growth rate if $\gamma > \delta$ (namely the movement from A to C). Also, a higher i continues to shift the line for the mutual R&D condition if $\gamma < \delta$, but the scale becomes larger, implying that the reduction in g_A (namely the movement from A to B) is larger. In other words, the overall effect of a higher nominal interest rate is to increase the product-variety growth rate at the expense of the product-quality growth

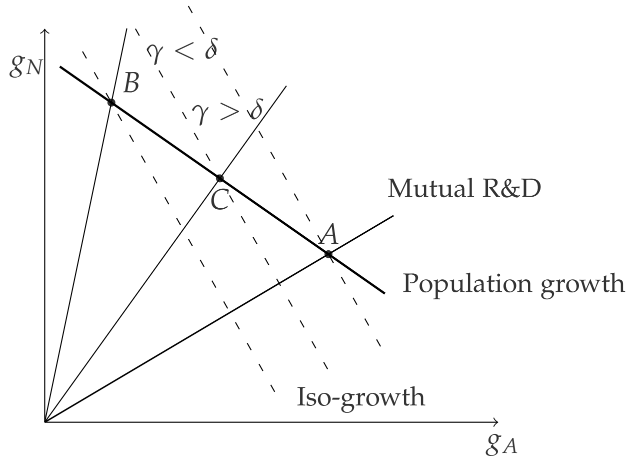

Fig. 29. The growth effect of a higher i with CIA constraint on vertical R&D.

rate, with a larger sacrifice in vertical innovation growth rate when $\gamma < \delta$. Combining (29) with (30) to express the aggregate economic growth rate exclusively as the vertical innovation growth rate such that $g = g_L + [1/(1-\alpha) - 1]g_A$. It states that a movement on the population-growth condition in the northwest direction(g_N increases and g_A decreases) is growth-retarding given $1 < 1/(1-\alpha)$. Therefore, a larger sacrifice in the product-quality growth rate g_A in the case of $\gamma < \delta$ implies a larger decrease in the aggregate economic growth rate comparing to the case of $\gamma > \delta$.

Lemma 4. *The model has a unique balanced-growth equilibrium. In the equilibrium with a CIA constraint on horizontal R&D only, a permanent increase in i (a) increases l_v and g_A if $\gamma > \delta$, and (b) yields an identical increasing and larger effect on l_v and g_A if $\gamma < \delta$.*

Proof. In an analogous fashion of the proof of Lemma 3, imposing $\xi_c = \xi_v = 0$ reduces (A.1.5), (A.1.8) and (A.1.9) to

$$g_N = \sigma \Psi^{\frac{\gamma}{\gamma-1}} (1 + \xi_h i)^{\frac{-\gamma}{1-\gamma}} l_v^{\frac{\gamma-\delta}{1-\gamma}} g_A, \tag{A.1.16}$$

$$g_A \left[\frac{1}{l_v} - 1 \underbrace{- \Psi^{\frac{1}{\gamma-1}} (1 + \xi_h i)^{\frac{-1}{1-\gamma}} l_v^{\frac{\gamma-\delta}{1-\gamma}}}_{(+)} - \frac{(1+\theta+\theta\alpha)(\Gamma-\sigma)}{\Gamma\delta\alpha} \right] = \frac{\sigma(1+\theta+\theta\alpha)(\rho+g_L)}{\Gamma\delta\alpha}, \tag{A.1.17}$$

and

$$g_L = \left[1 + \sigma \Psi^{\frac{\gamma}{\gamma-1}} (1 + \xi_h i)^{\frac{-\gamma}{1-\gamma}} l_v^{\frac{\gamma-\delta}{1-\gamma}} \right] g_A, \tag{A.1.18}$$

respectively. Equations (A.1.17) and (A.1.18) are graphed in Fig.30 given $\gamma > \delta$. There is a unique intersection of these two curves at point A, which pins down the balanced-growth equilibrium values of all endogenous variables. Thus, the model also has a unique balanced-growth equilibrium when $\gamma > \delta$. The effect of permanently increasing the nominal interest rate i is illustrated in Fig.30 by the movement from point A to point B. An increase in i unambiguously causes the curve for the R&D condition (A.1.17) to shift downward, and it unambiguously causes the curve for the population-growth condition (A.1.18) to shift upward. Hence, a higher i surely increases

l_v.

As to determine the effect on g_A, first, suppose that for some $\gamma > \delta$, an increase in i decreases (or does not change) g_A. Then, (A.1.18) implies that $(1+\xi_h i)^{-\gamma/(1-\gamma)} l_v^{(\gamma-\delta)/(1-\gamma)}$ increases (or remains unchanged) when i increases, from which it follows that $[(1+\xi_h i) l_v^{-1}]^{-\gamma/(1-\gamma)} l_v^{-\delta/(1-\gamma)}$ increases(or remain constant). Since l_v increases in response to an increase in i, $[(1+\xi_h i) l_v^{-1}]^{-\gamma/(1-\gamma)}$ increases and $[(1+\xi_h i) l_v^{-1}]$ decreases. According to (A.1.17), $\frac{1-l_v}{l_v} - \Psi^{\frac{1}{\gamma-1}}(1+\xi_h i)^{\frac{-1}{1-\gamma}} l_v^{\frac{\gamma-\delta}{1-\gamma}} = \frac{1}{1+\xi_h i}\left[\frac{(1+\xi_h i)(1-l_v)}{l_v} - \Psi^{\frac{1}{\gamma-1}}(1+\xi_h i)^{\frac{-\gamma}{1-\gamma}} l_v^{\frac{\gamma-\delta}{1-\gamma}}\right]$ must decrease and g_A must increase. This yields a contradiction. As a result, g_A must always increase in response to an increase i when $\gamma > \delta$.

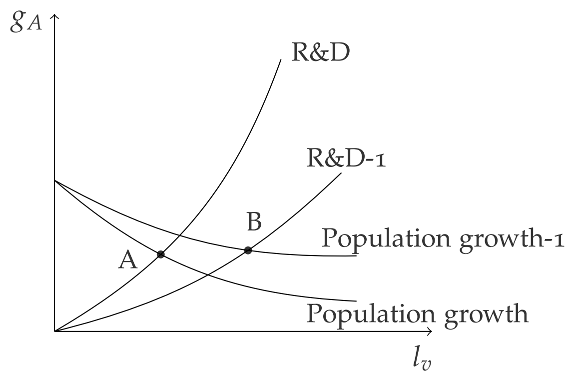

Fig. 30. The effect of a higher nominal interest rate for $\gamma > \delta$.

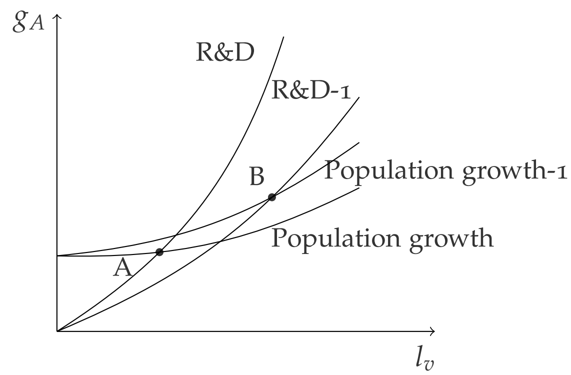

Fig. 31. The effect of a higher nominal interest rate for $\gamma < \delta$.

Equation (A.1.17) and (A.1.18) for $\gamma < \delta$ are graphed in Fig.31. There is also a unique intersection of these two curves at point A, and the model has a unique balanced-growth equilibrium when $\gamma < \delta$. The effect of permanently increasing the nominal interest rate i is illustrated in Fig.31 by the movement from point A to point B. An increase in i unambiguously causes the curve for the R&D condition (A.1.17) to shift downward, whereas it unambiguously shifts the curve for the population-growth condition (A.1.18) upward. Thus, a higher i unambiguously increases l_v. A similar proof applies for the change in g_A. The difference in $\gamma < \delta$ from $\gamma > \delta$ is that a higher i amplifies the positive effect on g_A in the former case, leading to a larger magnitude of the increase in g_A.

Proof of Proposition 3. The effects of a higher rate of nominal interest on the aggregate rate of economic growth g are displayed in Fig.32. An increase in the nominal interest rate i shifts down the line for mutual R&D condition (given by (A.1.16)), which eventually increases the vertical R&D growth rate if $\gamma > \delta$ (namely the movement from A to C). Also, a higher i shifts down the line for the mutual R&D condition if $\gamma < \delta$, but the scale becomes large, which implies that the increase in g_A is larger (namely the movement from A to B). In other words, the overall effect of a higher nominal interest rate is to increase the product-quality growth rate at the expense of the product-variety growth rate, with a larger sacrifice in horizontal innovation growth rate when $\gamma < \delta$. A combination of (29) and (30) yields $g = g_L + [1/(1-\alpha) - 1]g_A$, which implies that a movement on the population-growth condition in the southeast direction(g_A increases and g_N decreases) is growth-promoting given $1 < 1/(1-\alpha)$. Therefore, a larger sacrifice in the product-variety growth rate implies a larger increase in the aggregate economic growth rate when $\gamma < \delta$.

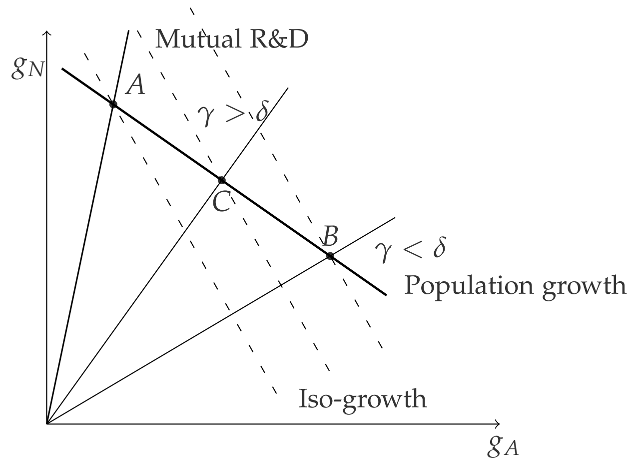

Fig. 32. The growth effect of a higher i with CIA constraint on horizontal R&D.

A.2 Proof of Proposition 4

To prove Proposition 4, the model is solved in a slightly different way. Given the equation (A.1.7), equation (A.1.2) is used to set up another correlation between l_{yt} and l_{vt}. To do this, ι needs to be eliminated. Rewriting the economic growth rate solely as the vertical innovation growth rate by combining (29) and (30) yields

$$g = g_L + \left(\frac{1}{1-\alpha} - 1\right) g_A.$$

Together with $g_A = \sigma\lambda l_v^{\delta}\iota$ and $g_N = \lambda l_h^{\gamma}\iota$, we obtain

$$g_L = \iota\lambda\left(\sigma l_v^{\delta} + \Omega^{\frac{\gamma}{\gamma-1}} l_v^{\frac{\gamma(\delta-1)}{\gamma-1}}\right). \tag{A.2.1}$$

Then, we reduce ι in (A.1.2) by making use of (A.2.1) to derive l_y as

$$\begin{aligned}
l_y &= \frac{(1+\xi_v i)[\rho + g_L + (\frac{1}{1-\alpha} - 1 + \frac{1}{\sigma}) g_A]}{\delta\alpha\Gamma\lambda\iota} l_v^{1-\delta} \\
&= \frac{(1+\xi_v i)(\rho+g_L)}{\delta\alpha\Gamma\lambda g_L}\left[\sigma\lambda l_v^{\delta} + \lambda\Omega^{\frac{\gamma}{\gamma-1}} l_v^{\frac{(\delta-1)\gamma}{\gamma-1}}\right] l_v^{1-\delta} + \frac{(1+\xi_v i)(\frac{1}{1-\alpha} - 1 + \frac{1}{\sigma})\sigma\lambda l_v^{\delta}\iota}{\delta\alpha\Gamma\lambda\iota} l_v^{1-\delta} \\
&= \frac{(1+\xi_v i)}{\delta\alpha\Gamma g_L}\Big(\rho\sigma + \Gamma g_L\Big) l_v + \frac{(1+\xi_v i)(\rho+g_L)}{\delta\alpha\Gamma g_L}\Omega^{\frac{\gamma}{\gamma-1}} l_v^{\frac{\delta-1}{\gamma-1}} \\
&= (1+\xi_v i)\left(\Theta l_v + \Lambda\Omega^{\frac{\gamma}{\gamma-1}} l_v^{\frac{\delta-1}{\gamma-1}}\right)
\end{aligned} \tag{A.2.2}$$

where $\Theta = \frac{\rho\sigma + g_L\Gamma}{\delta\alpha\Gamma g_L}$, and $\Lambda = \frac{\rho + g_L}{\delta\alpha\Gamma g_L}$. By plugging (A.2.2) into (A.1.4), the labor-market-clearing condition can be rewritten as

$$l_v[\Upsilon\Theta(1+\xi_v i)+1] + \Omega^{\frac{\gamma}{\gamma-1}} l_v^{\frac{1-\delta}{1-\gamma}} [\Upsilon\Lambda(1+\xi_v i) + \Omega^{-1}] = 1. \tag{A.2.3}$$

Finally, to find the relationship between i and g, we need to derive a function of g on l_v. By combining (29) with (30) and using the expression of ι, we obtain

$$g = g_L \left[1 + \frac{\left(\frac{1}{1-\alpha} - 1\right)\sigma}{\sigma + \Omega^{\frac{\gamma}{\gamma-1}} l_v^{\frac{\gamma-\delta}{1-\gamma}}}\right]. \tag{A.2.4}$$

Differentiating g with respect to i yields

$$\begin{aligned}
\frac{\partial g}{\partial i} &= \frac{-g_L(\frac{1}{1-\alpha} - 1)\sigma}{\left(\sigma + \Omega^{\frac{\gamma}{\gamma-1}} l_v^{\frac{\gamma-\delta}{1-\gamma}}\right)^2} \left\{ \frac{\gamma}{\gamma - 1} \Omega^{\frac{1}{\gamma-1}} \frac{\partial \Omega}{\partial i} l_v^{\frac{\gamma-\delta}{1-\gamma}} + \Omega^{\frac{\gamma}{\gamma-1}} \frac{\gamma-\delta}{1-\gamma} \frac{\partial l_v}{\partial i} l_v^{\frac{\gamma-\delta}{1-\gamma}-1} \right\} \\
&= \frac{\sigma g_L(\frac{1}{1-\alpha} - 1)\Omega^{\frac{1}{\gamma-1}} l_v^{\frac{\gamma-\delta}{1-\gamma}}}{(1-\gamma)\left(\sigma + \Omega^{\frac{\gamma}{\gamma-1}} l_v^{\frac{\gamma-\delta}{1-\gamma}}\right)^2} \left\{ \gamma\Psi \frac{\xi_h - \xi_v}{(1+\xi_v i)^2} + (\delta - \gamma)\Psi \frac{1+\xi_h i}{1+\xi_v i} \frac{\frac{\partial l_v}{\partial i}}{l_v} \right\} \\
&= \underbrace{\frac{\sigma\delta\Gamma g_L \Omega^{\frac{1}{\gamma-1}} l_v^{\frac{\gamma-\delta}{1-\gamma}} \left(\frac{1}{1-\alpha} - 1\right)}{(1-\gamma)(1+\xi_v i)^2 \left(\sigma + \Omega^{\frac{\gamma}{\gamma-1}} l_v^{\frac{\gamma-\delta}{1-\gamma}}\right)^2}}_{(+)} \left\{ (\xi_h - \xi_v) + (\delta - \gamma)(1+\xi_v i)(1+\xi_h i) \frac{\frac{\partial l_v}{\partial i}}{\gamma l_v} \right\}.
\end{aligned} \tag{A.2.5}$$

Therefore, the *sign* of $\partial g/\partial i$ depends on the *sign* of $\left[(\xi_h - \xi_v) + (\delta - \gamma)(1+\xi_v i)(1+\xi_h i)\frac{\partial l_v/\partial i}{\gamma l_v}\right]$. Differentiating (A.2.3) with respect to i to derive $\partial l_v/\partial i$ (note that Ψ, Θ, and Λ are unrelated to i) yields

$$\begin{aligned}
&\underbrace{\left\{ [\Upsilon\Theta(1+\xi_v i)+1] + \frac{1-\delta}{1-\gamma} \Omega^{\frac{1}{\gamma-1}} l_v^{\frac{\gamma-\delta}{1-\gamma}} [\Upsilon\Lambda(1+\xi_v i) + \Omega^{-1}] \right\}}_{\chi_1 > 0} \frac{\partial l_v}{\partial i} \\
&= \left\{ (\xi_h - \xi_v) \underbrace{\left[\frac{\gamma\Upsilon\Lambda + \gamma\Psi^{-1}(1+\xi_h i)}{(1-\gamma)(1+\xi_h i)} + \frac{1}{\Psi(1+\xi_h i)^2} \right]}_{\chi_2 > 0} - \underbrace{\Lambda[\theta\xi_c(1+\alpha)(1+\xi_v i) + \Upsilon\xi_v]}_{\chi_3 > 0} \right\} \Omega^{\frac{\gamma}{\gamma-1}} l_v^{\frac{1-\delta}{1-\gamma}} \\
&- \underbrace{\Theta[\theta\xi_c(1+\alpha)(1+\xi_v i) + \Upsilon\xi_v]}_{\chi_4 > 0} l_v
\end{aligned} \tag{A.2.6}$$

It is apparent that χ_2 monotonically decreases, while χ_3 and χ_4 monotonically increases. Thus for a positive χ_1, $\partial l_v/\partial i$ monotonically decreases as i increases. Therefore, $\partial g/\partial i$ eventually goes to negative(positive) on the condition of $\gamma < (>)\delta$. To see whether $\partial g/\partial i > 0$, one can substitute (A.2.6) into $\left[(\xi_h - \xi_v) + (\delta-\gamma)(1+\xi_v i)(1+\xi_h i)\frac{\partial l_v/\partial i}{\gamma l_v}\right]$ to show that

$$\begin{aligned}
&\left(\frac{\partial g}{\partial i}\right)_{i=0} > 0 \\
&\Leftrightarrow (\xi_h - \xi_v) + (\delta-\gamma)\left\{\frac{\Psi^{\frac{\gamma}{\gamma-1}} l_v^{\frac{\gamma-\delta}{1-\gamma}}}{\gamma\chi_1}[(\xi_h-\xi_v)\chi_2 - \chi_3] - \frac{\chi_4}{\gamma\chi_1}\right\}_{i=0} > 0 \\
&\Leftrightarrow (\xi_h - \xi_v) > \left\{\frac{(\delta-\gamma)\left(\chi_4 + \chi_3\Psi^{\frac{\gamma}{\gamma-1}} l_v^{\frac{\gamma-\delta}{1-\gamma}}\right)}{\gamma\chi_1 + (\delta-\gamma)\chi_2\Psi^{\frac{\gamma}{\gamma-1}} l_v^{\frac{\gamma-\delta}{1-\gamma}}}\right\}_{i=0} > 0,
\end{aligned} \tag{A.2.7}$$

where l_v is determined in the implicit function (A.2.3) evaluated at $i = 0$. Accordingly, a sufficiently large $(\xi_h - \xi_v)$ is a sufficient and necessary condition for a local maximum of function $g(i)$. It in turn implies that g increases as i becomes higher when $i < i^*$ and decreases as i increases when $i > i^*$, where i^* can be solved according to

$$(\xi_h - \xi_v) = \frac{(\delta-\gamma)\left(\chi_4 + \chi_3\Omega^{\frac{\gamma}{\gamma-1}} l_v^{\frac{\gamma-\delta}{1-\gamma}}\right)}{\gamma\chi_1 + (\delta-\gamma)\chi_2\Omega^{\frac{\gamma}{\gamma-1}} l_v^{\frac{\gamma-\delta}{1-\gamma}}}. \tag{A.2.8}$$

In addition, if $\gamma > \delta$, the condition (A.2.7) surely holds, because $(\xi_h - \xi_v) > 0$ while the RHS of (A.2.7) turns to negative. Therefore, $(\partial g/\partial i)_{i=0} > 0$. However, when $\gamma > \delta$, $\partial g/\partial i$ remains positive as i increases. Therefore, under the condition of $\gamma > \delta$, g is monotonically increasing in i.

A.3 Calibration Strategy

In this section, given all other parameters and values, we illustrate the strategy to calibrate σ and θ simultaneously to match the growth rate 1.25% and the standard time of employment $l = 0.33$. Using the individual's optimal decision on the labor-leisure choice (5), equilibrium final-good production function (24), the per-capita consumption share of outputs (25), the production-labor share of outputs (26), and $c_t = C_t/L_t$, we obtain

$$l = 1 - \theta(1+\alpha)(1+\xi_c i)l_y.$$

Together with (A.2.2), we derive the first equation for calibration such that

$$l = 1 - \theta(1+\alpha)(1+\xi_c i)(1+\xi_v i)\left\{\Theta l_v + \Lambda\Omega^{\frac{\gamma}{\gamma-1}} l_v^{\frac{1-\delta}{1-\gamma}}\right\}$$

Given all other parameters, there are three unknowns $\{\sigma, \theta, l_v\}$, and another two equations are needed for solution, which are (A.2.3) and (A.2.4), namely

$$l_v[\Upsilon\Theta(1+\xi_v i)+1]+\Omega^{\frac{\gamma}{\gamma-1}} l_v^{\frac{1-\delta}{1-\gamma}}[\Upsilon\Lambda(1+\xi_v i)+\Omega^{-1}]=1$$

and

$$g = g_L\left[1+\frac{\left(\frac{1}{1-\alpha}-1\right)\sigma}{\sigma+\Omega^{\frac{\gamma}{\gamma-1}} l_v^{\frac{\gamma-\delta}{1-\gamma}}}\right].$$

Finally, we have three equations to pin down the above three unknowns.

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