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26 February 2018

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Abstract

We generalize the results obtained by Braid (2008) and Beladi et al. (2014) for any non-negative, increasing, continuous function of distance as transportation cost function. By doing so, we show that in a duopoly, partial privatization does not change the socially optimal character of the Nash equilibrium location. Our results call for further research on testing their robustness under the existence of more than two competing firms.

JEL classification: L13; L32; L33; R32
Keywords: Privatization; Social optimality; Spatial competition; Transportation costs

1 Introduction

The role of the type of transportation costs has been widely highlighted in the literature of spatial competition. More specifically, d'Aspremont et al. (1979) showed that in the traditional Hotelling's model (Hotelling, 1929), the nature of travel costs is important for the existence of an equilibrium; an equilibrium exists when transportation costs are proportional to the square of distance while it doesn't when the travel costs are linear. Economides (1986) generalized the results of d'Aspremont et al. (1979) showing that equilibria exist for a family of non-linear transportation costs functions of the form $f(d) = d^{\alpha}, 1 \leq \alpha \leq 2$. It should be noted, however, that Economides' (weak) generalization refers only to the cases where the second-stage competition (in the first-stage firms compete in locations) is à la Bertrand

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(since it is based on the Hotelling's model). An alternative form of (second-stage) price competition is the so-called spatial price discrimination introduced by Hoover (1937) and Lerner and Singer (1937).[1] In this type of competition firms bear transportation costs and set delivered price schedules. Building upon this form of competition Braid (2008) constructed a duopoly model where firms enjoy monopoly power due to the fact that only two out of the three goods demanded by all consumers can be offered by a single firm. He demonstrated that under linear transportation costs the Nash equilibrium locations of the firms coincide with the socially optimal locations. Extending Braid's framework (and by keeping the assumption of linear transportation costs) to account for a partly privatized public company, Beladi et al. (2014) proved that the results are equivalent to those in Braid (2008) (i.e., the Nash equilibrium locations are socially optimal and equal to Braid's results) and as a consequence the degree of privatization of the public firm does not affect the equilibrium locations. In this paper, we attempt to answer the following main question: Are Braid (2008) and Beladi et al. (2014) findings sensitive to the type of transportation costs? Our analysis concludes that their results hold for any non-negative, increasing, continuous transportation costs function. Based on the discussion so far, this conclusion constitutes an indication that in this family of spatial competition models, the form of transportation costs is not as crucial as in models à la Hotelling.

2 Model and results

Our setting follows that of Braid (2008). We consider a duopoly, with a continuum of consumers uniformly distributed over the interval $[0, 1]$ of a linear city. Three products are offered to consumers; J and K from firm D_1 and K and L from firm D_2. Marginal costs of production are constant and without any loss of generality are set equal to 0. Let the fraction of consumers buying only good J equal that of those buying only good L equal to c.[2] Product K is bought by a fraction b of consumers. The above assumptions imply that

[1] This type of spatial price competition has been extensively used in the relevant literature (see Anderson et al. (1992)).

[2] It can be easily shown that our results hold even if the fraction of consumers buying good J is different from that of those buying only good L.

the two firms have monopoly power over the goods J and L. Let k denote the maximum reservation price that the consumers are willing to pay for a good. Evidently, D_1 and D_2 will charge a uniform price infinitesimally below k for J and L. Spatial price discrimination à la Lerner and Singer (1937) is assumed regarding product K, where Nash equilibrium in delivered price schedules exists.[3] The location of D_1 and D_2 over the interval $[0,1]$ is x and y, respectively. D_1 is privately owned whereas D_2 is partly privately owned and partly publicly owned in proportions a and $1-a$, respectively with $a \in [0,1]$. Transportation costs are equal to $f(d)$, where d is the distance shipped and f any non-negative, increasing, continuous function. D_1 and D_2 simultaneously choose their location in the market.

The aggregate shipping distance (for all locations z of consumers along the interval $[0,1]$) is equal to

$$\begin{aligned} T(x,y) &= c\left(\int_0^x f(x-z)dz + \int_x^1 f(z-x)dz\right) + b\left(\int_0^x f(x-z)dz + \int_x^s f(z-x)dz\right) \\ &\quad + b\left(\int_s^y f(y-z)dz + \int_y^1 f(z-y)dz\right) \\ &\quad + c\left(\int_0^y f(y-z)dz + \int_y^1 f(z-y)dz\right) \qquad (1) \end{aligned}$$

where s is the location of the indifferent consumer with $f(y-s) = f(s-x)$. Let $F(d) := \int f(d)$. Then (1) can be written as

$$\begin{aligned} T(x,y) &= c[-F(0) + F(x) + F(1-x) - F(0)] + b[-F(0) + F(x) + F(s-x) - F(0)] \\ &\quad + b[-F(0) + F(y-s) + F(1-y) - F(0)] \\ &\quad + c[-F(0) + F(y) + F(1-y) - F(0)] \qquad (1b) \end{aligned}$$

In order to find the socially optimal Nash equilibrium locations we have to minimize (1b) with respect to x and y taking into account that $f(y-s) = f(s-x)$. Hence, the socially

[3] This implies that the price charged for K by the firm that is closer to the consumer is equal to (or infinitesimally less than) the delivered cost of the firm that is further away. Delivered costs coincide with transportation costs due to the assumption of zero marginal production costs.

optimal locations satisfy the first order conditions:

$$\begin{aligned}\frac{\partial T(x,y)}{\partial x} &= b\frac{\partial s}{\partial x}[f(s-x)-f(y-s)]+b[f(x)+f(s-x)]+c[f(x)-f(1-x)] \\ &= b[f(x)-f(s-x)]+c[f(x)-f(1-x)]=0 \qquad (2)\end{aligned}$$

$$\begin{aligned}\frac{\partial T(x,y)}{\partial y} &= b\frac{\partial s}{\partial y}[f(s-x)-f(y-s)]+b[f(y-s)-f(1-y)]+c[f(y)-f(1-y)] \\ &= b[f(y-s)-f(1-y)]+c[f(y)-f(1-y)]=0 \qquad (3)\end{aligned}$$

Following Braid (2008), the profit functions of D_1 and D_2 when both firms are privately owned (i.e. when $a=1$) are:

$$\begin{aligned}\Pi_{D_1}(x,y) &= ck-c\left(\int_0^x f(x-z)dz+\int_x^1 f(z-x)dz\right) \\ &\quad +b\int_0^x [f(y-z)-f(x-z)]dz+b\int_x^s [f(y-z)-f(z-x)]dz \\ &= ck-c[F(x)+F(1-x)]+b[F(y)-F(x)]+b[-F(y-s)-F(s-x)] \qquad (4)\end{aligned}$$

$$\begin{aligned}\Pi_{D_2}(x,y) &= ck-c\left(\int_0^y f(y-z)dz+\int_y^1 f(z-y)dz\right) \\ &\quad +b\int_s^y [f(z-x)-f(y-z)]dz+b\int_y^1 [f(z-x)-f(z-y)]dz \\ &= ck-c[F(y)+F(1-y)]+b[-F(y-s)-F(s-x)] \\ &\quad +b[F(1-x)-F(1-y)] \qquad (5)\end{aligned}$$

Therefore, the Nash equilibrium locations when both firms are privately owned is given by the solution of the following system of equations:

$$\frac{\partial \Pi_{D_1}(x,y)}{\partial x} = -b[f(x) - f(s-x)] - c[f(x) - f(1-x)] = 0 \tag{6}$$

$$\frac{\partial \Pi_{D_2}(x,y)}{\partial y} = -b[f(y-s) - f(1-y)] - c[f(y) - f(1-y)] = 0 \tag{7}$$

Following Beladi et al. (2014), when $a \in [0,1)$, the profit function of D_2 is

$$\begin{aligned}
\hat{\Pi}_{D_2}(x,y) &= ck - c\left(\int_0^y f(y-z)dz + \int_y^1 f(z-y)dz\right) \\
&\quad + b\int_s^y [f(z-x) - f(y-z)]dz \\
&\quad + b\int_y^1 [f(z-x) - f(z-y)]dz + (1-a)g(x,y)
\end{aligned} \tag{8}$$

where

$$\begin{aligned}
g(x,y) &= ck - c\left(\int_0^x f(x-z)dz + \int_x^1 f(z-x)dz\right) \\
&\quad + b\int_0^x [f(y-z) - f(x-z)]dz \\
&\quad + b\int_x^s [f(y-z) - f(z-x)]dz + b\int_0^s [k - f(y-z)]dz \\
&\quad + b\int_s^1 [k - f(z-x)]dz \\
&= ck - c\left(\int_0^x f(x-z)dz + \int_x^1 f(z-x)dz\right) \\
&\quad - b\int_0^x f(x-z)dz - b\int_x^1 f(z-x)dz + b\int_0^1 kdz
\end{aligned} \tag{9}$$

At this point we have to note that D_2's profits are expressed as the weighted average of its profits and social welfare (the sum of consumers' surplus and firms' profits) with a being the weight term. Hence, $g(x,y)$ is the sum of consumers' surplus and D_1's profits.

The Nash equilibrium locations under $a \in [0,1)$ satisfy (6) and

$$\frac{\partial \hat{\Pi}_{D_2}(x,y)}{\partial y} = 0 \tag{10}$$

However, since (9) does not depend on y, $\frac{\partial \hat{\Pi}_{D_2}(x,y)}{\partial y} = \frac{\partial \Pi_{D_2}(x,y)}{\partial y}$.

It can be easily noted that the systems of (2) and (3), (6) and (7) and (6) and (10) are equivalent and therefore have the same solution.

The above analysis leads to the following propositions:

Proposition 1 *The Nash equilibrium locations for* $a \in [0,1]$ *are socially optimal under any non-negative, increasing, continuous transportation costs function.*

Proposition 2 *The degree of privatization does not affect the socially optimal Nash equilibrium locations under any non-negative, increasing, continuous transportation costs function.*

Proposition 3 *The Nash equilibrium locations for* $a \in [0,1)$ *are equal to those for* $a = 1$ *under any non-negative, increasing, continuous transportation costs function.*

Propositions 1, 2 and 3 apart from proving the results obtained by Braid (2008) and Beladi et al. (2014) at the same time in complete generality, they establish, most importantly, their independence from the linear nature of the original model.

The key observation behind the invariance of the socially optimal Nash equilibrium locations when firm D_2 is partly privatized is that the summand accounting for the welfare in its profit function, $\hat{\Pi}_{D_2}(x,y)$, is, in fact, independent of its location y regardless of the degree of privatization a.

Putting together the above results with the findings by Cremer et al. (1991), it emerges that these are duopoly results having nothing to do with the quadratic transportation costs considered in Cremer et al. (1991).[4]

3 Conclusion

We show that Braid (2008) and Beladi et al. (2014) conclusions are robust for any non-negative, increasing, continuous transportation costs function. As a result of this general-

[4]Cremer et al. (1991) do not assume a duopoly.

ization, we establish that the above conclusions are direct consequences of the duopolistic competition for differentiated products. Examining the robustness of our findings under a two-dimensional spatial framework with more than two competing firms constitutes a topic for future research.

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