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International Productivity Growth Differentials

Sectoral Analysis and Missing Productivity

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Abstract

In Jones (2002), Lucas (2009), and Lucas and Moll (2014), among others, growth is a function of new ideas, and reflects Kuznets (1960) *useful knowledge* (or testable knowledge) as a main driver of growth. In both Kuznets and Jones, the discovery of new ideas is tied to population growth. In the long run, the stock of ideas is proportional to the number of researchers, which is proportional to population. This is the scale factor, where essentially, long-run growth is tied to population growth in the advanced countries. The growth of knowledge due to new ideas depends on the number of people producing them and their productivity, essentially determines Total Factor Productivity growth (TFP). During the transition to the long-run, knowledge in excess of the long-run population growth, i.e., *excess knowledge*, explains most of the productivity growth. We use EUKLEMS 2017 data to show that the model explains 80 percent of the international productivity growth differentials because excess knowledge varies from one country to another. Effective world research efforts diffuse at a different pace from one country to another. We also modify the model and test hypotheses about sector-led growth such as finance and ICT. Finally, we shed light on the current missing productivity conundrum.

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1. Introduction

The causal relationship between population growth and per capita output growth in advanced countries is captured in Jones (2002) model of economic growth. The model is consistent with Kuznets' thesis (e.g., 1960) that population growth causes per capita output growth. In his view, among other channels, *useful knowledge* (or testable knowledge) could be a primary driver of per capita output growth.[i] See Mokyr (2002) for discussion about useful knowledge. Prescott (1998) calls it usable knowledge.

The idea that population growth causes per capita output growth has been controversial although it has been shown to hold under three conditions. First, the aggregate production function is an increasing return to scale; second, the increase in population growth is associated with a greater increase in the growth of capital stock; and third, the form of the production function changes when population increases.[ii] Today, the assumption of increasing returns is basic in all endogenous growth models.

In Jones (2002) just like in Lucas (2009) and Lucas and Moll (2014), long-run growth arises from the worldwide discovery of ideas, however in the former, researchers produce a flow of new ideas. In the long run, the stock of ideas is proportional to the number of researchers, which is proportional to population. This is the scale factor, where essentially, long-run growth is tied to population growth in the advanced countries. Constant growth could, temporarily, continue at a faster rate if research intensity, which creates the new ideas, rises steadily over time. The model distinguishes between two constant growth paths. At a balanced growth path, all variables in the economy grow at constant exponential rates forever. This growth rate is associated with the long-run steady state, and it is determined, essentially, by population growth and some deep parameters. Another constant path is associated with the transitional dynamics. This path is a function of factor of production, human capital, and excess ideas.

Excess ideas, or excess knowledge as we call it in this paper, is the deviations of the growth rate of TFP from steady-state growth. TFP growth is determined by the effective world research effort. As knowledge increases, it generates *a transition path growth effect* and *a level effect* on income. Per capita growth could settle down at a constant rate that is higher than its long-run rate. And, as the fraction of time that individuals spend accumulating skills and knowledge and the share of the labor force devoted to research level off, the economy's growth rate gradually decline to its long-run rate.

The similarities between Jones and Kuznets are intriguing. Essentially, a doubling of the population in the advanced countries – *ceteris paribus* – doubles the effective world research efforts, which causes the long-run level of income to increase by a certain positive fraction.

Jones (2002) provides time series evidence that growth accounting reveals that the factors determining transitional dynamic explain 80 percent of recent U.S. growth. Razzak *et al.* (2016) provides, a relatively, large cross-sectional evidence to Jones (2002) time series analysis. Neither one explains cross-country international productivity growth differentials.

Prescott (1998) shows that differences in factors growth rates cannot explain international differences in income per person growth rates. The same is true for savings and intangible capital differences, human capital or any other capital. This is the main objective of this paper.

First, this paper fits Jones (2002) model to10-EU advanced countries plus the U.S. using EUKLEMS (2017) data. We use the model to explain international productivity growth differentials. We show that differences in excess knowledge across countries explain 80 percent of the differences in productivity growth. The second objective is to modify the model to include a sector-effect then test the hypotheses about information technology-led growth; manufacturing -led growth; and finance-led growth. The model could be used to test any sector, but we focus on these three sectors. More literature that is recent emphasized ICT as a driver for economic growth (e.g., Jorgenson, 2001 and Jorgenson *et. al*, 2000). The idea is that ICT capital and investments cause economic growth in the same way different types of capital, e.g., human capital (Mankiw *et al.* 1992) and intangible capital (McGrattan and Prescott, 2010), also affect economic growth within the neoclassical growth framework. Information technology improves the firm's factor-input supply, final demand, marketing, management efficiency, and many aspects of the production process. A survey of this literature (Kretschmer, 2012) found that econometric estimates of the ICT elasticity vary considerably across studies. It concludes that a 10 percent increase in ICT results in a 0.5 – 0.6 percent increase in productivity growth. Stanley *et al.* (2015) undertakes a meta-regression analysis to 58 studies, document a specification bias and selection bias in favor of growth effect. For earlier studies see Oliner and Sichel (2000) who reported strong positive effects of various types of ICT capital on output and labor productivity growth during the 1990s in the United States.

Economists argued that manufacturing is the driver of growth, e.g., Verdoon's law, Verdoon (1980). It is a standard economic development argument that growth requires increasing the share of manufacturing and reducing the share of agriculture in developing countries. The idea is that manufacturing processes involve a relatively larger value added. The histories of industrial nations also indicate that they began to grow when manufacturing share began to increase. The recent Chinese economic growth is explained by the rise of manufacturing.

There is also an old and long literature on the relationship between finance and growth. Early writings include Schumpeter (1911), who argued that efficient financial markets, via the credit channel, help innovative entrepreneurs to embark on innovative business activities, and that's how the economy grows.[iii] In the 1990s endogenous growth models, such as Romer (1986), treated finance as an external effect on aggregate investment efficiency, which offsets the diminishing marginal product of capital, and sustain growth.[iv] This paper differs significantly from the literature on finance and growth in testing this hypothesis because the transitional growth path is a function of the sector's TFP growth rather than credit, money, and other financial indicators.

Finally, we address the missing productivity puzzle, which has been an issue of recent research (e.g. Feldstein, 2017). Since the model predicts that the long-run growth in any country in the sample is tied to all of the advanced countries population growth, the missing productivity growth could be explained by declining working age population in developed countries, declining fertility rate, declining youth population, and aging, which contribute to declining effective world research efforts.

Next, we describe the model; Section 3 includes the measurements and estimations of key parameters, computations of the key parameters and variables of model that affects growth, and the results. Section 4 concludes. There are two appendices, one for the modification of the model to account for the sectoral effect, and the other is a data appendix.

2. The Original model

We briefly describe Jones (2002) model, which encapsulates Kuznets' thesis about *useable knowledge*.

In each economy in the world, output of goods is produced by a Cobb-Douglas production function. The production function has three elements: the stock of physical capital, the total quantity of human capital, and the stock of ideas (knowledge). The production function is a constant return to scale in the physical capital and human capital, and an increasing return to scale in the three factors.

$$Y_t = A_t^{\sigma} K_t^{\alpha} H_{Y_t}^{1-\alpha} \ , \tag{1}$$

where Y_t is real output, K_t is physical capital, H_{Y_t} is the total quantity of human capital, and A_t is the accumulating stock of *knowledge* created in the world. It is assumed that $0 < \alpha < 1$, and $\sigma > 0$, which implies a constant return to scale in K, and H_Y, and an increasing return to scale in K, H_Y and A as $\alpha + 1 - \alpha + \sigma = 1 + \sigma > 1$. The subscript Y refers to the production of goods.

The physical stock of capital evolves as in the Perpetual Inventory equation, where people forgo some consumption, and the resulting saving out of output is invested. Capital depreciates at some constant exogenous depreciation rate.

Human capital is the product of human capital per person and the total amount of labor employed in the production of goods and services. It is produced when people forgo time working in the labor force. This is consistent with the (Mincer, 1974). The aggregate human capital employed to produce output is:

$$H_{Yt} = h_t \, L_{Yt}, \tag{2}$$

where h_t is the stock of human capital per person multiplied by the total amount of labor, which is produced by forgoing time in the labor force. The individual spends time in education, training …etc. This time is l_h, so h_t is:

$$h_t = e^{\psi\, l_{ht}}, \psi > 0, \tag{3}$$

Equation (3) is like the equation shown in Bils and Klenow (2000), where ψ is the rate of return on education.

Knowledge is non-rival and non-excludable. Once it is created it could be used by others (Romer, 1990). Knowledge created anywhere in the world is available for use by any country.[v] Thus, the stock of knowledge is the cumulative stock of knowledge anywhere in the world. Researchers produce Knowledge. There is a production function, where by the growth rate of knowledge is a function of the effective world research efforts, which is weighted sum of the number of researchers in each country. The weights adjust for human capital. Thus, the number of ideas produced at any point in time depends on the number of researchers and the existing stock of knowledge. Jones (2002) assumes that effective world research is produced in the 5 advanced countries in the world (the U.S., the U.K., Germany, France and Japan). This formulation is fully consistent with Kuznets (1960, p. 330). He says, "We should recognize that the creative and educated groups in developed countries – and they are the central reference point here – serve partly, and should serve more fully the economic needs of the whole world, not merely of their own countries."

$$\dot{A}_t = \delta H_{At}^{\lambda} A_t^{\phi} \quad A_0 > 0, \tag{4}$$

Where a dot on the variable denotes the growth rate, and H_{At} is the effective world research effort, and is given by:

$$H_{At} = \sum_{i=1}^{M} h_{it}^{\theta} L_{A,it}, \tag{5}$$

Where h_{it} is human capital and $L_{A,it}$ is the number of researchers. The subscript A in H_A refers to the human capital used in the production of knowledge as opposed to Y in H_Y, which is the human capital used in the production of goods (equation 1).

Finally, the model has a resource constraint on labor such that an economy populated by a number of identical, infinitely lived agents, whereby the number grows over time at a common and constant exogenous growth rate n. Each individual is endowed with one unit of time, which is divided among producing goods and services, ideas, and human capital.

Along the balanced growth path, capital-output ratio, labor used in production – total labor, and human capital are constant. The economy exhibits a stable balanced growth path, whereby all the variables grow at constant exponential rates forever. The allocations of factors along that path are also constant. The growth rate of output per worker is proportional to the growth rate of effective world research. Since human capital is constant along the balanced growth path, the growth of the effective number of world researchers is driven by population growth. The long-run per capita growth is ultimately tied to world population growth as in Kuznets (1960).

A key result of the model is that the long-run per capita growth is ultimately tied to population growth of the advanced countries. If the level of this population doubles, *ceteris-paribus*, the effective world research effort also doubles, which cause the long-run income level in each country increases by a factor 2^{γ}. The deep parameter γ is $\frac{\sigma}{1-\alpha}\frac{\lambda}{1-\phi}$.

The model results in the following growth-accounting equation:

$$\underbrace{\dot{y}_t = \frac{\alpha}{1-\alpha}(\dot{K}_t - \dot{Y}_t) + \dot{h}_t + \dot{l}_{Yt} + \underbrace{\left(\frac{\sigma}{1-\alpha}\dot{A}_t - \gamma n\right)}_{excess\,knowledge}}_{Transitional\,Dynamic} + \underbrace{\gamma n}_{Steady State}, \tag{6}$$

where $\dot{y}_t$ is real output per hour-worked growth rate (a dot on top of the variable refers to growth rates). The other variables are defined earlier. Contrary to all cross-country growth regressions, in this model the covariance between per capita growth and population growth is > 0, and not negative. Mankiw *et al.* (1992) interpret a negative covariance of per capita growth and population growth as reflecting the transition dynamics of the neoclassical growth model, i.e., a higher population growth reduces the steady-state capital-output ratio because more investments are required to maintain the existing capital-output ratio as population is growing. The first term of the equation captures this effect. The last term in the equation comes from the fact that the stock of ideas grows at a constant rate (and the stock is inferred from the flow).

The interpretation is that the RHS terms (except the last) are zero in the steady state. These terms represent the *transitional dynamic*. If an economy is close to its balanced growth path, the last term should account for most of growth. Therefore, the productivity growth transitional dynamic is a function of capital intensity, human capital growth, growth rate of labor allocated for the production of output, and excess knowledge $(\frac{\sigma}{1-\alpha}\dot{A}_t - \gamma n)$. Excess knowledge represents the growth rate of TFP, which is a function of effective world research efforts, in excess of population growth of the advanced countries.

Appendix I includes the solution of the modified model, which includes the sectoral effect. The assumption is that $A_{s_t} = \eta_0 A_t^{\eta}$, where the subscript s denotes the sector. The model boils down to the following growth-accounting equation:

$$\dot{y}_t = \underbrace{\frac{\alpha}{1-\alpha}(\dot{K}_t - \dot{Y}_t) + \dot{h}_t + \dot{l}_{Yt} + \underbrace{\left(\frac{\sigma}{\eta(1-\alpha)}\dot{A}_{s,t} - \gamma n\right)}_{excess\ knowledge}}_{Transitional\ Dynamic} + \underbrace{\gamma n}_{Steady State}, \qquad (7)$$

Where all the variables are defined earlier and the subscript s refers to sector.

3. Testing the prediction of the model

3.1 Measuring the parameters and the variables

We use EUKLEMS (2017) data to (1) test the model by examining how much of the productivity growth is explained by excess knowledge; (2) how much of the international productivity growth differentials is explained by excess knowledge differentials? and (3) test sector-led growth hypotheses. EUKLEMS (2017) include data for all the EU countries plus the United States. We only use 10 EU countries, which have all data needed to test the model. Most of the other countries do not have the required data to test the model. The countries are Austria, Belgium, Finland, France, Germany, Italy, the Netherland, Spain, Sweden, the U.K. and the U.S. The data are described in the data appendix II.

Measuring productivity growth and excess knowledge

We measure productivity growth $\hat{y}_t$ and excess knowledge $\frac{\sigma}{1-\alpha}\dot{A}_t$ for all 11 countries in the sample. Productivity is defined as real output per hours-worked. For real output put we use EUKLEMS value added measure (VA), which we deflate by the value added price, VA_P (2010=100). We then measure real value added per hours worked by dividing the real value added by total hours by persons – engaged (H_EMP). The growth rate of this would be our measure of productivity growth.

However, EUKLEMS provide two measures of value added and prices, a total economy and a market measure. We use the market economy measure, which excludes sectors whose outputs are "hard-to-measure." These are lines L, O, P, Q, T, and U, which are mainly services: Real Estate activity; Public Administration and Defense; Compulsory Social Security; Education, Health and Social Work; Activities of Households as Employers; Undifferentiated Goods- and Services-producing Activities of Households for own use. Data are in figure (1). We shall come to this measurement issue again in the last section.

Measuring excess knowledge

For excess knowledge, we also use the market economy data to measure $\dot{A}_t$, TFP as reported in EUKLEMS. The Penn World Table 9.0 reports time series for the share of labor so it is $1-\alpha$ in equation (6). We take the average value over the samples. However, the parameter σ is unidentifiable. Jones (2002) assumed that it is equal to $1-\alpha$ so that A is measured in units of Harrod-Neutral productivity. We use sensitivity analysis and calibrate the equation using a number of values. We that find $\sigma = 1$ provides the best fit for every country in the sample. Recall that the production function (eq. 1) exhibits increasing returns to scale.

The deep parameter γ is very difficult to measure. To measure $\frac{\sigma}{1-\alpha}\frac{\lambda}{1-\phi}$ we need values for λ and ϕ, but estimation of equation (4) is hampered by the discontinuity of the time series data for $L_{A,it}$ (the number of researchers). Jones (2002) provided more than one estimate for γ using different methods. He suggested that the value for the U.S. is between 0.05 and 0.30.

Dividing both sides of equation (4) by A_t gives:

$$\frac{\dot{A}_t}{A_t} = \delta\left(\frac{H_{A,t}^{\gamma}}{A_t}\right)^{1-\phi} \tag{8}$$

The equation says that the growth rate of TFP depends on the ratio of the quantity of human capital used in the production of knowledge to the level of productivity. Both $H_{A,t}$ and A_t are upward trending and the growth rate of TFP is I(0). Jones (2002) argues that the parameter γ de-trends the ratio $H_{A,t}^{\gamma} / A_t$ to give an I(0) TFP growth rate. Therefore, the parameter γ is equal to the ratio $\dot{H}_{A,t} / \dot{A}_t$, where $\dot{H}_{A,t}$ is the growth rate. The effective world research effort $H_{A,t}$ was defined in equation (5) as $\sum_{i=1}^{11} h_{it}^{\theta} L_{A,it}$, where h is human capital. The human capital index is reported in the Penn World Table 9.0, and it does vary significantly over time. The number of researchers L_{it} is reported in the World Bank data, but the time series has missing years across the panel. For this reason, we calculate $H_{A,t}$ for the year 1995 and the year 2014 then compute the weighted $H_{A,t}$ and the growth rate over that range. The value of the weight θ did not seem to matter for the computation of γ. We tried Zero, and the share of the number of researchers in the total number of researcher in these 11 countries. The results do not change.

Note that γn has to be small in magnitude since the growth rate of population in the EU and the U.S. is small, less than 1 percent on average. Thus, the size of γ does not matter a great deal for the computing excess knowledge.

Table (1) reports the values of σ , α , and γ , which we use in computing excess knowledge. Jones (2002) estimate for the U.S. was 0.30. We too have the exact estimate; however, for Finland we have 0.81 and -0.20 and -0.27 for Italy and Spain respectively because TFP growth over the sample was negative.

Population growth is measured by the growth rate of the labor force. We take the average of all 11 countries' labor force productivity growth, which is 0.81 percent (less than 1 percent). This the term γn - the steady state – growth is rather a small number.

Now we have all the data necessary to measure productivity growth and excess knowledge. Figures 2 plots the data for each country. The scattered plots around the 45° line indicate that excess knowledge explains real value added / hour-worked productivity rather very well in all countries, except in France and Spain. There is an obvious significant cross-country variation in the data. And, the effect of the global financial crisis and the Great Recession is evident, except for Germany. For the rest of the EU (2008-2009), Sweden (2008-2009, 2009-2010, and 2010-2011), and the U.S. (2007-2008) data are far from the 45° line.

3.2 International productivity growth differentials

The most important hypothesis is one regarding the international productivity growth differential. Do differences in excess knowledge explain differences in productivity growth? If so, then it would constitute a significant support for Kuznets (1960) and Jones (2002) models and it would answer Prescott's (1998) observation that factor inputs, tangible and intangible cannot account for the international productivity differentials. Conversely, Restuccia and Rogerson (2017) argue that misallocation of factors of production explains some of the productivity differences, albeit between the rich and the poor countries. As shown earlier, the model predicts that TFP growth is determined by effective world research efforts, which is ultimately tied by population growth in the advanced countries.

We define international differentials as the deviations of the magnitudes of each country of the 10-EU countries from the U.S. We measure the deviations of the *average* of each country's productivity growth and excess knowledge from the U.S. average magnitudes. Figure (3) is a scatter plot of these averages; only France and U.K. are off the 45° line. Excess knowledge differentials explain 80 percent of the differences in productivity growth on average. This result is remarkable new evidence. Our finding is somewhat consistent with the current literature that the frontier of technology and best practices diffuses at different rates across countries, including advanced countries.

Sector-led productivity growth

To examine the fit of the model in equation (7) we only need to measure η for each country.[vi] We assume $\eta_0 = 1$. We take the ratio of log TFP of the sector measured as Tornqvist indices to the market economy log TFP (eq. 2 Appendix I). We allowed that parameter to vary across the sample, hence η_{ijt}, for country i, sector j and year t even though it would not significantly change the result if we use the average value over the sample. Table (2) reports the values of η, which we used in estimating excess knowledge in the sector model. The rest of the parameters σ, α and γ are the same as reported in table (1).

Figures 4 plots the results country-by-sectors. We test three sectors: information and communication; manufacturing; and finance. The sectors excess knowledge varies significantly from one sector to another within each country, and across countries. The variations are larger than the ones we observed in the country's aggregate excess knowledge.

Figure (4.1) plots the Austrian data. The scattered points are far from the 45° line, except for a manufacturing (black). There is no support to neither the information nor the finance – led growth hypotheses. Belgium is quite different. Figure (4.2) shows Belgium's; with closer associations between the sector's excess knowledge and aggregate productivity growth for manufacturing and the information sectors around the 45° line. Even the finance sector seems to be closer to the 45° line if we ignore the global financial crisis periods.

Figure (4.3) plots the Finish data. There is much more association between excess knowledge in the sectors and productivity growth than in Austria and Belgium, crises and the Great Recession notwithstanding.

Figure (4.4) plots the French data, which display even tighter association between the sectors excess knowledge and aggregate productivity growth than in previous countries. Information, manufacturing, and even finance excess knowledge seem to be associated with productivity growth. Also a lot more strayed scatter dots associated with various crisis such as Dotcom (2001), the Asian (1997) and the global financial crises, and the Great Recession (2008). There is more support for the sectoral excess knowledge – productivity growth than to aggregate excess knowledge in Figure (5).

Figure (4.5) displays the German sectoral data. The global financial crisis had affected both the finance and the manufacturing sectors more than it affected the countries above. There are tight fluctuations around the 45° line nevertheless; not so for the information sector. There are a number of scatter points in the finance sector drifting away with extreme values from the 45° line in the years 2002-2004, which could be related to the Dotcom crisis in 2001-2002. Figure (4.6) plots the Italian sectors. The finance sector fluctuated widely, and away from productivity and the 45° line since the late 1990s, which coincides with the Asian financial crisis; it has no association with productivity growth. Even though the support for finance – led growth has been dismal so far, the hypothesis fails significantly in Italy. Manufacturing seems to have the highest association with productivity. We find little or no association between the information sector and productivity in Italy.

Figure (4.7) is for the Netherlands. The data provide more support to the sector-led growth hypothesis. The global financial crisis notwithstanding, all sectors excess knowledge seems to be associated with productivity growth. The financial crisis (2008-2009) affected all three sectors equally. Figure (4.8) displays the Spanish data, which are very similar to the Italian data. There is no support to the hypotheses that finance or/and information technology has any effect on productivity growth. Most of the support comes from the manufacturing sector. The black dots tightly fluctuate around the 45° line.

Figure (4.9) plots the Swedish data. The Swedish experience is similar to that of the Dutch. All sectors, global financial crisis notwithstanding, seem to contribute to productivity growth. Figure (4.10) for the U.K. shows more support for the manufacturing and finance sectors than to the information sector-led growth. Finally, figure (4.11) for the U.S. shows a huge variance, and a significant impact of the financial crisis, the dotcom crisis in 2001 and the Great Recession. Nevertheless, the scatter plots vary around the 45° line in support of the hypothesis of sector-led growth. It seems like all sectors have associations with productivity growth.

Generally speaking, the sectoral data seem to lend reasonable empirical support to the sector-led growth hypothesis, but the effects of the financial crisis, whether the global financial crisis or the Asian financial crisis, seem to manifest more in sectoral data than in the aggregate data. The finance-led growth hypothesis has a relatively less support by the data. There is some evidence for the information sector, and more evidence for manufacturing. We only examined three sectors, thus it remains possible that other sectoral changes have effects on productivity growth.

Figures (5) and (6) summarize the sectoral analysis. We plot the average excess knowledge by sector for each country in figure (25). The variation is much clearer in the finance and the information sectors, more so than in manufacturing. Germany has a negative average excess knowledge. France, the U.K. and the U.S. have the smallest excess average excess knowledge in the sample. We observe huge excess knowledge in the information sector, except in Austria, which is a small negative number. Figure (26) plots the average excess knowledge in the sectors against average productivity growth. Average excess knowledge in manufacturing across countries seem to be more correlated with productivity growth along the 45° line. Average excess knowledge in finance, however, does not seem less correlated with productivity growth than the manufacturing sector, but they have larger variance. For the finance sector, the global financial crisis and the Asian crisis before it had significant effects in Germany, Austria, Spain and maybe France. These red dots are farther away from the 45° line. For the information sector, Finland, Germany, and the U.K. are way off the 45° line. Therefore, in summary we would argue that the manufacturing sector-led growth hypothesis has more support in the data than finance and information-led growth, but volatility in these two sectors' excess knowledge and productivity was probably affecting the calculations.

3.3 The missing productivity

There has been a lot of concern about the decline in productivity in the developed countries lately. Feldstein (2017) argues that mismeasurement of the quality improvement in GDP explains the missing productivity level. Mismeeasurement is a plausible explanation. Relevant to this issue, generally speaking, is our use of *market* instead of *total* productivity data. Figure (1) showed real value added per hour growth rate (i.e., productivity growth rate) using the two measures, and the difference. The market economy measure yields a significantly higher productivity growth. This might be somewhat consistent with measurement explanation of the missing productivity puzzle. The market measure excludes services sectors, where output is hard to measure. Countries with relatively larger public services sectors such as the European countries miss significant productivity growth using the total economy measure; 1.5 percent in France, 0.78 percent in Finland, 1.06 percent in the Netherlands, and 0.81 percent in Sweden.

Syverson (2017) focuses on ICT, but shows evidence that the slowdown in productivity has occurred in many countries and not only in the U.S. and perhaps long before 2004.

Bloom *et al.* (2017) argue, using the same Johns model (2002) which we use here as a basis for analysis, that evidence at the micro level U.S. data suggest that there has been an increase in global research efforts (i.e., the number of people in research) coupled with a sharp decline in research productivity. They say that is getting harder to find a new idea. This is a controversial assumption. Lucas and Moll (2014), however, assumed that the stock of ideas is never exhausted. Luttmer (2012) assumes that productivity is subject to a small Brownian noise.

Fernald and Jones (2014) argued that the U.S. future economic growth is likely to slow down because educational attainment and population are likely to slowdown in the future. They also argue that the shape of the idea production function introduces uncertainty into the future growth. They also talk about the rise of China and India's research growth, artificial intelligence, climate change, income inequality, and health care.

Missing productivity seems consistent with the thesis of this paper. In our cross-sectional data, we show significant association between excess knowledge and productivity growth along the transitional growth path in countries other than the U.S. Figure (7) plots the average excess knowledge across all 11 countries, which seems to be trending down slightly. Italy and Spain have negative average excess knowledge growth over the sample. Figure (8) plots the average effective world research efforts, and figure (9) plots the growth rates. The *cross-sectional* trend is slightly downward slopping. Therefore, some of the missing productivity along the transitional growth path in the developed countries could be explained, by: (1) mismeasurement of productivity, which includes sectors with hard-to-measure output such as health, education, defense, and other services. A market measure such as the one we used in this paper reveals higher growth rates as shown in figure (1), especially in countries with relatively larger public sectors and social services programs, such as Finland, France, the Netherlands, and Sweden. (2) A decline in growth rate of excess knowledge in the advanced countries in general; and (3) a decline in the growth rate of effective world research efforts, which are due to decline in the supply of labor in the production of research that occurred after the Great Recession.

In the long run, declining population growth rate might put more downward pressure on productivity growth. There is a decline in working age population across the 11 developed countries (Figure 10); a stagnation and a decline in fertility rate (Figure 11); a decline in youth population (Figure 12). Aging population is a well-known issue in the advanced country.

4 Conclusion

Solow (1957) growth model is not a model for international productivity differentials; savings rates differentials are too small. Prescott (1998) showed that factor input growth differentials, savings, human capital and other intangible capital, could not explain international productivity growth differentials. To explain international productivity growth differentials we used Jones (2002) semi-endogenous growth model, which he tested for the U.S., to test the EUKLEMS (2017) data for 10 European countries and the U.S. The model's long run productivity growth is determined by population growth, which is consistent with Kuznets (1960). Along the transitional dynamic path, productivity growth is determined by the same neoclassical factor input growth rates, plus a new variable called excess ideas or excess knowledge. We showed that this variable explains 80 percent of the international productivity growth differentials.

In this mode, excess knowledge is TFP growth weighted by the ratio of the share of technical progress in the production function and the share of labor, minus *(or in excess of) long-run* population growth in the advanced countries weighed by a parameter that measures effective research efforts in these countries (see equation 6). And, the growth rate of TFP is itself a function of the research efforts in the advanced countries (see equation 8). This is essentially saying that countries differ in the amount of excess knowledge they possess, thus have different productivity growth rates. Most importantly is that TFP growth is a function of global research efforts, which are based on the stock of knowledge, the number of people producing research, and human capital in the advanced countries in the world.

We also modify the model to allow for excess knowledge at sectoral level. We tested hypotheses about sectoral-led growth for every country in the sample. We chose three sectors to test, manufacturing, finance and information. There is large literature on each of these hypotheses. We found more evidence for manufacturing-led productivity growth, on average, than for either finance or information sectors.

For finance-led growth hypothesis, unlike the existing literature we do not have money, credit and any of the typical variables used in testing this hypothesis. The association between excess knowledge in finance and aggregate productivity growth along the transitional dynamic path is highly affected by financial crisis such as the Asian crisis in 1997-1998 and the global financial crisis and the subsequent Great Recession between 2007 and 2009. We observe very large variations and extreme values. The same is true for the information sectors across countries in the sample. They too are affected by crisis such as the Dotcom. Nonetheless, excess knowledge in the finance and information sectors is closely correlated with productivity growth.

Finally, the model sheds light on the decline in productivity growth in the developed countries, especially the U.S. The model predicts that productivity growth is tied to population growth in the long run, and to excess knowledge (along with the other factors of production) along the transitional dynamic path. All the countries have declining working age population (15-65), fertility rates, and young population (0-14). Further, older population (65 and above) has been increasing. The average effective world research effort growth rate has been declining slightly over the period 1995 to 2016 across the developed countries (the EU 10 plus the U.S.) in the sample. These trends are consistent with the model's predictions.

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Table (1)
The values of the parameters used in the calibration of the growth equation

Country	σ	α	γ
Austria	1	0.41	0.402297
Belgium	1	0.37	0.283751
Finland	1	0.41	0.814458
France	1	0.38	0.122269
Germany	1	0.37	0.354242
Italy	1	0.47	-0.20123
The Netherlands	1	0.39	0.234447
Spain	1	0.37	-0.27976
Sweden	1	0.46	0.428839
U.K.	1	0.38	0.329777
U.S.	1	0.38	0.304491

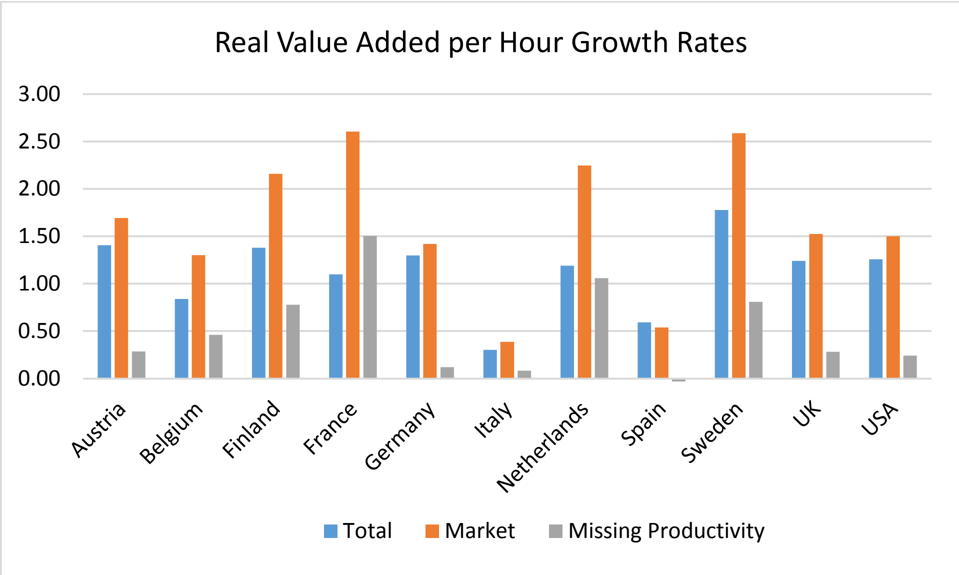
We conducted a sensitivity analysis for σ using values equal to $1-\alpha$ up to one and we found that a value of one gives the best fit.

Table (2)

The average value of η used in calibrating excess knowledge in the sector model

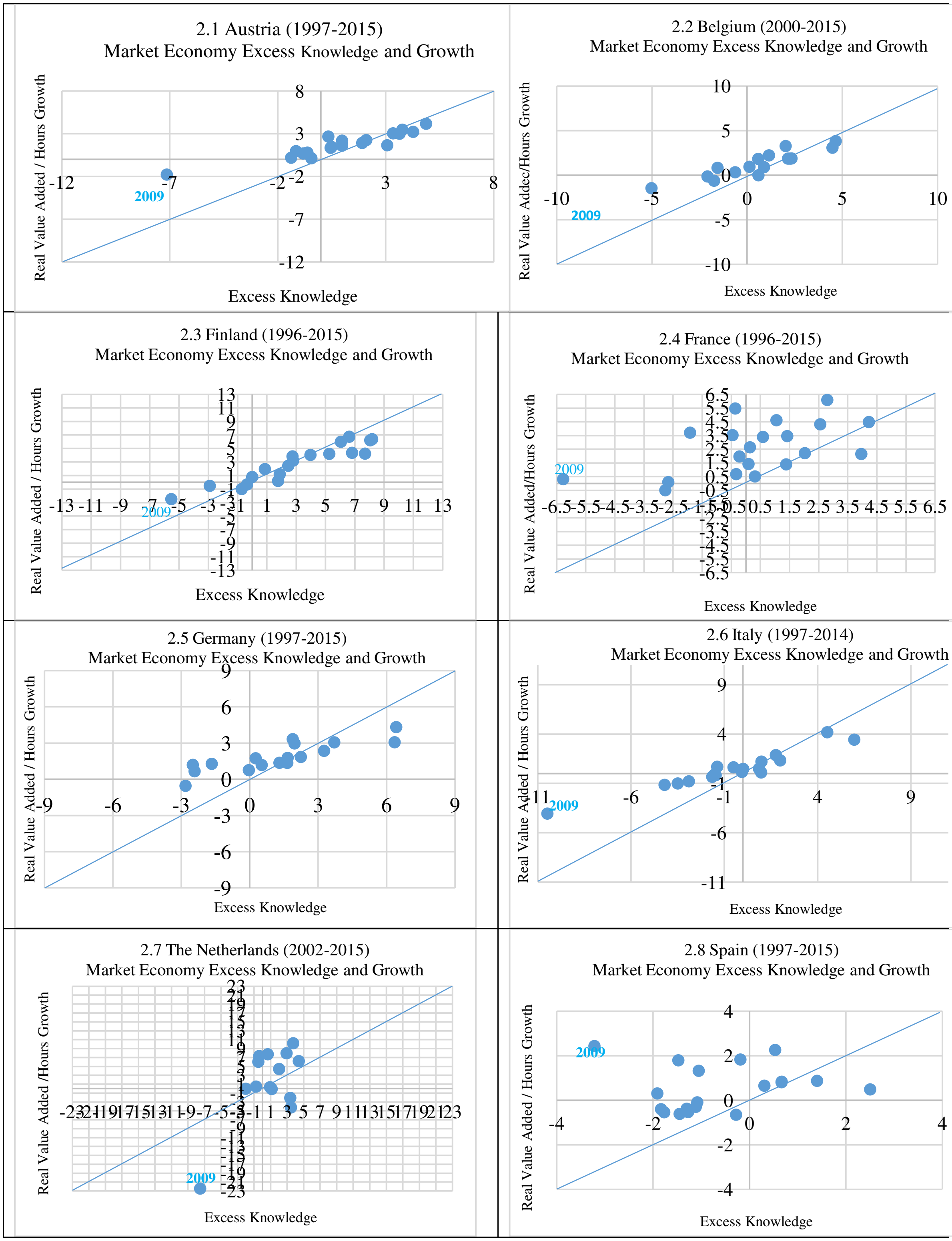
Country	Information Technology	Manufacturing	Finance
Austria	0.99	0.99	0.96
Belgium	0.99	0.99	0.99
Finland	0.98	0.98	0.99
France	0.99	0.98	0.99
Germany	0.87	0.98	0.98
Italy	0.96	0.99	0.97
The Netherlands	0.99	1.00	0.97
Spain	0.98	0.98	0.95
Sweden	0.97	0.97	1.00
U.K.	0.97	0.98	0.99
U.S.	0.97	0.98	0.99

Figure (1)



The sample is 1996-2015, except for Belgium (2000-2015) and the U.S. 1999-2015 because not all the data are available.

Figure (2)


2.1 Austria (1997-2015)
Market Economy Excess Knowledge and Growth
Real Value Added / Hours Growth
Excess Knowledge
2009
2.2 Belgium (2000-2015)
Market Economy Excess Knowledge and Growth
Real Value Addec/Hours Growth
Excess Knowledge
2009
2.3 Finland (1996-2015)
Market Economy Excess Knowledge and Growth
Real Value Added / Hours Growth
Excess Knowledge
2009
2.4 France (1996-2015)
Market Economy Excess Knowledge and Growth
Real Value Added/Hours Growth
Excess Knowledge
2009
2.5 Germany (1997-2015)
Market Economy Excess Knowledge and Growth
Real Value Added / Hours Growth
Excess Knowledge
2.6 Italy (1997-2014)
Market Economy Excess Knowledge and Growth
Real Value Added / Hours Growth
Excess Knowledge
2009
2.7 The Netherlands (2002-2015)
Market Economy Excess Knowledge and Growth
Real Value Added /Hours Growth
Excess Knowledge
2009
2.8 Spain (1997-2015)
Market Economy Excess Knowledge and Growth
Real Value Added / Hours Growth
Excess Knowledge
2009

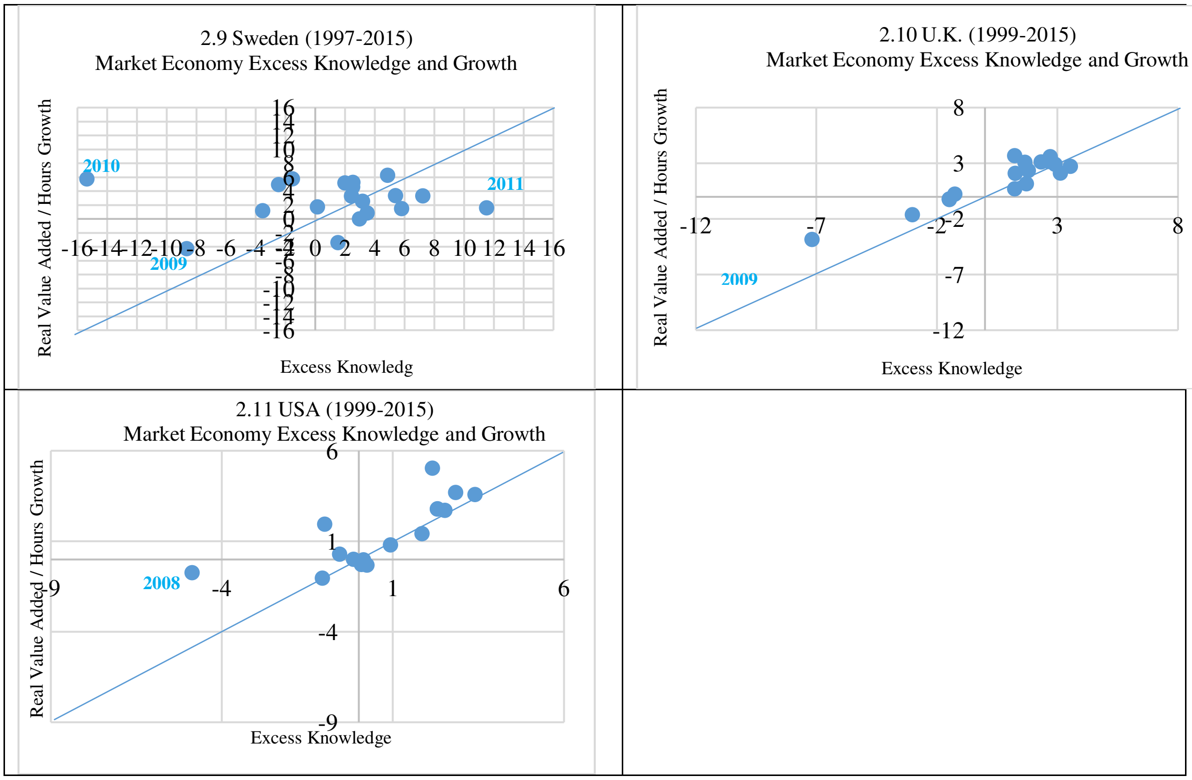

2.9 Sweden (1997-2015)
Market Economy Excess Knowledge and Growth
Real Value Added / Hours Growth
Excess Knowledg
2010
2009
2011
2.10 U.K. (1999-2015)
Market Economy Excess Knowledge and Growth
Real Value Added / Hours Growth
Excess Knowledge
2009
2.11 USA (1999-2015)
Market Economy Excess Knowledge and Growth
Real Value Added / Hours Growth
Excess Knowledge
2008

Figure (3)

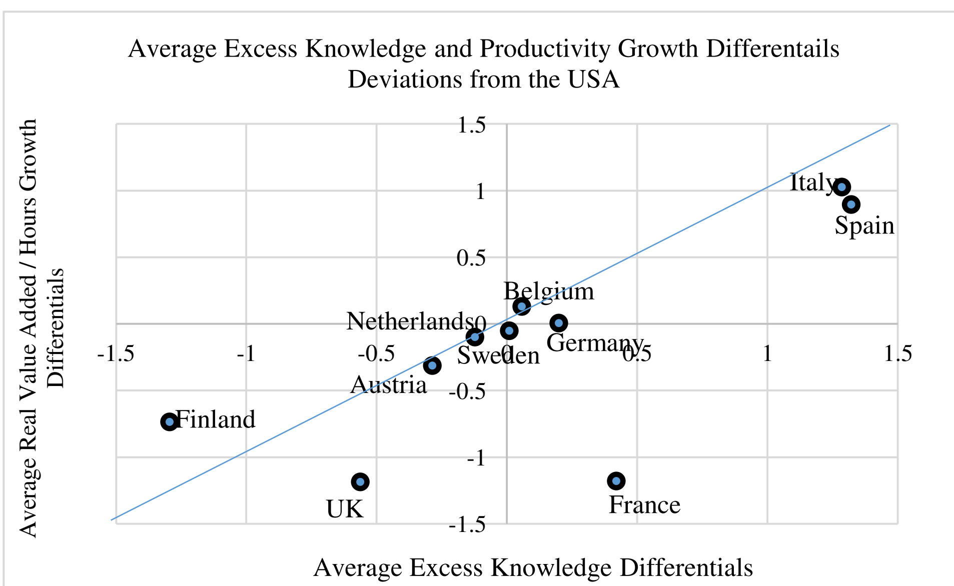
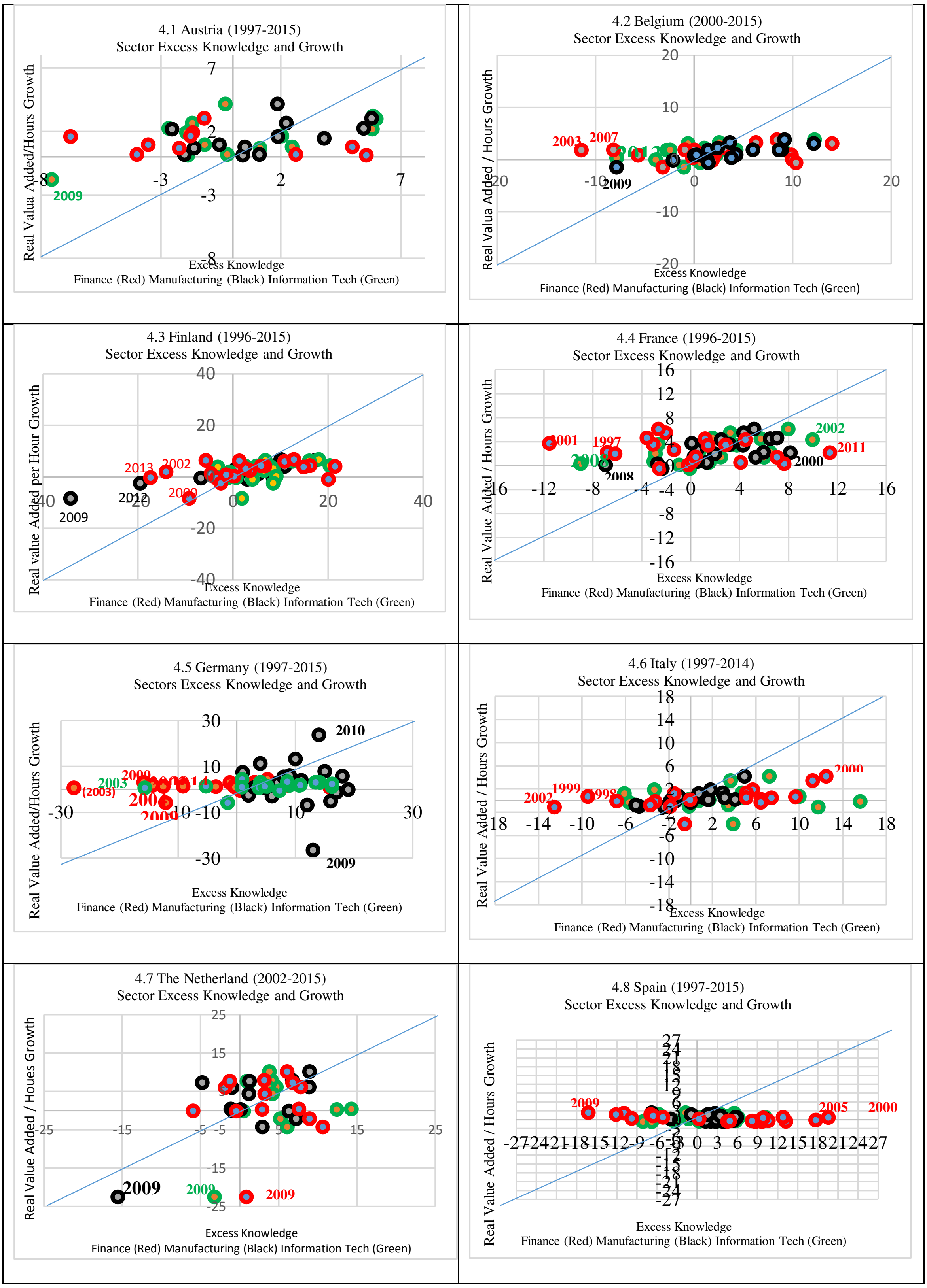

Average Excess Knowledge and Productivity Growth Differentails
Deviations from the USA
Average Real Value Added / Hours Growth
Differentials
Average Excess Knowledge Differentials
Italy
Spain
Belgium
Netherlands
Sweden
Germany
Austria
Finland
UK
France

Figure (4)


4.1 Austria (1997-2015)
Sector Excess Knowledge and Growth
Real Valua Added/Hours Growth
Excess Knowledge
Finance (Red) Manufacturing (Black) Information Tech (Green)
4.2 Belgium (2000-2015)
Sector Excess Knowledge and Growth
Real Valua Added / Hours Growth
Excess Knowledge
Finance (Red) Manufacturing (Black) Information Tech (Green)
4.3 Finland (1996-2015)
Sector Excess Knowledge and Growth
Real value Added per Hour Growth
Excess Knowledge
Finance (Red) Manufacturing (Black) Information Tech (Green)
4.4 France (1996-2015)
Sector Excess Knowledge and Growth
Real Value Added / Hours Growth
Excess Knowledge
Finance (Red) Manufacturing (Black) Information Tech (Green)
4.5 Germany (1997-2015)
Sectors Excess Knowledge and Growth
Real Value Added/Hours Growth
Excess Knowledge
Finance (Red) Manufacturing (Black) Information Tech (Green)
4.6 Italy (1997-2014)
Sector Excess Knowledge and Growth
Real Value Added / Hours Growth
Excess Knowledge
Finance (Red) Manufacturing (Black) Information Tech (Green)
4.7 The Netherland (2002-2015)
Sector Excess Knowledge and Growth
Real Value Added / Houes Growth
Excess Knowledge
Finance (Red) Manufacturing (Black) Information Tech (Green)
4.8 Spain (1997-2015)
Sector Excess Knowledge and Growth
Real Value Added / Hours Growth
Excess Knowledge
Finance (Red) Manufacturing (Black) Information Tech (Green)

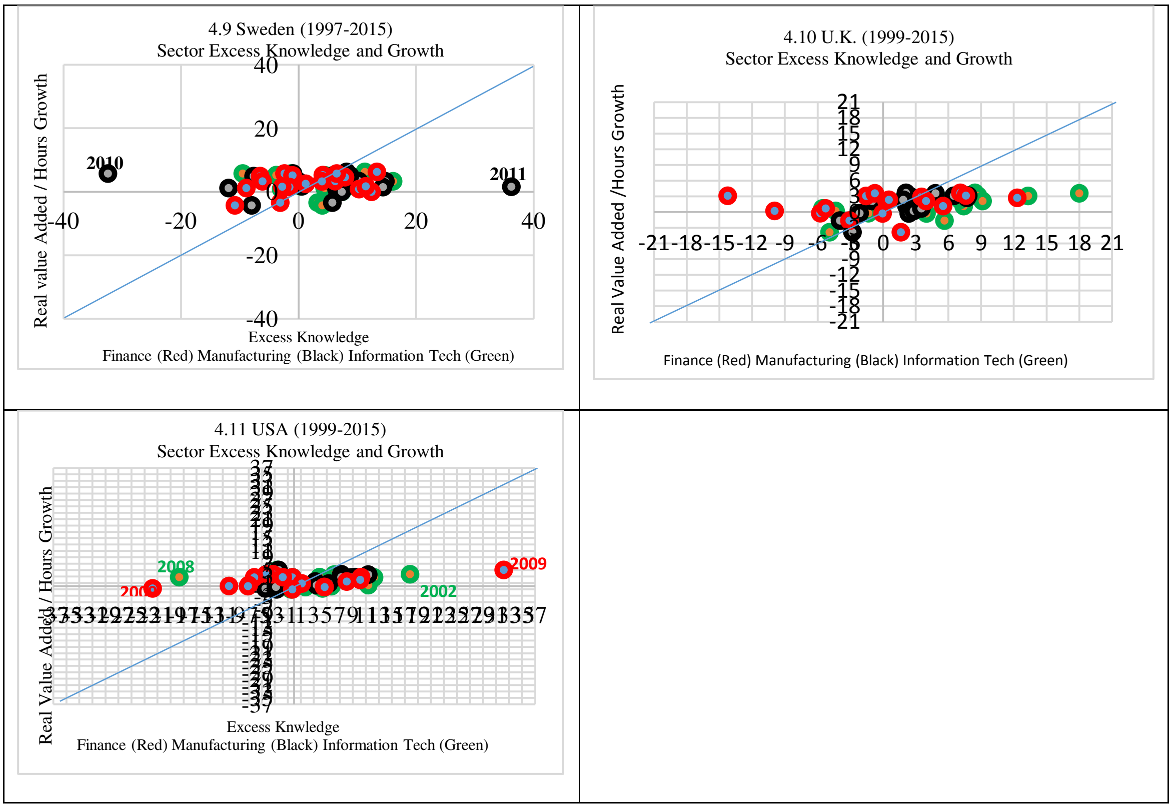

4.9 Sweden (1997-2015)
Sector Excess Knowledge and Growth
Real value Added / Hours Growth
Excess Knowledge
Finance (Red) Manufacturing (Black) Information Tech (Green)
2010
2011
40
20
0
-20
-40
4.10 U.K. (1999-2015)
Sector Excess Knowledge and Growth
Real Value Added /Hours Growth
Finance (Red) Manufacturing (Black) Information Tech (Green)
-21 -18 -15 -12 -9 -6 -3 0 3 6 9 12 15 18 21
4.11 USA (1999-2015)
Sector Excess Knowledge and Growth
Real Value Added / Hours Growth
Excess Knwledge
Finance (Red) Manufacturing (Black) Information Tech (Green)
2008
2009
2002

Figure (5)

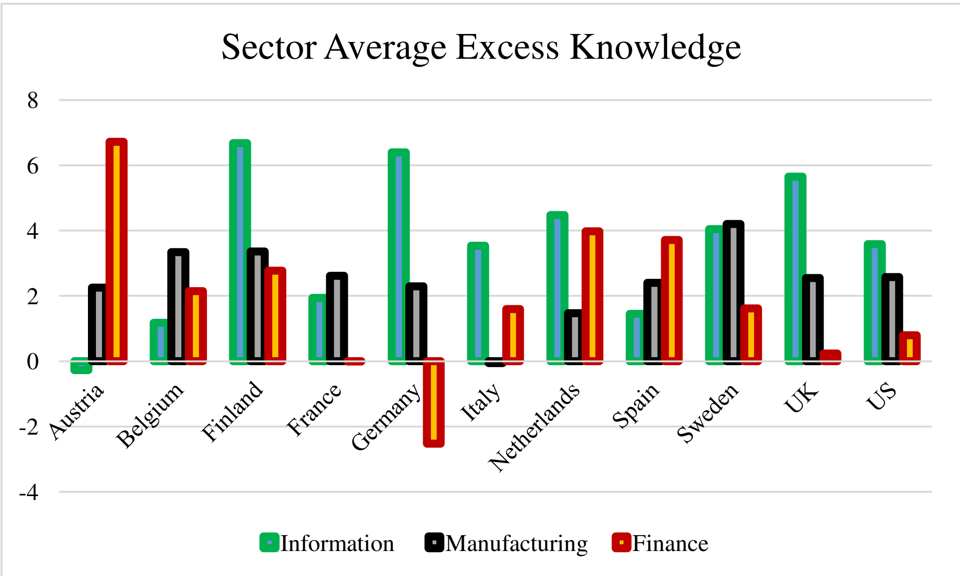

Sector Average Excess Knowledge
8
6
4
2
0
-2
-4
Austria
Belgium
Finland
France
Germany
Italy
Netherlands
Spain
Sweden
UK
US
Information
Manufacturing
Finance

Figure (6)

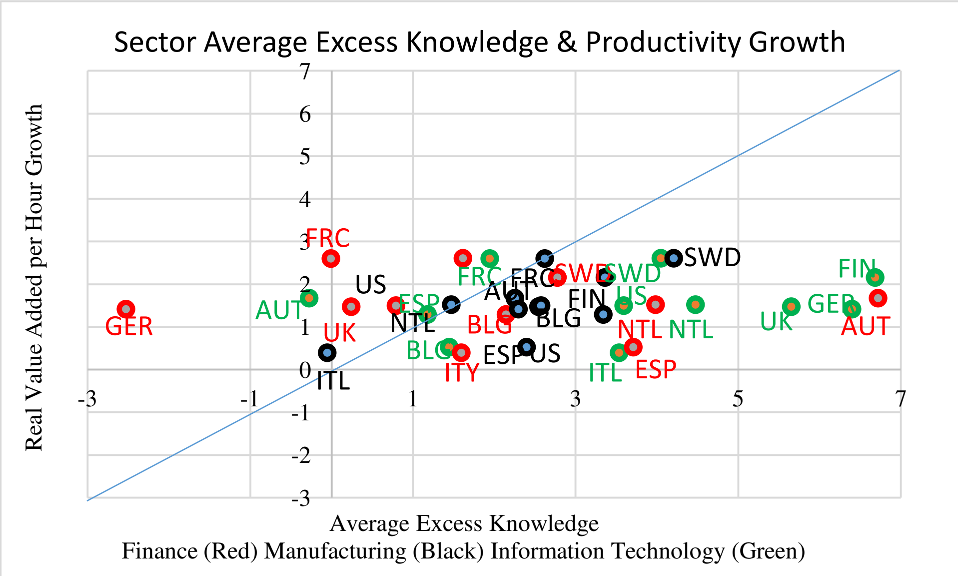


Figure (7)

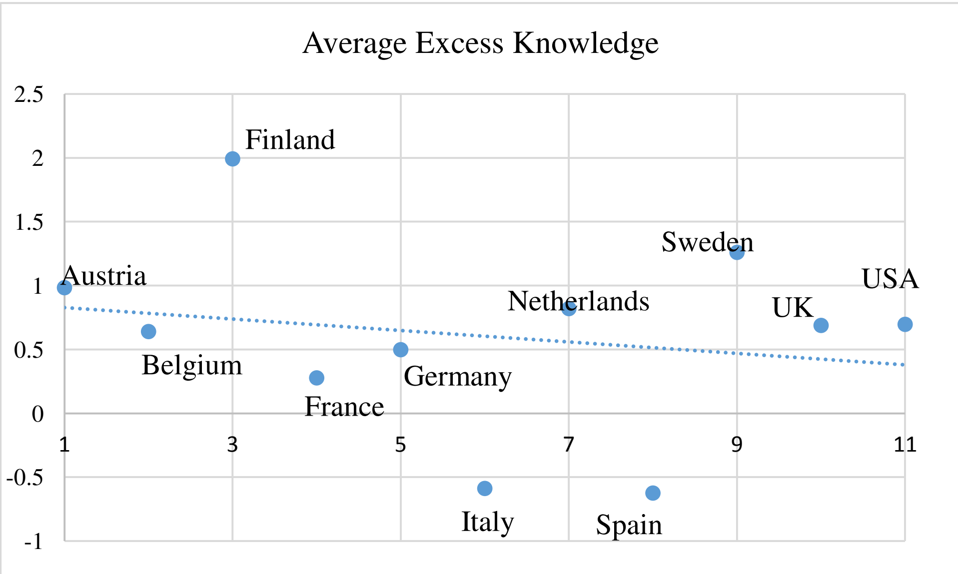


Figure (8)

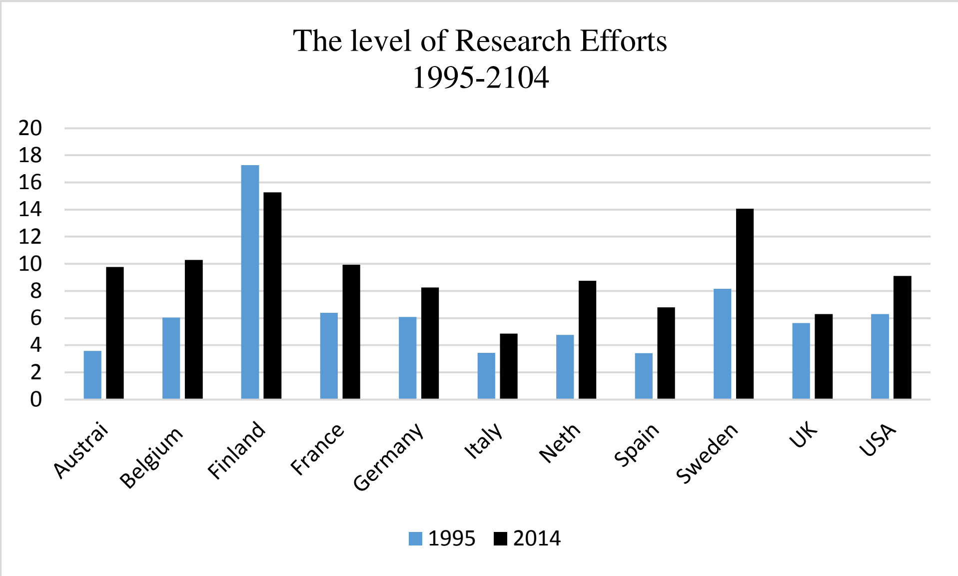


Figure (9)

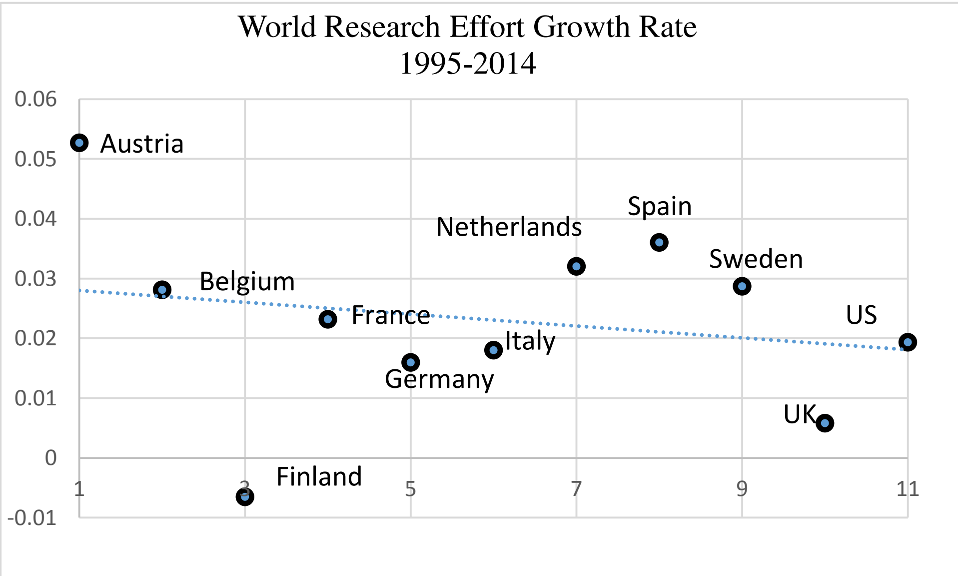


Figure (10)

Percentage of Working Age Populatio (15-64)

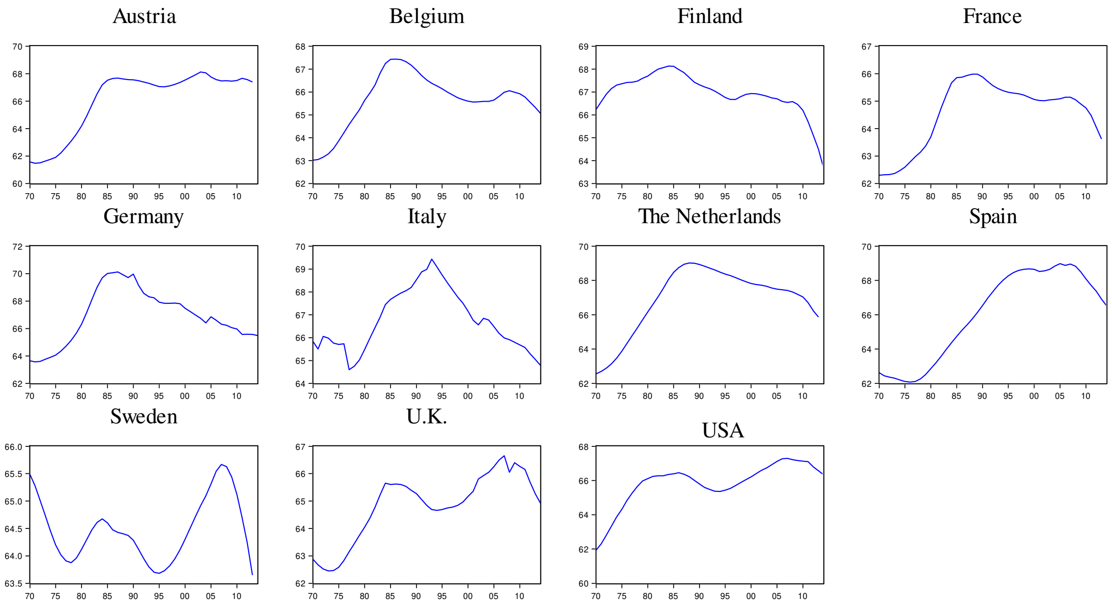


Figure (11)

Fertility Rate (child per Woman)

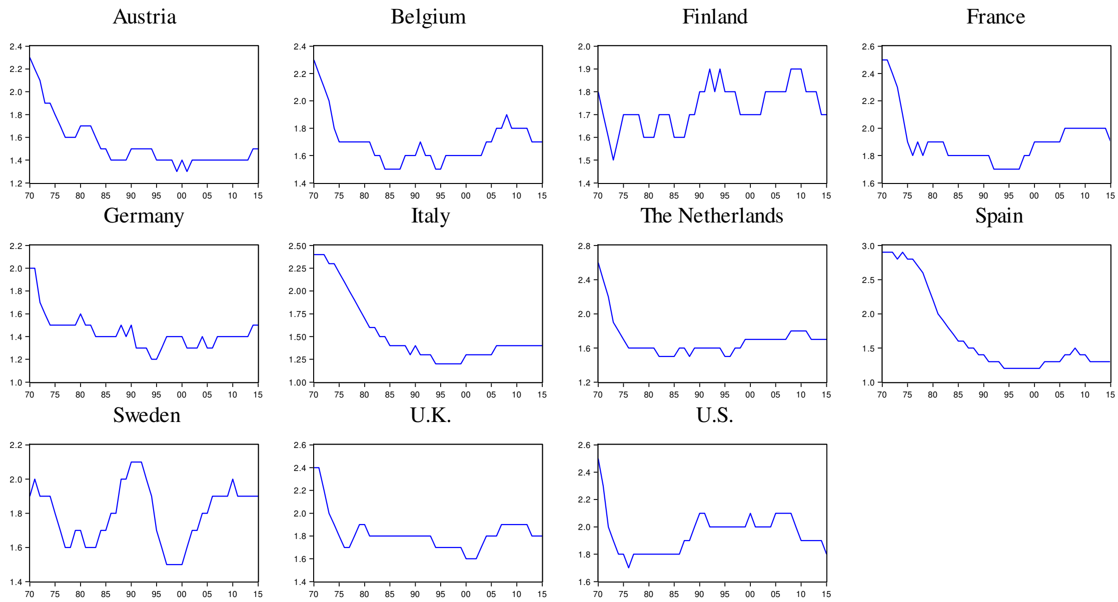
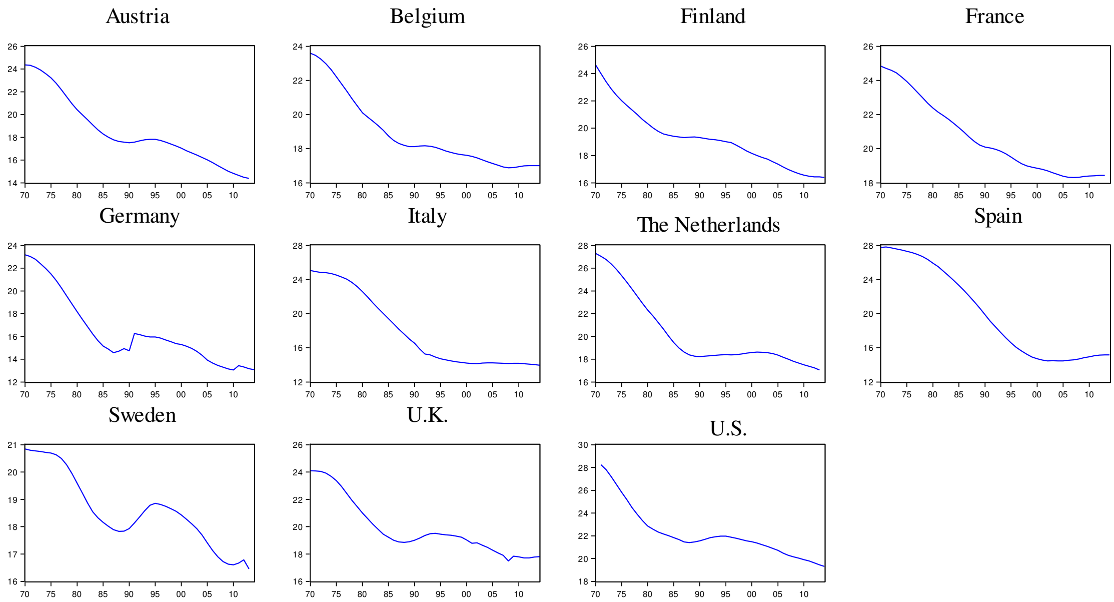


Figure (12)

Percent of Young Population (less than 15)



Appendix I

Modifying the model to includes sector growth.

$$Y_t = A_t^{\sigma} K_t^{\alpha} H_{Y_t}^{1-\alpha} \ , \qquad (1)$$

where K_t is physical capital, H_{Y_t} is the total quantity of human capital employed to produce output Y_t and A_t is the accumulating stock of *ideas* created in the world. It is assumed that $0 < \alpha < 1$, and $\sigma > 0$, which implies a constant return to scale in K and H_Y and an increasing return to scale in K, H_Y and A as $\alpha + 1 - \alpha + \sigma = 1 + \sigma > 1$

Sector assumption:

Let us assume that the stock of ideas in any sector of the economy s is A_{s_t} :

$$A_{s_t} = \eta_0 A_t^{\eta} \ , \qquad (2)$$

And the growth rate is g :

$$g_{As} = \eta \cdot g_A \qquad (3)$$

Substituting (2) in the production function (1) gives:

$$Y_t = \eta_0^{\frac{-\sigma}{\eta}} A_{st}^{\frac{\sigma}{\eta}} K_t^{\alpha} H_{Y_t}^{1-\alpha} \qquad (4)$$

Now we describe each element of the production function.

First, physical capital accumulates as usual:

$$\dot{K}_t = s_{K_t} Y_t - dK_t, \quad K_0 > 0 \qquad (5)$$

Where a dot over the variable denotes the variation between two consecutive points of time; s_{K_t} is the fraction of output that is invested; and $d > 0$ is a constant depreciation rate for all countries.[vii]

The aggregate human capital used in the production of output is:

$$H_{Y_t} = h_t L_{Y_t} \ , \qquad (6)$$

where L_{Y_t} is the number of workers creating output and,

$$h_t = e^{\varphi\, l_{ht}} \qquad (7)$$

is the human capital per person, where l_h is the time spent in accumulating capital (e.g., average years of schooling), and φ is the Mincerian rate of returns to education.

The final element in the production function of output is the stock of ideas A_t. Countries share ideas (there are no trade in goods and services in this model). Ideas created anywhere in the world are potentially available to be used in any other economy (Knowledge is non-rival and non-excludable). It follows that A_t corresponds to the cumulative stock of ideas created anywhere in the world and is common to all economies.

$$\dot{A}_t = \delta H_{A_t}^{\lambda} A_t^{\phi} \quad A_0 > 0. \qquad (8)$$

Let A_{s_t}, the ideas at the sectoral level be the same, where

$$H_{As_t} = \varepsilon\, H_{A_t}; \quad 0 \le \varepsilon \le 1\ , \qquad (9)$$

and $H_{A_t} = \sum_{i=1}^{M} h_{i_t}^{\theta} L_{A_{it}} = \frac{1}{\varepsilon} H_{As_t}$; H_{As_t} is the effective world research effort in the sector as a fixed proportion from the entire world research effort in the sector H_{A_t}, and $L_{A_{it}}$ is the number of researchers in economy i. Note that here we have a subscript i. The index i refers to the economies i to M. Assume that global research is the weighted sum of research conducted in the advanced countries, and assume $\theta \ge 0$, which means that the quality of research is constant across these advanced countries.

Let $L_{As_{it}} = a_i L_{it}; \quad 0 \le a_i \le 1$, then

$$H_{As_t} = \varepsilon \sum_{i=1}^{M} h_{it}^{\theta} \left(\frac{L_{As_{it}}}{a_i}\right), \qquad (10)$$

where $L_{As_{it}}$ is the number of researchers in the sector only in a given economy i.

From (2), $A_t = \eta_0^{-\frac{1}{\eta}} A_{s_t}^{\frac{1}{\eta}}$, thus

$$\dot{A}_t = \frac{1}{\eta}\dot{A}_{s_t}.$$

Substituting in (8),

$$\frac{1}{\eta}\dot{A}_{s_t} = \delta\left(\frac{1}{\varepsilon}H_{As_t}\right)^{\lambda}\left(\eta_0^{-\frac{1}{\eta}}A_{s_t}^{\frac{1}{\eta}}\right)^{\phi}.$$

So $\dot{A}_{s_t} = \underbrace{\eta\delta\varepsilon^{-\lambda}\eta_0^{-\frac{\phi}{\eta}}}_{\mu} H_{As_t}^{\lambda} A_{s_t}^{\frac{\phi}{\eta}}$.

Or, simply $\dot{A}_{st} = \mu H_{As_t}^{\lambda} A_{s_t}^{\frac{\phi}{\eta}}$, (11)

The growth of ideas in the sector produced at any point in time depends on the number of researchers and existing stock of ideas. Allowing $0 < \lambda \leq 1$ captures the possibility of duplication in research, i.e., a doubling of the number of researchers produces less than a doubling of the number of ideas. Jones also assumes that $\phi < 1$.

There is also a binding resource constraint on labor. Each economy is populated by N_t identical, infinitely-lived agents. The number of agents in each economy grows over time at a *common* and exogenous rate $n > 0$:

Population grows at natural rate n as follows:

$$N_t = N_0 e^{nt}, N_0 > 0 \tag{12}$$

Each individual is endowed with one unit of time, which is used to produce goods, ideas, and human capital. Because the time spent in school is excluded from labor force data, the labor constraints imply that each individual is endowed with one unit of time, divided among the production of goods, ideas, and human capital:

$$L_t = L_{A_t} + L_{Y_t} = \tfrac{1}{b}L_{A_{Ft}} + L_{Y_t} = (1 - l_{h_t})N_t \,, \tag{13}$$

l_{h_t} is the time spent producing human capital.

where $L_{As_t} = bL_{A_t}$ the number of researchers creating ideas in the sector globally as a proportion of L_{A_t}, $0 < b < 1$.

Let $y_t = \dfrac{Y_t}{L_t}$ the output per worker, $L_{Y\,t} = H_{Y_t} / h_t$ and $l_{y_t} = \dfrac{L_{Y_t}}{L_t}$, we get

$$y_t = \frac{Y_t}{L_t} = \frac{Y_t}{L_{Y\,t}} \cdot \frac{L_{Y\,t}}{L_t} = \frac{\eta_0^{\frac{-\sigma}{\eta}} A_{s\,t}^{\frac{\sigma}{\eta}} K_t^{\alpha} H_{Y_t}^{1-\alpha}}{H_{Y_t} / h_t} l_{y_t} = l_{y_t} h_t \eta_0^{\frac{-\sigma}{\eta}} A_{s\,t}^{\frac{\sigma}{\eta}} K_t^{\alpha} H_{Y_t}^{-\alpha} \quad (14)$$

Then from $Y_t = \eta_0^{\frac{-\sigma}{\eta}} A_{s\,t}^{\frac{\sigma}{\eta}} K_t^{\alpha} H_{Y_t}^{1-\alpha}$ we get

$$H_{Y_t}^{-\alpha} = Y_t^{-\alpha/(1-\alpha)} \eta_0^{\frac{-\alpha}{\eta(1-\alpha)}\sigma} A_{st}^{\frac{\alpha}{\eta(1-\alpha)}\sigma} K_t^{\frac{\alpha^2}{(1-\alpha)}} . \quad (15)$$

Substituting in y_t and simplifying, we get:

$$y_t = l_{y_t} h_t \eta_0^{\frac{-\sigma}{\eta(1-\alpha)}} A_{st}^{\frac{\sigma}{\eta(1-\alpha)}} (\frac{K_t}{Y_t})^{\frac{\alpha}{1-\alpha}} . \quad (16)$$

Solving for A_s we have

$$A_{s_t} = \frac{\eta_0 . y_t^{\eta\frac{1-\alpha}{\sigma}}}{\left(\frac{K_t}{Y_t}\right)^{\eta\frac{\alpha}{\sigma}} \left(l_{y_t} h_t\right)^{\eta\frac{1-\alpha}{\sigma}}} . \quad (17)$$

From (11), $\dot{A}_{s\,t} = \mu H_{As_t}^{\lambda} A_{s_t}^{\frac{\phi}{\eta}}, \quad A_{s_0} > 0$ we have:

$$g_{As} = \frac{\dot{A}_{st}}{A_{st}} = \mu . H_{As_t}^{\lambda} A_{st}^{\frac{\phi-\eta}{\eta}} . \quad (18)$$

Or also;

$$A_{s_t} = (\mu / g_{As})^{\frac{\eta}{\eta-\phi}} H_{A_{st}}^{\frac{\eta}{\eta-\phi}\lambda} . \quad (19)$$

So

$$A_{st}^{\frac{\sigma}{1-\alpha}} = (\mu / g_{A_s})^{\frac{\eta}{(\eta-\phi)}\frac{\sigma}{(1-\alpha)}} H_{A_{st}}^{\frac{\lambda\eta}{(\eta-\phi)}\frac{\sigma}{(1-\alpha)}} . \quad (20)$$

Let $\gamma = \frac{\lambda}{(\eta - \phi)} \frac{\sigma}{(1-\alpha)}$ then

$$A_{st}^{\frac{\sigma}{1-\alpha}} = (\mu / g_{A_s})^{\frac{\gamma}{\lambda}\eta} H_{A_{st}}^{\gamma\eta}. \tag{21}$$

From accumulating capital equation $\dot{K}_t = s_{K_t} Y_t - dK_t, K_0 > 0$ we get

$$g_k = \dot{K}_t - \dot{L}_t = s_{K_t}.(Y_t / K_t) - (d+n), \tag{22}$$

which gives

$$K_t / Y_t = \frac{s_{K_t}}{g_k + d + n}, \tag{23}$$

where g_k is the *constant* growth rate of *k=K/L.* Given equations (16), (21), and (23),

we get $y_t = \eta_0^{\frac{-\sigma}{\eta(1-\alpha)}} \left(\frac{s_{K_t}}{g_k + d + n} \right)^{\frac{\alpha}{1-\alpha}} l_{Y_t} h_t \left(\frac{\mu}{g_{A_s}} \right)^{\frac{\gamma}{\lambda}} H_{A_{st}}^{\gamma}.$ (24)

The stock of capital K and A_s grow at *constant rates*, which require H_{A_s} growing also at a *constant rate* (asterisk over variables mean that they grow at constant rate) we have:

$$y_t = \eta_0^{\frac{-\sigma}{\eta(1-\alpha)}} \left(\frac{s^*_{K_t}}{g_k + d + n} \right)^{\frac{\alpha}{1-\alpha}} l_{Y_t} h_t \left(\frac{\mu}{g_{A_s}} \right)^{\frac{\gamma}{\lambda}} H^{*\gamma}_{A_{st}} \tag{25}$$

On a balanced growth path, all variables grow at constant rate and the allocations of $S_{K,}, l_A, l_{A_s}, l_h$ and l_Y are exogenous, may differ across countries, and must be constant. The first term in parenthesis is capital-output ratio, which is proportional to the investment rate when the capital stock grows at a constant rate *a la* the Solow model. This economy exhibits a stable balanced growth path (possibly zero). Along that path, allocations must be constant. The growth rate of output per capita is proportional to the rate of growth of effective world research in the sector $H_{A_{st}}$. Since h must be constant along a balanced growth path, growth of the effective number of world researchers in that sector is driven by population growth. The following equations 26 to 29 show that.

Equation (16) gives

$$g_y = \frac{\sigma}{\eta(1-\alpha)} g_{A_s} \quad . \tag{26}$$

Also from $A_{st}^{\frac{\sigma}{1-\alpha}} = (\mu / g_{A_s})^{\frac{\gamma}{\lambda}\eta} H_{As_t}^{\gamma\eta}$ we arrive at the steady-state equation

$$\frac{\sigma}{1-\alpha} g_{A_s} = \gamma\eta . g_{H_{A_s}} \quad . \tag{27}$$

Finally, since h (human capital per person) must be constant along the steady state path, growth in the effective number of world researchers in the sector H_{As_t} is driven by population growth, so:

$$g_{H_{As}} = n \, , \tag{28}$$

then

$$g_y = \frac{\sigma}{\eta(1-\alpha)} g_{A_F} = \gamma n \tag{29}$$

Differentiating $y_t = l_{Y_t} h_t \eta_0^{\frac{-\sigma}{\eta(1-\alpha)}} A_{st}^{\frac{\sigma}{\eta(1-\alpha)}} (\frac{K_t}{Y_t})^{\frac{\alpha}{1-\alpha}}$ and add and subtract the steady-state term γn, we get the growth rate equation:

$$\underbrace{\dot{y}_t = \frac{\alpha}{1-\alpha}(\dot{K}_t - \dot{Y}_t) + \dot{h}_t + \dot{l}_{Yt} + \left(\frac{\sigma}{\eta(1-\alpha)} \dot{A}_{s,t} - \gamma n \right)}_{Transitional\,Dynamic} + \underbrace{\gamma n}_{SteadyState} \quad , \tag{30}$$

Appendix II

The data are from EUKLEMS (2017). The data set include all European countries and the United States. However, we only use the original EUKLEMS EU10 and the United States because the required data for the other countries are incomplete.

We measure productivity y_{it} by real value added per hours worked. We deflate the value added VA (Gross value added at current basic prices- in millions of national currency) by the price VA_P (Gross value added, price indices, 2010 = 100) then divide by hours worked H_EMP (Total hours worked by persons engaged in thousands). EU Stat defines gross Value Added (VA) as output value at basic prices less intermediate consumption valued at purchasers' prices. VA is calculated before consumption of fixed capital.

The aggregate TFP is Market Economy data. The Market Economy measure excludes lines L, O, P, Q, T, and U, which are the sectors real estate activity; Public administration and defense; compulsory social security; Education, Health and Social Work; and Activities of households as employers; undifferentiated goods- and services-producing activities of households for own use.

The data for the market value added output, prices, and hours are from 1995 to 2015. The *level* of market TFP and sectoral TFP's varies. For Austria (1996-2015); Belgium (1999-2015); Finland (1996-2015); France (1995-2015); Germany (1996-2015); Italy (1996-2015); The Netherlands (2001-2015); Spain (1996-2015); Sweden (1996-2015), U.K. (1998-2015); and the U.S. (1999-2015).

The share of labor / capital and the human capital index are taken from the Penn World Table 9.0.

Population is measured by the Labor Force as in Jones (2002), from OECD data.

The number of researchers is from the World Bank.

[i] Jones (2002) does not cite Kuznets.

[ii] See the mathematical representation provided by Richard E. Quandt in his discussion of Kuznets (1960) paper, page 340.

[iii] Similarly, Gurley and Shaw (1955), Goldsmith (1969) and Hicks (1969) argued that a well-developed financial system is important to stimulating economic growth. McKinnon (1973) and Shaw (1973) have contributed significantly to this literature with slightly different models. Bencivenga and Smith (1991), Roubini and Sala-i-Martin (1992), King and Levine (1993), and Mattesini (1996) are among a number of papers, which use endogenous growth models, though differ in many important aspects. For example, in Roubini and Sala-i-Martin (1992), just like Keynes (1939), financial repression is not ruled out. King and Levine (1993) have a Schumpeterian model of technical progress similar to Romer (1990) and Grossman and Helpman (1991), with a cost-reducing inventions applying to an intermediate product. Financial market affects technical progress by increasing the probability of having successful innovative projects, hence growth. The literature on the relationship between finance and growth is old and voluminous. Ang (2008), Trew (2005), Eschenbach (2004) and Levine (1997) are the most important surveys of the literature.

[iv] In theory, financial development affects economic growth via two channels: (1) capital accumulation and (2) technical progress. The capital accumulation channel is essentially a savings-investments-growth channel. A more efficient financial system mobilizes savings and channels them through the sectors of the economy in the form of productive investments, which is emphasized by, e.g., Wicksell (1935), Shaw (1955), and Tobin and Brainard (1963). Also efficient financial systems allow investors to diversify portfolios and hedge against risks (e.g., Diamond and Dybvig, 1983 and Bencivenga and Smith, 1991). Financial intermediaries manage and invest funds at a lower cost (e.g., Gurley and Shaw, 1960). Diamond (1984) also shows that that monitoring costs is reduced through efficient financial arrangements. The other channel works when innovative financial technologies lessen information-asymmetries, which adversely affect efficient allocations of savings and the monitoring of investment projects. See for example Townsend (1979), Greenwood and Javanovic (1990), King and Levine (1993 b).

[v] In Lucas and Moll (2014) knowledge is partially rival. In the short run it is rival because motivated individuals who want to increase their productivity must exert efforts to doing so, but knowledge in the long run is non-rival.

[vi] This term could be estimated econometrically because the data are available to estimate equation (2 or 3) in Appendix I.

[vii] The fraction of output that is spent is $1 - s_{Kt}$