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Abstract

This paper studies the macroeconomic consequences of interbank market disruptions caused by higher counterparty risk. I propose a novel, dynamic model of banking sector where banks trade liquidity in the frictional OTC market à la Afonso and Lagos (2015) that features counterparty risk. The model is then embedded into an otherwise standard New Keynesian framework to analyze the macroeconomic impact of interbank market turmoils: economy suffers from a prolonged slump and deflationary pressure during such episodes. I use the model to analyze the effectiveness of two policy measures: rise in the supply of central bank reserves and interbank market guarantees in mitigating the adverse effects of those disruptions.

Keywords: Financial crisis, Interbank market, Policy intervention, OTC market
JEL Classification: E44, E58, G21

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1 Introduction

Extreme levels of interbank lending rates were one of the hallmarks of the financial crisis of 2007-2008. A widely accepted interpretation of this phenomenon is that banks lending liquidity in the market began to worry whether borrowing counterparties will be able to repay loans.[1] Indeed, several episodes of the 2007-2008 turmoil seem to support the validity of this explanation. For example, the BNP Paribas announcement concerning its inability to value structured products and the associated freezing of redemptions for its investment funds triggered an illiquidity wave on the interbank market in August 2007.[2] As a result, the perceived default risks of banks increased substantially driving up the LIBOR (see Brunnermeier (2009)).

Interbank market turmoils have serious macroeconomic consequences. First, they weaken risk sharing between banks and lead to inadequate allocation of capital. Second, since interbank markets are essential for banks' liquidity management then tighter borrowing conditions in that market force banks to cut back on real-sector lending.

Despite the prominent role played by counterparty risk in interbank market turmoils and in spite of the macroeconomic importance of the latter, there are no works that analyze those issues within a single framework. This paper is intended to bridge this gap, by providing the first formal analysis of interbank market turmoils caused by a rise in counterparty risk in a DSGE environment. To this end, I develop a dynamic model of banking sector with banks trading liquidity in the frictional OTC market as in Afonso and Lagos (2015) which is then embedded into the standard New Keynesian framework. I use the model to analyze the impact of two policy measures: increase in central bank reserves and introduction of interbank market guarantees.

The contribution of this paper is twofold. First, on the methodological side, it is the first work that enables to organize our thinking about the macroeconomic consequences

[1] An alternative explanation of interbank market interest rate spikes emphasizes the role of liquidity hoarding: banks are not willing to lend even to credible counterparties because they prefer to store liquidity due to precautionary motives. The key insight of this theory is that bank's supply of interbank lending is determined by its own rollover risk (see Acharya and Skeie (2011)). Empirical work by Afonso et al. (2011) indicates that counterparty risk played a more important role than liquidity hoarding during the Fed funds market stress that followed Lehman Brothers collapse.

[2] Except for anecdotal evidence, there are works documenting those phenomena empirically. For instance, Afonso et al. (2011) analyze the situation on the Fed funds market following the collapse of Lehman Brothers and find that counterparty risk is a convincing explanation for the overnight interbank market stress following this event.

of the elevated levels of interbank market counterparty risk. Second, on the technical side, I modify the construct by Bianchi and Bigio (2014) in a substantial way so that it allows to formalize the notion of interbank market default risk within a dynamic model of banking sector.

My paper is related to several strands of the literature. First group of works discusses the impact of shifts in the interbank market counterparty risk on banking sector performance and liquidity dry-ups. For example, Heider et al. (2015) examine adverse selection that may lead to market collapse. In their model, information asymmetry worsens during a crisis when the fraction of risky banks rises and, at the same time, investors cannot differentiate among default risks of individual banks. Consequently, lenders demand higher interest payments to participate in the market. Similar mechanisms based on asymmetric information are present in works of Flannery (1996) and Freixas and Jorge (2008). I contribute to this literature by extending those models along two important dimensions. First, I replace the finite horizon optimization problems present in those works with an infinite one. Second, I introduce real sector so that I am able to study macroeconomic effects of interbank market disruptions. Furthermore, I formalize the notion of counterparty risk in a way that slightly differs from the concept of asymmetric information. More precisely, asymmetric information is a situation in which one party has more or better information than the other. In my model, for tractability purposes, I constrain the number of possible actions by assuming that lending in the interbank market takes place before certain proportion of banks that borrow funds in that market is affected by insolvency shock. This means that all borrowers are identical when interbank loans are made and hence those circumstances cannot be referred to as information asymmetry. At the same time, to guarantee that counterparty risk affects transactions in the interbank market, it is assumed that lending banks know the proportion of insolvent borrowers ex ante. However, they cannot distinguish between sound and risky borrowers when granting a loan as both types are virtually the same.

It seems that the closest paper to mine is Bianchi and Bigio (2014) who propose a model of banks' liquidity management and the credit channel of monetary policy and apply it to study the driving forces behind the decline in lending and liquidity hoarding by banks during the 2008 financial crisis. I modify the model of Bianchi and Bigio (2014) in a significant way. First, I extend the frictional OTC market in which banks trade liquidity which is present in their analysis by introducing counterparty risk. Second, I reformulate the model of Bianchi and Bigio (2014) by assuming that in the middle of the period each bank is divided into a continuum units that are heterogeneously affected by idiosyncratic shocks and which are again pooled into one bank by the midpoint of every consecutive period.[3] While retaining the intraperiod heterogeneity which gives rise to trade in the interbank market as in Bianchi and Bigio (2014), this large-bank construct simplifies the analysis in two ways. First, by eliminating the interperiod heterogeneity across banks it simplifies aggregation and allows the use of broader (than in Bianchi and Bigio (2014)) set of banker's utility functions under which the model remains solvable.[4] The latter is important as the liquidation protocols applied to insolvent bank units in my analysis require specific utility functions of bankers. Second, this modification enables embedding counterparty risk into the model in a tractable way and allows the use of standard solution techniques like perturbation methods. This second issue implies that the large-bank construct proposed in my work offers a tractable way of incorporating a dynamic and relatively realistic banking sector into the standard DSGE framework.

The remaining sections of the paper are organized as follows. Section 2 presents the model. In Section 3 I study the effects of an increase in the counterparty risk in the interbank market. Section 4 describes the role of several policy measures in mitigating the adverse consequences of interbank market turmoils. Section 5 concludes.

2 Model

2.1 Environment: general picture

Time is infinite and divided into discrete periods. Each period consists of two subperiods: day and night. The model is populated by the following types of agents: banks (bankers), households, firms and the consolidated government that collects taxes and conducts monetary policy. Banks channel funds between those who save (households) and those who take loans (firms). Households' need for services provided by bankers emerges because bank deposits are the only instrument available to shift purchase power

[3]This approach originates from Lucas (1990) and has been recently applied to households by, inter alia, Beaudry et al. (2014), Shi (2015), Cui and Radde (2016) and Negro et al. (2017).

[4]The analysis of Bianchi and Bigio (2014) remains feasible only when homothetic preferences are in place and resources available to bankers can be expressed as a linear function of equity.

across periods. On the asset side of bank balance sheet, a simple financial friction gives rise to demand for loans: firms have to pay workers before goods are produced and sold so they take loans to cover payroll costs.

2.2 Financial sector without counterparty risk

2.2.1 Banks

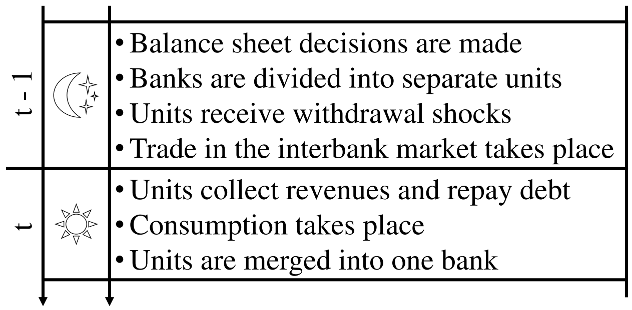
It is instructive to start with a simplified version of the banking sector without counterparty risk. There is a continuum of infinitely-lived banks of measure one. Each bank, in turn, consists of a continuum of units of measure one. Banks are divided into separate units at the beginning of night and are consolidated by pooling separated units at the end of day of the next period. Before the division, between day and night, each bank makes decision about the composition of its balance sheet: deposits d, loans l and reserves m and next period equity e given the value of equity chosen in the previous period.[5] This initial structure of balance sheet is then inherited by each unit and hence all units are identical at the beginning of the night. During the night, however, some bank units are subject to inflows and some experience outflows of proportion $\delta > -1$ of deposits. This process is accompanied by changes in reserve balances as shifts of deposits between two units are settled with reserves. The distribution of deposit withdrawals is identical across banks and is denoted by μ that satisfies:

$$\int \delta d\mu\left(\delta\right) = 0.$$

In other words, it is assumed that the circulation of deposits is closed within the banking sector. Similarly to Bianchi and Bigio (2014) and Freixas et al. (2011), the role of withdrawal shocks is to give rise to liquidity risk: between night and day, each unit is obligated to hold a stock of reserves which equals the proportion $\psi \geq 0$ of its current deposit holdings $(1 + \delta) \cdot d$ and hence bank units with reserve balances below this requirement have to borrow them at the end of night. Units with reserve deficits can borrow either from central bank or from units with surplus reserves in the interbank market. The effective real rate of borrowing reserves χ^{-} is defined later. Units with

[5] All balance sheet elements are expressed in real terms, i.e. in terms of consumption good which is a numerairé in the model.

Figure 1: Timeline for the banking sector



reserve surpluses can either deposit them at central bank or lend them to units with deficits. The effective real rate on surplus χ^+ is defined later, too.

At the beginning of next period's day, units collect revenues from loans to firms and reserves and repay deposits (the associated nominal interest rates are i_L, i_{ER} and i_D, respectively). Moreover, units experiencing reserve deficits at the end of night in previous period, bear costs of borrowing and those who had surpluses receive interest payments from lending in the interbank market or deposits at the Fed. Those resources are then divided between consumption $c(\delta)$ of unit affected by shock δ at the end of night, equity chosen in the previous period before the division into units and the penalty for the deviation of equity from the level imposed by regulator $\bar{e}_t$.[6] Finally, bank units are pooled into one bank and decisions concerning bank balance sheet are made. The order of events is presented in Figure 1.

Each unit values consumption $c(\delta)$, which can be thought of as a dividend, using a strictly increasing, concave and twice-differentiable function u.[7] Additionally, it is required that $u(0) = 0$ and $u'(c) \to +\infty$. First assumption simplifies the modeling of liquidation procedures discussed in Subsection 2.3. Second condition guarantees existence of the interior solution to the bank problem. Banks discount future utility

[6]The last element is introduced for several reasons. First, it enables to pin down the steady state level of equity in the model and guarantees the convergence of the model after a transitory shock. Second, it penalizes banks if the chosen level is lower than $\bar{e}_t$ (violation of capital requirement). If banks choose level that exceeds $\bar{e}_t$, it can be treated as tax on equity. More generally, such penalty makes banks decrease the volatility of equity which is consistent with the observations made by Adrian et al. (2012).

[7]Similarly to Bianchi and Bigio (2014), curvature in the objective function is introduced to generate dividend smoothing which is observed empirically.

streams with factor $\beta_B \in (0,1)$ and it is assumed that they are not able to transfer resources across units during the spells in which they are separated. Finally, it is assumed that banks formulate rational expectations about the evolution of aggregate state variables A captured by operator Γ:

$$A_t = \Gamma\left(A_{t-1}\right).$$

The following Bellman equation describes the problem of bank which is solved between day and night:

$$V\left(e_{t-1}, A_{t-1}\right) = \max_{x_t}\left\{\int_{-1}^{+\infty} u\left(c_t\left(\delta\right)\right) d\mu\left(\delta\right) + \beta_B \cdot \mathbb{E}V\left(e_t, A_t\right)\right\} \tag{1}$$

$$subject\ to:$$

$$\forall_\delta: \quad c_t\left(\delta\right) + e_t + \frac{\phi_E}{2} \cdot \left(e_t - \bar{e}_t\right)^2 = \frac{1 + i_{L,t-1}}{\Pi_t} \cdot l_{t-1} \tag{2}$$

$$+\frac{1 + i_{ER,t-1}}{\Pi_t} \cdot \underbrace{\left(m_{t-1} + \frac{1 + i_{D,t-1}}{1 + i_{ER,t-1}} \cdot \delta \cdot d_{t-1}\right)}_{\text{Reserve balances after withdrawal shock}} + \frac{1 + i_{D,t-1}}{\Pi_t} \cdot \underbrace{\left(d_{t-1} + \delta \cdot d_{t-1}\right)}_{\text{Deposits after withdrawal shock}}$$

$$+\left(\mathbb{I}_{\left\{\delta < \delta^*_{t-1}\right\}} \cdot \chi_t^- + \mathbb{I}_{\left\{\delta \geq \delta^*_{t-1}\right\}} \cdot \chi_t^+\right) \cdot \mathcal{M}\left(m_{t-1}, d_{t-1} | i_{D,t-1}, i_{L,t-1}, \delta\right)$$

$$l_{t-1} + m_{t-1} = e_{t-1} + d_{t-1} \tag{3}$$

$$A_t = \Gamma\left(A_{t-1}\right)$$

where $x_{t-1} = \left\{\left\{c_t\left(\delta\right)\right\}, l_{t-1}, m_{t-1}, d_{t-1}, e_t\right\}$, $\phi_E > 0$ is a parameter, V is value function associated with the maximization problem, Π_t is the ratio between prices of consumption goods p_t in period t and those observed in $t-1$ and $\mathcal{M}$ is surplus/deficit of reserves given by:

$$\mathcal{M}\left(m_{t-1}, d_{t-1} | i_{D,t-1}, i_{L,t-1}, \delta\right) = m_{t-1} + \frac{1+i_{D,t-1}}{1+i_{ER,t-1}} \cdot \delta \cdot d_{t-1} - \psi \cdot (1+\delta) \cdot d_{t-1}.$$

By $\mathbb{I}$ I denote the indicator function and δ^*_{t-1} is the threshold value of δ for which the surplus of reserves equals 0:

$$m_{t-1} + \frac{1+i_{D,t-1}}{1+i_{ER,t-1}} \cdot \delta^*_{t-1} \cdot d_{t-1} - \psi \cdot \left(1+\delta^*_{t-1}\right) \cdot d_{t-1} = 0.$$

First constraint of the maximization problem 1 is the budget constraint of a unit affected by shock δ. Second constraint is the balance sheet constraint for a bank in period $t-1$.

Several remarks are in order. First, notice that, similarly to Bianchi and Bigio (2014), I adopt a convention that a unit that issues deposit pays the interest for it - i.e., it settles the withdrawal of its deposits by transferring the corresponding amount of reserves plus rate i_D. Analogously, a unit that purchases reserves is entitled to interest paid by central bank on excess reserve holdings i_{ER}. These assumptions imply that a unit that receives shock δ gets amount $\frac{1+i_D}{1+i_{ER}} \cdot \delta \cdot d_t$ of reserves. Second remark concerns the sources of risk faced by bank units. Its primary source is the presence of idiosyncratic shocks δ: the possibility of deposit outflows and the associated costs of borrowing reserves discourages banks from choosing balance sheets characterized with high leverage which constrains the intermediation process. If I did not assume that banks cannot shift resources between units then it would be optimal to equalize $c(\delta)$ across them which, in turn, would eliminate the intermediation risk. Similarly, if I assumed that e_t can be chosen at the unit level then banks would set lower equity holdings for units affected by low values of δ and higher equity holdings for units with larger δ so that $c(\delta)$ would be equal across bank units and full insurance would be achieved.[8]

The presence of idiosyncratic risk faced by units coupled with conditions in the interbank market - captured by real rates χ^+ and χ^- - are essential for the level of prices at which banks transfer resources between savers (households) and borrowers (firms).

[8]Of course, to achieve this result regulator would need to impose capital requirements that depend on δ as well.

To see that, let us analyze two first order conditions associated with the maximization problem described by equation 1. First of them is related to the choice of m_{t-1} and reads:

$$i_{L,t-1} = i_{ER,t-1} + \Pi_t \cdot \frac{\mathbb{P}\left(\delta < \delta^*_{t-1}\right) \cdot \mathbb{E}_\delta\left(u'\left(c_t\left(\delta\right)\right)|\delta < \delta^*_{t-1}\right) \cdot \chi^-_t}{\mathbb{E}_\delta\left(u'\left(c_t\left(\delta\right)\right)\right)}$$
$$+\Pi_t \cdot \frac{\mathbb{P}\left(\delta \geq \delta^*_{t-1}\right) \cdot \mathbb{E}_\delta\left(u'\left(c_t\left(\delta\right)\right)|\delta \geq \delta^*_{t-1}\right) \cdot \chi^+_t}{\mathbb{E}_\delta\left(u'\left(c_t\left(\delta\right)\right)\right)} \tag{4}$$

where $\mathbb{E}_\delta$ is an operator associated with random variable δ. Equation 4 implies that conditions in the interbank market, described by effective real rates χ^+ and χ^-, translate directly into changes in i_L. Moreover, the lower bound on rate at which firms take loans i_L is pinned down by policy rate i_{ER}. Second condition is associated with deposits d_{t-1}:

$$i_{D,t-1} = i_{L,t-1} \tag{5}$$
$$+\Pi_t \cdot \frac{\mathbb{P}\left(\delta < \delta^*_{t-1}\right) \cdot \mathbb{E}_\delta\left(u'\left(c_t\left(\delta\right)\right) \cdot \left[\tilde{\delta}_{t-1} - \psi \cdot (1+\delta)\right]|\delta < \delta^*_{t-1}\right) \cdot \chi^-_t}{\mathbb{E}_\delta\left(u'\left(c_t\left(\delta\right)\right)\right)}$$
$$+\Pi_t \cdot \frac{\mathbb{P}\left(\delta \geq \delta^*_{t-1}\right) \cdot \mathbb{E}_\delta\left(u'\left(c_t\left(\delta\right)\right) \cdot \left[\tilde{\delta}_{t-1} - \psi \cdot (1+\delta)\right]|\delta \geq \delta^*_{t-1}\right) \cdot \chi^+_t}{\mathbb{E}_\delta\left(u'\left(c_t\left(\delta\right)\right)\right)}$$

where the sum of last two terms determines the spread between rate on loans i_L and rate on deposits i_D. To economize on notation I have introduced a new symbol:

$$\tilde{\delta}_t \equiv \frac{1+i_{D,t}}{1+i_{ER,t}} \cdot \delta.$$

Finally, by combining first order condition that corresponds to e_t with envelope condition I obtain the following Euler equation:

$$\left(1+\phi_E \cdot (e_t - \bar{e}_t)\right) \cdot \mathbb{E}_\delta\left(u'\left(c_t\left(\delta\right)\right)\right) = \beta_B \cdot \mathbb{E}\left[\left(\frac{1+i_{L,t}}{\Pi_{t+1}}\right) \cdot \mathbb{E}_\delta\left(u'\left(c_{t+1}\left(\delta\right)\right)\right)\right]. \tag{6}$$

2.2.2 Interbank market

Fed funds market opens after arrival of withdrawal shocks and closes at the end of night. It is specified as a frictional OTC market in which banks place borrowing and lending orders. Matching between orders is governed by a constant returns to scale technology M which implies matching probabilities for a bank with deficit of reserves ψ^- and for a bank with reserve surplus ψ^+ that depend on Fed funds market tightness θ_{FF} given by the ratio of aggregate deficit of reserves and aggregate surplus of reserves:

$$\theta_{FF,t} = \frac{\mathcal{D}_t}{\mathcal{S}_t} \tag{7}$$

where:

$$\mathcal{D}_t \equiv \left| \int_{-1}^{\delta_t^*} \left(m_t + \frac{1+i_{D,t}}{1+i_{ER,t}} \cdot \delta \cdot d_t - \psi \cdot (1+\delta) \cdot d_t \right) d\mu(\delta) \right|,$$

$$\mathcal{S}_t \equiv \int_{\delta_t^*}^{+\infty} \left(m_t + \frac{1+i_{D,t}}{1+i_{ER,t}} \cdot \delta \cdot d_t - \psi \cdot (1+\delta) \cdot d_t \right) d\mu(\delta).$$

It is assumed that orders unmatched in the market are automatically matched with central bank which occurs with probability $1-\psi^-$ for a borrowing order and with probability $1-\psi^+$ for a lending order. If a unit of excess reserves is matched with a borrowing order it earns Fed funds rate i_{FF} and it earns interest on excess reserves i_{ER} otherwise. If borrowing order is matched with lending order then the loan cost amounts to i_{FF}, and it equals to discount rate i_{DW} if it is traded with central bank. This implies the following values of real effective interest rates:

$$\chi_t^+ = \psi_{t-1}^+ \cdot \frac{i_{FF,t-1} - i_{ER,t-1}}{\Pi_t},$$

$$\chi_t^- = \psi_{t-1}^- \cdot \frac{i_{FF,t-1} - i_{ER,t-1}}{\Pi_t} + \left(1-\psi_{t-1}^-\right) \cdot \frac{i_{DW,t-1} - i_{ER,t-1}}{\Pi_t}.$$

Both i_{DW} and i_{ER} are set by the monetary authority. The value of i_{FF} is determined during the bargaining process, which is described in the Appendix. In the situation in which banks have equal bargaining weights, as it is assumed here, it takes the following, intuitive form:

$$i_{FF,t} = \frac{i_{ER,t} + i_{DW,t}}{2}.$$

2.3 Financial sector with counterparty risk

2.3.1 The counterparty risk shock

Let us turn to the situation in which bank units face counterparty risk when trading in the interbank market. The risk comes from the fact that at the beginning of day of period t, measure $1-S_{t-1}$ of bank units, which is distributed equally across all banks, faces problems with loan repayment. More precisely, firms repay all loans l_{t-1}

that are due but $(1 - S_{t-1}) \cdot l_{t-1}$ of them "disappear" on their way to bank units which means that proportion $1 - S_{t-1}$ of deposits $(1 + i_{D,t-1}) \cdot d_{t-1}$ cannot be repaid and thus those "risky" units become insolvent. This is the moment in which government steps in and the liquidation procedure begins. First, government confiscates equity e_t of risky units and it reduces their "dividends" $c_t(\delta)$ to zero. Second, it repays deposits of insolvent units as I assume that deposit insurance is in place and hence the possibility of bank runs is eliminated. In contrast to household deposits, government does not repay interbank loans during liquidation process.[9] This means that units which have reserve surpluses between periods $t-1$ and t are directly exposed to default risk as interbank loans are not collateralized (as it is the case in reality).

I assume that the value of shock S_{t-1} becomes known when banks choose their balance sheet structure in period $t-1$. This implies that S_{t-1} affects both the interbank trade during the night of period $t-1$ and balance sheet decisions made in the middle of period $t-1$. In particular, as we shall see below, the presence of $1-S_{t-1}$ of insolvent units in period t will affect the price of interbank loans $i_{FF,t-1}$. I adopt the convention that insolvent units are only among those with reserves deficits so I need to assume additionally that:

$$\forall_t: \;\; 1 - S_{t-1} < \mathbb{P}\left(\delta < \delta^*_{t-1}\right)$$

i.e., the proportion of risky units does not exceed the number of borrowers. The presence of insolvent units among loan takers gives rise to counterparty risk faced by units with reserve surpluses at the end of period $t-1$ as they cannot distinguish between safe and risky borrowers in the interbank market ex ante.

2.3.2 Banks

The maximization problem of a bank in period $t-1$ is characterized by the following Bellman equation:

$$V\left(e_{t-1}, S_{t-1}, A_{t-1}\right) = \max_{x_t} \left\{ \int_{\delta^*_{t-1}}^{+\infty} u\left(c_t\left(\delta\right)\right) d\mu\left(\delta\right) \right. \tag{8}$$

[9] By removing this assumption I study the role of interbank market guarantees.

$$+\frac{S_{t-1}-\mathbb{P}\left(\delta \geq \delta_{t-1}^{*}\right)}{\mathbb{P}\left(\delta<\delta_{t-1}^{*}\right)} \cdot \int_{-1}^{\delta_{t-1}^{*}} u\left(c_{t}(\delta)\right) d \mu(\delta)+\left(1-S_{t-1}\right) \cdot u(0)$$

$$+\beta_{B} \cdot \mathbb{E} V\left(e_{t}, S_{t}, A_{t}\right)\}$$

$$\textit{subject to}:$$

$$\forall_{\delta}: \quad c_{t}(\delta)+e_{t}+\frac{\phi_{E}}{2} \cdot\left(e_{t}-\bar{e}_{t}\right)^{2}=\frac{1+i_{L, t-1}}{\Pi_{t}} \cdot l_{t-1} \tag{9}$$

$$+\frac{1+i_{ER, t-1}}{\Pi_{t}} \cdot\left(m_{t-1}+\frac{1+i_{D, t-1}}{1+i_{ER, t-1}} \cdot \delta \cdot d_{t-1}\right)+\frac{1+i_{D, t-1}}{\Pi_{t}} \cdot(1+\delta) \cdot d_{t-1}$$

$$+\left(\mathbb{I}_{\left\{\delta<\delta_{t-1}^{*}\right\}} \cdot \chi_{t}^{-}+\mathbb{I}_{\left\{\delta \geq \delta_{t-1}^{*}\right\}} \cdot \chi_{t}^{+}\left(S_{t-1}\right)\right) \cdot \mathcal{M}\left(m_{t-1}, d_{t-1} | i_{D, t-1}, i_{L, t-1}, \delta\right)$$

$$l_{t}+m_{t}=S_{t-1} \cdot e_{t}+d_{t} \tag{10}$$

$$\left\{S_{t}, A_{t}\right\}=\Gamma\left(S_{t-1}, A_{t-1}\right)$$

There are several differences between the problem of a bank in environment without counterparty risk and the one characterized by equations 8-10. First, in addition to units' heterogeneity with respect to withdrawal shocks, Bellman equation 8 captures the fact that some units become insolvent. In particular, $\left[S-\mathbb{P}\left(\delta \geq \delta^{*}\right)\right] / \mathbb{P}\left(\delta<\delta^{*}\right)$ is the proportion of solvent units in the pool of those with reserve deficits. Second, equation 8 takes into account that there are units whose loan portfolios become non-performing which implies that they undergo the liquidation procedure. In particular, they do not receive consumption goods and hence the utility they derive is $u(0)$ which is well-defined and equal to 0 thanks to assumptions about u. Moreover, government confiscates equity of risky units. I assume that, after the liquidation process, insolvent bank units are pooled together with the remaining units and they continue to operate in the future.

Budget constraint of bank units with surplus reserves become slightly different, too. It is because, as we shall see in the coming subsection, the real effective rate on reserve surplus becomes dependent on S_{t-1}. This is intuitive as interbank market lenders face counterparty risk. Finally, notice that liquidation process affects the balance sheet constraint 10: since government confiscates equity of insolvent units then bank is left with $S_{t-1} \cdot e_t$ of capital between day and night of period t. This, in turn, hampers lending as banks have less resources to supply loans. It is useful to define aggregate equity of a bank E_t as:

$$E_t \equiv S_{t-1} \cdot e_t.$$

First order condition with respect to m_{t-1} becomes:

$$i_{L,t-1} = i_{ER,t-1} + \Pi_t \cdot \frac{\left(S_{t-1} - \mathbb{P}\left(\delta \geq \delta^*_{t-1}\right)\right) \cdot \mathbb{E}_\delta\left(u'\left(c_t\left(\delta\right)\right)|\delta < \delta^*_{t-1}\right) \cdot \chi^-_t}{\mathcal{Z}\left(\delta^*_{t-1}, S_{t-1}, \{c_t\left(\delta\right)\}_{\delta \in (-1,+\infty)}\right)}$$

$$+\Pi_t \cdot \frac{\mathbb{P}\left(\delta \geq \delta^*_{t-1}\right) \cdot \mathbb{E}_\delta\left(u'\left(c_t\left(\delta\right)\right)|\delta \geq \delta^*_{t-1}\right) \cdot \chi^+_t\left(S_{t-1}\right)}{\mathcal{Z}\left(\delta^*_{t-1}, S_{t-1}, \{c_t\left(\delta\right)\}_{\delta \in (-1,+\infty)}\right)}. \tag{11}$$

where $\mathcal{Z}$ is defined as:

$$\mathcal{Z}\left(\delta^*_{t-1}, S_{t-1}, \{c_t\left(\delta\right)\}_{\delta \in (-1,+\infty)}\right) \equiv \mathbb{P}\left(\delta \geq \delta^*_{t-1}\right) \cdot \mathbb{E}_\delta\left(u'\left(c_t\left(\delta\right)\right)|\delta \geq \delta^*_{t-1}\right)$$

$$+\left(S_{t-1} - \mathbb{P}\left(\delta \geq \delta^*_{t-1}\right)\right) \cdot \mathbb{E}_\delta\left(u'\left(c_t\left(\delta\right)\right)|\delta < \delta^*_{t-1}\right).$$

Equation that pins down the nominal rate i_D (i.e., the first order condition associated with d_{t-1}) becomes:

$$i_{D,t-1} = i_{L,t-1} \tag{12}$$

$$+\Pi_t \cdot \frac{\left(S_{t-1} - \mathbb{P}\left(\delta \geq \delta^*_{t-1}\right)\right) \cdot \mathbb{E}_\delta\left(u'\left(c_t\left(\delta\right)\right) \cdot \left[\tilde{\delta}_{t-1} - \psi \cdot (1+\delta)\right]|\delta < \delta^*_{t-1}\right) \cdot \chi^-_t}{\mathcal{Z}\left(\delta^*_{t-1}, S_{t-1}, \{c_t\left(\delta\right)\}_{\delta \in (-1,+\infty)}\right)}$$

$$+\Pi_t \cdot \frac{\mathbb{P}\left(\delta \geq \delta^*_{t-1}\right) \cdot \mathbb{E}_\delta\left(u'\left(c_t\left(\delta\right)\right) \cdot \left[\tilde{\delta}_{t-1} - \psi \cdot (1+\delta)\right]|\delta \geq \delta^*_{t-1}\right) \cdot \chi^+_t\left(S_t\right)}{\mathcal{Z}\left(\delta^*_{t-1}, S_{t-1}, \{c_t\left(\delta\right)\}_{\delta \in (-1,+\infty)}\right)}.$$

Finally, the Euler equation reads:

$$\left(1+\phi_E\cdot\left(e_t-\bar{e}_t\right)\right)\cdot\mathcal{Z}\left(\delta^*_{t-1},S_{t-1},\left\{c_t\left(\delta\right)\right\}_{\delta\in(-1,+\infty)}\right) \tag{13}$$

$$=\beta_B\cdot\mathbb{E}\left[\left(\frac{1+i_{L,t}}{\Pi_{t+1}}\right)\cdot S_t\cdot\mathcal{Z}\left(\delta^*_t,S_t,\left\{c_{t+1}\left(\delta\right)\right\}_{\delta\in(-1,+\infty)}\right)\right].$$

2.3.3 Interbank market

Let us discuss how the interbank market with counterparty risk works. Notice that the formula for market tightness 7 is not affected by changes in S. This is because all units are ex ante identical before receiving the withdrawal and insolvency shocks. Since ψ^+ and ψ^- are functions of θ_{FF} then they are not affected either. I assume that insolvent units are only among borrowers and hence the formula for effective rate χ^- remains unaffected, too. Formula for the real effective rate on surplus reserves $\chi^+(S)$ takes into account that some interbank loans will be repaid and some will be not:

$$\chi^+_t\left(S_{t-1}\right)=\underbrace{\frac{S_{t-1}-\mathbb{P}\left(\delta\geq\delta^*_{t-1}\right)}{\mathbb{P}\left(\delta<\delta^*_{t-1}\right)}}_{\substack{\text{Probability that}\\ \text{counterparty is}\\ \text{safe/solvent}}}\cdot\psi^+_{t-1}\cdot\frac{i_{FF,t-1}-i_{ER,t-1}}{\Pi_t}+\underbrace{\frac{1-S_{t-1}}{\mathbb{P}\left(\delta<\delta^*_{t-1}\right)}}_{\substack{\text{Probability that}\\ \text{counterparty is}\\ \text{risky/insolvent}}}\cdot\psi^+_{t-1}\cdot\left(-1\cdot\frac{1}{\Pi_t}\right).$$

Observe that if $S=1$ then $\chi^+(S)$ is identical to χ^+. If, however $S<1$ then with some probability bank that lends surplus reserves loses the entire amount $\mathcal{M}$ which gives rise to -1 in the formula above. Price index Π in denominator translates nominal terms into real terms.

As it is shown in the Appendix, the formula for Fed funds rate reads:

$$i_{FF,t}=\frac{i_{ER,t}+i_{DW,t}+\overbrace{\frac{1-S_t}{S_t-\mathbb{P}\left(\delta\geq\delta^*_t\right)}}^{\substack{\text{The counterparty}\\ \text{risk component}}}}{2}. \tag{14}$$

Intuitively, other things equal, lower proportion of safe units in the pool S increases the interbank lending rate.

2.4 Households

There is measure one of identical, infinitely-lived households. They value consumption good c by using felicity function $\tilde{u}$ which is increasing, strictly concave, twice-differentiable and satisfies the Inada conditions. Future utility streams are discounted with factor $\beta_H \in (0,1)$. There is no disutility from work so labor supply is perfectly elastic and hence employment n is driven solely by labor demand. Moreover, it is assumed that labor supply is bounded by unity. The only saving instrument available to households are bank deposits $\tilde{d}$ earning nominal interest i_D. Household income consists of firm profits π and labor income $w \cdot n$ where w is real wage. Each household pays lump sum tax τ levied by government.

Bellman equation which represents household problem reads:

$$W\left(\tilde{d}_{t-1}, S_t, A_t\right) = \max_{c_t, \tilde{d}_t} \left\{\tilde{u}\left(c_t\right) + \beta_H \cdot \mathbb{E}W\left(\tilde{d}_t, S_{t+1}, A_{t+1}\right)\right\} \tag{15}$$

$$subject\ to:$$

$$\tau_t + c_t + \tilde{d}_t = \frac{1 + i_{D,t-1}}{\Pi_t} \cdot \tilde{d}_{t-1} + w_t \cdot n_t + \pi_t \tag{16}$$

$$\{S_{t+1}, A_{t+1}\} = \Gamma\left(S_t, A_t\right)$$

where W is value function associated with maximization problem 15. Observe that timing of decisions is different than in case of bankers: households make decisions about period t variables in period t and not in $t-1$ as it is in case of banks. The associated Euler equation is:

$$\tilde{u}\left(c_t\right) = \beta_H \cdot \mathbb{E}\left[\frac{1 + i_{D,t}}{\Pi_{t+1}} \cdot \tilde{u}\left(c_{t+1}\right)\right]. \tag{17}$$

2.5 Retailers

The model is populated with perfectly competitive retailers who pack differentiated goods y_i produced by firms into baskets of consumption goods y using technology described by the Dixit-Stiglitz aggregator:

$$y_t = \left(\int_0^1 y_{i,t}^{1-\frac{1}{\gamma}} di\right)^{\frac{1}{1-\frac{1}{\gamma}}}$$

where $\gamma > 1$ is the elasticity of substitution between intermediate goods (varieties). Retailers choose $\{y_{i,t}\}$ to maximize profits:

$$p_t \cdot y_t - \int_0^1 p_{i,t} y_{i,t} di$$

where $p_{i,t}$ is price of variety produced by firm i in period t. The following equation describes first order condition of the retailer:

$$y_{i,t} = \left(\frac{p_{i,t}}{p_t}\right)^{-\gamma} \cdot y_t \tag{18}$$

where p_t reads:

$$p_t = \left(\int_0^1 p_{i,t}^{1-\gamma} di\right)^{\frac{1}{1-\gamma}}.$$

2.6 Firms

There is measure one of firms owned by households and indexed with $i \in [0, 1]$ that produce intermediate goods using linear technology with labor n as the only input. Firms are monopolistically competitive and set prices subject to quadratic price adjustment costs as in Rotemberg (1982) and subject to demand for their products given by equation 18. Future profit streams are discounted with factor Λ which depends on household's marginal utilities from consumption. It is assumed that firms operate under simple financial friction: workers demand payment for wages before output is generated and sold. This implies that firms have to take loans $\tilde{l}$ to finance payroll costs. Loans are repaid in the next period.

Bellman equation associated with firm's problem is:

$$F_i\left(p_{i,t-1}, \tilde{l}_{i,t-1}, S_t, A_t\right) = \max_{p_{i,t}, l_{i,t}, n_{i,t}, y_{i,t}} \left\{ \frac{p_{i,t}}{p_t} \cdot y_{i,t} - \frac{1 + i_{L,t-1}}{\Pi_t} \cdot \tilde{l}_{i,t-1} \right.$$

$$-\frac{\phi}{2}\left(\frac{p_{i,t}-p_{i,t-1}}{p_{i,t-1}}\right)^2\cdot y_t+\mathbb{E}\Lambda_t\cdot F_i\left(p_{i,t},\tilde{l}_{i,t},S_{t+1},A_{t+1}\right)\Bigg\} \tag{19}$$

$$subject\,to:$$

$$\tilde{l}_{i,t}=w_t\cdot n_{i,t} \tag{20}$$

$$y_{i,t}=n_{i,t}$$

$$y_{i,t}=\left(\frac{p_{i,t}}{p_t}\right)^{-\gamma}\cdot y_t$$

$$\{S_{t+1},A_{t+1}\}=\Gamma\left(S_t,A_t\right)$$

where $\phi>0$ is parameter, F_i is value function associated with the maximization problem of firm i and Λ is defined as:

$$\Lambda_t=\beta_H\cdot\frac{\tilde{u}'\left(c_{t+1}\right)}{\tilde{u}'\left(c_t\right)}.$$

In the symmetric equilibrium, in which all firms are identical and hence subscripts i can be omitted, first order condition that characterizes the optimal solution to problem 19 reads:

$$0=1-\gamma-\phi\cdot\left(\Pi_t-1\right)\cdot\Pi_t \tag{21}$$

$$+\mathbb{E}\Lambda_t\cdot\left(\gamma\cdot\frac{1+i_{L,t}}{\Pi_{t+1}}\cdot w_t+\phi\cdot\left(\Pi_{t+1}-1\right)\cdot\Pi_{t+1}\cdot\frac{y_{t+1}}{y_t}\right).$$

Finally, let us define the real value of firm's profits π:

$$\pi_t\equiv n_t-\frac{1+i_{L,t-1}}{\Pi_t}\cdot\tilde{l}_{t-1}-\frac{\phi}{2}\cdot\left(\Pi_t-1\right)^2\cdot n_t. \tag{22}$$

2.7 Government

Government consists of central bank and fiscal authority that are consolidated under single budget constraint:

$$\mathcal{C}_t+\frac{1+i_{ER,t-1}}{\Pi_t}\cdot S_{t-1}\cdot\tilde{m}_{t-1}=\tilde{m}_t+\tau_t+\left(S_{t-1}-\mathbb{P}\left(\delta\geq\delta^*_{t-1}\right)\right)+S_{t-1}\cdot\frac{\phi_E}{2}\cdot\left(e_t-\bar{e}_t\right)^2 \tag{23}$$

$$\times \left(1-\psi_{t-1}^{-}\right) \cdot \frac{i_{DW,t-1}-i_{IOR,t-1}}{\Pi_t} \cdot \left| \int_{-1}^{\delta_{t-1}^{*}} \mathcal{M}\left(m_{t-1}, d_{t-1} | i_{D,t-1}, i_{L,t-1}, \delta\right) \right| d\mu\left(\delta\right)$$

where $\tilde{m}$ is the amount of reserves supplied by central bank, $\mathcal{C}$ are costs associated with liquidation procedures. Equation 23 states that government issues reserves, collects taxes, penalties associated with violation of capital requirements and profits from discount window operations to cover costs of liquidation procedures and payments on excess reserves. Costs of compensations are given by:

$$\mathcal{C}_t = \underbrace{\left(1-S_{t-1}\right) \cdot \frac{1+i_{D,t-1}}{\Pi_t} \cdot d_{t-1}}_{\text{deposit insurance}} \tag{24}$$

$$- \underbrace{\left(1-S_{t-1}\right) \cdot \psi_{t-1}^{+} \cdot \int_{\delta_{t-1}^{*}}^{+\infty} \mathcal{M}\left(m_{t-1}, d_{t-1} | i_{D,t-1}, i_{L,t-1}, \delta\right).}_{\text{unsecured interbank lending}}$$

where the first component of compensations are deposits of liquidated bank units that are repaid by government to households. As it has been already mentioned, this is a standard deposit insurance that prevents from the emergence of bank runs. Costs of compensations are reduced by the amount of loans on which liquidated banks default. This happens because interbank lending is unsecured and government is not obliged to repay this debt after the takeover of bank units in course of the liquidation process.

Additionally, government sets policy rates i_{DW} and i_{ER} which are assumed to be constant in my analysis and adjusts taxes τ and reserves m to balance the budget. Finally government sets the reserve requirement parameter ψ and capital requirements that are assumed to take the following form:

$$\bar{e}_t = \xi \cdot \left(m_t + l_t\right). \tag{25}$$

In other words, banks are forced to hold equity equal to proportion $\xi \in (0,1)$ of their asset holdings.

2.8 Consistency and market clearing

The market clearing condition for consumption goods reads:

$$c_t + \int_{-1}^{+\infty} c_t(\delta)\, d\mu(\delta) + \frac{\phi}{2}(\Pi_t - 1)^2 \cdot y_t = y_t.$$

It is required that markets for deposits, loans and reserves clear, too:

$$d_t = \tilde{d}_t,\ l_t = \tilde{l}_t,\ m_t = \tilde{m}_t.$$

It is assumed that there is a constant returns to scale matching technology M that combines borrowing and lending orders in the interbank market. It implies the following values of probabilities ψ^+ and ψ^-:

$$\psi_t^+ = \frac{M(\mathcal{D}_t, \mathcal{S}_t)}{\mathcal{S}_t} = M(1, \theta_{FF,t}),$$

$$\psi_t^- = \frac{M(\mathcal{D}_t, \mathcal{S}_t)}{\mathcal{D}_t} = M\left(1, \frac{1}{\theta_{FF,t}}\right).$$

The last issue associated that needs to be addressed before defining equilibrium is associated with wage formation. Since labor supply is perfectly elastic then employment level is pinned down by demand for labor. This assumption is made to simplify the model and is motivated by results presented in Justiniano et al. (2010) who claim that labor supply is irrelevant for business cycle fluctuations. This implies that we have certain degree of freedom in setting wages so following Blanchard and Gali (2010), I assume that real wage is a function of labor productivity. As the latter is normalized and constant over time (the only exogenous shock in the model is S) then real wages w are constant over time and calibrated to match unemployment level in stationary equilibrium.[10]

Finally, vector of aggregate state variables A_t consists of employment at the beginning of period t, prices of consumption goods set in $t-1$ and equity chosen by banks in $t-1$:[11]

$$A_t = \{p_{t-1}, e_t\}.$$

[10] Real wage rigidities are important for obtaining a prolonged slump after the shock to S.

[11] For clarity, I do not include shock S in vector A.

2.9 Equilibrium

Having defined maximization problems of banks, households and firms, government budget constraint and market clearing conditions, we are in position to define recursive competitve equilibrium of the model:

Definition. *A recursive competitive equilibrium is: value functions V, W, F, policy functions x, c, $\tilde{d}$, p, $\tilde{l}$, n, y, real rates χ^+, χ^-, probabilities ψ^+, ψ^- implied by θ_{FF}, threshold value δ^*, profits π, discount factor Λ, aggregate price level index Π and nominal rates i_{FF}, i_L, i_D such that given taxes τ, supply of reserves $\tilde{m}$, reserve requirements ψ, capital requirements ξ, rates i_{DW}, i_{ER}, real wage w and path of shocks S:*

1. *Value function V solves the problem of bank (equation 8) given δ^*, S, i_L, i_D, i_{ER}, Π, χ^+, χ^- and x is vector of the associated policy functions,*
2. *Value function W solves the problem of household (equation 15) given Π, w, τ, i_D, π and c, $\tilde{d}$ are the associated policy functions,*
3. *Value function F solves the problem of firm (equation 19) given i_L, w, q, Λ and p, $\tilde{l}$, n, y are the associated policy functions,*
4. *Government runs a balanced budget,*
5. *Market clearing and consistency conditions hold,*
6. *Law of motion of aggregate state variables Γ is consistent with policies of banks and firms.*

2.10 Interbank market shutdown

Let us now discuss economic mechanisms behind the shutdown in the interbank market. The following proposition specifies the circumstances under which trade in the interbank market collapses:

Proposition. *If $S_t < S_t^*$ then the interbank market collapses (i.e., $\psi^+ = \psi^- = 0$), all borrowers use discount window facility to take loans and all excess reserves earn interest*

i_{ER}. *The threshold value* S^* *is given by:*

$$S_t^* = \frac{1 + \mathbb{P}\left(\delta \geq \delta_t^*\right) \cdot \left(i_{DW,t} - i_{ER,t}\right)}{1 + \left(i_{DW,t} - i_{ER,t}\right)}.$$

Intuition behind this result is straightforward. If S is lower or equal to S^* then Fed funds rate i_{FF} is higher or equals discount rate i_{DW}. This implies that banks with reserve deficits have no incentives to borrow at i_{FF} from banks with surpluses and they lend from the Fed. Lack of demand for loans forces liquid banks to store liquidity at the Fed which means that market collapses.

As it has been documented by Afonso et al. (2011), Fed funds market was not frozen even after the Lehman Brothers collapse so in the simulations I assume that:

$$\forall_t \, S_t^* < S_t \leq 1.$$

3 Quantitative analysis

3.1 Calibration

3.1.1 Functional forms

For convenience, I assume that measure μ consists of two points of equal mass situated at two values that are symmetric with respect to 0:

$$\mu\left(\delta\right) = \begin{cases} \frac{1}{2} & for \; \delta = \bar{\delta} \\ \frac{1}{2} & for \; \delta = -\bar{\delta} \end{cases} \tag{26}$$

where $\bar{\delta} \in (0, 1)$. Notice that this formulation implies that δ^* ceases to be relevant and in equilibrium it always satisfies:

$$-\bar{\delta} < \delta^* < \bar{\delta}.$$

Table 1: Parameter values

Parameter	Description	Value	Target
w	Real wage	0.91	Number of hours worked equal to 0.33
α	Parameter of matching technology M	68.9	Probabilities ψ^+ and ψ^- equal to 0.99
ϕ	Parameter of price adjustment costs	61.0	Share of adjustment costs in total revenue equal to 1.22%
i_{DW}	Discount window rate	0.0147	Annual discount window rate in 2006
i_{ER}	Interest on reserves	0	Annual rate on excess reserves in 2006
ξ	Capital requirement ratio	0.04	Assets to equity ratio of 4%
ψ	Reserve requirement	0.1	Reserve requirement equal to 10%
γ	Elasticity of substitution between varieties	11	Monopolistic markup equal to 10%
β_B	Banker's discount factor	0.997	Annual inflation rate equal to 2%
β_H	Household's discount factor	0.998	Euler equation consistency
ϕ_E	Equity adjustment cost	10	Discretionary choice, several other values checked
δ	Std. error of withdrawal shock	0.05	Evidence by Bianchi and Bigio (2014)
ρ	Parameter of banker's preferences	0.5	Discretionary choice, several other values checked
η	Risk aversion - households	2	Standard value in the literature
τ	Lump sum tax	-0.0002	Balanced budget
$\tilde{m}$	Supply of reserves	0.032	Value of tightness $\theta_{FF} = 1$

Utility function u describing preferences of bank unit is specified as:

$$u\left(c\left(\delta\right)\right) = c\left(\delta\right)^{\rho}$$

where $\rho \in (0,1)$. This guarantees that $u' > 0$, $u'' < 0$ and $u\left(0\right) = 0$ as required before. Utility function of households u is given by the standard CRRA function:

$$\tilde{u}\left(c\right) = \frac{c^{1-\eta} - 1}{1-\eta}.$$

Finally, matching technology in labor market M is specified as in Ramey et al. (2000):

$$M\left(z_1, z_2\right) = \frac{z_1 \cdot z_2}{\left(z_1^{\alpha} + z_2^{\alpha}\right)^{\frac{1}{\alpha}}}$$

where $\alpha > 1$.

3.1.2 Parameter values

The time period is a quarter. Targets of my calibration are moments characterizing the U.S. economy which correspond to values from the model when S is equal to one and it is constant over time.

Parameter α associated with matching technology is set to match rates ψ^+ and ψ^- equal to 0.99. This assumption is motivated by the fact that in normal times a vast majority of trade in reserves occurs in the interbank market. Real wage w is calibrated to equalize the number of hours worked equal to 0.33. The moment targeted by the value of ϕ is share of price adjustment costs in total annual revenue calculated in Zbaracki et al. (2004). Values of policy rates i_{DW} and i_{ER} in stationary equilibrium are 6% and 0% in annual terms which is consistent with Fed policy rates at the onset of the Great Recession. I set the value of capital requirement ξ at the level of 4% and the reserve requirement ψ at the level of 10%. Parameter γ that describes elasticity of substitution between product varieties is calibrated to match the value 1.1 of monopolistic markup. Banker's discount factor β_B targets the annual inflation rate of 2% and the analogous parameter for households - β_H - is set to be consistent with steady state value of interest rate on deposits (see equation 17). The value of parameter ϕ_E does not affect steady state allocation. It does, however, influence the transition path of economy. Additionally, its value can be treated as policy parameter that can be interpreted the restrictiveness of capital requirements. I have experimented with various values and I set $\phi_E = 10$ in my simulations. In practice I could report simulations for other values of ϕ_E but I think it is not a particularly interesting policy to be discussed here. I set $\delta = 0.05$ as Bianchi and Bigio (2014) and ρ equals 0.5. I assume that $\eta = 2$ - a standard value in the literature. Tax τ is calibrated to balance the budget. Finally, I set $\tilde{m}$ such that:

$$\tilde{m} = \psi \cdot d_{ss}. \tag{27}$$

This value implies that $\theta_{FF} = 1$ which reflects the assumption that central banks seeks to equalize the volumes of surplus and deficit reserves to enhance trade in the interbank market.

Figure 2: Baseline simulation: proportion of safe banks, GDP, inflation, interbank market rates

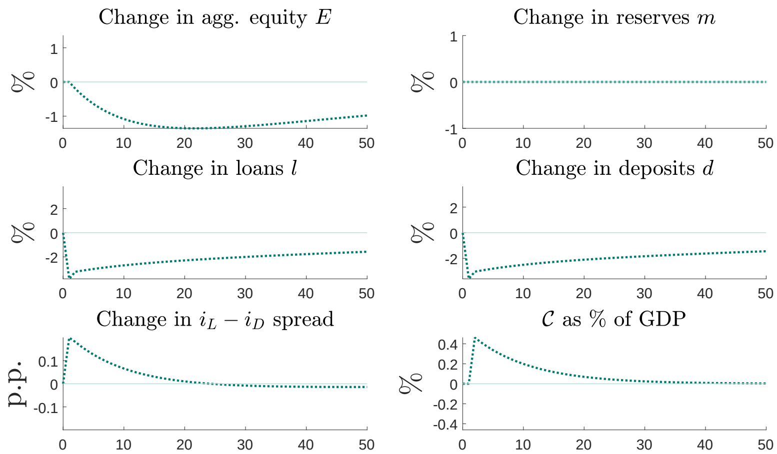
3.2 Simulations

In this part, I analyze the transition path of economy that is affected by negative shock to the proportion of safe banks S. More precisely, I assume that in periods $t = 0$ economy is in steady state. Next, in the middle of period $t = 1$, banks learn about the proportion of risky counterparties $1 - S_t$ which will default on their interbank loans in $t = 2$. The magnitude of shock $1 - S_t$ is 0.5% at its peak in $t = 1$ and its autocorrelation is 0.9. In what follows, I assume that the response of government to a drop in S is passive: neither interest rate target (pinned down by i_{DW} and i_{ER}), nor supply of reserves change during the crisis. Furthermore, taxes τ and compensations $\mathcal{C}$ adjust automatically according to 23 and 24. Assumption about passive government reaction is relaxed in Section 4.

I use standard solution methods based on the linear approximation around steady state because the magnitude of shock guarantees that market shutdown is avoided and hence the dynamic system preserves differentiability.

Figures 2 and 3 display simulation results. The top left panel of Figure 2 shows the path of proportion of safe banks S. Notice, that a drop in S in first period increases counterparty risk in the interbank market in period $t = 1$ as banks with reserve surpluses expect defaults on interbank loans in period $t = 2$. In particular, as formula 14 shows, a drop in S elevates fed funds rate via the counterparty risk effects in period $t = 1$

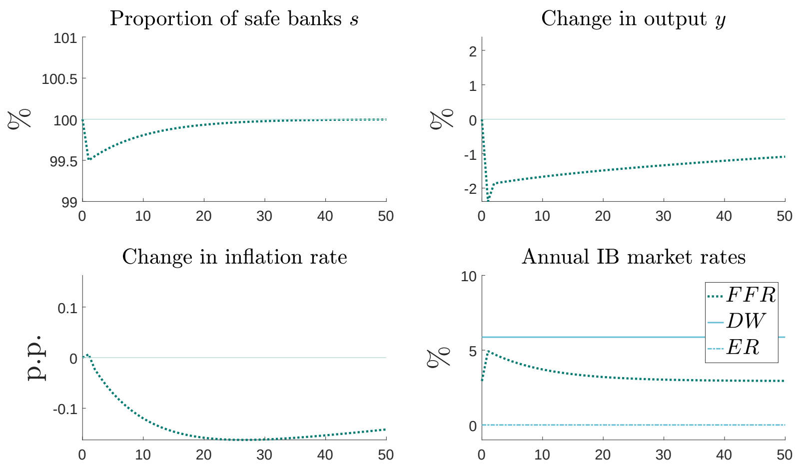
Figure 3: Baseline simulation: variables associated with the banking sector



which jumps from 3% to 5% in annual terms. This, in turn, affects borrowing costs in the interbank market which, combined with concavity of the utility function u and uninsured shocks δ, increases the riskiness of supplying loans with respect to holding reserves. As supply of the latter is constant in the baseline scenario price of loans i_L grows.

Since payroll costs are financed with loans then higher i_L decreases demand for loans and so l drops by 3% (see the middle left panel of Figure 3). Real wages are constant and therefore loans change one for one with employment and output which implies a drop in GDP by 3%. This, in turn, leads to lower household income. Due to household's consumption smoothing motives, agents decrease savings d (the bottom right panel of Figure 3) which leads to higher rates i_D. Despite this upward movement spread $i_L - i_D$ gets larger due to the increase in riskiness of financial intermediation when S contracts. Aggregate equity of the banking sector E lowers in period $t = 2$ and bounces back sluggishly. Lower equity cushion discourages banks from risky lending which leads to a prolonged slump in loan supply and output. On the top of that, interbank market turmoil is associated with persistent deflationary pressure which decreases annual inflation by about 0.3 percentage points. Amount of compensations $\mathcal{C}$ moves together with S as government guarantees deposits of insolvent bank units and it reaches the level of 0.45% of GDP.

Figure 4: Increase in reserve supply during the interbank market turmoil: proportion of safe banks, GDP, inflation, interbank market rates



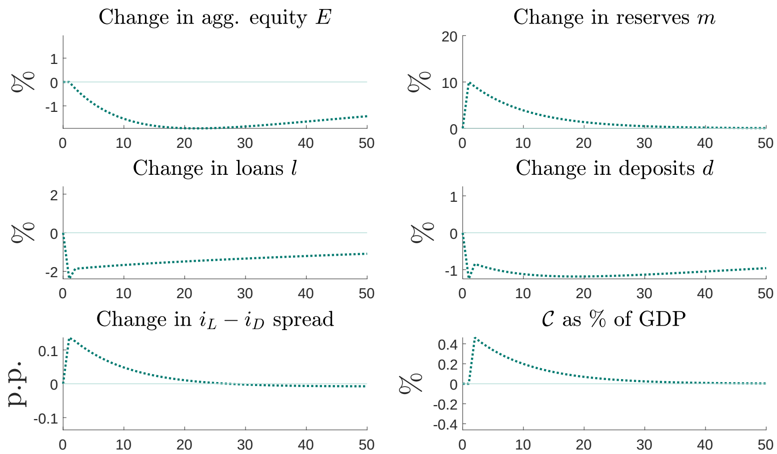
4 Policy interventions

4.1 Change in the supply of reserves

First policy measure considered here is change in reserves that accompanies the drop in the proportion of safe bank units. More precisely, in response to the contraction in S, central bank increases the supply of reserves by 10% and then reduces it at rate 0.9 to mimic the dynamics of shock S which is characterized with the same autocorrelation.

Figure 4 presents the behavior of main economic aggregates. It turns out that the intervention mitigates the macroeconomic impact of interbank market turmoil as output drops by 2.2% now in comparison to 3% in the baseline scenario. To understand this result, notice first that interbank market rate i_{FF} remains unaffected by this intervention as it does not decrease the counterparty risk since the number of insolvent counterparties remains constant. The effects of this policy measure are associated with the fact that liquidity becomes relatively cheaper as the supply of reserves grows which increases the probability ψ^{-} and thus reduces the intermediation risk. This implies that it is less expensive to self insure against withdrawal shocks which stimulates loan supply. As the system becomes more awash with liquidity, banks' self insurance motives leading to equity accumulation weaken and hence the drainage of capital cushion is more severe. The fact that idiosyncratic risk is mitigated by larger supply of liquidity

Figure 5: Increase in reserve supply during the interbank market turmoil: variables associated with the banking sector



is reflected by a less dynamic increase in $i_L - i_D$ spread. Finally, intervention tends to lower the deflationary pressure as the drop in inflation rate is almost two times smaller now.

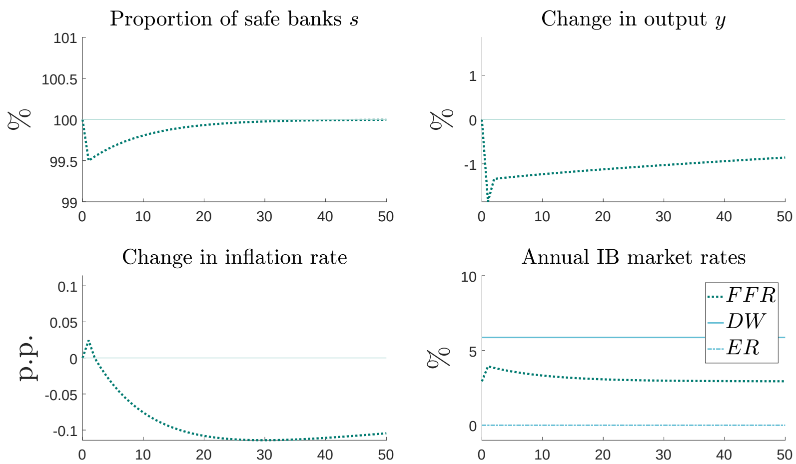
4.2 Interbank market guarantees

Let us analyze the effects of interbank market guarantees. This policy was introduced in several developed economies during the Great Recession. As reported by Heider et al. (2015), one example is Italy, where the Banca di Italia has established the Mercato Interbancario Collateralizzato (MIC). Despite the fact that its trading activity is actually collateralized, the Banca di Italia guaranteed repayment of all loans in the facility because of credit risk concerns associated with uncertain collateral values.

In what follows I assume that central bank sets parameter $\mathcal{P} \in (0, 1)$ which denotes the proportion of non-performing interbank loans that are guaranteed. This means that government commits itself to cover $100 \cdot \mathcal{P}$ % of losses generated by banks lending in the interbank market. The difference between compensations $\mathcal{C}$ in the baseline scenario and compensations $\mathcal{C}(\mathcal{P})$ paid when interbank guarantees are in place is:

$$\mathcal{C}_t(\mathcal{P}) - \mathcal{C}_t = \mathcal{P} \cdot (1 - S_{t-1}) \cdot \psi^{+}_{t-1} \cdot \mathcal{S}_{t-1} \cdot \frac{1 + i_{FF,t-1} - i_{ER,t-1}}{\Pi_t}$$

Figure 6: Interbank market guarantees: proportion of safe banks, GDP, inflation, interbank market rates



which is the proportion $\mathcal{P}$ of all interbank loans (with interest) that were not repaid.

More importantly, from the point of view of the impact on economy, guarantees affect borrowing costs in the interbank market:

$$i_{FF,t}\left(\mathcal{P}\right) = \frac{i_{ER,t} + i_{DW,t} + \frac{(1-\mathcal{P})\cdot(1-S_t)}{S_t + \mathcal{P}\cdot(1-S_t) - \mathbb{P}\left(\delta \geq \delta_t^*\right)}}{2}$$

which is derived in the Appendix. More precisely, guarantees decrease the counterparty risk component of the Fed funds rate. The last object which is affected by interbank market guarantees is the formula for $\chi^+\left(S\right)$ which now becomes:

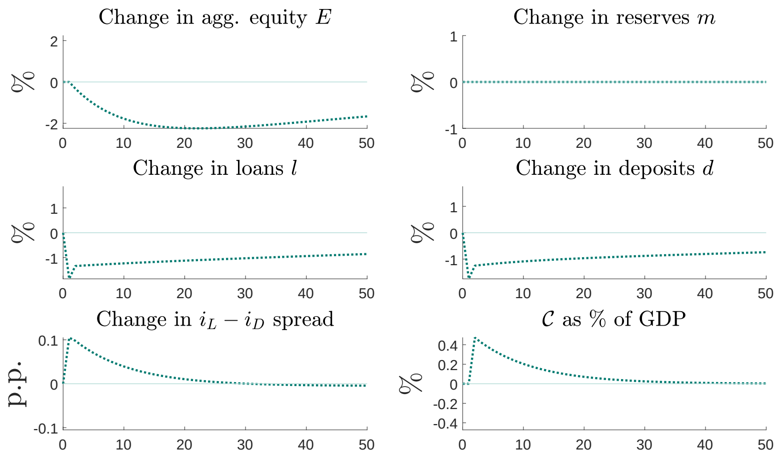
$$\chi_t^+\left(S_{t-1}\right) = \frac{S_{t-1} - \mathbb{P}\left(\delta \geq \delta_{t-1}^*\right) + \mathcal{P}\cdot\left(1 - S_{t-1}\right)}{\mathbb{P}\left(\delta < \delta_{t-1}^*\right)} \cdot \psi_{t-1}^+ \cdot \frac{i_{FF,t-1} - i_{ER,t-1}}{\Pi_t}$$

$$+ \frac{\left(1-\mathcal{P}\right)\cdot\left(1 - S_{t-1}\right)}{\mathbb{P}\left(\delta < \delta_{t-1}^*\right)} \cdot \psi_{t-1}^+ \cdot \left(-1 \cdot \frac{1}{\Pi_t}\right),$$

i.e., interbank market guarantees increase the probability of obtaining a return on lending which is associated with normal times (when $S = 1$).

Figures 6 and 7 display the results of the simulation for $\mathcal{P} = 0.5$. Introduction of interbank market guarantees leads to a substantial reduction in the severity of economic slump generated by the interbank market turmoil. More precisely, the drop in aggregate

Figure 7: Interbank market guarantees: variables associated with the banking sector



output is about 50% smaller than in baseline scenario. To see why this is the case observe that increase in $\mathcal{P}$ reduces Fed funds rate when $S < 1$ and hence its rise is much more moderate as it increases only up to 3.7% instead of reaching the level of 5%. This, in turn, lowers the cost of withdrawal risk which coupled with the fact that counterparty risk is already reduced leads to a less severe drop in the supply of loans. Moreover, as the risk faced by banks is smaller than in baseline scenario (which is captured by a less dynamic rise in spread $i_L - i_D$), banks' precautionary motives weaken which implies that they allow for a larger drainage of aggregate equity E. As it has been already mentioned, interbank market guarantees generate compensation costs $\mathcal{C}(\mathcal{P})$ that are strictly higher than $\mathcal{C}$ but the magnitude of this difference as a proportion of GDP is negligible because $\tilde{m}$ is a small proportion of d (according to 27) and hence the size of the interbank market is small in comparison to total bank assets. Introduction of interbank market guarantees leads to weaker deflationary pressure which is almost three times smaller than in baseline simulation.

5 Conclusions

In this paper I have studied the macroeconomic consequences of interbank market turmoils and the effects of two policy measures: higher supply of reserves and interbank market guarantees. To this end I have developed a novel dynamic model of banking sector in which banks trade liquidity in frictional OTC market as in Afonso and Lagos (2015) which has been embedded into an otherwise standard New Keynesian framework. I have found that even changes in the proportion of risky banks that are relatively small can have large and prolonged macroeconomic consequences as they elevate levels of interest rates in the interbank market which, in turn, translate into higher costs of managing liquidity risk. Finally, I have analyzed the effects of: a 10% increase in the supply of reserves and introduction of interbank market guarantees. It has turned out that both policy tools are effective in fighting the slump resulting from interbank market turmoils.

References

Acharya, V. V. and Skeie, D. (2011). A model of liquidity hoarding and term premia in inter-bank markets. *Journal of Monetary Economics*, 58(5):436–447.

Adrian, T., Colla, P., and Shin, H. S. (2012). Which Financial Frictions? Parsing the Evidence from the Financial Crisis of 2007-9. NBER Working Papers 18335, National Bureau of Economic Research, Inc.

Afonso, G., Kovner, A., and Schoar, A. (2011). Stressed, Not Frozen: The Federal Funds Market in the Financial Crisis. *Journal of Finance*, 66(4):1109–1139.

Afonso, G. and Lagos, R. (2015). Trade Dynamics in the Market for Federal Funds. *Econometrica*, 83:263–313.

Atkeson, A. G., Eisfeldt, A. L., and Weill, P. O. (2015). Entry and Exit in OTC Derivatives Markets. *Econometrica*, 83:2231–2292.

Beaudry, P., Galizia, D., and Portier, F. (2014). Reconciling Hayek's and Keynes Views of Recessions. NBER Working Papers 20101, National Bureau of Economic Research, Inc.

Bianchi, J. and Bigio, S. (2014). Banks, Liquidity Management and Monetary Policy. NBER Working Papers 20490, National Bureau of Economic Research, Inc.

Blanchard, O. and Gali, J. (2010). Labor Markets and Monetary Policy: A New Keynesian Model with Unemployment. *American Economic Journal: Macroeconomics*, 2(2):1–30.

Brunnermeier, M. K. (2009). Deciphering the Liquidity and Credit Crunch 2007-2008. *Journal of Economic Perspectives*, 23(1):77–100.

Cui, W. and Radde, S. (2016). Money and Asset Liquidity in Frictional Capital Markets. *American Economic Review*, 106(5):496–502.

Flannery, M. J. (1996). Financial Crises, Payment System Problems, and Discount Window Lending. *Journal of Money, Credit and Banking*, 28(4):804–824.

Freixas, X. and Jorge, J. (2008). The Role of Interbank Markets in Monetary Policy: A Model with Rationing. *Journal of Money, Credit and Banking*, 40(6):1151–1176.

Freixas, X., Martin, A., and Skeie, D. (2011). Bank Liquidity, Interbank Markets, and Monetary Policy. *Review of Financial Studies*, 24(8):2656–2692.

Heider, F., Hoerova, M., and Holthausen, C. (2015). Liquidity hoarding and interbank market rates: The role of counterparty risk. *Journal of Financial Economics*, 118(2):336–354.

Justiniano, A., Primiceri, G. E., and Tambalotti, A. (2010). Investment shocks and business cycles. *Journal of Monetary Economics*, 57(2):132–145.

Lucas, R. J. (1990). Liquidity and interest rates. *Journal of Economic Theory*, 50(2):237–264.

Negro, M. D., Eggertsson, G., Ferrero, A., and Kiyotaki, N. (2017). The Great Escape? A Quantitative Evaluation of the Fed's Liquidity Facilities. *American Economic Review*, 107(3):824–857.

Ramey, G., den Haan, W. J., and Watson, J. (2000). Job Destruction and Propagation of Shocks. *American Economic Review*, 90(3):482–498.

Rotemberg, J. J. (1982). Sticky Prices in the United States. *Journal of Political Economy*, 90(6):1187–1211.

Shi, S. (2015). Liquidity, assets and business cycles. *Journal of Monetary Economics*, 70(C):116–132.

Zbaracki, M. J., Ritson, M., Levy, D., Dutta, S., and Bergen, M. (2004). Managerial and Customer Costs of Price Adjustment: Direct Evidence from Industrial Markets. *The Review of Economics and Statistics*, 86(2):514–533.

Appendix

Bargaining in the market without counterparty risk

It is assumed that the size Δ of a borrowing (lending) order is infinitesimal. This guarantees that: i) a trader that trades it for a bank unit can ignore his own impact on other trades, ii) he can formulate his expectations about the results of other trades using the Law of Large Numbers, iii) the Nash bargaining problem has a tractable solution (see Bianchi and Bigio (2014) and Atkeson et al. (2015) for more details). By $\mathcal{V}$ let us denote the value of nominal earnings $\mathcal{E}$ for a bank unit at the end of night. Trade surplus of a borrowing order $\mathcal{S}^{-}(\Delta)$ is given by (notice that the outside option in the bargaining process is the trade with central bank):

$$\mathcal{S}^{-}(\Delta) = \mathcal{V}\left(\mathcal{E} - (i_{FF} - i_{ER}) \cdot \Delta\right) - \mathcal{V}\left(\mathcal{E} - (i_{DW} - i_{ER}) \cdot \Delta\right).$$

If it is divided by Δ and if we pass to the limit $\Delta \to 0$ we get:

$$\begin{aligned} \frac{\mathcal{S}^{-}(\Delta)}{\Delta} &= \frac{\mathcal{V}\left(\mathcal{E} - (i_{FF} - i_{ER}) \cdot \Delta\right) - \mathcal{V}(\mathcal{E}) - \mathcal{V}\left(\mathcal{E} - (i_{DW} - i_{ER}) \cdot \Delta\right) + \mathcal{V}(\mathcal{E})}{\Delta} \\ &= (i_{DW} - i_{ER}) \cdot \frac{\mathcal{V}(\mathcal{E}) - \mathcal{V}\left(\mathcal{E} - (i_{DW} - i_{ER}) \cdot \Delta\right)}{(i_{DW} - i_{ER}) \cdot \Delta} \\ &- (i_{FF} - i_{ER}) \cdot \frac{\mathcal{V}(\mathcal{E}) - \mathcal{V}\left(\mathcal{E} - (i_{FF} - i_{ER}) \cdot \Delta\right)}{(i_{FF} - i_{ER}) \cdot \Delta} \to (i_{DW} - i_{ER}) \cdot \mathcal{V}'(\mathcal{E}). \end{aligned}$$

Expression for $\frac{\mathcal{S}^{-}(\Delta)}{\Delta}$ is very useful when solving the Nash bargaining problem. A unit of surplus $\mathcal{S}^{+}(\Delta)$ reads:

$$\mathcal{S}^{+}(\Delta) = \mathcal{V}\left(\mathcal{E} + (i_{FF} - i_{ER}) \cdot \Delta\right) - \mathcal{V}\left(\mathcal{E} + (i_{ER} - i_{ER}) \cdot \Delta\right).$$

Again, we derive an expression which is very helpful afterwards - the limit of $\frac{\mathcal{S}^{+}(\Delta)}{\Delta}$:

$$\begin{aligned} \frac{\mathcal{S}^{+}(\Delta)}{\Delta} &= \frac{\mathcal{V}\left(\mathcal{E} + (i_{FF} - i_{ER}) \cdot \Delta\right) - \mathcal{V}(\mathcal{E})}{\Delta} \\ &= (i_{FF} - i_{ER}) \cdot \frac{\mathcal{V}\left(\mathcal{E} + (i_{FF} - i_{ER}) \cdot \Delta\right) - \mathcal{V}(\mathcal{E})}{(i_{FF} - i_{ER}) \cdot \Delta} \end{aligned}$$

$$\to (i_{FF} - i_{ER}) \cdot \mathcal{V}'(\mathcal{E}).$$

While solving the Nash bargaining problem observe that (I assume equal bargaining weights of $\frac{1}{2}$):

$$\arg\max_{i_{FF}} \left(\left[\lim_{\Delta \to 0} \left(S^- (\Delta) \right) \right]^{\frac{1}{2}} \cdot \left[\lim_{\Delta \to 0} \left(S^+ (\Delta) \right) \right]^{\frac{1}{2}} \right)$$

$$= \arg\max_{i_{FF}} \left(\left[\lim_{\Delta \to 0} \frac{(S^- (\Delta))}{\Delta} \right]^{\frac{1}{2}} \cdot \left[\lim_{\Delta \to 0} \frac{(S^+ (\Delta))}{\Delta} \right]^{\frac{1}{2}} \right)$$

$$= \arg\max_{i_{FF}} \left([(i_{DW} - i_{ER})]^{\frac{1}{2}} \cdot [(i_{FF} - i_{ER})]^{\frac{1}{2}} \right)$$

which implies the following first order condition:

$$i_{FF} = \frac{i_{DW} + i_{ER}}{2}.$$

Bargaining in the market with counterparty risk

Since there are no insolvent units in the pool of units with reserve surpluses, we can rewrite the formula for $\mathcal{S}^- (\Delta)$:

$$\mathcal{S}^- (\Delta) = \mathcal{V}(\mathcal{E} - (i_{FF} - i_{ER}) \cdot \Delta) - \mathcal{V}(\mathcal{E} - (i_{DW} - i_{ER}) \cdot \Delta).$$

Similarly, we can use previous computations to get:

$$\frac{\mathcal{S}^- (\Delta)}{\Delta} = (i_{DW} - i_{ER}) \cdot \mathcal{V}'(\mathcal{E}).$$

The expected surplus of a unit with positive reserve balances is now:

$$\mathcal{S}^+ (\Delta) = \underbrace{\frac{S - \mathbb{P}(\delta \geq \delta^*)}{\mathbb{P}(\delta < \delta^*)}}_{\substack{\text{Proportion of} \\ \text{safe borrowers}}} \cdot [\mathcal{V}(\mathcal{E} + (i_{FF} - i_{ER}) \cdot \Delta) - \mathcal{V}(\mathcal{E} + (i_{ER} - i_{ER}) \cdot \Delta)]$$

$$+ \underbrace{\frac{1 - S}{\mathbb{P}(\delta < \delta^*)}}_{\substack{\text{Proportion of} \\ \text{risky borrowers}}} \cdot [\mathcal{V}(\mathcal{E} - \Delta) - \mathcal{V}(\mathcal{E} + (i_{ER} - i_{ER}) \cdot \Delta)].$$

Observe that a unit with surplus loses entire sum of the interbank loan Δ if it lends to a risky/insolvent counterparty. Let us calculate $\mathcal{S}^{+}(\Delta)/\Delta$ now:

$$\frac{\mathcal{S}^{+}(\Delta)}{\Delta} = \frac{S-\mathbb{P}(\delta \geq \delta^{*})}{\mathbb{P}(\delta < \delta^{*})} \cdot (i_{FF}-i_{ER}) \cdot \frac{\mathcal{V}(\mathcal{E}+(i_{FF}-i_{ER})\cdot\Delta)-\mathcal{V}(\mathcal{E})}{(i_{FF}-i_{ER})\cdot\Delta}$$
$$+\frac{1-S}{\mathbb{P}(\delta < \delta^{*})} \cdot \left(-\frac{\mathcal{V}(\mathcal{E})-\mathcal{V}(\mathcal{E}-\Delta)}{\Delta}\right)$$
$$\rightarrow \frac{S-\mathbb{P}(\delta \geq \delta^{*})}{\mathbb{P}(\delta < \delta^{*})} \cdot (i_{FF}-i_{ER}) \cdot \mathcal{V}'(\mathcal{E}) - \frac{1-S}{\mathbb{P}(\delta < \delta^{*})} \cdot \mathcal{V}'(\mathcal{E})$$

when $\Delta \rightarrow 0$. Analogously to the case without counterparty risk:

$$\arg\max_{i_{FF}} \left(\left[\lim_{\Delta\rightarrow 0} \left(S^{-}(\Delta)\right)\right]^{\frac{1}{2}} \cdot \left[\lim_{\Delta\rightarrow 0} \left(S^{+}(\Delta)\right)\right]^{\frac{1}{2}} \right)$$
$$= \arg\max_{i_{FF}} \left(\left[\lim_{\Delta\rightarrow 0} \frac{(S^{-}(\Delta))}{\Delta}\right]^{\frac{1}{2}} \cdot \left[\lim_{\Delta\rightarrow 0} \frac{(S^{+}(\Delta))}{\Delta}\right]^{\frac{1}{2}} \right)$$
$$= \arg\max_{i_{FF}} \left([(i_{DW}-i_{ER})]^{\frac{1}{2}} \cdot \left[\frac{S-\mathbb{P}(\delta \geq \delta^{*})}{\mathbb{P}(\delta < \delta^{*})} \cdot (i_{FF}-i_{ER}) - \frac{1-S}{\mathbb{P}(\delta < \delta^{*})}\right]^{\frac{1}{2}} \right)$$

and the associated first order condition reads:

$$i_{FF} = \frac{i_{ER}+i_{DW}+\frac{1-S}{S-\mathbb{P}(\delta \geq \delta^{*})}}{2}.$$

Steady state of the model: algorithm

I present the algorithm that finds the steady state (i.e., for $S=1$) of the model and calibrates several parameters.

1. Guess i_L^{guess} and i_D^{guess}
2. Calibrate β_B consistent with i_L^{guess} and targeted inflation from the banker's Euler equation 6.

3. Calibrate β_H consistent with i_D^{guess} and targeted inflation from the household's Euler equation 17.

4. Use $\Lambda = \beta_H$, i_L^{guess} and Π induced by targeted inflation to calibrate w consistent with firm's FOC 21.

5. Compute l from the firm's constraint:

$$l = w \cdot n.$$

6. Use Π, targeted number of hours worked and i_L^{guess} to compute firm's profits π.

7. Construct a linear system of 7 equations: 2 for $\delta = \bar{\delta}$ and $\delta = -\bar{\delta}$, bank balance sheet constraint 3, household's budget constraint 16, relationship 27 pinned down by $\theta_{FF} = 1$, capital requirements 25 and government's constraint 23 with $\mathcal{C} = 0$ and solve for 7 unknowns: $c\left(\bar{\delta}\right)$, $c\left(-\bar{\delta}\right)$, c, m, d, e and τ.

8. Use 4 and 5 to compute i_L and i_D. Modify i_L^{guess} and i_D^{guess} towards i_L and i_D and start from the first step. Iterate until convergence.