Inequality and Real Interest Rates

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Abstract

We explore the relation between *income inequality* and *real interest rates* based on the marginal borrowing and saving rates of an heterogeneous population with respect to life-time income. We use an overlapping generations New Keynesian model with borrowing constraints and a bequest motive, to show how an increase of income inequality may trigger a permanent reduction of the real interest rate, (i) via a contraction of aggregate borrowing, when the marginal borrowing rate of the wealthier is lower than the one of the poorer with respect to income. (ii) We then show how an increase of inequality may trigger an expansion of savings through the channel of a bequest motive where generosity towards the next generation increases endogenously with lifetime income, so that the marginal savings rate of the wealthier is higher than the poorer.

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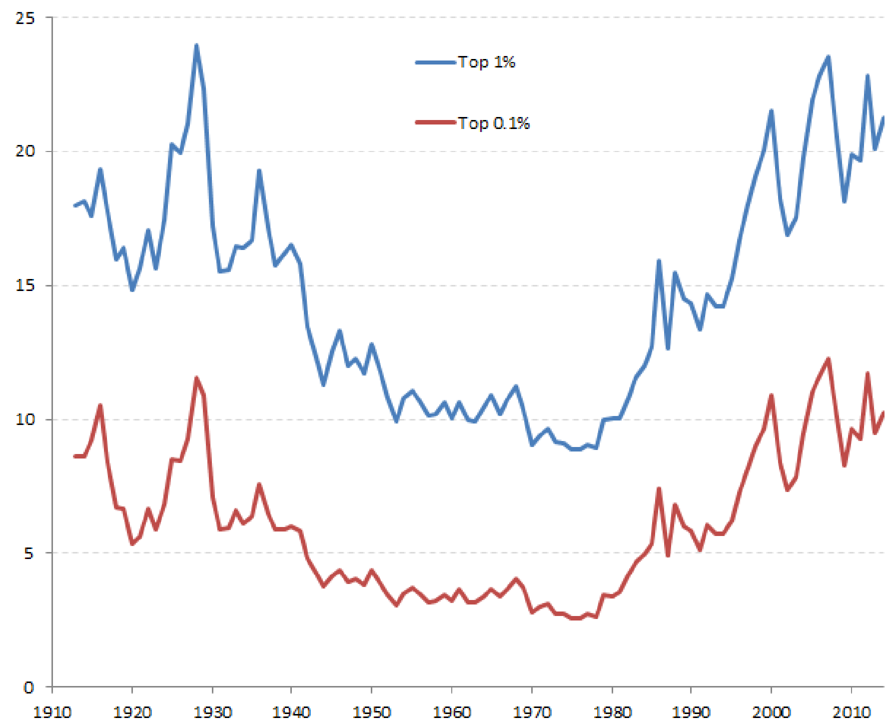
1 Introduction

Income inequality has been increasing since the early 80's in relevant world economies (Piketty [19]). For example, in US the income share of the 1% wealthier increased from around 10% in 1980 to more than 20% in 2014 (Figure:1). During the same period the real interest rates have been decreasing to levels close to zero. In this chapter we formally

Figure 1: US income shares with capital gains 1913-2014: Top 0.1% and 1%



Source: Piketty [19]; The World Wealth and Income Database - WID

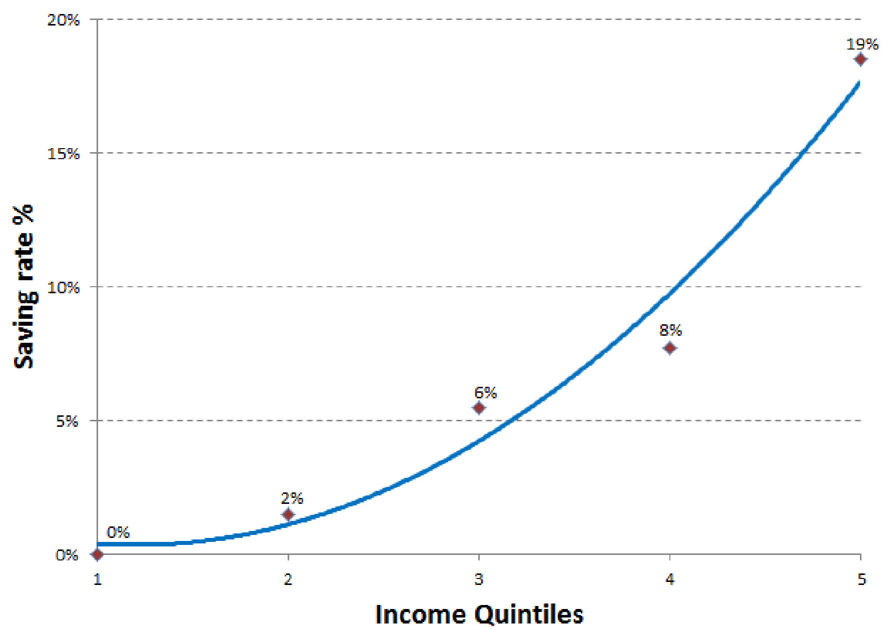
relate those two phenomena based on evidence from recent literature that the wealthier save marginally more and borrow marginally less than the poor, so that the net impact of an increase of income inequality would be an expansion of aggregate savings, together with a contraction of aggregate borrowing, thus dragging down the real interest rate.

On the savings side, Dynan et al. [11] besides empirically validating the fact that *the rich save more* (Figure:2), also found evidence that the *marginal savings rate* (MSR) is higher for high income households: In income quintiles 1 and 2 they estimate that MSR increases \$0.030 for each dollar of income increase, against \$0.429 for income quintiles 4 and 5. They also state that *higher savings rates for higher-income groups are consistent with an operative bequest motive*. Although, on another angle, bequests have also been considered in recent literature as a relevant source of income inequality (Galor [14], and Piketty [19]), in our work inequality is not endogenized, but it is the channel through which an endogenous bequest motive may become operative when parents expect their children to be less wealthy than themselves. So that when income inequality increases, high income type agents become even richer than the average, which further increases their endogenous generosity towards their children, whose expected wealth remains equal to population income mean, by increasing the level of bequests. The savings mechanism in our model relating inequality to real interest rates can then be summarized in the following way. Inequality turns on a bequest motive for the rich, and turns it off for the poor. The marginal savings rate with an operative bequest motive for the rich is greater than with an inoperative bequest motive for the poor, such that an increase of income inequality triggers an expansion of aggregate savings of the richer that prevails over the contraction of aggregate savings of the poor, resulting in a net expansion of total aggregate savings, that drags down the equilibrium real interest rate.

On the borrowing side, in a recent paper Mian and Sufi [18] found strong heterogeneity among *marginal propensities to borrow* (MPB) of households, by examining the impact of rising U.S. house prices on borrowing and spending from 2002 to 2006. They found evidence that the *rich* have a lower marginal propensity to borrow than the *poor*: Specifically, on average MPB increases by \$0.19 per dollar of home value increase, but by \$0.26 for the poorer households, and close to \$0 for the wealthier. Similarly, in a recent NBER working paper, Agarwal et al. [1] found empirical evidence that the *marginal propensity to borrow* is lower for higher credit scores, but in contrast with higher *marginal propensity to lend* (MPL) of the banks for higher credit scores. Taking this into account, in a credit constrained environment, the *marginal borrowing rate* (MBR) should be driven by banks marginal propensity to lend to the poorer, and by the marginal propensity to borrow of the unconstrained richer. This is a relevant factor to look at when modeling a borrowing constraint later in the paper.

In this paper we propose a formal framework to inspect the relation of increasing income inequality and persistently low real interest rates, based on decreasing marginal borrowing and increasing marginal saving rates with respect to income. So that an increase of income inequality triggers a net contraction of aggregate borrowing, as well as a net expansion of aggregate savings, resulting in a lower equilibrium real interest rate. We present a model where persistently low real interest rates are driven by increasing income inequality, by building and combining on some relevant topics in economic literature, namely on *Secular Stagnation* (Eggertsson and Mehrotra [12]), on the relation of marginal propensity to save and life-time income (Dynan et al. [11]), on the relation of marginal propensity to borrow with income and wealth (Mian and Sufi [18]) and (Agarwal et al. [1]), on inequality (Galor

Figure 2: Saving Rates and Income



Source: Dynan et al. [11], PSID

[14], and Piketty [19]), and bequests (Barro and Sala-i Martin [3] and Blanchard and Fischer [5]).

In what follows, in the second section we derive a formal general framework linking changes of real interest rates with increasing income inequality. We use the loan market equilibrium equation to derive an algebraic relation between (i) the partial derivative of the real interest rate with respect to the standard deviation of households' income distribution, (ii) and the marginal borrowing and saving rates of the wealthier and the poorer. We then use this relation to describe the mechanisms by which MBR decreases, and MSR increases with income, which respectively result in the contraction of aggregate borrowing, and the expansion of aggregate savings, when the standard deviation of income distribution increases. In the third section we formalize a more specific overlapping generations model with three periods based on the work of Eggertsson and Mehrotra [12], to explicitly describe the concepts presented before with a single agent type per generation, and observe that the real interest rate may decrease if the next generation is expected to be poorer.

In section 4, we introduce in the model income inequality among same generation households. We inspect how borrowing constraints alone can trigger a reduction of real interest rates when income inequality increases. Then, we use a *warm glow* bequest motive to start by showing that the equilibrium real interest rate can be persistently low, and materially lower than if the bequest motive is not operative. By adding income inequality into the model, and by assuming that generosity with respect to children is an exogenous positive function of income, we get an increasing marginal savings rate with income, causing the equilibrium real interest rate to decrease with an increase of income inequality. The mechanism is straightforward. If the rich get richer, then bequests increase more than proportionally to income: Bequests primarily increase because income increases, but also because households become more generous. Loan supply of the wealthier expands more than proportionally to the increase of income. The opposite is true for the poor, so that the net effect of increasing inequality, by causing a savings expansion of the rich which prevails over a savings contraction of the poor, results in a net aggregate savings expansion, and a reduction of the real interest rate. This mechanism is then endogenized by replacing bequests in household's preference function by the expected present value of children gross life-time income which includes the bequest received from their parents. We first consider a single agent income type to show that the marginal propensity to save is lower when parents expect their children to be wealthier than them, and higher otherwise. So that an intergenerational increase of income inequality, when parents expect children to be relatively poorer, expands aggregate savings and lowers real interest rates. The same is valid for intra-generational income inequality. Poorer households expect their descendants to be wealthier than them, so that their bequest motive endogenously becomes inoperative; the opposite is true for the wealthier. So the poor and the rich have respectively lower and higher marginal savings rate due to operative and non-operative bequest motives, so that the net effect of increasing income inequality is a net expansion of loans supply leading to a reduction of the real interest rate as well. We describe in appendix a similar result by considering the utility of consumption of children in the parent's preference function.

Finally in section 5 we calibrate a model with endogenous output and capital to estimate how much of the real interest rates reduction in recent years can be explained by the observed increase of income inequality in US, by using our model.

2 Inequality, Marginal Borrowing/Saving rates, Real Interest Rates

Imagine a closed economy, in the spirit of Eggertsson and Mehrotra [12], where households live for three periods, are borrowers in the first, savers in the second, and retired in the third. During the first period of their lives they consume by borrowing from the middle age. During the second period they receive an endowment income y, and eventually a bequest from their parents, they pay-back their loans to the retired, and save for retirement by lending to the young. In the third period they retire and use their savings to consume, and to possibly leave a bequest to their children. Middle age endowment is distributed according to the density function $f(y, \bar{y}, \sigma_y)$, where $\bar{y}$ and σ_y are respectively the mean and standard deviation of households' income. σ_y is here a measure of inequality, and $\bar{y}$ is assumed constant.

The real interest rate in this model is given by the equilibrium in the loan market such that aggregate demand equals aggregate supply of loans at any time t. At time t young households borrow $B_t^y(r_t, y_{t+1})$ to consume. Their borrowing level is a function of the real interest rate and of their expected income during the second period of their lives y_{t+1}, that we assume is known by lenders. In addition at time t, middle aged households save $-B_t^m(r_t, y_t)$ to be able to consume when old.

Let loan demand and supply per middle age household at time t be given by the following general expressions, which depend on the real interest rate and on the middle age endowment distribution:

$$L_t^d(r_t, \bar{y}_{t+1}, \sigma_{y_{t+1}}) = \int B_t^y(r_t, y_{t+1}) f(y_{t+1}, \bar{y}_{t+1}, \sigma_{y_{t+1}}) dy_{t+1} \tag{1}$$

$$L_t^s(r_t, \bar{y}_t, \sigma_{y_t}) = -\int B_t^m(r_t, y_t) f(y_t, \bar{y}_t, \sigma_{y_t}) dy_t \tag{2}$$

The equilibrium real interest rate r_t is a solution to loan market equilibrium such that:

$$L_t^e(r_t, \bar{y}_t, \sigma_{y_t}, \bar{y}_{t+1}, \sigma_{y_{t+1}}) = L_t^s(r_t, \bar{y}_t, \sigma_{y_t}) - L_t^d(r_t, \bar{y}_{t+1}, \sigma_{y_{t+1}}) = 0. \tag{3}$$

where we define L_t^e as excess savings at time t, and from where we derive the following equation in steady state by using the *Implicit Function Theorem*:

$$\frac{\partial L^e}{\partial r} dr + \frac{\partial L^e}{\partial \bar{y}} d\bar{y} + \frac{\partial L^e}{\partial \sigma_y} d\sigma_y = 0 \overset{d\bar{y}=0}{\Leftrightarrow} \frac{dr}{d\sigma_y} = -\frac{\frac{\partial L^e}{\partial \sigma_y}}{\frac{\partial L^e}{\partial r}} \tag{4}$$

If we assume that the denominator of the last expression $\frac{\partial L^e}{\partial r}$ is positive [1], then the real interest rate decreases with increasing income inequality if $\frac{\partial L^e}{\partial \sigma_y}$ is positive. For which it is sufficient that aggregate savings increases, and aggregate borrowing decreases with increasing

[1]In chapter 1 we show that to $\frac{\partial L^s}{\partial \sigma_y} > 0 \Leftrightarrow \frac{\partial L^s}{\partial r} > \frac{\partial L^d}{\partial r}$ can be a reasonable assumption.

income inequality:

$$\frac{\partial L^s}{\partial \sigma_y} > 0 \text{ and } \frac{\partial L^d}{\partial \sigma_y} < 0 \Rightarrow \frac{\partial L^e}{\partial \sigma_y} > 0 \Rightarrow \frac{\partial r}{\partial \sigma_y} < 0 \tag{5}$$

We now split household population in two sets of relative constant sizes. A set containing the low income types with relative size equal to $\eta = \int_l f(r_t, \bar{y}_t, \sigma_{y_t}) dy_t$, and a set containing the high income types with size equal to $1-\eta$. We assume that $\bar{y} = \eta \bar{y}_t^l + (1-\eta)\bar{y}_t^h$ is constant over time, where $\bar{y}_t^h = \frac{1}{1-\eta}\int_h y_t f(r_t, \bar{y}_t, \sigma_{y_t}) dy_t$ is, naturally, an increasing function of σ_{y_t}.

We can also express loan supply and demand as the weighted average of loan supply and demand of low and high types. Loan supply of high types is given by $L_t^{s,h} = \frac{1}{1-\eta}\int_h L_t^s(r_t, y_t) f(r_t, \bar{y}_t, \sigma_{y_t}) dy_t$, with similar expressions for $L_t^{s,l}$, $L_t^{d,h}$ and $L_t^{d,l}$. Note that[2]:

$$\frac{\partial L^s}{\partial \bar{y}^h} = \eta \frac{\partial L^{s,l}}{\partial \bar{y}^h} + (1-\eta)\frac{\partial L^{s,h}}{\partial \bar{y}^h} = (1-\eta)\left(\frac{\partial L^{s,h}}{\partial \bar{y}^h} - \frac{\partial L^{s,l}}{\partial \bar{y}^l}\right) = (1-\eta)\left(MSR^h - MSR^l\right) \tag{6}$$

$$\frac{\partial L^d}{\partial \bar{y}^h} = \eta \frac{\partial L^{d,l}}{\partial \bar{y}^h} + (1-\eta)\frac{\partial L^{d,h}}{\partial \bar{y}^h} = (1-\eta)\left(\frac{\partial L^{d,h}}{\partial \bar{y}^h} - \frac{\partial L^{d,l}}{\partial \bar{y}^l}\right) = (1-\eta)\left(MBR^h - MBR^l\right) \tag{7}$$

where $MBR^\theta \equiv \frac{\partial L^{d,\theta}}{\partial \bar{y}^\theta}$ and $MSR^\theta \equiv \frac{\partial L^{s,\theta}}{\partial \bar{y}^\theta}$ are respectively the marginal aggregate borrowing and saving rates with respect to average income of a given population segment θ.

The impact of increasing income inequality on the real interest rate is given by the sign of the marginal change of excess savings with respect to the standard deviation of endowments, which can be expressed in terms of marginal saving and borrowing rates in the following way:

$$\frac{\partial L^e}{\partial \sigma_y} = \frac{\partial L^e}{\partial \bar{y}^h}\frac{\partial \bar{y}^h}{\partial \sigma_y} = (1-\eta)\left(\triangle^{hl} MSR - \triangle^{hl} MBR\right)\frac{\partial \bar{y}^h}{\partial \sigma_y} \tag{8}$$

Where $\triangle^{hl} MSR = MSR^h - MSR^l$. Then, given that $\frac{\partial \bar{y}^h}{\partial \sigma_y} > 0$, and by assuming that $\frac{\partial L^e}{\partial r} > 0$, the real interest rate changes with income inequality according to the following equation:

$$\frac{\partial r}{\partial \sigma_y} < 0 \Leftrightarrow \triangle^{hl} MSR > \triangle^{hl} MBR \tag{9}$$

A sufficient condition for a decreasing real interest rate with increasing income inequality is that MSR increases and MBR decreases with income, as has been evidenced in recent economic literature. In particular, Mian and Sufi [18] found recently by examining the effects

[2] $d\bar{y}_t^h = -\frac{\eta}{1-\eta} d\bar{y}_t^l$

of rising U.S. house prices on borrowing and spending from 2002 to 2006, that on average the marginal propensity to borrow *MPB* increases \$0.19 per dollar of home value increase, but \$0.26 for the poorer households, and close to \$0 for the richer. Agarwal et al. [1] also found empirical evidence that the marginal propensity to borrow declines with credit scores, In their recent NBER working paper. In the savings side, besides empirically validating the fact that *the rich save more*, Dynan et al. [11] found evidence that the marginal propensity to save is higher for high income households: In income quintiles 1 and 2 they estimate that MPS increases \$0.030 for each dollar of income increase, against \$0.429 for income quintiles 4 and 5.

If empirical evidence suggests that marginal propensity to save is a positive function of income, $\triangle^{hl} MPS > 0$, and the marginal propensity to borrow is a negative function of income, $\triangle^{hl} MPB < 0$, and no borrowing or savings constraints were considered in our economy, then we could assume that $MSR = MPS$ and $MBR = MPB$ which would immediately imply by equation (9) that real interest rates would decrease with an increase of income inequality. We present next a simple model based on Eggertsson and Mehrotra [12] to inspect those mechanisms.

3 *Secular Stagnation* Endowment Economy Model with Bequests

We now formalize the framework described in the previous section in a specific model. We start by assuming that households have the same income type. More concretely a representative household of a generation born at time t has, for now, the following utility function[3]:

$$\max_{C_t^y, C_{t+1}^m, C_{t+2}^o, Q_{t+2}} \mathbb{E}_t \left\{ \log(C_t^y) + \beta \log(C_{t+1}^m) + \beta^2 \left[\log(C_{t+2}^o) + \frac{\log(Q_{t+2})}{1+\phi} \right] \right\} \tag{10}$$

Where C_t^y, C_t^m, and C_t^o are respectively the consumption of young, middle aged, and old; Q_t is a bequest transfered from old households to the next generation when middle aged,

[3]For now we derive the model with a log utility function because it improves the tractability of algebraic expressions, without loss of generality, as we show in the last section and in appendix A of this chapter, where we use a reasonable calibrated CRRA preference function and introduce endogenous output and capital in the model. We assume, as in the first chapter of this dissertation, that the slope of loan supply, if negative, would never be lower in absolute terms than the slope of loan demand, meaning that the elasticity of intertemporal substitution is assumed to be greater than a minimum threshold lower than one, and also lower than standard literature EIS levels[16]. A negative loan supply slope would mean that an income effect would prevail over the substitution effect given an increase of the real interest rate.

and $\phi \in]-1;+\infty[$ accounts for the fact that households when old may discount bequests differently than their consumption. For example if $\phi \to \infty$ parents are selfish in the sense they prefer an additional unit of old age consumption to any quantity of bequest left to children. Household budget constraints are then given by:

$$C_t^y = B_t^y \tag{11}$$

$$C_{t+1}^m = Y_{t+1}^m - (1+r_t)B_t^y + B_{t+1}^m + \frac{Q_{t+1}}{1+g_t} \tag{12}$$

$$C_{t+2}^o = -(1+r_{t+1})B_{t+1}^m - Q_{t+2} \tag{13}$$

$$(1+r_t)B_t^y \le D_t, \text{ where } D_t = \theta {Y_{t+1}^m}^{\mu} \tag{14}$$

$$0 \le Q_t \tag{15}$$

B_t is a one period risk-free bond at an interest rate r_t. Consumption of the young is constrained by the amount they can borrow (11). The budget constraint of the middle aged is given by equation (12); they receive and income Y^m, pay their loans with interest $(1+r)B^y$, save for retirement B^m, and receive from previous generation a bequest Q. Equation (13) is the budget constraint of old households that use their savings with interest $(1+r)B^m$ to consume, and transfer a positive bequest Q to the next generation. Inequality (15) correspond to the assumption that bequests are positive.

We assume for now that all households are credit constrained (14), and that the borrowing limit of the young is binding. Later we relax this assumption. We also assume that the borrowing limit is a positive function of expected lifetime income ($\mu > 0$ and $\theta > 0$), and known by the lenders when agents are young.

The equilibrium real interest rate of this model solves loan market equilibrium equation, requiring that the demand for loans L_t^d equals supply L_t^s at any time t, or that $N_t^y B_t^y$ is equal to $-N_t^m B_t^m =$, equivalent to:

$$L_t^d = (1+g_t)B_t^y = -B_t^m = L_t^s \tag{16}$$

where $N_t^m = N_{t-1}^y$ and $1+g_t = \frac{N_t^y}{N_{t-1}^y}$ is defined as the growth rate of births.

Loan Demand

From equation (11) and inequality (14) corresponding to the binding borrowing limit, we derive and expression for the consumption of the young given by:

$$C_t^y = \frac{D_t}{1+r_t} \tag{17}$$

Using inequality (14) and assuming the borrowing limit is binding, loan demand is given by:

$$L_t^d = (1 + g_t)B_t^y = \frac{1 + g_t}{1 + r_t} D_t \tag{18}$$

In order to model D_t as a binding borrowing limit for all households we need to ensure that the constrained demand for loans L_t^d is lower than the unconstrained $L_t^{d,u}$, which is a linear positive function of income:

$$L_t^d = \frac{1 + g_t}{1 + r_t} D_t \leq \frac{1 + g_t}{1 + r_t} \frac{Y_{t+1}^m}{1 + \beta + \beta^2} = L_t^{d,u} \Leftrightarrow D_t = \theta {Y_{t+1}^m}^{\mu} \leq \frac{Y_{t+1}^m}{1 + \beta + \beta^2} \tag{19}$$

By Setting $\mu \in]0; 1[$ and $\theta < \frac{Y_{min}^{1-\mu}}{1+\beta+\beta^2}$ we ensure that the limit is binding for all values of $Y \in]Y_{min}; Y_{max}[$. Those parameters also ensure that the marginal borrowing rate MBR is a negative function of income, consistent with what is empirically expected for the marginal propensity to borrow:

$$MBR = \frac{\partial L_t^d}{\partial Y_{t+1}} = \frac{1 + g_t}{1 + r_t} \theta \mu Y_{t+1}^{\mu-1}, \text{ and } \frac{\partial MBR}{\partial Y_{t+1}} < 0 \tag{20}$$

Later we endogenize the fact that $\frac{\partial MBR}{\partial Y} < 0$ by questioning the assumption $\mu \in]0; 1[$ and test an alternative where $\mu \geq 1$, and the binding borrowing limit assumption relaxed for the wealthier.

Loan Supply

The middle aged are at an interior solution and satisfy a consumption Euler equation given by:

$$\beta \mathbb{E}_t \frac{C_t^m}{C_{t+1}^o} = \frac{1}{1 + r_t} \tag{21}$$

Using equation (21) in the budget constraint of the old (13)[4], we get an expression for the consumption of the middle aged given by:

$$C_t^m = \frac{C_{t+1}^o}{\beta(1 + r_t)} = \frac{-(1 + r_t)B_t^m - Q_{t+1}}{\beta(1 + r_t)} = -\frac{B_t^m}{\beta} - \frac{Q_{t+1}}{\beta(1 + r_t)} \tag{22}$$

By combining this expression for C_t^m with the middle aged budget constraint (12) we derive an expression for Loan supply with bequests, which is greater or equal than the corresponding

[4]as well as the deterministic nature of the model.

expression with an inoperative bequest motive ($Q = 0$):

$$L_t^s = -B_t^m = \frac{\beta}{1+\beta}(Y_t^m - D_{t-1}) + \frac{\beta}{1+\beta}\left(\frac{Q_t}{1+g_t} + \frac{Q_{t+1}}{\beta(1+r_t)}\right) \tag{23}$$

Furthermore, the marginal savings rate MSR with an inoperative bequest motive is a positive function of income:

$$MSR = \frac{\partial L_t^s}{\partial Y_t} = \frac{\beta}{1+\beta}\left(1 - \theta\mu Y_t^{\mu-1}\right), \text{ and } \frac{\partial MSR}{\partial Y_t} > 0 \tag{24}$$

Note that the concavity of the marginal borrowing rate with respect to income is here explaining the convexity of the marginal savings rate with respect to income, which is itself the consequence of an exogenous parametrization of the borrowing limit given by assuming that $\mu < 1$. We later relax this assumption together with not requiring a binding borrowing limit for the wealthier households, and get a similar result. Furthermore we will analyze a mechanism only dependent on the loan supply side of the model based on bequests.

Bequests

We now inspect how the level of generosity of parents towards children affect the marginal savings rate and the natural rate of interest. In particular we compare expressions with and without an operative bequest motive. By combining the First Order Conditions for C_{t+2}^o and Q_{t+2} with the middle age budget constraint we derive the following expression for expected bequest of next period[5]:

$$\mathbb{E}_t Q_{t+1} = (1+r_t)\Psi\left(Y_t^m - D_{t-1} + \frac{Q_t}{1+g_{t-1}}\right) > 0 \tag{25}$$

The constant $\Psi = \frac{\beta}{\beta+(1+\phi)(1+\beta)} \in]0;1[$ for $\phi \in]-1;+\infty[$ can be interpreted as a generosity coefficient of parents towards children. The bequest to descendants at time $t+1$ is expected to be higher at time t for a higher interest rate at time t. Moreover, with this preference function the bequest motive is always strictly positive and operative unless $\phi = +\infty \Rightarrow \Psi = 0$. This becomes evident when loan market is in equilibrium and the real interest rate is the natural rate of interest r_t^n:

$$\mathbb{E}_t Q_{t+1}^n = \frac{1+g_t}{2+\phi} D_t \tag{26}$$

By replacing the bequest expression (25) in the loan supply expression (23), we derive a general expression for loan supply depending only on bequest received by the previous generation

[5]The endowment economy model with a warm glow bequest motive is derived in Appendix A.

during middle age, that we will use through the rest of the chapter[6]:

$$L_t^s = \frac{\beta + \Psi}{1+\beta}\left(Y_t^m - D_{t-1} + \frac{Q_t}{1+g_{t-1}}\right) \tag{27}$$

This is a useful expression in particular when we assume later on that children and parents income types are *iid*, so that the average expected bequest received by any income type is the same. In this single income type framework, we now derive the marginal savings rate when loan market is in equilibrium, from loan supply equilibrium, given by the following expression:

$$L_t^s = \frac{\beta + \Psi}{1+\beta}\left[Y_t^m - \frac{1+\phi}{2+\phi}D_{t-1}\right] > L_t^{s,Q=0} = \frac{\beta}{1+\beta}\left[Y_t^m - D_{t-1}\right] \tag{28}$$

As expected, loan supply in equilibrium expands with an operative bequest motive, which results in a lower natural rate of interest:

$$1 + r_t^n = \left(\frac{1+\beta}{\beta+\Psi}\right)\frac{(1+g_t)D_t}{Y_t^m - \frac{1+\phi}{2+\phi}D_{t-1}} < 1 + r_t^{n,Q=0} = \frac{1+\beta}{\beta}\frac{(1+g_t)D_t}{Y_t^m - D_{t-1}} \tag{29}$$

Regarding the marginal savings rate given by expression bellow we can observe that it increases when the bequest motive is operative, and also increases with income. [7]:

$$MSR_t = \frac{\beta+\Psi}{1+\beta}\left[1 - \frac{1+\phi}{2+\phi}D'_{t-1}\right] > \frac{\beta}{1+\beta}\left[1 - D'_{t-1}\right] = MSR_t^{Q=0} \tag{30}$$

$$MSR'_t = \frac{\beta+\Psi}{1+\beta}\frac{1+\phi}{2+\phi}(-D''_{t-1}) > 0 \tag{31}$$

But we can also observe that the convexity of MSR with respect to income is a direct consequence of the concavity of the marginal borrowing rate. Consequently with this preference function, an increase of income inequality would impact negatively the natural rate of interest from the savings side, only as long as the marginal borrowing rate is negative sloped with respect to income. Unless generosity with respect to the next generation increases when agents become wealthier. We explore those mechanisms in the next section by introducing income heterogeneity in the model, and inspect separately the impact of loans demand and supply sides on the natural rate of interest when inequality increases, respectively by

[6] As noted already, loan supply with a log utility function is inelastic with respect to the real interest rate, meaning that income and substitution effects cancel each-other. This makes algebraic expressions more tractable, without loss of generality, since for reasonable low EIS values (lower than one), when income effect prevails over the substitution effect, the negative slope of loan supply with respect to the real interest rate does not change the impact sign on the real interest rate of a loan supply expansion.

[7] $D'_{t-1} = \frac{\partial D_{t-1}}{\partial Y_t} = \theta\mu Y_t^{\mu-1} > 0$, $D''_{t-1} = \theta\mu(\mu-1)Y_t^{\mu-2} < 0$, and $MSR'_t = \frac{\partial MSR_t}{\partial Y_t}$.

Table 1: Summary of model conditions for increasing inequality to decrease r

General condition for $\frac{\partial r}{\partial \sigma_y} < 0 : \triangle^{hl} MSR > \triangle^{hl} MBR$		
Sufficient conditions for $\frac{\partial r}{\partial \sigma_y} < 0$:	$\triangle^{hl} MBR < 0$	$\triangle^{hl} MSR > 0$
Borrowing mechanism	✓	✓
- $\mu < 1$, and borrowing limit binding for all agents:	✓	✓
- or $\mu > 1$, and borrowing limit binding just for the poor:	✓	✓
Savings mechanism		✓
- $\phi'(y) < 0$: Generosity increases exogenously with income:		✓
- or bequest ZLB, $Q \geq 0$, becomes binding for the poor:		✓

canceling the bequest motive, and the borrowing limit concavity.

4 Decreasing real interest rates, with increasing income inequality

Inequality is introduced in the model by considering two types of agents with different endowments levels when middle-aged. We assume that the average endowment at time t is always constant, and given by:

$$Y_t^m = \eta Y_t^{m,l} + (1-\eta) Y_t^{m,h} = Y^m \tag{32}$$

η is the fraction of low income households. The standard deviation of this income distribution at time t and its derivative with respect to high type income are respectively given by the following two expressions:

$$\sigma_{y_t} = (Y_t^{m,h} - Y^m)\sqrt{\frac{1-\eta}{\eta}}, \text{ and } \frac{d\sigma_{y_t}}{dY_t^{m,h}} = \sqrt{\frac{1-\eta}{\eta}} \tag{33}$$

The natural rate of interest changes with an increase of income inequality according to:

$$\frac{\partial r}{\partial \sigma_y} = -\left(\frac{\sqrt{\eta(1-\eta)}}{\frac{\partial L^e}{\partial r}}\right)\left(\triangle MSR_t^{hl} - \triangle MBR_t^{hl}\right) \tag{34}$$

So the equilibrium real interest rate decreases with increasing income inequality if $\triangle MSR_t^{hl} > \triangle MBR_t^{hl}$. We now inspect separately the mechanisms respectively related to the borrowing and supply sides of the model, briefly summarized in table(1).

4.1 Borrowing constraints, inequality, and real interest rates

We cancel the bequest motive in this subsection by setting $\phi = +\infty$. From expressions (20) and (30) we derive $\triangle MSR_t^{hl}$ and MBR_t^{hl} and observe that the condition is verified, so that the real interest rate in this model decreases with an increase of income inequality:

$$\triangle MBR_t^{hl} = \frac{1+g_t}{1+r_t}(D_t^{h'} - D_t^{l'}) < 0 \tag{35}$$

$$\triangle MSR_t^{hl} = -\frac{\beta}{1+\beta}(D_t^{h'} - D_t^{l'}) > 0 \tag{36}$$

Note that $\mu < 1$ and $D_t^{h'} = \theta\mu Y_t^{h^{\mu-1}} < \theta\mu Y_t^{l^{\mu-1}} = D_t^{l'}$. It is the concavity of the borrowing limit that determines the above result, of an increase of income inequality lowering the equilibrium real interest rate. In fact, if $\mu = 0$ as in Eggertsson and Mehrotra [12], or $\mu = 1$ so that the borrowing limit is a linear function of expected income, then $\triangle MBR_t^{hl} = \triangle MSR_t^{hl} = 0$ and an increase of income inequality would not impact the real interest from the borrowing side of this economy.

i) Assumption that all agents are credit cosntrained when young

If we assume that all agents are borrowing constrained when young, it is also reasonable to assume the concavity of the borrowing limit, consistently with their marginal propensity to borrow, and in contrast with a constant borrowing limit presuming that lenders have no information whatsoever about borrowers income type, or, in the other extreme, in contrast with a linear function of expected income that would presume lenders have full information for a given lending motive.

ii) Assumption that only the low income type are credit constrained

We now recall the work of Agarwal et al. [1] who showed that credit card limits in US not only increase with credit scores, but so does the marginal propensity to lend from banks. The borrowing limit in our model would be an increasing convex function with respect to expected income, with $\mu > 1$, but would only bind for the low income types. We further assume that for average income $\bar{Y}^m$ constrained and unconstrained borrowing levels would be equal:

$$Y \leq \bar{Y} : L_t^d(Y) = \frac{1+g_t}{1+r_t}\theta Y^\mu \tag{37}$$

$$Y \geq \bar{Y} : L_t^{d,u}(Y) = \frac{1+g_t}{1+r_t}\frac{Y}{1+\beta+\beta^2} \tag{38}$$

$$Y = \bar{Y} : L_t^d(\bar{Y}) = L_t^{d,u}(\bar{Y}) \Leftrightarrow \theta = (1+\beta+\beta^2)^{-1} \tag{39}$$

Without loss of generality we assume that $\bar{Y} = 1$. Then for the borrowing limit to be binding for the low income type and non binding for the higher, we need $\mu > 1$:

$$L_t^{d,u}(Y_{t+1}^l) < L_t^d(Y_{t+1}^l) \Leftrightarrow \frac{1+g_t}{1+r_t}\theta(Y_{t+1}^l)^\mu < \frac{1+g_t}{1+r_t}\theta Y_{t+1}^l \Leftrightarrow (Y_{t+1}^l)^{\mu-1} < 1 \tag{40}$$

$$L_t^{d,u}(Y_{t+1}^h) > L_t^d(Y_{t+1}^h) \Leftrightarrow \frac{1+g_t}{1+r_t}\theta(Y_{t+1}^h)^\mu > \frac{1+g_t}{1+r_t}\theta Y_{t+1}^h \Leftrightarrow (Y_{t+1}^h)^{\mu-1} > 1 \tag{41}$$

From where the marginal borrowing rates of high and low types would be given by:

$$MBR_t^l = \frac{1+g_t}{1+r_t}\theta\mu {Y_{t+1}^l}^{\mu-1} \tag{42}$$

$$MBR_t^h = \frac{1+g_t}{1+r_t}\theta \tag{43}$$

The marginal borrowing rate of the low types is higher than the high types for reasonable values of μ and Y^l given by the condition $Y^l > h(\mu) = \left(\frac{1}{\mu}\right)^{\frac{1}{\mu-1}}$, where $h(\mu)$ is a positive function of μ[8]. In that case:

$$\triangle MBR_t^{hl} = -\frac{1+g_t}{1+r_t}\theta\left(\mu {Y_{t+1}^l}^{\mu-1} - 1\right) < 0 \tag{44}$$

$$\triangle MSR_t^{hl} = \frac{\beta}{1+\beta}\theta\left(\mu {Y_{t+1}^l}^{\mu-1} - 1\right) > 0 \tag{45}$$

Having inspected the impact of borrowing limits on inequality/real interest rate dynamics, we now look at the savings side of the model with an operative bequest motive.

4.2 Bequests, inequality, and real interest rates

In their paper *"Do the Rich Save More"* Dynan et al. [11] show that higher savings rates are associated to higher lifetime income. They also find evidence that the *marginal savings rate*(MSR) is a positive function of lifetime income. The results of previous sections are

[8] For reference $h(1^+) = 0.37$ and $h(4) = 0.63$

consistent with their findings, in particular due to the fact that the marginal savings rate of the rich are higher because their marginal borrowing rate are lower. Dynan et al. [11] inspect several factors explaining an increasing marginal savings rate with respect to income, that would result in the reduction of the natural rate of interest when income inequality increases. We next analyze the conditions under which a bequest motive is one of those factors. In order to do so, we cancel the concavity of the borrowing limit by setting $\mu = 0$ so that the marginal borrowing rate $MBR = 0$ as in Eggertsson and Mehrotra [12]. We also assume that the borrowing limit is binding for all households. Aggregate loan supply for each income type is given by:

$$L_t^{s,i} = \frac{\beta + \Psi}{1+\beta}\left(Y_t^{m,i} - D_{t-1} + \frac{Q_t}{1+g_{t-1}}\right) \tag{46}$$

We assume through the rest of the chapter that parents and children income types are *iid* so that on average the bequest received from parents is the same among income types, and has an average steady state expression independent of income types:

$$Q = \frac{Y^m - D}{\frac{1}{\Psi(1+r)} - \frac{1}{1+g}} \tag{47}$$

Note that average bequest naturally increase with generosity, and decrease with population growth since more children mean a lower parcel of the same bequest per child. Then in the present model the marginal savings rate is independent of income types, thus not triggering the expected mechanism:

$$MRS_t^i = \frac{\beta + \Psi}{1+\beta} \Rightarrow \triangle MSR_t^{hl} = 0 \tag{48}$$

An increase of inequality expands savings of the high type, and contracts savings of the low type. Because the marginal savings rate of high and low types are equal, the two effects cancel each other not affecting aggregate loan supply and equilibrium real interest rate. This model as is does not capture the fact that the poor may have less propensity to leave a bequest to their children because they prioritize consumption, or may expect their children to be better off. A straightforward way to account for this is to assume that the level of generosity of households with respect to the next generation increases exogenously with income.

i) Generosity increases exogenously with lifetime income

As seen earlier, the parameter ϕ is a measure of selfishness. When ϕ tends to infinity bequests tend to zero, as well as Ψ. The parameter Ψ can then be interpreted as a measure of generosity towards the next generation. We now assume that the level of generosity increases with income $Y_t^{m,i}$. Let $\Psi^i = \Psi(Y_t^{m,i}) = \psi Y_t^{m,i}$, where ψ is a positive constant. This is equivalent to set $\phi = \phi\left(Y_t^{m,i}\right) = \frac{\beta}{1+\beta}\left(\frac{1}{\psi Y_t^{m,i}} - 1\right)$ which, being a measure of selfishness, is now a decreasing function of lifetime income. The supply of loans for a given income type is given by:

$$L_t^{s,i} = \frac{\beta + \psi Y_t^{m,i}}{1+\beta}\left(Y_t^{m,i} - D_{t-1} + \frac{Q_t}{1+g_{t-1}}\right) \tag{49}$$

Now the steady state average bequest expression increases with increasing income inequality:

$$Q = (1+r)\frac{\psi({Y^h}^2 - {Y^l}^2) - \Psi D}{1 - \Psi\frac{1+r}{1+g}} = \frac{1}{\frac{1}{\Psi(1+r)} - \frac{1}{1+g}}\left[\frac{Y^h - \bar{Y}}{\eta} - D\right] \tag{50}$$

If the rich are more generous towards their descendants then the bequest increase of the wealthier prevails over the bequest contraction of the poorer when inequality increases. The same result is obtained for the marginal savings rate. Then $\triangle MSR_t^{hl} > 0$[9]:

$$\triangle MSR^{hl} = \frac{1}{1+\beta}\left[2(\Psi^h - \Psi^l) + \frac{\beta + \Psi^h}{1+g}\frac{\partial Q}{\partial Y^h} - \frac{\beta + \Psi^l}{1+g}\frac{\partial Q}{\partial Y^l}\right] > 0 \tag{51}$$

The fact that rich and poor discount bequests differently is sufficient to trigger the mechanism by which an increase of income inequality has a negative effect on the equilibrium real interest rate level. Next we endogenize this mechanism by making an adjustment of the bequest motive in the preference function, such that households leave bequests because they compare expected wealth of their descendants with themselves, and are willing to help if they expect children to be poorer, but not if they expect them to be wealthier.

ii) Generosity increases endogenously with lifetime income

We now assume that agents trade-off consumption by expected gross wealth of next generation, instead of bequests directly. They care for their children expected wealth relative to their own, in contrast of just caring for leaving a bequest to the next generation, independently of whether they need their help or not. By next generation expected gross wealth we mean the sum of their expected endowment and the bequest received from their parents. In this case the bequest zero lower bound given by (15) may become binding. The utility function of a representative household born at time t becomes:

$$\max_{C_t^y, C_{t+1}^m, C_{t+2}^o, W_{t+2}^m} \mathbb{E}_t\left\{\log(C_t^y) + \beta\log(C_{t+1}^m) + \beta^2\left[\log(C_{t+2}^o) + \frac{\log(W_{t+2}^m)}{1+\phi}\right]\right\} \tag{52}$$

[9]Note that $\frac{\partial Q}{\partial Y^h} > 0$ and $\frac{\partial Q}{\partial Y^l} < 0$. Furthermore $\Psi = \psi Y^m$, $\Psi^h = \psi Y^{m,h}$ and $\Psi^l = \psi Y^{m,l}$.

where $W_t^m = Y_t^m + \frac{Q_t}{1+g_{t-1}}$, and the utility is maximized subject to the same budget constraints (11),(12),(13) and inequalities (14) and (15). We use the same methodology as in previous sections to derive an expression for the bequest transfered to the next generation, when bequest zero lower bound is not binding:

$$Q_{t+1} = (1+r_t)\Psi_t \left(W_t^m - D_{t-1}\right) - (1-\Psi_t)(1+g_t)\mathbb{E}_t Y_{t+1}^m \tag{53}$$

where $\Psi_t = \frac{\beta}{\beta + \frac{(1+\phi)(1+\beta)}{1+g_t}} \in]0;1[$ for $\phi \in]-1;+\infty[$. The previous expression for Q_{t+1} may become negative if descendants endowment present value is expected to be higher than a threshold depending on the net wealth and generosity of their parents:

$$Q_{t+1} \geq 0 \Leftrightarrow \frac{\beta}{1+\beta}\frac{W_t^m - D_{t-1}}{1+\phi} \geq \frac{\mathbb{E}_t Y_{t+1}^m}{1+r_t} \tag{54}$$

Loan supply is conditional on the bequest being binding and can be expressed by:

$$L_t^s = \frac{\beta}{1+\beta}\left(W_t^m - D_{t-1}\right) + \frac{(1-\Psi_t)(1+g_t)}{1+\beta}\max\left[0, \frac{\beta}{1+\beta}\frac{W_t^m - D_{t-1}}{1+\phi} - \frac{\mathbb{E}_t Y_{t+1}^m}{1+r_t}\right] \tag{55}$$

In this economy, if the present value of children expected endowment is sufficiently low compared to the net wealth of their parents, and considering their level of generosity, then the bequest motive becomes operative and savings expand with respect to an inoperative bequest motive state. The expansion of parents savings increases further with expectations that children are poorer. Then the marginal savings rate when the bequest motive is operative is greater higher than when it is not:

$$MSR_t^Q = \frac{\beta + \Psi_t}{1+\beta} > \frac{\beta}{1+\beta} = MSR_t \tag{56}$$

This is a relevant expression when intra-generation inequality is introduced in the model, because wealthier agents tend to expect their children to be relative less wealthy, and then be more generous, in contrast with poorer households that expect their children to be better off than them, and then tend to be less generous. But before that we will still present the following expression for the natural rate of interest when the bequest motive is operative:

$$1 + r_t = \frac{1+\beta}{\beta + \Psi_t}\frac{(1+g_t)\left[D_t + \frac{1-\Psi_t}{1+\beta}Y_{t+1}^m\right]}{Y_t^m - D_{t-1} + \frac{Q_t}{1+g_{t-1}}} < 1 + r_t^n \tag{57}$$

Then, when the bequest motive is operative the natural rate of interest decreases if generosity towards children Ψ_t increases, or if expected income of descendants decreases. The natural rate of interest may increase until an upper bound given by its expression with an inoperative bequest motive, when children expected endowment increase above a given threshold.

Note that, an expected increase of inter-generation inequality reduces the *natural rate of interest* in this model.

If children are expected to become relatively poorer than parents, (or parents become relatively wealthier than children) then the supply of loans expands causing a reduction of the equilibrium real interest rate.

We now assume that there is intra-generation income inequality in our economy as in previous sections: $\eta_s Y_t^{m,l} + (1-\eta_s) Y_t^{m,h} = Y^m$, and that children and parents types are *iid*. We further assume that parents don't know their children income type when they make the bequest decision, so that children expected income is equal to Y^m. Then poorer households expect descendants to be richer than themselves, and the richer expect their children to be poorer. In this case the condition for a positive bequest is given by the following equation:

$$Q_{t+1}^{\gamma,i} \geq 0 \Leftrightarrow \frac{\beta}{1+\beta} \frac{W_t^{m,\gamma,i} - D_{t-1}}{1+\phi} \geq \frac{Y^m}{1+r_t} \tag{58}$$

$$\Leftrightarrow Y_t^{\gamma} + \frac{Q^i}{1+g_{t-1}} \geq \frac{(1+\beta)(1+\phi)}{\beta} \frac{Y^m}{1+r_t} + D_{t-1} \tag{59}$$

where $\gamma \in \{l, h\}$, and i is household's id. Note that this condition is the same for all households, since the right-hand side of the equation(59) does not depend on any income type in particular. For the subset above the threshold the marginal savings rate is equal to $\frac{\beta+\Psi_t}{1+\beta}$ and higher than the marginal savings rate of the ones in the subset below $\frac{\beta}{1+\beta}$. A sufficient condition for an operative bequest motive for all high income types is given by:

$$Y_t^h > \frac{(1+\beta)(1+\phi)}{\beta} \frac{Y^m}{1+r_t} + D_{t-1} \Rightarrow Q_{t+1}^{h,i} > 0 \tag{60}$$

If there exists at least one low type household for which the bequest zero lower bound binds then the aggregate marginal savings rate of low types is lower than the one of the high types, such that the conditions for the mechanism linking an increase of income inequality to lower interest rates are satisfied. In the next section and in appendix we show that under reasonable assumptions low types may have their bequest motive always inoperative, while always operative for high types. In that case we can derive a close form expression for the derivative of the natural rate of interest with respect to a measure of income inequality:

$$\frac{\partial r}{\partial \sigma_y} = -\left(\frac{\sqrt{\eta(1-\eta)}}{\frac{\partial L^e}{\partial r}}\right)\left(\triangle MSR_t^{hl} - \triangle MBR_t^{hl}\right) = -\left(\frac{\sqrt{\eta(1-\eta)}}{\frac{\partial L^e}{\partial r}}\right)\frac{\Psi_t}{1+\beta} < 0 \qquad (61)$$

where $\frac{\partial L^e}{\partial r} = \frac{1+g_t}{(1+r_t)^2}\left((1-\eta)Y^m\frac{1-\Psi_t}{1+\beta} + D_{t-1}\right) > 0$.

5 Quantitative calibration of the model

In previous sections we were able to formally capture some of the mechanisms supporting the relation between increasing income inequality and decreasing real interest rates, although with a simple and stylized OLG model. We nevertheless think it would be of value to try to estimate by how much our model, explicitly parametrized, could explain the reduction of real interest rates in recent years.

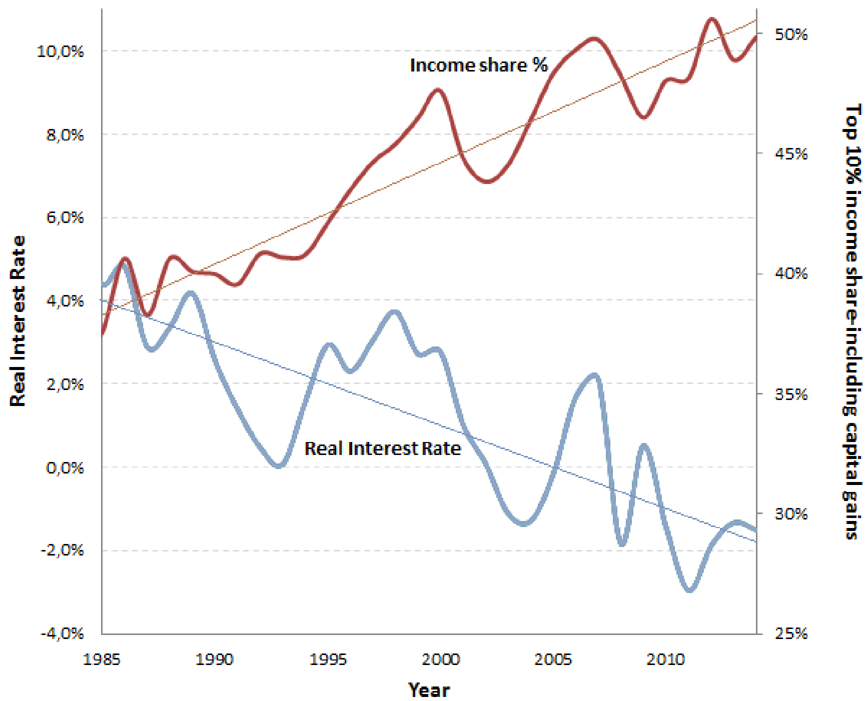
In the period between 1985 and 2005 the real interest rates in US have fallen from 4.4% to −0.2%, while the share of the wealthier population decile increased 10 percentage points, from 38% to 48% (Figure: 3)[10]. How much of the real interest rate reduction of −4.6% during that period is our model able to explain, is the question we try to answer next.

We start by verifying how much of the effective reduction of the real interest rate during the observation period is explained by a base case calibration of our model due to an increase of income inequality. We then test the robustness of the real interest rate reduction to calibration changes of relevant parameters.

The model used for calibration is the *warm glow* type version where generosity endogenously increases with agents lifetime income. The endogenous bequest motive is modeled by considering descendants expected wealth in agents preference function, rather than only the bequest itself. Expected wealth being equal to the sum of descendants expected lifetime income and bequest received from parents. It is assumed that labor endowment types of agents and their descendants are independent, so that expected income of descendants of the wealthier and poorer is the same, and equal to expected average income of all households of their generation. Agents expecting wealthier descendants than themselves have lower incentives to leave bequests, and would consequently save relatively less than wealthier agents that expect their descendants to be relatively poorer, and would then save more. We also

[10]Source: The World Wealth and Income Database - WID (Piketty); Fred

Figure 3: Real Interest Rates and Income Inequality in US



Source: The World Wealth and Income Database - WID (Piketty); Fred

introduce capital in the model as in Eggertsson and Mehrotra [12] [11], and allow for a CRRA utility function where the elasticity of inter-temporal substitution may be different than one. A steady state equilibrium for this economy is defined with the requirement that average total bequests received by middle-age household during one period is equal to the average total bequests left by the old during the next period.

The calibration of the benchmark model is described in Table 1, with usual annual parameters calibrated according to recent literature, namely the discount rate β[12], the elasticity of inter-temporal substitution $\frac{1}{\sigma}$, depreciation rate δ[13], and the loan collateral concavity μ. In the model, the discount rate β and the depreciation rate δ are adjusted for a period of 20 years. We match initial and final top decile income shares by adequately calibrating labor endowments of the wealthier and poorer at initial and final steady states. We then calibrate the discount factor β combined with the bequest discount rate (or selfish parameter ϕ), to

[11] Model with capital, a *warm glow* type bequest motive, and a CRRA utility function described in Appendix A

[12] β is calibrated at 0.571 for a period of 20 years, corresponding to 0.972 for a period of one year.

[13] δ is calibrated at 0.88 for a period of 20 years, corresponding to 0.10 for a period of one year.

Table 2: Parameters and Simulation Results

Description	Parameter	Value
Calibration:		
Discount rate	β	0.97
Intertemporal substitution	$\frac{1}{\sigma}$	0.5
Descendants wealth discount rate	ϕ	1.01
Collateral concavity ($D = \theta Y_i^{\mu}$)	μ	0.50
Depreciation rate (year)	δ	0.1
Population growth	g	0.7%
Low income type population share	η	90%
Average labor endowment	L	1
Low income type labor endowment	L^l	0.69
High income type labor endowment	L^h	3.80
Matching:		
High type population share	$1 - \eta$	10%
High type income share 1985	$\frac{(1-\eta)Y^h_{ini}}{Y}$	38%
High type income share 2005	$\frac{(1-\eta)Y^h_{ss}}{Y}$	48%
Mean initial estate size/lifetime income	Q/Y	3.6%
Debt to income ratio	D/Y	0.36
Capital to annual income ratio	k/y	3.0
Initial real interest rate	r_{ini}	4.4%
Results:		
Final real interest rate - model	r_{ss}	3.4%
real interest rate change - model	$\triangle_{mod} r$	-1.0%
real interest rate change - observed	$\triangle^{85-05}_{obs} r$	-4.6%

match the initial steady state values of the real interest rate, and average estate level as a proxy of the bequest level cap (Hendricks [17]), with the observed values in the beginning of the observation period. The collateral multiplier θ, is calibrated to obtain an initial reasonable debt limit to income ratio $\frac{D}{Y}$, after setting the collateral concavity.

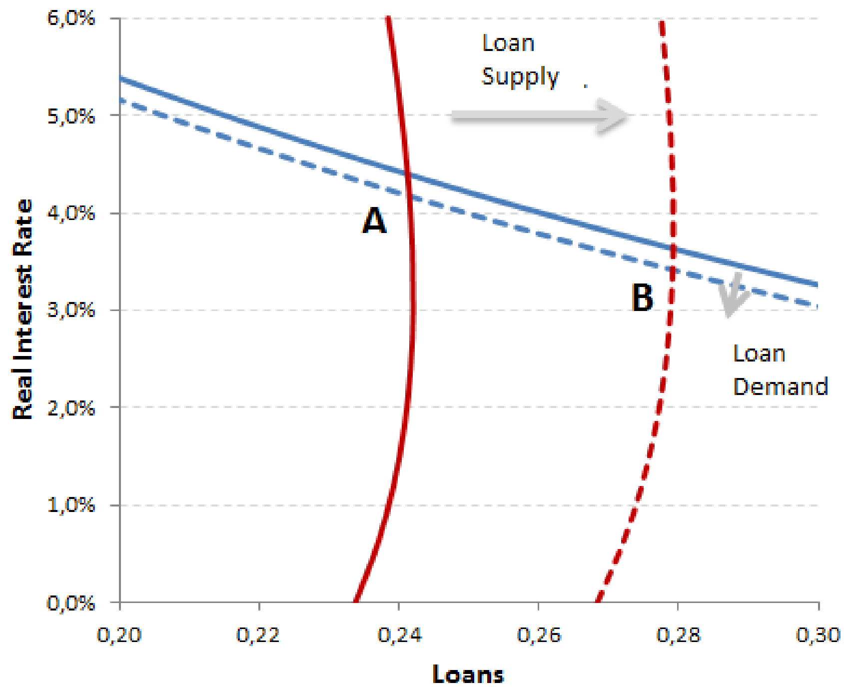
In an OLG model with three periods of 20 years each, the implied labor share of the Cobb-Douglas production function α_{20}, should be greater than the typical approximate value of two thirds. By setting $\alpha_{20} = 0.86 > \frac{2}{3}$, the capital to annual income ratio in the initial steady state is set to 3.0.

The benchmark model is able to explain around 22% of the effective real interest rate reduction during the observation period: An increase of the top decile income share from 38% to 48% would lead to a real interest rate reduction of 1.0%, from 4.4% to 3.4%, in contrast with an observed total reduction of 4.6%, from 4.4% to -0.2%. Low income agents do not leave bequests to their descendants in the initial and final steady states, which is consistent with recent literature (Benhabib et al. [4]). But high income types increase the level of bequests left at the end of their lives. The marginal increase in savings of high types, due to an increase of income inequality, prevails over the savings contraction of low types, resulting into a net expansion of loan supply, and a consequent reduction of the *natural rate of interest* (figure 4). Loan demand contracts because of the concavity of the borrowing limit.

We test the robustness of our results by changing some parameters one by one to commonly used values (Table: 2), and by checking the impact on the reduction level of the natural rate of interest, although ensuring the initial real interest rate and average estate values match observations in the beginning of the period. The change magnitude of the natural rate of interest due to an increase of 10 percentage points of top decile income share is generally robust to other parameter changes, in particular for significantly different initial bequest to income ratio levels.

Although the increase of income inequality between 1985 and 2005 only seems to account, according to our model, for around 22% of the reduction of the real interest rate during that period, it is nevertheless not an insignificant value. There are other relevant factors that certainly may explain the difference, namely age structure changes in population, like life expectancy and the retirement age (**?**]), or debt deleveraging, a birth rate slow-down, or a reduction of the price of investment (Eggertsson and Mehrotra [12]). Combining all those factors in a single multi-generations model in order to better understand the relative contributions of each of those factors in a consistent way is part of our work going forward.

Figure 4: Equilibrium in the Loan Market



Source: The World Wealth and Income Database - WID; FRED

6 Final remarks

In this paper, we formalize the relation between increasing income inequality and low real interest rates using an overlapping generations model with borrowing constraints and a bequest motive. The underlying mechanism in our model relating real interest rates and inequality is based on empirical evidence in recent literature that households' marginal borrowing and saving rates are respectively negative and positive functions of income, so that the net effect on aggregate borrowing and savings of a permanent increase of income inequality is respectively a net contraction and a net expansion, that may lead to a persistent reduction of the *natural rate of interest*.

In particular, the borrowing mechanism in our model is based on the concavity of the marginal propensity to borrow, and on binding borrowing constraints, both consistent with empirical observations in recent literature[1][18]. The saving mechanism is illustrated with an endogenous propensity of households to be more generous with respect to their children by leaving them greater bequests, when they are expected to be relatively poorer. In the opposite direction, if agents expect their direct descendants to be much wealthier than them, then the bequest motive may become inoperative. So that wealthier households are more

Table 3: Robustness analysis

INPUTS Description	parameter	benchmark	sensibility	OUTPUTS $\triangle r$
$\triangle$Real Int. Rate	$\triangle r$	-0.98%		-0.98%
EIS	$\frac{1}{\sigma}$	0.5	1.0	-0.64%
Depreciation (year)	δ	0.1	0.2	-1.02%
Population growth	g	0.7%	0.0%	-1.04%
Collateral multilier	θ	0.45	0.35	-1.04%
Collateral concavity	μ	0.5	0.0	-0.67%
Bequest/lifetime income	$\frac{Q}{Y}$	$3,6\%$	$[1.0\%; 10.0\%]$	$[-0.91\%; -1.13\%]$

generous, which makes their marginal savings rate higher than the poorer households that leave lower or no bequests at all, as also observed by Hendricks [17]. In this model the savings channel through which inequality affects the real interest rate is then the bequest motive, that is endogenously turned on or off respectively for the rich and the poor, making the marginal savings rate of the rich greater than the poor, so that an increase of inequality triggers a net expansion of aggregate savings that drags down the *natural rate of interest*.

Our main contribution in this paper is to present an explicit formalism linking low real interest rates with increasing income inequality, by gathering and building on some relevant topics in recent literature, namely increasing income inequality[19], bequests[2][5], the question of whether higher-lifetime income levels lead to higher marginal propensity to save[11] and lower marginal propensity to borrow[1][18], and *secular stagnation*[12][20].

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A Endogenous Output and Capital

Here we derive the model with endogenous output and capital, its equilibrium conditions, as well as operative bequest conditions for each income type. We assume that only the middle age supply labor and capital to competitive firms that take wages and rental capital rates as given, and maximize profits subject to a standard Cobb-Douglas production function:

$$Z_t = \max_{L_t, K_t} Y_t - w_t L_t - r_t^k K_t \tag{A.1}$$

$$\text{s.t.} Y_t = A_t K_t^{1-\alpha} L_t^{\alpha} \tag{A.2}$$

Firms labor and capital demand are given by:

$$L_t = \alpha \frac{Y_t}{w_t} \tag{A.3}$$

$$K_t = (1-\alpha) \frac{Y_t}{r_t^k} \tag{A.4}$$

From where $Z_t = 0$, and output supply can be expressed by $Y_t = w_t L_t + r_t^k K_t$. The Objective function and budget constraints of household i are given by:

$$\max_{C_t^y(i), C_{t+1}^m(i), C_{t+2}^o(i), W_{t+2}^m(i)} \mathbb{E}_t \left\{ U(C_t^y(i)) + \beta U(C_{t+1}^m(i)) + \beta^2 \left[U(C_{t+2}^o(i)) + \frac{U(W_{t+2}^m(i))}{1+\phi} \right] \right\} \tag{A.5}$$

$$\text{s.t. } C_t^y(i) = B_t^y(i) \tag{A.6}$$

$$C_{t+1}^m(i) = Y_{t+1}^m(i) - (1+r_t) B_t^y(i) + B_{t+1}^m(i) - K_{t+1}(i) + \frac{Q_{t+1}^o(j)}{1+g_t} \tag{A.7}$$

$$C_{t+2}^o(i) = -(1+r_{t+1}) B_{t+1}^m(i) + (1-\delta) K_{t+1}(i) - Q_{t+2}^o(i) \tag{A.8}$$

$$(1+r_t) B_t^y(i) \leq D_t^y(i), \text{ where } D_t^y(i) = \theta Y_{t+1}^m(i)^{\mu} \tag{A.9}$$

$$Q_{t+1}^o(j) \geq 0, \text{ where } (j) \text{ represents household's (i) parents.} \tag{A.10}$$

where $U(C_t) = \frac{C_t^{1-\sigma}-1}{1-\sigma}$, and $Y_t^m(i) = w_t L_t^m(i) + r_t^k K_t(i)$. $W_{t+2}^m = Y_{t+2}^m + \frac{Q_{t+2}^o}{1+g_{t+2}}$, where $Y_{t+2}^m(i)$ is household i children expected income. From the first order conditions os W_{t+2}^m and C_{t+2}^o we derive an expression for bequests:

$$W_{t+2}^m = \frac{C_{t+2}^o}{(1+\phi)^{\frac{1}{\sigma}}} \Leftrightarrow \tag{A.11}$$

$$Q_{t+2}^o = (1+g_{t+1}) \left[\frac{C_{t+2}^o}{(1+\phi)^{\frac{1}{\sigma}}} - Y_{t+2}^m \right] \tag{A.12}$$

The consumption Euler equation is given by[14]:

$$\mathbb{E}_t \frac{C^o_{t+1}}{1+r_t} = \beta_{r_t} C^m_t \tag{A.13}$$

where,

$$\beta_{r_t} = \beta^{\frac{1}{\sigma}} (1+r_t)^{\frac{1-\sigma}{\sigma}} \overset{(\sigma=1)}{=} \beta \tag{A.14}$$

Using the production function and the first order conditions for K_{t+1} we derive the following expressions for w_t and r^k_t:

$$w_t = \alpha A_t^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{r^k_t} \right)^{\frac{1-\alpha}{\alpha}} \tag{A.15}$$

$$r^k_t = \frac{r+\delta}{1+r} \tag{A.16}$$

In the case of the benchmark model we assume that children and parents labor endowment types are independent. Then for all household i we assume that, $Y^m_{t+2}(i) \equiv \bar{Y}^m_{t+2} = \frac{w_{t+2}}{\alpha} \bar{L}^m_{t+2}$. Using the expressions above we can derive the following expression for bequest if strictly positive, and equal to zero otherwise:

$$Q^o_{t+1}(i) = (1+r_t)\Psi_t \left[\alpha Y^m_t(i) - D_{t-1}(i) + \frac{Q^o_t(j)}{1+g_{t-1}} \right] - (1-\Psi_t)(1+g_t)\bar{Y}^m_{t+1} \tag{A.17}$$

where the constant $\Psi_t = \frac{\beta_{r_t}}{\beta_{r_t} + \frac{(1+\phi)^{\frac{1}{\sigma}}(1+\beta_{r_t})}{1+g_{t+1}}} \in]0;1[$ for $\phi \in]-1;+\infty[$.

We introduce income inequality in the model by assuming that there are two exogenous labor endowment types L^γ for the middle-aged, where $\gamma \in \{low, high\} \equiv \{l, h\}$. Loan supply per middle age household has then the following expression:

$$L^s_t = \eta L^{s,l}_t + (1-\eta) L^{s,h}_t \tag{A.18}$$

$$L^{s,\gamma}_t = -\bar{B}^{m,\gamma}_t \tag{A.19}$$

$$-B^{m,\gamma}_t(i) = \frac{\beta_{r_t}}{1+\beta_{r_t}} \left[Y^{m,\gamma}_t(i) - D^{y,\gamma}_{t-1}(i) + \frac{Q^o_t(j)}{1+g_{t-1}} - K^\gamma_t(i) \left(\frac{1-r^k_t}{\beta_{r_t}} + 1 \right) \right] + \frac{Q^{o,\gamma}_{t+1}(i)}{(1+\beta_{r_t})(1+r_t)} \tag{A.20}$$

[14] We can use this expression because the model is deterministic.

The marginal savings rates with an inoperative and operative bequest motive are given by:

$$\frac{\partial L_t^s}{\partial Y_t^m} = \frac{\beta_{r_t}}{1+\beta_{r_t}}\left[\alpha - (1-\alpha)\frac{1-\delta}{\beta_{r_t}(1+r_t)}\right] \tag{A.21}$$

$$\frac{\partial L_t^{s,Q}}{\partial Y_t^m} = \frac{\beta_{r_t}}{1+\beta_{r_t}}\left[\alpha - (1-\alpha)\frac{1-\delta}{\beta_{r_t}(1+r_t)}\right] + \frac{\alpha\Psi_t}{1+\beta_{r_t}} \tag{A.22}$$

If all high types leave a bequest to their descendants, and all low types leave none loan supply aggregation in steady state is straight forward, and the difference of marginal savings rate between high and low types is given by:

$$\triangle MSR_t^{hl} = \frac{\alpha\Psi_t}{1+\beta_{r_t}} > 0 \tag{A.23}$$

Although we do not enforce this mechanism we nevertheless present the conditions its verification. Given the bequest expression (A.17) it is possible to derive the value for the bequest received by one agent in period t when middle age, above which the bequest motive is operative for that agent during the next period:

$$Q_{min,t}^{m,\gamma} = \frac{1+\beta_{r_t}}{\beta_{r_t}}\frac{(1+\phi)^{\frac{1}{\sigma}}}{1+r_t}\bar{Y}_{t+1}^m - (\alpha Y_t^{m,\gamma} - D_{t-1}^{\gamma}) \tag{A.24}$$

Received bequests by any household have a maximum value which is the steady state bequest of a high agent type where all its ascendants where of the high type too. This expression is given by:

$$Q_{max}^m = \frac{1}{\frac{1+g}{\Psi(1+r)} - 1}\left[\left(\alpha Y^h - D^h\right) - \frac{1+\beta_r}{\beta_r}\frac{(1+\phi)^{\frac{1}{\sigma}}}{1+r}\bar{Y}^m\right] \tag{A.25}$$

In steady state, if $Q_{min,t}^{m,l} > Q_{max}^m$ then the bequest motive of all low income types is always inoperative, although they may receive some from their parents of high type. Moreover, if $Q_{min,t}^{m,h} < 0$ then all high type agents leave a bequest to their children independently of having received a bequest from their parents. Those two conditions are verified for any reasonable calibration of our model.

In this case the average steady state expressions for bequests of each type are given by:

$$\bar{Q}^{o,h} = \frac{1}{\frac{1}{\Psi(1+r)} - \frac{1-\eta}{1+g}}\left[\left(\alpha Y^h - D^h\right) - \frac{1+\beta_r}{\beta_r}\frac{(1+\phi)^{\frac{1}{\sigma}}}{1+r}\bar{Y}^m\right] \tag{A.26}$$

$$Q^{o,l} = 0 \tag{A.27}$$

$$\bar{Q}^o = (1-\eta)Q^{o,h} \tag{A.28}$$

B Intergenerations Utility of Consumption

We now consider the utility of children in the preference function. Utility is maximized subject to the same budget constraints as in previous sectins, (11), (12), (13), (14) and (15), for households born at time t, and $t+1$:

$$\max_{C_t^y, C_{t+1}^m, C_{t+2}^o, Q_{t+2}} \mathbb{E}_t \left\{ v_t + \frac{\beta}{1+\phi} v_{t+1} \right\} \tag{B.1}$$

$$\text{where } v_t = \log(C_t^y) + \beta \log(C_{t+1}^m) + \beta^2 \log(C_{t+2}^o) \tag{B.2}$$

Assuming that the bequest zero lower bound is not binding, from FOC Q_{t+2} we derive the following expression relating middle age consumption of two consecutive generations:

$$\mathbb{E}_t \frac{C_{t+1}^m}{C_t^m} = \frac{\beta}{1+\phi} \frac{1+r_t}{1+g_{t+1}} = \frac{1+r_t}{1+r_t^b} \tag{B.3}$$

Where $1 + r_t^b = (1+\phi)\frac{1+g_t}{\beta}$ .Then, in steady state, the equilibrium real interest rate with an operative bequest motive is given by:

$$1 + r = 1 + r^b = (1+\phi)\frac{1+g}{\beta} \tag{B.4}$$

From loan market equilibrium we derive a general expression for the equilibrium real interest rate in steady state, which is strictly lower than r_n when the bequest motive is operative, and $Q > 0$:

$$1 + r = \frac{1+\beta}{\beta} \frac{(1+g)D}{(Y^m - D) + \left(\frac{1}{1+g} + \frac{1}{\beta(1+r)}\right) Q} = 1 + r^b < 1 + r^n \tag{B.5}$$

In that case the bequest in steady state is given by:

$$Q = \left(\frac{\beta}{2+\phi}\right)(r^n - r^b)(Y^m - D) \tag{B.6}$$

Consequently, in steady state, the bequest motive is operative if and only if the no-bequest natural rate of interest $r^n = \frac{1+\beta}{\beta}\frac{(1+g)D}{(Y^m - D)}$ is greater than $r^b = (1+\phi)\frac{1+g}{\beta}$. Otherwise, when $r^n \leq r^b \Rightarrow Q = 0$ and $r = r^n$. Then, if the steady state no-bequest natural rate of interest r^n as in Eggertsson and Mehrotra [12] decreases below the threshold r^b the bequest motive of households become inoperative.

Inequality and low interest rates

We now consider two household income types h and l as in previous sections, and assume that average income is constant, so that $Y^m = \eta_s Y_t^{m,l} + (1-\eta_s) Y_t^{m,h}$. We further assume that parents born at time $t-1$ receive no bequests at time t, and children don't leave bequests to future generations at time $t+2$. From the budget constraints we derive the following expressions for middle age consumption at time t and $t+1$ for parent's and children of types i and j respectively:

$$(1+\beta)C_t^{m,i} = Y_t^{m,i} - D_{t-1} - \frac{Q_{t+1}^i}{1+r_t} \tag{B.7}$$

$$(1+\beta)C_{t+1}^{m,j} = Y_{t+1}^{m,j} - D_t + \frac{Q_{t+1}^i}{1+g_{t+1}} \tag{B.8}$$

Combining equation (B.3) with expressions(B.7),(B.8), Q_{t+1}^i is given by:

$$Q_{t+1}^i = \gamma \left[(1+r_t)(Y_t^{m,i} - D_{t-1}) - (1+r_t^b)(Y_{t+1}^{m,j} - D_t)\right] \tag{B.9}$$

where $\gamma = \frac{1}{1+\frac{1+\phi}{\beta}}$. Loan supply for household type i when the bequest motive is operative has the following expression:

$$L_t^{s,i} = \frac{\beta}{1+\beta}\left[(Y_t^{m,i} - D_{t-1})\left(1+\frac{\gamma}{\beta}\right) - \frac{\gamma}{\beta}\frac{1+r_t^b}{1+r_t}(Y_{t+1}^{m,j} - D_t)\right] \tag{B.10}$$

From where we can directly state that loan supply of an household expands if the descendants are expected to become poorer, or the agent becomes richer. The bequest motive is operative, $Q_{t+1}^i > 0$, if:

$$(1+r_t^b)\frac{Y_{t+1}^{m,j} - D_t}{Y_t^{m,i} - D_{t-1}} = 1 + r_t^{ij} < 1 + r_t \tag{B.11}$$

Note that $r_t^{ij} \leq r_t^{lh}$, so that when $r_t > r_t^{lh}$ bequest motives of all agents in this economy are operative. As long as this relation persists, aggregate loan supply and equilibrium real interest rate are not affected by a change in inequality as we can observe from expressions below:

$$L_t^s = \frac{\beta}{1+\beta}\left[(Y^m - D_{t-1})\left(1+\frac{\gamma}{\beta}\right) - \frac{\gamma}{\beta}\frac{1+r_t^b}{1+r_t}(Y^m - D_t)\right] \tag{B.12}$$

The corresponding equilibrium real interest is given by:

$$1 + r_t = \frac{1+\beta}{\beta}\frac{(1+g_t)D_t + \frac{\gamma(1+r_t^b)}{1+\beta}(Y^m - D_t)}{(Y^m - D_{t-1})\left(1+\frac{\gamma}{\beta}\right)} = 1 + r_t^* \tag{B.13}$$

Then,

$$\forall\{i;j\}, Q_t^i > 0 \text{ if } \frac{Y_{t+1}^{m,j} - D_t}{Y_t^{m,i} - D_{t-1}} < \frac{1+r_t^*}{1+r_t^b} \tag{B.14}$$

If children inherit their parents type, $Y_{t+1}^{m,j} = Y_t^{m,i}$, and we assume $D_t = D_{t-1} = D$ then $r_t^{ll} = r_t^{hh} = r_t^b$, the zero lower bound bequest threshold is the same for both types, and the equilibrium real interest rate $r_t = \min(r_t^n, r_t^*)$, is always unaffected by changes in income inequality.

Otherwise, if children and parents types are *iid*, and parents cannot predict their children type then the problem from the parent perspective is the same as the one previously specified, using an *average* income type for children with an endowment equal to the constant weighed average population endowment Y^m. Bequests are positive for both types if

$$\forall i Q_t^i > 0 \text{ if } \frac{Y^m - D_t}{Y_t^{m,i} - D_{t-1}} < \frac{1+r_t^*}{1+r_t^b} \Leftrightarrow \tag{B.15}$$

$$Y_t^{m,i} > Y_t^* = \frac{1+r_t^b}{1+r_t^*}(Y^m - D_t) + D_{t-1} \tag{B.16}$$

An increase of income inequality not affecting Y^m does not affect aggregate loan supply, demand, and the equilibrium real interest rate, as long as all endowments remain above the threshold Y_t^*.

If income inequality increases at time t so that the bequest zero lower bound for low income type becomes binding, or $Y_t^{m,l} < Y_t^* \Rightarrow Q^l = 0$ and $Y_t^{m,h} > Y_t^*$, then loan supply expressions for each household type are given by:

$$L_t^{s,l} = \frac{\beta}{1+\beta}(Y_t^{m,l} - D_{t-1}) \tag{B.17}$$

$$L_t^{s,h} = \frac{\beta}{1+\beta}\left[(Y_t^{m,h} - D_{t-1})\left(1+\frac{\gamma}{\beta}\right) - \frac{\gamma}{\beta}\frac{1+r_t^b}{1+r_t}(Y^m - D_t)\right] \tag{B.18}$$

Aggregate Loan supply becomes a positive function of the high type income, and expands with inequality increases:

$$L_t^s = \frac{\beta}{1+\beta}(Y^m - D_{t-1}) + (1-\eta_s)\frac{\gamma}{1+\beta}\left[(Y_t^{m,h} - D_{t-1}) - \frac{1+r_t^b}{1+r_t}(Y^m - D_t)\right] \tag{B.19}$$

The natural rate of interest, derived from equilibrium in the loan market, becomes a decreasing function of the high type income:

$$1 + r_t = \frac{1+\beta}{\beta} \frac{(1+g_t)D_t + (1-\eta_s)\frac{\gamma(1+r_t^b)}{1+\beta}(Y^m - D_t)}{(Y^m - D_{t-1}) + (1-\eta_s)\frac{\gamma}{\beta}\left(Y_t^{m,h} - D_{t-1}\right)} < 1 + r_t^* \tag{B.20}$$

When inequality increases above a certain level the bequest zero lower bound of poor becomes binding, and their marginal savings with respect to income decreases relative to the rich. Then the net effect on aggregate loan supply of an increase of income inequality will be positive, and negative on the equilibrium real interest rate. The mechanism is the same as the one presented in previous sections, but with a different preference function.