The Faustmann model under storm risk and price uncertainty: A case study of European beech in Northwestern France

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1 Introduction

French forest managers are deeply concerned by storm risks and price variations. For example, a recent national survey mentioned that storms are one the risks most feared by French private forest managers (RESOFOP, 2011). Their fears are justified by the number of significant storms experienced during this century. Gardiner et al. (2013) counted 130 storms over the last sixty years in Europe, that is an average of two per year. In particular, the storm Klaus in January 2009 induced an average cost of 1.5 billion euros on maritime pine in the South-West of France and damage from Lothar and Martin in December 1999 was estimated at about 6 billion euros for the whole of France (Peyron et al., 2009).

European beech (*Fagus Sylvatica*) represents 9 % of the total forest surface in France (IFN, 2012) and is the second hardwood species after oak. A decrease in timber prices for this species has heavily affected the entire French forest sector over the last 20 years, but the *Office National des Forêts* (ONF), which manages public forests in France, has especially suffered. Beech sales represented approximately 25.5 % of the ONF annual turnover in 1999 when beech prices were high against 12 % in 2015; this equates to a total accumulated loss of approximately one billion euros during the last sixteen years and an annual average loss of 62 million euros (ONF, 2016).

Faustmann (1849) proposed a simple deterministic economic model for evaluating the Land Expectation Value (LEV) over an infinite sequence of rotation. One of the fundamental assumptions of this model is that timber prices are constant over time and known with certainty. Costs and volume production are also considered to be fixed and without risk. This model makes it possible to calculate the optimal rotation age but does not treat the two above-mentioned concerns: fluctuating prices and natural risk. Reed (1984) introduced fire risk (into the equation) for optimal forest rotation and showed that taking risk into account is equivalent to increasing the discount rate. In Reed (1984), the tree damage is not modelled and thinnings are not considered. In the case of a storm, its occurrence depends on a probability density function but the intensity of damage is closely linked with the stand characteristics such as height and diameter (Loisel, 2014).

Concerning timber price, previous economic research has shown that it is an extremely variable parameter. The sources of this considerable variation in timber price have already been identified: time and space (Mitlin, 1987; Valsta, 1994), variations in supply and demand (Caurla et al., 2010), climatic policies (Buongiorno et al., 2011; Lecocq et al., 2011) and wood quality and characteristics (Cavaignac et al., 2006; Heshmatol Vaezin et al., 2008). However, several researchers have examined the effects of price uncertainty within the framework of a forest economic model like Faustmann's. From our point of view, timber price variation merits deeper research because it determines the profitability of forest management and is the main incentive to convince forest managers to reinvest in silviculture. Studying multiple risks inside economic models can also provide realistic assumptions concerning the value of adaptive policies in a context of climate change. As shown by Guo and Costello (2013), timber price variation is one of the most sensitive parameters when comparing climatic adaptation strategies such as reducing rotation age or changing species.

In the present work, our objective is to integrate multiple risks into the Faustmann model. In the first section, we review the literature focused on price uncertainty inside forest economic models similar to Faustmann's. In the second section, we present different models and the data we used to calculate the optimal Faustmann rotation with risks: economic criteria, storm risks, a storm damage function, price models and a growth model. In the third section, an empirical simulation on beech stands in northwestern France is presented in order to compare the situation with and without risks in two different conjunctures: high price values and low price values for beech. Finally, we discuss our results in terms of current forest management policies.

2 Review of the literature on price uncertainty

Due to some economic and political shocks experienced by the forest sector, some authors have begun to extend Faustmann's deterministic model in order to include the impact of price uncertainty. In this section, we review the models in each of three methodological families. We will see that conclusions diverge.

2.1 Discrete-time price processes

These models are based on the assumption that, due to price uncertainty, the best forest management strategy is to adapt harvesting decisions depending on the current price market. Results of optimization concern not only the LEV and the final rotation age but also a threshold price below which the forest manager decides to postpone the final harvest. This is known as the *Reservation Price Strategy* (RPS) or the "*flexible harvest policy*". Bhattacharyya and Snyder (1987) and Lohmander (1987) were among the first economists to integrate price uncertainty into the Faustmann model. They calculated two probabilities: the probability of a fixed price occurring year after year and the probability that a standing forest will survive from one year to the next. The first probability was calculated from price data. Survival probability depended on the price probability of the previous period and the reservation price was calculated by using a recursive simulation. Brazee and Mendelsohn (1988) conducted numerical simulations by using historical prices for Lobolly pine between 1970-1979 and Douglas fir between 1975-1984 in the US. They established that the reservation price decreases with standing age as forest growth slows down over time. They also found that the use of RPS increases forest value and optimal rotation age compared to the deterministic Faustmann model. Forboseh et al. (1996) pointed out that separately considering the reservation price of sawtimber and pulpwood can give the forest manager more flexibility . They noticed that variations in pulpwood prices had a more significant effect on the LEV than variations in sawtimber prices. Reeves and Haight (2000) found that a second-order autoregressive model was significant for monthly sawtimber prices and a first-order autoregressive model for monthly pulpwood prices. They showed that price uncertainty induces important effects on income and rotation age. Lu and Gong (2007) used the RPS with a discrete growth forest model which included thinning. In simulations for Scots pine in Northern Sweden, their results agreed with the majority of the previous RPS results: i.e. that incorporating flexibility due to price uncertainty increases the economic benefits. They pointed out the importance of taking thinning into account in the economic model. Lohmander (2007) argued that the RPS is particularly helpful to obey the constraints faced in the actively changing world such as the modification of harvesting capacity or the climate changes.

The RPS models create a net gain in LEV of between 8 % and 80 % depending on tree species, site quality, discount rate and degree of uncertainty (Gong and Löfgren, 2007). However, these models were criticized by

Gong and Löfgren (2007) who showed that the RPS has a tendency to reduce the timber supply and is only correct for short-run price variations. In the long term, this strategy generally has a low impact on forest NPV (Net Present Value: present value of an investment's expected cash inflow minus the cost of acquiring the investment) and no influence on consumer surplus. Another drawback of the RPS is that analytical causes and the process of timber price variation are not clearly analyzed and then, solutions for forest managers are difficult to characterize.

2.2 Continuous-time price processes

With these models, temporal variations in price are broken down into a continuous trend, a structural change and a random variation. The main difference compared with the RPS models is that timber price is assumed to be exogenous to the decision to harvest. Different functional forms are used in the literature.

The Wiener process or the geometric Brownian motion (GBM) is a common continuous-time price process to describe the dynamics of timber price. In this model, variations in price are broken down into two components: i) the trend, or drift rate, which describes the long-term change in price; and ii) an instantaneous deviation which follows a geometric Brownian motion. There are different extensions of this functional form. Thomson (1992) used a binomial probability for price and showed that the optimal rotation age increases compared with deterministic models. However, with low prices, he condoned converting land to another activity, and in that case, the optimal harvesting age is reduced. In Japan, Yoshimoto and Shoji (1992) and Yoshimoto (2002) used a Bernouilli probability for *Pinus densiflora*, *Cryptomeria japonica* and *Chamaecyparis obtuse* whose prices decreased between 1975 and 1998. They found that the optimal rotation age was delayed when the price level was crucially low. Navarrete (2012) analyzed the simultaneous impact of a diffusion process on stock and price functions for P*inus radiata* in Chile between 1985 and 2007. He showed that price drift rate can be assimilated to a decrease in the discount rate if the price drifts upwards. His results also revealed that price and wood stock volatility significantly increases physical stock but seems to have little influence on NPV and LEV compared with deterministic models.

The main advantage of using continuous-time price process model is linked to the possibility to completely characterize the optimal harvesting policy as a function of the discount rate and the drift of the price process.

Lohmander (2007) remarked however that these models are not relevant for forest sector as they are based on an assumption that price dynamics will definitely follow the same process over the rotation, ignoring the presence of economic, biological or climate shocks.

Other authors used price process models with fixed growth rates and structural changes whichare intermediate between discrete and continuous-time models. Newman et al.(1985) found that an exponentially increasing price model induced a 1 % to 3 % increase in the optimal rotation age compared with the Faustmann model. Sandhu and Philips (1991) showed that there are three types of changes in price: i) an exponentially increasing price such as Newman et. al (1985); ii) a progressive change with tree age but without any structural change under elastic demand; and iii) a progressive change with tree age mixed with a structural change under inelastic demand. They concluded that in both the first and second types, optimal rotation age decreases while it increases in the third type, when compared to the Faustmann model.

2.3 Price models with other risks

Holopainen et al. (2010) compared the sources of uncertainty inside the NPV with a GBM approach to price, inventory data errors and growth models for Scot pine, Norway spruce and birch in Finland. They concluded that the individual effect of timber price variation is not as important as errors in growth models in the LEV estimates.

Knoke and Wurm (2006) used a Monte-Carlo Simulation to combine the risk of fluctuating timber prices with natural risks (insect, snow and wind damage) on a mixed forest of Norway spruce and European beech in Germany and to calculate the NPV and the optimal rotation age. They showed that a flexible harvest policy increases the optimal proportions of European beech, compared with deterministic models. This result is due to the frequency of natural hazard risks for Norway spruce. However, European beech has a greater risk of negative NPV than does Norway spruce. The question of risk can also be linked to the risk neutrality policy of forest managers. Tahvonen and Kallio (2006) analyzed the utility function of forest managers who decide on the age classes of their forests They used a mean-reverting price function where harvest depended on price. With an assumption of risk neutrality, they found that price uncertainty lengthened the optimal rotation period. On the other hand, for forest owners who had small stands with diversified age classes and took into account time preference or health, or had non-forest financial assets, price uncertainty shortened the optimal rotation age.

3 Materials and methods

We explored LEV under multiple risks while respecting Faustmann's (1849) main assumption that the economic value of the next rotations be calculated with the same parameters as the first one. At each stand age, we studied the variations in price depending on wood characteristics, economic context and storm risk.

3.1 Models without storm risk

3.1.1 Timber price function

In a standing forest, we assumed a timber price function P_0 such that:

$$P_0 = P_0(s, q, a, E) \tag{1}$$

where s is the tree-basal area which gives the diameter of a tree measured at breast height (1.30 m from the ground), q is timber quality (form, presence of moisture, color …) which is a constant parameter over time, a is the year of sale, and E is the general economic context describing industrial demand, possible economic crisis, international trade levels, etc. We made an original assumption that, as for other authors seen in previous literature review, timber prices depend closely not only on time of sale and economic context but also on timber characteristic (diameter and quality) and then, indirectly on forest management.

3.1.2 Growth model

As did Loisel (2011), we considered an even-aged growth model at stand level where the stated variables were average height H, per hectare tree-density n and tree-basal area s measured at breast height (1.30 m from the ground). The average height H is given by a function of age t and the tree-basal area s is based on a differential equation.

$$H(t) = f(t) \tag{2}$$

$$\frac{ds}{dt} = U\big(H'(t)\big)\frac{g(n(t),s(t))}{n(t)} \tag{3}$$

Equation (2) describes a classic sigmoid growth curve with current stand age *t* and site fertility index, therefore $f'(t) > 0$. Equation (3) gives the growth in basal area of a tree which depends on maximum potential growth $U(.,.)$ and on a growth reduction function $g(.,.)$. Maximum potential growth is measured through height $H(t)$, which is an indicator of site fertility with $U'_H(H(t),t) \geq 0$. Due to density dependence, growth reduction decreases with the number of trees in a stand, giving $g'_n(n(t),s(t)) \leq 0$. Tree diameter is $d = \sqrt{4s/\pi}$ and tree stem volume is noted $\Upsilon(H(t),s(t))$.

At each thinning date u_k with a thinning rate h_k, the tree-density after thinning is:

$$n(u_k) = (1 - h_k)n(u_{k-1}) \tag{4}$$

The total harvested basal area at each thinning date u_k is calculated as follows:

$$X(u_k) = h_k n(u_{k-1}) s(u_k) \tag{5}$$

The total basal area stock $S(t)$at stand level is $S(t) = n(t)s(t)$.

3.1.3 The Faustmann value without risk

For a given cutting age *T* and a discount rate δ, the Faustmann value W_0 of a stand, taking into account thinning income, is:

$$W_0 = -c_0 - \frac{c_y}{\delta} + \sum_{i=1}^{+\infty}(I_0(0,T) - c_0)e^{-i\delta T} = -c_0 - \frac{c_y}{\delta} + \frac{I_0(0,T) - c_0}{e^{\delta T} - 1} \tag{6}$$

where c_0 is the initial cost of regeneration (which represents the forest investment), c_y is the per year annual cost and I_0 is the total income on $[0,T]$. The total income I_0 is composed of the sum of thinning incomes $\mathcal{H}_0(0,T)$ and the final income $V(T)$ minus the silvicultural costs $\daleth(0,T)$ summed on $[0,T]$ and actualized at time *T*.

$$I_0(0,T) = \mathcal{H}_0(0,T) + V(T) - \daleth(0,T) \tag{7}$$

Let $\mathbf{R}(P_0,t)$ be the potential income at time *t*, which depends on the timber price P_0 defined in Eq. (1); *N* the number of thinnings; u_k the thinning dates such that $0 < u_1 < u_2 \ldots < u_N < T$; and $\mathbf{h}_k$ the vector of the corresponding thinning rate at those dates, which will reduce the density as defined in Eq. (3). Hence, the thinning income on $[0,T]$ actualized to time T is: $\mathcal{H}_0(0,T) = \sum_{k=1}^{N} \mathbf{R}_{\boldsymbol{k}}\,(P_0, u_k).\,\mathbf{h}_{\boldsymbol{k}} e^{\delta(T-u_k)}$

and the final income is: $V(T) = \mathbf{R}(P_0, T).\,\mathbf{1}$.

Compared with Loisel (2014), we consider a more realistic forest economic income by considering the actualized sum of the cost of each silvicultural task noted C_j which takes place at a date v_j such that $0 < v_1 < v_2 \ldots < v_{N_w} < T$. The silvicultural cost is then $\daleth(0,T) = \sum_{j=1}^{N_w} C_j e^{\delta(T-v_j)}$.

This gives a total income:

$$I_0(0,T) = \sum_{k=1}^{N} \mathbf{R}_{\boldsymbol{k}}\,(P_0, u_k).\,\mathbf{h}_{\boldsymbol{k}} e^{\delta(T-u_k)} + \mathbf{R}(P_0, T) - \daleth(0,T) \quad (8)$$

This total income can also be rewritten as follows*:*

$$I_0(0,T) = \sum_{k=1}^{N} P_0\,(s(u_k)).\,h_k.\,n_k.\,\Upsilon(H(u_k), s(u_k)) e^{\delta(T-u_k)} + P_0\big(s(T)\big).\,n(T).\,\Upsilon(H(T), s(T)) - \sum_{j=1}^{N_w} C_j e^{\delta(T-v_j)} \quad (9)$$

Here, the first term of the equation represents the thinning income, derived from the sum of unit price $P_0(.)$ Multiplied by the harvested volume ϒ *(.,.)*. Harvested volume depends on height *H (.)* and tree basal area *s (.)* at thinning dates u_k actualized at time *T*. The second term is the final income at time *T* calculated with the same method. The third term is the silvicultural cost.

3.2 Models with storm risk

3.2.1 Storm risk and damage functions

As in Reed (1984) and Loisel (2014), we assumed that storm occurrence is compatible with a Poisson process, i.e. that storms occur independently of one another, and randomly in time. Let τ_i be the time span between the establishment of the stand and the first storm event in the stand. Thus, the distribution of the times between successive storms is an exponential value with a mean of $1/\lambda$ and a distribution of $F(\tau_i) = 1 - \exp(-\lambda\tau_i)$, where λ is the expected number of storms per unit time.

The proportion of damaged trees θ_{τ_i} (Eq. (5)) is assumed to be positively correlated with wind speed. Moreover, stand characteristics may contribute to damage intensity (Albrecht et al., 2010). Storms have less impact on young stands (Bock et al., 2006; Schmidt et al., 2010). Following Bock et al. (2006), we propose the following damage function:

$$\theta_{\tau_i} = \begin{cases} 0 & if \quad H(\tau_i) \leq H_L \\ \frac{\exp(-\alpha+\beta H(\tau_i))}{\exp(-\alpha+\beta H(\tau_i))+1} & if \quad H(\tau_i) > H_L \end{cases} \tag{10}$$

where $H(\tau_i)$ is the average tree height in the stand and H_L is the height limit reached at time t_L with $H_L = H(t_L)$. Eq. (10) states that if a storm takes place at time $\tau_i \leq t_L$, the storm has no impact; otherwise, for a storm occurring at time $\tau_i > t_L$, there is an impact whose intensity follows a logistic function with height $H(\tau_i)$.

3.2.2 Model with timber price depreciation

Due to the timber influx on the market following a storm, we observed a depreciation in timber price. Let $\mathcal{E}_\tau$. be the rate of depreciation in timber price, which is positively correlated with the severity and the extent of the storm in an area. As in Loisel (2014), $\mathcal{E}_\tau$ is calculated as.

$$\mathcal{E}_\tau(\sigma(t)) = \alpha_d \left(1 - \frac{\sigma(t)^4}{\Delta^4}\right) \tag{11}$$

where α_d is the expected rate of depreciation --in timber price, and σ is the number of years between present time *t* and the previous storm at time τ_i; therefore, $\sigma(t) = t - \tau_i$ and Δ is the length of time the timber market is disturbed after a storm.

The timber price function in Eq. (1) thus becomes:

$$P_1 = (1 - \mathcal{E}_\tau(\sigma(t)))P_0 \tag{12}$$

3.2.3 The Faustmann value with risk

In the presence of storm risk, net economic return y_i (thinning incomes, final income minus costs) actualized at final time *T* is as follows:

$$y_i = \begin{cases} \mathcal{H}_1(0,T,\sigma) + V(T) - \daleth(0,T) - c_0 & \tau_i = T \\ \mathcal{H}_1(0,\tau_i,\sigma) + V_1(\theta_\tau, \mathcal{E}_\tau, \tau_i) - \daleth(0,\tau_i) - c_0 - C_n(\theta_\tau,\tau_i) & t_L < \tau_i < T \end{cases} \quad (13)$$

where τ_i is the random date of a storm occurring at each time i with $\tau \leq T$. If no storm occurs until the final stand age $\tau_i = T$ (first line of Eq.13), the net present value (NPV) is calculated without storm risk. If a storm occurs after t_L corresponding to the height limit and before the final stand age T, the expression of the NPV changes. Thinning income $\mathcal{H}_1(0,\tau_i,\sigma)$ is defined as:

$$\mathcal{H}_1(0,\tau_i,\sigma) = \sum_{k=1}^{N} \mathbf{R}_{\boldsymbol{k}}(P_1,u_k).\mathbf{h}_{\boldsymbol{k}} e^{\delta(\tau_i-u_k)} = \sum_{k=1}^{N}(1-\mathcal{E}_\tau(\sigma(\tau_i))P_0(s(u_k)).h_k.\Upsilon(H(u_k),s(u_k))e^{\delta(\tau_i-u_k)} \quad (14)$$

Final income $V_1(\theta_\tau,\mathcal{E}_\tau,\tau_i)$, which depends on the proportion of damaged trees θ_τ and the rate of timber price depreciation after a storm $\mathcal{E}_\tau$ is as follows:

$$V_1(\theta_\tau,\mathcal{E}_\tau,\tau_i) = \mathbf{R}(P_1,\tau_i).\mathbf{1} = (1-\theta_\tau)(1-\mathcal{E}_\tau(\sigma(\tau_i))P_0(s(\tau_i)).n(\tau_i).\Upsilon(H(\tau_i),s(\tau_i)) \quad (15)$$

$C_n(\theta_\tau,\tau_i)$ represents clearing costs based on the proportion of damaged trees θ_τ. These clearing costs are assumed to depend linearly on the volume of damaged trees. Silvicultural costs function and regeneration cost are assumed to be unchanged compared with the situation without storm risk.

Finally, assuming that storms occur independently of one another and randomly in time and that the agent is neutral to risk, the expression of the Faustmann Value including annual costs can be rewritten as:

$$W_1 = E\left(\sum_{i=1}^{+\infty} e^{-\delta(\tau_1+\tau_2+\cdots+\tau_i)} y_i\right) - c_0 - \frac{c_y}{\delta} = \sum_{i=1}^{+\infty}\prod_{j=1}^{i-1} e^{-\delta\tau_j}.E\left(e^{-\delta\tau_i} y_i\right) - c_0 - \frac{c_y}{\delta} \quad (16)$$

This expression can be simplified as follows:

$$W_1 = \frac{E(e^{-\delta\tau}y)}{1-E(e^{-\delta\tau})} - c_0 - \frac{c_y}{\delta} \quad (17)$$

3.3 Optimization

The objective is to maximize the economic criteria:

$$\max_{u_k,h_k,T} W_0 \qquad \text{or} \qquad \max_{u_k,h_k,T} W_1$$

and has three technical constraints:

1) $u_k - u_{k-1} \geq \Delta t$;

2) $0 \leq h_k \leq \bar{h}$;

3) $n(T) \leq n_T$.

The first constraint imposes a minimum number of years Δt between two thinning operations. The second constraint means that the thinning rate h_k cannot exceed a maximum rate noted $\bar{h}$. The third constraint represents a guarantee of final minimal tree-density fixed by the forest manager.

We simulated the foregoing optimization model with the *EvaSylv* software developed by INRA – Mistea in France. The software uses a classic Nelder Mead algorithm (Nelder and Mead, 1965) to optimize the chosen economic criteria.

4 Application

We applied our methodology to an example of a beech stand in Northwestern France.

4.1 Price data

We compiled data on the timber prices for beech between 1974 to 2014 from private experts, the "Cabinet Chavet" agency (Chavet, 1974-2014). Price data was available for seven diameter classes from 20 cm to more than 81 cm, and for two qualities: 1st choice quality corresponded to high quality, perfect stems used mainly for staircases, and 2nd choice was standard quality with potentially physical defaults (noodle, color variance …). Changes in average beech price (€/m3) for standard quality timber are presented in Figure 1. Prices were deflated according to the price and purchasing power index from the French National Institute of Statistics and Economic Studies (Insee, 2015).

Figure 1 around here.

The average price was 64.13 €/m3 but it was extremely variable and fluctuated widely between 7.5 €/m3 and 190.97 €/m3. A first peak occurred around 1979 due to an increase in demand caused by an oil crisis (Nellen, 2012) and a second peak occurred before 1999 due to a high demand from Asian countries (ONF, 1998). Though beech prices are generally declining, the rate of decline is not similar across diameter classes. Prices for large diameter trees decreased 4 % per year during this period. While for diameter classes under 40 cm, prices declined only 1 % per year. However, it should be noted that data were unavailable in 1999 after the Lothar and Martin storms and from 2001-2008 for small diameters under 22.5 cm.

Our objective was to estimate beech price function following Eq. (1). The most significant model we found with the data available to us is the following multiple linear regression:

$$P_0 = E(P_0) + \varepsilon = \beta_0 + \beta_1 s + \beta_2 q + \beta_3 a + \beta_4 E + \varepsilon \qquad (18)$$
$$\varepsilon \sim N(0, \sigma^2)$$

where P_0 is the average timber price (€/m3, constant in 2011); *d* is the average diameter of each class (cm), which in turn gives tree base area *s; q* is wood quality (0 if for 1st choice and 1 for 2nd choice, as classified by experts); *a* is the year of sale between 1974-2014; and *E* is a dummy variable describing the economic conjuncture (0 before 1999 and 1 after that date). The estimated coefficients (with R- 3.1.2 software) $\beta_0, \beta_1, \beta_2, \beta_3$ and $\beta_4$are presented in table 1. ε is an error vector, which follows a Gaussian distribution $N(0, \sigma^2)$.

Table 1 around here

A statistic test was performed and the model residuals show no serious violation of normality (see the Table 1). The Shapiro-Wilk test of normality gave a non-significant p-value of 0.05.

As expected, the coefficients for diameter and quality were significant since these are the main characteristics that customers consider when buying wood products in France (Cavaignac et al., 2006; Heshmatol Vaezin et al., 2008; Pajot, 2006). The diameter coefficient was positive: for every 1-cm increase in diameter, price increased by 1.68 €/m3, all other variables being equal. The quality coefficient was negative and has the highest value, which indicates that wood quality is an important variable influencing timber price. More precisely, the quality coefficient of -24.10 €/m3 gives the average difference between the price of the 1st and 2nd choice quality, all other variables being equal.

The time coefficient, linked to year of sale, was negative, demonstrating that there was a significant decreasing trend for beech prices in France. Each year the average price decreased by -0.99 €/m3, all other variables being equal. This result echoes the presence of a trend or drift rate as found in other price models (Insley and Rollins, 2005; Navarrete, 2012; Thomson, 1992; Yoshimoto, 2002).

In accordance with Sandhu and Philips (1991), the period coefficient reflecting economic conjuncture was significant, thus validating the presence of a structural change in the beech market. The negative sign shows that timber prices decreased by 15.44 €/m3 during the post-1999 period, all other variables being equal. This is an original result compared with previous timber functions applied to coniferous species (Knoke et al., 2001; Lu and Gong, 2005; Thomson, 1992).

Due to the small proportion of high quality timber in our sample, we limited our simulation to standard quality timber prices. All the parameters (except diameter) in our timber price model were fixed so we retained three periods corresponding to different economic conjunctures for the numerical simulation:

- From 1974 to 1999, which corresponds to a period of high demand: the average price (€/m3) for European beech followed the relation: $P_0 = 1.68d - 21.60$ depending on diameter. Trees of less than 18 cm in diameter were sold at approximately 10 €/m3.
- From 2000 to 2013, which corresponds to a period of lower demand: the average price (€/m3) followed the relation: $P_0 = 1.68d - 56.76$. Tree of less than 39 cm in diameter were sold at approximately 10 €/m3.
- In 2014, the demand for beech was low: prices of all trees (€/m3) followed the relation: $P_0 = 0.48d + 2.50$.

The coefficients for the model incorporating storm risk (Eq (11)) were also calibrated with data after the 1999 storms. In that case, the expected rate of depreciation α_d was 30% and the duration of disturbance after a storm Δ was 8 years.

4.2 Silvicultural costs

To calculate the cost of silvicultural operations, we used the current ONF referential for beech forest management in the North-West of France (Pilard-Landeau and Simon, 2008) which fixes the task for each site type. Data on cost was updated in 2014 with internal data from the ONF. We focused on the most frequent situations, that is flat sites where mechanization is possible, soil is not compacted and natural regeneration is implemented with no specific constraints on the young growth stage. The resulting regeneration costs c_0 and the silvicultural costs $\daleth(0, T)$ are given in Table 2. We estimated the annual management cost c_y at 60 €/ha/year (Sardin and Mothe, 2010). Thinning and final harvesting costs were 200 €/ha; this corresponds approximately to a half-day's work of a forest worker. Clearing costs after a storm C_N was estimated by forest experts at 500 €/ha (personal communication).

Table 2 around here

4.3 Growth model

The growth model we used is a simplified version from Le Moguedec and Dhôte (2011): following these authors, we estimated the main variables each year at stand scale. Height growth depends on the age of the stand *t* and on site fertility H_{100}. The latter variable corresponds to the expected average height at 100 years. For beech in the Northwestern France, height growth in Eq (2) is:

$$f(t) = H_{100}\exp(-(0.47 + 0.01t)^{-\frac{1}{0.517}}) \qquad (19)$$

The most frequent fertility index used for beech in Northwestern France is H_{100} = 32 m (Pilard-Landeau and Simon, 2008). Le Moguedec and Dhôte (2011) show that the potential increase linked to soil fertility is $U(H'(t)) = 0.15 + 2.15H'(t)$ while $g(n(t),s(t))$ is the density dependent effect which limits tree basal growth (for more details see Le Moguedec and Dhôte (2011)). We assumed an initial tree number $n(0)$ of 66,987 stems/ha and an initial tree basal area $s(0)$ of $0.000171\ m^2$.There was a first culling at 15th year with a 93 % rate, then a second one at 33th year with a 38 % rate.

4.4 Other parameters

The discount rate $\delta$was 2 % as advised by the French government for long-term public investment (Quinet, 2013). Storm frequency based on a Poisson process was fixed at 1 % as suggested by Loisel (2014). We used French data on European beech after storms Lothar and Martin in 1999 to determine the tree height from which storm impact was started to occur: H_L= 23 m (Bock et al., 2006). The same reference allowed us to calculate the parameters related to the damage function: $\alpha = 4$ and $\beta = 0.08$. Following advice from ONF forest experts, we set the minimum duration between two thinning harvests Δt at 10 years, maximum thinning rate $\overline{h}$ at 61 % and the tree-density at final cutting n_T at 70 stems per hectare.

5 Results

For each of the three economic conjunctures (1974-1999, 2000-2013 and 2014 alone), we estimated the optimal expected land value, rotation age and thinning regime, with and without storm risks.

Table 3 around here

Table 3 presents the estimated results for a beech stand located in Northwestern France. For each of the three economic context periods, the corresponding optimal rotation age with or without storm risk and the expected land value without (W_0) or with storm risk (W_1) are shown. In all cases, the rotation age has been shorten by 7-8 years to take into account storm risks. The average difference between W_0 and W_1 was around 33%, indicating that storm risks significantly reduce the expected value of a forest. This result is coherent with most of the literature (Loisel, 2014; Price, 2011; Reed, 1984).

For a high timber price context as the period between 1974 and 1999, the expected land values for both W_0 and W_1 were positive, suggesting that forest investment was a profitable activity at that time. For an intermediate price situation as the 2000-2013 period, W_0 and W_1 remain positive. Nevertheless W_0 without storm risk (respectively W_1 with storm risk) decreases abruptly by 69 % (respectively 97 %) with W_0 value in the 1974-1999 period. In 2014, timber prices were low without or with storm risks, implying negative economic values for both W_0 and W_1. These results show that timber price variation induces a higher economic loss than does storm risk. However, as Gong and Löfgren (2007) we found, that price uncertainty does not imply a significant effect on the optimal rotation age; rotation ages in Table 3 were not sensitive to price variations over time.

To better understand the management implications of multiple risks, we also examined the optimal date and intensity rate of thinning for this example stand. Figure 2 presents the simulation results associated with the three economic contexts. The optimal solutions suggest four thinning times: at 33, 39, 45 and 51 years. Following Reeves and Haight (2000), we obtain the optimal harvest rate of each thinning at the maximal authorized rate 61%. Our results show that, to maximize profitability in beech silviculture, management should be intensive during early stages when a stand is not vulnerable to wind, under the height limit of 23.5 m for European beech. This would quickly produce intermediate income (Figure 2) and reduce thinning costs but would also increase the diameter of the remaining standing trees and result in a higher expect price at the end of the management cycle. In contrast to Lu and Gong's (2005) results for Scots pine and Norway spruce, our optimal dates and thinning intensity for beech did not vary widely among different price scenarios. Though price variation had a heavy impact on expected thinning income, it did not change the thinning program.

Figure 2 around here

Figure 3 shows that all thinning harvests should end by the 51st year, which corresponds to the height limit H_L =23 m in the storm damage function. This result is in accordance with previous results (Loisel, 2014; Price, 2011). Above this height limit, the rate of damage increases from 10 % to 20 % with a 1 % probability of a storm occurring each year.

Figure 3 around here

6 Conclusions

This paper presents an empirical study which integrates multiple risks into a commonly used Faustmann model to help make forest management decisions. We first reviewed different discrete models such as RPS and continuous models such as GBM which introduce timber price variation processes over time into forest economic models. We showed that these studies do not provide clear advice for forest managers and rarely include simultaneous multiple risks. We then presented analytical expressions of the Faustmann criteria including storm risks and price variation. We applied our models to real data for European beech in Northwestern France. We estimated different timber price functions depending on diameter, wood quality, year of sale and economic conjuncture, and included a damage function based on the impact and costs incurred by the Lothar and Martin storms in 1999. Our results demonstrate that: i) price uncertainty induces an important impact on LEV and that the structural economic change after 1999 incurred high economic loss for beech in France; and ii) storm risk shortens the optimal rotation age, limits the number of thinning operations and increases economic loss, though it is a less important factor than price variation.

If we compare these results with current ONF management guidelines for beech in Northwestern France, we can say that the current referential is near the optimal solution in terms of final rotation age. However, to optimize silvicultural management, simulation results suggest reducing the number of thinnings from 8 to 4 and concentrating them during the young forest stage. Forest managers should take timber price variations into account in their decisions. Furthermore, one priority of forest sector policy is to develop products and markets for beech. When analyzing the significant drop in beech sales of these last sixteen years (approximately 62 million euros per year for French public forest), different strategies such as developing new technologies and wood products, reducing industrial costs, optimizing transport and wood production, or subsidizing the wood industry, must be undertaken to enhance the value of this natural resource. However, global warming may cause an increase in storm frequency and severity (Haarsma et al., 2013) and must be taken into account.

In this paper our approach was limited to timber production. The next challenge will be to develop a model which includes non-timber benefits. For example, in view of recreational concerns (mushroom picking, landscape aesthetics…), a forest manager may prefer to maintain some over-mature stands despite their declining market value and susceptibility to storm risk. Economic models for mixed or irregular forests are also interesting in the current context of biodiversity conservation and climate change.

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Table 1 : Estimated coefficients of timber price for beech in France

Explicative Variables	Estimated Coefficients	Standard error	t value	Pr(>|t|)	Significance codes
Quality	-24.10	1.58	-15.28	< 2e-16	***
Time	-0.99	0.12	-7.91	0.00	***
Diameter	1.68	0.04	43.62	<2e-16	***
Period	-15.44	3.15	-4.90	0.00	***
Intercept	1960.68	247.41	7.93	0.00	***

Residual standard error: 16.92 on 457 degrees of freedom
(45 observations deleted due to missing values)
Multiple R-squared: 0.8418, Adjusted R-squared: 0.8405
F-statistic: 608.1 on 4 and 457 DF, p-value: < 2.2e-16

Table 2: Silvicultural costs for European beech in Northwestern France

Stand age	Task description	Average cost (€/ha)
0	Regeneration cost with soil preparation (c_0)	460
2	Mechanical or chemical cleaning	370
4	Creating racks	149
4	Manual cleaning	435
10	Rack maintenance	117
33	Paint marking	128

Table 3: Optimal rotation age and Faustmann values with respect to thinning rates

Economic context period	Scenario	Rotation age (year)	Faustmann Values W_0 or W_1 (euros/ha)	Difference with W_0 in 1974-1999 (%)
1974-1999	Without storm risk	109	3 449.40	
	With storm risk	102	2 327.46	-33 %
2000-2013	Without storm risk	116	1 085.38	-69 %
	With storm risk	108	90.13	-97 %
2014	Without storm risk	108	- 1,894.19	-155 %
	With storm risk	100	- 2,297.90	-167 %