Social capital, human capital and fertility

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Abstract

We develop an overlapping generations model to study how the interplay between social and human capital affects fertility. In a framework where families face a trade-off between the quantity and quality of children, we incorporate the assumption that social capital plays a key role in the accumulation of human capital. We show how the erosion of social capital can trigger a chain of reactions leading households to base their childbearing decisions on quantity, instead of quality, resulting in higher fertility.

Keywords: fertility, quantity-quality trade-off, human capital, education, social capital, trust.

JEL Classification: J13, Z1, Z13.
1 Introduction

The trade-off between offspring quantity and quality (hereafter QQ) is commonly acknowledged as one of the driving forces of human fertility. The seminal work of Gary Becker (Becker and Lewis, 1973; Becker and Tomes, 1976) was the first to provide a rationale for the declining fertility rates and rising income levels observed in developed countries after industrialization. According to Becker, parents derive utility from both offspring quantity and quality. Given budget constraints, however, they choose between the number of children and their human capital in response to the economic incentives deriving, for example, from income prospects, relative prices, and technological change.

In Becker’s model, rising income levels stimulate an increase in fertility via a substitution effect, because at higher levels of income the elasticity of income to the quality of children outweighs that related to quantity. Galor and Weil (2000) and Galor and Moav (2002) argued that the trade-off crucially depends on technological change, which makes human capital more profitable thereby leading parents to prefer quality over quantity. This literature overall implies that economic growth could be detrimental to fertility via the QQ trade-off.

In these frameworks, the choice between fertility and investments in children’s human capital happens endogenously, with simultaneity and omitted variables bias being the main sources of endogeneity. As a result, empirical tests struggled identifying causality in the relationship between economic variables and childbearing decisions (Fernihough, 2017). Not surprisingly, theories of the QQ trade-off can only partially explain the still declining trend in fertility during the recent times of crisis and cannot easily be reconciled with recent empirical studies suggesting that the economic insecurity associated to low growth-scenarios can be a deterrent to childbearing (Adserà, 2011; Modena and Sabatini, 2012; Modena et al., 2014). Overall, the available evidence suggests that variables so far unexplored by theoretical and empirical research can play a role in shaping the QQ trade-off in households’ fertility choices.

This paper offers a new perspective on the trade-off by investigating the role of an economic variable that has so far not been explored in the fertility literature: social capital. We argue that families’ demand for human capital depends not only on conjunctural factors such as income and technological innovations but also on the economy’s stock of social capital, which takes decades, or even centuries, to accumulate, is generally persistent over the short run, and only partially reacts to economic shocks (Guiso et al., 2016).
Social capital is a key factor in fostering investments in human capital. Knack and Keefer [1997] illustrated why trusting societies have higher returns to the accumulation of human capital. Since trust improves access to credit, enrollment in higher education may be higher (Karlan et al. [2009]).

Trust and civic involvement are linked to better performance of government institutions, including publicly provided education (Coleman [1988], Putnam et al. [1993]), thereby raising the quality of schools and increasing the return to education. By facilitating the enforcement of contracts, both spontaneously and through a higher efficiency of enforcement institutions, trust also increases the return to specialized and vocational education (Knack and Keefer [1997], Guiso et al. [2010]). Finally, in trusting societies, hiring decisions are more likely to be influenced by talent and effort instead of the personal attributes of applicants, such as blood ties and personal knowledge - which are common surrogates of trustworthiness in low-trusting societies — thereby further increasing the returns to educational achievements (Knack and Keefer [1997], Alesina and La Ferrara [2005]).

Households’ investment decisions in the human capital of children depend on social capital both directly and indirectly. The direct channel of transmission is related to the above mentioned social capital’s ability to increase the returns to education. The indirect transmission relies on the fact that parents’ decisions also depend on their own human capital and on the economy’s average endowment of human capital, which in turn are affected by social capital.

To show how these two channels of transmission work, we develop an overlapping generations model that incorporates the QQ trade-off and the assumption that social capital plays a key role in the creation of human capital.

As in traditional QQ frameworks, in our model agents derive utility from the number of children and their quality, but we also add social interactions to the sources of utility. Following Galor and Weil (2000), we assume that the trade-off is influenced by the profitability of investments in education. Differently from them, we posit that returns to the human capital accumulated by the offspring depend on the existing stock of social capital. In this scenario, social capital is a public good that incidentally arises as a by-product of other activities and, as any public good, can be underproduced by private agents interacting in markets.

We find that a reduction in the level of social capital triggers a chain of reactions affecting fertility. The erosion of the stock of social capital can lead the economy into a “social poverty trap” (Antoci et al. [2005], Antoci et al. 2011, 2013), in which no one spends time on social interaction and human
capital can be stuck at a low level or, in the best case scenario, grows at a slow rate. On the other hand, if the productivity of human capital is low, the economy will follow a path ending in a “development trap” (Yakita, 2010) independently of the initial stock of social capital. In both the types of trap, incentives lead agents to prefer quantity over the quality of the offspring, resulting in higher fertility.

Our contribution bridges two strands of literature. The first broadly includes research on the determinants of fertility. After the seminal work of Becker and Lewis (1973) and Becker and Tomes (1976), several studies in this field have investigated the substitution mechanism inherent in the QQ trade-off (Becker et al., 2010; Yakita, 2010; Fernihough, 2017). Others analyzed the role of education (McCrary and Royer, 2011; Duflo et al., 2015; Hansen et al., 2017), child policies (Fanti and Gori, 2012, 2014), economic insecurity (Adsera, 2011; Modena et al., 2014), and technology adoption (Basso and Cuberes, 2017). We add to this literature by studying the role of social capital.

The second strand includes studies that have investigated the long run effects of social capital on economic outcomes such as access to credit (Karlan et al., 2009), financial development (Guiso et al., 2004), mitigation of agency problems (Costa and Kahn, 2003), political accountability (Nannicini et al., 2013), and growth (Knack and Keefer, 1997; Growiec and Growiec, 2014), just to name a few. We add to these studies by providing a theoretical testable prediction of how social capital shapes the QQ trade-off thereby influencing fertility choices.

The paper proceeds as follows. Section 2 illustrates the set up of the model. Section 3 presents the dynamics of the system. Sections 4 and 5 contain a discussion of results and some concluding remarks.

2 The model

We consider a production economy populated by overlapping generations of agents who live for three periods: childhood, adulthood, and old age. Time is discrete and indexed by \( t = 0, 1, 2, \ldots, n \).

2.1 Individuals

All decisions are made by adults, who choose how much time to devote to working, rearing children, and social participation - for example through civic engagement and interpersonal interactions. Parents are “altruistic”, as they care for the potential earnings of children in time \( t + 1 \) and invest in
their education accordingly (Becker and Tomes [1976]). The rearing time per child is constant. The higher the number of children the less time parents ceteris paribus will be able to devote to work. As a result, a higher number of children will entail a lower income, lower expenditure in the education of the offspring, and lower savings. For simplicity, we assume that agents only consume after retirement. Consumption is financed by returns to the savings accumulated during adulthood (see for example Galor and Weil [2000]). In addition, we assume that all goods are perishable and that agents can only transfer value across time by means of capital markets.

To summarize, adults derive utility from the consumption they will enjoy in the future, from the number of children and their “quality”, and from the social interactions enjoyed during the time left from work and child rearing, which hereafter we will call “leisure time”. We finally assume that the utility of leisure time depends on the stock of social capital and on the amount of leisure time that is on average enjoyed in society. In fact, spending time on social participation is more rewarding in a trusting society where the social environment offers better opportunities of engagement, e.g. if it is richer in civic networks, the cultural supply is higher and other people devote more time to social interactions (Antoci et al. [2005]; Antoci et al. [2011]; [2013])

The lifetime utility of an individual of generation $t$ is represented by the following function:

$$U_t = \rho \ln C_{t+1} + \gamma \ln n_t + \beta \ln h_{t+1} + K s_t \omega \ln \left(1 + l_t^{\frac{1}{t^2}}\right)$$

Where $C_{t+1}$ is consumption during retirement, $n_t$ is the number of children, $h_{t+1}$ is the human capital of the offspring, $K s_t$ is the stock of social capital at time $t$, $l_t$ is the leisure time of adults, and $l_t$ is the average time a society devotes to leisure. Parameters $\rho$, $\gamma$, $\beta$, and $\omega$ are strictly positive, while $\pi \in (0, 1)$.

The use of this specification of the utility function respect to the role of leisure time relies on two reasons. First, it allows the choice of $l_t = 0$ in the allocation of time (see Antoci et al. [2011]; [2013]; [2015]); on the other hand, the constant 1 inside the logarithmic function allows to avoid paradoxical results. In fact, removing 1 from the expression, since $l_t \in (0, 1)$, it would be $K s_t \omega \ln \left(l_t^{\frac{1}{t^2}}\right) < 0$, i.e. the time needed to produce social capital would decrease with the stock of social capital.

The budget constraint of adults is:

$$s_t = w_t (1 - n_t z - l_t) h_t - e_t n_t$$

Where $s_t$ are agents’ life-cycle savings, $w_t$ is the wage rate for labor, $z > 0$ is the time devoted to rearing children, and $e_t$ is per child educational
expenditure. Given parents’ salary and the interest rate, the cost of education distracts resources from future consumption, the number of children, and leisure time.

Denoting $r_{t+1}$ the interest rate in the period $t + 1$, the budget constraint of the elderly thus is:

$$C_{t+1} = r_{t+1} \left[ w_t (1 - n_t z - l_t) h_t - e_t n_t \right]$$

(3)

Following De La Croix and Doepke (2003), Yakita (2010), De La Croix (2013) and Hirazawa and Yakita (2017), we assume that education consists of two parts: one is the fruit of spillover effects from parents. This component does not require additional time to parents and basically depends on their human capital. The second part relies on the offspring’s learning from educational institutions, and thereby entails an expenditure that is subject to budget constraints. In other words, parents delegate the formal education of the offspring to the educational system (see for example De La Croix, 2013). The average level of human capital in the economy, $\bar{h}_t$, also plays a role due to spillover effects. In addition, following the social capital literature (e.g. Coleman, 1988; Knack and Keefer, 1997), we assume that social capital fosters the accumulation of human capital because it improves the returns to education, as explained in the Introduction. The human capital of an individual working in time $t + 1$ is thus produced according to the following function:

$$h_{t+1} = \varepsilon \left( h_t \theta + K s_t \phi e_t \right)^{\delta} \bar{h}_t^{1-\delta}$$

(4)

With $\varepsilon, \theta > 0$ and $\phi, \delta \in (0, 1)$. The parameter $\varepsilon$ indicates the technology of production of human capital, $\delta$ indicates the role of the human capital of parents and their expenditure for the education of the offspring, $1 - \delta$ is the productivity of the average level of human capital in the economy, and $\phi$ expresses the importance of social capital in increasing the effectiveness of education.

This formulation implies that social capital plays its role at advanced stages of development, when a positive share of income is devoted the education of the offspring. If agents do not invest in education, social capital does not play a role in the accumulation of human capital.

The problem for the individual of generation $t$ is to choose savings $s_t$, leisure time $l_t$, the number of children $n_t$, old age consumption $C_{t+1}$ and the educational expenditure $e_t$ in order to maximize his lifetime utility $U_t$ defined in (1), subject to (2), (3), (4), and considering $h_t$ and $K s_t$ as given. As in De La Croix and Doepke (2003), De La Croix (2013) and Yakita (2010),
in order to guarantee the sufficient optimality conditions, we assume that \( \gamma > \beta \delta \). The first-order conditions for maximization and the post-optimality condition \( \bar{h}_t = h_t \) give the following solutions.

\[
\begin{align*}
CS_1: \quad & \left\{ \begin{array}{l}
e_t = 0; \quad l_t = 0 \\
1 = \frac{\gamma}{z(\gamma + \rho)}; \quad s_t = \frac{\rho h_t w_t}{\gamma + \rho} \\
C_{t+1} = \frac{\rho h_t w_t}{\gamma + \rho} r_{t+1}
\end{array} \right. \quad \text{if } w_t < \bar{w}_t, \; K_s < \bar{K}s
\end{align*}
\]

(5)

\[
\begin{align*}
CS_2: \quad & \left\{ \begin{array}{l}
e_t = 0; \quad l_t = \frac{\pi K s_t^\rho - \gamma - \rho}{\pi K s_t^\rho + \gamma + \rho} \\
n_t = \frac{2w_t}{\pi K s_t^\rho + \gamma + \rho}; \quad s_t = \frac{2h_t \rho w_t}{\pi K s_t^\rho + \gamma + \rho} \\
C_{t+1} = \frac{2h_t \rho w_t}{\pi K s_t^\rho + \gamma + \rho} r_{t+1}
\end{array} \right. \quad \text{if } w_t < \bar{w}_t, \; K_s > \bar{K}s
\end{align*}
\]

(6)

\[
\begin{align*}
CS_3: \quad & \left\{ \begin{array}{l}
e_t = h_t \frac{\beta \delta w_t z - K s_t^\rho \theta^\gamma}{\gamma - \beta \delta} \\
n_t = \frac{w_t K s_t^\rho (\gamma - \beta \delta)}{(\gamma + \rho) \pi K s_t^\rho w_t - \theta^\gamma}; \quad s_t = \frac{h_t \rho w_t}{\gamma + \rho} \\
C_{t+1} = \frac{h_t \rho w_t}{\gamma + \rho} r_{t+1}
\end{array} \right. \quad \text{if } w_t > \bar{w}_t, \; K_s < \bar{K}s
\end{align*}
\]

(7)

\[
\begin{align*}
CS_4: \quad & \left\{ \begin{array}{l}
e_t = h_t \frac{\beta \delta w_t z - K s_t^\rho \theta^\gamma}{\gamma - \beta \delta} \\
n_t = \frac{2w_t K s_t^\rho (\gamma - \beta \delta)}{\pi K s_t^\rho w_t + \theta (\gamma + \rho) K s_t^\rho - \theta (\pi K s_t^\rho + \gamma + \rho)}; \quad s_t = \frac{2h_t \rho w_t}{\pi K s_t^\rho + \gamma + \rho} \\
C_{t+1} = \frac{2h_t \rho w_t}{\pi K s_t^\rho + \gamma + \rho} r_{t+1}
\end{array} \right. \quad \text{if } w_t > \bar{w}_t, \; K_s > \bar{K}s
\end{align*}
\]

(8)

Where \( \bar{K}s := \frac{\gamma + \rho}{\pi} \) and \( \bar{w}_t := \frac{h_t \rho w_t}{\gamma + \rho} \) are critical levels of the stock of social capital and wage, respectively. On the one hand, agents invest in the education of children if and only if the wage is sufficiently high (that is \( w_t > \bar{w}_t \)). On the other hand, they devote time to social interactions if and only if the stock of social capital is above the threshold (\( K_s > \bar{K}s \)).

As in De La Croix and Doepke (2003), De La Croix (2013) and Yakita (2010), education increases with income. It is worth noting that this relation is dynamic, in that it refers to the time-varying condition \( w_t > \bar{w}_t \). As in Antoci et al. (2011), 2013, the time spent on social interaction increases with the stock of social capital. In a non-trusting society, where people engage less in interpersonal interactions and public affairs, social participation is less rewarding, and individuals prefer to spend their time working, as suggested by Antoci et al. (2005) and Antoci et al. (2011).

In case (5), both the wage rate and social capital fall below the critical thresholds. Agents do not invest in the quality of the offspring, and neither do they devote time to social relations.
If the wage rate lies below the critical threshold but there is enough social capital in the economy (6), agents do not invest in the quality of the offspring, yet they devote a positive amount of time to social relations, thereby contributing to the accumulation of social capital.

In case (7), the wage rate is higher than the critical threshold but the stock of social capital is low. Agents are incentivized to work more, invest resources in the human capital of children and do not devote time to social relations.

If the wage rate and social capital are higher than their critical thresholds (8), then agents will find it rewarding to invest both in the quality of children and in social relations. In case (8), agents spend more resources on education than in case (7), due to the higher returns to education determined by the stock of social capital.

Figure 1 shows the location of the solutions to the first order conditions in the space defined by \((K_s, \omega_t)\).

![Figure 1: Solutions to the optimization problem respect to \(\omega_t\) and \(K_s\).](image)

Apart from the case \(CS_1\) in which fertility is constant, fertility decreases with social capital.

The level of social capital affects fertility choices through two channels. First, it is the effect of a time constraint. If the stock of social capital is high, then social participation is more rewarding and agents spend more time on it (Antoci et al., 2011). As less time is available for rearing children, households will prefer quality - which, in our model, is not time consuming -
over quantity. Secondly, social capital increases the productivity of education in the accumulation of the human capital. As the returns to education are higher, altruistic parents are encouraged to invest resources in the education of children. Given budget and time constraints, this entails a preference for quality over quantity.

2.2 Firms

We assume that there are many competitive producers with the constant-returns-to-scale production technology. Production in time $t$ employs physical capital $K_t$ and labor $L_t$. Denoting $Y_t$ the aggregate output, the aggregate technology of the economy can be represented by the following production function:

$$F(K_t, L_t) = AK_t^\alpha L_t^{1-\alpha} \quad (9)$$

Where $A$ is the productivity of physical capital. The profit maximization conditions are given as:

$$w_t := A(1-\alpha)K_t^\alpha L_t^{-\alpha}, \quad (10)$$

$$r_t := A\alpha K_t^{\alpha-1} L_t^{-\alpha-1} \quad (11)$$

2.3 Social capital

Agents consider the stock of social capital as a public good and do not internalize its accumulation. By deciding the amount of time to devote to leisure, however, they unintentionally create social capital as a by-product, as suggested by Coleman (1988). Social capital evolves according to the following dynamic:

$$K_{st+1} = B(K_{st})^{\lambda l_t^{1-\lambda} + (1 - \zeta)K_{st}} \quad (12)$$

Where $\zeta \in (0, 1)$, $\lambda \in (0, 1)$, and $B > 0$ is a parameter capturing the productivity of social participation in the accumulation of social capital.

As the time spent on interpersonal interactions creates social capital as a by-product, social participation prevents the erosion of the stock of social capital in the long run, as in Antoci et al. (2011) and Bilancini and D’Alessandro (2012). By contrast, when the environment is adverse to the accumulation of social capital (i.e. $K < K_{st}$), its stock constantly reduces at a rate $(1 - \zeta)$ until complete erosion.

Given the assumptions regarding the formation of human capital, indefinite growth paths of $K_t, N_t$ and $h_t$ are possible. Unlike several previous
studies (e.g. [Agenor and Dinh 2015]), the assumption $\lambda < 1$ prevents a perpetual growth of the stock of social capital. The concept of social capital, in fact, has mainly been operationalized as trust and civic engagement in the empirical literature (see [Guiso et al. 2010] for a review). At the macro level, trust is commonly measured as the share of trusting people in a given population (see for example [Knack and Keefer 1997] and [Algan and Cahuc 2010]). Civic engagement is measured as the density of civic association (e.g. [Putnam et al. 1993] and [Guiso et al. 2016]). Both these dimensions are clearly subject to saturation and their perpetual growth would be implausible.

### 2.4 Market clearing conditions

The equilibrium condition in the labor market is:

$$L_t = N_t(1 - n_t z - l_t)h_t$$

Where $N_t$ is the population of generation $t$. Population evolves according to the following:

$$N_{t+1} = n_t N_t.$$  

The equilibrium condition in the capital market is:

$$K_{t+1} = n_t s_t$$

Equilibrium dynamics are described by a four-dimensional system in which $n_t, s_t, e_t, l_t$ are determined as in $CS_1, CS_2, CS_3$ and $CS_4$ and the equations (4), (12), (13), (14), and (15) hold. Depending on the value of the four state variables $K_t, K_{st}, N_t, h_t$, we have four different dynamical systems corresponding to the solutions (5), (6), (7), and (8) to the first order conditions.

### 3 Dynamical systems

From the inspection of the equations describing the equilibrium dynamics, it can be noted that the dimension of the system can be reduced by introducing the variable $v_t$ defined as the ratio between physical and human capital:

$$v_t := \frac{K_t}{h_t N_t} = \frac{k_t}{h_t}$$

As a result, we obtain a two-dimensional dynamical system in the variables $v_t$ and $K_{st}$. Moreover, $K_{st}$ evolves independently from $v_t$. In particular

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For a critical review on the measurement of social capital see for example [Sabatini 2007].
at a generic fixed point of the map \((v^*, Ks^*)\) with \(v^* > 0\) and \(Ks^* \geq 0\) we obtain a balanced growth path at which the physical-human capital ratio (per agent in the adulthood) and the stock of social capital stay constant. Moreover by noting that the wage rate can be expressed as
\[
w_t = \frac{A(1-\alpha)v_t^\alpha}{(1-n_tz-l_t)^\rho}
\] (17)
it is easy to check that along a balanced growth path also \(n_t, l_t, w_t\) and \(\frac{v_t}{K_t}\) remain constant and the common growth rate for physical and human capital is determined by (4). In the case in which agents spend positive amounts on the education of children, we obtain
\[
\frac{k_{t+1}}{k_t} - 1 = \frac{h_{t+1}}{h_t} - 1 = \left[ \varepsilon \left( \theta + \frac{(K_s^*)^\phi \beta \delta w_{ss}z - \gamma \theta}{\gamma - \beta \delta} \right)^\delta \right] - 1
\] (18)
with \(w_{ss}\) being the stationary state value of the wage rate; while if agents do not invest in education, we have
\[
\frac{k_{t+1}}{k_t} - 1 = \frac{h_{t+1}}{h_t} - 1 = \left[ \varepsilon \theta^\delta \right] - 1.
\] (19)
It is worth noting that, in both cases, the long run growth rates are positive if and only if the expressions inside squared brackets are higher than 1. In addition, in both cases the rate defined in the expression (18) is higher than the one defined in (19).

By considering the expression in (17), a new threshold \(\bar{v}_t\), corresponding to the threshold \(\bar{w}_t\), can be defined:
\[
\bar{v}_t = \begin{cases} \bar{v}_1^1 := \frac{(\gamma \theta A \beta \delta z (1-\alpha))^{\dagger}}{\rho} & \text{if } Ks_t < \bar{K}s \\ \bar{v}_2^2 := \frac{(1-n_tz-l_t)^\gamma \theta (Ks^{\omega} + \gamma + \rho)}{Ks^{\omega} \beta \delta \pi (\pi Ks^{\omega} - \gamma + \rho)} & \text{if } Ks_t > \bar{K}s \end{cases}
\] (20)
According to (20), we obtain the following dynamical systems:

System 1

If both the wage rate and the stock of social capital fall below the respective critical threshold (allowing for positive investments in the human capital of
children and social participation), case $CS_1$ holds and equilibrium dynamics are described by the following system:

$$
S_1 : \begin{cases} 
v_{t+1} = \frac{A(1-\alpha)v_1^\alpha(\frac{\gamma+\rho}{\theta})^\alpha}{\theta^\beta} v_t \\
K_{s_{t+1}} = (1-\zeta)K_{s_t} 
\end{cases}
$$

(21)

valid in the space $V_1 := \{(v_t, K_{s_t}) \in R^2_+ : v_t < v_1^1$ and $K_{s_t} < K_s\}$. Neglecting the state constraints, it is trivial to verify that there exists a unique (virtual) steady state $(v_1^*, K_{s_1}^*)$ for (21), with $v_1^* = \left(\frac{A\rho_z(1-\alpha)}{\theta^\beta}\right)^{\frac{1}{1-\alpha}}$ and $K_{s_1}^* = 0$. In addition, because of the definition of $\overline{w}_t$, $(v_1^*, K_{s_1}^*)$ is feasible, (that is $(v_1^*, K_{s_1}^*) \in V_1$).

From the monotonicity of $v_{t+1}$ respect to $v_t$ and the decreasing dynamics of $K_{s_t}$ converging to zero, we have that the steady state $(v_1^*, K_{s_1}^*)$ is locally asymptotically stable.

**System 2**

Considering the case in which $(i)$ the wage rate falls below the threshold allowing investment in the human capital of children $w_t$ and $(ii)$ social capital is above the threshold allowing the choice of a positive level of social participation $\overline{K}c$, $CS_2$ holds and equilibrium dynamics are governed by the following map:

$$
S_2 : \begin{cases} 
v_{t+1} = zA(1-\alpha)^{\beta_1-2\alpha}(\pi K_{s_t}^{\gamma+\rho})^\alpha v_t \\
K_{s_{t+1}} = BK_{s_t}^{\frac{\pi K_{s_t}^{\gamma+\rho}}{\pi K_{s_t}^{\gamma+\rho}}} + (1-\zeta)K_{s_t} 
\end{cases}
$$

(22)

that is valid in the space $V_2 := \{(v_t, K_{s_t}) \in R^2_+ : v_t < v_2^1$ and $K_{s_t} > K_s\}$. Virtual stationary states for map $S_2$ are of the type $(v_2^*, (K_{s_t})^*)$, with $(K_{s_t})^*$ being a positive solution of the equation $K_s = \frac{\overline{B}}{\zeta} \frac{\pi K_{s_t}^{\gamma+\rho}}{\pi K_{s_t}^{\gamma+\rho}}$. From the analysis of the social capital dynamics, it follows that there is a threshold value of $B$, $\overline{B}$, such that for $B < \overline{B}$ no fixed points exist for the map $S_2$, while for $B > \overline{B}$ two virtual fixed points exist, as illustrated in Figure 2. Such virtual fixed points are feasible if they belong to $V_2$.

Given the monotonicity of $(i)$ $v_{t+1}$ respect to $v_t$ and $(ii)$ $K_{s_{t+1}}$ respect to $K_{s_t}$, if two fixed points exist, then the one associated with the lowest (respectively highest) value of $K_s$ is unstable (respectively locally asymptotically stable). Fixed points with a positive stock of social capital are possible only to the extent to which social capital productivity is high enough to counterbalance its depreciation.
Figure 2: The mechanism leading to the birth of two steady states when the productivity of social participation is high ($B_h > B$), or no steady state when the productivity of social participation is low ($B_l < B$).

System 3

This case is related to $CS_3$ (the wage rate is above the threshold and social capital is below it). In this configuration, equilibrium dynamics are governed by the map:

$$S_3: \begin{cases} v_{t+1} = \frac{\rho A(1-\alpha)}{((\gamma+\rho)\alpha n\tau)(1-n\tau-l_t)^\alpha} \left( \frac{\gamma-\beta \delta}{AK_s t v_{t,z}(1-\alpha)(1-n\tau-l_t)^{-\alpha+\theta}} \right) v_t^\delta \\ K_{s,t+1} = (1-\zeta)K_{s,t} \end{cases}$$

valid in the space $V_3 := \{(v_t, K_{s,t}) \in \mathbb{R}^2_+: v_t > \tau_t^1 \text{ and } K_{s,t} < \bar{K}_s\}$ with $l_t$ and $n_t$ defined in $CS_3$.

In this case, there are no feasible fixed points, in that, imposing the stationary state conditions, no positive value of the investment in education can be associated to $K_{s,t} = 0$. This system only describes a transitory phase in which, at a time $t$, the stock of social capital decreases below the threshold allowing for investments in education. From that time on, equilibrium dynamics will be described by System 1.
System 4

This is related to CS4, in which both \( w_t \) and \( Ks_t \) are above the threshold allowing for investments in human capital and social capital, respectively. Equilibrium dynamics are governed by the following equation:

\[
S_4: \begin{cases} 
    v_{t+1} = \frac{2\rho A(1-\alpha)}{(1-n_t z-t_l)^{\gamma}(\pi Ks_t^{\alpha}+\gamma+\rho)z^n_t} \left( \frac{\gamma-\beta \delta}{\Delta Ks_t^{\alpha} v_t^z z(1-\alpha)(1-n_t z-t_l)^{-\alpha+\theta}} \right)^{\frac{\delta}{v_t^2}} \\
    Ks_{t+1} = BKs_t^{\lambda} \frac{\pi Ks_t^{\alpha-\gamma-\beta}}{\pi Ks_t^{\alpha+\gamma+\rho}} + (1-\zeta)Ks_t 
\end{cases}
\]

valid in the space \( V_4 := \{ (v_t, Ks_t) \in \mathbb{R}_+^2 : v_t > \bar{v}_t^2 \text{ and } Ks_t > \overline{Ks} \} \) with \( l_t \) and \( n_t \) defined in CS4.

We can notice that the second equation in \( S_4 \) coincides with the second one in \( S_2 \). Therefore we can conclude that the threshold value detected for the system \( S_2 \) applies for \( S_4 \). In particular, for \( B < B \) no fixed points exist for the map \( S_4 \), while for \( B > B \) two virtual fixed points exist (see Figure 2). Such virtual fixed points are feasible if they belong to \( V_4 \). If two fixed points exist, the one associated to the lowest value of \( Ks \) is unstable. From the properties of social capital dynamics and of the first derivative of \( v_{t+1} \) in the stationary state, and given the monotonicity of \( v_{t+1} \) respect to \( v_t \), we have that the stability of the fixed point associated to the highest value of \( Ks \) is guaranteed by the condition \((1-\delta)\alpha \leq 1 - \left( \frac{\theta}{((Ks_t)^{\gamma}w_{ss} z)_{Ks}} \right)\).

4 Discussion

The analysis of dynamics shows that the initial stock of social capital plays a fundamental role in the QQ trade-off, due to its influence on the accumulation of human capital. If the stock of social capital is below the threshold \( Ks \), the economy will not be able to experience a path in which agents invest in the human capital of the offspring.

In systems 1 and 3, condition \( Ks_t < \overline{Ks} \) holds and agents do not invest their time in social relations. Due to the different speeds in the accumulation of physical, human, and social capital, the economy can experience transitory phases in which agents invest positive amounts in the education of children. However, in the long run, the erosion of the stock of social capital will lead the economy along a dynamic path ending into a social poverty trap characterized by high fertility and low levels of social and human capital. In the trap, fertility rises as a result of two transmission mechanisms. The lack of social capital decreases the returns to education thereby encouraging
parents to prefer quantity over quality. In addition, as social interaction is less rewarding, the incentive to spend time on rearing children becomes stronger. In this scenario, the accumulation of human capital only relies on the spillover effects from parents to children, and no specific investment is made for increasing the potential earnings of the offspring in the future.

An initially high endowment of social capital $K_s > K_s$, however, does not guarantee that the economy will follow a path resulting in lower fertility and positive expenditure for education, as there still are further forces that can lead the economy into a trap. First, the public good nature of social capital prevents agents from internalizing its positive externalities. The resulting underinvestment could cause a reduction in the stock of social capital from generation to generation until its complete erosion, which in turn will cause a reduction to zero of the investments in human capital in the long run. This last scenario is illustrated in Figure 3, obtained by considering this parameter set: $A = 60; \alpha = 0.33; \beta = 0.1; \gamma = 0.3; \delta = 0.7; \epsilon = 1; \zeta = 0.25; \theta = 1; \lambda = 0.3; \phi = 0.2; \pi = 0.52; \rho = 0.1; \omega = 0.6; B = 1.3; z = 0.1$. In this case the equation of social capital has two fixed points $K_s^* \simeq 0.844$ and $K_s^* \simeq 6.24$, and the initial conditions are the following: $K_0 = 0.3, N_0 = 1, K_s_0 = 0.8 (< K_s^*), h_0 = 15.5$.

In addition, the productivity of human capital also plays a decisive role. Given our assumptions on the accumulation of human capital (see equation 4), social capital displays its effect only at an advanced stage of development, in which agents devote resources to the formal education of the offspring. If the productivity of human capital is too low, no one will spend resources on education and such an advanced stage of the economy could not even be reached.

To summarize, social capital can support endogenous growth through the key role it plays in the accumulation of human capital. However, a critical level of human capital is in turn needed to switch on the effect of social capital. In other words, the economy needs human capital to undertake an endogenous growth path and it needs social capital to make growth sustainable in the long run, resulting in lower fertility. This result is compatible with early sociological theories claiming that human capital (in the form, for example, of education) is needed to switch on the effect of social capital. According to Bourdieu (1986), investments in social capital require human capital, always in highly specific forms. “The reproduction of social capital presupposes an unceasing effort of sociability, a continuous series of exchanges in which recognition is endlessly affirmed and reaffirmed. This work, which implies expenditure of time and energy and so, directly or indirectly, of economic capital, is not profitable or even conceivable unless one
invests in it a specific competence and an acquired disposition to acquire and maintain this competence” (Bourdieu, 1986, p. 250, italic ours). For example, human capital is needed to understand the potential benefits of trust and to build bridging and liking connections that can in turn be used to strengthen the accumulation of all the forms of capital (Bourdieu, 1982; 1986).

Under the same parameter set, if the initial conditions are as follows: $K_0 = 0.3; N_0 = 1; Ks_0 = 3.4; h_0 = 15.5$, the performance of the economy over a horizon of 35 generation is described in Figure 4. The economy initially experiences a period in which no investments are made in education (the first generation), followed by a persistent phase characterized by positive educational expenditures for children. Along the trajectory approaching the stable balanced growth path, identified by $(Ks^*_2, v^*_2) \simeq (6.23, 1.42)$, there is a reduction in fertility rates, an increase in the time devoted to social participation, and an increase in the amount spent on education, which is due to the effect of social capital on the returns to education.
In addition, if the economy starts from the system $S_2$ that admits only one non-feasible solution $v^*_2((Ks_1)^*, (Ks_1)^*)$, i.e. $(v^*_2((Ks_1)^*) > v^*_2)$, then there will be a point at which the economy switches from a regime where no investments in human capital are made to an endogenous growth phase with the engine of human capital, or what Galor and Weil (1999, 2000) referred to as the switch from the “Malthusian” to the “modern growth regime”. This is because, by making investments in education more profitable and encouraging the accumulation of human capital, social capital causes an increase in wages above the threshold that allows for investments in education, resulting in a situation in which $w_t > \bar{w}_t$, $Ks_t > \bar{K}s$ and fertility is durably low.

5 Conclusion

In this paper we developed an overlapping generations model to study the role of an economic variable that has so far not been explored in the fertility literature: social capital. Our results highlight two channels through which a
high stock of social capital might reduce fertility. First, social capital makes investments in education become more profitable. The higher returns to education orient the QQ trade-off in favor of quality. This channel is related to a side effect of social capital on the accumulation of human capital that has been claimed in several studies (e.g. Coleman, 1988; Knack and Keefer, 1997; Bofota et al., 2016) but was never theoretically analyzed in the literature. Second, the higher productivity of social interaction related to the $K_{St} > K_S$ scenario creates the incentive to devote more time to social participation, which has to be detracted from child rearing. This channel is related to the ability of social capital to make social interaction more rewarding, which was previously analyzed in growth (Antoci et al., 2011; 2013) and evolutionary frameworks (Antoci et al., 2005; Antoci and Sabatini, 2018).

The activation of these channels of transmission depends on the structural parameters of the economy. If the stock of social capital is low, the economy will fall into a social poverty trap where agents do not invest in the human capital of their children and fertility rates remain high. A high level of social capital, however, does not necessarily lead the economy to an endogenous growth path. Investments in social participation must be high enough to counterbalance the underinvestment related to the public good nature of the stock of social capital. In addition, social capital can display its effect only in the context of a “modern growth regime” characterized by positive investments in education. If the productivity of human capital is not high enough and agents have no incentive to invest in the education of children, the potential role of social capital in the accumulation of human capital is neutralized and the economy will be stuck in a “development trap” as in Yakita (2010).

Our work not only contributes to the understanding of the determinants of fertility. The analysis of dynamics also adds to the literature on the long run effects of social capital because it reveals that its stock might affect economic development through a new mechanism that deserves further research and policy attention. On the one hand, we provide a theory illustrating how the QQ trade-off might work as a vehicle of the impact of social capital on the accumulation of human capital. This contribution has policy implications in light of the effect of human capital on economic growth (e.g. Barro, 1991; Growiec, 2010; Ketteni et al., 2011). On the other hand, the possible role of social capital in the reduction of fertility rates equally has implications for policy and development in light of the relationship between fertility and growth (Becker et al., 1990; Barro, 1991; Galor and Weil, 1999; Nakamura and Seoka, 2013; Strulik, 2018). Future empirical research exploiting longitudinal data is needed to test our predictions and to better understand the
conditions under which social capital actually displays its effect on human capital, fertility, and growth.

References


