Monetary policy with non-homothetic preferences

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Abstract

This paper studies the role of non-homothetic preferences for monetary policy from both a positive and a normative perspective. It draws on a dynamic stochastic general equilibrium model characterized by preferences with a variable elasticity of substitution among goods and with price adjustment costs à la Rotemberg. These preferences - introduced by Cavallari and Etro (2017) in a setup with flexible prices - have remarkable implications for monetary policy. Three main results stand out from a comparison of models with an increasing and a constant elasticity. First, an increasing elasticity induces novel intertemporal substitution effects that amplify the propagation of monetary and technology shocks. Second, it weakens the ability of a simple Taylor rule to attain a given level of macroeconomic stabilization. Third, the smallest welfare losses can be attained by stabilizing both inflation and output, in contrast to the prevailing view - based on models with a constant elasticity - that the best thing the monetary authority can do is to control inflation only.

Keywords: non-homothetic preferences; monetary policy; output stabilization; inflation stabilization; Taylor rule; new-Keynesian model; time-varying elasticity.

JEL classification: E12; E32; E52

1 Introduction

The new Keynesian model, a dynamic stochastic general equilibrium model with monopolistic competition and sticky prices, has become the workhorse for the analysis of monetary policy. A basic framework consisting of a dynamics IS equation, a forward-looking Phillips curve, and a Taylor rule is often used for discussing optimal policy (e.g., the textbooks of Galì, 2015 and Walsh, 2010; see also Clarida, Galì and Gertler, 2000), while forecasting and the evaluation of monetary policy are mainly conducted in medium-scale models accounting for a variety of frictions and other features (for instance, Christiano, Eichenbaum and Evans, 2005; Smets and Wouters, 2007). The typical new Keynesian model - either in the basic or in the extended version - assumes homothetic preferences, generally in the form of a constant elasticity of substitution among goods (CES). This assumption has several (unappealing) consequences. The demand for any two given goods depends only on their relative price and is independent of income. This stands in contrast with well-established empirical regularities in the behavior of consumers suggesting that income affects the share of expenditure devoted to different types of goods.[1] Moreover, the marginal propensity to consume out of disposable income seems to change systematically with the level of income and also with changes in income, being higher for individuals experiencing income decreases and lower for individuals experiencing income increases (Attanasio and Weber, 2010). These behaviors - neglected in model based on homothetic preferences - may indeed affect the way shocks are transmitted in the economy, and the main purpose of my analysis is to explore to what extent they do so. [2]

This paper puts to scrutiny the way monetary policy propagates its effects and the way it should be conducted, focusing on the role of preferences. For this purpose, it considers a dynamic stochastic general equilibrium model with non-homothetic preferences belonging to the class of symmetric, directly additive preferences already used by Dixit and Stiglitz (1977), and with price adjustment costs à la Rotemberg (1982). The model draws on Cavallari and Etro (2017) for the specification of preferences with a variable elasticity of substitution among different varieties of goods. For expositional convenience the elasticity is assumed to be increasing in the level of consumption, yet the model easily nests the cases of a decreasing and a constant elasticity.

I start by illustrating the mechanics of monetary transmission in a basic model without capital, comparing the propagation of monetary and technology shocks under an increasing and a constant elasticity. An increasing elasticity amplifies the real effects of a shock to the Taylor rule, while dampening the effects on prices. An unexpected increase in the nominal interest rate leads to

[1] The Engel's law, for example, says that the proportion of expenditure devoted to food is a decreasing function of total expenditure (Jorgenson, 1997). In general, evidence based on consumers' behavior suggests that the demand elasticity is indeed time-varying (e.g. Attanasio and Browning, 1995).

[2] A key limitation of macroeconomic models based on homothetic preferences is that aggregate demand is independent of the distribution of income, overlooking the distributive consequences of monetary policy. These questions are beyond the scope of the paper.

a fall in output that is twice as large in the model with an increasing elasticity compared to a constant elasticity. The reason is a weak incentive to smooth out consumption in the presence of output fluctuations. With an increasing elasticity, in fact, the cost of trading-off maintaining a high level of consumption today (when income declines most), for a slightly declining consumption path in the future is relatively high and the consumer will rather prefer to reduce consumption on the impact of the shock (when the elasticity is low), while maintaining a consumption path as high as possible in the future (when the elasticity is high). On the other side, the shock implies a smaller fall in inflation, for firms take advantage of a low elasticity in the early part of the transition to revise their desired markups upwards. For similar motives, an increasing elasticity amplifies both the output and the price effects of shocks, like technology shocks, that move prices and quantities in opposite directions.

I then extend the model to account for the role of capital in a setup with an increasing elasticity. The introduction of capital has the usual effect of maintaining consumption as smooth as possible, and let output fluctuations be absorbed almost entirely by investments. A time-varying elasticity is key for the timing and magnitude of the propagation of shocks: both monetary and technology shocks have larger real effects when the elasticity is increasing, for reasons similar to those considered in the model without capital. In addition, their effects are much more persistent, since changes in the demand elasticity induce intertemporal substitution of expenditure decisions that favour the accumulation of capital.

Finally, I address the question of how monetary policy should be conducted when the elasticity is time-varying, and how a simple Taylor rule performs relative to the optimal rule. The evaluation is based on a second-order approximation to the utility losses experienced by the representative consumer as a consequence of deviations from the efficient allocation, in the spirit of Rotemberg and Woodford (1999). Several results stand out. First, the Taylor rule appears to be less effective in stabilizing the economy: an increasing elasticity, by amplifying business cycle fluctuations, requires a far more aggressive policy to obtain a given level of stabilization. Second, the usual policy trade-off between output stabilization on the one hand and stabilization of inflation and the output gap on the other hand - emerging when the elasticity is constant - disappears with an increasing elasticity. In this case, stabilizing output is equivalent to stabilizing the output gap, and a sufficiently large gain in terms of output stabilization may more than compensate the loss in terms of inflation. Numerical exercises show that a moderate motive for output stabilization can improve welfare when it is combined with a strong anti-inflationary stance. This constitutes an important departure from the conventional view - based on preferences with a constant elasticity - that the best thing the monetary authority can do is to respond to changes in inflation only. With an increasing elasticity, the smallest welfare losses can be attained when the monetary authority responds to both inflation and output.

2 The basic model without capital

The basic model has the purpose of illustrating the mechanics of monetary transmission in a framework with non-homothetic preferences. To set notation, the exposition is based on Cavallari and Etro (2017). The economy is populated by a unitary mass of agents, who are consumers, workers and entrepreneurs. The labor input L_t is entirely employed by a perfectly competitive sector producing an intermediate good with a linear technology. The intermediate good can be used to produce a variety of downstream final goods of unit mass, each of them produced with an identical linear technology. The utility function is:

$$U = \mathbb{E}\left[\sum_{t=1}^{\infty}\beta^{t-1}\left(\log \mathbb{U}_t - \frac{\upsilon L_t^{1+\varphi}}{1+\varphi}\right)\right] \tag{1}$$

where $\mathbb{E}[\cdot]$ is the expectations operator, $\beta \in (0,1)$ is the discount factor, L_t is labor supply, $\varphi \geq 0$ is the inverse of the Frisch elasticity, $\upsilon \geq 0$ is a scale parameter for the disutility of labor and $\mathbb{U}_t$ is a utility functional of the consumption of final goods, which is assumed symmetric on the mass of goods. This intratemporal utility is directly additive:

$$\mathbb{U}_t = \int_0^1 u(C_{jt})dj \tag{2}$$

where the subutility $u(C)$ satisfies $u'(C) > 0$ and $u''(C) < 0$. Following Cavallari and Etro (2017), I depart from the traditional specification used in macroeconomics, which is based on "log-CES" homothetic preferences, and consider a polynomial specification combining a linear and a power function[3]:

$$u(C) = \gamma C + \frac{\theta}{\theta-1}C^{\frac{\theta-1}{\theta}} \quad \text{with } \theta > 1 \tag{3}$$

The elasticity of substitution between goods is $\vartheta(C) = \theta(1+\gamma C^{\frac{1}{\theta}})$ and is increasing (decreasing) in consumption if $\gamma > (<)\ 0$. Of course this subutility reduces to the CES case for $\gamma = 0$. Although both cases are in principle possible, herein I focus on an increasing elasticity of substitution, IES for short.

The starting point of the analysis is the system of equations describing the general equilibrium, with the new-Keynesian Phillips curve (NKPC) already in its linearized form around the zero steady-state inflation rate. The derivation of the system from first principles can be found in Cavallari and Etro (2017), while the derivation of the Phillips curve is in the Appendix. The general equilibrium is given by:

$$\upsilon L_t^{\varphi} = \frac{\theta(\gamma C_t^{\frac{1}{\theta}}+1)-1}{\theta(\gamma C_t^{\frac{1}{\theta}}+\frac{\theta}{\theta-1})C_t}W_t \tag{4}$$

[3] These preferences belong to the more general class of symmetric implicit CES utility, see Etro (2018).

$$\frac{\theta(\gamma C_t^{\frac{1}{\theta}}+1)-1}{(\gamma C_t^{\frac{1}{\theta}}+\frac{\theta}{\theta-1})C_t}=\beta\mathbb{E}[\frac{1+i_t}{1+\pi_{t+1}}\frac{\theta(\gamma C_{t+1}^{\frac{1}{\theta}}+1)-1}{(\gamma C_{t+1}^{\frac{1}{\theta}}+\frac{\theta}{\theta-1})C_{t+1}}] \tag{5}$$

$$i_t = i+\tau\pi_t+\xi_t \tag{6}$$

$$\pi_t=\beta\mathbb{E}[\pi_{t+1}]-\Psi\left(\mathfrak{m}_t-\mathfrak{m}_t^d\right) \tag{7}$$

$$Y_t = C_t \tag{8}$$

where C_t is aggregate consumption, W_t is the real wage, i_t is a one-period nominal interest rate, π_t is the inflation rate between periods t-1 and t, ξ_t is a mean-zero monetary policy shock, $\mathfrak{m}_t$ is the (linearized) markup, and $\mathfrak{m}_t^d$ is the (linearized) desired markup. Equation (4) is the consumer's first order condition for labor, equation (5) is the Euler equation for a one-period nominal bond, equation (6) is the Taylor rule, equation (7) is the NKPC, where $\Psi\equiv\frac{\vartheta(C)-1}{\kappa}>0$, and $\kappa\geq0$ is the Rotemberg cost parameter, and equation (8) is the resource constraint. Variables without a time subscript denote steady-state values.

In a setup with a time-varying elasticity, also markups are time-varying. In particular, the desired markup over marginal costs defined as $\mathfrak{M}_t^d=\frac{\vartheta(C_t)}{\vartheta(C_t)-1}$ is increasing (decreasing) with the level of consumption when $\gamma<(>)\ 0$. In linearized form it is given by:

$$\mathfrak{m}_t^d=\frac{\gamma C^{\frac{1}{\theta}}}{\vartheta(C)(1-\vartheta(C))}c_t$$

where lowercase letters denote percentage deviations from steady state, e.g. $c_t\equiv(C_t-C)/C$. The desired markup is distinct from the actual markup $\mathfrak{M}_t$, which is inversely related to real marginal costs, in linearized form $\mathfrak{m}_t=-w_t$. Price setting frictions imply a wedge between actual and desired markups, precisely:

$$\mathfrak{M}_t=\frac{\mathfrak{M}_t^d}{\left(1-\frac{\kappa}{2}\pi_t^2-\frac{\kappa}{1-\vartheta(C)}\left(1+\pi_t\right)\pi_t\right)}$$

This wedge in turn affects inflation through the Phillips curve (7): inflation will be positive when firms expect actual markups to be below their desired level, for in that case firms are willing to pay the adjustment cost and set higher prices to realign their markup to the desired level.

The system (4)-(8) can be conveniently reduced to get two equations in two endogenous variables, y_t and π_t. In linearized form around a steady state with $C=Y=1$, it reads:

$$\eta y_t=\eta\mathbb{E}y_{t+1}+\tau\pi_t-\mathbb{E}\pi_{t+1}+\xi_t \tag{9}$$

$$\pi_t=\beta\mathbb{E}[\pi_{t+1}]-\Omega y_t \tag{10}$$

where $\eta \equiv -1+\gamma \left[\frac{1}{\theta(\gamma+1)-1} - \frac{1}{\theta(\gamma+\frac{\theta}{\theta-1})} \right] < 0$ and $\Omega \equiv \Psi \left(1+\varphi+\frac{\gamma}{\theta(\gamma+\frac{\theta}{\theta-1})} \right) > 0$. The slope of the Phillips curve is an increasing function of the parameter γ: given all other parameters, an increasing elasticity, namely a positive γ, implies a smaller trade-off between inflation and output.

The exogenous factor, and unique state variable, is the monetary policy shock ξ_t. The system (9)-(10) can be solved by the method of undetermined coefficients. Assume that the equilibrium decision rule and pricing function are linear functions of the state variable:

$$y_t = a\xi_t \quad \text{and} \quad \pi_t = b\xi_t$$

where a and b are unknown. Suppose that the monetary policy shock follows a stationary AR(1) process:

$$\xi_{t+1} = \rho\xi_t + \varsigma_{t+1} \tag{11}$$

where ς_{t+1} is an innovation. Substituting the guesses into the system (9)-(10), and evaluating the expectations using the AR(1) process gives unique equilibrium coefficients:

$$\begin{aligned} a &= -\frac{(1-\beta\rho)}{\eta(\rho-1)(1-\beta\rho)+\Omega(\tau-\rho)} < 0 \\ b &= -\frac{\Omega}{\eta(\rho-1)(1-\beta\rho)+\Omega(\tau-\rho)} < 0 \end{aligned} \tag{12}$$

A, say, positive shock to the nominal rate (an increase in ξ_t) leads to a decline in both inflation and output. The former is a consequence of the Taylor rule: on the impact of the shock, inflation must decline to absorb the spike in the nominal interest rate. Over time, the pressure on the nominal rate will gradually die off and the inflation rate will return to the steady-state level. The dynamics of output is governed by the Phillips curve (7): output declines whenever inflation is below the expected rate and markups are above their desired level.

An increasing elasticity of substitution (i.e. a positive γ) affects the equilibrium coefficients (12) through the terms η and Ω. The former reflects intertemporal substitution of expenditure decisions and is governed by the marginal utility of income. A high γ implies a weak incentive to smooth out consumption over time (small η in absolute value). Consider, for instance, a decline in output (income). With a constant elasticity, the consumer will trade-off maintaining a relatively high level of consumption today (when income declines most), for a slightly declining consumption path in the immediate future. With an increasing elasticity, however, the cost of doing so is relatively high and the consumer will have an incentive to reduce consumption on the impact of the shock (when the elasticity is low), while maintaining a consumption path as high as possible in the future (when the elasticity is high). Clearly, this works in the direction of increasing the responsiveness of output to the shock (high a in absolute value).

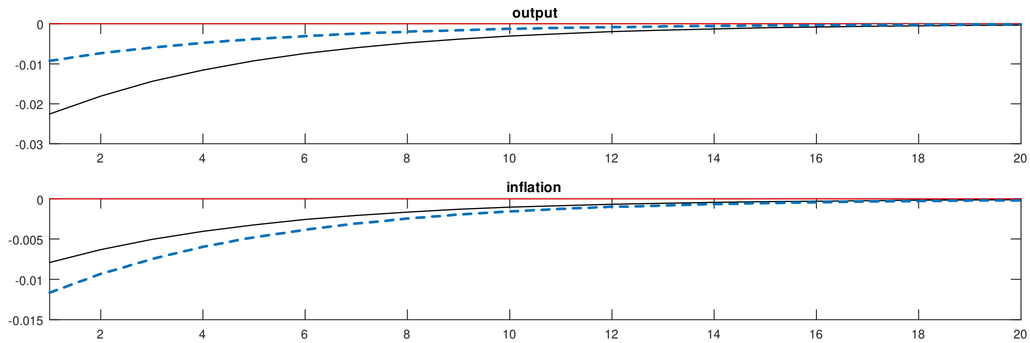

Figure 1: IRF to a one percent rise in the nominal interest rate in the IES model (solid line) and in the CES model (dashed line).

The second term reflects optimal pricing decisions. An increasing elasticity implies a weak incentive for firms to adjust prices in response to shocks which move prices and quantities in the same direction (the opposite is true for supply shocks). An unexpected, say, fall in aggregate demand, by reducing the demand elasticity today compared to tomorrow, induces firms to set temporarily high markups. This in turn generates a lower pressure on reducing prices when the shock hits (low b in absolute value). An increasing elasticity therefore implies more quantity movements and less price movements compared to a constant elasticity in response to demand shocks.

I now proceed numerically to illustrate the quantitative implications of an increasing elasticity, comparing the macroeconomic dynamics in the CES model $(\gamma = 0)$ and in the IES model $(\gamma > 0)$. To facilitate the comparison the preference parameters reflect a steady-state markup of 25 percent in both models, in line with the average markup found in US data. In the CES model, this implies $\theta = 5$. In the IES model, where the markup depends on both θ and γ (recall that steady-state consumption is equal to one), $\gamma = 1.8$ and $\theta = 1.79$. [4] The remaining parameters are fairly standard: the discount factor is $\beta = 0.99$, the Frisch elasticity is $\varphi = 1$, the persistence of the monetary shock is $\rho = 0.8$, the coefficient of inflation in the Taylor rule is $\tau = 1.5$, and the Rotemberg cost is set to mimic an average duration of price rigidity of 3.3 quarters as in US data.[5]

In Figure 1 a one percent positive shock to the Taylor rule reduces output on

[4] These values are the posterior mean of the distribution of the parameters of a DSGE model with IES preferences estimated by Cavallari and Etro (2017).

[5] For the calibration of κ I use the standard mapping between Rotemberg and Calvo pricing:

$$\frac{(1-\phi)(1-\phi\beta)}{\phi} = \frac{\vartheta(C)-1}{\kappa}$$

where ϕ is the percentage of firms whose prices are fixed in each period. The number of periods prices are on average fixed is $(1-\phi)^{-1}$.

impact by roughly one percent in the CES model and by more than 2 percent in the IES model. The effect on inflation is, on the contrary, almost twice as high in the CES (1.3 percent) compared to the IES model (less than 0.7 percent). As already mentioned, time-varying elasticity (and markups) amplify the real effects of price stickiness: a declining demand elasticity in the early part of the transition, in fact, induces firms to temporarily increase their markups while consumers have an incentive to postpone their expenditures in the future. These behaviors result in a small effect on inflation and a large effect on output. Opposite conclusions would hold with a decreasing elasticity.

It is illustrative to consider two extreme cases of price rigidity. First, assume that prices are perfectly flexible ($\kappa = 0 \Rightarrow \Omega \to \infty$). The equilibrium coefficients become $a \to 0$ and $b \to -\frac{1}{(\tau-\rho)}$: output is independent of the monetary shock while monetary policy is in complete control of inflation.[6] Under flexible prices, the system (9)-(10) becomes recursive and the classical neutrality holds. A time-varying elasticity has no impact on the equilibrium of the model in this case. On the opposite extreme, when prices are perfectly fixed ($\kappa = \infty \Rightarrow \Omega \to 0$), inflation is independent of the monetary shock while monetary policy is in complete control of output (the equilibrium coefficients become $a \to -\frac{1}{\eta(\rho-1)}$ and $b \to 0$). An increasing elasticity leads to a larger output effect (low η).

2.1 The business cycle

For comparison with business cycle models, it is instructive to consider the implications of an increasing elasticity for the propagation of technology shocks.[7] Assume that labor productivity A_t follows a stationary AR(1) process:

$$A_t = \varrho A_{t-1} + a_{t+1} \tag{13}$$

where a_{t+1} is an innovation. A rise in labor productivity reduces the real marginal cost and increases the markup, i.e. $\mathfrak{m}_t = -w_t + A_t$, thereby reducing inflation for a given level of output. The Phillips curve modifies as follows:

$$\pi_t = \beta \mathbb{E}[\pi_{t+1}] - \Omega y_t - \Psi \varphi A_t$$

Figure (2) compares the impulse response functions to a 1 percent rise in labor productivity under an increasing and a constant elasticity. These simulations are obtained for the same parameterization as before, while the new parameter measuring the persistence of the productivity shock is set at $\varrho = 0.95$.

[6] Equilibrium inflation can also be derived as the forward solution of (9) for $y_t = 0$, yielding:

$$\pi_t = -\frac{1}{\tau} \sum_{j=1}^{\infty} \left(\frac{1}{\tau}\right)^j E_t \xi_{t+j} = -\frac{1}{\tau - \rho} \xi_t$$

Under flexible prices, the Fisherian principle holds and inflation is only determined by the expected path of the monetary shock.

[7] Cavallari and Etro (2017) study the macroeconomic implications of an increasing elasticity in a framework with perfectly flexible prices.

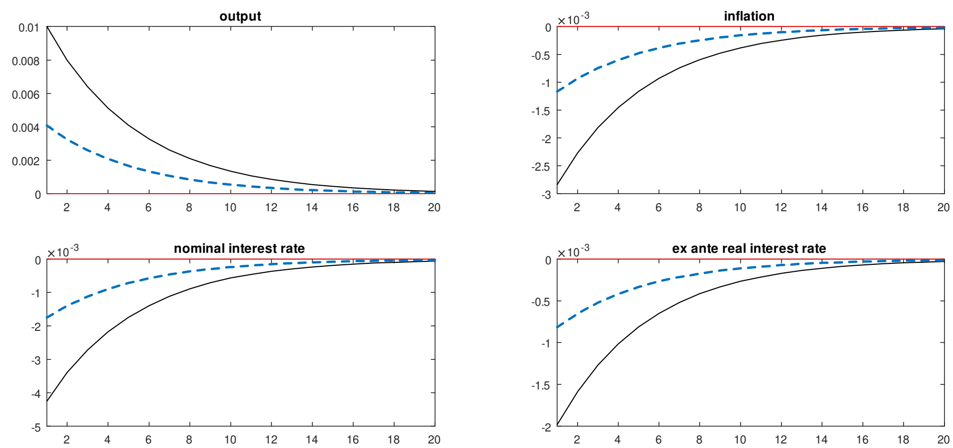

Figure 2: IRF to a one percent rise in labor productivity in the IES model (solid line) and in the CES model (dashed line).

In contrast to what happens after a monetary shock, where an increasing elasticity amplifies the output effects while reducing the impact on inflation, now the responses of both output and inflation are larger in the IES model than in the CES model. With a constant elasticity, the shock drives markups above the (constant) desired level, inducing firms to pay the adjustment cost and reduce prices in the attempt to realign the markup to the desired level. This incentive is particularly strong when the elasticity is increasing, for the desired markup will decline on the impact of the shock and will stay below the steady-state level for a while, widening the gap between actual and desired markups.[8] Notice that the Taylor rule contributes to amplifying the propagation of the shock: the decline in inflation leads to a drop in both the nominal and the ex-ante real interest rate, $r_t \equiv i_t - \mathbb{E}[\pi_{t+1}]$, accommodating the expansionary impulse of the shock. For the reasons already mentioned, this effect is larger when the elasticity is increasing.

3 The model with capital

When capital K_t is introduced into the model, the general equilibrium modifies as follows:

[8] The decline in $\mathfrak{m}_t^d$ more than compensates the fact that actual markups react less in the IES model compared to the CES model. Mechanically, the impact on $\mathfrak{m}_t$ is proportional to η, which is smaller (in absolute value) when the elasticity is increasing. As already mentioned, η reflects intertemporal substitution effects, namely the incentive for consumers to anticipate expenditure in periods where prices (and markups) are low. Hence, the productivity rise leads to a lower supply of labor, lower wages and smaller markups in the IES model.

$$\upsilon L_t^{\varphi} = \frac{\theta(\gamma C_t^{\frac{1}{\theta}} + 1) - 1}{\theta(\gamma C_t^{\frac{1}{\theta}} + \frac{\theta}{\theta-1})C_t} W_t \tag{14}$$

$$\frac{\theta(\gamma C_t^{\frac{1}{\theta}} + 1) - 1}{(\gamma C_t^{\frac{1}{\theta}} + \frac{\theta}{\theta-1})C_t} = \beta\mathbb{E}[\,(1 + R_{t+1} - \delta)\, \frac{\theta(\gamma C_{t+1}^{\frac{1}{\theta}} + 1) - 1}{(\gamma C_{t+1}^{\frac{1}{\theta}} + \frac{\theta}{\theta-1})C_{t+1}}] \tag{15}$$

$$\frac{\theta(\gamma C_t^{\frac{1}{\theta}} + 1) - 1}{(\gamma C_t^{\frac{1}{\theta}} + \frac{\theta}{\theta-1})C_t} = \beta\mathbb{E}[\frac{1 + i_t}{1 + \pi_{t+1}} \frac{\theta(\gamma C_{t+1}^{\frac{1}{\theta}} + 1) - 1}{(\gamma C_{t+1}^{\frac{1}{\theta}} + \frac{\theta}{\theta-1})C_{t+1}}] \tag{16}$$

$$Y_t = K_t^{\alpha} L_t^{1-\alpha} \text{ with } \alpha \in (0,1) \tag{17}$$

$$\frac{W_t}{R_t} = \frac{1-\alpha}{\alpha}\left(\frac{K_t}{L_t}\right) \tag{18}$$

$$\mathfrak{M}_t = \left(\frac{R_t}{\alpha}\right)^{-\alpha}\left(\frac{W_t}{1-\alpha}\right)^{\alpha-1} \tag{19}$$

$$i_t = i + \tau\pi_t + \xi_t \tag{20}$$

$$\pi_t = \beta\mathbb{E}[\pi_{t+1}] - \Psi\left(\mathfrak{m}_t - \mathfrak{m}_t^d\right) \tag{21}$$

$$Y_t = C_t + K_{t+1} + (1-\delta)\,K_t \tag{22}$$

where $\delta \in (0,1)$ is the depreciation rate and R_t is the rental rate of capital. Two new equations add the two new endogenous K_t and R_t: (15) is the Euler equation for capital and (18) is the condition for the optimal mix of capital and labor in the production function (17), which equates the marginal rate of substitution between labor and capital to the relative factor prices. The accumulation of capital has important effects for the propagation of shocks. First, it provides a natural means for the economy to smooth out consumption in the presence of fluctuations in income: in the resource constraint (22) investments allow consumption to move away from output. Second, it affects the Phillips curve via its impact on markups.

The system (14)-(22) can be reduced to four equations for the endogenous c_t, y_t, k_t, and π_t, which when linearized around a steady state with $L = 1$ become:

$$\eta c_t = \eta\mathbb{E}c_{t+1} + \tau\pi_t - \mathbb{E}\pi_{t+1} + \xi_t \tag{23}$$

$$\eta c_t = \eta\mathbb{E}c_{t+1} + R\mathbb{E}(-\eta c_{t+1} + \frac{\varphi+1}{1-\alpha}y_{t+1} - \frac{\alpha\varphi+1}{1-\alpha}k_{t+1}) \tag{24}$$

$$\pi_t = \beta\mathbb{E}[\pi_{t+1}] - \Psi\left(\eta c_t - \frac{\varphi+\alpha}{1-\alpha}y_t + \frac{\alpha\,(\varphi+1)}{1-\alpha}k_t - \Theta c_t\right) \tag{25}$$

$$y_t = \frac{C}{Y} c_t + \frac{K}{Y} \left(k_t - (1-\delta) k_{t-1} \right) \tag{26}$$

where $\eta = -1 + \gamma C^{\frac{1}{\theta}} \left[\frac{1}{\theta(\gamma C_t^{\frac{1}{\theta}}+1)-1} - \frac{1}{\theta(\gamma C_t^{\frac{1}{\theta}}+\frac{\theta}{\theta-1})} \right] < 0$, and $\Theta = \frac{\gamma C^{\frac{1}{\theta}}}{\vartheta(C)(1-\vartheta(C))} < 0$. Here, (23) is the same as in the basic model without capital (recall that $c_t = y_t$ in the basic model), (25) and (26) are also the same once we posit $\alpha = 0$ and $k_t = 0$. The key difference is the Euler equation for capital (24), where the presence of capital breaks the close connection between output and consumption, i.e. $c_t \neq y_t$, and allows to smooth out consumption in the presence of output fluctuations as in a standard real business cycle model. Here, changes in consumption over time, $\eta c_t - \eta \mathbb{E} c_{t+1}$, reflect changes in the ex-ante real interest rate (the term in brackets): a high rate implies an incentive to trade-off a reduction in consumption today for a growing path of consumption in the immediate future (recall that η is negative). Notice that movements in the real rate transmit to consumption in proportion to the steady-state value of the real rate, which is typically small in standard calibrations ($R = \frac{1}{\beta} - (1-\delta) = 0.0351$ in the baseline parameterization).

To gauge the macroeconomic implications of an increasing elasticity, I proceed numerically using the same parameterization as in the model without capital, except for the preference parameters in the IES model, as now the steady-state value of consumption is equal to 1.98.[9] Figure 3 shows the impulse responses in the IES and the CES models in the wake of a positive shock to the Taylor rule.

The shock reduces prices and quantities as in the model without capital. The decline in output, however, is absorbed almost entirely by investments, especially on impact when the response of consumption is close to zero. As already mentioned, the presence of capital allows consumers to maintain consumption as smooth as possible and this is true independently of fluctuations in the demand elasticity. An increasing elasticity (and countercyclical markups) are key for the timing and magnitude of the propagation: the monetary shock has larger and more persistent real effects when the elasticity is increasing. The reasons are similar to those considered in the model without capital: a relatively low elasticity on the impact of the shock provides an incentive to reduce current expenditures in exchange for an increasing expenditure path in the future. In addition, the presence of capital shifts the burden of adjustment on investments (rather than consumption), making the whole transition much more persistent (capital is a state variable). With an increasing elasticity, investments take almost twice as long to converge to the steady state compared to the case of a constant elasticity. In contrast to the model without capital, an increasing elasticity has only minor consequences for inflation. The reason is a strong consumption smoothing, which dampens the variability of the desired markup and therefore aligns the dynamics of prices to the case of a constant elasticity.

An increasing elasticity has a major impact on the propagation of technology shocks (see Figure 4). Assume that total factor productivity is an AR(1) process

[9] Precisely, I set $\gamma = 1$ and $\theta = 2.1$ so that $\vartheta(1.98) = 5$.

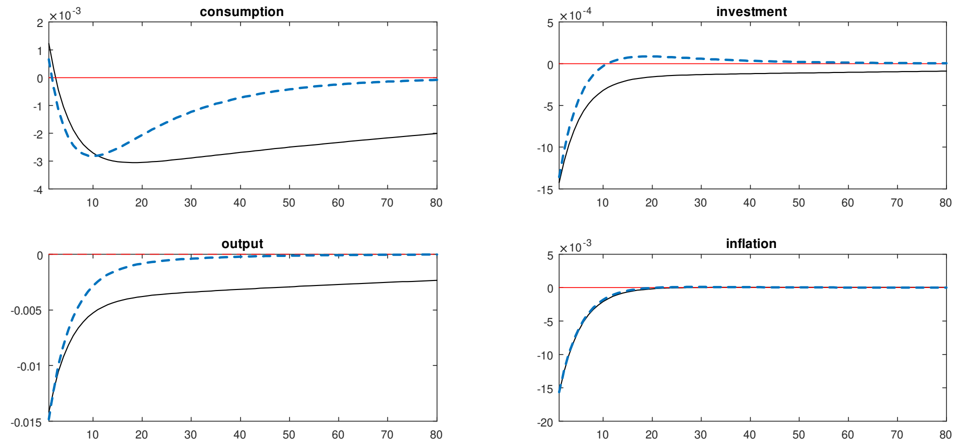

Figure 3: IRF to a one percent rise in the nominal interest rate in the IES model (solid line) and in the CES model (dashed line).

given by (13). The shock affects the dynamics of factor prices through its impact on the marginal products of labor and capital, modifying the Euler equation for capital and the Phillips curve as follows:

$$\eta c_t = \eta \mathbb{E} c_{t+1} + R\mathbb{E}(-\eta c_{t+1} + \frac{\varphi+1}{1-\alpha}(y_{t+1} - A_{t+1}) - \frac{\alpha\varphi+1}{1-\alpha}k_{t+1}) \tag{27}$$

$$\pi_t = \beta\mathbb{E}[\pi_{t+1}] - \Psi\left(\eta c_t - \frac{\varphi+\alpha}{1-\alpha}(y_t - A_t) + \frac{\alpha(\varphi+1)}{1-\alpha}k_t - \Theta c_t\right) \tag{28}$$

In (27), a positive shock tends to reduce the ex-ante real rate given all other conditions, thereby boosting current consumption. On the impact of the shock, when the stock of capital is given, the effect is larger the higher η in absolute value (and this explains a high initial response when the elasticity is constant in Figure 4). Over time, it depends positively on the accumulation of capital, which works in the direction of reducing the real rate, and is therefore larger with an increasing elasticity. The shock affects the Phillips curve through the dynamics of markups. With a constant elasticity, falling marginal costs drive markups above the constant desired level (the first three addends in (28)), inducing firms to reduce prices and realign their markups to the desired level. Consequently, inflation declines. With an increasing elasticity, the sign of the response is a priori ambiguous as the fall in marginal costs is accompanied by a drop in the desired markups (the fourth addend in (28)).

In Figure 4 the productivity rise boosts output and its components, and the more so with an increasing elasticity, similarly to what happens in the model

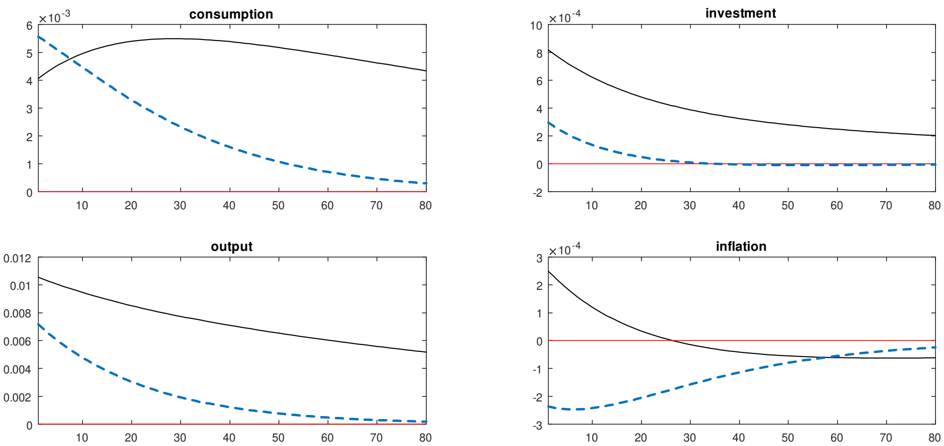

Figure 4: IRF to a one percent rise in total factor productivity in the IES model (solid line) and in the CES model (dashed line).

without capital. As already noted, the presence of capital shifts the burden of adjustment from consumption to investments, increasing macroeconomic persistence. In contrast to the model without capital, inflation rises on the impact of the shock when elasticity is increasing and remains positive for quite a long time before converging to the steady state from below. The reason is a decline in the desired markup in the early part of the transition when the elasticity stays above the steady-state level.

4 Monetary policy design

The analysis in the preceding sections has established that a time-varying demand elasticity (and time-varying markups) can have relevant consequences for the propagation of shocks. Precisely, an increasing elasticity amplifies the real effects of monetary shocks while dampening the effect on prices. Moreover, it amplifies the effects on both output and inflation in response to technology shocks. I now address the question of how monetary policy should be conducted when the elasticity is time-varying, and how a simple Taylor rule performs relative to the optimal rule. First I characterize the efficient allocation and optimal monetary policy when the economy is hit by a technology shock. Then, I consider a simple Taylor rule as a candidate rule for implementing the optimal policy and evaluate its performance relative to the optimal policy. The evaluation is based on a second-order approximation to the utility losses experienced by the representative consumer as a consequence of deviations from the efficient allocation, in the spirit of Rotemberg and Woodford (1999). Finally, I consider the performance of the Taylor rule in the wake of a monetary policy shock.

The efficient allocation can be determined by solving the problem of a benevolent social planner seeking to maximize the representative consumer's welfare, given technology and preferences, subject to the resource constraints. As is well-known the efficient allocation coincides with the equilibrium allocation under flexible prices, also called the natural allocation, once an optimal subsidy is assumed that exactly offsets the market power distortion. In these conditions, a policy that stabilizes firms' marginal costs at a level consistent with their desired markup, at unchanged prices can attain the efficient allocation.

In the model with capital the efficient allocation is given by:

$$\upsilon L_t^{\varphi} = \frac{(\gamma C_t^{\frac{1}{\theta}} + 1)}{(\gamma C_t^{\frac{1}{\theta}} + \frac{\theta}{\theta-1}) C_t} W_t \tag{29}$$

$$\frac{(\gamma C_t^{\frac{1}{\theta}} + 1)}{(\gamma C_t^{\frac{1}{\theta}} + \frac{\theta}{\theta-1}) C_t} = \beta \mathbb{E}[\,(1 + R_{t+1} - \delta)\, \frac{(\gamma C_{t+1}^{\frac{1}{\theta}} + 1)}{(\gamma C_{t+1}^{\frac{1}{\theta}} + \frac{\theta}{\theta-1}) C_{t+1}}] \tag{30}$$

$$\frac{(\gamma C_t^{\frac{1}{\theta}} + 1)}{(\gamma C_t^{\frac{1}{\theta}} + \frac{\theta}{\theta-1}) C_t} = \beta \mathbb{E}[\frac{1 + i_t}{1 + \pi_{t+1}} \frac{(\gamma C_{t+1}^{\frac{1}{\theta}} + 1)}{(\gamma C_{t+1}^{\frac{1}{\theta}} + \frac{\theta}{\theta-1}) C_{t+1}}] \tag{31}$$

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \text{ with } \alpha \in (0,1)$$

$$W_t = (1-\alpha)\, A_t \left(\frac{K_t}{L_t}\right)^{\alpha} \tag{32}$$

$$R_t = \alpha A_t \left(\frac{K_t}{L_t}\right)^{\alpha - 1} \tag{33}$$

$$i_t = i + \tau \pi_t + \xi_t \tag{34}$$

$$Y_t = C_t + K_{t+1} + (1-\delta)\, K_t \tag{35}$$

In linearized form around the steady state with $L = 1$, the system (29)-(35) reads:

$$\eta \widetilde{c}_t = \eta \mathbb{E} \widetilde{c}_{t+1} + \tau \pi_t - \mathbb{E} \pi_{t+1} + \xi_t \tag{36}$$

$$\eta \widetilde{c}_t = \eta \mathbb{E} \widetilde{c}_{t+1} + R \mathbb{E}(\widetilde{y}_{t+1} - k_{t+1}) \tag{37}$$

$$\eta \widetilde{c}_t = \frac{\varphi + \alpha}{1 - \alpha} (\widetilde{y}_t - A_t) - \frac{\alpha\,(\varphi + 1)}{1 - \alpha} \widetilde{k}_t \tag{38}$$

$$\widetilde{y}_t = \frac{C}{Y} \widetilde{c}_t + \frac{K}{Y} \left(\widetilde{k}_t - (1-\delta) \widetilde{k}_{t-1}\right) \tag{39}$$

where a tilde denotes a variable in the efficient allocation. The natural equilibrium is recursive, (37), (38) and (39) determine $\widetilde{c}_t$, $\widetilde{y}_t$, and $\widetilde{k}_t$ while inflation is pinned down from (36). Therefore, monetary policy is neutral and monetary shocks affect only inflation. The optimal policy requires that $\pi_t = 0$ and $y_t = \widetilde{y}_t$ in each period.

The utility losses experienced by the representative consumer as a consequence of deviations from the efficient allocation are expressed in terms of the equivalent permanent consumption decline, measured as a fraction of steady-state consumption. The second-order approximation to these losses yields the (average) welfare loss function, given by a linear combination of the variances of the output gap and inflation:

$$\mathfrak{L}_t = \frac{1}{2}\left[\left(1 + \frac{\varphi + \alpha}{1 - \alpha}\right) var(y_t - \widetilde{y}_t) + \frac{\vartheta\,(C)}{\Psi} var(\pi_t)\right]$$

Notice that the weight of output gap fluctuations in the loss is increasing in the "curvature parameters" φ and α, because large values of these parameters amplify the effect of deviations from the efficient allocation. The weight of inflation fluctuations is instead increasing in the steady-state elasticity of substitution among goods, for a high elasticity amplifies the consumption effect of any given price dispersion. The weight of inflation is also increasing with the degree of price stickiness κ (which is inversely related with Ψ), since the latter amplifies the price dispersion associated with any deviation from the optimal inflation rate.

Consider the following interest rate rule

$$i_t = i + \tau \pi_t + \iota y_t$$

where $\iota > 0$.

Table 1 reports the standard deviation of output, the output gap and inflation (all expressed in percentage terms) for different configurations of the parameters τ and ι, as well as the welfare loss implied by the deviations from the efficient allocation. The top panel refers to the IES model while the bottom panel reports results for the CES model. In both cases, the analysis is conducted conditional on the technology shock, where the standard deviation of the innovation in the technology process is set to one percent. In each panel, the first column reports results for the baseline calibration of the Taylor rule, which assigns a zero weight to output stabilization. The second column refers to the original calibration proposed by Taylor (1993), the third and fourth columns are based on rules involving, respectively, either a strong motive for output stabilization ($\iota = 1$) or a strong anti-inflationary stance ($\tau = 5$), and the fifth column combines both anti-inflationary and output stabilization motives. The remaining parameters are calibrated at the values reported above for the model with capital.

$Table 1$ Evaluation of Taylor rule (technology shock)

IES Model					
	$\tau = 1.5$	$\tau = 1.5$	$\tau = 1.5$	$\tau = 5$	$\tau = 5$
	$\iota = 0$	$\iota = 0.125$	$\iota = 1$	$\iota = 0$	$\iota = 0.125$
$\sigma(y)$	9.64	8.68	4.17	9.57	9.44
$\sigma(\pi)$	0.01	2.16	8.04	0.01	0.29
$\sigma(y - \widetilde{y})$	7.21	6.10	0.99	7.14	7.0
$\mathfrak{L}$	0.76	1.42	12.3	0.73	**0.71**
CES Model					
	$\tau = 1.5$	$\tau = 1.5$	$\tau = 1.5$	$\tau = 5$	$\tau = 5$
	$\iota = 0$	$\iota = 0.125$	$\iota = 1$	$\iota = 0$	$\iota = 0.125$
$\sigma(y)$	2.82	2.61	1.67	2.84	2.82
$\sigma(\pi)$	0.16	0.77	3.24	0.02	0.11
$\sigma(y - \widetilde{y})$	0.98	1.20	2.14	0.96	0.99
$\mathfrak{L}$	0.02	0.13	2.05	**0.01**	0.02

The table reports the standard deviation of a variable x $\sigma(x)$ in percentage terms.

Several results stand out. For given parameters τ and ι, the Taylor rule appears to be less effective in stabilizing the economy in the IES model compared to the CES model: an increasing elasticity, by amplifying business cycle fluctuations, requires a far more aggressive policy to obtain a given level of stabilization. Second, the usual policy trade-off between output stabilization on the one hand and stabilization of inflation and the output gap on the other hand emerges when elasticity is constant (CES model): increasing ι leads to a reduction in the volatility of output and to an increase in the volatility of inflation and the output gap, and hence to larger welfare losses. The best-performing rule is characterized by a strong anti-inflationary stance ($\tau = 5$) and no motive for output stabilization ($\iota = 0$). This rule is very close to the optimal policy, implying a permanent reduction in consumption relative to the efficient allocation as low as 0.01 percent.

Remarkably, this trade-off disappears with an increasing elasticity, and stabilizing output is equivalent to stabilizing the output gap. The monetary authority can in fact weaken the amplification brought about by an increasing elasticity by reducing the fluctuations of output and its components. In so doing, she will help align markups to their desired level and hence stabilize natural output. A sufficiently large gain in terms of output stabilization may more than compensate the loss in terms of inflation, so that an increase in ι might turn welfare-improving (with a time-varying elasticity the welfare function becomes a non-monotonic function of ι). In the baseline calibration, this happens, for instance, when a moderate motive for output stabilization is combined with a strong anti-inflationary stance. Among the rules considered in Table 1, the best approximation to the optimal policy is a rule with $\tau = 5$ and $\iota = 0.125$, leading to a permanent reduction in consumption relative to the efficient allocation of around 0.7 percent. This constitutes an important departure from the conventional wisdom - based on preferences with a constant elasticity - that the best thing the monetary authority can do is to respond to changes in inflation only. In my setup with an increasing elasticity, instead, the smallest welfare losses can be attained when the monetary authority responds to both inflation and output. [10]

I now turn to evaluate the performance of the simple Taylor rule in the wake of a shock that moves prices and quantities in the same direction (a demand shock). The analysis is conducted conditional on the shock to the Taylor rule (11), where the standard deviation of the innovation is set to one percent. Table 2 reports the results.

$Table2$ Evaluation of Taylor rule (nominal shock)

IES Model				
	$\tau = 1.5$	$\tau = 1.5$	$\tau = 1.5$	$\tau = 5$
	$\iota = 0$	$\iota = 0.125$	$\iota = 1$	$\iota = 0$
$\sigma(y)$	5.15	4.28	1.63	0.79
$\sigma(\pi)$	2.77	2.27	1.54	0.43
$\sigma(y - \widetilde{y})$	5.15	4.28	1.63	0.79
$\mathfrak{L}$	1.83	1.24	0.49	**0.04**
CES Model				
	$\tau = 1.5$	$\tau = 1.5$	$\tau = 1.5$	$\tau = 5$
	$\iota = 0$	$\iota = 0.125$	$\iota = 1$	$\iota = 0$
$\sigma(y)$	2.83	2.40	1.13	0.43
$\sigma(\pi)$	2.70	2.24	0.96	0.43
$\sigma(y - \widetilde{y})$	2.83	2.40	1.13	0.43
$\mathfrak{L}$	1.50	1.03	0.19	**0.04**

When nominal shocks are the source of fluctuations, the policy trade-off disappears also in the CES model as natural output remains unchanged, and output stabilization is equivalent to output gap stabilization. Increases in either τ or ι are effective in reducing the volatility of, respectively, inflation and output, and reducing welfare losses in both models. As before and essentially for the same reasons, the degree of stabilization attained with a given rule is much smaller with an increasing than with a constant elasticity.

5 Concluding remarks

This paper has evaluated the role of non-homothetic preferences for monetary policy from both a positive and a normative perspective, drawing on a dynamic stochastic general equilibrium model characterized by preferences with a variable elasticity of substitution among goods. These preferences - introduced by Cavallari and Etro (2017) in a setup with flexible prices - have remarkable implications for monetary policy. Three main results stand out. First, an increasing elasticity amplifies the propagation of monetary and technology shocks, for it induces intertemporal substitution effects that reduce consumption smoothing and affect firms' desired markups. Second, an increasing elasticity weakens the

[10] Experimentations with low values for κ and θ, which work in the direction of increasing the relative weight of output in the loss function, show that the region of non-monotonicity increases substantially relative to the baseline calibration.

ability of a simple Taylor rule to attain a given level of macroeconomic stabilization. Third, the monetary authority can attain the smallest welfare losses by stabilizing both inflation and output, in contrast to the conventional wisdom - based on models with a constant elasticity - suggesting that the best thing the monetary authority can do is to control inflation only.

The specification of preferences proposed in the paper lends itself to several applications. The model can be easily extended to account for features - like endogenous firm entry and bank credit - that affect monetary transmission, opening the way to novel interactions between market demand and the dynamics of firms and the credit channel. Moreover, the monetary setting can be amended to incorporate the presence of a zero lower bound and allow a role for unconventional policies.

Second, the intertemporal effects arising in my setup with a time-varying elasticity can play a role also for macroeconomic interdependence and the international monetary transmission, shedding new light on traditional policy issues, as the choice of the exchange rate regime and the gains to international monetary policy coordination. Finally, the introduction of non-homothetic preferences allows to address important questions about the distribution of income and the distributive consequences of monetary policy.

References

[1] Attanasio Orazio and Martin Browning, 1995, Consumption over the Life-cycle and over the Business cycle", *American Economic Review* vol. 85, 5, 1118-37

[2] Attanasio Orazio and Guglielmo Weber, 2010, Consumption and Saving: Models of Intertemporal Allocation and their Implications for Public Policy, *Journal of Economic Literature* 48, 693-751

[3] Cavallari Lilia and Federico Etro, 2017, Demand, Markups and the Business Cycle: Bayesian Estimation and Quantitative Analysis in Closed and Open Economies, Department of Economics working paper #9, Ca' Foscari University of Venice

[4] Clarida Richard, Jordi Galì and Mark Gertler, 2000, Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory, *Quarterly Journal of Economics* 105, 1, 147-180

[5] Christiano Lawrence, Martin Eichenbaum and Charles Evans, 2005, Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy, *Journal of Political Economy* 113, 1, 1-45

[6] Dixit Avinash and Joseph Stiglitz, 1977, Monopolistic Competition and Optimum Product Diversity, *American Economic Review*, 67, 297-308

[7] Etro, Federico, 2018, Symmetric Implicit CES, unpublished manuscript, University of Ca' Foscari

[8] Jordi Galì, 22015, *Monetary Policy, Inflation, and the Business Cycle*, second edition, Princeton University Press

[9] Jorgenson Dale, 1997, *Welfare - Vol. 1 Aggregate Consumer Behavior*, MIT Press

[10] Rotemberg Julio, 1982, Monopolistic Price Adjustment and Aggregate Output, *Review of Economic Studies* 49, 517-531

[11] Rotemberg Julio and Michael Woodford, 1999, The Cyclical Behavior of Prices and Costs, Ch. 16 in *Handbook of Macroeconomics*, J. B. Taylor and M. Woodford Eds., Elsevier, 1, 1051-135

[12] Smets Frank and Rafael Wouters, 2007, Shocks and Frictions in US Business Cycles: A Bayesian DSGE approach, *American Economic Review*, 97, 3, 586-606

[13] Carl Walsh, 2000, *Monetary Theory and Policy*, second edition, MIT Press

6 Appendix

This appendix derives the new-Keynesian Phillips curve in the model with capital. Capital and labor input are entirely employed by a perfectly competitive sector producing an intermediate good with the Cobb-Douglas production function (17). The intermediate good can be used to produce a variety of downstream final goods, each with an identical linear technology, or to invest in the accumulation of capital.

Each firm producing a variety i has profits:

$$d_{it} = (p_{it} - \chi_t)\, C_{it} - pac_{it}$$

where χ_t is the real marginal cost, and pac_{it} are price adjustment costs at the firm level. These costs are assumed to be proportional to firms' real revenues:

$$pac_{it} = \frac{\kappa}{2}(\frac{p_{it}}{p_{it-1}} - 1)^2 p_{it} C_{it}$$

where $\kappa \geq 0$. Price adjustment costs are higher the higher the change in the firm's price between any two periods, and the higher is the parameter κ. Flexible prices are given by $\kappa = 0$.

The first order condition for each firm requires:

$$\frac{\delta d_{it}}{\delta p_{it}} = 0 \tag{40}$$

where the derivative of profits with respect to the product price - equivalent to the derivative of profits with respect to the quantity consumed (see Cavallari and Etro, 2017 for a detailed derivation of the optimal price under monopolistic competition and a time-varying elasticity) - is:

$$\frac{\delta d_{it}}{\delta p_{it}} = (1 - \vartheta(C_{it}))\, C_{it} + \chi_t \frac{C_{it}}{p_{it}} \vartheta(C_{it}) - \frac{\delta pac_{it}}{\delta p_{it}} \tag{41}$$

and

$$\frac{\delta pac_{it}}{\delta p_{it}} = \kappa \frac{p_{it}}{p_{it-1}} \left(\frac{p_{it}}{p_{it-1}} - 1 \right) C_{it} + \frac{\kappa}{2} (1 + \pi_t^i)^2 C_{it} \,(1 - \vartheta(C_{it})) \tag{42}$$

where $\pi_t^i = \frac{p_{it} - p_{it-1}}{p_{it}}$ is firm-level inflation between periods t and t-1. Substituting (41) and (42) into (40) and solving for $p_{it} = p_t$ in a symmetric equilibrium gives:

$$p_t = \frac{\mathfrak{M}_t^d}{\left(1 - \frac{\kappa}{2}\pi_t^2 - \frac{\kappa}{1-\vartheta(C)}\,(1 + \pi_t)\,\pi_t\right)} \chi_t$$

Clearly, the pricing condition above implies that markups are at the desired level when prices are flexible. The new-Keynesian Phillips curve is the linearized price markup written in terms of inflation:

$$\mathfrak{M}_t = \frac{\mathfrak{M}_t^d}{\left(1 - \frac{\kappa}{2}\pi_t^2 - \frac{\kappa}{1-\vartheta(C)}\,(1 + \pi_t)\,\pi_t\right)}$$