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Abstract: Polarized interest groups compete to influence a decision-maker through monetary contributions. This decision-maker chooses a one-dimensional policy and has private information about his ideal point. Competition between interest groups under asymmetric information yields a rich pattern of equilibrium strategies and payoffs. Policies are systematically biased towards the decision-maker's ideal point and it may sometimes lead to a "laissez-faire" equilibrium. Either the most extreme decision-makers or the most moderate ones may get information rent depending on the importance of their ideological bias. The market for influence may exhibit segmentation with interest groups keeping an unchallenged influence on ideologically close-by decision-makers. Indeed, interest groups stop contributing when there is too much uncertainty on the decision-maker's ideology and when the latter is ideologically too far away.

Keywords: Lobbying Competition, Common Agency, Asymmetric Information, Contributions.

JEL Classification : D72; D82.

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1 Introduction

The pluralistic view of politics pushed forward by political scientists in last four decades has highlighted the important role played by special interests in shaping political decision-making.[1] One major thrust of this literature is that competition between interest groups should induce efficient and balanced policies and that, as a result, a large political representation of private interests ensures that policies are better aligned with social welfare. The paradigm to model pluralistic politics namely the "common agency model" of policy formation provides a theoretical support to this conclusion. This model was cast earlier by Bernheim and Whinston (1986) in a complete information abstract framework and then adapted by Grossman and Helpman (1994) and others towards various political economy applications (international trade, tax policies, regulation, etc.). Under common agency, competing lobbying groups (the principals) design non-cooperatively contributions to influence a decision-maker (the agent). At equilibrium, all organized interest groups actively contribute whatever their ideological distances to the decision-maker. This decision-maker chooses which contributions to accept and the policy to implement. Under complete information, this decentralized political process is efficient, i.e., the aggregate payoff of the grand-coalition made of all principals and their common agent is maximized.

However, many economists have forcefully argued that politics is plagued with transaction costs resulting both from asymmetric information and the limited ability to enforce contributions.[2] All forms of casual and empirical evidence show that interest groups have limited knowledge on legislators' preferences and that this imperfect knowledge determines whether groups contribute or not and, if they do so, the size of their contributions. For instance, Kroszner and Stratman (1999) and Stratman (2005) pointed out that interest groups adopt different attitudes vis-à-vis young legislators whose preferences are quite unknown compare to older legislators whose ideology has been better revealed by their past response to earlier PACs contributions. Moreover, those contributions seem to increase over time as legislators clarify their ideologies. Among others, Kroszner and Stratman (1998) and Wright (1996) also provided strong evidence suggesting that the ideological distance between an interest group and a decision-maker is key to assess the importance of contributions.Wright (1996), for example, noticed that the National Automobile Dealers Association (NADA) contributed far more heavily to conservatives (78,1 percent) than to liberals (about 12 percent) during the election cycles 1979-1980 and 1981-1982. He suggested that such pattern might be explained by the close ideological connection between members of NADA who are generally pro-business and conservative politicians.

[1] Dahl (1961), Lowi (1979), Moe (1981), Truman (1952) and Wilson (1973) among others.
[2] Dixit (1996).

In this paper, we revisit the common agency model of pluralistic politics but, to account for reported evidence, we introduce asymmetric information between interest groups and decision-makers whose ideologies are privately known. Asymmetric information creates transaction costs in the relationships between interest groups and the decision-makers. Limited activism by some interest groups, segmentation of the market for influence and weak contributions reflect these existing transaction costs. Our theory provides thus a richer pattern of equilibrium behaviors than predicted by complete information models; a pattern better suited to reconcile theory with the evidences stressed above.

First, far from aggregating the preferences of interest groups efficiently, equilibrium policies might not be as responsive to private interests as in a frictionless world. This phenomenon can be so pronounced that a "laissez-faire" equilibrium might arise with the decision-maker, free from any influence, choosing his own ideal policy. Second, due to frictions caused by asymmetric information, interest groups may choose to target only decision-makers who are ideologically close and thus easier to influence. They eschew contributions for ideologically distant decision-makers because of high transaction costs. This feature explains the prevalence of one-lobby influence in environments where interest groups are sufficiently polarized.

To obtain these results, we consider two interest groups and a decision-maker who all have quadratic spatial preferences with ideal points in a one-dimensional policy space.[3] Interest groups have ideal points located on both sides of the policy space. These assumptions allow us to parameterize equilibrium patterns with respect to firstly, the ideological distances between interest groups and the decision-maker; and secondly, the degree of polarization between groups. The decision-maker has private information on his ideal point. This is meant to capture the existing uncertainty on the ideological bias of key decision-makers in the political process and the limited knowledge that interest groups may have on the decision-maker's ideology. Groups non-cooperatively design contributions not only to influence the decision-maker's choice as in the case of complete information, but also to elicit revelation of his preferences. The decision-maker gives different weights to his ideological concerns and to the contributions he receives from interest groups.

In a delegated common agency model, all principals observe and contract on the same variable, namely the policy chosen by the agent, but the agent is free to choose any subset of contracts. Competing interest groups offer contributions not only to elicit the decision-maker's true preferences but also to shift policies towards their own ideal points. Since a given interest group does not internalize the impact of others modifying their own contribution to extract the decision-maker's information rent, there is excessive rent extraction in equilibrium. The marginal contributions of an interest group then no longer reflects its

[3] The assumption that policies result from the influence of two competing groups is common in the empirical literature. See for instance Kroszner and Stratman (1999) and Gawande et al. (2005).

marginal utility among alternative policies as in the case of complete information; they are always too low and sometimes can even be null. As a result, equilibrium policies are pushed towards the decision-maker's ideal point. Lobbying competition, far from reaching efficiency, shifts the balance of power significantly towards the decision-maker. The decision-maker may thus find more worthwhile to refuse contributions, especially when he is ideologically too distant from the contributing interest group.

Altogether, asymmetric information and competition thus provide a much less optimistic view of the political process than predicted by complete information models of pluralistic politics. Equilibrium patterns under asymmetric information might significantly differ according to the importance the decision-maker attributes to his own ideology, the extent of ideological uncertainty, and the degree of polarization between competing interest groups. When the decision-maker's ideological bias is large enough and groups are sufficiently polarized, there exists a unique equilibrium. This equilibrium is characterized by larger contributions and information rents for extreme decision-makers but also by segmented areas of influence for interest groups. When polarization between interest groups is small compared to ideological uncertainty, it is quite likely that interest groups end up being closer to each other than to the decision-maker. Their only concern is then to coordinate contributions. Multiple equilibria may arise from miscoordination problems. Contributions overlap and the more moderate decision-makers secure greater contributions and more information rent. Counter-lobbying always arises in equilibrium.

Following Grossman and Helpman (1994), Dixit, Grossman and Helpman (1997) and others, the bulk of the common agency literature explaining interest groups' behavior has focused on complete information. Fewer contributions have explicitly analyzed the transaction costs of contracting due to asymmetric information between those principals and their agent and how these costs affect the pattern of contributions and the political landscape. Under moral hazard, i.e., when the decision-maker's action (or effort) is non-verifiable, Dixit (1996) argued that a bureaucracy subject to the conflicting influences of various legislative committees/interest groups may end up having very low incentives for exerting effort but, because of an implicit focus on models of intrinsic common agency, groups do not eschew contributions.

Le Breton and Salanié (2003) considered a decision-maker who has private information on the weight he gives to social welfare in his objective function. A discrete political decision may be favored by some groups while others oppose it. Groups only contribute for their most preferred option. In such contexts, an interest group is active only upon learning that it is not too costly to move the decision-maker away from social welfare maximization.

Epstein and O'Halloran (2004) also studied a common agency model with spatial preference similar to ours but with the decision-maker's ideal point taking only two possible values. Restricting the analysis to direct mechanisms, they gave much attention to the case of several interest groups and their incentives to collude.

Restricted participation of interest groups and low equilibrium contributions have already found other rationales in the literature. Mitra (1999) (under complete information) and Martimort and Semenov (2007b) (under symmetric but incomplete information) investigated how the equilibrium payoffs from the common agency game where interest groups play with public officials determine whether the interest groups find worth to enter the political arena if the groups face some exogenous fixed-cost of organization. Merging a common agency model of lobbying with legislative bargaining, Helpman and Persson (2001) demonstrated that equilibrium contributions may be quite small and still have a significant impact on policies. Lastly, Felli and Merlo (2006) argued that some interest groups may not participate in the lobbying process in a model with active voters and candidates choosing from which lobbies they want support. They also found some tendency towards moderate policies but for different reasons from ours.

Section 2 presents the model and the complete information benchmark. Section 3 analyzes a hypothetical benchmark where interest groups cooperatively design their contribution. Section 4 deals with the case of competing interest groups and characterize various equilibrium patterns. Section 5 summarizes the main results. Proofs are relegated to an Appendix.

2 The Model and Complete Information Benchmark

Two polarized interest groups P_1 and P_2 (also referred to as the principals) simultaneously offer contributions to influence a political decision-maker (common agent). Let $q \in \mathbb{R}$ be a one-dimensional policy parameter over which the decision-maker has control. Interest group P_i $(i = 1, 2)$ has a quasi-linear and quadratic utility function over policies and monetary transfers t_i which is given by:

$$V_i(q, t_i) = -\frac{1}{2}(q - a_i)^2 - t_i.$$

The parameter a_i is P_i's ideal point in the one-dimensional policy space. We assume that principals' ideal points are symmetrically located around the origin, $a_1 = -a_2 = a > 0$.

The decision-maker has similar quasi-linear preferences given by:

$$U\left(q, \sum_{i=1}^{2} t_i, \theta\right) = -\frac{\beta}{2}(q - \theta)^2 + \sum_{i=1}^{2} t_i,$$

where $\beta \geq 0$.

The decision-maker has private information on his ideal policy θ. This parameter is uniformly distributed on a set $\Theta = [-\delta, \delta]$ centered around zero with δ representing the degree of ideological uncertainty. Since for our analysis and for interpretation of results only the ratio $\frac{a}{\delta}$ matters, we in what follows put $a = 1$. The parameter $\frac{1}{\delta}$ can then be viewed as the degree of polarization between the two organized groups on the particular policy.

Interest groups influence the decision-maker by offering nonlinear non-negative contributions $t_i(q) \geq 0$, which specify a monetary transfer depending on the decision q that the agent takes. The set of feasible transfers is assumed to consists of continuous, piece-wise differentiable functions of the policy variable q.

Timing: The timing of the game unfolds as follows:

- The decision-maker learns his ideal point θ;
- Interest groups non-cooperatively offer contributions $\{t_1(q), t_2(q)\}$ to the agent;
- The decision-maker decides whether to accept or refuse each of these offers. If he refuses all offers, he gets his status quo payoff normalized at zero;
- Finally, the decision-maker chooses the policy q and receives the corresponding payments from the interest groups whose offers have been accepted.

The timing of the game is of interim contracting when the contract is done only when the privately informed party observes its type. Note that since the transfers assumed to be non-negative, it is weakly dominant strategy for the decision-maker to accept all offers.[4] Thus although the decision-maker formally has right to choose any set of offers (*delegated common agency*) in equilibrium he accepts all of them. The agent's outside opportunity if he refuses all contributions is in fact his payoff if he chooses his own ideal policy. We will refer to this setting as a "laissez-faire" outcome.

The equilibrium concept we employ is a subgame perfect Nash equilibrium (SPNE).

Interpretation of variables: The principals can be thought of as two legislative Committees willing to influence a regulatory agency or as two lobbying groups dealing with an elected political decision-maker. Then the policy variable q can be a regulated price, an import tariff, a wage level or a number of permits depending on the application under scrutiny.

In our paper we depart from the literature on common agency with asymmetric information where the principals are aligned in their interests and the issue was not competition

[4] We thank referee for noticing that.

but coordination. Competition between lobbies assumes some degree of conflict between them. In a unidimensional framework this conflict is easily captured by diametrically opposed principals. Trade policy gives an interesting example of such diametrically opposed preferences for interest groups. Typically, upstream producers may want tariffs for the downstream product to be low whereas downstream producers want them to be high.[5]

The parameter β characterizes how the agent trades off contributions against his own ideological bias. As β increases, the agent values less monetary transfers and puts more emphasis on ideology. Principals have thus to spend more to influence the agent. A stronger ideological bias is expected on issues of much relevance for the public at large because the decision-maker's ideology plays then an important role for reelection. Macroeconomic issues such as unemployment, debt, inflation may fall into this category. A weaker ideological bias is more likely in the case of issues which might appear too technical to the general public. Regulatory and trade policies are relevant examples. Goldberg and Maggi (1999) estimated β in the U.S. patterns of protection to be between 50 and 88. Bradford (2003) suggested a lower value of β but still higher than one.

Lobbying groups usually have publicly known ideal policies. In some extent these ideal policies determine the very nature of special interest groups. Politician may have bias towards some policies which can be observed through his history of voting. However, the mere existence of lobbying competition assumes the uncertainty of the decision-maker's position. We model this in a crude way: The agent is moderate when his ideal point lies near the origin and more extreme otherwise. Following the earlier works of Niskanen (1971) and Laffont and Tirole (1993) among others, we thus envision the bureaucrat (or the policy-maker) as having private information. In Niskanen (1971), information is related to the production function of the bureaucrat's services. In Laffont and Tirole (1993) information is related to the monitoring task that this bureaucrat undertakes.Stratman (2005) argued that interest groups face significant uncertainty on the preferences of legislators and that this uncertainty can only be alleviated over time by seeing how legislators respond through their voting behavior to PACs' contributions. Higher uncertainty may thus arise for younger legislators who have not yet revealed much on their preferences.

Justification of assumptions of the model: The quadratic form of utilities is a standard assumption in the political economy literature.[6] This assumption can be relaxed by considering concave-single peaked utilities. However, sharp predictions will be lost and this generalization will come at cost of significant complexity.

Symmetry of the principals' ideal points makes the analysis simpler. Most of our results can be generalized to the case of asymmetric principals at the cost of more cumbersome

[5] Gawande et al. (2005) provided an interesting empirical study along these lines.

[6] Austen-Smith and Banks (2000).

notations.[7] In tandem with quadratic preferences, taking a uniform distribution of ideal points yields tractability and thus sharp predictions on equilibrium policies and payoffs.

The agent's ideal point θ being privately known, contributions also serve (as usual in the screening literature) to elicit this parameter.In economic applications it is often assumed that the transfers from the principals to the agent may not be negative. In the companion paper[8] to include the interesting cases of regulatory capture on the inception stage, we assume for the *ex ante* contracting that the transfers are non negative in the expected terms.

The most troublesome issue is related to the timing of the lobbying game. In the absence of formally written contract and in the absence of Court who can enforce an informal agreement the assumption that payments can be enforced is quite strong. In the lobbying literature following Grossman and Helpman (1994) this assumption is motivated by an informal reputation argument that such payments are enforceable because of prospective future interactions. Evidences in support of this model in trade can be found in Goldberg and Maggi (1999) and Bradford (2003). These evidences suggest that the heuristic reputation argument seems to work, though it of course requires a formal treatment in future.

To simplify the analysis, the set of feasible policies is the whole real line $\mathbb{R}$. This assumption is not restrictive: In all equilibria below, the range of policies is located within the extreme ideal points of the players. Thus, considering the reduced form of the model where the decision variable is free to change we, in fact, ex post satisfy the constraints on q in a sense that it belongs to the region between the ideal points of the players.

Common Agency under Complete Information: In a complete information framework the efficient policy will be implemented.[9] The efficient policy $q^{FB}(\theta)$ maximizes the aggregate payoff of the grand coalition made of both principals and their common agent:

$$q^{FB}(\theta) = \arg\max_{q\in\mathbb{R}} \left\{ \sum_{i=1}^{2} V_i(q, t_i) + U\left(q, \sum_{i=1}^{2} t_i, \theta\right) \right\} = \frac{\beta\theta}{\beta+2}. \tag{1}$$

As the decision-maker's ideological bias is stronger (i.e., β increases), the optimal policy is shifted towards his own ideal point. Nevertheless, this policy always reflects both existing groups' preferences. This efficient policy is implemented at an equilibrium of the common agency game provided that the interest groups commit to the following truthful concave

[7] In that case we will have to include a new parameter characterizing the degree of asymmetry between principals.

[8] Martimort and Semenov (2007b).

[9] Bernheim and Whinston (1986).

contributions:

$$t_i(q) = \max\left\{0, -\frac{1}{2}(q-a_i)^2 - C_i\right\},$$

for some constant C_i and where we make explicit the dependence of this schedule on θ since this parameter is common knowledge. The agent is always (at least weakly) better off accepting all such non-negative schedules, and with such payments, each interest group P_i makes the decision-maker a residual claimant for the payoff of the bilateral coalition they form altogether. This ensures an efficient aggregation of preferences.

Although strict concavity of the objective functions ensures uniqueness of the efficient policy, many possible distributions of equilibrium payoffs might be feasible. A payoff vector for the interest group corresponds to a pair (C_1, C_2) which ensures that the agent is at least as well-off by taking both contracts than accepting only a contribution from one group.

3 The Benchmark of a Coalition of Interest Groups

Under asymmetric information, the decision-maker might get some information rent even when contributions are cooperatively designed had interest groups merged into a single entity. Of course, the optimal policy is no longer efficient as a result of a trade-off between rent extraction and allocative efficiency.

The merged entity has now an objective which can be written as:

$$V_M(q,t) = -\frac{1}{2}\{(q-1)^2 + (q+1)^2\} - t,$$

where t is now the groups' joint contribution. The merged entity's ideal point is located at zero. This principal gives more weight to ideology than each interest group separately.

Instead of using the truthful direct revelation mechanism the principal can give up any communication and offer the agent a nonlinear contribution schedule $t(q)$.[10] Let us denote by $U(\theta)$ the agent's payoff when he accepts the contribution $t(q)$ and $q(\theta)$ the corresponding optimal policy. By definition, we have:

$$U(\theta) = \max_{q\in\mathbb{R}}\left\{t(q) - \frac{\beta}{2}(q-\theta)^2\right\} \text{ and } q(\theta) = \arg\max_{q\in\mathbb{R}}\left\{t(q) - \frac{\beta}{2}(q-\theta)^2\right\}.$$

The following Lemma characterizes the implementable profiles $\{U(\theta), q(\theta)\}$.

Lemma 1 *: $U(\theta)$ and $q(\theta)$ are almost everywhere differentiable with, at any differentiability point,*

$$\dot{U}(\theta) = \beta\left(q(\theta) - \theta\right), \tag{2}$$

[10] Taxation Principle, see Rochet (1985).

$$\dot{q}(\theta) \geq 0. \tag{3}$$

The set of incentive constraints in the decision-maker's problem is equivalent to conditions (2) and (3). Condition (2) is obtained from the first-order conditions of the agent's optimization problem (local optimality). Condition (3) guarantees the global optimality of the allocation. Together, these conditions fully describe the set of implementable allocations. Hence, under interim contracting, the merged entity solves the following problem:

$$(\mathcal{P}^M): \quad \max_{\{q(\cdot),U(\cdot)\}} \int_{-\delta}^{\delta} \left(-\frac{1}{2}(q(\theta)-1)^2 - \frac{1}{2}(q(\theta)+1)^2 - \frac{\beta}{2}(q(\theta)-\theta)^2 - U(\theta) \right) \frac{d\theta}{2\delta},$$

$$\text{subject to } \dot{U}(\theta) = \beta(q(\theta) - \theta); \quad (2)$$

$$\dot{q}(\theta) \geq 0; \quad (3)$$

$$\text{and } U(\theta) \geq 0, \quad \text{for all } \theta \in \Theta. \tag{4}$$

As usual in screening models, the subset of types where the interim participation constraint (4) binds plays an important role. Contrary to standard screening models where agents have monotonic preferences in terms of the policy choice, the slope of the agent's rent does not necessarily keep a constant sign and the participation constraint may not necessarily bind at the end-points $\pm\delta$.[11] To get a full description of the optimum and limit technicalities associated to that non-standard feature of the screening problem, we will rely on the quadratic utility functions and the fact that the distribution of the agent's ideal point is uniform.

Proposition 1 *: Assume that interest groups jointly design contributions. The optimal policy $q^M(\theta)$ and the decision-maker's information rent $U^M(\theta)$ both depend on the decision-maker's ideological bias.*

Weak Ideological Bias, $0 < \beta < 2$**.**

- *The optimal policy is inefficient and distorted towards the decision-maker's ideal point: $q^M(\theta) = \frac{2\beta\theta}{\beta+2}$, for any $\theta \in \Theta$;*

- *Only moderate decision-makers get information rent, extreme ones don't. This information rent is non-negative, zero at both endpoints $\pm\delta$, and strictly concave:*

$$U^M(\theta) = \frac{\beta(2-\beta)}{2(\beta+2)}\left(\delta^2 - \theta^2\right), \textit{ for any } \theta \in \Theta;$$

[11] This is reminiscent to the analysis of countervailing incentives explored by Lewis and Sappington (1989) and Maggi and Rodriguez-Clare (1995).

- *The coalition of interest groups offers a positive and strictly concave contribution on its positive part which is maximized for the most moderate decision-maker:*

$$t^M(q) = \max\left\{0, -\frac{(2-\beta)}{4}q^2 + \frac{\beta(2-\beta)}{2(\beta+2)}\delta^2\right\}, \text{ for any } q.$$

Strong Ideological Bias, $\beta \geq 2$**.**

- *The optimal policy always coincides with the decision-maker's ideal point:* $q^M(\theta) = \theta$*, for any* $\theta \in \Theta$*;*

- *The decision-maker's information rent is always zero:* $U^M(\theta) = 0$*, for any* $\theta \in \Theta$*;*

- *There is no contribution.*

Under asymmetric information, policies are inefficient and are always *closer* to the agent's ideal policy than under complete information. For intuition, consider the hypothetical case where the merged entity still wants to reward the decision-maker for implementing an efficient policy $q^{FB}(\theta) = \frac{\beta\theta}{\beta+2}$, exactly as under complete information. To induce the decision-maker to reveal his ideal point, he must receive some information rent. The corresponding rent profile is rather concave with a steep increasing part on $[-\delta, 0]$ (with slope $\frac{2\beta\theta}{\beta+2}$) and a steep decreasing part on $[0, \delta]$ (with slope $-\frac{2\beta\theta}{\beta+2}$).[12] Of course, this rent is viewed as costly from the point of view of the coalition of interest groups. Reducing this rent is done by making it somewhat flatter, i.e., by better aligning the policy $q(\theta)$ with the decision-maker's ideal point so that the quantity $q(\theta) - \theta$ is reduced in (2). Shifting the optimal policy towards the agent's ideal point and making it more sensitive to his ideological preferences reduces the agent's rent.

The impact of asymmetric information under monopolistic screening is thus akin to an increase of the agent's bargaining weight within the grand-coalition, making his preferences more relevant to evaluate policy outcomes.

The decision-maker's rent decreases with his ideological distance with the merged principal. This is an important feature of optimal contracting under monopolistic screening. Remember that the coalition of interest groups has preferences which are aligned with those of a decision-maker lying in the middle of the ideology space. Inducing less moderate types to adopt a policy closer to that of the coalition requires giving them enough contributions and that may be found attractive by the more moderate types. Those types have incentives to pretend being more extreme. To avoid this problem, the coalition of

[12] The nonlinear contribution $t^{FB}(q) = \frac{\beta\delta^2}{\beta+2} - \beta q^2$ generates this profile and ensures that all types participate with the most extreme decision-makers being just indifferent between participating or not.

interest groups concedes that the policy implemented will be inefficient and thus closer to that of more extreme decision-makers. Second, more moderate decision-makers receive an information rent to avoid taking more extreme stances.

When the decision-maker has a sufficiently strong ideological bias ($\beta \geq 2$), it becomes too costly for interest groups to move the decision-maker away from his ideal point. The merged entity prefers not contributing at all and always let the agent choose his ideal point. Influence has to be easy to buy to allow the coalition of interest groups to overcome the cost of asymmetric information and have access to the decision-maker. Otherwise, transaction costs of asymmetric information keep the coalition of interest groups outside of the political process. This effect will be significantly magnified when interest groups compete since it appears for lower values of β.

4 Competition Between Interest Groups

So far, our analysis has only emphasized informational asymmetries as the only potential source of rent for the decision-maker. Competition between interest groups and their conflicting desires to influence the decision-maker introduces another source of rent under competitive screening.

We focus on pure strategy equilibria of the common agency game. A deterministic contract between the interest group P_i and the decision-maker is a nonlinear contribution $t_i(q)$ which maps the decision-maker's choice of the policy to the transfer paid by P_i. The Taxation Principle says that for any direct incentive compatible mechanism there exists a contribution schedule $t_i(q)$ that implements the same outcome. Contrary to the standard version of the Revelation principle, this Taxation Principle is still useful in non-cooperative screening environments. Peters (2001) and Martimort and Stole (2002) show that there is no loss of generality in restricting the analysis to the case where principals compete through nonlinear contributions $t_i(q)$ ($i = 1, 2$).

Let us denote by $U(\theta)$ the decision-maker's payoff when he accepts both contributions $\{t_1(q), t_2(q)\}$ and let $q(\theta)$ be the chosen policy. The rent-policy profile $\{U(\theta), q(\theta)\}$ which is implemented by the pair of contributions $\{t_1(q), t_2(q)\}$ satisfies:

$$U(\theta) = \max_{q \in \mathbb{R}} \left\{ \sum_{i=1}^{2} t_i(q) - \frac{\beta}{2}(q-\theta)^2 \right\} \text{ and } q(\theta) = \arg\max_{q \in \mathbb{R}} \left\{ \sum_{i=1}^{2} t_i(q) - \frac{\beta}{2}(q-\theta)^2 \right\}.$$

Similarly, the rent-policy profile $\{U_i(\theta), q_i(\theta)\}$ that is implemented had the agent only accepted principal P_i's contribution is defined as:

$$U_i(\theta) = \max_{q \in \mathbb{R}} \left\{ t_i(q) - \frac{\beta}{2}(q-\theta)^2 \right\} \text{ and } q_i(\theta) = \arg\max_{q \in \mathbb{R}} \left\{ t_i(q) - \frac{\beta}{2}(q-\theta)^2 \right\}.$$

For a given contribution $t^*_{-i}(q)$ offered by principal P_{-i}, principal P_i's best-response solves now the following problem:

$$(\mathcal{P}_i^C): \max_{\{q(\cdot),U(\cdot)\}} \int_{-\delta}^{\delta} \left(-\frac{1}{2}(q(\theta)-a_i)^2 - \frac{\beta}{2}(q(\theta)-\theta)^2 + t^*_{-i}(q(\theta)) - U(\theta)\right)\frac{d\theta}{2\delta},$$

$$\text{subject to } \dot{U}(\theta) = \beta(q(\theta)-\theta); \quad (2)$$

$$\dot{q}(\theta) \geq 0; \quad (3)$$

$$U(\theta) \geq 0; \quad (4)$$

$$\text{and } U(\theta) \geq U_{-i}(\theta) \quad \text{for all} \quad \theta \in \Theta. \tag{5}$$

The new participation constraint (5) ensures that the decision-maker prefers taking both contributions rather than accepting only P_{-i}'s contract. The immediate consequence of the non-negativity of transfers is that the ex post zero participation constraint (4) is always implied by (5).

Definition 1 *: A subgame-perfect equilibrium of the common agency game under asymmetric information is a pair of contributions $\{t_1^*(\cdot), t_2^*(\cdot)\}$ which implement a rent-policy profile $\{U^*(\theta), q^*(\theta)\}$ solving both $(\mathcal{P}_1^C)$ and $(\mathcal{P}_2^C)$.*

To solve the problem $(\mathcal{P}_i^C)$ given the contribution schedule $t^*_{-i}(q)$ we form a Hamiltonian of the system:

$$H_i(U, q, \lambda_i, \theta) = \left\{-\frac{1}{2}(q-a_i)^2 - \frac{\beta}{2}(q-\theta)^2 + t^*_{-i}(q) - U\right\}\frac{1}{2\delta} + \lambda_i\beta(q-\theta),$$

where λ_i is the co-state variable. Maximizing the Hamiltonian with respect to the control variable q leads to the following first-order condition:

$$-(\beta+1)q(\theta) + a_i + \beta\theta + t^{*\prime}_{-i}(q(\theta)) + 2\lambda_i(\theta)\beta\delta = 0. \tag{6}$$

Different kinds of differentiable equilibria may be sustained depending on the parameter values for β and δ. Those equilibria are characterized by different areas where the participation constraint (5) binds. To compute the interest groups' equilibrium payoffs, we need to determine the decision-maker's payoff when taking only one contract. To do so requires to extend contributions for off the equilibrium outputs as long as these contributions remain non-negative.[13] Following Martimort and Stole (2007), we shall focus on equilibria with *natural contributions* which keep then the same analytical expressions

[13] This is a familiar argument from Bernheim and Whinston (1986), Klemperer and Meyer (1989) or Martimort and Stole (2003) who reduced the problem of the multiplicity of equilibria in multiprincipals settings by putting restrictions on out-of-equilibirum strategies.

(6) both on and off the equilibrium path as long as they are non-negative. One motivation for focusing on this particular extension is that the marginal contributions in those equilibria are kept unchanged as the support of the type distribution is slightly enlarged still keeping a uniform distribution on this enlarged support. In the rest of our analysis, equilibria should be understood as being natural equilibria.

4.1 The "Laissez-Faire" Equilibrium: No Influence

First, let us investigate whether there might exist an equilibrium such that none of the interest groups ever contributes. As a result, the decision-maker always chooses his ideal point: a "laissez-faire" outcome. In this case, and although they are both organized, both groups simultaneously eschew any contribution and look as being inactive.

Proposition 2 : Strong Ideological Bias/Large Ideological Uncertainty/Small Polarization. *Assume that $\beta \geq 1$ and that $1 \leq \delta$. There is no contribution by either interest group in the unique equilibrium. The decision-maker always chooses his ideal point $q^*(\theta) = \theta$ and gets zero information rent $U^*(\theta) = 0$ for all $\theta \in \Theta$.*

Lobbying competition under asymmetric information significantly erodes the interest groups' influence when the decision-maker puts enough weight on ideology and there is enough ideological uncertainty, i.e., when the interest groups' ideal points both lie within the interval defined by the most extreme views of the agent.[14] Everything happens then as if the decision-maker was free from any influence and could always choose his most preferred policy.

The same zero-contribution outcome already occurred under merged contracting when $\beta \geq 2$ (see Proposition 1). The point is that the non-cooperative behavior of principals exacerbates this effect which now arises also for lower levels of β as well, i.e., even when the decision-maker gives only a moderate weight to his ideology.

To give some intuition on this result, let us think about the case with only one interest group, say P_1 and let us look for the optimal contribution that P_1 offers in such a hypothetical monopolistic screening environment. Compared to the analysis performed in Section 3, this single principal is now biased on one side of the policy space: He prefers a higher policy than the decision-maker's average ideal point. Also, ideology matters less at the margin for this principal than for the merged entity since P_1 does not care about P_2's utility. Even if both this biased principal and the merged entity have to give up the same information rent in order to implement a given policy profile and should shift the optimal policy towards the decision-maker's ideal point by the same amount, P_1 suffers

[14] It can be verified that this result is robust to introducing some asymmetry between interest groups.

more from that increase than the merged entity. This implicit increase in the weight given to the agent's ideology erodes all P_1's bargaining power when there is enough ideological uncertainty. P_1 no longer influences the agent's choice. Considering now the case of two groups, an equilibrium where both principals do not contribute arises.

This result may explain the apparent nearly welfare-maximizing behavior of U.S. policy-makers in some areas, especially trade policy. Using a complete information model, Gawande at al. (2005) argued for instance that the competition between interest groups (final producers and intermediate ones) whose impacts cancel out might explain that the estimated implicit weight given to contributions in the decision-maker's preferences is low. They nevertheless concluded their study by noticing that their estimated parameter is still excessively high. Introducing asymmetric information in such analysis would magnify the policy bias towards the agent's ideal point (who may have as ideal point a social-welfare maximizing policy) and might help to solve this empirical puzzle.

4.2 Partial Influence: Non-Overlapping Areas

In the "laissez-faire" equilibrium, none of the interest groups secures any area of influence. Even when the ideological distance between the agent and an interest group is small, this principal cannot make sure that the decision-maker will only follow his own recommendation because there is too much uncertainty on the decision-maker's preferences which may be too far away from that of the group.

We now investigate conditions under which such unchallenged influence occurs instead. The market for influence is then segmented with interest groups on both sides of the political spectrum being linked in exclusive relationships with decision-makers who are ideologically sufficiently close. We call such a pattern of influence a *partition equilibrium of type 1.* From Proposition 2, the degree of polarization between interest groups (resp. the ideological uncertainty) must increase (resp. decrease) for such pattern to arise.

Definition 2 *: In a partition equilibrium of type 1, principal P_i offers a positive contribution only on a non-empty subset Ω_i of Θ. Moreover, the principals' areas of influence are disconnected, i.e., $\Omega_1 \cap \Omega_2 = \emptyset$. A partition equilibrium of type 1 is symmetric ($SPE1$) when there exists $\tau \in (0, \delta)$ such that $\Omega_2 = [-\delta, -\tau]$ and $\Omega_1 = [\tau, \delta]$. We also denote by $\Omega_0 = [-\tau, \tau]$ the area where none of the principals contribute.*

We provide below conditions ensuring existence and uniqueness of a $SPE1$.

Proposition 3 *:* **Strong Ideological Bias/Intermediate Ideological Uncertainty.** *Assume that $\beta > 1$ and $\delta < 1 \leq \beta\delta$. The unique equilibrium of the common agency game is a $SPE1$. Given that $\tau = \tau_1 = -\tau_2 = \frac{\beta\delta - 1}{\beta - 1}$, this equilibrium entails:*

- *A policy $q^*(\theta)$ which reflects the preferences of the contributing interest group only for the most extreme realizations of θ and otherwise is equal to the decision-maker's ideal point*

$$q^*(\theta) = \begin{cases} \frac{2\beta\theta-(\beta-1)\tau_i}{\beta+1} & \text{if} \quad \theta \in \Omega_i, \\ \theta & \text{if} \quad \theta \in \Omega_0; \end{cases}$$

- *The decision-maker's information rent $U^*(\theta)$ is convex and null only on Ω_0*

$$U^*(\theta) = \begin{cases} \frac{\beta(\beta-1)}{2(\beta+1)}(\theta-\tau_i)^2 & \text{if} \quad \theta \in \Omega_i, \\ 0 & \text{if} \quad \theta \in \Omega_0; \end{cases}$$

- *Contributions are piecewise continuously differentiable and convex*

$$t_1^*(q) = \begin{cases} \frac{(\beta-1)}{4\beta}(q-\tau)^2 & \text{if} \quad q \geq \tau, \\ 0 & \text{otherwise}; \end{cases}$$

$$t_2^*(q) = \begin{cases} \frac{(\beta-1)}{4\beta}(q+\tau)^2 & \text{if} \quad q \leq -\tau, \\ 0 & \text{otherwise.} \end{cases}$$

On Figures 1a and 1b, we have represented respectively equilibrium policy and contributions with disconnected areas of influence for parameters $\beta = 2$ and $\delta = 0.7$.

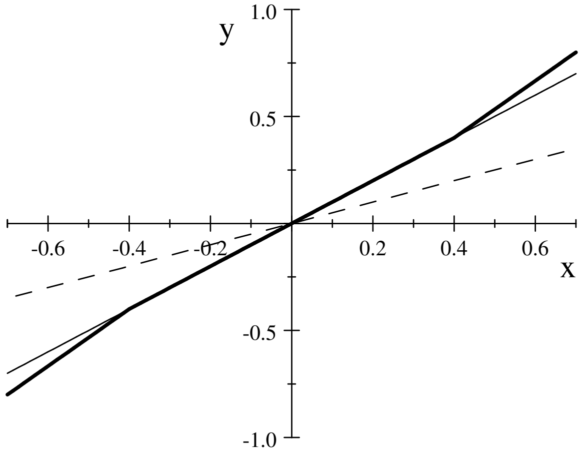

Figure 1a. Policy $q(\theta)$ - thick line, "laissez-faire" policy - thin line, efficient policy - dashed line.

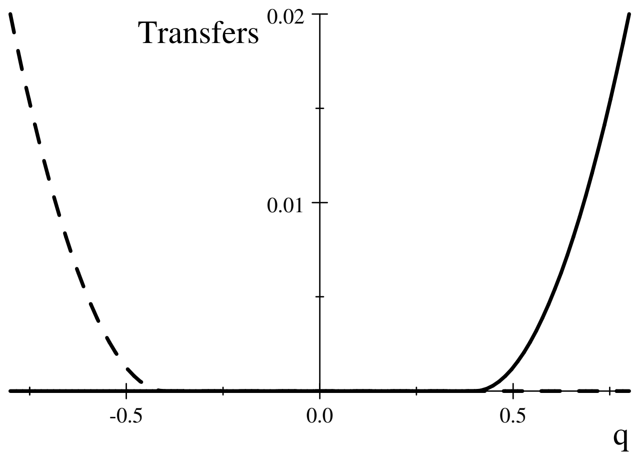

Figure 1b. Disconnected transfers: $t_1^*(q)$ - solid line, $t_2^*(q)$ - dashed line.

A partition equilibrium shares some common features with the "laissez-faire" equilibrium. In both cases the decision-maker might be freed from the principals' influence but now this occurs only when the decision-maker is moderate enough. Interest groups are now able to secure unchallenged influence when their ideological distance with the agent is sufficiently small. The most extreme decision-makers are thus linked in exclusive relationships with their near-by groups.

To understand the shape of contributions and the equilibrium pattern, it is important to first think about the case where one principal, say P_1, is alone and has more extreme views than the agent ($1 \geq \delta$). Under complete information, the optimal policy that such bilateral coalition would implement would be an average of the respective ideal points of the interest group and the decision-maker, namely $q_1^*(\theta) = \frac{\beta\theta+1}{\beta+1}$. The key observation is that, when $\beta > 1$, this policy schedule is very sensitive to the agent's ideal point. The same is true of course for the policy that such bilateral coalition would implement under asymmetric information. This sensitivity means that, as the agent becomes more extreme, the difference between his own ideal point and what the principal would like to implement increases significantly. Filling this gap requires an increasingly higher marginal contribution as policies become more extreme and come closer to the principal's ideal point. This explains why equilibrium contributions are in fact convex and increase as the ideological distance between the agent and the principal diminishes.

When ideology matters greatly to the decision-maker, the most extreme types obtain a positive rent by pretending being more moderate than what they really are. To reduce this rent, interest groups reduce contributions for more moderate types. Doing so is of course constrained by the fact that the decision-maker may refuse any contribution and choose his own ideal point. Contributions are thus null for moderate decision-makers who are ideologically too far away. The limiting case being given by the "laissez-faire" equilibrium of Proposition 2.

4.3 Partially Overlapping Areas of Influence

Key to the result of Proposition 3 is the fact that interest groups are sufficiently far apart to secure an unchallenged influence on an ideologically adjacent decision-maker. Suppose now that interest groups are more polarized, or alternatively that ideological uncertainty diminishes. Transaction costs of contracting under asymmetric information diminish and both interest groups suffer less from not knowing the decision-maker's preferences. Moderate decision-makers receive now positive contributions from both interest groups. We will call such a pattern of influence a *partition equilibrium of type 2* .

Definition 3 *: In a partition equilibrium of type 2, principal P_i offers a positive contri-*

bution only on a non-empty subset Ω_i of Θ. Moreover, the principals' areas of influence overlap, i.e., $\Omega_1 \cap \Omega_2 = \Omega_0 \neq \emptyset$ where Ω_0 is the area where both principals simultaneously intervene. A partition equilibrium of type 2 is symmetric ($SPE2$) when there exists $\tau \in (0, \delta)$ such that $\Omega_2 = [-\delta, \tau]$ and $\Omega_1 = [-\tau, \delta]$.

We provide below the conditions ensuring existence of an $SPE2$. As in the case of disconnected areas of influence this equilibrium is unique.[15]

Proposition 4 : Strong Ideological Bias, More Polarization/Smaller Ideological Uncertainty. *Assume that $\beta > 1$ and $\beta\delta < 1 \leq \frac{\beta(2\beta+1)}{(\beta+2)}\delta$. The unique equilibrium of the common agency game is a $SPE2$. Denoting $\gamma = 1 - \beta\delta$, and $\tau = \tau_1 = -\tau_2 = \frac{(\beta+2)\gamma}{\beta(\beta-1)}$, this equilibrium entails:*

- *A policy $q^*(\theta)$ which reflects the preferences of all groups only for moderate decision-makers and is otherwise biased towards the preferences of the nearby group for extreme ones*

$$q^*(\theta) = \begin{cases} \frac{2\beta\theta+\gamma}{\beta+1} & \text{if} \quad \theta \in [\tau, \delta], \\ \frac{3\beta\theta}{\beta+2} & \text{if} \quad \theta \in [-\tau, -\tau], \\ \frac{2\beta\theta-\gamma}{\beta+1} & \text{if} \quad \theta \in [-\delta, -\tau]; \end{cases}$$

- *The decision-maker's information rent $U^*(\theta)$ is convex and minimized for the most moderate type with $U^*(0) = \frac{3\gamma^2}{\beta-1} > 0$;*

- *Contributions are convex and continuously differentiable*

$$t_1^*(q) = \begin{cases} 0 & \text{if} \quad q \leq \frac{-3\gamma}{\beta-1}, \\ \frac{(\beta-1)}{6}q^2 + \gamma q + \frac{3\gamma^2}{2(\beta-1)} & \text{if} \quad q \in [\frac{-3\gamma}{\beta-1}, \frac{3\gamma}{\beta-1}], \\ \frac{(\beta-1)}{4}q^2 + \frac{\gamma}{2}q + \frac{9\gamma^2}{4(\beta-1)} & \text{if} \quad q \geq \frac{3\gamma}{\beta-1}; \end{cases}$$

$$t_2^*(q) = \begin{cases} 0 & \text{if} \quad q \geq \frac{3\gamma}{\beta-1}, \\ \frac{(\beta-1)}{6}q^2 - \gamma q + \frac{3\gamma^2}{2(\beta-1)} & \text{if} \quad q \in [\frac{-3\gamma}{\beta-1}, \frac{3\gamma}{\beta-1}], \\ \frac{(\beta-1)}{4}q^2 - \frac{\gamma}{2}q + \frac{9\gamma^2}{4(\beta-1)} & \text{if} \quad q \leq \frac{3\gamma}{\beta-1}. \end{cases}$$

On Figures 2a and 2b, we have represented respectively the equilibrium policy and contributions with disconnected areas of influence for parameters $\beta = 2$ and $\delta = 0.47$.

[15] Note that the set of parameter values corresponding to both types of equilibria do not overlap.

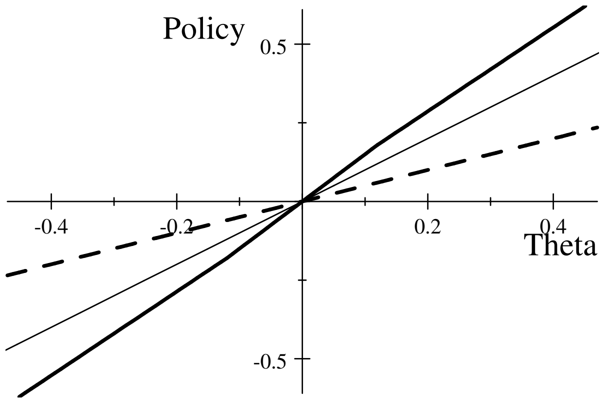

Figure 2a. Policy $q(\theta)$ - thick line, "laissez-faire" policy - thin line, efficient policy - dashed line.

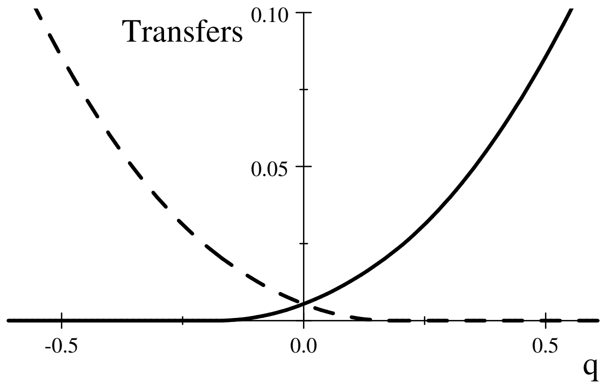

Figure 2b. Partially overlapping transfers: $t_1^*(q)$ - solid line, $t_2^*(q)$ - dashed line.

As polarization increases, each interest group finds it relatively easy to influence nearby types and is ready to give them a positive rent. The point is that, for the rival interest group on the other side of the ideological space, policies led by those far away types may be excessively biased towards opposite views. To counter this effect, this second principal must himself reward the agent even if the latter is on the opposite side of the ideological spectrum. This strategy is of course valuable as long as the ideological distance with the agent is not too large. In such cases, areas of influence start overlapping for the most moderate decision-makers.[16]

4.4 Fully Overlapping Areas of Influence

We distinguish now between two cases depending on whether or not the ideology matters greatly in the decision-maker's utility function.

4.4.1 Strong Ideological Bias

Proposition 4 already shows that, as ideological uncertainty decreases, the interest groups' areas of influence start overlapping. When uncertainty is small enough or alternatively when the degree of polarization is sufficiently large, both groups are able to always influence the decision-maker whatever his own ideal point is as shown in the next proposition.

[16] This result is due to our assumption of interim contracting, i.e., the decision-maker accepts contributions after knowing his own ideology. Moving policy towards his own ideal point requires that a principal offers a positive contribution to the informed agent. Under competition, contributions must thus be piled up for the most moderate types to keep them choosing a balanced policy.

Proposition 5 : Strong Ideological Bias, Large Polarization/Small Ideological Uncertainty. *Assume that $\beta > 1$ and $1 > \frac{(2\beta+1)}{(\beta+2)}\beta\delta$. The interest groups' areas of influence fully overlap in the unique equilibrium. Still denoting $\gamma = 1-\beta\delta$, this equilibrium entails:*

- *A policy $q^*(\theta)$ which is more biased towards the decision-maker's ideal point than when groups cooperate*

$$q^*(\theta) = \frac{3\beta\theta}{\beta+2} \text{ with } |q^*(\theta)-\theta| \leq |q^M(\theta)-\theta|, \text{ for all } \theta \in \Theta; \tag{7}$$

- *A convex rent profile $U^*(\theta)$ which is strictly positive everywhere*

$$U^*(\theta) \;=\; \frac{\beta(\beta-1)}{\beta+2}\theta^2 - 2C, \text{ for all } \theta \in \Theta;$$

- *Contributions are convex and positive everywhere*

$$t_1^*(q) = \max\left\{\frac{\beta-1}{6}q^2 + \gamma q - C, 0\right\}, \; t_2^*(q) = \max\left\{\frac{\beta-1}{6}q^2 - \gamma q - C, 0\right\}$$

$$\text{with } C = \frac{3}{2}\left(\frac{\beta^2\delta^2}{\beta+2} - \frac{1}{2\beta+1}\right) < 0.$$

On Figures 3a and 3b, we have represented respectively the equilibrium policy and contributions with overlapping areas of influence for $\beta = 2$ and $\delta = 0.2$.

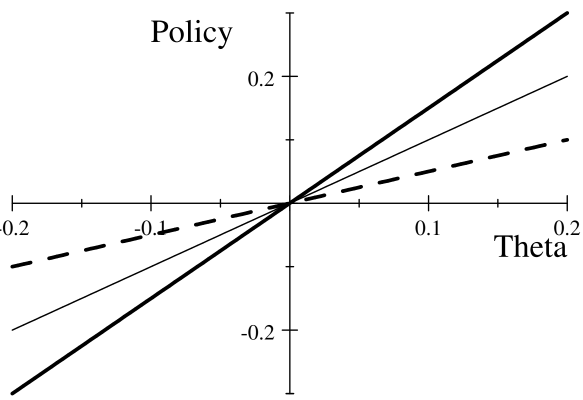

Figure 3a. Policy $q(\theta)$ - thick line, "laissez-faire" policy - thin line, efficient policy - dashed line.

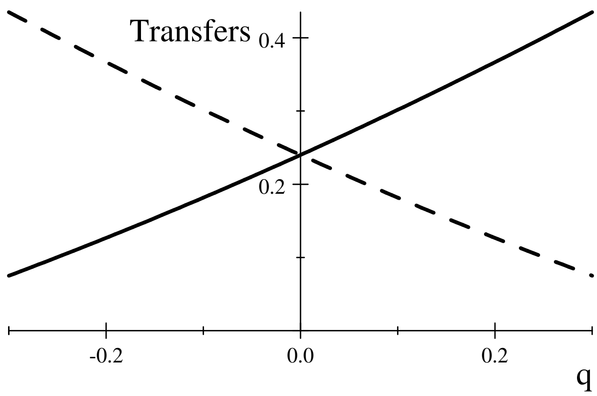

Figure 3b. Fully overlapping transfers: $t_1^*(q)$ - solid line, $t_2^*(q)$ - dashed line.

Altogether, Propositions 2, 3, 4 and 5 provide some interesting comparative statics on the role of ideological uncertainty. The decision-maker's rent is null only when there is enough ideological uncertainty compared with the degree of polarization between groups. As ideological uncertainty diminishes, each principal secures an area of influence if the decision-maker is ideologically sufficiently close and let the other principal enjoy unchallenged influence if he is further away. A moderate decision-maker receives positive contributions only when ideological uncertainty is sufficiently small and groups find it worthwhile to compete more head-to-head for the agent's services, thereby raising contributions and giving up some positive rent. Fully overlapping areas of influence arise when ideological uncertainty is very small. Interest groups compete now for all types and, whatever his type, the decision-maker gets a positive rent.

This rent has now two sources. First, interest groups find worthwhile bidding for the agent's services as his ideology is more certain. But now, an extra source of information rent comes from the possibility for an extreme decision-maker to behave as being more moderate. In fact by doing so, he would increase the ideological distance with both principals and grasp greater contributions.

4.4.2 Weak Ideological Bias

When $\beta < 1$, the decision-maker's ideology becomes less of a concern and interest groups can now quite easily influence his decision. Intuitively, one should expect that the resulting equilibrium policy to be less sensitive to ideology than before. This means that as the agent becomes more extreme, the difference between his ideal point and what the nearby principal would like to implement decreases. Filling this gap requires an increasingly lower marginal contribution as policies become more extreme. The equilibrium contribution of a given group is now concave and increasing as his own ideal point comes closer to that of the decision-maker.

The next two propositions describe the different equilibrium patterns in distinguishing, as we usually do, between the cases of a large and a small ideological uncertainty. We comment below on the differences and similarities between those two cases.

Proposition 6 : Weak Ideological Bias, Small Polarization/Large Ideological Uncertainty. *Assume that $\beta < 1$ and $1 \leq (2\beta+1)\delta\sqrt{\frac{\beta}{3(\beta+2)}}$. Then, there exists $\lambda_1^* \leq \frac{1}{2}$ such that, for each $\lambda_1 \in [-\frac{1}{2}, \lambda_1^*)$, such that $1 < (2\beta+1)\delta\sqrt{\frac{\beta}{3(\beta+2)}} - \beta\delta(1+2\lambda_1)$, there exists a continuum of equilibria which have fully overlapping influences and entail:*

- *A policy $q^*(\theta) = \frac{3\beta\theta}{\beta+2}$ is again more biased towards the decision-maker's ideal point than when groups cooperate;*
- *A strictly concave rent profile $U^*(\theta)$ which is zero only at both endpoints $\pm\delta$ and*

always lower than when groups cooperate

$$U^*(\theta) \quad = \quad \frac{\beta(1-\beta)}{\beta+2}(\delta^2-\theta^2) \le U^M(\theta) \ \textit{for all}\ \theta \in \Theta;$$

- *Contributions are strictly concave on their positive part*

$$t_1^*(q) = \max\left\{0, -\frac{1-\beta}{6}q^2 + \gamma q - C_1\right\}\ ,\ t_2^*(q) = \max\left\{0, -\frac{1-\beta}{6}q^2 - \gamma q - C_2\right\}, \tag{8}$$

where $\gamma = 1 + 2\beta\delta\lambda_1$*. There exists an interval of possible values for the constants* (C_1, C_2)*. Those constants satisfy the following linear constraints:*

$$C_1 + C_2 = -\frac{\beta(1-\beta)}{\beta+2}\delta^2,\ \textit{and}\ C_i \ge \frac{3(\beta\delta+\gamma)^2}{2(2\beta+1)} - \frac{\beta\delta^2}{2},\ i = 1, 2.$$

Turning now to the case of a small ideological uncertainty, we obtain again the uniqueness of the equilibrium. When the ideological uncertainty is small the issue of competition between principals dominates coordination.

Proposition 7 : Weak Ideological Bias, Large Polarization/Small Ideological Uncertainty. *Assume that* $\beta < 1$ *and* $1 > (2\beta+1)\delta\sqrt{\frac{\beta}{3(\beta+2)}}$*. The unique equilibrium has fully overlapping influences and entails:*

- *An equilibrium policy* $q^*(\theta) = \frac{3\beta\theta}{\beta+2}$*;*
- *A strictly concave rent profile* $U^*(\theta)$ *which is strictly positive at both endpoints* $\pm\delta$

$$U^*(\theta) \quad = \quad \frac{\beta(1-\beta)}{\beta+2}(\delta^2-\theta^2) - 2C \ \textit{for all}\ \theta \in \Theta;$$

- *Contributions are strictly concave on their positive part*

$$t_1^*(q) = \max\left\{0, -\frac{1-\beta}{6}q^2 + \gamma q - C\right\}\ ,\ t_2^*(q) = \max\left\{0, -\frac{1-\beta}{6}q^2 - \gamma q - C\right\}, \tag{9}$$

where $\gamma = 1 - \beta\delta$ *and* $C = \frac{3}{2}\left(\frac{\beta^2\delta^2}{\beta+2} - \frac{1}{2\beta+1}\right)$.

On Figure 4 we draw the equilibrium policy in the case of $\beta = \frac{1}{2}$ and $\delta = 1$.

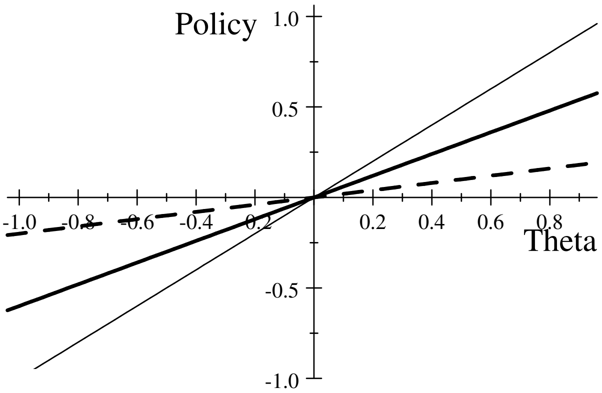

Figure 4. Policy $q(\theta)$ - thick line, "laissez-faire" policy - thin line, efficient policy - dashed line.

When $\beta < 1$, the equilibrium shares some common features with the optimal contribution and policy achieved had interest groups cooperated in designing contributions: Everything happens as if the decision-maker's ideology had a greater implicit weight in the policy process. However, this effect is magnified compared to the cooperative outcome. The equilibrium policy comes closer to the decision-maker's ideal point. Because $|q^*(\theta) - \theta| \leq |q^M(\theta) - \theta|$, the rent profile under lobbying competition is now flatter. To reduce the agent's information rent, *each* interest group needs to shift the policy towards the agent's ideal point and, to do so, offers a relatively flat contribution. However, a given interest group does not take into account that his rival also offers such a flat contribution so that the equilibrium policy is already significantly shifted towards the decision-maker's ideal point. The rent profile is excessively flat compared with merged contracting.[17] This contractual externality between principals leads to an excessive bias towards the agent's ideal point compared to the case where contributions are jointly designed.

When ideology does not matter greatly to the decision-makers, the moderate ones receive more rent than extreme ones. Exactly as when groups collude, those types may want to look more extreme than what they really are to raise contributions. Two cases may then arise.

If polarization is strong or ideological uncertainty is small: Head-to-head competition between interest groups ensures that the equilibrium payoffs of all players are uniquely defined. Even the most extreme types get a positive rent out of the principals' aggressive bidding for their services.

[17] This revisits in the context of spatial preferences a result already found in other common agency games with public screening devices in case where principals have monotonic preferences (see for instance Martimort and Stole (2007)).

If polarization is weak or ideological uncertainty sufficiently large: Interest groups now become more congruent. They both want to extract as much rent as possible from the agent. Because a moderate decision-maker can look more extreme, he must obtain some rent. This congruence between competing interest groups creates a coordination problem leading to multiple equilibria. Multiplicity affects both the levels of contributions and their margins. First, there are different ways of designing contributions so that interest groups collectively extract the rent of the most extreme types and prevent those types from offering services exclusively to the closer interest group. This coordination problem affects only the level of contributions. Second, for a given amount of ideological uncertainty, interest groups compete more fiercely for the services of the most extreme decision-maker; with the group the further away from an extreme decision-maker having to concede the most to get influence. Indeed, an extreme agent finds it more attractive to take only the contribution of the nearby group. This hardens his participation constraint and makes him a tougher bargainer with the opposite interest group.[18] As competition for the services of an extreme decision-maker becomes tougher, the screening possibilities of the interest group which is on the other side of the ideological space become more limited. This increases the marginal contribution of this group.

5 Conclusion and Directions for Future Research

Let us briefly recapitulate the main results of our analysis. In passing, we suggest some testable implications that immediately follow from our work.

Inefficient Policies: Under asymmetric information, competition between interest groups leads to huge inefficiencies in policy choices. There always exists a strong bias towards the decision-maker's ideal point. If ideological uncertainty is too important, transaction costs become too large. Interest groups might prefer to eschew any contribution and leave the decision-maker free to pursue his own ideological views.

Contributions and Segmentation of the Market for Influence: When his ideological bias is strong and there is enough uncertainty, interest groups may choose not to contribute to a decision-maker whose ideal point lies too far away from their own preferences. The market for influence is segmented with exclusive relationships between interest groups and legislators on their side of the political spectrum. This case is likely to occur for general policies that have a broad appeal to the public and that decision-makers value greatly (maybe for electoral concerns), for young legislators who have not yet revealed much on their preferences through past voting behavior, and for those who have not shown

[18] There remains some freedom in choosing the corresponding multiplier λ_1 with that multiplier being greater if the reservation payoff that the agent gets by taking only one contract is steeper.

any expertise or interest in the field under scrutiny.

As ideological uncertainty decreases, the areas of influence of competing groups start to overlap. More extreme legislators keep collecting most of the contributions though they may still receive contributions from opposite groups. One should thus expect older decision-makers whose preferences are better known to gain more support from both sides of the political spectrum.

If the decision-maker's ideological bias is not so strong, maybe because the policy at stake is sector specific and has little appeal for the general public, interest groups will always contribute a positive amount. However, the nature of competition is highly dependent on the amount of uncertainty. Interest groups are more congruent when facing much ideological uncertainty since it becomes quite likely that their ideal policies stand both on the same side of the decision-maker's own ideal point. The main features of the pattern of contributions then seem very much as though groups had cooperated in designing contributions. Moderate legislators collect the bulk of contributions and rent. Instead, with less uncertainty, competition raises contributions even for the most extreme decision-makers.

Uniqueness of equilibria: One striking feature of the common agency game with opposed principals is that we are able to pin down the unique natural equilibrium of the game in many cases. We will see that when the competition between the principals is the issue this competition leads to the unique equilibrium. In case of congruent principals where coordination is the issue there is still an infinity of equilibria even in the case of asymmetric information. The uniqueness is in contrast with the common agency game with complete information which is plagued by multiplicity of equilibria.

Extensions: Let us briefly stress a few possible extensions of our framework. A first one would be to investigate what would happen with more than two interest groups. Incentives of interest groups biased in similar directions to coalesce might be worth investigating in such framework. On a related vein, it could be interesting to introduce some asymmetry between principals, both in terms of how distant their ideal points are from the agent's average ideology and also in terms of their capacity to influence the decision-maker.[19]

Our view of the political process has also been overly simplified by focusing on a one-dimensional policy space. More complex multi-dimensional policy spaces and spatial preferences could be investigated. Interest groups may tailor their contributions to the particular policy dimensions they are interested in or they may make contributions conditional on the whole array of policies. It would be worthwhile investigating whether the strong bias towards the agent's ideal point that arises under interim contracting also occurs in those more general environments. From the earlier common agency literature

[19] See Epstein and O'Halloran (2004) for some steps in those directions.

under asymmetric information,[20] it is well-known that the pattern of policy distortions and rent distributions may depend on the interest groups' ability to contract or not on the whole array of policies.

It is also worthwhile investigating dynamic extensions of the model. By imposing limits on commitment and imperfect information learning over time, such extensions may introduce further frictions that may affect patterns of contributions in interesting ways.

Finally, we have simplified our modelling of the political process assuming a unique decision-maker. A less abstract description of legislative organizations and regulatory agencies would require opening the black-box of those organizations. Multiagent/multiprincipals models which are not yet developed might have some strong appeal in that respect.

All those are extensions that we hope to undertake in future studies.

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[20] Martimort (1992) and Stole (1991).

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Appendix

• **Proof of Lemma 1** The proof is standard and thus omitted. See Laffont and Martimort (2002) for instance. ∎

• **Proof of Proposition 1:** To characterize the optimum with a merged principal, we shall apply the techniques of optimal control. The implementability conditions for a profile $\{U(\theta), q(\theta)\}$ are given by (2) and (3). We ignore the monotonicity constraint (3) in $(\mathcal{P}^{\mathcal{M}})$ and consider the reduced problem $(\widetilde{\mathcal{P}}^M)$ with the state variable $U(\theta)$ and control variable $q(\theta)$. The monotonicity constraint (3) is checked ex post on the solution. The program of the merged entity is now:

$$(\widetilde{\mathcal{P}}^M): \quad \max_{\{q(\cdot),U(\cdot)\}} \int_{-\delta}^{\delta} \left\{ -\frac{1}{2}(q(\theta)-1)^2 - \frac{1}{2}(q(\theta)+1)^2 - \frac{\beta}{2}(q(\theta)-\theta)^2 - U(\theta) \right\} \frac{d\theta}{2\delta},$$

$$\text{subject to } \dot{U}(\theta) = \beta(q(\theta)-\theta); (2) \text{ and } \dot{q}(\theta) \geq 0 \ (4).$$

Denoting by λ the co-state variable for (2), the Hamiltonian of $(\widetilde{\mathcal{P}}^M)$ can be written as:

$$H(U,q,\lambda,\theta) = \left\{ -\frac{1}{2}(q-1)^2 - \frac{1}{2}(q+1)^2 - \frac{\beta}{2}(q-\theta)^2 - U(\theta) \right\} \frac{1}{2\delta} + \lambda\beta(q-\theta).$$

Since $H^*(U,\lambda,\theta) = \max_{q\in\mathbb{R}} H(U,q,\lambda,\theta)$ and the state constraint (2) are both linear in U, the problem is concave in U. Therefore, the sufficient conditions for optimality with pure state constraints (see Seierstad and Sydsaeter (1987, Theorem 1, p. 317-319)) are also necessary. To write these conditions, consider the following Lagrangian:

$$L(U,q,\lambda,p,\theta) = H(U,q,\lambda,\theta) + pU,$$

where p is the Lagrange multiplier associated with the state constraint (4). Let $\{U(\theta), q(\theta)\}$ be an admissible pair which solves $(\widetilde{\mathcal{P}}^M)$. The sufficient conditions for optimality are:

$$\frac{\partial H}{\partial q} = 0, \text{ for almost all } \theta; \tag{A1}$$

$$\dot{U}(\theta) = \beta(q(\theta)-\theta); \tag{A2}$$

$$\dot{\lambda}(\theta) = \frac{1}{2\delta} - p(\theta); \tag{A3}$$

$$p(\theta)U(\theta) = 0, p(\theta) \geq 0, \ U(\theta) \geq 0; \tag{A4}$$

$$\lambda(-\delta) = \lambda(\delta) = 0; \tag{A5}$$

where $p(\theta)$ and $\lambda(\theta)$ are piecewise continuous and $\lambda(\theta)$ may have jump discontinuities at τ_j, such that, $-\delta \leq \tau_j \leq \delta$ $(j = 1, ..., n)$, and for these jumps:

$$\lambda\left(\tau_j^-\right) - \lambda\left(\tau_j^+\right) = \epsilon_j, \ \epsilon_j \geq 0, \tag{A6}$$

with

$$\epsilon_j = 0 \text{ if } \begin{cases} \text{either} & a) \ U(\tau_j) > 0, \\ \text{or} & b) \ U(\tau_j) = 0 \text{ and } q(\theta) \text{ is discontinuous at } \tau_j. \end{cases} \tag{A7}$$

From (A1), the optimal policy is

$$q(\theta) = \frac{\beta\theta + 2\beta\delta\lambda(\theta)}{\beta + 2}, \text{ for almost all } \theta. \tag{A8}$$

This expression together with (A6) leads to the following useful Lemma (whose proof is immediate):

Lemma 2 *: The equilibrium policy $q(\theta)$ and the co-state variables $\lambda(\theta)$ are continuous at any interior point $\theta \in (-\delta, \delta)$. $\lambda(\theta)$ is continuous at $\theta = \pm\delta$ only if $q(\theta)$ is discontinuous at those end-points and $U(\pm\delta) = 0$.*

We are now ready to prove Proposition 1. Using the sufficient conditions for optimality given above, let us guess the form of the solution and check that it satisfies all the conditions for optimality (A1) to (A6). Two cases are possible:

Zero rent on all Θ: $p(\theta) > 0$ for all $\theta \in \Theta$ implies $U(\theta) = 0$. Then, it is easy to see that the quadruple

$$(U(\theta), q(\theta), \lambda(\theta), p(\theta)) = \left(0, \theta, \frac{\theta}{\beta\delta}, \frac{\beta - 2}{2\beta\delta}\right)$$

satisfies (A1) to (A6) if and only if $\beta \geq 2$. Note that $\lambda(\cdot)$ is discontinuous at both endpoints $\tau_1 = -\tau_2 = \delta$, with $\epsilon_1 = \epsilon_2 = \frac{1}{\beta} > 0$.

Zero rent only at endpoints: To find the solution when $\beta < 2$, let us set $p(\theta) = 0$ for all θ in Θ. The quadruple

$$(U(\theta), q(\theta), \lambda(\theta), p(\theta)) = \left(\frac{\beta(\beta - 2)}{2(\beta + 2)}\left(\theta^2 - \delta^2\right), \frac{2\beta\theta}{\beta + 2}, \frac{\theta}{2\delta}, 0\right),$$

satisfies (A1) to (A6) if and only if $0 \leq \beta \leq 2$. Note again that $\lambda(\cdot)$ is discontinuous at both endpoints $\tau_1 = -\tau_2 = \delta$, with $\epsilon_1 = \epsilon_2 = \frac{1}{2} > 0$. ■

• **Lobbying Competition. Preliminaries:** To characterize equilibria under interim contracting, we first consider the reduced problem $(\widetilde{\mathcal{P}}_i^C)$ where the monotonicity constraint (3) is ignored. This constraint will be checked ex post on the solution of the relaxed problem. This is now an optimal control problem with a unique pure state constraint whose Lagrangian can be written as:

$$L_i(U, q, \lambda_i, p_i, \theta) = H_i(U, q, \lambda_i, \theta) + p_i(U - U_{-i}(\theta)),$$

where λ_i is the co-state variable, p_i is the multiplier of the pure state constraint (5). The Hamiltonian of $(\widetilde{\mathcal{P}}_i^C)$ is:

$$H_i(U, q, \lambda_i, \theta) = \left\{-\frac{1}{2}(q - a_i)^2 - \frac{\beta}{2}(q - \theta)^2 + t_{-i}^*(q) - U\right\}\frac{1}{2\delta} + \lambda_i\beta(q - \theta).$$

To characterize the solution of $(\widetilde{\mathcal{P}}_i^C)$, we shall apply the sufficient conditions for optimality (see Seierstad and Sydsaeter (1987, Chapter 5, Theorem 1, p. 317)) which hold when $\hat{H}_i(U, \lambda_i(\theta), \theta) = \max_q H_i(U, q, \lambda_i(\theta), \theta)$ is concave in U. First, we show:

Lemma 3 *: The pair $\{t_1^*(q), t_2^*\}$ is an equilibrium only if it generates a rent-policy profile $\{U^*(\theta), q^*(\theta)\}$ which solves problems (P_i^C) for $i = 1, 2$. Moreover, if $t_i^{*\prime\prime}(q) < \beta + 1$ $i = 1, 2$, then $H_i(U, q, \lambda_i(\theta), \theta)$ is concave in q.*

Proof: If, for the transfers $t_i^*(q)$, the property stated in Lemma 3 is true, then the objective functions in problems (P_i^C) are concave in q. It can be verified ex post (i.e., once the equilibrium schedules $t_i^*(\cdot)$ are obtained) that the condition of Lemma 3 holds so that the Hamiltonian is indeed concave in q. ■

The sufficient conditions for optimality ensure that there exists a pair of piecewise continuous functions $\lambda_i(\theta)$ and $p_i(\theta)$, and constants $\epsilon_k \geq 0$, $k = 1, ..., n$, such that:

$$\dot{U}(\theta) = \beta(q(\theta) - \theta), \tag{A9}$$

$$\frac{\partial H_i}{\partial q}(U, q, \lambda_i, \theta) = 0, \text{ for almost all } \theta \in \Theta, \tag{A10}$$

$$\dot{\lambda}_i(\theta) = \frac{1}{2\delta} - p_i(\theta), \text{ for almost all } \theta \in \Theta, \tag{A11}$$

$$p_i(\theta)(U(\theta) - U_{-i}(\theta)) = 0, \; p_i(\theta) \geq 0, \; U(\theta) \geq U_{-i}(\theta), \tag{A12}$$

$$\lambda_i(-\delta) = \lambda_i(\delta) = 0. \tag{A13}$$

Moreover, $\lambda_i(\theta)$ may have jump discontinuities at points τ_k, for $-\delta \leq \tau_k \leq \delta$, $(k = 1, ..., n)$ such that

$$\lambda_i\left(\tau_k^-\right) - \lambda_i\left(\tau_k^+\right) = \epsilon_k \geq 0, \tag{A14}$$

and

$$\epsilon_k = 0 \text{ if } \begin{cases} \text{either} & a)\; U(\tau_k) > U_{-i}(\tau_k), \\ \text{or} & b)\; U(\tau_k) = U_{-i}(\tau_k) \text{ and } q(\theta) \text{ is discontinuous at } \tau_k. \end{cases} \tag{A15}$$

Let us now use the above conditions to derive some properties of the equilibria.

Properties of the Policy Profile: Assuming strict concavity of the agent's objective function,[21] the corresponding first-order condition for the agent's behavior is written as:

$$\sum_{i=1}^{2} t_i'(q(\theta)) = \beta(q(\theta) - \theta). \tag{A16}$$

[21] This property will be checked ex post once the equilibrium contributions are derived.

From (A10), we immediately get:

$$-(\beta+1)\,q(\theta)+a_i+\beta\theta+t_{-i}^{*\prime}(q(\theta))+2\lambda_i(\theta)\,\beta\delta=0. \tag{A17}$$

Using (A17), a similar equation obtained by permuting indices and (A16), we get the following expression of an equilibrium policy:

$$q(\theta)=\frac{\beta\theta+2\,(\lambda_1(\theta)+\lambda_2(\theta))\,\beta\delta}{\beta+2},\text{ for almost all }\theta\in\Theta. \tag{A18}$$

Also, we can generalize Lemma 2 to the case of competing principals:

Lemma 4 *: The equilibrium policy $q(\theta)$ is everywhere continuous and $\lambda_i(\theta)$ is continuous at any interior point $\theta\in(-\delta,\delta)$. If $\lambda_i(\theta)$ is discontinuous at $\theta=\delta$ or at $\theta=-\delta$, then $q(\theta)$ is continuous and the state constraint is binding at this point.*

Proof: Suppose that at an interior point $\tau\in(-\delta,\delta)$ $\lambda_i(\cdot)$ is discontinuous with $\varepsilon_i=\lambda_i(\tau^-)-\lambda_i(\tau^+)>0$. Then, from (A15), $q(\theta)$ must be continuous at τ. From (A18) it is possible only if $\lambda_{-i}(\cdot)$ is discontinuous at τ with $\lambda_{-i}(\tau^-)-\lambda_{-i}(\tau^+)=-\varepsilon_i$. But $-\varepsilon_i<0$, a contradiction with (A14). ■

Properties of the Rent Profile: Let us now give a first property of the rent profile that helps limiting the investigation of the different equilibrium patterns.

Lemma 5 *: The shape of the agent's information rent depends on whether β is greater or less than one:*

- *If $\beta\leq 1$ (resp. <1), the agent's information rent $U(\theta)$ is concave (resp. strictly concave) in θ.*
- *If $\beta>1$, the information rent $U(\theta)$ is strictly convex on a non-empty interval $[\delta_1,\delta_2]$ if and only if $p_1(\theta)+p_2(\theta)<\frac{\beta-1}{\beta\delta}$ for all $\theta\in[\delta_1,\delta_2]$.*
- *If $\beta=1$, the agent's information rent $U(\theta)$ is linear on a non-empty interval $[\delta_1,\delta_2]$ if and only if $p_i(\theta)=0$ on this interval.*

Proof: Using (2) and differentiating w.r.t. θ yields

$$\ddot{U}(\theta)=\beta\,(\dot{q}(\theta)-1)\,.$$

From (A11) and (A18), we get

$$\dot{q}(\theta)=\frac{\beta+2\beta\delta\left(\frac{1}{\delta}-p_1(\theta)-p_2(\theta)\right)}{\beta+2}=\frac{3\beta-2\beta\delta\,(p_1(\theta)+p_2(\theta))}{\beta+2}.$$

Thus, we have:

$$\ddot{U}(\theta) = \frac{2\beta(\beta - 1 - \beta\delta(p_1(\theta) + p_2(\theta)))}{\beta + 2}.$$

Since $p_i(\theta) \geq 0$, Lemma 5 is proved. ■

The positiveness of the Lagrange multipliers $p_i(\theta)$ $(i = 1, 2)$ has an important impact on the properties of equilibria. If $p_i(\theta) > 0$ on non-degenerate interval, the corresponding state constraint $U(\theta) \geq U_{-i}(\theta)$ is binding on that interval, and, consequently, the transfer $t_i(q(\theta))$ is identically equal to zero there.

From (A17), we obtain an expression for $t_i'(\cdot)$ that does not depend on $t_{-i}(\cdot)$. For each configuration of parameters that we consider below, (A17) uniquely defines the derivative of the equilibrium schedule in the equilibrium range. This leads to the unique equilibrium up to some constants of integration depending of the different possible configurations for the sign of those Lagrange multipliers of the problems $(\mathcal{P}_i^C)$, $i = 1, 2$. This is what we will do in the next proofs. ■

• **Proof of Proposition 2:** Suppose that $t_2^*(q) = 0$ for all q so that $U_2(\theta) = 0$ for all $\theta \in \Theta$. Then, from (A18), we get that P_1's best-response is to induce $q(\cdot)$ such that:

$$-(\beta + 1)q(\theta) + 1 + \beta\theta + 2\lambda_1(\theta)\beta\delta = 0.$$

The "laissez-faire" policy $q(\theta) = \theta$ is optimal when

$$\lambda_1(\theta) = \frac{\theta - 1}{2\beta\delta}.$$

From (A11), we get

$$p_1(\theta) = \frac{\beta - 1}{2\beta\delta} \geq 0 \text{ if and only if } \beta \geq 1.$$

so that the participation constraint is everywhere binding, $U(\theta) = U_2(\theta) = 0$ on all Θ.

Finally, we must check the transversality conditions (A13) with the possible discontinuities at end-points given by (A14). Note that

$$\lambda_1(\delta^-) - \lambda_1(\delta^+) = \frac{\delta - 1}{2\beta\delta} = \epsilon \geq 0 \text{ if and only if } \delta \geq 1,$$

$$\lambda_1(-\delta^-) - \lambda_1(-\delta^+) = \frac{\delta + 1}{2\beta\delta} = \epsilon' \geq 0.$$

Since $U(\theta) = U_2(\theta) = 0$ on all Θ, $t_1^*(q) = 0$ for all q. Proceeding similarly for P_2 ends the proof. ■

• **Proof of Proposition 3:** Denote by $\Omega_0 = [-\tau, \tau]$, the symmetric interval where both principals offer null contributions:[22] $U(\theta) = U_1(\theta) = U_2(\theta) = 0$ and thus $q(\theta) = \theta$ on this interval.

[22] It is easy to show that the interval Ω_0 is indeed symmetric following the same reasoning as in the proof of Lemma 3.

Using (A17) for principal P_1, we obtain:

$$\lambda_1(\theta) = \frac{\theta - 1}{2\beta\delta} \text{ for } \theta \in \Omega_0. \tag{A19}$$

Instead, we have on $\Omega_1 = [\tau, \delta]$, $U(\theta) = U_1(\theta) > U_2(\theta)$. From that, we deduce that $p_1(\theta) = 0$ on Ω_1. Using the transversality condition (A13) and integrating (A11) yields:

$$\lambda_1(\theta) = \frac{\theta - \delta}{2\delta} \text{ for } \theta \in \Omega_1. \tag{A20}$$

Using the continuity of $\lambda_1(\cdot)$ at τ yields

$$\tau = \frac{\beta\delta - 1}{\beta - 1}.$$

Rewriting the condition that τ should belong to $[0, \delta]$ yields that $\delta < 1 < \beta\delta$.

Finally inserting the expression of $\lambda_1(\cdot)$ found in (A20) into (A18) yields:

$$q(\theta) = \frac{2\beta\theta + 1 - \beta\delta}{\beta + 1} \text{ for } \theta \in \Omega_1. \tag{A21}$$

A similar and symmetric expression is obtained for $\theta \in \Omega_2 = [-\delta, -\tau]$.

To find the expression of the equilibrium contribution, note that, on Ω_1, we have:

$$t_1^{*'}(q(\theta)) = \beta(q(\theta) - \theta) = \frac{\beta((\beta - 1)\theta + 1 - \beta\delta)}{\beta + 1} \text{ for } \theta \in \Omega_1.$$

Manipulating yields the expression in the text. ■

• **Proof of Proposition 4:** Consider again $\Omega_0 = [-\tau, \tau]$, the symmetric interval where both principals offer non-negative contributions. On this interval, it must be that $U(\theta) \geq \max\{U_1(\theta), U_2(\theta)\}$. Moreover $U(\theta) = U_1(\theta)$ (resp. $U(\theta) = U_1(\theta)$) only at $-\tau$ (resp. τ).

We have $p_1(\theta) = 0$ on $(-\tau, \delta]$. Using the transversality condition (A13) and integrating (A11) yields:

$$\lambda_1(\theta) = \frac{\theta - \delta}{2\delta} \text{ for } \theta \in (-\tau, \delta]. \tag{A22}$$

Similarly, we have $p_2(\theta) = 0$ on $[-\delta, \tau)$ and:

$$\lambda_2(\theta) = \frac{\theta + \delta}{2\delta} \text{ for } \theta \in [-\delta, \tau). \tag{A23}$$

Using (A18) for principal P_1 and (A22), we obtain:

$$-(1 + \beta)q(\theta) + 1 + \beta\theta + t_2^{*'}(q(\theta)) + \beta(\theta - \delta) = 0 \text{ for } \theta \in (-\tau, \delta]. \tag{A24}$$

Similarly, using (A18) for principal P_2 and (A23), we obtain:

$$-(1 + \beta)q(\theta) - 1 + \beta\theta + t_1^{*'}(q(\theta)) + \beta(\theta + \delta) = 0 \text{ for } \theta \in [-\delta, \tau). \tag{A25}$$

Summing (A24) and (A25) and using the agent's first-order condition yields:

$$q(\theta) = \frac{3\beta\theta}{\beta+2} \text{ for } \theta \in (-\tau, \tau). \tag{A26}$$

Using (A24) and taking into account that $t_2^*(q(\theta)) = 0$ on Ω_1 gives the following expression of the policy on that interval:

$$q(\theta) = \frac{2\beta\theta + 1 - \beta\delta}{\beta+1} \text{ for } \theta \in [\tau, \delta]. \tag{A27}$$

Continuity of $\lambda_2(\cdot)$ at τ is ensured when:

$$\frac{2\beta\tau + 1 - \beta\delta}{\beta+1} = \frac{3\beta\tau}{\beta+2}$$

or when

$$\tau = \frac{(2+\beta)(1-\beta\delta)}{\beta(\beta-1)}. \tag{A28}$$

If $\beta > 1$, τ belongs to $(0, \delta]$ when $\beta\delta < 1 \le \frac{\beta(2\beta+1)}{(\beta+2)}\delta$.

To find the expression of the equilibrium contribution $t_1^*(q)$, note that using (A25) yields for $\theta \in \Omega_0$:

$$t_1^{*\prime}(q) = \begin{cases} 0 & \text{if} \quad q \le \frac{-3\beta\tau}{2+\beta}, \\ \frac{\beta(\beta-1)}{\beta+2}\tau + \frac{(\beta-1)}{3}q & \text{if} \quad q \in [\frac{-3\beta\tau}{2+\beta}, \frac{3\beta\tau}{2+\beta}], \\ \beta(\beta-1)\left(q + \frac{\tau}{\beta+2}\right) & \text{if} \quad q \ge \frac{3\beta\tau}{2+\beta}. \end{cases}$$

Note that $t_1(q(-\tau)) = 0$ because $U(-\tau) = U_2(-\tau)$ by definition of Ω_2. Integrating and taking into account that $t_1(\cdot) = 0$ is continuous at $q(\tau) = \frac{3\beta\tau}{2+\beta}$ yields thus the expression in the text.

It is important to note that $U^*(\cdot)$ is convex and piecewise continuously differentiable profile with:

$$\dot{U}^*(\theta) = \begin{cases} \frac{\beta(\beta-1)}{\beta+1}\left(\theta - \frac{\tau\beta}{\beta+2}\right) & \text{if} \quad \theta \le -\tau, \\ \frac{2\beta(\beta-1)}{\beta+2}\theta & \text{if} \quad \theta \in [-\tau, \tau], \\ \frac{\beta(\beta-1)}{\beta+1}\left(\theta - \frac{\tau\beta}{\beta+2}\right) & \text{if} \quad \theta \ge \tau. \end{cases}$$

$U^*(\cdot)$ is thus minimum at zero with:

$$U^*(0) = \frac{3\beta^2(\beta-1)\tau^2}{(\beta+2)^2} > 0.$$

■

• **Proof of Proposition 5:** We are now looking for an equilibrium such that the participation constraint $U(\theta) \geq U_2(\theta)$ (resp. $U(\theta) \geq U_1(\theta)$) binds only at the end-point $-\delta$ (resp. δ). We have thus $p_1(\theta) = 0$ on $(-\delta, \delta]$. Using the transversality condition (A13) and integrating (A11) yields:

$$\lambda_1(\theta) = \frac{\theta - \delta}{2\delta} \text{ for } \theta \in (-\delta, \delta]. \tag{A29}$$

Similarly, we have $p_2(\theta) = 0$ on $[-\delta, \delta)$. Using the transversality condition (A13) and integrating (A11) yields:

$$\lambda_2(\theta) = \frac{\theta + \delta}{2\delta} \text{ for } \theta \in [-\delta, \delta). \tag{A30}$$

Using (A18) for principal P_1 and (A29), we obtain:

$$-(1+\beta)q(\theta) + 1 + \beta\theta + t_2^{*'}(q(\theta)) + \beta(\theta - \delta) = 0 \text{ for } \theta \in (-\delta, \delta]. \tag{A31}$$

Similarly, using (A18) for principal P_2 and (A30), we obtain:

$$-(1+\beta)q(\theta) - 1 + \beta\theta + t_1^{*'}(q(\theta)) + \beta(\theta + \delta) = 0 \text{ for } \theta \in [-\delta, \delta). \tag{A32}$$

Summing (A31) and (A32) and using the agent's first-order condition yields:

$$q^*(\theta) = \frac{3\beta\theta}{\beta + 2} \text{ for } \theta \in (-\delta, \delta). \tag{A33}$$

Hence,

$$\dot{U}^*(\theta) = \frac{2\beta(\beta - 1)}{\beta + 2}\theta \text{ for } \theta \in (-\delta, \delta). \tag{A34}$$

Integrating yields the expression of $U^*(\theta)$ in the text.

Similarly using (A32) and (A34) yields

$$t_1^{*'}(q) = \frac{(\beta - 1)}{3}q + \gamma q \text{ for } q \in \left(-\frac{3\beta\delta}{\beta + 2}, \frac{3\beta\delta}{\beta + 2}\right). \tag{A35}$$

Integrating and keeping the non-negative part yields the expression of $t_1^*(q)$ in the text. $t_2^*(q)$ is obtained similarly.

Note that the condition $U^*(-\delta) = U_2^*(-\delta)$ can be rewritten as:[23]

$$\frac{\beta(\beta - 1)}{\beta + 2}\delta^2 - C_1 - C_2 = \max\left\{0, -C_2 + \max_q \frac{(\beta - 1)}{6}q^2 - \gamma q - \frac{\beta}{2}(q + \delta)^2\right\}, \tag{A36}$$

or

$$\frac{\beta(\beta - 1)}{\beta + 2}\delta^2 - C_1 - C_2 = \max\left\{0, -C_2 + \frac{3}{2(2\beta + 1)} - \frac{\beta\delta^2}{2}\right\}. \tag{A37}$$

[23] A similar condition is obtained by writing the boundary condition $U^*(\delta) = U_1^*(\delta)$.

But, because we must have $U^*(0) = -C_1 - C_2 \geq 0$, the only possibility to solve (A37) is:

$$C_1 = C_2 = \frac{3\beta^2\delta^2}{2(\beta+2)} - \frac{3}{2(1+2\beta)} \tag{A38}$$

which is a negative number (as requested by the condition $U^*(0) = -C_1 - C_2 \geq 0$) when $1 \geq \beta\delta\sqrt{\frac{2\beta+1}{\beta+2}}$ but this latter condition holds since $1 \geq \frac{(2\beta+1)\beta\delta}{\beta+2}$ and $\beta > 1$. ■

• **Proofs of Propositions 6 and 7:** We consider an equilibrium such that the participation constraints (5) are binding only at endpoints. For all $\theta \in (-\delta, \delta)$ we have thus $p_1(\theta) = p_2(\theta) = 0$. The co-state variables $\lambda_i(\theta)$ are continuous for all $\theta \in (-\delta, \delta)$ but may still have discontinuities at end-points. From (A11), we get by integrating:

$$\lambda_i(\theta) = \frac{\theta}{2\delta} + \lambda_i, \tag{A39}$$

where λ_1 and λ_2 and are some constants. Using (A18) yields

$$q(\theta) = \frac{3\beta\theta + 2(\lambda_1 + \lambda_2)\beta\delta}{\beta+2}. \tag{A40}$$

Therefore, we get:

$$\dot{U}(\theta) = \frac{2\beta(\beta-1)}{\beta+2}\theta + \frac{2(\lambda_1+\lambda_2)\beta^2\delta}{\beta+2}. \tag{A41}$$

and thus

$$U(\theta) = \frac{2\beta}{\beta+2}\left[\frac{(\beta-1)(\theta^2-\delta^2)}{2} + \beta\delta(\lambda_1+\lambda_2)\theta\right] - 2C \tag{A42}$$

where C is a constant of integration.

Because of the symmetry of the model, we focus on symmetric rent profile such that

$$U(\delta) = U(-\delta). \tag{A43}$$

Using (A42), and (A43) yields necessarily:

$$\lambda_1 + \lambda_2 = 0. \tag{A44}$$

From (A40), we conclude that

$$q(\theta) = \frac{3\beta\theta}{\beta+2}. \tag{A45}$$

To satisfy the transversality conditions (A13) it must be that $\lambda_i \in \left[-\frac{1}{2}, \frac{1}{2}\right]$. Then, observe that both co-state variables may have jumps at endpoints. Those jumps corresponds to the binding participation constraints $U(\delta) = U_2(\delta) = U_1(\delta)$ and $U(-\delta) = U_2(-\delta) = U_1(-\delta)$. Denote by $-C = U(\delta) = U(-\delta)$ the common utility level at both endpoints $\pm\delta$. Using (A42), the agent's information rent becomes:

$$U(\theta) = \frac{\beta(1-\beta)}{\beta+2}\left(\delta^2 - \theta^2\right) - 2C.$$

It is non-negative and concave only if $\beta \leq 1$ and $C \leq 0$.

From (A17) and (A18), we get the expression of the marginal contributions offered by both principals at any equilibrium policy:

$$t_1'(q) = -\frac{1-\beta}{3}q + (2\beta\delta\lambda_1 + 1) \quad \text{and} \quad t_2'(q) = -\frac{1-\beta}{3}q - (2\beta\delta\lambda_1 + 1) \tag{A46}$$

where $\lambda_1 \in \left[-\frac{1}{2}, \frac{1}{2}\right]$ is arbitrary. Let us define $\gamma = 2\beta\delta\lambda_1 + 1$

Integrating (A46) yields the following expressions of contributions up to some constants (C_1, C_2):

$$t_1(q) = -\frac{1-\beta}{6}q^2 + \gamma q - C_1, \quad \text{and} \quad t_2(q) = -\frac{1-\beta}{6}q^2 - \gamma q - C_2. \tag{A47}$$

Those expressions are of course valid as long as contributions are positive. The corresponding schedules are thus defined as:

$$t_1(q) = \max\left\{0, -\frac{1-\beta}{6}q^2 + \gamma q - C_1\right\} \text{ and } t_2(q) = \max\left\{0, -\frac{1-\beta}{6}q^2 - \gamma q - C_2\right\}. \tag{A48}$$

From this, it is easy to obtain the policies chosen by the agent when he contracts only with one of the principals and the corresponding contribution is positive. These policies are respectively given by:

$$q_1(\theta) = \frac{3}{2\beta+1}\left[\beta\theta + \gamma\right] \quad \text{and} \quad q_2(\theta) = \frac{3}{2\beta+1}\left[\beta\theta - \gamma\right].$$

Observe that $q_1(\theta) > q(\theta)$, and $q_2(\theta) < q(\theta)$.

Let us turn now to the characterization of the pairs (C_1, C_2). Because (5) is binding at both endpoints, those constants must satisfy the condition

$$\max_q\left\{-\frac{1-\beta}{3}q^2 - \frac{\beta}{2}(q \pm \delta)^2\right\} - C_1 - C_2$$

$$= \max\left\{0, \max_q\left\{-\frac{1-\beta}{6}q^2 + \gamma q - \frac{\beta}{2}(q-\delta)^2 - C_1\right\}, \max_q\left\{-\frac{1-\beta}{6}q^2 - \gamma q - \frac{\beta}{2}(q+\delta)^2 - C_2\right\}\right\}. \tag{A49}$$

Two cases should be considered depending on whether $2C = \frac{\beta(1-\beta)}{\beta+2}\delta^2 + C_1 + C_2$ is negative or null.

Zero rent at endpoints (Proposition 6), $C = 0$: Then (A49) can be rewritten as a pair of conditions. The first condition $U(\pm\delta) = 0$ becomes:

$$C_1 + C_2 = -\frac{\beta(1-\beta)}{\beta+2}\delta^2. \tag{A50}$$

To satisfy the second condition $U(\delta) = U_1(\delta) = 0$ (resp. $U(-\delta) = U_2(-\delta) = 0$), we must also have:[24]

$$C_i \geq \max_q \left\{ -\frac{1-\beta}{6} q^2 + \gamma q - \frac{\beta}{2}(q-\delta)^2 \right\} = \frac{3(\gamma + \beta\delta)^2}{2(2\beta+1)} - \frac{\beta\delta^2}{2}. \tag{A51}$$

The linear constraints (A50) and (A51) are compatible when:

$$1 \leq (2\beta+1)\delta\sqrt{\frac{\beta}{3(\beta+2)}} - \beta\delta(2\lambda_1 + 1)$$

which gives the upper bound in the text.

Positive rent at endpoints (Proposition 7), $C < 0$**:** The conditions $U(\delta) = U_1(\delta) = -C > 0$ and $U(-\delta) = U_2(-\delta) = -C > 0$ yield:

$$-\frac{\beta\,(1-\beta)}{\beta+2}\delta^2 - C_1 - C_2 = \frac{3(\beta\delta+\gamma)^2}{2(2\beta+1)} - \frac{\beta\delta^2}{2} - C_1 = \frac{3(\beta\delta+\gamma)^2}{2(2\beta+1)} - \frac{\beta\delta^2}{2} - C_2 > 0.$$

This can be rewritten as

$$C_i = \frac{3}{2}\left(\frac{\beta^2\delta^2}{\beta+1} - \frac{(\gamma+\beta\delta)^2}{2\beta+1}\right). \tag{A52}$$

Note that, because $U(\delta) = U_1(\delta) = -C > U_2(\delta) \geq 0$ we must have $\lambda_1(\delta) = 0$ which means that necessarily $\lambda_1 = -\frac{1}{2}$ and $\gamma = 1 - \beta\delta$. ■

[24]It is easy to show that $U(\delta) = U_1(\delta) = 0$ implies also $U(-\delta) = U_1(-\delta) = 0$ and $U(-\delta) = U_2(-\delta) = 0$ implies $U(\delta) = U_2(\delta) = 0$.