Health and Innovation in a Monetary Schumpeterian Growth Model

He, Qichun

Central University of Finance and Economics

March 2018

Online at https://mpra.ub.uni-muenchen.de/85218/
MPRA Paper No. 85218, posted 20 Mar 2018 05:39 UTC
Health and Innovation in a Monetary Schumpeterian Growth Model*

Qichun He†
(Central University of Finance and Economics, Beijing, China)
March, 2018

Abstract
This study explores a novel channel—endogenous health investment—through which monetary policy impacts growth and welfare. We use a scale-invariant Schumpeterian growth model with a cash-in-advance (CIA) constraint on R&D investment. We find that the effect of an increase in the nominal interest rate on long-run growth crucially depends on the form of the CIA constraint. When the CIA constraint does not apply to medical expenditure, long-run growth does not depend on the nominal interest rate. The result remains robust when health capital does not need medical expenditure to produce (i.e., health capital only needs leisure to produce). By contrast, when the CIA constraint applies to medical expenditure, an increase in the nominal interest rate leads to a decrease in R&D and health investment, which in turn reduces the long-run growth rates of technology and output. Nevertheless, welfare is always a decreasing function of the nominal interest rate, and the welfare loss is larger under the CIA constraint on medical expenditure. The results hold up with the health-in-the-utility function (HIU).

JEL Classification: O30, O40, E41; I15
Keywords: Monetary policy; innovation; health; economic growth; welfare

*I am grateful to Angus Chu for comments that substantially improved this paper. I also thank Xuezheng Qin and seminar participants at Peking University for helpful comments.

†Associate Professor in Economics, China Economics and Management Academy, Central University of Finance and Economics, No. 39 South College Road, Haidian District, Beijing, China. 100081. Email: qichunhe@gmail.com, heqichun@cufe.edu.cn.
1 Introduction

There is a large existing literature on the role of health in the process of economic development (see elaboration below). In this paper we contribute by analyzing the effect of monetary policy on health capital accumulation in a Schumpeterian growth model. In so doing, we reveal a novel channel for monetary policy to impact economic growth and welfare. We follow Chu and Cozzi (2014) and Chu et al. (2015) to model money demand via a cash-in-advance (CIA) constraint on R&D investment. We find that the effect of an increase in the nominal interest rate on long-run growth crucially depends on the form of the CIA constraint. When the CIA constraint does not apply to medical expenditure, long-run growth does not depend on the nominal interest rate, but welfare depends negatively on the nominal interest rate. The result remains robust when health capital does not need medical expenditure to produce (i.e., health capital only needs leisure to produce). By contrast, under the CIA constraint on medical expenditure, an increase in the nominal interest rate leads to a decrease in R&D and health investment, which in turn reduces the long-run growth rates of technology and output. The welfare loss is also larger. The results hold up with the health-in-the-utility function (HIU).

It is generally the case that with a CIA constraint on R&D investment, a higher nominal interest rate would decrease R&D labor and thereby the growth rate of output (see e.g., Chu and Cozzi, 2014; Chu et al., 2017). For instance, Chu et al. (2017) illustrate that the accumulation of human capital amplifies the negative effect of a higher nominal interest rate on growth. The mechanism of our finding is as follows.

To remove the scale effect in the innovation process to be consistent with the relative constant growth rate of total factor productivity in developed countries, authors usually assume a rising R&D difficulty. For instance, as discussed in Chu et al. (2017), Segerstrom (1998) has considered an industry-specific index of R&D difficulty; Venturini (2012) has provided empirical evidence based on industry-level data that supports the presence of increasing R&D difficulty. We also assume that there is an increasing-complexity effect of technology. Specifically, the aggregate arrival rate of innovation depends on the ratio of the level of health to the level of technology (to capture the increasing-complexity effect of technology and remove the scale effect of health capital on innovation) as well as the share of R&D employment (as in Schumpeterian growth models, see Aghion and Howitt, 1998, ch. 2; Chu and Cozzi, 2014).

In our model, the accumulation of health capital is not affected by the increase in the nominal interest rate. That is, the share of effective labor devoted to leisure does not depend on the nominal interest rate. When there is an increase in the nominal interest rate, the share of effective labor employed by R&D decreases, all else equal. This happens because of the CIA constraint on R&D. Because entrepreneurs have to borrow
from households to pay the wage bill of R&D workers, an increase in the nominal interest rate would increase the borrowing cost of entrepreneurs, ending up shifting labor from R&D to manufacturing. As labor reallocates from R&D to manufacturing, the aggregate arrival rate of innovation decreases, which in turn reduces the aggregate level of technology. When the ratio of the level of health to the level of technology increases, the complexity of technology relative to human health decreases, ending up increasing the aggregate arrival rate of innovation. On the balanced growth path, the effect of the decrease in R&D share of labor due to an increase in the nominal interest rate on the growth rate of technology is totally offset by that of the increase in the ratio of the level of health to the level of technology, leaving the growth rates of technology and output unchanged. However, an increase in the nominal interest rate decreases the initial level of output, ending up lowering welfare.

Our results concerning growth are in contrast to the findings of Chu et al. (2017). The difference in findings can be explained as follows. In Chu et al. (2017), an increase in the nominal interest rate would also decrease the share of skilled labor (i.e., human capital) employed by R&D. However, when the demand for unskilled labor increases, the return to unskilled labor relative to skilled labor increases, people would devote less time to education that produces skilled labor. As a result, the accumulation of human capital is negatively affected by the increase in the nominal interest, amplifying the effect of an increase in the nominal interest rate on innovation. By contrast, the allocation of time (or health capital) between work (either in manufacturing or in R&D) and leisure is not affected by the increase in the nominal interest rate. Thus, the accumulation of health capital is not affected by the increase in the nominal interest rate, decreasing the relative complexity of innovation and thereby increasing the aggregate arrival rate of innovation, which in turn helps to nullify the negative effect of an increase in the nominal interest rate on R&D. The result holds up when the accumulation/production of health needs both medical expenditure and leisure.

However, the results will differ when the medical expenditure is subject to the CIA constraint. In other words, when the medical expenditure is a cash good instead of a credit good, it will amplify the negative effect of an increase in the nominal interest rate. An increase in the nominal interest rate raises the cost of medical expenditure via the CIA constraint and leads to a reallocation of output from medical expenditure to consumption. Moreover, the share of effective labor supply devoted to leisure decreases as the nominal interest rate increases. This is because leisure and medical expenditures are complementary in producing health. The decrease in one input would decrease the marginal product of the other input, and agents would invest less in the other input as well. Therefore, the growth rate of health capital decreases. As the growth rate of output is twice those of health capital and technology, the decrease in the growth rate of output
is twice as large as those of health capital and technology. Moreover, the welfare loss would also be larger.

This study relates to the large literature on health (e.g., Grossman, 1972; Ehrlich and Chuma, 1990; van Zon and Muysken, 2001; Huang, He, and Hung, 2013; Huang and He, 2015; Halliday et al., 2017; Kelly, 2017). Early studies, such as Grossman (1972), have discussed the inputs of health status and the effect of health for consumers. Since then a growing literature has emerged. Generally speaking, health generates utility for consumers (i.e., HIU) (see e.g., Hall and Jones, 2007); health could increase labor supply by decreasing sick time (see e.g., Huang and He, 2015; Halliday et al., 2017); health could help productivity increase as in van Zon and Muysken (2001); health could also increase life expectancy or survival probability as in Hall and Jones (2007). The seminal study of Hall and Jones illustrates that the marginal utility of health increases while the marginal utility of consumption decreases as people get richer, which causes the sharp rise in health spending. Kelly (2017) studies health insurance via a neoclassical approach. Bloom et al. (2003) find that health measured as life expectancy significantly promotes growth. Madson (2017) uses a unique annual dataset covering the period 1800–2011 for 21 OECD (Organisation for Economic Co-operation and Development) countries and finds that health improvements can account for approximately a third of the productivity advances in the OECD countries since 1865, and that these improvements have been influential for enhancement in education, savings, innovations, life expectancy, and democracy.

This study also relates to the literature on inflation/monetary policy and endogenous growth (for recent studies, see e.g., Chu and Cozzi, 2014; Chu et al., 2015; He, 2015; Huang, Chang, and Ji, 2015; He and Zou, 2016; Arawatari, Hori, and Mino, 2017; Chu et al., 2017; Huang, Yang, and Cheng, 2017; Chu et al., 2018; He, 2018). Chu et al. (2017) study human capital accumulation in a scale-invariant monetary Schumpeterian growth model. As discussed in many existing studies (e.g., van Zon and Muysken, 2001), health and human capital share many similar features such as being beneficial for productivity growth. Therefore, our work together with Chu et al. enhances our understanding the role of health and human capital in the process of economic development.

The rest of this paper is organized as follows. Section 2 sets up the monetary Schumpeterian growth model. Section 3 analyzes the growth and welfare effects of monetary policy. The final section concludes.

2 A Monetary Schumpeterian Model with Health

For monetary policies to play a role, we need to introduce money into the Schumpeterian growth models. Built on existing studies (e.g., Chu and Cozzi, 2014; Chu et al., 2017),
we model money with a CIA constraint on R&D expenditure.

In this section, we follow the human capital setup in Chu et al. (2017). That is, we assume that health only needs leisure time to produce. We present the basic model for the following reasons. One the one hand, health is in many aspects like human capital. For instance, both health and human capital can increase effective labor supply. They both need time to produce. On the other hand, the basic model allows us to see clearly the effect of monetary policy on health accumulation and long-run growth.

2.1 The Households

At time $t$, the population size of each household is fixed at 1. There is a unit continuum of identical households, who have a lifetime utility function as

$$U = \int_0^\infty e^{-\rho t} \ln(c_t) \, dt,$$

where $c_t$ is per capita real consumption of final goods (numeraire) at time $t$. $\rho > 0$ is the rate of time preference.

Each household maximizes her lifetime utility given in equation (1) subject to the asset-accumulation equation given by

$$\dot{a}_t + \dot{m}_t = r_t a_t + w_t N_t - c_t - \pi_t m_t + i_t b_t + \tau_t,$$

where $a_t$ is the real value of equity shares in monopolistic intermediate goods firms owned by each member of households; $r_t$ and $w_t$ are the rate of real interest and wage respectively; $N_t$ is effective labor supplied to production and R&D. $c_t$ is per capita consumption. $m_t$ is the real money balance held by each person, and $\pi_t$ is the cost of holding money (i.e., the inflation rate). In (2), each person also receives a per capita lump-sum transfer of the seigniorage revenue $\tau_t$ from the government (or pay a lump-sum tax if $\tau_t < 0$). The CIA constraint is given by $b_t \leq m_t$, where $b_t$ is the amount of money borrowed by entrepreneurs to finance R&D, and the rate of return is $i_t$ (the nominal interest rate).

Following the existing literature (e.g., Grossman, 1972; Ehrlich and Chuma, 1990; van Zon and Muysken, 2001; Hall and Jones, 2007; Huang, He, and Hung, 2013; Halliday et al., 2017), we assume that the level of health $h_t$ increases the amount of effective labor services a person can supply:

$$N_t + l_t = 1 \cdot h_t = h_t,$$

where $l_t$ is the amount of effective labor allocated to leisure, and each person has one unit of time endowment. The accumulation equation of health status/stock $h_t$ is given by

$$\dot{h}_t = \xi l_t,$$
where $\xi$ is the productivity parameter for health capital investment. Chu et al. (2017) made a similar assumption on human capital accumulation. This assumption is justified as follows. People spend their leisure time on physical exercises, relaxation, and meditation. This is similar to Chu et al. (2017) who assume that people spend their leisure time studying. Physical exercises build up health (both physical and mental) and relaxation and meditation are conducive to mental health, just as studying accumulates human capital.

We can derive the no-arbitrage condition (i.e., the Fisher equation) $i_t = \pi_t + r_t$ (see the Appendix for the derivation of a more complete model in Section 3.2). The optimality condition for consumption is

$$\frac{1}{c_t} = \mu_t,$$

where $\mu_t$ the Hamiltonian co-state variable on (2). The intertemporal optimality condition is

$$-\frac{\mu_t}{\mu_t} = r_t - \rho.$$  

We also have the arbitrage condition between investment in asset holding and that in health:

$$r_t = \xi + \frac{w_t}{w_t}.$$  

### 2.2 The Final Goods Sector

The final goods sector is competitive. The production function of the final goods firms is given by

$$y_t = \exp \left( \int_0^1 \ln x_t(j) \, dj \right),$$

where $x_t(j)$ denotes intermediate goods $j \in [0, 1]$. The final goods firms maximize their profit, taking the price of each intermediate good $j$, denoted $p_t(j)$, as given. The demand function for $x_t(j)$ is

$$x_t(j) = \frac{y_t}{p_t(j)}.$$  

### 2.3 The Intermediate Goods Sector

As clearly elaborated in Chu and Cozzi (2014), there is a unit continuum of industries producing differentiated intermediate goods. Each industry is temporarily dominated by an industry leader until the arrival of the next innovation, and the owner of the new innovation becomes the next industry leader. The leader in industry $j$ has the following production function:

$$x_t(j) = \gamma^{x_t(j)} N_{x_t}(j).$$
The parameter $\gamma > 1$ is the step size of a productivity improvement, and $q_t(j)$ is the number of productivity improvements that have occurred in industry $j$ as of time $t$. $N_{x,t}(j)$ is production labor in industry $j$. As Chu and Cozzi (2014) point out, equation (10) adopts a cost-reducing view of vertical innovation. Given $q_t(j)$, the marginal cost of production for the industry leader in industry $j$ is $mc_t(j) = w_t/\gamma q_t(j)$.

Standard Bertrand price competition leads to a profit-maximizing price $p_t(j)$ determined by a markup $\gamma$ (the step size of innovation) over the marginal cost. The amount of monopolistic profit is

$$\Pi_t(j) = \left(\frac{\gamma - 1}{\gamma}\right) p_t(j) x_t(j) = \left(\frac{\gamma - 1}{\gamma}\right) y_t. \quad (11)$$

The labor income from production is

$$w_t N_{x,t}(j) = \left(\frac{1}{\gamma}\right) p_t(j) x_t(j) = \left(\frac{1}{\gamma}\right) y_t. \quad (12)$$

### 2.4 Research Arbitrage

Research arbitrage is similar to Chu and Cozzi (2014). In a symmetric equilibrium, we have $\Pi_t(j) = \Pi_t$. We denote $v_t(j)$ as the value of the monopolistic firm in industry $j$. In a symmetric equilibrium, $v_t(j) = v_t$. The no-arbitrage condition for $v_t$ is

$$r_t v_t = \Pi_t + \dot{v}_t - \lambda_t v_t. \quad (13)$$

Equation (13) says that the return of holding an innovation, $r_t v_t$, equals the sum of the flow profit of innovation, $\Pi_t$, and potential capital gain ($\dot{v}_t$), less the expected capital loss, $\lambda_t v_t$, where $\lambda_t$ is the arrival rate of the next innovation.

With the CIA constraint on R&D, following Chu and Cozzi (2014), the zero-expected-profit condition of R&D firm $k \in [0, 1]$ in each industry is

$$\lambda_t(k) v_t = (1+i) w_t N_{r,t}(k), \quad (14)$$

where $N_{r,t}(k)$ is the amount of labor hired by R&D firm $k$, and the firm-level innovation rate per unit time (i.e., $\lambda_t(k)$) is $\lambda_t(k) = h_t N_{r,t}(k)/Z_t$, where $Z_t$ is the aggregate level of technology. This assumption is made to remove the scale effect of health capital on steady-state growth (see also Chu et al., 2017). The aggregate arrival rate of innovation is

$$\lambda_t = \int_0^1 \lambda_t(k) \, dk = \varphi \frac{N_{r,t}}{Z_t} = \frac{h_t}{Z_t} \frac{N_{r,t}}{h_t} = \Psi n_{r,t}, \quad (15)$$

where we define a transformed variable $\Psi \equiv \varphi h_t/Z_t$, and another transformed variable
\( n_{r,t} \equiv N_{r,t}/h_t \) (the share of R&D labor in total effective labor supply). Similarly, the share of production labor would be \( n_{x,t} = N_{x,t}/h_t \).

### 2.5 The Monetary Authority

The monetary authority exogenously chooses the monetary growth rate \( \dot{M}_t/M_t \). As discussed in Chu and Cozzi (2014) and Chu et al. (2017), it is equivalent to the case in which the nominal interest rate is chosen as the policy instrument because \( i_t = \dot{M}_t/M_t + \rho \).

### 2.6 The General Equilibrium

As in Chu and Cozzi (2014), the general equilibrium is a time path of prices \( \{p_t(j), r_t, w_t, i_t, v_t\} \) and allocations \( \{c_t, m_t, b_t, y_t, l_t, h_t, x_t(j), N_t, N_{x,t}(j), N_{r,t}(k)\} \), which satisfy the following conditions at each instance of time:

- households maximize utility taking prices \( \{r_t, w_t, i_t\} \) as given;
- competitive final-goods firms maximize profit taking \( \{p_t(j)\} \) as given;
- monopolistic intermediate-goods firms choose \( \{N_{x,t}(j), p_t(j)\} \) to maximize profit taking \( \{w_t\} \) as given;
- R&D firms choose \( \{N_{r,t}(k)\} \) to maximize expected profit taking \( \{w_t, i_t, v_t\} \) as given;
- labor market clears (that is, \( N_t + l_t = N_{x,t} + N_{r,t} + l_t = h_t \));
- final goods market clears (that is, \( y_t = c_t \));
- the value of monopolistic firms adds up to the value of households’ assets (i.e., \( v_t = a_t \));
- the real money balance borrowed by R&D entrepreneurs from the household is \( b_t = w_t N_{r,t} \).

### 3 The Effect of Monetary Policy

Because balanced growth rate is pinned down by the share of labor employed by R&D firms, we solve for the equilibrium labor allocation. The equilibrium labor allocation is stationary on a balanced growth path. Using conditions \( \Pi_t/\Pi_t = g, \lambda \Pi_t = (1 + i)(\rho + \lambda) w_t N_{r,t}, \) (11), (12), (14), and (15), we end up with

\[
(\gamma - 1) n_x = (1 + i)(n_r + \rho/\Psi).
\]
The labor market clearing condition is

\[ n_r + n_x = 1 - \frac{l}{h}. \]  

(17)

Solving (16)-(17) yields the equilibrium labor allocation as

\[ n_r = \frac{(\gamma - 1)}{\gamma + i} \left( 1 - \frac{l}{h} + \frac{\rho}{\Psi} \right) - \frac{\rho}{\Psi}, \]  

(18)

\[ n_x = \frac{1 + i}{\gamma + i} \left( 1 - \frac{l}{h} + \frac{\rho}{\Psi} \right). \]  

(19)

In this paper we focus exclusively on the balanced growth path along which each variable grows at a constant rate. Plugging equation (10) into (8), we have

\[ y_t = \exp \left( \int_0^1 q_t(j) dj \ln \gamma \right) N_x = \exp \left( \int_0^t \lambda_v dv \ln \gamma \right) N_x = Z_t N_x, \]  

(20)

where \( Z_t \equiv \exp \left( \int_0^t \lambda_v dv \ln \gamma \right) \) is the level of aggregate technology. The growth rate of \( Z_t \) is

\[ g_z = \lambda_t \ln \gamma = \Psi n_{r,t} \ln \gamma, \]  

(21)

which is linear in the share of labor employed by R&D firms, as in standard Schumpeterian growth models (see e.g., Aghion and Howitt, 1998, ch. 2; Chu and Cozzi, 2014). On the balanced growth path, the constant \( g_z = (\Psi \ln \gamma) n_{r,t} \) implies that \( \Psi \) is a constant. Therefore, \( h_t \) and \( Z_t \) must grow at the same rate: \( g_z = g_h \). Equation (20) shows that \( g_y = g_z + g_h \). Therefore, we have

\[ g_y = 2g_h = 2\xi \frac{l}{h}, \]  

(22)

where we have used equation (4). Therefore, the balanced growth rate is a constant if \( l/h \) is stationary. Combining equations (5), (6), (7), and (22) yields

\[ g_h = \frac{h_t}{h_t} = \xi - \rho. \]  

(23)

Combining equations (23) and (22) yields

\[ \frac{l}{h} = \frac{\xi - \rho}{\xi}. \]  

(24)

Equation (24) shows that the value of \( l/h \) is stationary. Moreover, the stationary value
of \( l/h \) is not a function of the nominal interest rate. The growth rate of output would be

\[
g_y = 2(\xi - \rho),
\]

which shows that the growth rate of output is not a function of the nominal interest rate. Given that \( g_z = g_h = g_y/2 \), both the growth rates of health capital and technology are not a function of the nominal interest rate.

**Proposition 1** *In our model, the growth rates of output, health capital and aggregate technology do not depend on the nominal interest rate.*

**Proof:** Proven in text. Q.E.D.

Our results are in contrast to the findings of Chu et al. (2017) and Chu and Cozzi (2014). It is generally the case that with a CIA constraint on R&D investment, a higher nominal interest rate would decrease R&D labor and thereby the growth rate of output. However, when there is health capital, the negative effect of an increase in the nominal interest rate is offset by the accumulation of health capital. To explain the intuition of the above results, we first show the following. We have

\[
(\Psi \ln \gamma) \left[ \frac{(\gamma - 1)}{\gamma + i} \left( 1 - \frac{l}{h} + \frac{\rho}{\Psi} \right) - \frac{\rho}{\Psi} \right] = g_z = g_h = \xi - \rho,
\]

which would enable us to solve for \( \Psi \) (the ratio of health capital to technology) as

\[
\Psi = \frac{h_t}{Z_t} = \frac{(\xi - \rho)(\gamma + i) + \rho \ln \gamma (1 + i)}{(1 - \xi + \rho)(\gamma - 1) \ln \gamma},
\]

which illustrates that \( \Psi \) (the ratio of health capital to technology) is an increasing function of the nominal interest rate.

As discussed in Chu et al. (2017), Segerstrom (1998) has considered an industry-specific index of R&D difficulty; Venturini (2012) has provided empirical evidence based on industry-level data that supports the presence of increasing R&D difficulty. We also follow Chu et al. (2017) to capture an increasing-complexity effect of technology. Doing so serves to remove a scale effect of health capital in the innovation process (see Jones, 1999, and Laincz and Peretto, 2006, for a discussion of scale effects in R&D-based growth models). The reason of doing so is similar to the argument of Chu et al. (2017): The level of human health has been increasing in many developed countries, but this increase in the level of human health is not accompanied by a rise in the growth rate of total factor productivity.

Specifically, the firm-level arrival rate of innovation is \( \lambda_t(k) = \frac{\varphi}{Z_t} N_{r,t}(k) = \frac{h_t}{Z_t} \frac{N_{r,t}(k)}{h_t} \). Therefore, the aggregate arrival rate of innovation is \( \lambda_t = \frac{h_t}{Z_t} \frac{N_{r,t}}{h_t} \). We have defined
Given $\Psi \equiv \varphi \frac{h_t}{Z_t}$ and $n_{r,t} \equiv \frac{N_{r,t}}{n_t}$. Therefore, the aggregate arrival rate of innovation positively depends on the ratio of the level of health to the level of technology ($\frac{h}{Z}$) as well as the share of effective labor in R&D ($n_{r,t}$). In our model, the accumulation of health capital is not affected by the increase in the nominal interest rate. That is, the share of effective labor devoted to leisure does not depend on the nominal interest rate. When there is an increase in the nominal interest rate, the share of effective labor employed by R&D (i.e., $n_{r,t}$) decreases, as illustrated in (18), all else equal. This happens because of the CIA constraint on R&D. Because entrepreneurs have to borrow from households to pay the wage bill of R&D workers, an increase in the nominal interest rate would increase the borrowing cost of entrepreneurs, ending up shifting labor from R&D to manufacturing. As labor reallocates from R&D to manufacturing, the aggregate arrival rate of innovation decreases, which in turn reduces the aggregate level of technology. When the ratio of the level of health to the level of technology ($\frac{h}{Z}$) increases, the complexity of technology relative to human health decreases, ending up increasing the aggregate arrival rate of innovation. This effect is illustrated in (27), where $\Psi$ is an increasing function of the nominal interest rate. On the balanced growth path, the effect of the decrease in R&D share of labor due to an increase in the nominal interest rate on the growth rate of technology is totally offset by that of the increase in the ratio of the level of health to the level of technology, leaving the growth rates of technology and output unchanged.

Our results are in contrast to the findings of Chu et al. (2017) who find that an increase in the nominal interest rate would decrease the growth rates of technology, human capital and output. The difference in findings can be explained as follows. In Chu et al., (2017), an increase in the nominal interest rate would also decrease the share of skilled labor (i.e., human capital) employed by R&D. However, when the demand for unskilled labor increases, the return to unskilled labor relative to skilled labor increases, people would devote less time to education to produce skilled labor. As a result, the accumulation of human capital is negatively affected by the increase in the nominal interest, amplifying the effect of an increase in the nominal interest rate on innovation. By contrast, the allocation of time (or health capital) between work (either in manufacturing or in R&D) and leisure is not affected by the increase in the nominal interest rate. Thus, the accumulation of health capital is not affected by the increase in the nominal interest rate, decreasing the relative complexity of innovation and thereby increasing the aggregate arrival rate of innovation, which in turn helps to nullify the negative effect of an increase in the nominal interest rate on R&D due to the CIA constraint.

To summarize, the return to education and thereby the accumulation of human capital in Chu et al. (2017) is affected by the relative return between skilled and unskilled labor. Such relative return between skilled and unskilled labor is absent in our model, which is reasonable because an individual, be it skilled or unskilled worker, needs the same amount...
of leisure to stay healthy or accumulate health capital. Therefore, the amplification of
the negative effect of an increase in the nominal interest rate is absent in our model. Our
finding is not only consistent with the empirical evidence in Chu and Lai (2013) and Chu
et al. (2015), who provide empirical evidence for a negative relationship between inflation
and R&D, but also consistent with the mixed evidence on the effect of the inflation rate
on the growth rate in existing studies.\footnote{The balanced growth rate and thereby the real interest rate does not depend on the nominal interest rate in our model. The Fisher equation $i = r + \pi$ indicates that the nominal interest rate and the inflation rate are positively correlated.} For instance, some authors find a negative effect of inflation on growth (e.g., Chu et al., 2014), while others find a positive effect of inflation on growth (e.g., Bullard and Keating, 2005; He and Zou, 2016).

**Proposition 2** The welfare is a decreasing function of the nominal interest rate.

**Proof.** Imposing balanced growth on (1) yields

$$U = \frac{1}{\rho} \left[ \ln \left( Z_0 N_{x,0} \right) + \frac{g}{\rho} \right],$$

where $Z_0$ is the aggregate technology at time 0, and $N_{x,0}$ is the production labor at time 0. Given Proposition 1 (i.e., the growth rate does not depend on the nominal interest rate), the nominal interest rate impacts the welfare through the level effect on the initial output level $y_0 = Z_0 N_{x,0}$. That is, we have

$$\text{sign} \left( \frac{\partial U}{\partial i} \right) = \text{sign} \left( \frac{\partial \ln (y_0)}{\partial i} \right).$$

As we discussed above, the accumulation of health is independent of the nominal interest rate, therefore, $h_0$ is not affected by the nominal interest $i$. Therefore, we have

$$y_0 = \left( \frac{Z_0 N_{x,0}}{h_0} \right) h_0^2 = \left( \frac{\varphi}{\Psi_0 n_{x,0}} \right) h_0^2 \Rightarrow \ln (y_0) = - \ln \Psi_0 + \ln n_{x,0} + \ln \varphi + 2 \ln h_0.$$

Therefore, we have

$$\frac{\partial \ln (y_0)}{\partial i} = \frac{\partial \ln n_{x,0}}{\partial i} - \frac{\partial \ln (\Psi_0)}{\partial i} = \frac{1}{n_{x,0}} \frac{\partial n_{x,0}}{\partial i} - \frac{1}{\Psi_0} \frac{\partial \Psi_0}{\partial i}. \tag{32}$$

Using equations (17), (24), and (26), it can be shown that $\partial n_{x,0}/\partial i > 0$ and

$$\frac{\partial \Psi_0}{\partial i} = n_{r,0} \frac{\partial n_{x,0}}{\partial i}. \tag{33}$$
Plugging (33) into (32) yields

$$\frac{\partial \ln(y_0)}{\partial i} = \left( \frac{1}{n_{x,0}} - \frac{1}{n_{r,0}} \right) \frac{\partial n_{x,0}}{\partial i} < 0,$$

where the last inequality holds because production/manufacturing labor is usually much larger than the R&D labor. Therefore, welfare is a decreasing function of the nominal interest rate. Q.E.D.

The intuition can be seen from (28). When the nominal interest rate increases, the CIA constraint on R&D investment would shift labor away from R&D to manufacturing, ending up increasing $N_{x,0}$. However, this effect is dominated by the decrease in the arrival rate of innovation because less labor employed by R&D decreases the arrival rate of innovation (i.e., the decrease in the initial level of technology $Z_0$). Although the growth rate remains constant, the initial level of output decreases, ending up lowering the welfare.

Our model is similar to Chu et al. (2017). Therefore, it becomes very important to explain the key areas where the present paper is deviating from their model, the rationale for making those changes in assumptions and what it leads to in terms of the results. For instance, throughout Chu et al. (2017), it is maintained that there is a CIA constraint on consumption along with R&D investment. However, in the present paper there is no CIA constraint on consumption. It is worth explaining why we are doing away with the assumption of a CIA constraint on consumption and whether it has any bearing on the results.

We can show that Propositions 1 and 2 still hold up when the CIA constraint also applies to consumption (i.e., the CIA constraint is $c_t + b_t \leq m_t$). In this case, only equation (2) needs to be modified as

$$\frac{1}{c_t} = \mu_t (1 + i),$$

while the rest of the model remains the same. Therefore, Propositions 1 and 2 still hold up. The intuition is as follows. Without leisure in the utility function (i.e., labor supply is inelastic), the effect of the nominal interest rate operates only through the CIA constraint on R&D investment. That is, under inelastic labor supply, the CIA constraint on consumption has no effect on labor supply and labor allocation (between manufacturing and R&D) (see also discussions in Chu and Cozzi, 2014). This is because the channel for the CIA constraint on consumption to impact the economy is through the consumption-leisure choice that is absent in our model. Therefore, the nominal interest rate through the CIA constraint on consumption has no effect on growth and welfare. To summarize, the difference between our findings and those in Chu et al. (2017) is not driven by the CIA constraint on consumption.
It is worth discussing that our predictions in Propositions 1 and 2 remain robust to the consideration of the fact that the accumulation (production) of health needs both medical expenditure (a sacrifice of consumption) and leisure, as elaborated on in the following section.

3.1 Medical Expenditure and Leisure as Two Inputs of Health Production

Now each household maximizes her lifetime utility given in equation (1) subject to the asset-accumulation equation given by

$$\dot{a}_t + \dot{m}_t = r_t a_t + w_t N_t - c_t - e_t - \pi_t m_t + i_t l_t + \tau_t,$$

(36)

where $c_t$ and $e_t$ are per capita consumption and medical expenditure, respectively. The other variables are the same as before.

Labor market condition is still (3). However, now $l_t$ (the amount of effective labor allocated to leisure) would be combined with medical expenditure to produce a high level of health (see e.g., van Zon and Muysken, 2001; Huang, He, and Hung, 2013; Huang and He, 2015; Halliday et al., 2017; Kelly, 2017). The accumulation equation of health status/stock $h_t$ is given by

$$h_t = \xi l_t^\alpha \left( \frac{e_t}{w_t} \right)^{1-\alpha},$$

(37)

where $\xi$ is the productivity parameter for health capital investment, and $\alpha \in (0,1)$ is a parameter governing the share of leisure in producing health. Medical expenditure is scaled by the wage rate to remove the scale effect in health capital accumulation. Otherwise, the growth rate would not be a constant (which violates the balanced growth rate observed in advanced economies).

The optimal condition for consumption and the intertemporal optimality condition remain the same as in equations (5) and (6). The optimal condition for the ratio of medical expenditure to the market value of leisure (the product of the wage rate and leisure) is

$$\frac{e_t}{w_t l_t} = \frac{1 - \alpha}{\alpha},$$

(38)

which shows that the ratio of medical expenditure to the market value of leisure is stationary, which is not a function of the nominal interest rate.

We also have the arbitrage condition between investment in asset holding and that in health:

$$r_t = \xi \alpha \left( \frac{1 - \alpha}{\alpha} \right)^{1-\alpha} + \frac{w_t}{w_t l_t},$$

(39)

The growth rate of $Z_t$ is still $g_z = \lambda_t \ln \gamma = \Psi n_{r,t} \ln \gamma$, which is linear in the share
of labor employed by R&D firms. On the balanced growth path, the constant \( g_z = (\Psi \ln \gamma) n_{r,t} \) implies that \( \Psi \) is a constant, which yields \( g_z = g_h \). Equation (20) shows that \( g_y = g_z + g_h \). Therefore, we have

\[
g_y = 2g_h = 2\xi \frac{l}{h} \left( \frac{1 - \alpha}{\alpha} \right)^{1-\alpha}, \tag{40}
\]

where we used equations (37) and (38). Therefore, the balanced growth rate is a constant if \( l/h \) is stationary. Combining equations (40) and (39) yields

\[
2\xi \frac{l}{h} \left( \frac{1 - \alpha}{\alpha} \right)^{1-\alpha} + \rho = \xi \alpha \left( \frac{1 - \alpha}{\alpha} \right)^{1-\alpha} \frac{\dot{w}_t}{w_t}. \tag{41}
\]

On the balanced growth path, we have \( \dot{w}/w = \dot{h}/h \), given \( g_y = 2g_h \). Therefore, equation (41) becomes

\[
\xi \frac{l}{h} \left( \frac{1 - \alpha}{\alpha} \right)^{1-\alpha} + \rho = \xi \alpha \left( \frac{1 - \alpha}{\alpha} \right)^{1-\alpha}, \tag{42}
\]

which solves for the stationary value of \( l/h \), which is not a function of the nominal interest rate. The growth rate of output would be

\[
g_y = 2 \left[ \xi \alpha \left( \frac{1 - \alpha}{\alpha} \right)^{1-\alpha} - \rho \right], \tag{43}
\]

which is independent of the nominal interest rate. Given \( g_z = g_h = g_y/2 \), the growth rates of health capital and technology are not a function of the nominal interest rate.

**Proposition 3** Even if health capital needs medical expenditure and leisure to produce, the growth rates of output, health capital and aggregate technology still do not depend on the nominal interest rate. Moreover, welfare is still a decreasing function of the nominal interest rate.

**Proof:** The growth path is proven in text. The proof of the welfare part remains identical to that in Proposition 2. Q.E.D.

Concerning growth, the mechanism and its difference from that in Chu et al. (2017) remain similar to those at end of Proposition 1. Generally speaking, the accumulation of health capital is not affected by the increase in the nominal interest rate, decreasing the relative complexity of innovation and thereby increasing the aggregate arrival rate of innovation, which in turn helps to nullify the negative effect of an increase in the nominal interest rate on R&D due to the CIA constraint.
It is worth discussing the following. Although our predictions in Propositions 1 and 2 do not depend on the accumulation/production of health, they crucially depend on the form of the CIA constraint, as elaborated on in the following section.

### 3.2 CIA on Medical Expenditure

Building on Section 3.1, now the CIA constraint is given by $\theta c_t + b_t \leq m_t$, where the parameter $\theta > 0$ determines the strength of the CIA constraint on medical expenditure. Here we assume that the medical expenditure is subject to the CIA constraint.

The optimal condition for consumption and the intertemporal optimality condition remain the same as in equations (5) and (6). The optimal condition for the ratio of medical expenditure to the market value of leisure (the product of the wage rate and leisure) is (see the Appendix for derivation)

$$\frac{e_t}{w_t l_t} = \frac{1 - \alpha}{\alpha} \frac{1}{(1 + \theta i_t)}, \quad (44)$$

which shows that the ratio of medical expenditure to the market value of leisure is a decreasing function of the nominal interest rate. This is expected because medical expenditure is subject to the CIA constraint.

We also have the arbitrage condition between investment in asset holding and that in health:

$$r_t = \xi \alpha \left( \frac{e_t}{w l_t} \right)^{1 - \alpha} + \frac{w_t}{w_t}.$$  

We still have $g_z = g_h = g_y/2$, and

$$g_y = 2g_h = 2\xi \frac{1}{h} \left( \frac{e_t}{w l_t} \right)^{1 - \alpha}, \quad (46)$$

where we used equation (37). Equation (44) pins down the stationary value of $e/wl$. Therefore, the balanced growth rate is a constant if $l/h$ is stationary.

Combining equations (46) and (45) yields

$$2\xi \frac{l}{h} \left( \frac{e_t}{w l_t} \right)^{1 - \alpha} + \rho = \xi \alpha \left( \frac{e_t}{w l_t} \right)^{1 - \alpha} + \frac{w_t}{w_t}. \quad (47)$$

On the balanced growth path, we have $\dot{w}/w = \dot{h}/h$, given $g_y = 2g_h$. Therefore, equation (47) becomes

$$\xi \frac{l}{h} \left( \frac{e_t}{w l_t} \right)^{1 - \alpha} + \rho = \xi \alpha \left( \frac{e_t}{w l_t} \right)^{1 - \alpha}, \quad (48)$$

15
which solves for the stationary value of $l/h$, which is a decreasing function of the nominal interest rate. As the nominal interest rate increase, the share of effective labor supply devoted to leisure decreases. The growth rate of output would be

$$g_y = 2 \left[ \xi \alpha \left( \frac{1 - \alpha}{\alpha} \frac{1}{(1 + \theta_i)} \right)^{1-\alpha} - \rho \right], \quad (49)$$

Equation (49) shows that the growth rate of output is a decreasing function of the nominal interest rate. Given that $g_z = g_h = g_y/2$, both the growth rates of health capital and technology are a decreasing function of the nominal interest rate.

**Proposition 4** Under the CIA constraint on medical expenditure, the growth rates of output, health capital and aggregate technology are all a decreasing function of the nominal interest rate.

**Proof:** Proven in text. Q.E.D.

The intuition of the above results can be explained as follows. There is a CIA constraint on R&D investment and medical expenditure. Health capital/health status needs medical expenditure (a sacrifice in consumption) and leisure to produce. The benefit of good health is that it increases effective labor supply. An increase in the nominal interest rate raises the cost of medical expenditure via the CIA constraint and leads to a reallocation of output from medical expenditure to consumption. Moreover, $l/h$ (the share of effective labor supply devoted to leisure) decreases as the nominal interest rate increases. This is because leisure and medical expenditures are complementary in producing health. The decrease in one input would decrease the marginal product of the other input, and agents would invest less in the other input as well. Therefore, the growth rate of health capital decreases. As the growth rate of output is twice those of health capital and technology, the decrease in the growth rate of output is twice as large as those of health capital and technology. On the flip side, a decrease in the nominal interest rate would increase the growth rates of health capital and technology, and the increase in output growth would be twice as large. This happens because output is a product of technology and production labor. An increase in health increases the effective amount of labor used in R&D and decreases the relative complexity of innovation, ending up increasing the growth rate of technology. Additionally, the increase in health increases the effective amount of labor used in production, doubling the effect of an increase in health. In other words, when the medical expenditure is a cash good instead of a credit good, it will amplify the negative/positive effect of an increase/decrease in the nominal interest rate.

Concerning welfare, considering Proposition 2 and equation (28), one can see that an increase in the nominal interest decreases the initial level of output besides the growth
rate of output. Therefore, with the CIA constraint on medical expenditure, the welfare loss would be larger.

3.3 Health-in-the-Utility Function

Building on Section 3.2., at time $t$, the population size of each household is fixed at 1. There is a unit continuum of identical households, who have a lifetime utility function as

$$U = \int_{0}^{\infty} e^{-\rho t} \left[ \ln (c_t) + j \ln (h_t) \right] dt,$$

(50)

where $h_t$ is per capita health stock at time $t$. $j > 0$ governs the preference for health relative to consumption.

The arbitrage condition between investment in asset holding and that in health becomes

$$r_t = \frac{j}{h \eta} + \xi \alpha \left( \frac{e_t}{wl_t} \right)^{1-\alpha} + \frac{\dot{w}_t}{w_t},$$

(51)

where $\eta$ is the co-state variable on (36).

We still have $g_z = g_h = g_y/2$, and

$$g_y = 2g_h = 2 \frac{l}{h} \left( \frac{e_t}{wl_t} \right)^{1-\alpha}.$$  

(52)

Combining equations (52) and (51) yields

$$2 \frac{l}{h} \left( \frac{e_t}{wl_t} \right)^{1-\alpha} + \rho = \frac{j}{h \eta} + \xi \alpha \left( \frac{e_t}{wl_t} \right)^{1-\alpha} + \frac{\dot{w}_t}{w_t}.$$  

(53)

On the balanced growth path, we have $\dot{w}/w = \dot{h}/h$, given $g_y = 2g_h$. Therefore, equation (53) becomes

$$\xi \frac{l}{h} \left( \frac{e_t}{wl_t} \right)^{1-\alpha} + \rho = \frac{j}{h \eta} + \xi \alpha \left( \frac{e_t}{wl_t} \right)^{1-\alpha}.$$  

(54)

The growth rate of output would be

$$g_y = 2 \left[ \frac{j}{h \eta} + \xi \alpha \left( \frac{e_t}{wl_t} \right)^{1-\alpha} - \rho \right].$$

(55)

Comparing to cases where health is not in the utility function, the balanced growth rate is higher with the HIU. This is because with the HIU (i.e., $j > 0$), investing in health has an additional benefit (i.e., having a good health directly increases utility level). Therefore,
consumers would like to invest more in health, ending up pushing up the balanced growth rate.

However, it is still obvious that growth of output is still a decreasing function of the nominal interest rate when the CIA constraint applies to the medical expenditure. Using (55), the growth rate of output would be

\[ g_y = 2 \left[ \frac{j}{h \eta} + \xi \alpha \left( \frac{1 - \alpha}{\alpha \cdot (1 + \theta i)} \right)^{1-\alpha} - \rho \right], \]

which is a decreasing function of the nominal interest rate.

By contrast, when there is no CIA constraint on the medical expenditure, combining (38) and (55) yields the growth rate of output as

\[ g_y = 2 \left[ \frac{j}{h \eta} + \xi \alpha \left( 1 - \frac{\alpha}{\alpha} \right)^{1-\alpha} - \rho \right], \]

which is not a function of the nominal interest rate.

4 Conclusions

In this study we investigate the role of endogenous health investment in a monetary Schumpeterian growth model. In so doing, we reveal a novel channel through which monetary policy could impact economic growth and welfare. We find that the effect of an increase in the nominal interest rate on long-run growth crucially depends on the form of the CIA constraint. When the CIA does not apply to medical expenditure, long-run growth does not depend on the nominal interest rate. The result remains robust when health capital does not need medical expenditure to produce (i.e., health capital only needs leisure to produce). By contrast, when the CIA applies to medical expenditure, an increase in the nominal interest rate leads to a decrease in R&D and health investment, which in turn reduces the long-run growth rates of technology and output. Concerning welfare, it is always a decreasing function of the nominal interest rate. However, with the CIA constraint on medical expenditure, the welfare loss is larger. The results hold up with the HIU.

As discussed in the introduction, health and human capital share many similar features. Therefore, our work together with Chu et al. (2017) enhances our understanding the role of health and human capital in the process of economic development. The policy implication is as follows. Health capital is essential for promoting economic growth, but people’s health investment may be affected by monetary policy. In our study, health capital needs medical expenditure and leisure to produce, and medical expenditure may...
also be subject to the CIA constraint. When this happens, a higher nominal interest rate (an expansionary monetary policy) would incur an additional cost for health capital investment. A resultant lower rate of health capital accumulation may retard growth and reduce welfare. This important channel should be taken into account in evaluating the effects of monetary policy on economic growth and social welfare. There may be other important issues concerning the role of health in the process of economic development, and we leave them to future research.

Appendix.

Households’ dynamic optimization: The Hamiltonian is

\[ H_t = \ln c_t + \mu_t \left[ r_t a_t + w_t (h_t - l_t) - c_t - e_t - \pi_t m_t + i_t b_t + \tau_t \right] + v_t (m_t - b_t - \theta e_t) + \eta_t \xi_t^{\alpha} \left( \frac{e_t}{w_t} \right)^{1-\alpha}. \]

The first-order conditions include

\[
\begin{align*}
\frac{\partial H_t}{\partial c_t} &= \frac{1}{c_t} - \mu_t = 0, \quad (58) \\
\frac{\partial H_t}{\partial e_t} &= -\mu_t - \theta v_t + (1 - \alpha) \eta_t \xi_t^{\alpha-1} e_t^{-\alpha} w_t^{\alpha-1} = 0, \quad (59) \\
\frac{\partial H_t}{\partial l_t} &= -\mu_t w_t + \alpha \eta_t \xi_t^{\alpha-1} \left( \frac{e_t}{w_t} \right)^{1-\alpha} = 0, \quad (60) \\
\frac{\partial H_t}{\partial b_t} &= \mu_t i_t - v_t = 0, \quad (61) \\
\frac{\partial H_t}{\partial a_t} &= \mu_t r_t = \rho \mu_t - \dot{\mu}_t, \quad (62) \\
\frac{\partial H_t}{\partial m_t} &= -\mu_t \pi_t + v_t = \rho \mu_t - \dot{\mu}_t, \quad (63) \\
\frac{\partial H_t}{\partial h_t} &= \mu_t w_t = \rho \eta_t - \dot{\eta}_t. \quad (64)
\end{align*}
\]

(58) yields

\[ c_t = \frac{1}{\mu_t}. \quad (65) \]

(62) gives the intertemporal optimality condition:

\[ -\frac{\dot{\mu}_t}{\mu_t} = r_t - \rho. \quad (66) \]

Combining (59), (60) and (61) yields

\[ \frac{e}{w_t} = \frac{1 - \alpha}{\alpha} \frac{1}{(1 + \theta t_t)}. \quad (67) \]
Substituting (61) into (63) and equating it to (62) yield \( i_t = \pi_t + r_t \), where \( i_t \) is the nominal interest rate. Taking the logarithm of (60) and differentiating it with respect to \( t \) yield

\[
\frac{\dot{i}_t}{i_t} + \frac{\dot{w}_t}{w_t} = \frac{\dot{\eta}_t}{\eta_t} + (1 - \alpha) \frac{(e/wl)}{e/wl},
\]

(68)

Substituting (62), (64) and (60) into (68) yields

\[
r_t = \xi \alpha \left( \frac{e_t}{wl_t} \right)^{1-\alpha} + \frac{\dot{w}_t}{w_t} - (1 - \alpha) \frac{(e/\dot{w}l)}{e/wl} = \xi \alpha \left( \frac{e_t}{wl_t} \right)^{1-\alpha} + \frac{\dot{w}_t}{w_t},
\]

(69)

where the last equality uses (67).

References


