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Abstract

We propose a theory based on the firm's hiring behavior that rationalizes the observed decline of callback rates for an interview, exit rates, and reemployment wages over unemployment duration. We build a directed search model with symmetric incomplete information on worker types and non-sequential search by firms. Sorting due to firms' testing of applicants in the past makes expected productivity fall with unemployment duration, which induces firms to rank applicants by duration. In equilibrium callback and exit rates both fall with duration. In our numerical exercise using US data we show that our model can replicate quite well the observed falling patterns, and that the effects of the firm's ranking decision can be sizable.

Keywords: Ranking; Sorting; Directed Search; Exit Rates; Reemployment Wages; Unemployment Duration

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1 Introduction

It is well documented that exit rates from unemployment strongly fall with unemployment duration. Most explanations of this pattern have focused on the supply side, assigning a very limited role to firms. A primary reason for this has been the scarcity of empirical evidence about the hiring process.[1] However, recent field experiments provide strong indication that firms make use of the information that unemployment duration conveys for their recruiting decisions. By submitting applications of fictitious workers to job postings, Oberholzer-Gee (2008), Kroft, Lange, and Notowidigdo (2013) and Eriksson and Rooth (2014) find that the rate at which an applicant is called back for an interview (i.e. the callback rate) significantly declines with unemployment duration.

When unemployment duration is informative about workers' expected productivity and firms meet several applicants at once, recruiting firms have incentives to rank applicants by duration when calling them for interviews. Note, however, that the firm's ranking reduces the chances to fill a vacancy as those job-seekers who anticipate that they will be discriminated against will apply somewhere else unless wages compensate for the additional unemployment risk. Thus, the ranking of candidates that firms choose will depend on both expected productivity and wages.

In this paper, we analyze the firm's optimal hiring decisions, and how these decisions determine the duration dynamics of callback rates for an interview, exit rates from unemployment and reemployment wages. We show that firms rank applicants by unemployment duration in equilibrium, which endogenizes the ranking mechanism first introduced by Blanchard and Diamond (1994). We also show that our theory rationalizes the observed severe fall of callback and exit rates as well as the mild decline of wages over duration. Furthermore, we find that the ranking margin is crucial for these results, and that its quantitative effects can be sizable.

To explicitly model the ranking strategies of employers, we set up a directed search model of the labor market with two key ingredients. First, skilled workers are both more productive and more likely to be suitable for any job than the unskilled, but information about workers' type is symmetric and incomplete. Second, firms can screen out unsuitable applicants, and can discriminate among observationally different workers both through wages and by deciding on a ranking of the candidates to be tested.

In our model, unobserved heterogeneity together with the firms' testing in the past leads to sorting as in the seminal paper by Lockwood (1991). In effect, unemployment duration

[1] See Oyer and Schaefer (2011) for a discussion.

conveys information about expected productivity because skilled workers leave the unemployment pool faster. Firms set wages to solve the trade-off spelled out above between attracting more applicants and testing them according to a chosen ranking. Notice that if firms committed to a single wage, as in Lang, Manove, and Dickens (2005) and Peters (2010), the ranking by unemployment duration would be the obvious optimal decision ex-post, but firms would attract fewer applicants. In contrast, we allow firms to commit to a menu of wages, potentially contingent on expected productivity. Equilibrium wages pay a share of worker's productivity net of the value of the next best candidate, thereby making the most productive applicants the most profitable ones as well as ensuring that all workers obtain their market value when queueing for the job. As a result, the labor market is non-segmented in equilibrium because firms find it optimal to attract workers of all durations to increase their job-filling rate and to save on wages as the presence of more competitors reduces the marginal value of any given worker.

Why do exit rates fall as unemployment duration increases? First, because applicants are tested only if no worker with shorter duration either applied for the job or was found to be suitable, thereby leading callback rates for an interview to fall with duration. Second, exit rates also fall due to sorting as it more and more reduces the probability of succeeding at the test.[2]

We then investigate the effects of the firm's ranking decision by comparing to an alternative economy that only differs from the benchmark in that firms are forced to test all applicants. We refer to it as the *NR economy.* In this alternative setting, callback rates are constant in duration by assumption, and, although firms can still discriminate among observationally different workers through wages, the labor market is segmented in equilibrium.[3] The elimination of the ranking margin makes it too costly for firms to attract all applicants: the gains from the higher job-filling rates when also attracting workers with longer unemployment spells are offset by the lower expected productivity per filled vacancy and the higher wage costs necessary to compensate the more productive workers for their lower matching rates. When firms can discriminate in favor of the more productive workers instead, such costs do not exist. Likewise, we find that while in our benchmark economy

[2]Notice that the declining pattern over unemployment duration hinges on the labor market being non-segmented in equilibrium. If it were segmented to some degree with different submarkets targeting different subsets of unemployment duration, then callback rates would exhibit upward jumps when moving from one submarket to the subsequent one as applicants would go from being ranked last to being ranked first.

[3]This is in line with the results obtained in Menzio and Shi (2010) with on-the-job directed search and heterogeneous workers. Furthermore, Lang, Manove, and Dickens (2005) and Peters (2010) show that the wage margin is also central for the market to be non-segmented by studying an economy in which firms commit to a single wage but can rank applicants.

exit rates always fall with duration but wages might be non-monotone, wages in the NR economy continuously fall and the duration dynamics of exit rates are ambiguous.

We evaluate the quantitative properties of our model to gauge how well it can replicate the data, and to illustrate the effects of the firms' ranking decision. We calibrate the model to the U.S. labor market. We target several moments of the duration distribution of exit rates that we construct from the Current Population Survey (CPS), whereas no information from wages and callback rates is used. The simulated data replicates very well the actual distribution of exit rates. In addition, model wages show a declining pattern over unemployment duration very close to the actual one we obtain from the CPS. Callback rates also significantly drop with duration in line with the empirical evidence from the aforementioned field experiments. Moreover, our numerical exercise suggests that ranking strongly amplifies the fall of exit rates along duration, even if the degree of ex-ante heterogeneity is very small.

Our main contribution to the macroeconomic literature is to provide a theory that jointly rationalizes the observed duration patterns of callback rates, exit rates, and wages. As we briefly discuss below, previous work has primarily focused on mechanisms that theoretically explain the decline of job-finding rates along unemployment duration, giving little role to wages and, more generally, to the firms' hiring behavior. In contrast, our theory is based on firm's endogenous ranking induced by sorting, which together with the equilibrium wages firms commit to ensures a non-segmented labor market.

Our ranking result is a form of rational stigma based on the information conveyed by the sorting of unemployed workers as time passes. Lockwood (1991) first modeled rational stigma associated with long-term unemployment as a duration cut-off rule within a random search framework.[4] Two recent pieces in this vein that have a quantitative approach are Jarosch and Pilossoph (2015), who also study the duration dynamics of callback rates, and Doppelt (2015). Search is random and wages do not play an allocative role in these papers since they are derived from a surplus-sharing rule. To gain tractability, firms have all the bargaining power in Lockwood (1991) and Jarosch and Pilossoph (2015), but thereby making reemployment wages constant in duration as in the ranking model of Blanchard and Diamond (1994). In Doppelt (2015), workers learn about their types during both employment and unemployment spells, and, according to his calibration exercise, his stigma mechanism also accounts for a sizable part of the decline in exit rates, with equilibrium wages varying non-monotonically with duration.

We also contribute to the directed search literature that started with Peters (1991) and

[4] Vishwanath (1989) is an earlier contribution on stigma using a partial equilibrium search model. The importance of unobserved heterogeneity in the decline of exit rates over duration is well known since the work of Lancaster (1979) and Heckman and Singer (1984).

Moen (1997). Within this literature, ranking as a decision of firms has first been modeled in a static setting by Shi (2002) and Shimer (2005a).[5] Their primary focus is on the assignment of exogenously heterogeneous workers to a distribution of firms. Instead, we investigate the effects of ranking on the duration dynamics of exit rates and wages when the distribution of expected productivities is endogenous. Furthermore, and in contrast to their models, we argue that the equilibrium allocation is not constrained efficient in our setting due to an intertemporal information externality.

In closely related work, Gonzalez and Shi (2010) analyze an economy with bilateral meetings, where equally productive workers learn about their matching ability (akin to our concept of suitability) from their own search. In contrast to our benchmark model, a segmented labor market arises endogenously because staying unemployed one more period makes workers search for lower wage jobs, which are easier to obtain. Therefore, reemployment wages decline with duration in equilibrium, but exit rates need not fall because of sorting. Flemming (2015) also rationalizes the different sensitivity of exit rates and wages to unemployment duration in a directed search model with learning by doing.

As the assumption of non-sequential search of firms is key for our results, we now briefly summarize the empirical evidence in its support. Using Dutch data, van Ours and Ridder (1992) and Abbring and van Ours (1994) conclude that firms' search is non-sequential, unlike workers' search. Consistent with these findings, van Ommeren and Russo (2009) find that whether firms' search is sequential depends on the search method. In particular, firms search non-sequentially when they use formal methods such as advertising.

Finally, there is a number of theories, complementary to ours, that model a causal effect of duration on exit rates. Workers may get discouraged if the returns to their search fall with unemployment duration due to for example skill attrition (Pissarides (1992)). Fewer job opportunities also arise as unemployment progresses in stock-flow search models (Coles and Smith (1998)) and models with informational networks deteriorating along unemployment duration (Calvo-Armengol and Jackson (2004)).

The paper proceeds as follows. In Section 2, we analyze the benchmark economy and characterize the equilibrium. Section 3 studies the economy without ranking. In Section 4, we undertake a numerical exercise. Finally, Section 5 concludes and discusses some central assumptions of the model. All proofs and data work are relegated to the Appendix.

[5] Moen (1999) constructs a model of ranking by education showing that human capital investments prior to matching are undertaken not only to raise future wages, but also employment prospects.

2 Benchmark Model

This section presents a directed search model of the labor market in which firms may discriminate among applicants in wages and in the hiring decision.

2.1 Environment

Time is discrete and continues forever. There is a unit mass of infinitely lived workers and a large continuum of identical firms. The mass of new firms is determined by free entry every period. All agents are risk neutral and discount future payoffs at a common factor β. The focus of this paper is on the steady-state allocation; hence, time indices are suppressed.

Workers can be either skilled (type h) or unskilled (type ℓ).[6] There is a mass $\mu \in (0,1)$ of skilled workers, and $1-\mu$ of unskilled workers. Worker types differ both by their market productivity and by their idiosyncratic suitability at any given firm. A type-i job-seeker turns out to be suitable for the job at hand with probability λ_i, with $i \in \{\ell, h\}$. If a worker is not suitable for a given job, the match is not productive and the worker is not hired. A type-i suitable applicant produces y_i units of output if hired. We assume that skilled workers have higher chances of being suitable and perform strictly better at the production stage, i.e. $0 < \lambda_\ell < \lambda_h \leq 1$ and $y_\ell < y_h$.[7]

At the beginning of every period, workers can be either employed or unemployed. The unemployed seek job opportunities and derive utility from home production, $b < y_\ell$. Let τ denote their elapsed duration of unemployment, with $\tau = 1, 2, ..., T$. The value $\tau = T$ stands for unemployment durations greater than or equal to T. That is, workers with $\tau = T$ form the homogeneous group of the long-term unemployed. Unemployment duration is public information.[8] In contrast, there is symmetric incomplete information on the type of the worker. That is, the worker's type is unobservable to both the worker herself and potential employers. Concretely, we make two assumptions on what information is held and acquired in each period to ensure that current unemployment duration is the only observable

[6] Although the analysis is done with two types of workers for expositional simplicity, the model can easily be extended to any finite number of types.

[7] Our suitability concept is in line with the frictions proposed by Petrongolo and Pissarides (2001, p.402). We further comment on this in Section 4. It could also be interpreted as an extreme form of match-specific productivity if output is the product of a match-specific component and worker's time-invariant productivity. We would say that a worker is unsuitable for a given firm if the match-specific term were zero. Suitability may also represent a reduced-form of modeling the scope of tasks at a given job, where skilled workers can do a larger number of tasks. The central assumption in our setting is that skilled workers perform better at the testing stage.

[8] Kroft, Lange, and Notowidigdo (2013) report that 75% of the actual resumes they collected from job boards did specify the year and month the last job of the candidate had ended.

characteristic.

First, we assume that information about a worker's type reduces to her current unemployment spell. Specifically, the worker herself only knows the information of the current spell. This assumption is made for tractability since the whole labor history of each worker would be informative to herself and recruiting firms otherwise. In support of this, Eriksson and Rooth (2014) find that past, unlike contemporary, unemployment does not drive the recruiting decisions of firms. Second, for tractability and since our main interest is the hiring behavior of firms, we abstract from the worker's learning, which has been modeled in Gonzalez and Shi (2010), and assume that job-seekers do not learn from their own search experience. Thus, a worker's search influences only their contemporaneous prospects in the labor market, but do not affect their continuation value of unemployment if failing to find a job.[9]

Consequently, workers with unemployment duration τ are all observationally identical, also to themselves. They are suitable for any job with probability

$$p_\tau = \frac{\lambda_\ell u_\ell(\tau) + \lambda_h u_h(\tau)}{u_\ell(\tau) + u_h(\tau)}, \tag{1}$$

where $u_i(\tau)$ denotes the measure of unemployed workers of type i and duration τ at the beginning of a given period. The unemployment distribution $u \equiv \{u_i(\tau)\}_{i,\tau}$ is the aggregate state variable in this economy. The expected match productivity of a suitable candidate of duration τ is determined by

$$\overline{y}_\tau = \frac{y_\ell \lambda_\ell u_\ell(\tau) + y_h \lambda_h u_h(\tau)}{\lambda_\ell u_\ell(\tau) + \lambda_h u_h(\tau)}, \tag{2}$$

Each period consists of four stages: the separation stage, the job-posting stage, the application and meeting stage, and the hiring and production stage. At the beginning of the period, in the first stage, idiosyncratic job-separation shocks hit ongoing matches with probability δ. In those cases, the worker becomes newly unemployed and the firm exits.

The Job-posting Stage. In the second stage, firms decide whether to enter the labor market or not. As is common in the search literature, each firm posts a single vacancy. Firms incur a cost k when posting the vacancy. To ensure existence of equilibria, we assume that vacancy creation costs are low relative to the discounted net productivity of unskilled workers, i.e. $k < \frac{y_\ell - b}{1-\beta(1-\delta)}$.

[9] We discuss the sensitivity of the main results to the information structure in Section 5.

Recruiting firms announce and fully commit to a contract. A contractual offer consists of a menu of wages, which may be contingent on expected productivity.[10] Let $\omega \equiv \{w_\tau\}_{\tau \leq T} \in [0, y_h]^T$ denote a job offer. We shall refer to a submarket as the marketplace defined by a given contract. Two remarks are in order. First, we could allow for wage contracts to specify a continuous function of expected productivity. Since in our model only a finite number of expected productivity levels prevail, we can restrict the set of contracts to finite wage schemes without loss of generality. Second, we index wages by unemployment duration instead of expected productivity. This notation does not mean that wages are contingent on duration, as it may be against the law. This is for notational simplicity as expected productivity given by expression (2) maps bijectively to unemployment duration.

The Application and Meeting Stage. Unemployed workers observe all job offers and submit one application. Search is directed in the sense that those vacancies that promise a higher expected value will attract a larger number of applicants. However, firms set a menu of wages not only to trade off higher wages with a higher rate of applications (extensive margin), but also to influence the relative number of candidates of a given expected productivity (intensive margin).

Meetings are multilateral in the sense that any given firm may receive several applications. As is standard to assume in the literature for large economies, (observationally) identical workers use identical mixed application strategies, and, hence, the realized number of applications a firm receives for any given unemployment duration is a Poisson random variable under the assumption that actual applications are independent across workers. The key characteristic of a multilateral meeting technology is that it enables firms to compare applicants.

The Hiring and Production Stage. In the hiring stage, firms make three decisions if receiving any application. First, they rank candidates according to their expected profitability. Second, they test those who are ranked first. Third, they select a suitable applicant, if there is any. In this last step, firms are assumed to randomize among suitable workers who are observationally identical (i.e. with the same unemployment duration), leading to the so-called coordination frictions. If no candidate is suitable, then they continue testing in the chosen order. The firm vanishes if there is no suitable candidate queueing for the job.

Firms have access to a simple testing technology: A firm observes a private, match-specific

[10] In particular, wages cannot be contingent on ex-ante unobservables such as the ex-post revealed actual productivity or the outcome of the meeting process (that is, e.g., how many applications the firm received of each type). We think of those events as unverifiable by a third party and therefore non-enforceable.

signal, which perfectly identifies unsuitable matches. Unsuitable candidates are discarded as the match turns out to be unproductive. Other than determining suitability, firms have no means to identify the type of the applicants when making the hiring decision. We assume that the test outcome is not verifiable and cannot be traded. Thus, neither workers nor other firms learn the private signals of any given firm. Furthermore, the testing expenses are included in the vacancy creation cost k.

Firms decide on an order to start testing applicants.[11] Formally, a firm sets a ranking rule σ. This ordering must be such that, for any $\tau, \tau' \in \{1, ..., T\}$,

$$\sigma(\tau) < \sigma(\tau') \text{ iff either } J_\tau(\omega) > J_{\tau'}(\omega) \text{ or } J_\tau(\omega) = J_{\tau'}(\omega) \text{ and } \tau < \tau', \tag{3}$$

where $J_\tau(\omega)$ stands for the type-ω firm's expected discounted value when filling the vacancy with a suitable worker of duration τ. We have imposed that if two different durations correspond to the same expected profitability, then workers with the shorter spell are ranked higher by assumption. For a given value of the state variable u, the permutation σ is bijective by construction.

We are interested in the equilibrium allocation in which firms rank candidates by unemployment duration. Therefore, for expositional simplicity, we guess that the ranking rule $\sigma(\tau) = \tau$ is consistent with the profit-maximizing behavior of firms, and, later on, Proposition 2.3 will verify this guess.[12] The intuition is that expected productivity declines with unemployment duration irrespective of the ranking rule firms have chosen in the past as the more productive workers are more likely to leave unemployment at any duration. Thus, our guess on the optimal ranking rule must be read as follows: firms rank candidates by expected productivity because the more productive applicants are also the more profitable ones. All agents rationally anticipate that recruiting firms will optimally rank workers by expected productivity (or, equivalently, duration) at any point in time. Finally, production takes place.The worker and all other agents in the economy only observe the hiring decision.[13]

[11] Since testing costs are included in the vacancy creation costs k, firms could test all applicants at once instead of proceeding according to some order, and then rank suitable candidates by profitability. The economy that results from this alternative interpretation is completely equivalent to ours, except for the fact that the rate at which an applicant is called back for an interview would be 1 for all applicants by construction. However, our setting can be defended on the grounds that a tiny testing cost would make firms establish some order in the testing process.

[12] In contrast to Lang, Manove, and Dickens (2005), Shimer (2005a), Peters (2010) and our setting, Shi (2002) allows firms to commit to a menu of wages and also to a ranking rule ex-ante. In this alternative contracting space, he shows that it is also ex-ante optimal to rank workers by productivity.

[13] Consistent with our information assumptions, we can assume that either the actual worker's productivity is never learned by employers, or it is instantaneously learned upon hiring but firing is not allowed because of full commitment.

Matching Probabilities. A firm posting a job ω expects $q_\tau(\omega)$ suitable applicants of duration τ. To simplify notation, we will omit the dependence of the expected queue length q on the contract ω hereafter, unless needed for clarity. Define $q^\tau \equiv (q_1, q_2, ..., q_\tau)$. For each firm, the probability of filling a job ω with a suitable worker of duration τ is

$$\eta_\tau(q^\tau) = e^{-\sum\limits_{\tau'<\tau} q_{\tau'}} \left(1 - e^{-q_\tau}\right). \tag{4}$$

The first factor of this expression stands for the Poisson probability that no worker with a higher ranking than a candidate of duration τ either applies to the firm or, if applies, is found suitable for the job.[14] The second term is the Poisson probability that the firm receives at least one suitable application from workers of duration τ. Note that this expression captures both the firm's ranking strategy and the fact that unsuitable workers are never hired.

Since the measures of newly employed workers and filled vacancies of each duration must coincide, it must be the case that $\nu_\tau(q^\tau)q_\tau = \eta_\tau(q^\tau)$, where $\nu_\tau(q^\tau)$ denotes the job-finding probability for a worker of duration τ conditional on applying to a type-ω firm and being suitable for the job. That is,

$$\nu_\tau(q^\tau) = e^{-\sum_{\tau'<\tau} q_{\tau'}} \frac{1 - e^{-q_\tau}}{q_\tau}. \tag{6}$$

Therefore, the actual matching probability for a worker of duration τ is then defined as

$$h_\tau(q^\tau) = p_\tau \nu_\tau(q^\tau) = e^{-\sum_{\tau'<\tau} q_{\tau'}} p_\tau \frac{1 - e^{-q_\tau}}{q_\tau} \tag{7}$$

[14] The application and meeting process can be also thought of as the limit to the large economy of its counterpart in a finite game with the urn-ball meeting protocol. In this sense, we can derive the first factor of expression (4) as the limit of the probability of receiving no suitable application of a shorter duration in the finite economy. That is, we take the limit of the probability of receiving at least one suitable application as $\{u_i(\tau)\}_i$ and v (the mass of vacancies) go to infinity, while keeping the ratio $\frac{\sum_i \lambda_i u_i(\tau)}{v}$ constant. Let $\rho_\tau(\omega)$ be the probability with which a worker of duration τ applies to a job offering contract ω. A firm receives no suitable applications of unemployment duration τ with probability

$$\Big(1 - p_\tau \rho_\tau(\omega)\Big)^{\sum_i u_i(\tau)}. \tag{5}$$

As the economy gets large, and assuming symmetric mixed strategies, i.e. $\rho_\tau = \frac{1}{v}$, we obtain

$$\lim_{\{u_i(\tau)\}_{i,\tau}, v\to\infty} \prod_{\tau'<\tau} \Big(1 - p_{\tau'}\rho_{\tau'}(\omega)\Big)^{\sum_i u_i(\tau')} = e^{-\sum_{\tau'<\tau} q_{\tau'}}.$$

Finally, the second factor of expression (4) is obtained by taking the limit of expression (5).

The exit rate from unemployment has three components. The first factor incorporates the firm's ranking strategy. We refer to it as the *callback rate* as it is the probability of being called for an interview, which occurs if no suitable worker with a shorter spell queued up for the same job. The second term captures how the composition of the unemployment pool evolves with unemployment duration, i.e. the sorting mechanism. The third term results from the standard coordination frictions among applicants of duration τ.

Value Functions. Let us proceed with the value functions of workers and firms. An employed worker derives utility from wages until the arrival of an idiosyncratic job-termination shock, which occurs with probability δ. Her value function is defined by

$$E_\tau(\omega) = w_\tau + \beta\big(\delta U_1 + (1-\delta)E_\tau(\omega)\big) \tag{8}$$

An unemployed worker of duration τ has value U_τ. She may apply to any posted job offer ω, and becomes employed with probability $h_\tau(q^\tau(\omega))$, in which case she receives the value $E_\tau(\omega)$. Otherwise, she produces b at home and remains unemployed one more period. A worker of duration τ applies to a job offering contract ω if the expected value derived from ω equates the unemployment value U_τ for a positive $q_\tau(\omega)$. Otherwise, no worker of duration τ applies to such a job, and $q_\tau(\omega) = 0$. Expectations about $q_\tau(\omega)$ are thus pinned down on and off the equilibrium. The following equilibrium condition summarizes this logic.

$$\begin{aligned} U_\tau \quad &\geq \quad h_\tau(q^\tau(\omega))\left(E_\tau(\omega) - b - \beta U_{\tau+1}\right) + b + \beta U_{\tau+1} \\ \text{and} \quad &q_\tau(\omega) \geq 0, \quad \text{with complementary slackness,} \end{aligned} \tag{9}$$

where $U_{T+1} \equiv U_T$.

Now, the expected value of a firm offering contract ω and filling its vacancy with a worker of duration τ amounts to the worker's expected productivity net of wages until the arrival of a job-destruction shock, when it exits the economy. Its value function is defined by

$$J_\tau(\omega) = \overline{y}_\tau - w_\tau + \beta(1-\delta)J_\tau(\omega). \tag{10}$$

Firms write contracts to maximize profits. A firm incurs a recruitment cost k when posting a vacancy. A job with contract ω is filled with a candidate of duration τ with probability $\eta_\tau(q^\tau(\omega))$, and then the firm obtains the expected value $J_\tau(\omega)$. The value function of a vacant firm is then defined by

$$V = \max_{\omega} \Big\{ -k + \sum_{\tau=1}^{T} \eta_\tau(q^\tau(\omega)) J_\tau(\omega) \Big\}. \tag{11}$$

2.2 Equilibrium

Next, we define the symmetric directed search equilibrium in the steady state. We use the term *symmetric* to refer to the case where identical agents make identical decisions. In particular, all firms commit to the same contract.

Definition 1 *A steady state symmetric directed search equilibrium consists of a distribution of unemployed workers $u \in [0,1]^{2\times T}$, value functions $J_\tau, E_\tau : [0, y_h]^T \to \mathcal{R}_+$, and $V, U_\tau \in \mathcal{R}_+$, $\forall \tau \in \{1, ..., T\}$, a menu of contracts $\omega \in [0, y_h]^T$, an expected queue length function $Q \equiv (Q_\tau)_\tau : [0, y_h]^T \to {\mathcal{R}_+}^T$, and the ranking rule $\sigma(\tau) \equiv \tau$ such that:*

i) Given Q and u, the value functions satisfy the Bellman equations (8)-(11).

ii) Firms' profit maximization and zero-profit condition:

- *Given u and ω, the ranking rule σ satisfies condition (3).*
- *Given $(U_\tau)_\tau$, Q, and u, ω is the profit-maximizing contract, and expected profits become zero at ω:*

$$\forall \omega' \in [0, y_h]^T, \; -k + \sum_{\tau=1}^{T} \eta_\tau(Q(\omega')) J_\tau(\omega') \le V = 0, \; \text{with equality for } \omega' = \omega.$$

iii) Workers direct their search:
$\forall \omega' \in [0, y_h]^T$ and $\forall \tau \in \{1, ..., T\}$, $Q_\tau(\omega')$ satisfies the complementary slackness condition (9).

iv) Recursivity condition:
Let $q_\tau \equiv Q_\tau(\omega)$. The distribution of workers recursively satisfies

$$\begin{aligned} u_i(\tau) &= u_i(\tau-1)\left(1 - \lambda_i \nu_{\tau-1}(q^{\tau-1})\right), \forall \tau \in \{2, ..., T-1\}, \\ u_i(T) &= u_i(T-1)\left(1 - \lambda_i \nu_{T-1}(q^{T-1})\right) + u_i(T)\left(1 - \lambda_i \nu_T(q^T)\right), \\ &\text{and} \\ u_i(1) &= \delta\Big(\mu_i - \sum_{\tau=2}^{T} u_i(\tau)\Big), \end{aligned} \tag{12}$$

v) Resource constraints:

$$\frac{q_\tau}{\sum_i \lambda_i u_i(\tau)} = \frac{q_1}{\sum_i \lambda_i u_i(1)}, \quad \textit{if } q_\tau \neq 0. \tag{13}$$

Firms maximize profits, which equal zero in equilibrium because of free entry. To rank applicants by unemployment duration is profit maximizing. The third equilibrium condition is required to pin down rational expectations on queue lengths out of the equilibrium. This condition determines the expected queue length for any given contract attractive to workers by making them indifferent between the equilibrium contract and this other offer.

The aggregate state variable u is determined by the history of all agents' equilibrium decisions. The Law of Large Numbers ensures that a measure $u_i(\tau)\lambda_i\nu_\tau(q^\tau)$ of type i workers with duration τ becomes employed. Thus, the fourth condition determines the law of motion for the state variable. Finally, the set of constraints (13) result from the requirement that in a symmetric equilibrium the ratio of suitable workers to the queue length must be the same across durations.

We now rewrite expression (11) as the profit maximization problem of a representative firm. For any given pair $(u, (U_\tau)_\tau)$, the firm's program is

$$\begin{aligned} \max_{\omega, q^T} \quad & \textstyle\sum_\tau \eta_\tau(q^\tau) J_\tau(\omega) - k \\ \text{s. to} \quad & p_\tau \eta_\tau(q^\tau)\big(E_\tau(\omega) - b - \beta U_{\tau+1}\big) + q_\tau\big(b + \beta U_{\tau+1}\big) = q_\tau U_\tau, \quad \forall \tau \end{aligned} \tag{14}$$

That is, firms choose the pair (ω, q^T) that maximizes their profits, rationally anticipating the relationship between wages and queue lengths that arises from the optimal search behavior of workers. Indeed, the constraints are the equilibrium complementary slackness conditions (9). For later use, it is convenient to define $\Delta_\tau \equiv J_\tau(\omega) + E_\tau(\omega) - b - \beta U_{\tau+1} = \frac{\overline{y}_\tau + \beta\delta U_1}{1-\beta(1-\delta)} - b - \beta U_{\tau+1}$. It is the net value of a match with a worker of duration τ. The following proposition states that there exists a solution for the firm's problem. We also provide a sufficient condition for uniqueness. Moreover, we show that it is optimal for firms to attract applicants of all durations in equilibrium.

Proposition 2.1 *Given the state variable u and the unemployment values U_τ for $\tau \in \{1, ..., T\}$, there exists a solution (ω, q^T) for the firm's program. If Δ_τ falls with duration, then the firm's problem has a unique solution. Furthermore, in any symmetric equilibrium, all queue lengths are strictly positive, $q_\tau > 0$ $\forall \tau$.*

The next proposition establishes existence of symmetric equilibrium by showing that it can be formulated as a fixed point problem, and then applying the Brouwer fixed-point theorem.

Proposition 2.2 *There exists a symmetric directed search equilibrium.*

This result together with Proposition 2.1 implies that workers of all unemployment durations search in the same labor market. Firms find it profitable to attract all workers in order to both increase the probability of filling their vacancies and extract a higher share of the surplus from a given match.[15] The latter relies on the firms' ability of treating different applicants differently through wages and ranking. To see this, notice that, because of the ranking by expected productivity, the presence of workers with longer unemployment durations queueing for the job hedges the firm's risks of ending up with the vacancy not filled by any worker of duration τ. As a result, the standard trade-off between wages and job-filling rates is weakened for applicants with shorter durations, so that firms lower the wage promises to these workers.

A natural question is why there is no profitable deviation targeting only a subset of workers. This is a fair critique to random search models with ranking wherein all job-seekers are concentrated in a single market by assumption, like Blanchard and Diamond (1994).[16] The intuition underlying this critique is that if workers could direct their search, the ranking strategy would no longer take place in equilibrium. This is because firms could profitably deviate by targeting only workers with a given duration τ and saving on wages by offering them a higher job-finding rate. In contrast, this logic does not hold in our setting because the deviating firms would find it more profitable to attract not only all workers with durations longer than τ for the two reasons listed above, but also workers with shorter durations as they are more profitable.[17] Therefore, deviations with such a recruiting strategy cannot be profitable.

2.3 Equilibrium Duration Dynamics

In this section, we look at the duration dynamics of the equilibrium variables. A key result is stated in the following proposition: although the distribution of expected productivities is an endogenous outcome, expected productivity falls with unemployment duration regardless of

[15] Recall that all workers are employable as their market productivity is strictly higher than their home productivity by assumption.

[16] See e.g. Shi (2002)

[17] To see this formally, notice that deviating firms must also solve problem (14) because its formulation allows for workers of a given duration τ not being targeted (i.e. $q_\tau = 0$).

how firms ranked candidates in the past. The underlying intuition is that skilled candidates are more likely to be suitable for any job than their unskilled counterparts at any duration and, hence, the relative mass of skilled job-seekers declines as the duration of unemployment increases. Therefore, ordering candidates by expected productivity is equivalent to ordering them by unemployment duration. It turns out that the most productive candidates are the most profitable ones in equilibrium as the fall in the expected productivity is not offset by the wage changes over duration. We give more intuition for why wages decline by less than expected productivity when we discuss equilibrium wages further below in this section. As a result, it is optimal for firms to rank candidates by their unemployment duration, confirming that the imposed ranking rule is consistent with profit-maximizing behavior.

Proposition 2.3 *Independently of the ranking rule, $\overline{y}_\tau$ falls with unemployment duration τ. Furthermore, the value of an active firm $J_\tau(\omega)$ declines with τ in equilibrium.*

The continuous decline in expected productivity suggests that the employment prospects of a worker also deteriorate over time. Proposition 2.4 shows that the equilibrium unemployment value, U_τ, declines with duration. More importantly for us, callback rates for an interview and exit rates from unemployment also fall with duration. Both rates fall because of firms' ranking decision, but also because of the labor market being non-segmented in equilibrium. To see this, consider a labor market segmented by different subsets of unemployment duration. If firms ranked workers by duration given this market structure, callback rates would exhibit upward jumps as workers of duration τ would be ranked last in one submarket whereas workers of duration $\tau + 1$ would be ranked first in another submarket. Moreover, since the probability of being suitable would monotonically decline with duration, the slope of the duration profile of exit rates would be ambiguous at those points.

It is worth underscoring that sorting is the primary factor underlying the negative relationship between unemployment duration and both callback and exit rates. To be more precise, the ranking by expected productivity relies on the information flows generated by the sorting mechanism. If there were no sorting, i.e. if $\lambda_\ell = \lambda_h$, the probability p_τ would be constant and unemployment duration would not be informative about the applicants' expected productivity, thereby eliminating the rational grounds of ranking.[18] The equilibrium allocation would not differ from the standard setting with homogeneous workers, in which all agents meet in the same market and exit rates and re-employment wages are constant in unemployment duration. However, arbitrarily small differences in suitability rates across

[18]If $\lambda_\ell = \lambda_h$ and firms ranked by unemployment duration, firms targeting workers who are discriminated against would make strictly positive profits.

types would make the component p_τ roughly constant over unemployment duration, while they would translate into large falls in callback and exit rates because of the amplification through ranking. Likewise, if there were no productivity differences across workers ($y_\ell = y_h$) and, hence, firms did not rank applicants, exit rates would still decline because of sorting.[19]

Proposition 2.4 *The callback rate, the job-finding rate and the value of unemployment for a worker fall with unemployment duration.*

We now turn to the determination of equilibrium wages. Wages obtain directly by manipulating the first order conditions of the firms' problem (14) and the complementary slackness conditions (9). They are determined by the following expression.

$$\frac{w_\tau + \beta\delta U_1}{1-\beta(1-\delta)} = \frac{q_\tau e^{-q_\tau}}{1-e^{-q_\tau}}\left(\Delta_\tau - \sum_{\tau'=\tau+1}^{T} e^{-\sum_{\tau''=\tau+1}^{\tau'-1} q_{\tau''}}(1-e^{-q_{\tau'}})\Delta_{\tau'}\right) + b + \beta U_{\tau+1}, \tag{15}$$

where $U_{T+1} \equiv U_T$. The left hand side of this expression is the employment value $E_\tau(\omega)$. In search models, workers are paid a share of the match surplus on top of their unemployment value. In directed search models, the worker's share is determined in equilibrium as the elasticity of the job-filling probability, which is the fraction in the first term on the right hand side of equation (15). In our setting with multilateral meetings, it can also be interpreted as the probability that the applicant is the only one of duration τ conditional on the firm receiving at least one suitable application of that duration. This term is multiplied by the net value of the match minus the expected net value derived from any other potential match with workers of higher durations.[20] In other words, workers are rewarded according to their marginal value relative to the next best alternative. Now, for workers of low durations the value of this next best alternative is larger than for workers of high durations, which leads to a wage profile that is less steep than the productivity profile. This explains the result in Proposition 2.3 that ex-post profits decline in duration.

Do equilibrium wages decline with unemployment duration? They need not. The following proposition claims that wages do not always fall with duration if worker types are sufficiently similar in at least one dimension.

[19] It is very easy to show that p_τ continuously falls with duration.

[20] Notice that if there were another applicant of duration τ, the marginal value of the worker would be zero. Since contractual offers cannot be made contingent on the number of applications received, firms commit to the expected marginal value of the applicant by averaging over these two events. As Shimer (2005a) and others have pointed out, equilibrium wages would equal the expected compensation a worker would obtain if the firm sold the job to the worker by using a second price sealed bid auction.

Proposition 2.5 *There exists $\epsilon > 0$ such that $w_T > w_{T-1}$ if either $y_h - y_\ell < \epsilon$ or $\lambda_h - \lambda_\ell < \epsilon$.*

To better understand the wage dynamics, consider the $T = 2$ economy, in which job-seekers can be either short-term ($\tau = 1$) or long-term ($\tau = T = 2$) unemployed. We rewrite equation (15) for the $T = 2$ case as

$$E_1(\omega) = \frac{q_1 e^{-q_1}}{1 - e^{-q_1}} \left(\Delta_1 - (1 - e^{-q_2})\Delta_2\right) + b + \beta U_2, \;\; E_2(\omega) = \frac{q_2 e^{-q_2}}{1 - e^{-q_2}} \Delta_2 + b + \beta U_2$$

It follows that

$$\begin{aligned} w_1 \geq w_2 \;\; &\Leftrightarrow \;\; \frac{q_1 e^{-q_1}}{1 - e^{-q_1}} \left(\Delta_1 - (1 - e^{-q_2})\Delta_2\right) \geq \frac{q_2 e^{-q_2}}{1 - e^{-q_2}} \Delta_2 \\ &\Leftrightarrow \;\; \frac{q_1 e^{-q_1}}{1 - e^{-q_1}} \left(\frac{\overline{y}_1 - \overline{y}_2}{1 - \beta(1-\delta)}\right) \geq e^{-q_2} \Delta_2 \left(\frac{q_2}{1 - e^{-q_2}} - \frac{q_1 e^{-q_1}}{1 - e^{-q_1}}\right) \end{aligned} \tag{16}$$

Consider first the limit case: either $y_\ell = y_h$ or $\lambda_\ell = \lambda_h$. Then, the left hand side of the inequality is zero because $\overline{y}_1 = \overline{y}_2$. However, the right hand side is strictly positive because the first term within the parenthesis is greater than one whereas the second is lower than one. Thus, $w_1 < w_2$. Consider now an arbitrarily small gap in either productivity or suitability rates between types so that the difference in expected productivity between short- and long-term workers $\overline{y}_1 - \overline{y}_2$ is also arbitrarily close to zero. Then, the above inequality is violated, and wages are higher for the long-term unemployed.

The intuition is as follows. If worker types were sufficiently close in either dimension and were perfectly observable, then their unemployment values should also be close. Since in our setting types are not observable, but unemployment duration is, then the value of unemployment could not fall significantly with unemployment duration in that case. As firms discriminate against long-term unemployed workers in the hiring stage in equilibrium, w_2 would have to be larger than w_1 to compensate for the lower matching probability. Notice that this result of increasing wages holds even if types are very dissimilar in the other dimension because this large difference is not translated into productivity differences over unemployment duration. That is, if skilled workers were much less likely to be screened out, but almost equally productive as the unskilled, expected productivity would decline very little as the productivity of the unskilled would be a lower bound. Instead, if suitability rates were arbitrarily close and skilled workers were much more productive, the impact of sorting would be very weak and the expected productivity decline would also be tiny.

It is apparent from the previous reasoning that expected productivity must fall sufficiently with unemployment duration to make wages decline. This decline in expected productivity requires sufficiently large differences in both productivity and suitability rates between worker types. However, this may not suffice for the $T > 2$ case as applicants of longer durations may also queue for the same job and, hence, some degree of convexity in the endogenous distribution of expected productivities seems necessary for falling wages. This is because firm's ranking decision implies that workers are worth a share of their marginal value instead of their productivity, and marginal values may not be monotonically decreasing even if expected productivities are. Our numerical work for high T values indicates that wages monotonically decline for a broad range of parameter values, although not in all instances.[21]

To conclude, unlike in models with bilateral meetings, where workers get a share of their productivity net of their own outside option, wages in our model reward the workers' productivity net of workers outside option and the firm's next best alternative. As a result, the ranking mechanism in our framework with multilateral meetings compresses wage differences across workers of different durations.

2.4 Constrained Efficiency

The equilibrium in the static models with observable worker productivities of Shi (2002) and Shimer (2005a) is constrained efficient. For tractability reasons, we study the efficiency properties of our model within a simplified two-period version, the details of which we delegate to Appendix 6.3. In contrast to their static models, we find that constrained efficiency is not attained in our setting because of a twofold intertemporal inefficiency.[22]

The intuition for this result is as follows. Due to sorting, vacancy creation in period one affects job creation in the second period through two margins: a vacancy posted in period one both lowers the expected number of suitable candidates queueing for jobs, and reduces the expected productivity of the matches in period two. The second externality has been previously identified by Albrecht, Navarro, and Vroman (2010) and Chéron, Hairault, and Langot (2011).[23]

To understand this inefficiency result, it is instructive to study the limit cases. First, if the testing technology is almost not informative ($\lambda_h \sim \lambda_\ell$) and, hence, neither sorting

[21] While the effect of the next best worker on wages can be significant, the reduction in value due to the long-term unemployed applicants, which have a low matching probability, tends to be quite small. Blanchard and Diamond (1994) find a similar result.

[22] Contrary to the informational cascade literature, firms do use their private signals and complement them with the publicly available information in our model. Thus, the inefficiency result found in this literature, which is due to the herding behavior of the agents who decide not to use their own information, is not present in our setting. See e.g. Bikhchandani, Hirshleifer, and Welch (1992).

[23] These two models are built within a random search framework with wage-bargaining. They find that the so-called Hosios condition does not suffice to attain constrained efficiency as wages are negotiated after learning the characteristics of the worker at hand, while firms' entry decision hinges on the workers' average productivity.

nor ranking take place, our setting turns out to be equivalent to the complete information economy. Put differently, symmetric incomplete information by itself does not generate inefficiencies. The limit case with $\lambda_h = 1$ and λ_ℓ arbitrarily small is of particular interest as the equilibrium is constrained inefficient even though the second externality is not present because all unskilled workers are unsuitable and automatically discarded. Why is this the case? If candidates' types are (arbitrarily close to be) perfectly learned through testing, but workers remain uninformed, recruiting firms benefit from the information asymmetry: the unemployment value is lower than optimal, firms post inefficiently low wages, and firms' entry becomes inefficiently large because they do not internalize the effects on period-two firms. In contrast, the social planner only cares about the mass of suitable workers, and pays no attention to the information asymmetry. For values of λ_ℓ between 0 and λ_h, the two intertemporal externalities are not fully internalized in equilibrium. We show that constrained efficiency can be attained in the market economy by properly taxing the entry of firms.

3 No-ranking Economy

In the benchmark economy, firms have two instruments to discriminate among observationally different candidates. They can offer different wages to different candidates, but also rank them according to expected profitability in the hiring stage. To understand the importance of ranking, we study an alternative setting, to which we refer as the *NR economy*, wherein firms can only use wages as a discrimination instrument, and are forced to test all candidates and randomize among suitable applicants in the hiring stage. Callback rates are, thus, constant over unemployment duration by construction. Moreover, recruiting firms that attract applicants of different expected productivities must form rational expectations about the proportions of each type. Other than that, the economy is identical to the benchmark.

The probability of becoming employed conditional on being suitable for the job ω does not vary with the worker' unemployment duration by assumption, and, hence, it amounts to $\nu(q(\omega)) = \frac{1-e^{-q(\omega)}}{q(\omega)}$, where $q(\omega)$ denotes the expected number of applicants to job ω.[24] The exit rate from unemployment still depends on the length of the unemployment spell because

[24] Although this conditional probability formula may be intuitive given firms' randomization among suitable applicants, it can be formally derived as follows (we are thankful to Steffen Grønneberg for giving key insights). Let $q^T \equiv (q_1, ..., q_T)$ and $q \equiv \sum_\tau q_\tau$. Since the probability distribution that governs the arrival of applications of any type is Poisson, a firm hires a type τ worker with probability

$$\eta_\tau(q^T) = e^{-q} \sum_{m_s \geq 0; 1 \leq s \leq T} \frac{m_\tau}{\sum_s m_s} \frac{q_\tau^{m_\tau}}{m_\tau!} \prod_{s \neq \tau} \frac{q_\tau^{m_s}}{m_s!}, \tag{17}$$

of the sorting factor, $h_\tau(q(\omega)) = p_\tau \nu(q(\omega))$. Consistently, the probability of filling a job equals $\eta(q(\omega)) = 1 - e^{-q(\omega)}$.[25]

When posting job ω, firms form rational expectations about the probability of filling the vacancy with a worker of duration τ. Let $\rho(\omega) \equiv \big(\rho_1(\omega), ..., \rho_T(\omega)\big)$ be a point of the unit (T-1)-simplex that denotes such expectations. The expected value of a vacant firm is

$$V = \max_{\omega} \Big\{ -k + \eta(q(\omega)) \sum_{\tau} \rho_\tau(\omega) J_\tau(\omega) \Big\} \tag{18}$$

Likewise, the unemployment value for a worker with unemployment duration τ applying to job ω is

$$\begin{aligned} h_\tau(q(\omega))\,(E_\tau(\omega) - b - \beta U_{\tau+1}) + b + \beta U_{\tau+1} \quad &\leq U_\tau, \quad \text{and } \rho_\tau(\omega) \geq 0, \\ \text{with complementary slackness,} \quad &\text{and} \quad q(\omega) > 0 \text{ iff } \max_\tau \rho_\tau(\omega) > 0 \end{aligned} \tag{19}$$

Notice that these two expressions are the counterparts of expressions (11) and (9) in the NR economy, respectively, and only differ in the object ρ. The expectations on the queue length q and proportions ρ are also pinned down off the equilibrium path to help determine the equilibrium allocation.

3.1 No-ranking Equilibrium

We next define a symmetric equilibrium in the steady state. We use the term *symmetric* to refer to the allocation wherein workers who are observationally identical (i.e. workers with the same unemployment duration) make identical decisions. However, firms may commit to different contracts, unlike in the benchmark, and, hence, there may be a number of submarkets open in equilibrium.

Definition 2 *A steady state no-ranking symmetric directed search equilibrium consists of a distribution of unemployed workers* $u \in [0,1]^{2\times T}$, *value functions* $J_\tau, E_\tau : [0, y_h]^T \to \mathcal{R}_+$,

where m_s denotes the number of applications received from type s workers, and the first term after the sum symbol stands for the randomization strategy in the selection process. Again, the conditional job-finding probability must satisfy $\nu_\tau(q^T) = \eta_\tau(q^T)/q_\tau$. We use expression (17) to obtain a similar formula for ν_τ. By manipulating conveniently the subindices, we obtain that $\nu \equiv \nu_\tau = \nu_{\tau'}$ for all τ, τ'. Then, from $\eta_\tau(q^T) = q_\tau \nu(q^T)$, it follows that $\eta(q^T) \equiv \sum_\tau \eta_\tau(q^T) = q\nu(q^T)$. Given that $\eta(q^T) = 1 - e^{-q}$, we obtain $\nu(q^T) = \frac{\eta(q^T)}{q} = \frac{1-e^{-q}}{q}$.

[25] In effect, the NR economy is equivalent to an economy with a bilateral meeting technology that matches vacancies and suitable workers. For a related study of the relationship between the set of equilibria and the meeting technology in an asymmetric information environment see Eeckhout and Kircher (2010).

and $V, U_\tau \in \mathcal{R}_+$, $\forall \tau \in \{1, ..., T\}$, *a distribution of vacancies* $\mathcal{F}$ *over the set* $[0, y_h]^T$ *with support* Ω*, an expected queue length function* $Q : [0, y_h]^T \to \mathcal{R}_+$*, and an expectation function* $\rho : [0, y_h]^T \to \{x \in [0,1]^T | \sum_t x_t = 1\}$ *such that:*

i) Given Q *and* u*, the value functions satisfy the Bellman equations (8), (10), (18) and (19).*

ii) Firms' profit maximization and zero-profit condition:

Given $(U_\tau)_\tau$*,* Q*,* ρ *and* u*, firms maximize profits at any contract in* Ω*, and expected profits are zero:*

$$\forall \omega \in [0, y_h]^T, \ -k + \eta(q(\omega)) \sum_\tau \rho_\tau(\omega) J_\tau(\omega) \leq V = 0, \ \text{with equality for } \omega \in \Omega$$

iii) Workers direct their search:

$\forall \omega \in [0, y_h]^T$ *and* $\forall \tau \in \{1, ..., T\}$*,* $Q(\omega)$ *and* $\rho(\omega)$ *satisfy the complementary slackness condition (19).*

$\forall \omega \in \Omega$*,* $Q(\omega) > 0$*, and* $\exists \tau \in \{1, ..., T\}$ *such that* $\rho_\tau(\omega) > 0$*.*

iv) Recursivity condition:
The distribution of workers recursively satisfies

$$\begin{aligned} u_i(\tau) &= u_i(\tau-1)\Big(1 - \lambda_i \int_\Omega \rho_\tau(\omega)\nu(Q(\omega))d\mathcal{F}(\omega)\Big), \ \forall \tau \in \{2, ..., T-1\}, \\ u_i(T) &= u_i(T-1)\Big(1 - \lambda_i \int_\Omega \rho_{T-1}(\omega)\nu(Q(\omega))d\mathcal{F}(\omega)\Big) + \qquad (20) \\ &\quad + u_i(T)\Big(1 - \lambda_i \int_\Omega \rho_T(\omega)\nu(Q(\omega))d\mathcal{F}(\omega)\Big), \qquad (21) \end{aligned}$$

and

$$u_i(1) = \delta\Big(\mu_i - \sum_{\tau=2}^{T} u_i(\tau)\Big),$$

v) Market clearing:

$$\int_\Omega \rho_\tau(\omega)Q(\omega)d\mathcal{F}(\omega) = \lambda_\ell u_\ell(\tau) + \lambda_h u_h(\tau) \qquad (22)$$

When designing profit-maximizing contracts, firms form rational expectations about both the queue length and the proportion of suitable applicants from each duration that apply to the job. The third equilibrium condition establishes that the expected queue length at any job ω equals the maximum of the q values that guarantees workers of any duration τ to obtain the unemployment value U_τ if applying to contract ω. The intuition is as follows:[26] consider workers of two different unemployment durations τ and τ' applying to contract ω, and let q and q', with $q < q'$, denote the queue lengths that secure their unemployment value. Then, if q were the equilibrium expected queue length, there would be a flow of workers of duration τ' applying to ω, increasing the ratio of applicants per vacancy, and workers of duration τ would no longer apply. Moreover, firms form rational expectations about the proportion of suitable applicants from each duration that apply to the job because the labor market may be segmented, unlike in the benchmark. The last equilibrium condition ensures that the sum of suitable applicants of a given duration across markets must be equal to the total supply of suitable unemployed workers of that duration.

We now turn to the firm's problem. Given the vector $(u, (U_\tau)_\tau)$, a firm chooses the combination of an expected queue length and a wage scheme to maximize its expected discounted profits. By choosing this pair, the firm is indeed deciding on what type of workers to target. Since it may not be optimal for the firm to attract all workers as it will not be able to discriminate among them ex-post, the optimal application condition (19) will only hold for the targeted durations. We can rewrite the counterpart of the firm's problem (14) as

$$\begin{aligned} \max_{q,\{w_\tau, r_\tau\}_\tau} \quad & \eta(q) \sum_\tau r_\tau \frac{\overline{y}_\tau - w_\tau}{1-\beta(1-\delta)} \\ \text{s. to} \quad & p_\tau \eta(q) \frac{w_\tau + \beta\delta U_1}{1-\beta(1-\delta)} \geq qU_\tau - q\left(1 - p_\tau \nu(q)\right)\left(b + \beta U_{\tau+1}\right), \ \forall \tau \mid r_\tau > 0 \\ & r_\tau \geq 0, \sum_\tau r_\tau = 1 \end{aligned} \tag{23}$$

Notice that this problem is linear in r_τ. Profit-maximization requires that no firm attracts workers of a duration τ if the expected profits are below the maximum. That is, a firm will

[26] This thought experiment is similar to the one in Guerrieri, Shimer, and Wright (2010) in a setting with asymmetric information. They show that, under some sorting condition, the equilibrium is separating. Chang (2012) shows that heterogeneity other than productivity is needed to obtain semi-pooling equilibria with asymmetric information.

obtain $M \equiv \max_{\tau} M_\tau$ by making $r_\tau = 0$ and $w_\tau = 0$ for all τ such that $M_\tau < M$, where

$$\begin{aligned} M_\tau \quad &\equiv \max_{q_\tau, w_\tau} \quad \eta(q)\frac{\overline{y}_\tau - w_\tau}{1-\beta(1-\delta)} \\ &\text{s. to} \quad p_\tau \eta(q_\tau)\frac{w_\tau + \beta\delta U_1}{1-\beta(1-\delta)} \geq q_\tau U_\tau - q_\tau\left(1 - p_\tau \nu(q_\tau)\right)\left(b + \beta U_{\tau+1}\right) \end{aligned} \tag{24}$$

Thus, the firm's problem reduces to allocating a positive weight to duration τ if the net returns from attracting only workers of duration τ, M_τ, attain the maximum M and a zero weight otherwise. If workers of different durations yielded the maximum return, then firms would be indifferent between attracting applicants from one or several durations. Submarkets are linked both through the recursivity condition of the state variable u, which helps to determine both the expected productivity $\overline{y}_\tau$ and the suitability probability p_τ, and the unemployment continuation values $\{U_\tau\}_\tau$.

Using Brouwer's fixed point theorem, we show existence of an equilibrium allocation with a labor market fully segmented by unemployment duration. The following proposition states this result, and characterizes the equilibrium. Notice that any other symmetric equilibrium must be observationally equivalent to this one. It also states that there is no equilibrium with a single labor market because the $\tau = T$ and $\tau = T-1$ workers will always search in different submarkets as ex-post profits are strictly lower if forming a match with the former than with the latter.

Proposition 3.1 *There exists an equilibrium in which the labor market is segmented by unemployment duration and $\Omega = \{\omega_1, ..., \omega_T\}$, where ω_τ consists of a zero wage for all durations but τ. The equilibrium queue length q_τ and wage w_τ in submarket τ satisfy the following conditions*

$$\frac{w_\tau + \beta\delta U_1}{1-\beta(1-\delta)} = \frac{e^{-q_\tau} q_\tau}{1-e^{-q_\tau}}\Delta_\tau + b + \beta U_{\tau+1} \tag{25}$$

$$k = \eta(q_\tau)\left(1 - \frac{e^{-q_\tau} q_\tau}{1-e^{-1_\tau}}\right)\Delta_\tau \tag{26}$$

Furthermore, there does not exist an equilibrium with a non-segmented market.

The equilibrium equation (26) is the zero-profit condition, whereas expression (25) is the standard determination of wages and is derived from the first order condition of program (24).[27] Notice the difference between the benchmark equilibrium wages, determined

[27]Notice that, after replacing wages using the constraint, the objective function of problem (24) is strictly concave in q. Therefore, the necessary first order condition is also sufficient, and there exists a unique

in expression (15), and the wages in the NR economy. When ranking is allowed, wages are reduced relative to the NR wages by the expected value of filling the vacancy with a suitable candidate of a longer unemployment duration. Therefore, conditional on the same queue lengths, equilibrium wages in the benchmark would be lower than in the NR economy.

3.2 Equilibrium Duration Dynamics

We turn now to study the dynamics of the equilibrium variables over unemployment duration. Proposition 2.3 states that expected productivity declines with duration. However, we show below that ex-post profits J_τ need not decline with τ in the NR economy. Still, a worker's unemployment value falls as the length of the unemployment spell increases if sorting takes place. Moreover, in contrast to the benchmark, reemployment wages always fall with duration.

Proposition 3.2 *The value of unemployment U_τ and reemployment wages w_τ fall with unemployment duration.*

What can be said about the duration dynamics of exit rates from unemployment? If $T = 2$, it can be shown that exit rates decrease with duration.[28] However, in the general case, this is more difficult to establish as the two components of the exit rates in the NR economy, $h_\tau(q_\tau) = p_\tau \nu(q_\tau)$, can move in opposite directions. The first component is related to the sorting mechanism and always declines with unemployment duration if $\lambda_\ell < \lambda_h$. The second factor also falls if q_τ increases, which requires that the joint value Δ_τ or, equivalently, ex-post profits decline with duration, according to the zero-profit condition (26). However, the profits from hiring a worker may increase with the length of her unemployment duration. For example, consider the limit case of $y_\ell = y_h$. Expected productivity $\overline{y}_\tau$ remains constant as duration increases, and the joint value Δ_τ and the profits $\left(1 - \frac{e^{-q_\tau} q_\tau}{1-e^{-q_\tau}}\right)\Delta_\tau$ increase because of the steady decline in the unemployment value U_τ. In this case, the second component of the exit rates, $\nu(q_\tau)$, increases because of a larger firm entry generated by the increase in Δ_τ. Therefore, the duration pattern of the exit rates is ambiguous as the two factors move in opposite directions as duration increases.

solution to the program.

[28]When $T = 2$, the labor market is fully segmented as the short-term unemployed workers are more profitable than their long-term counterparts, i.e. $\Delta_1 > \Delta_2$. As a result, the zero-profit condition implies that $q_1 < q_2$, and, hence, exit rates fall with unemployment duration. Proposition 3.2 states that equilibrium wages w_τ also fall. Notice that this contrasts with the increasing wages over unemployment duration in the benchmark if worker types are sufficiently close in one dimension as stated in Proposition 2.5

3.3 The Qualitative Effects of Ranking by Duration

In our model, sorting leads to an endogenous preference for workers with short unemployment spells. The multilateral meeting technology together with their ability to discriminate at the hiring stage permits firms to treat different applicants differently. Against the backdrop of our discussion of the NR economy we now highlight the role of the firms' ranking decision.

First, note that the ranking result is independent of whether firms have the ability to commit to wages contingent on expected productivity or not. If firms can only commit to a single wage contract as in Lang, Manove, and Dickens (2005) and Peters (2010), ranking by productivity is an obvious outcome. In our setting, like in Shi (2002) and Shimer (2005a), as contingent wages reward worker's marginal value, they are compressed enough so that the ex-post profits fall with expected productivity, and, hence, ranking by expected productivity is an optimal decision.

The firm's ability of establishing a hiring order and offering different wages is instrumental in the labor market being non-segmented in equilibrium. By attracting all workers, firms increase the chances to fill their vacancies. Moreover, the ranking of applicants allows firms to further reduce the wage bill for any unemployment duration as they no longer face the standard trade-off between a lower wage and a lower job-filling probability, since it is now attenuated by the presence of applicants with longer unemployment spells. As a result, exit rates fall because of sorting, which lowers the probability of passing the test as unemployment duration increases, and the ranking decision of firms, which makes callback rates fall with duration.

When ranking is not permitted instead, the market is segmented in equilibrium. To better understand this, consider the $T = 2$ case. When firms have only wages as an instrument to discriminate among applicants, they find it too costly to attract observationally different workers at the same time. Suppose that a firm targeting only workers of duration $\tau = 1$ offered the job also to workers with longer unemployment spells to increase the probability of filling the vacancy. In equilibrium, this positive effect on expected profits would be dominated by two negative effects. First, the firm would have to compensate the $\tau = 1$ applicants with higher wages because of the reduction in their job-finding probability. Second, conditional on filling the vacancy, expected profits would be lower because expected productivity declines over unemployment duration as stated in Proposition 2.3. In contrast, all firms find it optimal to attract all types of workers in the benchmark economy because the two negative effects do not occur when firms can discriminate among candidates in the hiring stage.

Notice that the result of ranking by duration is preserved even for arbitrarily small differences across worker types. Put differently, ranking has the potential to significantly amplify the effects of worker differences in either productivity or suitability rates on exit rates. We discuss the magnitudes of the ranking effect in the following section.

4 Quantitative Exploration

We next calibrate the model to the U.S. economy to gauge how well the benchmark model captures the data, and to illustrate the quantitative effects of the ranking mechanism. To better summarize these effects, we often focus on the values at 3 months of unemployment.[29] Furthermore, we use the calibrated model to explore how the duration dynamics of callback rates, exit rates and wages are affected by changes in aggregate productivity.

4.1 Calibration and Results

For this numerical illustration, we use publicly available data from Current Population Survey (CPS) when possible. Our CPS data work is postponed to Appendix 6.1. We now outline our calibration strategy.

We set a period to be a week despite transition rates in and out of unemployment being estimated at a monthly frequency. The reason for this choice is twofold. First, unemployment duration is reported in weeks by the subjects surveyed in the CPS, and, as Figure 5 shows, most of the action takes place within the first 15 weeks of unemployment. Second, it alleviates the time aggregation bias of the transition rates as we can construct the model counterparts of actual monthly aggregates. We comment on this issue further below. The weekly discount factor β matches a yearly interest rate of 5%. The parameter defining the state of long-term unemployment, T, is set equal to 52 weeks.[30] We normalize the market productivity of the skilled workers to one, $y_h = 1$.

We then jointly calibrate the parameters b, k, δ, λ_h, λ_ℓ, μ, and y_ℓ. We follow Hall and Milgrom (2008) and set 71% of average worker productivity as the target for b. Parameter k is calibrated to match the ratio of vacancy costs to the simulated average quarterly wage per hire, which is estimated to be 13% by Abowd and Kramarz (2003).[31]

[29]The reason to choose this specific duration is twofold: it is approximately the average unemployment duration in our sample, and marks a clear change in the slope of the duration distribution of exit rates. Moreover, only approximately 17% of the observations in our sample correspond to unemployment durations longer than 6 months.

[30]In our dataset 97% of the transitions from unemployment to employment correspond to spells shorter than one year. Moreover, we find that our results are robust to changes in T. For example, an increase (decrease) of T by 50% changes the fall of the exit rate at duration 3 months, relative to the first week, from 51.68% in the benchmark calibration to 52.30% (50.49%) after recalibrating.

[31]The number for this target that Hall and Milgrom (2008) use is 14%.

Table 1: Calibration

Parameter	Description	Value	Target
Exogenously Set Parameters			
β	Discount factor	0.999	Annual interest rate of 5%
T	LTU-defining duration	52	-
y_h	Productivity of skilled	1.0	(Normalization)
Jointly Calibrated Parameters			
δ	Job-separation rate	0.0033	Predicted monthly job-separation rate
k	Vacancy cost	0.7071	13% avg. quarterly wage per hire
b	Home productivity	0.6968	71% of avg. productivity
λ_h	Skilled suitability prob.	0.3195	Avg. duration prior to E within next month
λ_ℓ	Unskilled suitability prob.	0.0832	St. dev., skewness and kurtosis of monthly exit rates
μ	Share of skilled	0.3439	
y_ℓ	Productivity of unskilled	0.9712	

The remaining parameters are related to labor market transitions. We use targets from the predicted distributions obtained from our data set as described in Section 6.1 in the Appendix.[32] The target for the job-separation rate δ is the average monthly transition rate from employment (E) to unemployment (U). The predicted EU transition rate has a period average of 0.009.[33]

Petrongolo and Pissarides (2001) find that the simple version of the urn-ball matching technology implies too low an unemployment duration for a given ratio of vacancies to unemployment. To improve the fit to the data they suggest to add an additional friction in the form of match-specific suitability. This is indeed the role played by λ_h in our setting (whereas $\frac{\lambda_\ell}{\lambda_h}$ accounts for the relative suitability of the unskilled). Therefore, we set λ_h to match an average unemployment duration of 12.57 weeks.[34]

The remaining parameters, λ_ℓ, μ, and y_ℓ, are related to the unobserved heterogeneity across workers. Because differences in types translate into differences in job-finding rates, our strategy is to make the model replicate the actual duration distribution of monthly exit rates.[35] Specifically, we target the standard deviation (0.0801), skewness (0.7725), and

[32] Recall expectations are quite common in the U.S. Since we do not model recall, workers expecting to be rehired by a former employer are not counted as unemployed in the calibration targets we compute from our predicted data. See e.g. Pries and Rogerson (2005) and Bils, Chang, and Kim (2011) for a similar approach. In the words of Blanchard, Diamond, Hall, and Murphy (1990), recalls do not require the posting of vacancies. Fujita and Moscarini (2012) find that 85% of workers in temporary layoff are rehired. This significantly affects our estimates of the transition rates.

[33] This estimate is much lower than the standard one, reported e.g. in Shimer (2005b) for the 1951-2003 period. Yet, it is close to the numbers found by Fujita and Moscarini (2012), also for the period 1994-2012 and conditioning on permanent separations, and close to the estimate in Pries and Rogerson (2005).

[34] To be precise, average duration is computed as the average length of unemployment spells conditional on finding a job within the next month both for actual and simulated data.

[35] As the number of worker types is fixed and sufficiently small, it can be shown that the distribution

kurtosis (2.3267) of the distribution of the predicted monthly exit rates.

Time aggregation bias in the transition rates among employment states has been found to be quantitatively important, particularly when estimating EU transition rates. The standard correction for this bias assumes a duration-independent exit rate, see Shimer (2005b). Obviously, this assumption fails to hold in our setting. Instead of correcting for it in the data, we simulate data from the model and aggregate appropriately to obtain the model counterparts of the monthly numbers from the data. For example, for the monthly job-separation rates, we compute, out of the total mass of workers who are employed at the beginning of a given period, how many have become unemployed four periods later. We do take into account that displaced workers can find new jobs in each of the interim periods, but we do not consider further rounds in and out of unemployment as their occurrence is negligible.

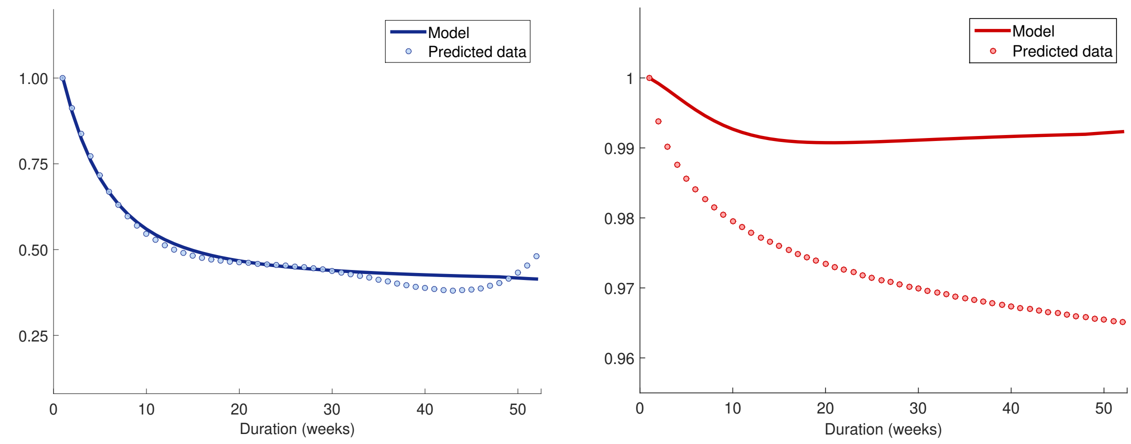
Table 1 summarizes our procedure and estimates. All the targets are very accurately matched. One out of three workers in our economy is skilled, and they account for about 11% of the unemployed. The skilled are about four times more likely to be suitable, and almost 3% more productive when employed.[36] We can compare our model outcomes to other available data moments not targeted in the calibration. First, the model steady state unemployment rate is 3.79, slightly below its data counterpart of 4.18. Second, the number of the tested unemployed per vacancy is 5.17 in the model, close to the 6.33 documented by Barron, Bishop, and Dunkelberg (1985) for the U.S. 1980 EOPP survey, but far away from the 12 reported by van Ours and Ridder (1992) using Dutch 1987 data.

We now turn to the duration profiles of exit rates, callback rates, and wages.[37] Figure 1(a) depicts normalized simulated data of the monthly exit rates from our calibrated model together with the actual values. The model matches the actual duration function of the exit rates from unemployment remarkably well. The monthly job-finding rate after 13 weeks of

by unemployment duration of exit rates along with the job-separation rate provide sufficient information to identify the values of these parameters. In our numerical illustration, we do not aim at identifying the unobserved distribution of types itself.

[36]Our interpretation of the small difference in productivities together with the large difference in suitability rates is as follows. The suitability friction has a direct effect on the duration distribution of exit rates. In contrast, productivity differences only affect them indirectly through its effects on vacancy creation, and such effects are attenuated because of the ranking decision of firms. That is, regardless of how low the expected productivity is, the exit rate of a suitable candidate with a 10-week unemployment spell is primarily reduced by the presence of other applicants with shorter durations who are ranked higher. In contrast, in the NR economy, productivity has a much stronger effect on vacancy creation for all durations and thus productivity differences need to be larger to match a given duration profile of exit rates. Specifically, when the NR economy is calibrated to the same set of targets (as reported in Appendix 6.4), productivity differences are much larger, while the suitability rates are marginally different from the ones in the baseline calibration.

[37]The net value Δ_τ falls with duration in equilibrium, and, hence, the firm's problem has a unique solution.



(a) Monthly Exit Rates

(b) Reemployment Wages

Figure 1: Distributions over Unemployment Duration: Model and Data.

Note: Exit rates from unemployment are the transition rates from unemployment to employment within the following month. Wages are the expected wage conditional on being employed after four weeks. All values are normalized by the value at the first week.

unemployment, roughly the average unemployment duration, is 51.68% of the rate at the first week (vs. 50% in the data). As in the actual data, the simulated duration function shows a severe decline over the first 3 months and flattens out from then on. Furthermore, we also closely match the level of the non-normalized exit rates not targeted in the calibration. The simulated monthly job-finding probability at the first week of unemployment is 0.43, whereas it is 0.44 in the data.

Moreover, our calibrated model delivers a distribution of callback rates. As shown in Figure 2, these rates strongly fall across duration.[38] Specifically, callback rates fall by 24% from one month of unemployment to six months. The field experiments conducted in Oberholzer-Gee (2008) for Switzerland, Kroft, Lange, and Notowidigdo (2013) for the U.S. and Eriksson and Rooth (2014) for Sweden also show a steep decline of callback rates along duration. In particular, in Kroft, Lange, and Notowidigdo (2013) fictitious job-seekers of different unemployment duration each submit a single application and wait for an interview call for up to 6 weeks, although most calls arrive within the first two weeks. They find that their estimated callback rates fall by 20% from one month of unemployment to six months.

Next, we consider reemployment wages. Recall that we do not use any target related to wages in our calibration. Figure 1(b) displays the normalized simulated wages together with the normalized predicted wages estimated in Section 6.1 in the Appendix. To be consistent with the data counterpart, simulated wages for any duration τ are computed as the expected

[38]In this figure, weekly rates are reported.

wage conditional on being employed after 4 weeks. While we show the wages on a magnified scale for better visibility, the model distribution of wages over unemployment duration is very close to the actual one. Indeed, model wages are compressed slightly more than what we observe in the data. They fall by 0.8% after three months of unemployment (vs. 2.3% in the data) and very slightly increase towards the end. Another way to compare the two wage series is to compute the duration elasticity of the simulated wages by applying the regression procedure that we used for the actual data in Section 6.1. We obtain that the data and simulated duration elasticities are -0.0090 and -0.0032, respectively.

Why is our model able to generate the diverging pattern of exit rates and wages present in the data? Notice that workers are heterogenous along two dimensions, which both affect wages and exit rates. The productivity difference, $y_h - y_\ell$, is fairly small in our calibration and thus expected productivity and wages do not fall much along duration. In contrast, the large difference in the suitability parameters λ_h and λ_ℓ allows for a large variation of the probability of being suitable, p_τ, across durations. This makes the fall in exit rates strong. As we argue below, the ranking component also substantially contributes to explaining the different slopes of exit rates and wages along duration.

Decomposing the Exit Rate Dynamics. We now decompose the exit rates from our calibrated economy into its three components determined in expression (7), i.e. the callback rate ($e^{-\sum_{\tau'<\tau} q_{\tau'}}$), the probability of passing the suitability test (p_τ), and the pure coordination friction for duration-τ workers ($(1-e^{-q_\tau})/q_\tau$). Figure 2 plots the job-finding rate as well as its components, all normalized by the respective first value. As the components cannot be individually time-aggregated we report weekly values. Both the callback rates, which capture the ranking decision, and the probability of being suitable first rapidly decline and then flatten out. The latter declines fast initially because of the large differences in suitability rates leading to a strong sorting effect that becomes less pronounced as the pool of unemployed more and more consists only of unskilled applicants. Further, due to sorting and hiring, the pool of unemployed, and thus the ratio q_τ, becomes smaller with duration. This implies that a worker of low duration has relatively many competitors from the next lower duration, whereas this is not as pronounced for workers with long spells. Thus, the callback rate initially declines fast and then becomes flatter with duration. Finally, as q_τ decreases the coordination friction becomes less severe and therefore slightly increases with duration. Unlike the first two components, the pure coordination friction does not contribute much to the duration dynamics in quantitative terms.[39]

[39] In terms of levels, only the sorting factor is significantly below one at the first week and thus is the dominating component for determining the level of the job-finding rate at the first week. Recall that the

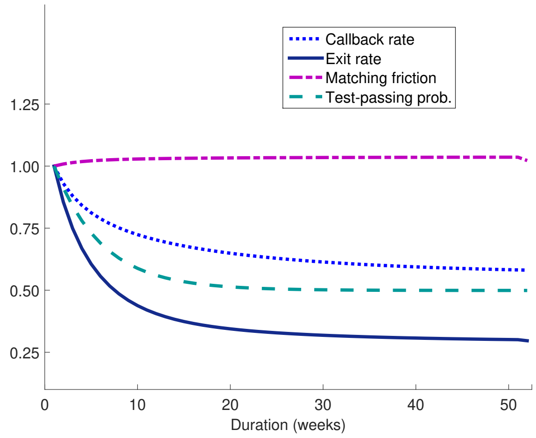


Figure 2: Components of Weekly Exit Rates

Notes: All values are determined on a weekly basis and normalized by the value at the first week.

4.2 The Quantitative Effects of Ranking by Duration

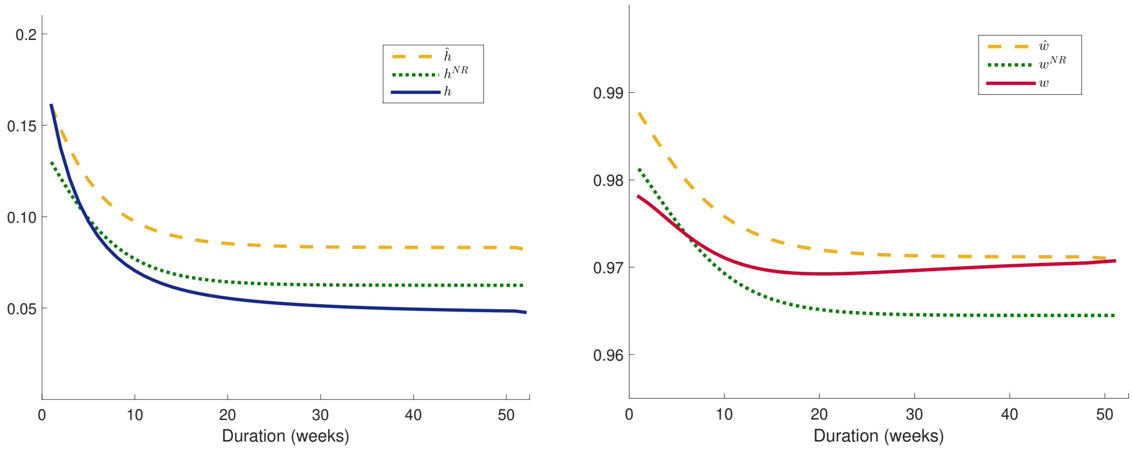
To provide further insights on the quantitative effects of ranking, Figure 3(a) plots the exit rate of the benchmark model with (h) and without ($\hat{h}$) the callback rate component as well as the exit rates of the NR economy (h^{NR}) using the same calibrated parameters.[40] Going from the overall exit rate h to the counterfactual rate $\hat{h}$, we see that without the ranking effect the duration profile becomes much flatter. This difference is due to the amplifying effect of ranking on the fall of the exit rate. Next, the move from $\hat{h}$ to h^{NR} captures the additional general equilibrium effects of ruling out ranking. While the slope of the duration profile stays similar to the one of $\hat{h}$, there is a downward shift of the whole distribution in addition. As a result, without ranking workers of low duration have a lower exit rate, whereas it is the opposite for workers with high duration. In the benchmark economy, workers with short unemployment spells face more job opportunities than in the NR economy because of a larger entry of firms due to higher expected profits and being ranked ahead. In contrast, applicants with longer unemployment spells are discriminated against in the benchmark, which is not the case in the NR economy. Our calibration suggests that the difference between the two environments regarding the decline of the exit rates is quantitatively significant. The fall in the monthly exit rate after 3 months relative to the first week is 48% in the benchmark -which is closely matching the fall in the data-, and only 37% in the NR economy.[41]

callback rate at the first week is one by construction.

[40] To see the shift in levels the values are not normalized.

[41] While the distributions of the exit rates strongly differ across economies, the average monthly job-finding rates and implied unemployment rates are quite similar. They are 26.4% and 3.8% for the benchmark economy and 26.6% and 3.6% for the NR economy, respectively. Further, the vacancy rates are 0.72% for

We can also show the effects of ranking on wages. Similar to the above comparison we first compute counterfactual wages by omitting the summation term from expression (15). As explained above, this summation term is due to the ability of the firm to rank applicants and represents the expected value of the next best alternative. To capture the general equilibrium effect we compare to the wages of the NR economy, again using the parameters of the calibrated benchmark model. Figure 3(b) shows the wages for the benchmark (w), the counterfactual wages without the ranking component ($\hat{w}$), and the NR wages (w^{NR}), which fall by 1.3% after three months of unemployment.



(a) Exit Rates (b) Wages

Figure 3: The Effects of Ranking

Note: Exit rates and wages are determined on a weekly basis and reported as non-normalized values. Symbols h and w refer to the equilibrium exit rates and wages in the benchmark economy, whereas $\hat{h}$ and $\hat{w}$ denote the same rates and wages without the ranking factor. Symbols h^{NR} and w^{NR} denote the equilibrium exit rates and wages in the NR economy.

Wages of low durations are increased more than wages for high duration when we remove the ranking component of wages, i.e. when moving from w to $\hat{w}$. This is due to the fact that the expected value of the next best alternative is higher for applicants of low duration. If in addition general equilibrium effects are taken into account, wages w^{NR} still show a steeper decline, and are shifted downwards so that only workers with short spells get a higher wage in the case of no ranking. Wages across duration are therefore more compressed in the presence of ranking.

If we instead recalibrate the NR economy using the same targets, exit rates will match closely the targeted empirical exit rates, but now wages will decline much more strongly than in the data because the difference in the productivity values across types is now much

the benchmark and 0.75% for the NR economy, respectively.

larger. See Appendix 6.4 for details. Thus, the ranking element helps to capture the duration profiles of both exit rates and wages in the data.

The previous analysis has decomposed the duration dynamics of exit rates and wages into its main driving factors, namely ranking and sorting. Recall that ranking is an endogenous response to sorting in our model and therefore cannot be considered as an independent factor. In particular, the relationship between ranking and sorting is very non-linear. When there are no differences in suitability rates, unemployment duration is not informative, and hence firms have no reason to rank candidates. However, even if there is an arbitrarily small difference in suitability rates, firms will rank, thereby largely affecting callback rates. To give an example, in our calibrated benchmark, the weekly rate of being suitable declines by 45% after 3 months, whereas callback rates fall by 31%. Now, if we decrease λ_h to a value of just 5% above λ_ℓ, the fall in the suitability factor reduces to less than a tenth of a percent, whereas callback rates still decline by 27%.[42] Thus, even a small amount of unobserved heterogeneity can lead to a large effect on callback rates.

4.3 Comparative Statics of Productivity Changes

We can use the calibrated model to explore the impact of changes in aggregate productivity on the duration dynamics of callback rates, exit rates and wages. This is interesting as the effects of productivity changes are the result of opposing forces coming from the sorting and ranking components.

To begin with, we look at exit rates, $h_\tau(q^\tau) = p_\tau \nu_\tau(q^\tau)$. On the one hand, notice that when vacancies are relatively more abundant due to higher aggregate productivity, sorting is stronger, workers with long unemployment spells are more likely to be unskilled and, hence, function p_τ falls more steeply over duration. On the other hand, the relative decrease of the callback rates across durations becomes less pronounced as more vacancies are offered because job-seekers with long spells have relatively fewer competitors of shorter duration. In addition, at each duration, there is a lower degree of coordination friction as more vacancies are available. As we cannot ascertain theoretically which effect dominates, we compare the calibrated economy with one in which both productivities, y_h and y_ℓ, are increased by 3%. Figure 4 plots the ratio of exit rates and its components for the two economies. For the overall exit rate, the ratio remains above one for all durations and is relatively higher for the longer spells. By looking at the ratios of the individual components we can see that the effect of callback rates is positive and increasing along duration, whereas the effect of the probability

[42] Wages are not affected much. The fall is reduced from 1% to approximately 0%

of being suitable is negative and non-monotone. The effect on the coordination friction is negligible. Overall, the sorting effect is outweighed by the ranking mechanism. That is, an increase in aggregate productivity makes the decline in exit rates along unemployment duration less severe due to the effect stemming from the callback rates.

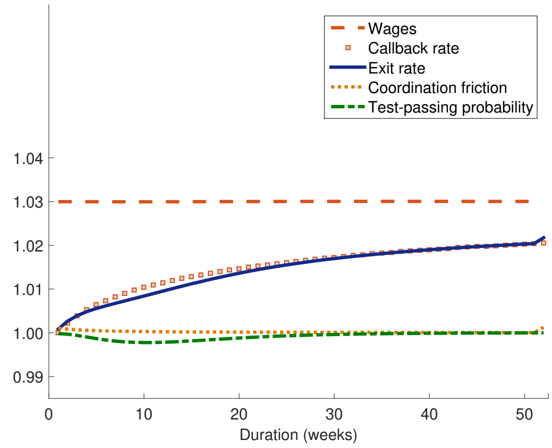


Figure 4: Effects of an Increase of both y_l and y_h by 3%.

Note: The plotted values represent the ratios of non-normalized exit rate and its components as well as non-normalized wages relative to the benchmark of an economy wherein y_l and y_h are 3% higher than the calibrated parameters.

We now turn to the comparative static outcome for wages. Recall that in the equilibrium wage equation (15), the first factor stands for the conditional probability that a given worker is the only applicant for a given duration, whereas the second term is the marginal value of the worker. The conditional probability is higher when there are more vacancies because the queue length q is lower. Regarding the second factor, on the one hand, the marginal value depends on the likelihood of replacing the given candidate with another one with a longer spell, which is lower when vacancies are less scarce. On the other hand, the sorting component implies that the expected productivity declines more severely over duration in this case. The total effect across durations is again difficult to determine. Figure 4 shows that wages for increased productivities are systematically higher at all durations, whereas there is no visible effect on the duration dynamics of wages.

5 Discussion and Conclusion

This article analyzes the effects of firm's recruiting decisions in a labor market in which information on worker type is symmetric and incomplete and firms test applicants. Our composite mechanism combines sorting and ranking, and there exists a single labor market in equilibrium. The model rationalizes the joint declining pattern of callback rates for an interview, exit rates from unemployment and reemployment wages.

A natural question is how sensitive the main results are to the information structure of the model conditional on information being symmetrically held. Although these assumptions are made for tractability, we conjecture that if either complete worker histories were observable or the continuation value depended on the worker's search, ranking by expected productivity would still take place in equilibrium because of the two incentives firms have to attract many applicants, namely to increase the job-filling probability and to reduce wages because the marginal value of a worker is reduced by the presence of other applicants. However, if the whole labor history were observable, expected productivity would be inferred from the whole history of the worker instead of from the current spell and the single-market equilibrium feature would be preserved. We conjecture that, if workers did learn from their job-search experience, e.g. from receiving test calls or from the expected queue length associated to their individual application, multiple labor submarkets would be active in equilibrium and ranking by unemployment duration would prevail in each submarket.

Furthermore, by reducing the whole learning process to a test, probationary periods are ruled out in our model. Our intuition is that firms would continue to rank applicants by expected productivity if probation and more complex wage schemes were allowed. This is because in any case firms would prefer to fill the vacancy with candidates of higher expected productivity. If instead wages could not be re-adjusted when new information is acquired, then modeling worker's type as an experience good would make dismissals informative, which would get us back to the previous discussion about the importance of the whole labor history of workers. In contrast, in our setting worker's productivity is an inspection good, thereby limiting the information to the unemployment spell.

Finally, although this paper has focused on the steady-state equilibrium, notice that the information conveyed by the length of joblessness spells varies over the cycle. In tight labor markets, with a large number of vacancies per job-seeker, unemployment duration is more informative about the expected skills of the applicants than in slack markets. Although our comparative static analysis suggests that the ranking by duration dominates the sorting effects, a more comprehensive business cycle study is needed, which we leave for future research.

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6 Appendix

6.1 Data

In this section, we use data from the Current Population Survey (CPS) to construct the empirical counterparts of our model distributions. Thus the main purpose of the following estimation procedures is to filter out observable heterogeneity and within-duration-group dispersion that are present in the data, but cannot be accounted for in our model. We make use of the longitudinal feature of this dataset to keep track of the employment status of the interviewed agents as well as to record wages at re-employment.

The Bureau of Labor Statistics (BLS) collects data on employment- and earnings-related issues since 1940 by means of the CPS. Individuals stay in the survey for two sets of four consecutive months with an eight month period in between. The fourth and eighth interviews constitute the so-called outgoing rotation group (ORG) and earnings-related questions are asked only then. The dataset we consider is formed by observations from the ORG for the 1994-2011 period. Surveys from June to September 1995 are excluded due to methodological changes that prevent us from tracing individuals over time. With this rich microeconomic dataset we can analyze the dependence of the transitions from unemployment to employment and vice versa as well as reemployment wages on unemployment duration, which is self-reported in weeks.

We limit our sample to individuals aged 20 to 60 years who reported being unemployed in the previous month.[43] Individuals reporting either not to be actively searching for a job or expecting a recall in the previous month are excluded as their employment prospects

[43] Individuals with unemployment durations longer than one year are discarded. See the details on the analysis of job-separation rates below.

differ from the regular unemployed job-seekers.[44] Furthermore, farming-, army- and public-administration-related (un)employment is also removed from the pool. To keep track of individuals over two consecutive months, we use the household identifier, person line number, and month-in-sample, and check for consistency by comparing sex, race and age variables across months. We use the ORG weights.

6.1.1 Job-finding Rates.

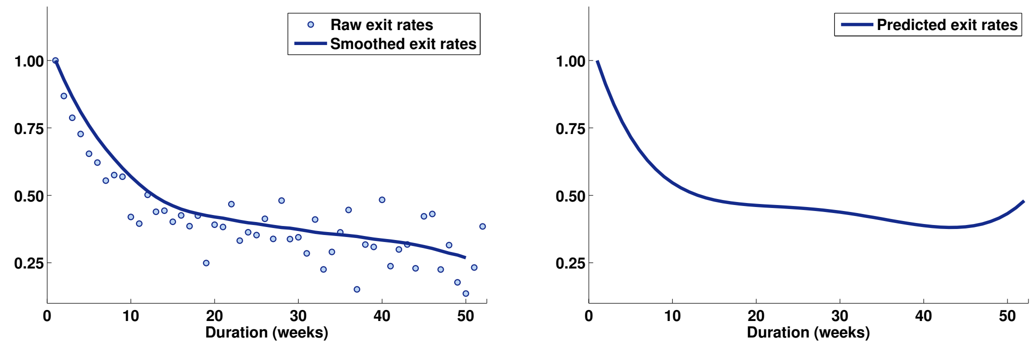
The above filters leave us with 73109 observations of exit rates from unemployment. Figure 5(a) depicts the monthly job-finding rates normalized by the value corresponding to unemployment duration of one week. These rates are non-treated data. It shows the exit rates smoothed by a Kernel-weighted local method, and the non-smoothed data, which are depicted by dots. Since unemployment duration is reported in weeks, the horizontal axis is on a weekly basis.

To remove the effects of observable characteristics, we use a Probit model to estimate the probability of transiting from unemployment to employment with a quartic polynomial of the unemployment duration (see the Probit details below). We assign to each individual the predicted monthly exit rate by replacing the duration variable by her actual duration and the remaining variables by their means. This constitutes the empirical counterpart of the model distribution of exit rates shown in Figure 5(b) in terms of the normalized values. Regardless of whether the data are treated or not, the unemployment-to-employment transition rate falls rapidly with unemployment duration, with the monthly exit rate declining by approximately 50% after 3 months and flattening out from then on.

Regarding the Probit specification to estimate the empirical exit rate from unemployment, the set of regressors is formed by a yearly time variable, the log of the unemployment rate, a quadratic polynomial of imputed experience, along with monthly, sex, race, marital status, industry, occupation, state, education and reason of unemployment dummies.[45] In addition, following Addison, Portugal, and Centeno (2004), we have a quartic polynomial in unemployment duration. Table 2 shows the statistics (and standard errors in parenthesis) of the Probit estimation.

[44]See e.g. Blanchard, Diamond, Hall, and Murphy (1990) for a similar approach. Furthermore, Katz and Meyer (1990) show how the search effort depends on recall expectations and estimate the mean weekly income loss at 14.44% upon a job switch, and at 5.73% after a recall.

[45]A dummy to control for the potential effects of unemployment benefits exhaustion at the 26th week is included, although turns out not to be statistically significant. For the experience imputation, we follow the usual procedure of subtracting 6 and the schooling years from the workers' age. See Jaeger (1997).



(a) Raw and smoothed data (b) Predicted values

Figure 5: Normalized monthly exit rates

Table 2: Transition Probabilities from Unemp. to Emp.

Probit regression	Number of obs = 73109
	Wald chi2(114) = 5314.89
	Prob> chi2 = 0.0000
Log pseudolikelihood = -39427.739	Pseudo R2 = 0.0897
exit	Coef. (se)
unemp. duration	-.1202 (.0069)
unemp. duration^2	.0067 (.0000)
unemp. duration^3	-.0002 (.0000)
unemp. duration^4	1.43e-06 (.0000)

Notes: CPS data for 1994-2011, individuals aged 20-60 years.

6.1.2 Other Data

Job-separation Rates. In the calibration exercise, we target the average job-termination rate. Our dataset for the analysis of job destruction is formed by individuals who reported to be employed in the previous interview, and stay active in the labor market at the interviewing time. We consider that a job has terminated when the worker reports to be unemployed, and not expecting a recall. The remaining above restrictions also apply. Our dataset is formed by 831328 observations. We run a Probit regression to estimate the empirical job-separation rate. The same set of regressors as for the analysis of job-finding rates is used, except for the variables related to individual's unemployment experience. The predicted probabilities have a mean of .009 and a standard deviation of .008.

Hourly Wages. Although we do not target wages, we compare the model duration distribution of re-employment wages with its empirical counterpart. We now estimate such wages at each unemployment duration.

In 1994 a major redesign of the CPS took place concerning both question wording and data processing, which particularly affected the earnings variable (see US-Census-Bureau (2006) and Polivka (1996)). These changes make it difficult to compare hours worked and hourly wage before and after 1994. Prior to this methodological change, individuals were asked to report their earnings on a weekly basis, including overtime, tips, and commissions. In addition, they were asked to report their usual working hours at all jobs. After 1994, interviewees were allowed to report their earnings at an hourly basis, being labeled "hourly workers".[46] Such workers report their hourly rate at their main job, excluding overtime, tips, and commissions. These extra payments may be also reported at the weekly basis. Remaining interviewees report total earnings (including extra payments), which are converted to weekly rates.

We focus on the hourly wage of hourly workers at their main job. This may have some caveats, particularly if there is a selection effect from excluding non-hourly workers, or if the extra-payments are a substantial component of earnings. An alternative is to analyze weekly earnings (extra payments included) divided by usual hours (see Schmitt (2003) for a discussion). There are, however, some reasons in favor of our procedure: First, hourly workers amount to 80% of the employed pool in our sample, and, for hourly workers, weekly earnings obtain from multiplying the hourly wage and the usual number of working hours (adding the overtime, tips and commissions, if reported). There are very few observations with extra payment for hourly workers, however. Second, as opposed to working time at all jobs, since 1994 usual working time at the main job has been directly addressed, and double checked. Further, *hours vary* was introduced as a new answer in the working time question. Imputation strategies may be undertaken given that such a response accounts for just 6 to 7% of the total employment (see Schmitt (2003)). However, the percentage rises to over 12% in our subsample of newly employed. Further, Abraham and Haltiwanger (1995) emphasize the problem of workers' over-reporting their working time.

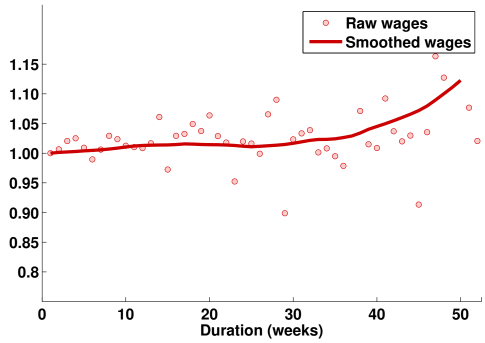
There are some common issues related to processing the earnings data. Regarding trimming, i.e. how to best deal with implausibly high hourly wages, we follow Schmitt (2003) and keep only hourly rates between \$1 and \$100. Less than 0.2% of our weekly earnings observations are top-coded, which suggests that our filter is likely to be costless. Another potential issue is the allocated earnings. BLS uses the cell hot decking procedure to impute earnings to those missing responses.[47] The allocated responses account for over 30% of the outgoing rotation group after 1994, and about 25% in our subsample of newly employed

[46] To be precise, those individuals who prefer to report on a non-hourly basis, but declare to be paid hourly are also labeled as "hourly workers".

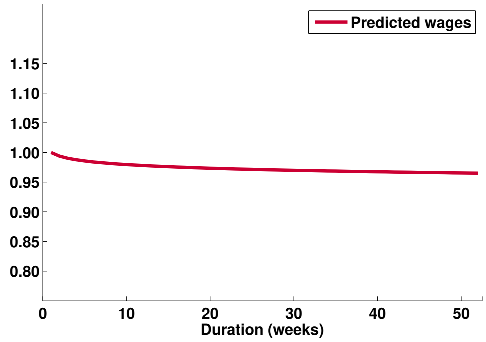
[47] According to the U.S.-Census-Bureau, the weekly earnings hot deck is defined by age, race, sex, usual hours, occupation, and educational attainment.

workers. We have performed the analysis excluding those imputed earnings and end up with 12192 observations.[48]

Wages are deflated using the U.S. city average CPI-U. Analogously to the treatment to exit rates, we plot the non-treated (smoothed and non-smoothed) average re-employment wage at each unemployment duration normalized by the value at duration one week in Figure 6(a), and the predicted wages from a log-linear regression in Figure 6(b). The elasticity of hourly wages with respect to unemployment duration is statistically significant and amounts to -0.009. Analogously to the case of exit rates, in order to remove the heterogeneity linked to observable characteristics and the within-duration-group heterogeneity, we assign to each individual her predicted wage obtained by replacing all the variables by their means, and duration by the reported number of weeks. After normalizing, we obtain the following duration function $w(\tau) = \tau^{-0.009}$.[49] In sharp contrast, wages stay constant or weakly increase with unemployment duration.



(a) Raw and smoothed data



(b) Predicted values

Figure 6: Re-employment wages

Regarding the econometric approach, we estimate a Mincerian regression of log hourly wages. The set of regressors largely coincides with the one for job-finding rates, but following Addison, Portugal, and Centeno (2004) we include the log of unemployment duration instead of a quartic polynomial.

The results are fairly robust relative to the data choices. For example, if allocated earnings are included, the estimate of the log of unemployment duration is -0.0055 and statistically significant at 10%. If non-hourly workers are also considered,[50] then the elasticity amounts

[48] For methodological reasons, there are no imputed earnings from January 1994 to August 1995.

[49] An alternative approach would be to look at the modified residuals, which are the residuals added to the duration component, i.e. $\widetilde{\epsilon_i} \equiv \log(\tau_i)\widehat{\beta_\tau} + \widehat{\epsilon}_i$. We find that the averages of the modified residuals also show a constant pattern in unemployment duration.

[50] If the interviewee responds that hours vary at the main job, we use the reported number of hours in the last week.

Table 3: Log Hourly Reemployment Wages

Linear regression	Number of obs = 12192
	F(107, 12084) = 48.067
	Prob > F = 0.0000
	R-squared = .3942
	Root MSE = .34399
log exit wage	Coef. (se)
log unemp duration	-.0090 (.0036)

Notes: CPS data for 1994-2011, individuals aged 20-60 years.

to -0.01411, and the re-employment wage fall after three months of unemployment is about 3.5%. Finally, if both allocated earnings and non-hourly workers are in the sample, then the elasticity amounts to -0.0104 and the wage fall to almost 3%. In the last two cases, the elasticity is statistically significant at 1%.

6.1.3 Data discussion

The data work is robust to different time periods. In particular, omitting the latest recession has no significant impact. The normalized exit rate at 13 weeks amounts to 0.5, whereas its wage counterpart is 0.975, for the 1994-2007 period. If we looked at 1996 instead, the corresponding normalized exit rate would be 0.442, and the wage ratio 0.959, with the duration elasticity not statistically significant in this case.

A number of papers also use CPS data, and although the analyzed samples differ from ours, the findings related to exit rates from unemployment are very close. Shimer (2008) obtains a very similar duration function, with a 30-40% fall in the first month of unemployment and leveling out after 3 months with a 50% fall relative to the first week of unemployment. The comparison with the numbers in Figure 15 in Elsby, Hobijn, and Sahin (2010) is not so direct because they group observations into longer periods. At first glance, the exit rate for the 3-6 month unemployed workers relative to that for the 1 month unemployed seems approximately 60-70%, yet their numbers get closer to ours once we take into account the significant decline in the first month of unemployment. Farber and Valletta (2013) measure unemployment duration in months. According to their Figure 3, the fall in the exit rate is smaller, but not significantly so, particularly when correcting for transitions from unemployment to employment which are likely due to reporting errors.

Regarding reemployment wages, the empirical literature mostly aims to evaluate the effects of unemployment duration on the earnings change with respect to the pre-unemployment spell. Methodologically, this implies that worker fixed effects are taken into account. In contrast, unobserved heterogeneity is present in our model mechanism and, therefore, we cannot clean the data of unobserved characteristics. Nonetheless, our estimates of re-employment wages over unemployment duration do not differ much from, for example, those of Gregory and Jukes (2001), who use UK data for the 1984-1994 period. They find that the duration of the most recent unemployment spell reduces wages by 0.8% in the case of a spell of one month and by 5.1% after 6 months. In our dataset, these numbers amount to approximately 1% and 3%, respectively. Flemming (2015) also estimates the effects of duration on reemployment wages using both CPS and PSID data, and her results are very much in line with ours. Furthermore, a number of studies investigate the effects of a variation of the generosity of unemployment insurance and find small or no effects on re-employment earnings and significant effects on unemployment duration. See e.g. Card, Chetty, and Weber (2007) for Austria, and Van Ours and Vodopivec (2008) for Slovenia. This evidence suggests that wages may be not very sensitive to unemployment duration. Addison, Portugal, and Centeno (2004) analyze the unemployment duration effects on the two variables for the same European dataset. They also find that the impact of duration on exit rates is strong, mostly due to a steep decline in the arrival of job offers over duration, whereas the effects on accepted wages are much milder.

6.2 Proofs

Proof of Proposition 2.1

We first show that there exists a solution to the firm's problem (14). After some manipulations and substituting out the wages from the complementary slackness conditions (9), the firm's problem can be rewritten as

$$\max_{q^T} \sum_{\tau=1}^{T} \left(\eta_\tau(q^\tau)\Delta_\tau - q_\tau \frac{U_\tau - b - \beta U_{\tau+1}}{p_\tau} \right)$$

where $\Delta_\tau = \frac{\bar{y}_\tau + \beta\delta U_1}{1-\beta(1-\delta)} - b - \beta U_{\tau+1}$. If a sufficiently low wage w_τ for some τ is offered, then $q_\tau = 0$ and the firm derives no profits from type τ applicants.

Notice that the profit function is continuous in q^T. The set of plausible vectors q^T can be restricted to a compact set since they must be non-negative and the complementary slackness conditions (9) puts an upper bound for any duration τ. Therefore, the Weierstrass Theorem ensures the existence of a solution.

Second, we prove by contradiction that if there exists a symmetric equilibrium, then $q_\tau > 0$ for all τ. Suppose that there exists at least one duration group of workers such that its associated queue is 0. Let us denote by τ_0 the first duration for which the queue length is 0. To be consistent with the guess that profitability falls with unemployment duration, which is confirmed in Proposition 2.3, all queues associated with longer durations must also be 0. Then, the unemployment value of workers with unemployment duration greater than or equal to τ_0 must be $b/(1-\beta)$ as they will remain unemployed forever. Let ω be the profit-maximizing contract. Given that $y_\ell > b$, there exists an arbitrarily small, but positive ϵ such that $b + \epsilon < \overline{y}_{\tau_0}$. Consider now the alternative contract ω' that stipulates the same wages as ω, but offers type τ_0 workers a wage $b + \epsilon$. Because of the ranking strategies, workers of duration τ_0 do not crowd out the candidates with higher expected productivities, and imply expected positive profits for the firm. Because the alternative contract ω' delivers strictly higher profits than ω, this cannot be profit-maximizing. Therefore, $q_\tau > 0$ for all τ.

Finally, we show that if Δ_τ falls with τ, then the solution of the firm's problem is unique, and is characterized by the first order conditions.

The Hessian of function F is $D^2F = (h_{ij})_{i,j}$, where for any given pair (i, j), with $i \leq j$,

$$h_{ij} \equiv \sum_{\tau \geq j} \frac{\partial^2 \eta_\tau(q^\tau)}{\partial q_i \partial q_j} \Delta_\tau = -\sum_{s=j}^{T-1} e^{-\sum_{\tau \leq s} q_\tau}(\Delta_s - \Delta_{s+1}) - e^{-\sum_{\tau \leq T} q_\tau} \Delta_T$$

As Δ_τ declines with τ, all the coefficients of the Hessian matrix are negative, $h_{ij} < 0$. To show that the Hessian is negative definite, we prove that the leading principal minors of the Hessian alternate signs. Notice that h_{ij} does not depend on i. Thus, the Hessian has a very particular form as $h_{ij} = h_{i'j}$ for all $i, i' \leq j$ and, obviously, $h_{ij} = h_{ij'}$ for all $j, j' \leq i$. Let $z_j \equiv h_{jj}$ and $|H_j|$ denote the leading principal minor of the Hessian with the first j rows and columns.

$$|H_j| = \begin{vmatrix} z_1 & z_2 & z_3 & \dots & z_j \\ z_2 & z_2 & z_3 & \dots & z_j \\ z_3 & z_3 & z_3 & \dots & z_j \\ \dots & \dots & \dots & \dots & \dots \\ z_j & z_j & z_j & \dots & z_j \end{vmatrix} = \begin{vmatrix} z_1 - z_2 & 0 & 0 & \dots & 0 \\ z_2 - z_3 & z_2 - z_3 & 0 & \dots & 0 \\ z_3 - z_4 & z_3 - z_4 & z_3 - z_4 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ z_j & z_j & z_j & \dots & z_j \end{vmatrix},$$

where the second determinant obtains from subtracting each row of $|H_j|$ to its previous one. Then,

$$\begin{aligned} |H_j| &= z_j \prod_{s<j} (z_s - z_{s+1}) = z_j \prod_{s<j} \left(-e^{-\sum\limits_{\tau \leq s} q_\tau} (\Delta_s - \Delta_{s+1}) \right) = \\ &= (-1)^{j-1} z_j \prod_{s<j} \left(e^{-\sum\limits_{\tau \leq s} q_\tau} (\Delta_s - \Delta_{s+1}) \right) \end{aligned}$$

The sign of the expression is positive if j is even and negative otherwise. Therefore, the

Hessian is negative definite and the FOC are also sufficient. Furthermore, since this is the case at any point, the objective function is strictly concave and there is a unique global maximum. ‖

Proof of Proposition 2.2

We show existence of equilibrium. Let $f : K \subset \mathbb{R}_+^{3\times T} \to K$ be defined as the composite correspondence $f \equiv \phi \circ \psi$, where ϕ and ψ are defined below. Let K be defined as

$$K \equiv \left\{ (u, U') \in [0,1]^{2\times T} \times \left[\frac{b}{1-\beta}, \frac{y_h}{1-\beta}\right]^T \mid \forall \tau,\ \ u_i(\tau+1) \leq u_i(\tau) \text{ and } u_h(\tau+1)u_\ell(\tau) \leq u_h(\tau)u_\ell(\tau+1) \right\}$$

In words, K is the nonempty set of pairs formed by worker distributions in the two dimensional duration-productivity space, u, and today's expectations on tomorrow's unemployment value $(U'_\tau)_\tau$. Notice that u uniquely pins down the expected productivity $\overline{y}_\tau$ and the test-passing probability p_τ for all duration τ. The defining constraints ensure that the distribution of workers is characterized by both a declining expected productivity and a declining mass of each worker type over time. Obviously, the set K is compact.

We start with correspondence ψ. We define $\psi(u, (U'_\tau)_\tau)$ as the set of triples $(\omega, q^T, (U_\tau)_\tau)$ that satisfy the system of resource constraints (13) and the zero-profit condition as well as solve the firms' profit maximization program (14). To determine the image we proceed in several steps. First, we can write the equilibrium condition (9) as

$$\eta_\tau(q^\tau) \max\left\{E_\tau(\omega) - b - \beta U'_{\tau+1}, 0\right\} = q_\tau \frac{U_\tau - b - \beta U'_{\tau+1}}{p_\tau} \tag{27}$$

This implies that either $U_\tau > b + \beta U'_{\tau+1}$ and $q_\tau > 0$ or $U_\tau = b + \beta U'_{\tau+1}$ and $q_\tau = 0$. Then, by replacing wages using this expression, the objective function of the firm's problem becomes

$$\sum_{\tau=1}^{T} \left(\eta_\tau(q^\tau)\Delta_\tau - q_\tau \frac{U_\tau - b - \beta U'_{\tau+1}}{p_\tau} \right) - k \tag{28}$$

where $\Delta_\tau = \frac{\overline{y}_\tau + \beta\delta U'_1}{1-\beta(1-\delta)} - b - \beta U'_{\tau+1}$. We obtain the first order conditions by differentiating with respect to q_τ:

$$e^{-\sum_1^\tau q'_\tau}\Delta_\tau - \sum_{\tau'=\tau+1}^{T} \eta_{\tau'}(q^{\tau'})\Delta_{\tau'} \leq \frac{U_\tau - b - \beta U'_{\tau+1}}{p_\tau}, \tag{29}$$
$$\text{and } q_\tau \geq 0, \text{ with complementary slackness}$$

We now distinguish between two cases.

Case 1: $q_\tau > 0$ for all τ.

Since $q_\tau > 0$, expression (27) implies that $U_\tau > b + \beta U'_{\tau+1}$, which in turn leads to positive profits linked to duration τ, $\eta_\tau(q^\tau)\Delta_\tau - q_\tau \frac{U_\tau - b - \beta U'_{\tau+1}}{p_\tau} \geq 0$, (and also $\Delta_\tau > 0$) as otherwise firms would set $q_\tau = 0$. We use the first order conditions to rewrite expected profits (28) as

$$\sum_{\tau=1}^{T}\left(\eta_\tau(q^\tau)\Delta_\tau - q_\tau e^{-\sum_1^\tau q'_\tau}\Delta_\tau + q_\tau \sum_{\tau+1}^{T} \eta_{\tau'}(q^{\tau'})\Delta_{\tau'}\right) - k \tag{30}$$

We now impose the system of resource constraints (13) together with the zero-profit condition, and, after rearranging the terms, we obtain the following equation with the only unknown q_1.

$$\sum_{\tau=1}^{T} \Delta_\tau e^{-q_1 \sum_{\tau'=1}^{\tau-1} \alpha'_\tau}\left(\left(1 - e^{-q_1\alpha_1}\right) q_1 \sum_{\tau'=1}^{\tau-1} \alpha_{\tau'} + 1 - e^{-q_1\alpha_\tau}(1 + q_1\alpha_\tau)\right) = k \tag{31}$$

where $\alpha_\tau = \frac{\lambda_h u_h(\tau) + \lambda_\ell u_\ell(\tau)}{\lambda_h u_h(1) + \lambda_\ell u_\ell(1)}$ as defined by (13).

We are to show that this equation has a unique solution. Let us refer to the left hand side of this equation as $F(q_1)$. Notice that $\lim\limits_{q_1 \to 0} F(q_1) = 0$, which indicates that firm's expected profits (without considering the vacancy costs) are 0 if there are no applicants to the job. Likewise, $\lim\limits_{q_1 \to \infty} F(q_1) = \frac{\overline{y}_1 - b}{1 - \beta(1-\delta)}$ because firms find a worker of duration $\tau = 1$ with probability 1, and $w_1 = b$ and $U_\tau = \frac{b}{1-\beta}$ for all durations.

Because of our assumption $\frac{\overline{y}_1 - b}{1-\beta(1-\delta)} > \frac{y_\ell - b}{1-\beta(1-\delta)} > k$ and F being a continuous function of q_1, there exists a solution to equation (31). To see that the solution is unique, we now show that the derivative of F is positive.

$$\begin{aligned} F'(q_1) &= q_1 \sum_{\tau=1}^{T} \Delta_\tau e^{-q_1 \sum_{\tau'=1}^{\tau-1} \alpha'_\tau}\left(e^{-q_1\alpha_\tau}\left(\sum_{\tau'=1}^{\tau} \alpha_{\tau'}\right)^2 - \left(\sum_{\tau'=1}^{\tau-1} \alpha_{\tau'}\right)^2\right) \\ &= q_1 \sum_{\tau=2}^{T} \tilde{F}(\tau)\left(\left(\sum_{\tau'=1}^{\tau} \alpha_{\tau'}\right)^2 - \left(\sum_{\tau'=1}^{\tau-1} \alpha_{\tau'}\right)^2\right) \end{aligned} \tag{32}$$

where

$$\tilde{F}(\tau) \equiv e^{-\sum_{\tau'=1}^{\tau} q_{\tau'}}\Delta_\tau - \sum_{\tau'=\tau+1}^{T} \eta_{\tau'}(q^{\tau'})\Delta_{\tau'}$$

The first equality is straightforward, while the second one obtains after some rearrangements. Notice that $\tilde{F}(\tau)$ is the left hand side of the first order condition (29). Since $q_\tau > 0$ the inequality is indeed an equality and the right hand side is strictly positive, and, hence, so is

$\tilde{F}(\tau)$. This ensures that the function F is increasing. As a result, there exists a unique value q_1 that solves equation (31). Using the system of conditions (13), we obtain the solution vector q^T. The vector $(U_\tau)_\tau$ is now derived from the first order conditions (29). Finally, wages ω obtain from the set of constraints of the firm's problem.

Case 2: $q_\tau = 0$ for some τ.

The equilibrium resource constraints (13) together with Proposition 2.1 imply that queue lengths are zero for all durations, i.e. $q_\tau = 0$. It follows from expression (27) that $U_\tau = b + \beta U'_{\tau+1}$ for all τ, and let $w_\tau = b$ for all durations. Therefore, there is no vacancy creation in equilibrium.

Since the triple $(\omega, q^T, (U_\tau)_\tau)$ is uniquely determined in both cases, ψ is a function. Furthermore, ψ is a continuous function within each case. To see that there is no discontinuity at $q_\tau = 0$ for some τ, notice that if q_τ goes to 0 so do all the other queue lengths by construction of $\tilde{u}$.

Second, let ϕ be a function defined as $\phi(\omega, q^T, (U_\tau)_\tau) \equiv (\tilde{u}, (\tilde{U}_\tau)_\tau)$. The first variable $\tilde{u}$ is uniquely determined by the equilibrium recursivity condition (12). That is, given q^T, $\tilde{u}$ is the unique solution of that system of equations. Furthermore, we define $\tilde{U}_\tau = U_\tau$ for all τ.

Notice that $f(u, U') \in K$ as the inequalities defining K hold by construction of $\tilde{u}$. Finally, the equilibrium allocation can be identified with the solution of a fixed point of correspondence f. As ψ and ϕ both are continuous functions, so is f. Therefore, the Brouwer Fixed Point Theorem applies to ensure the existence of a fixed point of f.‖

Proof of Proposition 2.3

We refer to the event of a worker being suitable for the job as $\mathcal{T}$. Let $P(i|\tau, \mathcal{T})$ denote the probability that a worker of duration τ is of type i conditional on the event $\mathcal{T}$. We show that $P(h|\tau, \mathcal{T})$ is decreasing in τ. By using Bayes' rule,

$$P(h|\tau, \mathcal{T}) > P(h|\tau + 1, \mathcal{T})$$

$$\Leftrightarrow$$

$$\frac{P(h)P(\tau, \mathcal{T}|h)}{P(h)P(\tau, \mathcal{T}|h) + (1 - P(h))P(\tau, \mathcal{T}|\ell)} > \frac{P(h)P(\tau + 1, \mathcal{T}|h)}{P(h)P(\tau + 1, \mathcal{T}|h) + (1 - P(h))P(\tau + 1, \mathcal{T}|\ell)}$$

Now, after some manipulations and taking into account that $P(\tau, \mathcal{T}|i) = \lambda_i P(\tau|i)$, this inequality holds if and only if

$$\frac{P(\tau|\ell)}{P(\tau|h)} < \frac{P(\tau + 1|\ell)}{P(\tau + 1|h)} = \frac{P(\tau|\ell)(1 - \lambda_\ell + \lambda_\ell P(\tau + 1|\tau, \mathcal{T}))}{P(\tau|h)(1 - \lambda_h + \lambda_h P(\tau + 1|\tau, \mathcal{T}))} \Leftrightarrow \lambda_h > \lambda_\ell$$

where $P(\tau+1|\tau,\mathcal{T})$ stands for the probability of staying unemployed one more period conditional on being suitable.

The expected productivity can be rewritten as $\overline{y}_\tau = y_\ell + (y_h - y_\ell)P(h|\tau,\mathcal{T})$. Since $y_h > y_\ell$ by assumption, the expected productivity falls with τ provided that type-ℓ workers are less likely to be suitable than their type-h counterparts.

The proof of declining values $J_\tau(\omega)$ follows closely Shimer (2005). So, it is omitted.‖

Proof of Proposition 2.4

Let us first show that U_τ decreases in τ. For simplicity, we set $b=0$. We can rewrite the first order condition of the firm's program (14) with respect to q_τ as

$$U_\tau = \beta U_{\tau+1} + p_\tau e^{-\sum_1^\tau q_{\tau'}} \left(\sum_{\tau'=\tau}^{T-1} e^{-\sum_{\tau''=\tau+1}^{\tau'} q_{\tau''}} (\Delta_{\tau'} - \Delta_{\tau'+1}) + e^{-\sum_{\tau''=\tau+1}^{T} q_{\tau''}} \Delta_T \right)$$

where once again $U_{T+1} \equiv U_T$. We now proceed by backward induction. The first inequality in the following two cases comes from $p_\tau > p_{\tau+1}$ for all τ.

Case $T-1$:

$$U_{T-1} - U_T > p_T e^{-\sum_1^{T-1} q_{\tau'}} (\Delta_{T-1} - \Delta_T) = p_T e^{-\sum_1^{T-1} q_{\tau'}} \frac{\bar{y}_{T-1} - \bar{y}_T}{1-\beta(1-\delta)},$$

and the final expression is strictly positive.

Case τ:

$$\begin{aligned} U_\tau - U_{\tau+1} > \quad & \beta(U_{\tau+1} - U_{\tau+2}) + p_{\tau+1} e^{-\sum_1^\tau q_{\tau'}} (\Delta_\tau - \Delta_{\tau+1}) = \\ & = \beta(U_{\tau+1} - U_{\tau+2})(1 - p_{\tau+1} e^{-\sum_1^\tau q_{\tau'}}) + p_{\tau+1} e^{-\sum_1^\tau q_{\tau'}} \tfrac{\bar{y}_\tau - \bar{y}_{\tau+1}}{1-\beta(1-\delta)}, \end{aligned}$$

which is also strictly positive by induction and the outcome of falling expected productivities.

To prove that the exit rate declines with unemployment duration, we first show that the conditional probability ν_τ is a decreasing function of τ. That follows if and only if, for any τ,

$$e^{q_\tau} \frac{1 - e^{-q_\tau}}{q_\tau} \geq \frac{1 - e^{-q_{\tau+1}}}{q_{\tau+1}}$$

Notice that the left hand side always lies above 1, whereas the unity is an upper bound for the right side. The inequality becomes equality if and only if we are in the limit case $q_\tau = q_{\tau+1} = 0$.

Now, to show that the exit rate also declines with τ, we just need to remember that $h_\tau(u, q^\tau) = p_\tau(u)\nu_\tau(q^\tau)$ and that the two factors have been proved to decline with τ.‖

Proof of Proposition 2.5.

Consider first the case of small differences in the suitability rate, $\lambda_h = \lambda_\ell + \epsilon$, with $\epsilon > 0$ arbitrarily small. After some simple simplifications, we can write the difference in expected productivities as

$$\overline{y}_{T-1} - \overline{y}_T \leq \overline{y}_{T-1}\left(1 - \left(1 + \epsilon\frac{\frac{\nu_{T-1}(q^{T-1})}{1-\lambda_h\nu_{T-1}(q^{T-1})}}{y_h\lambda_h u_h(T-1) + y_\ell\lambda_\ell u_\ell(T-1)}\right) \times \frac{\lambda_h u_h(T-1) + \lambda_\ell u_\ell(T-1)}{\lambda_h u_h(T-1) + \lambda_\ell u_\ell(T-1)\left(1 + \frac{\epsilon\nu_{T-1}(q^{T-1})}{1-\lambda_h\nu_{T-1}(q^{T-1})}\right)}\right)$$

Notice that the right hand side vanishes as ϵ goes to 0. Therefore, the left hand side of the last inequality of the counterpart of expression (16) for durations $T-1$ and T is arbitrarily close to 0, whereas the right hand side is strictly positive, which implies $w_{T-1} < w_T$.

The case with arbitrarily small differences in productivity is obvious because the expected productivity $\overline{y}_\tau$ is a convex combination of y_ℓ and y_h, and, hence, $\overline{y}_{T-1} - \overline{y}_T < y_h - y_\ell$.$\|$

Proof of Proposition 3.1

Consider the case in which all recruiting firms target a single duration by posting contract ω_τ, which has all its components equal to 0 except the wage w_τ. Let a submarket be defined by the pair (q_τ, ω_τ) that is the maximizer of the firm problem (24). Given u and $(U_\tau)_\tau$, such a solution is pinned down by equations (25) and (26), which correspond to the first order condition and the zero-profit condition, respectively. We define $Q(\omega_\tau) = q_\tau$ and $\rho_\tau(\omega'_\tau) = \mathcal{I}_\tau(\tau')$, where $\mathcal{I}_\tau(\tau')$ is an indicator function that values 1 if $\tau = \tau'$ and 0 otherwise. Furthermore, for any other $\omega \in [0, y_h]^T$, $Q(\omega)$ is pinned down by the maximum queue length that makes workers of all durations indifferent to the equilibrium contract to which they apply to. It is easy to show that it satisfies all the remaining equilibrium conditions.

Now, before showing the existence of the fully segmented equilibrium, we prove that there cannot be an equilibrium with a single labor market. If there were a pooling equilibrium, then all workers would face the same expected queue length. Proposition 2.3 states that expected productivity falls with unemployment duration. Therefore, $\Delta_{T-1} > \Delta_T$, and the zero-profit condition (26) implies that $q_{T-1} < q_T$. That is, workers with unemployment duration $T-1$ and T must search in different submarkets in equilibrium.

We next show the existence of a fully segmented equilibrium, which resembles the proof of Proposition 2.2. Let $f : K \to K$, where $K \equiv [0,1]^{2\times T} \times [\frac{b}{1-\beta}, \frac{y_h}{1-\beta}]^T$ is a compact set. We define f as the composite correspondence $f \equiv \phi \circ \psi$, where ϕ and ψ are defined as follows. First, let $z \equiv (u, (U'_\tau)_\tau)$ and $\psi(z)$ be defined as the set of elements $\{q_\tau, w_\tau, U_\tau\}_\tau$ that satisfy the zero-profit conditions (26), solve the firms' profit maximization program (24) and U_τ is obtained as a new iteration using equation (19). Second, let ϕ be a function defined as $\phi(\omega, q^T, (U_\tau)_\tau) \equiv (\tilde{u}, (\tilde{U}_\tau)_\tau)$, where $\tilde{u}$ is uniquely determined by the equilibrium recursivity condition (20), and $\tilde{U}_\tau = U_\tau$ for all τ. Notice that the equilibrium allocation can be identified with the solution of a fixed point of correspondence f. We are to show that f is a continuous function, and, then, Brouwer's Fixed Point Theorem applies to ensure the existence of a fixed point of f.

To show that f is a continuous function, it suffices to show that $\psi(z)$ is singleton and continuous for every $z \in K$ as its other component is obviously a continuous function. Notice that the objective function of program (24), after replacing wages, is strictly concave in q. Therefore ψ is a function. The Maximum Theorem ensures that ψ is continuous in $z \in K$. Therefore, the composite function is also a continuous function.$\|$

Proof of Proposition 3.2.

We first show that the unemployment value falls with duration. The proof is by backwards induction. Recall that the value of unemployment can be written as

$$U_\tau = h_\tau(q_\tau)\gamma(q_\tau)\Delta_\tau + b + \beta U_{t+1} = p_\tau e^{-q_\tau}\Delta_\tau + b + \beta U_{t+1}$$

First, we show that $U_T < U_{T-1}$.

$$U_T < U_{T-1} \Leftrightarrow p_T e^{-q_T}\Delta_T < p_{T-1}e^{-q_{T-1}}\gamma(q_{T-1})\Delta_{T-1}$$

Notice that $\Delta_T \leq \Delta_{T-1}$ and $p_T < p_{T-1}$ if $\lambda_\ell < \lambda_h$. Then, the zero-profit condition (26) implies $q_{T-1} \leq q_T$. Therefore, $U_T < U_{T-1}$.

Second, we assume that $U_{t+1} < U_t$ for $\tau \leq t \leq T-1$, and show that $U_\tau < U_{\tau-1}$. Let $\underline{m} \equiv \min\{p_{\tau-1}e^{-q_{\tau-1}}, p_\tau e^{-q_\tau}\}$ and $\overline{m} \equiv \max\{p_{\tau-1}e^{-q_{\tau-1}}, p_\tau e^{-q_\tau}\}$

$$\begin{aligned}
U_{\tau-1} - U_\tau &= p_{\tau-1}e^{-q_{\tau-1}}\frac{\overline{y}_{\tau-1} + \beta\delta U_1}{1-\beta(1-\delta)} - p_\tau e^{-q_\tau}\frac{\overline{y}_\tau + \beta\delta U_1}{1-\beta(1-\delta)} + \\
&\quad + \beta U_\tau\big(1 - p_{\tau-1}e^{-q_{\tau-1}}\big) - \beta U_{\tau+1}\big(1 - p_\tau e^{-q_\tau}\big) + \\
&\geq \underline{m}\frac{\overline{y}_{\tau-1} - \overline{y}_\tau}{1-\beta(1-\delta)} + \\
&\quad + \big(1 - \overline{m}\big)\beta\big(U_\tau - U_{\tau+1}\big) > 0
\end{aligned}$$

where the last inequality stems from the induction assumption and productivity difference if any.

Now, we turn to prove that wages always fall with unemployment duration. Consider first the case in which the joint value Δ_τ also falls. Then, the expected queue length q_τ must increase for the zero-profit condition (26) to hold in equilibrium. As all the terms in the equilibrium wage equation (25) fall, so do wages. Consider now the case in which Δ_τ increases with unemployment duration. Then, equation (26) implies that q_τ declines with τ. Now, we can rewrite the zero-profit condition as $k = \eta(q_\tau)\frac{\overline{y}_\tau - w_\tau}{1-\beta(1-\delta)}$. Since the second factor on the right hand side must increase with τ because the first factor decreases. Proposition 2.3 establishes that $\overline{y}_\tau$ decreases and, hence, so must w_τ for the ex-post profits to increase.$\|$

6.3 Constrained Efficiency in a Two-Period Model

We study constrained efficiency of the equilibrium allocation in the simplest economy comparable to the benchmark. We only describe those features that differ from the model set in Section 2.

Consider an economy that lasts for two periods, $t \in \{1, 2\}$. A unit mass of risk-neutral, unemployed workers are born every period. Unemployed workers are identified by a pair (τ, i), where $\tau = 1$ if short-term unemployed and $\tau = 2$ if long-term unemployed. Index $i \in \{\ell, h\}$ stands for their market skills. A type i worker is suitable for any given job with probability λ_i, with $\lambda_h = 1$ and $\lambda \equiv \lambda_\ell < 1$. The market productivity of a type i worker suitable for the applied job amounts to y_i, with $y_h > y_\ell = 0$. A worker is born skilled with probability $\mu \in (0, 1)$. Let $u_i^t(\tau)$ denote the measure of unemployed workers of type (τ, i) at the beginning of period t.[51] We normalize home productivity to 0. Employment is an absorbing state, i.e. $\delta = 0$.

A worker who has been unemployed for τ periods expects to be suitable for the job in period t with probability

$$p_{t,\tau} = \frac{u_h^t(\tau) + \lambda u_\ell^t(\tau)}{u_h^t(\tau) + u_\ell^t(\tau)}. \tag{33}$$

The expected productivity of suitable type τ candidates in period t is determined by the counterpart of expression (2):

$$\overline{y}_{t,\tau} = \frac{y_h u_h^t(\tau) + y_\ell \lambda u_\ell^t(\tau)}{u_h^t(\tau) + \lambda u_\ell^t(\tau)}, \tag{34}$$

whereas the actual productivity of the selected applicant is instantaneously revealed upon hiring.

[51]For notational consistency, $u_i^1(2) = 0$ for $i \in \{\ell, h\}$.

6.3.1 First Period Search and Value Functions

For convenience, we omit the reference to the duration of unemployment, τ, and the state variable u in the first period.

Firms post and fully commit to job offers. A job offer consists of a single wage w_1 to be paid in each period. Let q denote the expected queue length at any given job. The probability of filling the job is $\eta_1(q) = 1-e^{-q}$, whereas the job-finding probability conditional on being suitable becomes $\nu_1(q) = \frac{\eta_1(q)}{q}$. The actual job-finding probability, hence, refers to the composite event of being suitable and being selected for the job, $h_1(q) = p_1\nu_1(q)$.

The complementary slackness condition, counterpart of condition (9), is

$$\begin{aligned} U_1 \quad &\geq \quad h_1(q_1(w_1))\left(w_1(1+\beta) - \beta U_2(2)\right) + \beta U_2(2) \\ \text{and} \quad q_1(w_1) &\geq 0, \quad \text{with complementary slackness.} \end{aligned}$$

The value of a firm in period 1 is

$$V_1(w_1) = -k + \eta_1(q_1)\left(\overline{y}_1 - w_1\right)(1+\beta)$$

6.3.2 Second Period Search and Value Functions

As in the main model, firms can treat observationally different workers differently. Therefore, contractual job offers in period two may stipulate wages contingent on expected productivity, $\omega_2 = \{w_{2,\tau}\}_{\tau \leq 2}$. Each firm posting a job ω_2 expects $q_2(\omega_2, \tau)$ suitable applicants of duration τ. Let $q^2(\omega_2) \equiv (q_{2,1}(\omega_2), q_{2,2}(\omega_2))$. We will omit the reference to ω_2 unless necessary. Because Proposition 2.3 holds also in this environment, firms discriminate against long term unemployed workers in the hiring stage. Therefore, the probability of filling a job with a type τ worker is

$$\eta_{2,\tau}(q^2) = e^{-\sum_{\tau' < \tau} q_{2,\tau'}}\left(1 - e^{-q_{2,\tau}}\right).$$

The actual matching probability for a worker of duration τ is defined as $h_{2,\tau}(q^2) = p_{2,\tau}\nu_{2,\tau}(q^2)$, where the conditional probability $\nu_{2,\tau}(q^2) = \frac{\eta_{2,\tau}(q^2)}{q_{2,\tau}}$. For each duration τ in period two,

$$U_{2,\tau} \quad \geq \quad h_{2,\tau}(q^2(\omega_2))w_{2,\tau} \text{ and } q_{2,\tau}(\omega_2) \geq 0, \text{ with complementary slackness,}$$

The value of a firms in period two is

$$V_2(\omega_2) = -k + \sum_{\tau} \eta_{2,\tau}(q_{2,\tau}(\omega_2))\left(\overline{y}_{2,\tau} - w_{2,\tau}\right)$$

There are two resource constraints in this second period. First, the law of motion of available labor resources is

$$u_i^2(2) = u_i^1(1)\left(1 - \lambda_i \nu_1(q_1)\right), \forall i \in \{\ell, h\}. \tag{35}$$

Second, the mass of vacancies does not depend on the unemployment duration of applicants because in a symmetric equilibrium there is a single labor market in the second period. It follows that

$$\frac{q_{2,2}}{u_h^2(2) + \lambda u_\ell^2(2)} = \frac{q_{2,1}}{\mu + \lambda(1-\mu)} \tag{36}$$

6.3.3 Equilibrium

A symmetric directed search equilibrium consists of unemployment values $U_1, \{U_{2,\tau}\}$, distributions of unemployed workers $u_t \in [0,1]^{2\times 2}$, wage contracts $w_1 \in [0, y_h]$ and $\omega_2 \in [0, y_h]^2$, and expected queue length functions $Q_1(\cdot) : [0, y_h] \to \mathcal{R}_+$ and $Q^2 : [0, y_h]^2 \to \mathcal{R}_+^2$, such that firms maximize expected profits at contracts w_1 and ω_2, expected profits are zero, workers search optimally, and the two resource constraints (35) and (36) hold.

Next, we characterize the equilibrium allocation. We solve out the firm's profit-maximizing problem subject to the optimal search behavior of job-seekers. Given the unemployment value of workers, the firm's problem (P_t) in period t becomes

$$\begin{aligned}(P_1) \quad & \max_{q_1, w_1} \quad \eta_1(q_1)\left(\overline{y}_1 - w_1\right)(1+\beta) \\ & \text{s. to} \quad q_1 U_1 \leq q_1 h_1(q_1)\Big(w_1(1+\beta) - \beta U_{2,2}\Big) + q_1 \beta U_{2,2}\end{aligned}$$

$$\begin{aligned}(P_2) \quad & \max_{q^2, \omega_2} \quad \sum_\tau \eta_{2,\tau}(q^2)\left(\overline{y}_{2,\tau}(u) - w_{2,\tau}\right) \\ & \text{s. to} \quad q_{2,\tau} U_{2,\tau} \leq q_{2,\tau} h_{2,\tau}(u, q^2) w_{2,\tau}\end{aligned}$$

The following proposition establishes the existence of equilibrium as well as a unique solution for the two firms' problems. The proof is omitted.

Proposition 6.1 *Given $U_1, \{U_{2,\tau}\}$, problems (P_1) and (P_2) have a unique solution. Furthermore, second period firms find it optimal to employ workers of all unemployment durations.*

There exists a symmetric equilibrium.The equilibrium wages are

$$w_1(1+\beta) = \frac{q_1 e^{-q_1}}{1-e^{-q_1}} \left(\overline{y}_1(1+\beta) - \beta U_{2,2}\right) + \beta U_{2,2} \tag{37}$$

$$w_{2,1} = \frac{q_{2,1} e^{-q_{2,1}}}{1-e^{-q_{2,1}}} \left(\overline{y}_{2,1} - (1-e^{-q_{2,2}})\overline{y}_{2,2}\right) \tag{38}$$

$$w_{2,2} = \frac{q_{2,2} e^{-q_{2,2}}}{1-e^{-q_{2,2}}} \overline{y}_{2,2} \tag{39}$$

We now derive the private net returns of a vacancy in period one. To obtain the profits that firms make in equilibrium, we substitute out the equilibrium wages into the firm's value function and the continuation value of unemployment $U_{2,2}$. The expected discounted profits are

$$V_1^* = \eta_1(q_1)\overline{y}_1(1+\beta) - k - q_1 \tfrac{\partial \eta_1(q_1)}{\partial q_1} \overline{y}_1(1+\beta) + p_{2,2} \tfrac{\partial \eta_{2,2}(q^2)}{\partial q_{2,2}} q_1^2 \tfrac{\partial \nu_1(q_1)}{\partial q_1} \beta \overline{y}_{2,2} \tag{40}$$

The interpretation of expression (40) is straightforward. The first two terms amount to the expected discounted output net of the vacancy cost. The remaining two terms are the wage bill the firm incurs. The first part of these wage costs stands for the externality the marginal firm creates on the other vacancies in period one, whereas the second one captures the intertemporal effects on period-two firms. Free entry implies $V_1^* = 0$ in equilibrium.

6.3.4 Constrained Efficiency

We argue that a benevolent social planner can improve upon the decentralized equilibrium. It can be shown that the second period equilibrium allocation is constrained efficient, conditional on an efficient entry of firms in the first period. This is not surprising because the economy starting in the second period does not differ from Shi (2002) and Shimer (2005a) conditional on the first period decisions. Since the inefficiency outcome results from the dynamic externalities it suffices to show that the private and social gains of period-one firms do not coincide with each other.

Having determined the profits of period-one firms in the previous section, we now turn to the social planner problem. As is standard, given risk neutrality of the workers' preferences, the goal of the planner is to maximize total output net of recruitment costs. The planner sets the mass of vacancies $\{v_t\}_t$ and the hiring strategies given the heterogeneity in productivity for each period. The planner is subject to the same constraints specified above for the decentralized economy. First, as the planner cannot assign workers to jobs, coordination frictions arise. Second, the planner faces the same incomplete information problem and has access to the same testing technology. Aiming to maximize output, the planner also discriminates against candidates with longer unemployment spells because of sorting. The planner's problem is

$$\max_{\{v_t\}_t} \quad v_1\Big(\eta_1(q_1)\overline{y}_1(1+\beta)-k\Big)+v_2\beta\Big(\textstyle\sum_{\tau=1}^{2}\eta_{2,\tau}(q^2)\overline{y}_{2,\tau}-k\Big)$$
$$s.\ to \quad v_t=\tfrac{\mu+\lambda(1-\mu)}{q_{t,1}}, \text{ and the resource constraints (35) and (36).}$$

The following result follows. The proof is omitted.

Lemma 6.2 *The social returns of the marginal vacancy in period one are*

$$\begin{aligned}\hat{V}_1 = \ & \eta_1(q_1)\overline{y}_1(1+\beta)-k-q_1\tfrac{\partial\eta_1(q_1)}{\partial q_1}\overline{y}_1(1+\beta) \\ & -\beta\tfrac{q_1^2}{q_{2,1}}\Big(\eta_{2,2}(q^2)\tfrac{\partial\overline{y}_{2,2}}{\partial q_1}+\tfrac{\partial\eta_{2,2}(q^2)}{\partial q_1}\overline{y}_{2,2}\Big)\end{aligned} \tag{41}$$

The first two terms stand for the expected output net of vacancy costs. The third term is the standard negative externality on contemporaneous firms as they are less likely to fill their vacancies. When comparing with expression (40) for the private returns, we see that this effect is internalized in equilibrium as it is usually the case in directed search models. The last term of expression (41) captures the reduction in the expected returns of period-two firms. This intertemporal effect occurs through two channels that correspond to the two addends within this last term. First, the intensive margin: posting one more vacancy affects negatively the composition of the pool of unemployed and reduces the expected returns in period two. Second, the extensive margin: the marginal firm reduces the mass of suitable job-seekers in period two, making it more difficult to fill jobs in that period.

After some simplifications, the difference between the private and social returns of a marginal vacancy becomes

$$\begin{aligned}V_1^*-\hat{V}_1 = \ & \beta\tfrac{q_1^2}{q_{2,1}}\Big(\eta_{2,2}(q^2)\tfrac{\partial\overline{y}_{2,2}}{\partial q_1}+\tfrac{\partial\eta_{2,2}(q^2)}{\partial q_1}\overline{y}_{2,2}+q_{2,1}\overline{y}_{2,2}\tfrac{\partial\eta_{2,2}(q^2)}{\partial q_{2,2}}\tfrac{d\nu_1(q_1)}{dq_1}p_{2,2}\Big) \\ = \ & -\beta\tfrac{q_1^2}{q_{2,1}}\tfrac{d\nu_1(q_1)}{dq_1}\Big(\eta_{2,2}(q^2)\tfrac{\mu(1-\mu)(1-\lambda)\lambda y_h}{(u_h^2(2)+\lambda u_\ell^2(2))^2}+q_{2,1}\overline{y}_{2,2}\tfrac{\partial\eta_{2,2}(q^2)}{\partial q_{2,2}}\Big(\tfrac{\mu+\lambda^2(1-\mu)}{\mu+\lambda(1-\mu)}-p_{2,2}\Big)\Big)\end{aligned} \tag{42}$$

The next proposition is based on the fact that this difference is positive as the last term between brackets is positive. Thus, there is excessive entry of firms in equilibrium. Furthermore, notice that the efficient allocation can be implemented through a tax on the entry cost, or equivalently on firms' profits, equal to the amount in (42). The following proposition states these results.

Proposition 6.3 *Constrained efficiency is not attained in the market economy. There are too many vacancies in equilibrium. By implementing a tax on firms' profits or a fee on posting vacancies, the equilibrium allocation becomes constrained efficient.*

6.3.5 Understanding the Inefficiency

To understand the inefficiency result, it is instructive to look at the two limit cases in which unemployment duration becomes worthless information. If λ is either arbitrarily small or close to 1, the efficiency loss associated with the intensive margin is negligible, $\frac{\partial \overline{y}_{2,2}(u)}{\partial q_1} \sim 0$, and the first term of expression (42) becomes 0. That is, there are no composition effects as either firms almost perfectly detect the skilled applicants, if $\lambda \sim 0$, or the signal is barely informative, if $\lambda \sim 1$. In the latter case, i.e. when firms cannot detect unskilled workers, the loss in period-two output due to the marginal increase of the mass of period-one vacancies is neutralized by the increase in total output in period one. As a result, the equilibrium is constrained efficient. Notice that, in this case, the setup does not differ from the standard directed search model. In other words, symmetric incomplete information by itself does not generate inefficiencies.

However, if the testing technology is almost perfect, $\lambda \sim 0$, the equilibrium is still not constrained efficient even though there are no efficiency losses on the intensive margin. The inefficiency outcome results from the fact that the intertemporal effects on the extensive margin are not totally captured by the equilibrium wages. When posting vacancies, the planner looks at the expected number of suitable units of labor, whereas firms are subject to the worker's unemployment value constraint. The difference between these two economies is captured by the last term of expression (42). If λ is arbitrarily close to 0, the first probability within brackets of this last term is 1, whereas the second probability becomes strictly lower than 1.

Put differently, in the directed search framework, competition comes from firms taking the market value of workers as given, which amounts to the continuation value they can obtain elsewhere. If firms perfectly learn the applicant's type and, hence, unskilled workers are not employable, the unskilled do not alter the applicants' expected productivity, but do affect their unemployment value. In contrast, the planner decision is not affected by the unskilled as vacancy creation is determined by the number of effective working units. Thus, firms in the market economy benefit from this asymmetry of information leading to inefficiently low wages and an excessively large entry. Notice that the equivalent setting to the planner's problem would be a market economy in which there would be only skilled workers. In this alternative world, both the planner and firms in the market economy would behave identically, and the last term of expression (42) would vanish.

For intermediate values of λ, the inefficiency result is due to the intertemporal externalities on both the intensive and extensive margins not captured by the equilibrium wages.

6.4 Calibration of the NR economy

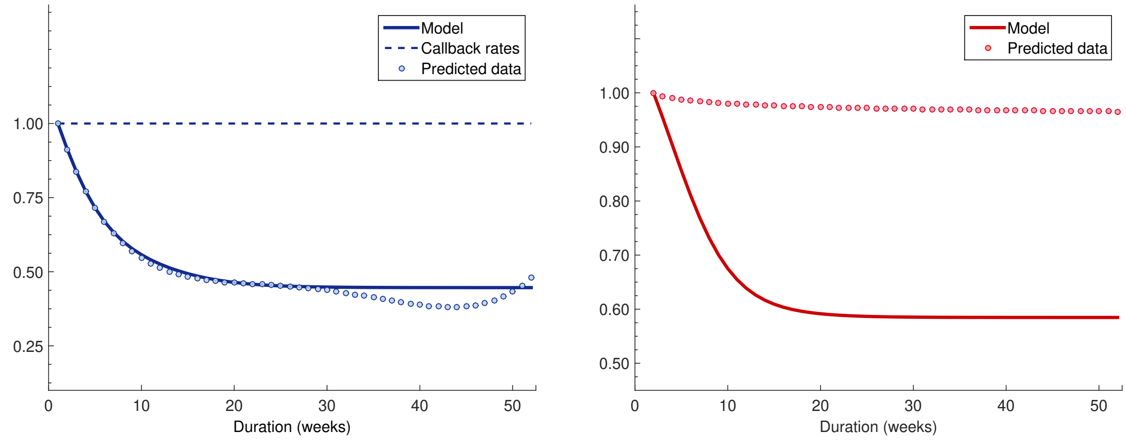
In this Appendix, we report the results when calibrating the NR economy to the same targets as used for the baseline calibration. The calibrated parameters are summarized in the following table:

Table 4: Calibration of the NR economy

Parameter	**Description**	**Value**	**Target**
Exogenously Set Parameters			
β	Discount factor	0.999	Annual interest rate of 5%
T	LTU-defining duration	52	-
y_h	Productivity of skilled	1.0	(Normalization)
Jointly Calibrated Parameters			
δ	Job-separation rate	0.0032	Predicted monthly job-separation rate
k	Vacancy cost	0.3888	13% avg. quarterly wage per hire
b	Home productivity	0.4741	71% of avg. productivity
λ_h	Skilled suitability prob.	0.3302	Avg. duration prior to E within next month
λ_ℓ	Unskilled suitability prob.	0.0685	St. dev., skewness and kurtosis of monthly exit rates
μ	Share of skilled	0.3551	
y_ℓ	Productivity of unskilled	0.4741	

Compared to the results for the calibration of the ranking case shown in Table 1, the main differences correspond to parameters k, b, and y_ℓ. In particular, y_ℓ is now much lower, while the suitability parameters remain at similar levels. That is, once the ranking mechanism is eliminated, larger productivity differences are necessary to match the duration distribution of exit rates. On the aggregate level the calibrated parameters imply an unemployment rate of 3.74%, a vacancy rate of 1.96%, and an average monthly exit rate of 26.68 %. As the exit rates are directly targeted, these numbers are similar to the corresponding values of the ranking economy.

The normalized exit rates and wages are compared in the following figure:



(a) Monthly Exit Rates

(b) Reemployment Wages

Figure 7: Distributions over Unemployment Duration: Model and Data.

Note: Exit rates from unemployment are the transition rates from unemployment to employment within the following month. Model callback rates are depicted in the left panel. Wages are the expected wage conditional on being employed after four weeks. All values are normalized by the value at the first week.

The exit rates closely match the data. After 3 months, the fall in exit rates of the model is 49% whereas it is 50% in the data. By construction, callback rates are flat in duration. Wages, however, are declining much faster over duration: the fall after 3 months is 38%, whereas it is 2.3% in the data (recall that wages are not targeted). The steep fall in wages reflects the large difference in productivity over types and the segmentation of markets over duration.