Optimal operational monetary policy rules in an endogenous growth model: a calibrated analysis

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abstract

We construct an endogenous growth model with new Keynesian-type sticky prices and wages. In this model, monetary policy affects long-run output growth. We characterize the optimal operational monetary policy rule in this economy. We find that even though stabilization of output growth increases long-run output growth, the optimal monetary policy rule is the rule that makes interest rate respond to price and wage actively and output growth mutely, similar as in exogenous growth models. We also find that the optimal monetary policy rule virtually maximizes mean growth. These results suggest that although long-run growth is important for welfare, new Keynesian's claim that monetary policy should stabilize nominal variables is highly robust.

Keywords: Monetary policy, Sticky price and wage, Business cycle fluctuations, Productivity growth

JEL classification: E31, E32, E52, O41

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1 Introduction

Our aim in this paper is to give some answers to the following questions. How much does monetary policy affect long-run growth? Does the long-run growth effect change the features of the optimal monetary stabilization policy rule? In order to highlight the growth and welfare effect of monetary policy, we incorporate Calvo (1983)-type sticky prices and wages to the two-capital convex model of endogenous growth. We here consider two types of growth effects, that is, the growth effect caused by the changes of long-run target rate of inflation under the deterministic environment and the effect arisen from uncertainty and monetary stabilization policy rules such as the Taylor rule. We shall call the former *the deterministic growth effect* and the latter *the stochastic growth effect*. Our motivations and findings of this research are as follows.

We first explain the deterministic growth effect. Empirical studies using cross-country data claim that long-run growth and inflation have a negative relationship. Calibrating the model, we find that in steady state, price stickiness cause negative long-run relationship between growth and inflation under positive inflation rate and that the magnitude of this relationship strongly depends on the degree of price stickiness. In the existence of price stickiness, non-zero inflation is a source of distortion which comes from price dispersion This distortion reduces resources which can be used for investment and so decreases output growth. This finding suggests that the various strength of price stickiness account for the difference of the relationship across countries.

Second, for the stochastic growth effect, Jones et al. (2005b) show that this type of model has the effect of uncertainty on long-run growth. Incorporating nominal rigidities into their model can cause changes of long-run growth rate through changes of interest rate policy rules. Moreover, It is also known that this class of model improves over simple (exogenous growth) RBC models.[1] Hence, the endogenous growth models with nominal rigidities has the potential abilities accounting the business cycle properties better than existing New Keynesian models. The relationship between fluctuations and growth is also important from normative perspective. In his seminal work, Lucas (1987) shows that the cost of business cycles is much less than that of growth. It is well known that his claim is strongly robust,[2] but it does not imply that fluctuations are negligible for the macroeconomics at all, because even if fluctuations itself has the small welfare effect, fluctuations can affect the long-run growth through the optimization of the economic

[1]See Jones et al. (2005a). Comin and Gertler (2006) also show that other endogenous growth model accounts for the medium term properties of business cycles well.

[2]The excellent surbeys in the literature are Lucas (2003) and Barlevy (2004a).

agents(Barlevy, 2004b).

For these reasons, endogenizing productivity growth can be thought to be important for the analysis of business cycle and stabilization policy. Most of studies about short-run monetary policy (New Keynesian approach), however, have been ignored the effect of monetary policy on long-run economic growth. We conjecture that the reason is purely technical issue. In most of New Keynesian studies, they approximate to the policy functions around non-stochastic steady-state up to first-order. In linear models, unconditional mean of endogenous variables are idendical to the non-stochastic steady-state value. Hence, even if long-run growth rate is endogenous, higher-order approximation is needed for the model to show the growth effect. We apply the numerical computation method which approximating to the policy function up to second-order developed by Schmitt-Grohe and Uribe (2004). Their numerical method enables us to address the relationship between stabilization policy and long-run growth because endogenous growth models with nominal rigidities approximated up to second-order do not hold certainty equivalence so that long-run growth rate is no longer identical to non-stochastic steady-state growth rate.

Solving our model by second-order approximation, we obtain some findings about the stochastic growth effect and about the optimal operational monetary policy as follows. First, in our model with stochastic disturbances, the long-run rate of output growth is affected by the monetary stabilization policy rules, especially policy rule responding to output though, under simple Taylor rule, deviation of annual growth rate from deterministic balanced growth path is very small, about -10^{-3} percent. Second, The effect of volatility of inflation on long-run growth is not clear because of existence of wage stickiness. We think it as a reason why empirical evidence about the correlation between inflation volatility and growth is unclear. Third, we characterize the optimal operational monetary policy rules and find that the features of the optimal operational policy is not turned from exogenous growth New Keynesian models. This result implies that the growth effect of Barlevy (2004b) which is caused by investment adjustment costs and nominal rigidities do not have a strong tradeoff. Finally, We find the optimal operational monetary policy rule is virtually identical to the growth-maximizing operational monetary policy rule in the sense that the growth-maximizing policy rule attains virtually the same levels of walfare and growth rate as the optimal policy rule. This finding suggests that the monetary authorities can virtually optimal allocation under price- and wage-stickiness only by resolving the tradeoff between price- and wage- stabilization even if monetary policy affects long-run growth. This result is also consistent with Blackburn and Pelloni (2005). As long as we know, it is the unique study about the relationship between the optimal monetary stabilization policy and long-run growth. They show analytically that optimal monetary policy is identical to the growth-maximizing policy in the endogenous growth model with neoclassical-type nominal wage rigidity.

This paper is organized as follows. In the next section, we present the model into which stochastic endogenous growth and nominal rigidities are fused, and calibrate the model. Section 3 analyzes the steady state and shows some results about the deterministic growth effect. Section 4 consider the stochastic growth and welfare effect under versions of Taylor rule. Section 5 considers the optimal operational interest-rate feedback rule. Section 6 concludes this paper.

2 The model

The model is a two-capital convex model of endogenous growth by Jones et al. (2005a) incorporating Calvo-type sticky prices and wages, physical and human capital investment adjustment costs, and habit persistence.

2.1 Households

The representative families, across whose menbers consumption and hour worked are identical, have preferences which are described by the following utility function,

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t - bC_{t-1}, 1 - n_t),$$

with

$$U(C_t - bC_{t-1}, 1 - n_t) \equiv \begin{cases} \frac{(C_t - bC_{t-1})^{1-\sigma}(1-n_t)^{\psi(1-\sigma)}}{1-\sigma} & \text{when } \sigma \neq 1 \\ \log(C_t - bC_{t-1}) + \psi \log(1 - n_t) & \text{when } \sigma = 1, \end{cases}$$

where E_t is the standard expectations operator conditional on information at time t, C_t denotes per capita consumption, n_t represents per capita labor supply, β, b, and σ are a subjective discount factor, the habit formation parameter, and the curvature parameter of utility, respectively. Households can consume the single final good, and the final good also can be used for human and physical capital investment. The final good is a composite good made of a continuum of defferenciated goods Y_{it} indexed by $i \in [0, 1]$, by

Dixit-Stiglitz aggregator. Hence the demand for Y_{it} is given by

$$Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\theta} Y_t,$$
$$Y_t = C_t + I_t^K + I_t^H \tag{1}$$

and price index P_t is

$$P_t = \left(\int_0^1 P_{it}^{1-\theta} di\right)^{\frac{1}{1-\theta}}, \tag{2}$$

where Y_t is aggregate absorption, P_{it} denotes the price of good i, and I_t^K and I_t^H represent physical and human capital investment per capita, respectively.

Households' expenditures on consumption goods are subject to a cash-in-advance constraint

$$M_t^h \geq \nu^h C_t, \tag{3}$$

where M_t^h denotes real money balances holding by households in period t and ν^h is a parameter.

Households own human capital H_t, and physical capital K_t. Capital accmulation equations are assumed as follows.

$$K_{t+1} = (1-\delta_K)K_t + I_t^K - \frac{a_K}{2}\left(\frac{I_t^K}{I_{t-1}^K} - \eta_K^F\right)^2 I_t^K \tag{4}$$

$$H_{t+1} = (1-\delta_H)H_t + I_t^H - \frac{a_H}{2}\left(\frac{I_t^H}{I_{t-1}^H} - \eta_H^F\right)^2 I_t^H \tag{5}$$

where δ_K and δ_H denote the depreciation rates with respect to physical and human capital, and a_K and a_H represent the investment adjustment cost parameters for physical and human capital,[3] respectively.

Following Schmitt-Grohe and Uribe (2005) (henceforth SGU), labor supply is decided by "a union", which supplies labor monopolistically to a continuum of labor markets indexed by $j \in [0,1]$. As we shall see below, the demand for labor in the labor market j is

$$n_{jt} = \left(\frac{W_{jt}}{W_t}\right)^{-\tilde{\theta}} n_t^d. \tag{6}$$

[3]This type of investment adjustment cost function is assumed in Christiano et al. (2005) for physical capital investment. We applies this specification also to human capital investment because we cannot find the emperical evidence about the form of human capital invenstment technology. The study about it remains for future research.

and nominal wage index is

$$W_t = \left(\int_0^1 W_{jt}^{1-\tilde{\theta}} dj \right)^{\frac{1}{1-\tilde{\theta}}} \tag{7}$$

where n_t^d is the aggregate labor demand and W_{jt} denotes the nominal wage rate in the labor market j. We define real wage index, $w_t \equiv W_t/P_t$, and real wage rate in labor market j, $w_{jt} \equiv W_{jt}/P_t$, respectively. The resource constraint of labor supply is

$$n_t = \int_0^1 n_{jt} dj, \tag{8}$$

From (7) and (8), a resource constraint which the union faces is obtained as

$$n_t = n_t^d \int_0^1 \left(\frac{w_{jt}}{w_t} \right)^{-\tilde{\theta}} dj. \tag{9}$$

We assume that households can access to a complete set of nominal state-contingent claims and that the effective labor is defined as product of hour worked and human capital, so that households' intertemporal budget constraint is

$$\begin{aligned} & E_t d_{t,t+1} \frac{X_{t+1}}{P_t} + M_t^h + C_t + I_t^K + (1+\tau^h) I_t^H \\ &= \frac{X_t}{P_t} + \frac{P_{t-1}}{P_t} M_{t-1}^h + (1+\tau^h) \int_0^1 \left(\frac{w_t^j}{w_t} \right)^{-\tilde{\theta}} n_t^d H_t w_t^j dj + r_t^K K_t + \Phi_t + T_t, \end{aligned} \tag{10}$$

where $d_{t,s}$ is nominal stochastic discount factor, X_t is nominal payment in period t, r_t^K denotes real rental rate on physical capital, Φ_t is profits received from firms, and T_t denotes the transfer from the government. τ^h represents human capital investment tax rate and wage subsidy rate to eliminate distortion which comes from monopolistic competition in labor markets. We assume $\tau^h = 1/(\tilde{\theta} - 1)$.

We assume wage stickiness following Calvo (1983) and SGU, that is, in each period the union cannot reoptimize the nominal wage in a fraction $\xi_w \in [0, 1)$ of randomly chosen labor markets. Following SGU, in these non-optimized markets, the nominal wages are (fully or partially) indexed to the nominal good-price inflation in the previous period.[4] Therefore, the nominal

[4] In some models where labor productivity grows exogenously such as SGU, the nominal wage rates in the non-reoptimizing labor markets are also indexed to average real wage growth. In our model, however, average real wage growth is zero because growth of wage payment comes from human capital accumulation.

wage setting rule is

$$W_{jt} = \begin{cases} \tilde{W}_t & \text{if the wage can be re-optimized,} \\ \pi_{t-1}^{\tilde{\chi}} W_{j,t-1} & \text{otherwise,} \end{cases}$$

where $\tilde{W}_t$ is the optimal nominal wage rate in period t, $\pi_t \equiv P_t/P_{t-1}$ is nominal good-price inflation rate. Defining the optimal real wage rate as $\tilde{w}_t \equiv \tilde{W}_t/P_t$, We can rewrite the rule as

$$w_{jt} = \begin{cases} \tilde{w}_t & \text{if the wage can be re-optimized,} \\ \frac{\pi_{t-1}^{\tilde{\chi}}}{\pi_t} w_{j,t-1} & \text{otherwise.} \end{cases} \tag{11}$$

Households maximize (2.1) subject to (3), (4), (5), (9), (10), the sticky wage assumption (11), and the no-Ponzi game condition, choosing processes for C_t, n_t, X_{t+1}, M_t, I_t^K, I_t^H, K_{t+1}, H_{t+1}, and w_t^j. Let us define the Lagrange multipliers associated with (3), (4), (5), (9), (10) as $\beta^t \Lambda_t \zeta_t$, $\beta^t \Lambda_t q_t^K$, $\beta^t(1+\tau^h)\Lambda_t q_t^H$, $\frac{\beta^t \Lambda_t w_t H_t}{\tilde{\mu}_t}$, and $\beta^t \Lambda_t$ respectively. Restricting our attention to the strictly-positive nominal interest rate equilibria, we obtain the first-order conditions with respect to $C_t, n_t, X_{t+1}, M_t, I_t^K, I_t^H, K_{t+1}, H_{t+1}$ as follows.

$$C_t: \ \Lambda_t(1+\nu^f \zeta_t) = (C_t - bC_{t-1})^{-\sigma}(1-n_t)^{\psi(1-\sigma)} - \beta b E_t (C_{t+1} - bC_t)^{-\sigma}(1-n_{t+1})^{\psi(1-\sigma)}, \tag{12}$$

$$n_t: \ \frac{\Lambda_t w_t H_t}{\tilde{\mu}_t} = \psi (C_t - bC_{t-1})^{1-\sigma}(1-n_t)^{\psi(1-\sigma)-1}, \tag{13}$$

$$X_{t+1}: \ d_{t,t+1} = \frac{\beta \Lambda_{t+1}}{\Lambda_t \pi_{t+1}}, \tag{14}$$

$$M_t: \ \Lambda_t(1-\zeta_t) = \beta E_t \frac{\Lambda_{t+1}}{\pi_{t+1}}, \tag{15}$$

$$\begin{aligned} I_t^K: \ \Lambda_t &= \Lambda_t q_t^K \left[1 - \frac{a_K^F}{2}\left(\frac{I_t^K}{I_{t-1}^K} - \eta_K^F\right)^2 - a_K^F \left(\frac{I_t^K}{I_{t-1}^K} - \eta_K^F\right)\frac{I_t^K}{I_{t-1}^K}\right] \\ &+ \beta E_t \Lambda_{t+1} q_{t+1}^K a_K^F \left(\frac{I_{t+1}^K}{I_t^K} - \eta_K^F\right)\left(\frac{I_{t+1}^K}{I_t^K}\right)^2, \end{aligned} \tag{16}$$

$$\begin{aligned} I_t^H: \ \Lambda_t &= \Lambda_t q_t^H \left[1 - \frac{a_H^F}{2}\left(\frac{I_t^H}{I_{t-1}^H} - \eta_H^F\right)^2 - a_H^F \left(\frac{I_t^H}{I_{t-1}^H} - \eta_H^F\right)\frac{I_t^H}{I_{t-1}^H}\right] \\ &+ \beta E_t \Lambda_{t+1} q_{t+1}^H a_H^F \left(\frac{I_{t+1}^H}{I_t^H} - \eta_H^F\right)\left(\frac{I_{t+1}^H}{I_t^H}\right)^2, \end{aligned} \tag{17}$$

$$K_{t+1}: \ \Lambda_t q_t^K = \beta E_t \Lambda_{t+1}[r_{t+1}^K + q_{t+1}^K(1-\delta_K)], \tag{18}$$

$$H_{t+1}: \ \Lambda_t q_t^H = \beta E_t \Lambda_{t+1}[w_{t+1} n_{t+1}^d + q_{t+1}^H(1-\delta_H)]. \tag{19}$$

From (14) and the definition of nominal interest rate, $1/R_t = E_t d_{t,t+1}$, we obtain the well-known fisher relationship,

$$\frac{1}{R_t} = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t \pi_{t+1}}. \tag{20}$$

and, from (15) and (20), the cost of holding money ζ_t is representing that

$$\zeta_t = 1 - R_t^{-1}. \tag{21}$$

Next, we consider optimal wage setting behavior. The parts of Lagrangian that are relevant for wage setting is

$$\begin{aligned} L_t^w = E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \Big[(1+\tau^h)\Lambda_{t+s} n_{t+s}^d H_{t+s} \tilde{w}_t \tilde{X}_{t,t+s} \Big(\frac{\tilde{w}_t \tilde{X}_{t,t+s}}{w_{t+s}} \Big)^{-\tilde{\theta}} \\ - \frac{\Lambda_{t+s} w_{t+s} H_{t+s}}{\tilde{\mu}_{t+s}} n_{t+s}^d \Big(\frac{\tilde{w}_t \tilde{X}_{t,t+s}}{w_{t+s}} \Big)^{-\tilde{\theta}} \Big], \end{aligned}$$

where,

$$\tilde{X}_{t,t+s} = \begin{cases} 1 & s = 0, \\ \frac{\pi_t^{\tilde{\chi}}}{\pi_{t+1}} \cdot \cdots \cdot \frac{\pi_{t+s-1}^{\tilde{\chi}}}{\pi_{t+s}} & s = 1, 2, \cdots. \end{cases}$$

The first-order conditions with respect to $\tilde{w}_t$ is

$$E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \Lambda_{t+s} n_{t+s}^d H_{t+s} \left(\frac{\tilde{w}_t}{w_{t+s}} \right)^{-\tilde{\theta}} \tilde{X}_{t,t+s}^{-\tilde{\theta}} \left[(1+\tau^h) \frac{\tilde{\theta}-1}{\tilde{\theta}} \tilde{w}_t \tilde{X}_{t,t+s} - \frac{w_{t+s}}{\tilde{\mu}_{t+s}} \right] = 0.$$

Define F_t^1 and F_t^2 as

$$\begin{aligned} F_t^1 &= (1+\tau^h) \frac{\tilde{\theta}-1}{\tilde{\theta}} \tilde{w}_t E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \Lambda_{t+s} n_{t+s}^d H_{t+s} \left(\frac{\tilde{w}_t}{w_{t+s}} \right)^{-\tilde{\theta}} \tilde{X}_{t,t+s}^{1-\tilde{\theta}}, \\ F_t^2 &= E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \Lambda_{t+s} n_{t+s}^d H_{t+s} \left(\frac{\tilde{w}_t}{w_{t+s}} \right)^{-\tilde{\theta}} \tilde{X}_{t,t+s}^{-\tilde{\theta}} \frac{w_{t+s}}{\tilde{\mu}_{t+s}}, \end{aligned}$$

respectively, and we obtain the recursive formulation of optimal wage setting behavior,

$$F_t^1 = (1+\tau^h) \frac{\tilde{\theta}-1}{\tilde{\theta}} \Lambda_t n_t^d H_t \left(\frac{\tilde{w}_t}{w_t} \right)^{-\tilde{\theta}} \tilde{w}_t + \beta\xi_w E_t \left(\frac{\tilde{w}_t}{\tilde{w}_{t+1}} \right)^{1-\tilde{\theta}} \left(\frac{\pi_t^{\tilde{\chi}}}{\pi_{t+1}} \right)^{1-\tilde{\theta}} F_{t+1}^1, \tag{22}$$

$$F_t^2 = \Lambda_t n_t^d H_t \left(\frac{\tilde{w}_t}{w_t} \right)^{-\tilde{\theta}} w_t \tilde{\mu}_t^{-1} + \beta\xi_w E_t \left(\frac{\tilde{w}_t}{\tilde{w}_{t+1}} \right)^{-\tilde{\theta}} \left(\frac{\pi_t^{\tilde{\chi}}}{\pi_{t+1}} \right)^{-\tilde{\theta}} F_{t+1}^2, \tag{23}$$

$$F_t^1 = F_t^2. \tag{24}$$

2.2 Firms

Each good i is produced by a single firm indexed by $i \in [0,1]$ in monopolistically competitive good market. The production technology of Firm i is represented by the following Cobb-Douglas production function,

$$Y_{it} = A_t K_{it}^{\alpha} Z_{it}^{1-\alpha},$$

where A_t represents exogenous aggregate productivity, K_{it} and Z_{it} denotes physical capital and the effective labor demanded by firm i, respectively.

We impose a cash-in-advance constraint for wage payments

$$M_{it}^{f} = \nu^f w_t Z_{it}, \tag{25}$$

where M_{it}^{f} denotes the demand for real money balances by firm i and ν^f is a parameter. The cost of holding money is $(1 - R_t^{-1})M_{it}^{f}$, hence the nominal total production cost of firm i is represented as

$$r_t^K K_{it} + w_t Z_{it} + (1 - R_t^{-1})M_{it}^{f}.$$

The first-order conditions of the cost minimization problem are given by

$$r_t^K = \alpha A_t K_{it}^{\alpha-1} Z_{it}^{1-\alpha} mc_t, \tag{26}$$

$$w_t[1 + \nu^f(1 - R_t^{-1})] = (1-\alpha) A_t K_{it}^{\alpha} Z_{it}^{-\alpha} mc_t, \tag{27}$$

$$\left(\frac{P_{it}}{P_t}\right)^{-\theta} Y_t = A_t K_{it}^{\alpha} Z_{it}^{1-\alpha}, \tag{28}$$

where mc_t represents real marginal cost. Note that the real marginal cost is in common among the firms, hence the subscript representing the firm index i is dropped.

We assume price stickiness following Calvo (1983) and Yun (1996), that is, each period a fraction $\xi_p \in [0,1)$ of randomly chosen firms cannot reoptimize the nominal price of their producted good. Formally, the firms set their nominal prices according to the following rule,

$$P_{it} = \begin{cases} \tilde{P}_t & \text{if the firm can set their price optimally,} \\ \pi_{t-1}^{\chi} P_{i,t-1} & \text{otherwise.} \end{cases} \tag{29}$$

Hence, profit maximization problem are formulated as

$$\max \; E_t \sum_{s=0}^{\infty} d_{t,t+s} P_{t+s} \xi_p^s \left[(1+\tau^Y) \left(\frac{X_{t,t+s}\tilde{P}_t}{P_{t+s}}\right)^{1-\theta} Y_{t+s} - \left(\frac{X_{t,t+s}\tilde{P}_t}{P_{t+s}}\right)^{-\theta} Y_{t+s} mc_{t+s} \right].$$

where,

$$X_{t,t+s} = \begin{cases} 1 & s = 0, \\ \pi_t^{\chi} \cdot \dots \cdot \pi_{t+s-1}^{\chi} & s = 1, 2, \cdots, \end{cases}$$

and where $\tau^Y = 1/(\theta - 1)$ represents production subsidy rate to eliminate distortion which comes from monopolistic competition in good markets.

The first-order condition with respect to $\tilde{P}_t$ is

$$E_t \sum_{s=0}^{\infty} d_{t,t+s} \xi_p^s Y_{t+s} \left[(1+\tau^Y)(\theta-1) X_{t,t+s}^{1-\theta} \left(\frac{\tilde{P}_t}{P_{t+s}} \right)^{-\theta} - \theta X_{t,t+s}^{-\theta} \left(\frac{\tilde{P}_t}{P_{t+s}} \right)^{-\theta-1} mc_{t+s} \right] = 0.$$

Define X_t^1 and X_t^2 as

$$X_t^1 \equiv E_t \sum_{s=0}^{\infty} d_{t,t+s} \xi_p^s Y_{t+s} X_{t,t+s}^{-\theta} \left(\frac{\tilde{P}_t}{P_{t+s}} \right)^{-\theta-1} mc_{t+s},$$
$$X_t^2 \equiv E_t \sum_{s=0}^{\infty} d_{t,t+s} \xi_p^s Y_{t+s} X_{t,t+s}^{1-\theta} \left(\frac{\tilde{P}_t}{P_{t+s}} \right)^{-\theta},$$

respectively, and we obtain the recursive formulation of optimal price setting behavior,

$$X_t^1 = \tilde{p}_t^{-\theta-1} Y_t mc_t + \xi_p E_t d_{t,t+1} (\pi_t^{\chi})^{-\theta} \left(\frac{\tilde{p}_t}{\pi_{t+1} \tilde{p}_{t+1}} \right)^{-\theta-1} X_{t+1}^1, \tag{30}$$

$$X_t^2 = \tilde{p}_t^{-\theta} Y_t + \xi_p E_t d_{t,t+1} (\pi_t^{\chi})^{1-\theta} \left(\frac{\tilde{p}_t}{\pi_{t+1} \tilde{p}_{t+1}} \right)^{-\theta} X_{t+1}^2, \tag{31}$$

$$\theta X_t^1 = (1+\tau^Y)(\theta-1) X_t^2, \tag{32}$$

where we define $\tilde{p}_t \equiv \tilde{P}_t / P_t$.

Finally, we show that labor demand in the market j and nominal wage index are described as the form of (6) and (7), respectively. The effective labor input of firm i, Z_{it}, is assumed to be a composite effective labor made by the following aggregator

$$Z_{it} = \left[\int_0^1 (Z_{it}^j)^{\frac{\tilde{\theta}-1}{\tilde{\theta}}} dj \right]^{\frac{\tilde{\theta}}{\tilde{\theta}-1}},$$

where Z_{it}^j denotes the effective labor demand by firm i in effective labor market j. We also assume that the effective labor in the market j is defined as the product of labor forces in the market j and human capital, which is assumed to be identical across the family menbers. More formally, $Z_{it}^{j} \equiv n_{it}^{j} H_t$, where n_{it}^{j} represents the labor demand by firm i in labor market j. Hence, the "composite labor" demand by firm i, n_{it}, can be described as

$$n_{it} = \left[\int_0^1 (n_{it}^{j})^{\frac{\tilde{\theta}-1}{\tilde{\theta}}} dj \right]^{\frac{\tilde{\theta}}{\tilde{\theta}-1}},$$

and

$$Z_{it} = n_{it} H_t, \tag{33}$$

that is, the composite effective labor input can be written as the product of the composit labor and human capital.

The cost minimization problem of firm i with respect to labor is

$$\min \int_0^1 W_{jt} n_{it}^{j} dj$$

$$\text{s.t. } n_{it} = \left[\int_0^1 (n_{it}^{j})^{\frac{\tilde{\theta}-1}{\tilde{\theta}}} dj \right]^{\frac{\tilde{\theta}}{\tilde{\theta}-1}}.$$

Define the Lagrange multiplier as W_t, which represents nominal wage index because it is marginal cost of composite labor, and we obtain the following equations

$$n_{it}^{j} = \left(\frac{W_{jt}}{W_t} \right)^{-\tilde{\theta}} n_{it} \tag{34}$$

$$W_t = \left[\int_0^1 (W_{jt})^{1-\tilde{\theta}} dj \right]^{\frac{1}{1-\tilde{\theta}}} \tag{35}$$

The equation (35) is identical to the equation (7). By defining aggregate labor demand in market j and aggregate composite labor demand as

$$n_{jt} \equiv \int_0^1 n_{it}^{j} di$$

and

$$n_t^d \equiv \int_0^1 n_{it} di, \tag{36}$$

respectively. From (34), we obtain the labor demand equation as the form of (6),

$$n_{jt} = \int_0^1 n_{it}^{j} di = \int_0^1 \left(\frac{W_{jt}}{W_t} \right)^{-\tilde{\theta}} n_{it} di = \left(\frac{W_{jt}}{W_t} \right)^{-\tilde{\theta}} n_t^d. \tag{37}$$

2.3 Government

Our study focuses only on the monetary policy, hence, for simplicity we assume that the government does not perchase the final good and that all seigniorage is transfered to households. Therefore the intertemporal budget constraint of the government is

$$(1+\tau^h)I_t^H + \left(M_t - \frac{M_{t-1}}{\pi_t}\right) = T_t + (1+\tau^h)w_t n_t^d H_t + (1+\tau^Y)Y_t,$$

where M_t denotes aggregate real money supply. In equilibrium, therefore, it holds that

$$M_t = M_t^h + M_t^f = \nu^h C_t + \nu^f w_t n_t^d H_t, \tag{38}$$

where $M_t^f \equiv \int_0^1 M_{it}^f di$.

Following some rules described below, monetary authority sets the process for the nominal interest rate, R_t.

2.4 Aggregation, Market Clearing, and Exogenous process

2.4.1 Price and wage indexes

By equations (2) and (29), we obtain

$$1 = (1-\xi_p)\tilde{p}_t^{1-\theta} + \xi_p \left(\frac{\pi_{t-1}^{\chi}}{\pi_t}\right)^{1-\theta}. \tag{39}$$

By equations (7) and (11), it holds that

$$w_t^{1-\tilde{\theta}} = (1-\xi_w)\tilde{w}_t^{1-\tilde{\theta}} + \xi_w \left(\frac{\pi_{t-1}^{\tilde{\chi}}}{\pi_t}\right)^{1-\tilde{\theta}} w_{t-1}^{1-\tilde{\theta}}. \tag{40}$$

2.4.2 Final-good markets

Market clearing condition in good market i is written as

$$A_t K_{it}^{\alpha} Z_{it}^{1-\alpha} = \left(\frac{P_{it}}{P_t}\right)^{-\theta} Y_t. \tag{41}$$

By equations (33), (36), the resouce constraint with respect to capital

$$\int_0^1 K_{it} di = K_t,$$

and the fact that, in equilibrium, capital to effective labor ratios are identical across the firms because the firm's production function is homogeneous of degree one, we can integrate the equation (41) over all good markets. As the result we obtain aggregate resouce constraint

$$A_t K_t^{\alpha} (n_t^d H_t)^{1-\alpha} = Y_t s_t, \tag{42}$$

where we define

$$s_t \equiv \int_0^1 \left(\frac{P_{it}}{P_t} \right)^{-\theta} di,$$

or as recursive representation,

$$s_t = (1-\xi_p) \tilde{p}_t^{-\theta} + \xi_p \left(\frac{\pi_{t-1}^{\chi}}{\pi_t} \right)^{-\theta} s_{t-1}. \tag{43}$$

s_t denotes the inefficiency by the price dispersion.

By equations (26), (27), and the fact that, the equilibrium capital to effective labor ratios are identical across the firms, we obtain

$$r_t^K = \alpha A_t K_t^{\alpha-1} (n_t^d)^{1-\alpha} H_t^{1-\alpha} mc_t, \tag{44}$$

$$w_t[1+\nu^f(1-R_t^{-1})] = (1-\alpha) A_t K_t^{\alpha} (n_t^d)^{-\alpha} H_t^{-\alpha} mc_t. \tag{45}$$

2.4.3 labor markets

By aggregating (37) over all labor markets, we obtain aggregate resouce constraint with respect to labor,

$$n_t = n_t^d \tilde{s}_t, \tag{46}$$

where $\tilde{s}_t \equiv \int_0^1 \left(\frac{w_{jt}}{w_t} \right)^{-\tilde{\theta}} dj$ denotes the inefficiency by the wage dispersion. We can write the difinition as recursive representation,

$$\tilde{s}_t = (1-\xi_w) \left(\frac{\tilde{w}_t}{w_t} \right)^{-\tilde{\theta}} + \xi_w \left(\frac{w_{t-1}}{w_t} \right)^{-\tilde{\theta}} \left(\frac{\pi_{t-1}^{\tilde{\chi}}}{\pi_t} \right)^{-\tilde{\theta}} \tilde{s}_{t-1}. \tag{47}$$

2.4.4 Exogenous process

The law of the motion of aggregate productivity A_t is assumed to be given by the following exogenous stochastic process

$$\log \left(\frac{A_t}{\bar{A}} \right) = \rho \log \left(\frac{A_{t-1}}{\bar{A}} \right) + \sigma_{\epsilon} \epsilon_t, \tag{48}$$

$$0 \le \rho < 1,\ \epsilon_t \sim N(0,1),$$

where σ_{ϵ} is standard deviation of the stochastic shock.

2.5 Stationary Competitive Equilibrium

In the environment described above, some of endogenous variables are not stationary along the balanced-growth path because the economy shows endogenous growth by the same mechanism as Jones et al. (2005b).[5] We therefore rewrite these variables to be stationary. For this purpose, we categorize the nonstationary variables into three groups and divide these variables in each group by the appropriate factors. The first group are composed of C_t, Y_t, I_t^K, I_t^H, X_t^1, X_t^2, K_t, and M_t, divided by H_t. The second group consists in F_t^1 and F_t^2, divided by $H_t^{1-\sigma}$. A variable in the third group is Λ_t, divided by H_t^σ. We denote the corresponding stationary variables with lower letters. Finally, we define the growth rate of human capital as $\gamma_t^H \equiv H_t/H_{t-1}$.

We define a stationary competitive equilibrium as the set of stationary processes γ_t^H, c_t, λ_t, ζ_t, n_t, $\tilde{\mu}_t$, π_t, y_t, i_t^K, i_t^H, x_t^1, x_t^2, mc_t, $\tilde{p}_t$, k_t, n_t^d, s_t, r_t^K, w_t, f_t^1, f_t^2, $\tilde{w}_t$, $\tilde{s}_t$, q_t^K, q_t^H, and m_t satisfying the equilibrium conditions (1), (4), (5), (12), (13), (16)-(21), (22)-(24), (30)-(32), (38)-(40), (42)-(47) written in terms of the stationary variables, given the nominal interest rate policy process R_t, exogenous aggregate productivity stochastic process A_t, and initial conditions γ_0^H, c_{-1}, π_{-1}, i_{-1}^K, i_{-1}^H, k_0, s_{-1}, w_{-1}, and $\tilde{s}_{-1}$.

2.6 Calibration

The parameter values are summarized in Table 1. The deep parameters are calibrated by the following way. The time unit is assumed to be one quarter. We assume cashless economy so that ν^h and ν^f is zero. σ is set to be 1, that is, we assume that log utility. Given that, we divide the parameters to be calibrated into three groups. For parameters in the first group, b, δ_K, δ_H, α, θ, $\tilde{\theta}$, ξ_p, ξ_w, χ, $\tilde{\chi}$, and ρ, we draw on related studies. ρ are set to be 0.95, which are in the range of the value used in RBC literature. We set b to be 0.69, δ_K to be 0.025, α to be 0.36, $\tilde{\theta}$ to be 21, ξ_p to be 0.8, and ξ_w to be 0.69, following Altig et al. (2005). δ_H is assumed to be $1-(1-0.025)^{1/4}$, following Jones et al. (2005a). θ is assumed to be 6, such that the steady-state markup in product good markets is 20 percent. Following SGU and the empirical fact found by Levin et al. (2005), we set χ and $\tilde{\chi}$ to be 0 and 1, respectively.

Second, given the values set above, parameters in the second group, which is composed of ψ, η, $\bar{A}$, are calibrated by using steady-state conditions and some restrictions. We think zero-inflation steady-state and assume that steady-state output growth rate is 0.45 percent per quarter, following SGU. In the deterministic steady state, we assume labor supply, n, to be one half,[6]

[5] See Jones and Manuelli (1990), for more general formulation.

[6] Given log utility, $n = 0.5$ implies unit Frisch elasticity of labor supply.

Table 1: Deep Structural Parameters

Parameter	Value		Description
	Endogenous growth	Exogenous growth	
σ	1	1	Preference parameter
β	0.99455	0.99455	Subjective discount rate
ψ	2.3776	0.92976	Preference parameter
b	0.69	0.69	Degree of habit persistence
δ_K	0.025	0.025	Depreciation rate of physical capital
δ_H	$1-(1-0.025)^{\frac{1}{4}}$	—	Depreciation rate of human capital
a^K	0.036527	2.79	Physical capital IAC parameter
a^H	0.025380	—	Human capital IAC parameter
η	1.0045	1.0045	IAC Parameter
α	0.36	0.36	Cost Share of physical capital
$\bar{A}$	0.064307	1	Production function parameter
θ	6	6	Price elasticity of good demand
$\tilde{\theta}$	21	21	Wage elasticity of labor demand
ξ_p	0.8	0.8	Degree of nominal good price rigidities
ξ_w	0.69	0.69	Degree of nominal wage rigidities
χ	0	0	Degree of price indexation
$\tilde{\chi}$	1	1	Degree of nominal wage indexation
ν^h	0	0	Parameter of firm's CIA constraint
ν^f	0	0	Parameter of households' CIA constraint
ρ	0.95	0.95	Serial correlation of productivity shock
σ_ϵ	0.0053317	0.007	Scaling parameter of uncertainty
γ^H	(endogenous)	1.0045	Growth rate of Human capital

quarterly real interest rate, r^K, to be one percent, each price of physical and human capital, q^K and q^H, to be 1, respectively. Under these assumptions, we obtain the values of ψ, η, and $\bar{A}$ by using steady-state conditions.

Finally, parameters in the third group, a^K, a^H, and σ_ϵ, are calibrated such that the second moment of key variables in the model match the U.S. business cycle fact. We set these values such that standard deviation of output growth rate is 0.84, that standard deviation of physical capital investment growth is three times larger than that of output growth, and that standard deviation of broad consumption[7] growth is as half as that of output growth under simple Taylor rule with zero inflation target, $\log(R_t/\bar{R}) = 1.5 \log \pi_t$.

For comparative purpose, we also consider the counterpart exogenous growth model. In the exogenous growth model, the growth rate of human capital, γ_t^H, is exogenously 1.0045 in all t and human capital investment is zero. a^H no longer affects the equilibrium. a^K is set to be 2.79, following

[7]Following Jones et al. (2005a), we refer the sum of consumption and human capital investment as 'broad consumption'.

SGU. σ_ϵ is set to be 0.007, which is standard RBC literature. Given that, We calibrate the parameters of the exogenous growth model by the same steady-state restrictions as the endogeous growth model.[8]

3 Steady State Analysis

3.1 Optimal Long-run Inflation Rate

Given our assumptions of cashless economy, tax on human capital investment, subsidies on employment and production, no price indexation, and full wage indexation, we ensure that real allocation of the deterministic steady state at zero inflation is the same as steady-state real allocation of social planner solution. Therefore, the deterministic steady state at zero inflation is Pareto optimal.

3.2 Growth and Welfare effect of Long-run Inflation

Despite some empirical studies claim the importance of the negative correlation between long-run growth and inflation, it is underestimated in theoritical literature using nominal frictions such as cash-in-advance constraints. For example, Kormendi and Meguire (1985) estimated their correlations using cross-country data and found that a decreasing of inflation by 2% would rise the growth rate by 1% per annum, but in theorictical analysis, computed growth effect caused by 10% increasing of annual inflation rate lowers the annual growth rate by only 0.06% in Gomme (1993) and 0.3% in Jones and Manuelli (1995). In contrast, the sticky price model has the more significantly negative growth and welfare effect of inflation than their flexible price models. Figure 1 represents the relationship between growth and inflation in the deterministic steady steady state. In our baseline calibration, a increasing of inflation from 0% to 10% lowers growth by about 1% per annum. Moreover, we find that in our model, welfare cost of inflation is also large. a increasing of inflation by 10% per annum decreases economic welfare by more than 30% consumption measured at zero-inflation (Pareto-optimal) steady state.

Our result obtained above shows that price stickiness brings the significant growth and welfare effect.[9] The degree of price stickiness, however,

[8]The scaling parameter of production function, $\bar{A}$, is arbitary in the exogenous model. Without loss of generality, we set $\bar{A} = 1$.

[9]Wage stickiness does not cause growth and welfare effects in steady state of our model because we assume full nominal wage indexation so that all labor markets is not distorted

Fig. 1: The effects of long-run inflation

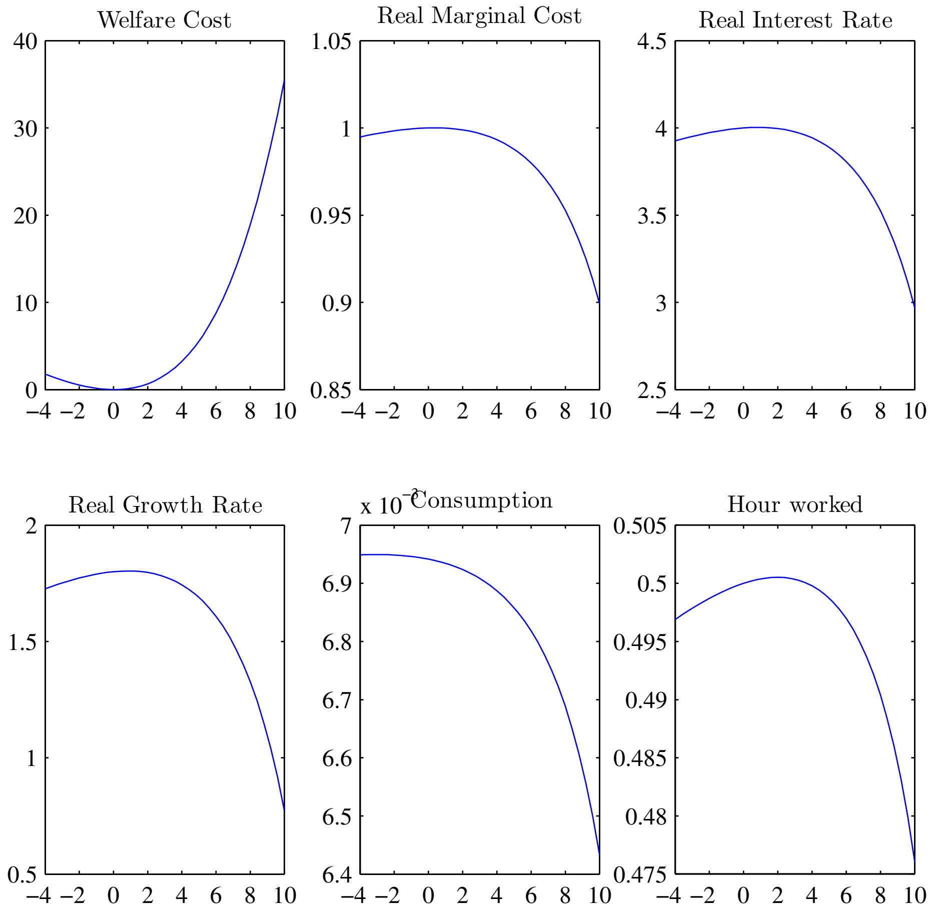

Table 2: The growth and welfare effect of 10% annual inflation

ξ_p	0.2	0.4	0.6	0.8 (baseline)
Growth effect	-0.007	-0.0257	-0.102	-1.03
Welfare cost	0.255	0.983	3.92	35.5

Note: i) Growth rate is described in net growth per year. ii) Welfare cost is measured by percentage of consumption in zero-inflation steady state.

would be highly difference across countries. We then do a sensitivity analysis for the degree of price stickiness, ξ_p. Table 2 represents growth and welfare effect of 10% annual inflation when $\xi_p = 0.2, 0.4, 0.6, 0.8$. The result shows that the low degree of price stickiness lower the growth effect of inflation.

Note that growth and welfare effect is nonlinear in our model. The higher inflation is, the more marginal growth decreasing is. This nonlinearity is not consistent with recent empirical evidence. We conjecture that our nonlinearity result is caused by the time-dependent sticky price assumption. In the assumption, high inflation bring severe distortion of price stickiness. If the assumption of sticky price was state-dependent pricing such as Dotsey et al. (1999), high inflation would lower the degree of price stickiness and might be consistent with empirical studies.

4 Equilibrium Dynamics

4.1 Solution Method

Because of the complexity of the model, an exact numerical solution does not exist. The log-linearization method, however, eliminates the growth effects which comes from uncertainty because unconditional means of endogenous variables are identical to the values at deterministic steady state in log-linearized model. We then approximate the equilibrium conditions and the conditional welfare measure to second-order accuracy by using the computation method developed by Schmitt-Grohe and Uribe (2004).

4.2 Monetary Policy Rules

We consider the equilibrium dynamics and welfares under simple Taylor rule and optimal operational monetary policy rules below. In this subsection, we present the definitions of Taylor rules.

in steady state with *any* inflation rate.

The Simple Taylor Rule First, as a benchmark, we apply the simple Taylor rule responding only to inflation,

$$\log\left(\frac{R_t}{R^*}\right) = 1.5\log\left(\frac{\pi_t}{\pi^*}\right), \tag{49}$$

where variables with asterisks denote their values at the zero-inflation deterministic steady state.

The Taylor Rule In order to investigate the effect of growth by responding to output, we use the standard Taylor rule also responding to cyclical components of output,

$$\log\left(\frac{R_t}{R^*}\right) = 1.5\log\left(\frac{\pi_t}{\pi^*}\right) + 0.5\log\left(\frac{y_t}{y^*}\right). \tag{50}$$

4.3 Results

4.3.1 Growth effects of Monetary Policy

Our numerical result under the simple and standard Taylor rules are represented in Table 3. According to this, under baseline parameters the simple Taylor rule lowers the long-run growth rate by 3×10^{-3} percent per year. Furthermore, the rule responding to output has more significantly negative growth effect. the rule lowers the long-run growth rate by 7×10^{-2} percent per year. It is seen to be small for policy implication but we think it as not to be negligible for the literature of the relationship between growth and fluctuations. For example, Jones et al. (2005b) show that the convex models of endogenous growth without nominal rigidities has positive or negative growth effect of uncertainty. Under reasonable parameter values, the growth effects of uncertainty in their model increase long-run growth rate by at most 7×10^{-2} percent per year. Hence, the growth effects of monetary policy at least offset their growth effect when the economies have nominal rigidities. Jones et al. (2005b) claim that the differences of long-run growth across countries may be able to contribute to sharper estimations of deep parameters such as intertemporal elasticity of substitution. Our result, however, suggests that given the existence of nominal rigidities and the differences of policy rules across countries, we can not ignore the growth effect of the differences of monetary policy rules to estimate those ones.

4.3.2 Relationship between Inflation Volatility and Growth

Dotsey and Sarte (2000) show that financial imperfection cause the positive relationship between inflation volatility and growth. Empirical evidences,

Fig. 2: The inflation volatility effects on long-run growth

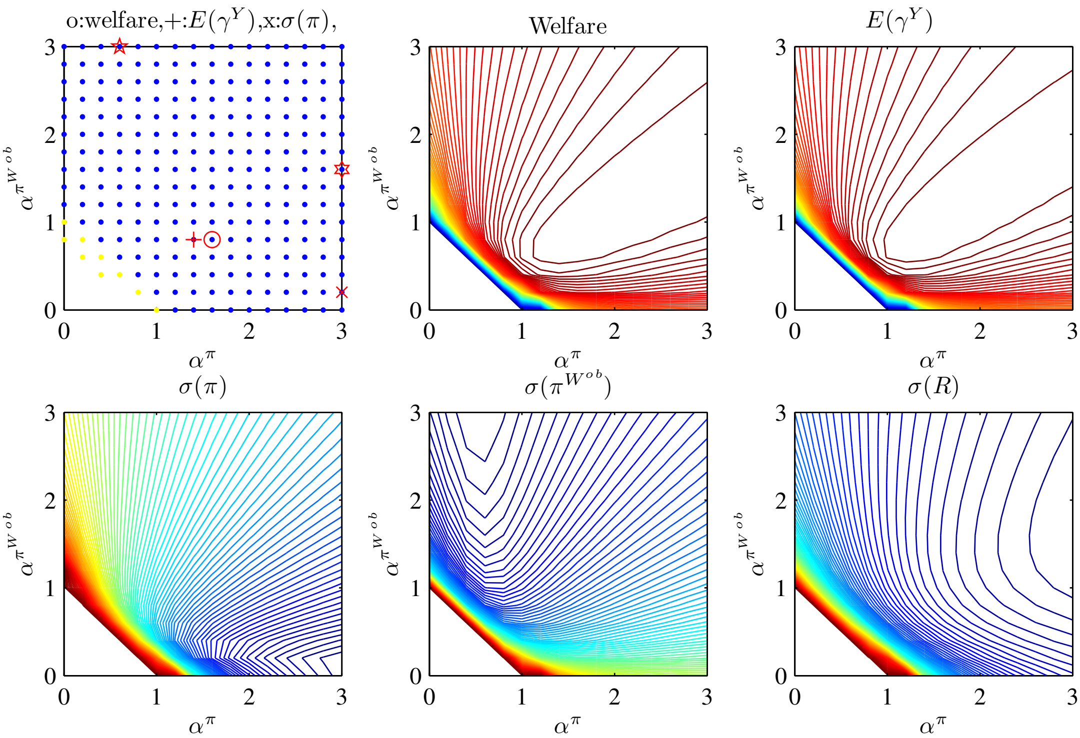

Table 3: The Taylor Rules
Simple Taylor Rule: $\log(R_t/R^*) = 1.5\log(\pi_t/\pi^*)$
Taylor Rule - Output: $\log(R_t/R^*) = 1.5\log(\pi_t/\pi^*) + 0.5\log(y_t/y^*)$

	Welfare Cost	σ_π	$\sigma_{\pi^{W^{ob}}}$	σ_R	$E(\hat{\gamma}^Y)$
A: Endogenous Growth New Keynesian Model					
Baseline					
Simple Taylor Rule	0.073	0.38	1.12	0.57	-2.61×10^{-3}
Taylor Rule - Output	4.005	5.56	5.52	5.32	-7.74×10^{-2}
High IACs ($a^K = a^H = 10$)					
Simple Taylor Rule	0.058	0.86	0.58	1.29	-2.81×10^{-3}
Taylor Rule - Output	4.300	5.77	5.77	6.12	-8.41×10^{-2}
B: Standard New Keynesian Model					
Baseline					
Simple Taylor Rule	0.011	0.79	0.64	1.19	—
Taylor Rule - Output	3.189	7.99	8.01	8.30	—
High IAC ($a^K = 10$)					
Simple Taylor Rule	0.029	1.09	0.73	1.63	—
Taylor Rule - Output	3.042	7.83	7.87	8.29	—

Note: i) For definition of welfare cost, see appendix. ii) σ^x represents the standard deviation of x. σ^π, σ^R denote on annual rates. iii) $E(\hat{c})$ represents unconditional mean of percentage deviation of c from deterministic steady state. $E(\hat{\gamma}^Y)$ denotes unconditional mean of percentage deviation of γ^Y from deterministic steady state on annual rate.

however, implies unclear correlations across countries. In our model, their relationship depends on the monetary policy rules. We here consider the following rule,

$$\log\left(\frac{R_t}{R^*}\right) = \alpha^\pi \log\left(\frac{\pi_t}{\pi^*}\right) + \alpha^{\pi^{W^{ob}}} \log\left(\frac{\pi_t^{W^{ob}}}{\pi^{W^{ob*}}}\right), \tag{51}$$

where we define *the observable nominal wage inflation,* $\pi_t^{W^{ob}}$, as

$$\pi_t^{W^{ob}} \equiv \frac{W_t H_t}{W_{t-1}H_{t-1}} = \pi_t \gamma_t^H \frac{w_t}{w_{t-1}}.$$

Figure 2 shows that the relationship between policy coefficients which govern response to price- and wage-inflation and the moments of relavant endogenous variables. When $\alpha^{\pi^{W^{ob}}}$ is high, long-run growth has negative relationship to inflation volatility. However, When $\alpha^{\pi^{W^{ob}}}$ is relatively low, the two has positive relationship because then the higher α^π is, the smaller $\sigma(\pi)$ is but the greater volatility of wage inflation $\sigma(\pi^{W^{ob}})$ is. Therefore, the existence of wage stickiness cause either positive or negative relationship between inflation volatility and growth.

Table 4: Optimal Operational Monetary Policy Rules

	Policy Coefficients				σ_π	$\sigma_{\pi^{W^{ob}}}$	σ_R	$E(\hat{\gamma}^Y)$
	α^π	α^{γ^Y}	$\alpha^{\pi^{W^{ob}}}$	α^R				
A: Endogenous Growth New Keynesian Model								
Baseline	1.70	0.00	0.89	0.00	0.28	0.42	0.27	1.7×10^{-4}
High IACs $a^K = a^H = 10$	6.52	0.00	6.28	0.00	0.41	0.34	1.30	-1.1×10^{-3}
B: Standard New Keynesian Model								
Baseline	2.70	0.00	0.15	0.04	0.52	0.57	1.39	—
High IACs $a^K = 10$	3.68	0.00	2.31	0.00	0.59	0.52	1.41	—

Note: i) For definition of policy rule, see equation (52). ii) σ^x represents the standard deviation of x. σ^π, σ^R denote on annual rates. iii) $E(\hat{c})$ represents unconditional mean of percentage deviation of c from deterministic steady state. $E(\hat{\gamma}^Y)$ denotes unconditional mean of percentage deviation of γ^Y from deterministic steady state on annual rate.

5 Optimal Operational Monetary Policy Rules

5.1 Definition

The operational monetary policy rules are defined as the rules satisfying the following four *operational conditions* similar to SGU. First, monetary orthority must set nominal interest rate according to the following interest rate feedback rules,

$$\log\left(\frac{R_t}{R^*}\right) = \alpha^\pi \log\left(\frac{\pi_t}{\pi^*}\right) + \alpha^{\gamma^Y} \log\left(\frac{\gamma_t^Y}{\gamma^{Y*}}\right) + \alpha^{\pi^{W^{ob}}} \log\left(\frac{\pi_t^{W^{ob}}}{\pi^{W^{ob*}}}\right) + \alpha^R \log\left(\frac{R_{t-1}}{R^*}\right), \tag{52}$$

Second, coefficients of the interest rate rules, α^π, α^{γ^Y}, $\alpha^{\pi^{W^{ob}}}$, and α^R, must be in the ranges of $[0, 10]$. Third, The equilibrium must be uniquely determined, that is, equilibrium under operational policy must not have fluctuation driven by agents' expectations. Fourth, The standard deviation of nominal rate of interest must be less than a half of steady state value of nominal interest rate, that is, $2\sigma_R < \log R^*$, that is, the possibilities that nominal interest rate hits its zero-lower bound must be kept to be low.

In this section, we consider optimal operational monetary policy rules of our model. Assume that the economy is in the deterministic steady state at the beginning of period 0. The optimal policy (α^π α^{γ^Y} $\alpha^{\pi^{W^{ob}}}$ α^R) maximizes the welfare measure conditional on the deterministic steady state, v_t, defined as

$$v_t = \frac{\beta}{1-\beta} \log \gamma_{t+1}^H + \log(c_t - bc_{t-1}/\gamma_t^H) + \psi \log(1-n_t) + \beta E_t v_{t+1}, \quad (53)$$

subject to the operational conditions defined above. This conditional welfare measure is derived from the utility function of representative household. The details are in appendix. We analyze the optimal policy under baseline structural parameter values.

Moreover, we compute the optimal policy under the high investment adjustment cost case, $a^K = a^H = 10$. The reasons is as follows. First, we calibrate the parameters which govern the degrees of investment adjustment cost to match the ratio of volatility of physical capital investment and the ratio of volatility of broad consumption to the volatility of output. However, we have not empirical evidence enough to estimate the degrees of investment adjustment cost sharply, especially human capital investment adjustment cost. Hence, we would have to do sensitivity analysis about their parameter. Second, more importantly, investment adjustment costs theirselves have the growth effect of investment volatility. Barlevy (2004b) shows that in the endogenous growth models with concave capital production technologies, equivalently convex investment adjustment costs, investment volatilities lower long-run growth even if unconditional means of investment levels unchange. Therefore, if the model has a negative relationship between price- or wage-stabilization and investment stabilization, Optimal policy rules may respond to output when Barlevy's effect is strong. The socond reason is that we would like to investigate whether optimal monetary policy is changed by Barlevy's effect.

5.2 Results

Our numerical results are shown in Table 4. Under the baseline calibration, the response of optimal monetary policy rules to price and wage inflation are positive, and the one to output growth is mute. This result is similar to the exogenous growth model with sticky price and wage. SGU and Schmitt-Grohe and Uribe (2007) show that optimal operational monetary policy should respond to inflation and should not respond to output or output growth. The reason is that the larger policy coeffcient with respect to output growth, α^{γ^Y}, is, the greater volatility of inflation is so that the distortion of price dispersion becomes higher. We see the fact that this intuition is right by the analogy to Table 3.

Our second finding is that optimal monetary policy virtually does *not* depends on the degree of investment adjustment cost. Even if the degree of

Table 5: Growth-Maximizing Monetary Policy Rules

	Policy Coefficients				Welfare	σ_π	$\sigma_{\pi^{Wob}}$	σ_R	$E(\hat{\gamma}^Y)$
	α^π	α^{γ^Y}	$\alpha^{\pi^{Wob}}$	α^R	Cost $(100 \times \Lambda^c)$				
Baseline	1.37	0.00	0.83	0.00	4.4×10^{-4}	0.25	0.46	0.26	1.9×10^{-4}
High IACs $a^K = a^H = 10$	5.27	0.00	4.88	0.00	8.4×10^{-5}	0.43	0.33	1.27	-1.1×10^{-3}

Note: i) For definition of policy rule, see equation (52). ii) σ^x represents the standard deviation of x. σ^π, σ^R denote on annual rates. iii) $E(\hat{c})$ represents unconditional mean of percentage deviation of c from deterministic steady state. $E(\hat{\gamma}^Y)$ denotes unconditional mean of percentage deviation of γ^Y from deterministic steady state on annual rate.

adjustment costs are extremely high, $a^K = a^H = 10$, the features of optimal policy regime that monetary authority should respond to price- and wage-inflation and not to real activity does not change.

These two findings suggest that growth effect itself have only weak tradeoff between price- and wage- inflation stabilization descrived in the previous subsection because optimal policy rules feature only price- and wage-stabilization. In order to ensure that growth effect of investment adjustment costs, we do an exercise seeking operational monetary policy rules maximize the unconditional mean of output growth. The result represents in Table. 5. We find that the growth-maximizing policy rule responds only to price- and wage-inflation and not to output growth and this rule atteins virtually identical long-run output growth rate to that of optimal policy rule. This result suggests that our conclusion that Barlevy's effect has not strong tradeoff between nominal and investment stabilization would be right.

6 Conclusion

This paper analyses the effects of monetary policy in an endogenous growth model. In this paper, we obtain some positive and normative implications about inflation and growth. First, in steady state, sticky price distortion have the negative growth and welfare effect and this effect is highly sensitive to the degree of price stickiness. This sensitivity may account for the difference of growth rate across countries. Of cource, price stickiness would change along the economic environment and time so that we need further theoretical and empirical studies about price stickiness.

Second, in stochastic environment, the long-run rate of output growth is affected by the monetary stabilization policy rules. Especially, policy rule responds to output lower the long-run growth rate strongly. The empirical study by Orphanides (2003) suggests the possiblities that output response of monetary policy in *the Great Inflation* period brings stagflation. Our numerical result is consistent with his findings. The simulation analysis using our model about the periods would be useful to understand the mechanism of stagflation.

Third, The effect of volatility of inflation on long-run growth is not clear. This result is consistent with empirical evidence. The source of unclearness in our model is the existence of wage stickiness. Our conclusion should be tested by an empirical exercise about the relationship between inflation volatility and growth, in addition to wage volatility.

Fourth, in spite of above various results that growth and nominal variables, the features of the optimal operational policy is not turned from exogenous growth New Keynesian models. Our numerical exercises demonstrate that resolution of price- and wage- tradeoff virtually maximize the long-run growth. This result implies that the tradeoff between stabilization and growth does not exist or is weak. This conclusion is also very similar to Blackburn and Pelloni (2005). They show that in an endogenous growth model with sticky wage, growth-maximizing policy is optimal. Our result implies that growth is important from positive perspective but monetary authorities do not need to consider growth in practice.

Finally, note that because of complexity of the model, we do not find accurate mechanism of growth effect. Even in the two-capital convex model of endogenous growth without nominal rigidities developed by Jones et al. (2005b), they does not find exact mechanism about growth effect of uncertainty. We should analyze the mechanism of growth effect of nominal rigidities more analytically, even in simpler model such as AK model with single nominal rigidity.

Appendix A: Equilibrium Conditions

Equilibrium conditions

$$\Lambda_t[1+\nu^h(1-R_t^{-1})] = (C_t - bC_{t-1})^{-\sigma}(1-n_t)^{\psi(1-\sigma)} - \beta b E_t (C_{t+1} - bC_t)^{-\sigma}(1-n_{t+1})^{\psi(1-\sigma)}$$

$$\frac{1}{R_t} = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t \pi_{t+1}}$$

$$Y_t = C_t + I_t^K + I_t^H$$

$$\theta X_t^1 = (\theta - 1) X_t^2$$

$$X_t^1 = \tilde{p}_t^{-\theta-1} Y_t mc_t + \xi_p \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \left(\frac{\pi_t^{\chi}}{\pi_{t+1}}\right)^{-\theta} \left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{-\theta-1} X_{t+1}^1$$

$$X_t^2 = \tilde{p}_t^{-\theta} Y_t + \xi_p \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \left(\frac{\pi_t^{\chi}}{\pi_{t+1}}\right)^{1-\theta} \left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{-\theta} X_{t+1}^2$$

$$1 = (1-\xi_p)\tilde{p}_t^{1-\theta} + \xi_p \left(\frac{\pi_{t-1}^{\chi}}{\pi_t}\right)^{1-\theta}$$

$$A_t K_t^{\alpha} (n_t^d)^{1-\alpha} H_t^{1-\alpha} = Y_t s_t$$

$$s_t = (1-\xi_p)\tilde{p}_t^{-\theta} + \xi_p \left(\frac{\pi_{t-1}^{\chi}}{\pi_t}\right)^{-\theta} s_{t-1}$$

$$r_t^K = \alpha A_t K_t^{\alpha-1} (n_t^d)^{1-\alpha} H_t^{1-\alpha} mc_t$$

$$w_t[1+\nu^f(1-R_t^{-1})] = (1-\alpha) A_t K_t^{\alpha} (n_t^d)^{-\alpha} H_t^{-\alpha} mc_t$$

$$F_t^1 = \frac{\tilde{\theta}-1}{\tilde{\theta}} \Lambda_t n_t^d H_t \left(\frac{\tilde{w}_t}{w_t}\right)^{-\tilde{\theta}} \tilde{w}_t + \beta \xi_w E_t \left(\frac{\tilde{w}_t}{\tilde{w}_{t+1}}\right)^{1-\tilde{\theta}} \left(\frac{\pi_t^{\tilde{\chi}}}{\pi_{t+1}}\right)^{1-\tilde{\theta}} F_{t+1}^1$$

$$F_t^2 = n_t^d \left(\frac{\tilde{w}_t}{w_t}\right)^{-\tilde{\theta}} \psi (C_t - bC_{t-1})^{1-\sigma}(1-n_t)^{\psi(1-\sigma)-1} + \beta \xi_w E_t \left(\frac{\tilde{w}_t}{\tilde{w}_{t+1}}\right)^{-\tilde{\theta}} \left(\frac{\pi_t^{\tilde{\chi}}}{\pi_{t+1}}\right)^{-\tilde{\theta}} F_{t+1}^2$$

$$F_t^1 = F_t^2$$

$$n_t = n_t^d \tilde{s}_t$$

$$\tilde{s}_t = (1-\xi_w)\left(\frac{\tilde{w}_t}{w_t}\right)^{-\tilde{\theta}} + \xi_w \left(\frac{w_{t-1}}{w_t}\right)^{-\tilde{\theta}} \left(\frac{\pi_{t-1}^{\tilde{\chi}}}{\pi_t}\right)^{-\tilde{\theta}} \tilde{s}_{t-1}$$

$$w_t^{1-\tilde{\theta}} = (1-\xi_w)\tilde{w}_t^{1-\tilde{\theta}} + \xi_w \left(\frac{\pi_{t-1}^{\tilde{\chi}}}{\pi_t}\right)^{1-\tilde{\theta}} w_{t-1}^{1-\tilde{\theta}}$$

$$M_t = \nu^h C_t + \nu^f w_t n_t^d H_t$$

$$K_{t+1} = (1-\delta_K)K_t + I_t^K - \frac{a_K^F}{2}\left(\frac{I_t^K}{I_{t-1}^K} - \eta_K^F\right)^2 I_t^K$$

$$H_{t+1} = (1-\delta_H)H_t + I_t^H - \frac{a_H^F}{2}\left(\frac{I_t^H}{I_{t-1}^H} - \eta_H^F\right)^2 I_t^H$$

$$\Lambda_t = \Lambda_t q_t^K \left[1 - \frac{a_K^F}{2}\left(\frac{I_t^K}{I_{t-1}^K} - \eta_K^F\right)^2 - a_K^F \left(\frac{I_t^K}{I_{t-1}^K} - \eta_K^F\right) \frac{I_t^K}{I_{t-1}^K}\right] + \beta E_t \Lambda_{t+1} q_{t+1}^K a_K^F \left(\frac{I_{t+1}^K}{I_t^K} - \eta_K^F\right)\left(\frac{I_{t+1}^K}{I_t^K}\right)^2$$

$$\Lambda_t = \Lambda_t q_t^H \left[1 - \frac{a_H^F}{2}\left(\frac{I_t^H}{I_{t-1}^H} - \eta_H^F\right)^2 - a_H^F\left(\frac{I_t^H}{I_{t-1}^H} - \eta_H^F\right)\frac{I_t^H}{I_{t-1}^H}\right] + \beta E_t \Lambda_{t+1} q_{t+1}^H a_H^F \left(\frac{I_{t+1}^H}{I_t^H} - \eta_H^F\right)\left(\frac{I_{t+1}^H}{I_t^H}\right)^2$$

$$\Lambda_t q_t^K = \beta E_t \Lambda_{t+1}[r_{t+1}^K + q_{t+1}^K(1-\delta_K)]$$

$$\Lambda_t q_t^H = \beta E_t \Lambda_{t+1}[w_{t+1} n_{t+1}^d + q_{t+1}^H(1-\delta_H)]$$

Equilibrium conditions written by stationary variables

$$\gamma_{t+1}^H k_{t+1} = (1-\delta_K)k_t + i_t^K - \frac{a_K^F}{2}\left(\frac{\gamma_t^H i_t^K}{i_{t-1}^K} - \eta_K^F\right)^2 i_t^K$$

$$\gamma_{t+1}^H = 1 - \delta_H + i_t^H - \frac{a_H^F}{2}\left(\frac{\gamma_t^H i_t^H}{i_{t-1}^H} - \eta_H^F\right)^2 i_t^H$$

$$\lambda_t = \lambda_t q_t^K \left[1 - \frac{a_K^F}{2}\left(\frac{\gamma_t^H i_t^K}{i_{t-1}^K} - \eta_K^F\right)^2 - a_K^F\left(\frac{\gamma_t^H i_t^K}{i_{t-1}^K} - \eta_K^F\right)\frac{\gamma_t^H i_t^K}{i_{t-1}^K}\right]$$
$$+ \beta E_t(\gamma_{t+1}^H)^{-\sigma}\lambda_{t+1} q_{t+1}^K a_K^F \left(\frac{\gamma_{t+1}^H i_{t+1}^K}{i_t^K} - \eta_K^F\right)\left(\frac{\gamma_{t+1}^H i_{t+1}^K}{i_t^K}\right)^2$$

$$\lambda_t = \lambda_t q_t^H \left[1 - \frac{a_H^F}{2}\left(\frac{\gamma_t^H i_t^H}{i_{t-1}^H} - \eta_H^F\right)^2 - a_H^F\left(\frac{\gamma_t^H i_t^H}{i_{t-1}^H} - \eta_H^F\right)\frac{\gamma_t^H i_t^H}{i_{t-1}^H}\right]$$
$$+ \beta E_t(\gamma_{t+1}^H)^{-\sigma}\lambda_{t+1} q_{t+1}^H a_H^F \left(\frac{\gamma_{t+1}^H i_{t+1}^H}{i_t^H} - \eta_H^F\right)\left(\frac{\gamma_{t+1}^H i_{t+1}^H}{i_t^H}\right)^2$$

$$\lambda_t[1 + \nu^h(1 - R_t^{-1})] = (c_t - bc_{t-1}/\gamma_t^H)^{-\sigma}(1-n_t)^{\psi(1-\sigma)} - \beta b E_t(\gamma_{t+1}^H c_{t+1} - bc_t)^{-\sigma}(1-n_{t+1})^{\psi(1-\sigma)}$$

$$\lambda_t q_t^K = \beta E_t(\gamma_{t+1}^H)^{-\sigma}\lambda_{t+1}[r_{t+1}^K + q_{t+1}^K(1-\delta_K)]$$

$$\lambda_t q_t^H = \beta E_t(\gamma_{t+1}^H)^{-\sigma}\lambda_{t+1}[w_{t+1} n_{t+1}^d + q_{t+1}^H(1-\delta_H)]$$

$$\frac{1}{R_t} = \beta E_t \frac{(\gamma_{t+1}^H)^{-\sigma}\lambda_{t+1}}{\lambda_t \pi_{t+1}}$$

$$y_t = c_t + i_t^K + i_t^H$$

$$\theta x_t^1 = (\theta - 1)x_t^2$$

$$x_t^1 = \tilde{p}_t^{-\theta-1} y_t mc_t + \xi_p \beta E_t \frac{\lambda_{t+1}}{\lambda_t}(\gamma_{t+1}^H)^{1-\sigma}\left(\frac{\pi_t^\chi}{\pi_{t+1}}\right)^{-\theta}\left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{-\theta-1} x_{t+1}^1$$

$$x_t^2 = \tilde{p}_t^{-\theta} y_t + \xi_p \beta E_t \frac{\lambda_{t+1}}{\lambda_t}(\gamma_{t+1}^H)^{1-\sigma}\left(\frac{\pi_t^\chi}{\pi_{t+1}}\right)^{1-\theta}\left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{-\theta} x_{t+1}^2$$

$$1 = (1-\xi_p)\tilde{p}_t^{1-\theta} + \xi_p\left(\frac{\pi_{t-1}^\chi}{\pi_t}\right)^{1-\theta}$$

$$A_t k_t^\alpha (n_t^d)^{1-\alpha} = y_t s_t$$

$$s_t = (1-\xi_p)\tilde{p}_t^{-\theta} + \xi_p\left(\frac{\pi_{t-1}^\chi}{\pi_t}\right)^{-\theta} s_{t-1}$$

$$r_t^K = \alpha A_t k_t^{\alpha-1}(n_t^d)^{1-\alpha} mc_t$$

$$w_t[1 + \nu^f(1 - R_t^{-1})] = (1-\alpha)A_t k_t^\alpha (n_t^d)^{-\alpha} mc_t$$

$$f_t^1 = \frac{\tilde{\theta}-1}{\tilde{\theta}}\lambda_t n_t^d \left(\frac{\tilde{w}_t}{w_t}\right)^{-\tilde{\theta}} \tilde{w}_t + \beta\xi_w E_t \left(\frac{\tilde{w}_t}{\tilde{w}_{t+1}}\right)^{1-\tilde{\theta}} \left(\frac{\pi_t^{\tilde{\chi}}}{\pi_{t+1}}\right)^{1-\tilde{\theta}} (\gamma_{t+1}^H)^{1-\sigma} f_{t+1}^1$$

$$f_t^2 = n_t^d \left(\frac{\tilde{w}_t}{w_t}\right)^{-\tilde{\theta}} \psi(c_t - bc_{t-1}/\gamma_t^H)^{1-\sigma}(1-n_t)^{\psi(1-\sigma)-1} + \beta\xi_w E_t \left(\frac{\tilde{w}_t}{\tilde{w}_{t+1}}\right)^{-\tilde{\theta}} \left(\frac{\pi_t^{\tilde{\chi}}}{\pi_{t+1}}\right)^{-\tilde{\theta}} (\gamma_{t+1}^H)^{1-\sigma} f_{t+1}^2$$

$$f_t^1 = f_t^2$$

$$n_t = n_t^d \tilde{s}_t$$

$$\tilde{s}_t = (1-\xi_w)\left(\frac{\tilde{w}_t}{w_t}\right)^{-\tilde{\theta}} + \xi_w \left(\frac{w_{t-1}}{w_t}\right)^{-\tilde{\theta}} \left(\frac{\pi_{t-1}^{\tilde{\chi}}}{\pi_t}\right)^{-\tilde{\theta}} \tilde{s}_{t-1}$$

$$w_t^{1-\tilde{\theta}} = (1-\xi_w)\tilde{w}_t^{1-\tilde{\theta}} + \xi_w \left(\frac{\pi_{t-1}^{\tilde{\chi}}}{\pi_t}\right)^{1-\tilde{\theta}} w_{t-1}^{1-\tilde{\theta}}$$

$$m_t = \nu^h c_t + \nu^f w_t n_t^d$$

$$\log\left(\frac{R_t}{\bar{R}}\right) = \alpha_\pi \log\left(\frac{\pi_t}{\bar{\pi}}\right) + \alpha_{\gamma_Y} \log\left(\frac{\gamma_t^Y}{\bar{\gamma}^Y}\right) + \alpha_R \log\left(\frac{R_{t-1}}{\bar{R}}\right)$$

$$\gamma_t^Y = \gamma_t^H \frac{y_t}{y_{t-1}}$$

Deterministic steady state

$$(\gamma^H - 1 + \delta_K)k = i^K \left[1 - \frac{a_K^F}{2}(\gamma^H - \eta_K^F)^2\right]$$

$$\gamma^H = 1 - \delta_H + i^H \left[1 - \frac{a_H^F}{2}(\gamma^H - \eta_H^F)^2\right]$$

$$1 = q^K \left[1 - \frac{a_K^F}{2}(\gamma^H - \eta_K^F)^2 - a_K^F(\gamma^H - \eta_K^F)\gamma^H\right] + \beta(\gamma^H)^{2-\sigma} q^K a_K^F(\gamma^H - \eta_K^F)$$

$$1 = q^H \left[1 - \frac{a_H^F}{2}(\gamma^H - \eta_H^F)^2 - a_H^F(\gamma^H - \eta_H^F)\gamma^H\right] + \beta(\gamma^H)^{2-\sigma} q^H a_H^F(\gamma^H - \eta_H^F)$$

$$\lambda[1 + \nu^h(1 - R^{-1})] = c^{-\sigma}(1-n)^{\psi(1-\sigma)}(\gamma^H - b)^{-\sigma}[(\gamma^H)^\sigma - \beta b]$$

$$q^K = \beta(\gamma^H)^{-\sigma}[r^K + q^K(1-\delta_K)]$$

$$q^H = \beta(\gamma^H)^{-\sigma}[wn^d + q^H(1-\delta_H)]$$

$$R = \frac{(\gamma^H)^\sigma \pi}{\beta}$$

$$y = c + i^K + i^H$$

$$x^1 = \tilde{p}^{-\theta-1} ymc + \xi_p \beta(\gamma^H)^{1-\sigma}\pi^{-\theta(\chi-1)}x^1$$

$$x^2 = \tilde{p}^{-\theta} y + \xi_p \beta(\gamma^H)^{1-\sigma}\pi^{(1-\theta)(\chi-1)}x^2$$

$$\theta x^1 = (\theta-1)x^2$$

$$1 = (1-\xi_p)\tilde{p}^{1-\theta} + \xi_p \pi^{(1-\theta)(\chi-1)}$$

$$\bar{A}k^\alpha (n^d)^{1-\alpha} = ys$$

$$s = (1-\xi_p)\tilde{p}^{-\theta} + \xi_p \pi^{-\theta(\chi-1)} s$$
$$r^K = \frac{\alpha ysmc}{k}$$
$$w[1+\nu^f(1-R^{-1})] = \frac{(1-\alpha)ysmc}{n^d}$$
$$f^1 = \frac{\tilde{\theta}-1}{\tilde{\theta}} \lambda n^d \left(\frac{\tilde{w}}{w}\right)^{-\tilde{\theta}} \tilde{w} + \beta\xi_w \pi^{(\tilde{\theta}-1)(1-\tilde{\chi})} (\gamma^H)^{1-\sigma} f^1$$
$$f^2 = n^d \left(\frac{\tilde{w}}{w}\right)^{-\tilde{\theta}} \psi c^{1-\sigma} (\gamma^H)^{\sigma-1} (\gamma^H - b)^{1-\sigma} (1-n)^{\psi(1-\sigma)-1} + \beta\xi_w \pi^{\tilde{\theta}(1-\tilde{\chi})} (\gamma^H)^{1-\sigma} f^2$$
$$f^1 = f^2$$
$$n = n^d \tilde{s}$$
$$\tilde{s} = (1-\xi_w)\left(\frac{\tilde{w}}{w}\right)^{-\tilde{\theta}} + \xi_w \pi^{\tilde{\theta}(1-\tilde{\chi})} \tilde{s}$$
$$w^{1-\tilde{\theta}} = (1-\xi_w)\tilde{w}^{1-\tilde{\theta}} + \xi_w \pi^{(\tilde{\theta}-1)(1-\tilde{\chi})} w^{1-\tilde{\theta}}$$
$$\gamma^Y = \gamma^H$$
$$m = \nu^h c + \nu^f w n^d$$

Appendix B: Welfare Cost Measure

CRRA case

$$\begin{aligned} V_t &\equiv E_t \sum_{s=0}^{\infty} \beta^s \frac{(C_{t+s} - bC_{t+s-1})^{1-\sigma}(1-n_{t+s})^{\psi(1-\sigma)}}{1-\sigma} \\ &= \frac{(C_t - bC_{t-1})^{1-\sigma}(1-n_t)^{\psi(1-\sigma)}}{1-\sigma} + \beta E_t V_{t+1} \end{aligned}$$

Dividing it by $H_t^{1-\sigma}$ and defining $v_t = \frac{V_t}{H_t^{1-\sigma}}$, we rewrite the welfare function as the recursive formulation by the stationary variables:

$$v_t = \frac{(c_t - bc_{t-1}/\gamma_t^H)^{1-\sigma}(1-n_t)^{\psi(1-\sigma)}}{1-\sigma} + \beta(\gamma_{t+1}^H)^{1-\sigma} E_t v_{t+1}.$$

log case ($\sigma = 1$)

$$\begin{aligned} V_t &\equiv E_t \sum_{s=0}^{\infty} \beta^s [\log(C_{t+s} - bC_{t+s-1}) + \psi \log(1-n_{t+s})] \\ &= \frac{1}{1-\beta}\log H_t + E_t \sum_{s=0}^{\infty} \left[\frac{\beta}{1-\beta}\log \gamma_{t+1+s}^H + \log(c_{t+s} - bc_{t+s-1}/\gamma_{t+s}^H) + \psi \log(1-n_{t+s})\right] \end{aligned}$$

Defining v_t as:

$$v_t \equiv V_t - \frac{1}{1-\beta} \log H_t,$$

we obtain the recursive formulation:

$$v_t = \frac{\beta}{1-\beta} \log \gamma_{t+1}^H + \log(c_t - bc_{t-1}/\gamma_t^H) + \psi \log(1-n_t) + \beta E_t v_{t+1}.$$

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