Ranking Supply Function and Cournot Equilibria in a Differentiated Product Duopoly with Demand Uncertainty

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Abstract. In this paper, we provide a welfare ranking for the equilibria of the supply function and quantity competitions in a differentiated product duopoly with demand uncertainty. We prove that the expected consumer surplus is always higher under the supply function competition. By numerical simulations, we also show that if the degree of product substitution is extremely low, then the supply function competition can become a superior form of competition for the duopolistic producers, as well. However, if the degree of product substitution is not extremely low, then the expected producer profits under the supply function competition can be lower than under the quantity competition in situations where the size of the demand uncertainty is below a critical level. We find that this critical level is non-decreasing in the degree of product substitution, while non-increasing both in the marginal cost of producing a unit output and in the own-price sensitivity of each inverse demand curve. Our results imply that in electricity markets with differentiated products, the regulators should not intervene to impose the quantity competition in favor of the supply function competition unless the degree of product substitution is sufficiently high and the predicted demand fluctuations are sufficiently small.

Keywords: Supply function competition; Cournot competition; duopoly; differentiated products; uncertainty

JEL Codes: D43; L13

1 Introduction

The supply function competition that was originally developed by Grossman (1981) could find applications in oligopolistic industries only after Klemperer and Meyer (1989), who eliminated the problems with the multiplicity of supply function equilibria by introducing an exogenous uncertainty about the demand functions faced by oligopolists. Definitely, the best known application has been observed in the deregulated electricity markets, where the supply function competition can model the strategic interplay among power generators more successfully than the price competition of Bertrand (1883) and the quantity competition of Cournot (1838) (see, for example, Green and Newbery 1992, Rudkevich and Duckworth 1998, and Rudkevich et al. 1998). Since Klemperer and Meyer (1989), an appraisal of the supply function competition in terms of the expected welfares of producers and/or consumers has been made both in the absence and the presence of demand uncertainty.

In the absence of demand uncertainty, the welfare analysis of Klemperer and Meyer (1989) concluded that profits of oligopolistic producers under the supply function competition are intermediate between Cournot and Bertrand competition profits. This conclusion was recently challenged by Delbono and Lambertini (2016), who showed that with quadratic costs price competition yields multiple equilibria, which can be separated into three groups depending upon whether the profits under the price competition are (i) above the profits under the quantity competition, (ii) below the profits under the supply function competition, or (iii) in between the profits under the quantity and supply function competitions. Delbono and Lambertini (2016) also showed that at any price competition equilibrium within the first group mentioned above the social welfare would be even lower than at the quantity competition equilibrium. On the other hand, Monden (2017) showed that in a vertical market where an upstream firm sequentially contract with two downstream firms the social welfare under the supply function competition may be lower than under the quantity and price competitions.

In the presence of demand uncertainty, a welfare comparison between the quantity and supply function competitions was very recently made by Saglam (2018), who showed that in an oligopolistic industry with a homogeneous product the supply function competition is always ex-ante superior to the quantity competition from the viewpoint of consumers independent of the size of the demand uncertainty. Saglam (2018) also found that if the demand uncertainty in the industry is sufficiently large with respect to the number of firms, the size of the product markets, and the marginal cost of the unit output, then the supply function competition can be ex-ante more desirable for the producers, as well. In this paper, we study the question as to whether the results of Saglam (2018) extend to the case of differentiated products. We believe that this question is important especially for the analysis of electricity markets where not only the supply function competition has a wide application but also the product differentiation, as recently argued in a comprehensive survey of Woo et al. (2014), can be considered as a very meaningful concept.[1]

[1] Woo et al. (2014) support their argument by pointing to several distinct attributes of electricity –such as quality, reliability, time of use, consumption volume, maximum demand, and environmental impact– that can be packaged at alternative proportions.

We answer our research question by borrowing from Klemperer and Meyer (1989) a duopolistic industry setting with product differentiation and demand uncertainty. For this setting we characterize the unique and symmetric Nash (1950) equilibrium obtained under the supply function and quantity competitions, and show that the expected consumer surplus under the supply function competition is always higher than under the quantity competition independent of the size of the demand uncertainty and any other attributes of the industry. By numerical simulations, we also show that if the degree of product substitution is extremely low, then the supply function competition can become a superior form of competition for the duopolistic producers, as well. However, if the degree of product substitution is not extremely low, then the expected producer profits under the supply function competition can be lower than under the quantity competition in situations where the size of the demand uncertainty is below a critical level. We find that this critical level is non-decreasing in the degree of product substitution, while non-increasing both in the marginal cost of producing a unit output and in the own-price sensitivity of each inverse demand curve.

The rest of the paper is organized as follows: Section 2 introduces a duopoly model with differentiated products and demand uncertainty. Section 3 presents the results and Section 4 concludes.

2 Model

Borrowing from Klemperer and Meyer (1989), we consider a duopolistic industry, where each firm produces a differentiated product under demand uncertainty and faces a continuum of identical consumers. Each firm producing a quantity of output $q \geq 0$ incurs the cost

$$C(q) = c\, q^2/2 \tag{1}$$

with $c > 0$. On the other hand, the representative consumer maximizes the utility

$$U(q_1, q_2) - \sum_{i=1}^{2} p_i q_i, \tag{2}$$

where $q_i \geq 0$ denotes the quantity produced by firm $i = 1, 2$ and $p_i \geq 0$ denotes the price of its product. It is assumed that the function U is given by

$$U(q_1, q_2) = \alpha(q_1 + q_2) - \frac{1}{2}(\beta q_1^2 + 2\gamma q_1 q_2 + \beta q_2^2), \tag{3}$$

where $\beta > \gamma > 0$. Because of the assumption $\gamma > 0$, the products of the two firms are always substitutes. Moreover, in the above function α is a scalar random variable representing an ex-ante unobservable shock to the utility of consumers. This shock variable is distributed with density $f(\alpha)$ that is strictly positive everywhere on the support $[0, \infty)$. Given this density distribution, let $E[\alpha]$ and $\sigma(\alpha)$ respectively denote the mean and the standard deviation of α.

Given (2) and (3), the solution to the maximization problem of the representative consumer yields the inverse demands $P_i(q_i, q_j),\ i = 1, 2$, satisfying

$$P_i(q_i, q_j) = \alpha - \beta\, q_i - \gamma\, q_j. \tag{4}$$

In the region of quantity space where the prices p_i and p_j of firms i and j are non-negative, the demand curve of firm i can then be derived as follows:

$$D_i(p_i, p_j) = a - b\, p_i + g\, p_j, \tag{5}$$

where $a = \alpha/(\beta + \gamma)$, $b = \beta/(\beta^2 - \gamma^2)$, and $g = \gamma/(\beta^2 - \gamma^2)$. It is assumed that the form of the cost, demand and inverse demand curves, the parameters c, β, γ, b, and g, the density $f(\alpha)$ and its support are commonly known by both firms.

3 Results

For the industry described above, we will characterize and analyze, in Sections 3.1 and 3.2, the equilibria under the quantity and supply function competitions respectively. Later, in Section 3.3, we will make a welfare comparison between these two forms of competition for consumers and producers.

3.1 Quantity Competition

In this form of competition firms set fixed quantities before production takes place, without knowing the realization of the demand shock. That is, a strategy for firm $i = 1, 2$ is a nonnegative quantity of output, $q_i \in [0, \infty)$. Firms simultaneously determine their strategies to maximize their expected profits. We say that a pair of quantities $\hat{q}_1$ and $\hat{q}_2$ constitutes a Nash equilibrium if for each $i, j \in \{1, 2\}$ with $j \neq i$ the quantity $\hat{q}_i$ maximizes the expected profits of firm i when firm j sticks to the quantity $\hat{q}_j$. That is, the quantity $\hat{q}_i$ solves

$$\max_{q_i \geq 0} E\Big[P_i(q_i, \hat{q}_j) q_i - \frac{c}{2} q_i^2\Big]. \tag{6}$$

Proposition 1. *In the studied duopolistic industry with differentiated products and demand uncertainty, there exists a unique symmetric (Cournot) Nash equilibrium in quantities where each firm chooses the quantity given by*

$$q^C = \left(\frac{1}{2\,\beta\, + \gamma + c}\right) E[\alpha]. \tag{7}$$

Proof. Using (4) at $q_j = \hat{q}_j$, the optimization problem in (6) can be written as

$$\max_{q_i \geq 0} E\Big[(\alpha - \beta\, q_i - \gamma\, \hat{q}_j)\, q_i - \frac{c}{2}\, q_i^2\Big]. \tag{8}$$

Differentiating (8) with respect to q_i we obtain the first-order necessary condition

$$E[\alpha] - 2\,\beta\, q_i - \gamma\, \hat{q}_j - c\, q_i = 0. \tag{9}$$

If the strategy pair $(\hat{q}_i, \hat{q}_j)$ is a Nash equilibrium, then $q_i = \hat{q}_i$ must satisfy the above first order condition. Moreover, if this equilibrium is symmetric we must have $\hat{q}_i = \hat{q}_j \equiv q^C$. Inserting these into (9) we obtain

$$q^C = \left(\frac{1}{2\,\beta\, + \gamma + c}\right) E[\alpha]. \tag{10}$$

Finally, for the problem in (8) the second-order sufficiency condition holds since

$$\frac{\partial^2 E[\pi_i(\alpha)]}{\partial (q_i)^2} = -2\beta - c < 0. \tag{11}$$

So, the choice of quantities (q^C, q^C) constitutes a Nash equilibrium. □

Note that using (4) and (7) we can calculate the equilibrium price at any realization of α as follows:

$$p^C(\alpha) = P_i(q^C, q^C) = \alpha - (\beta + \gamma)\, q^C = \alpha - \left(\frac{\beta + \gamma}{2\,\beta\, + \gamma + c}\right) E[\alpha] \tag{12}$$

Accordingly, given any α the equilibrium profits of each firm become

$$\begin{aligned} \pi^C(\alpha) &= p^C(\alpha)\, q^C - \frac{c}{2}\,(q^C)^2 = \alpha\, q^C - (\beta + \gamma)(q^C)^2 - \frac{c}{2}\,(q^C)^2 \\ &= \alpha \left(\frac{1}{2\,\beta\, + \gamma + c}\right) E[\alpha] - \left(\beta + \gamma + \frac{c}{2}\right) \left(\frac{1}{2\,\beta\, + \gamma + c}\right)^2 (E[\alpha])^2. \end{aligned} \tag{13}$$

It follows that the expected equilibrium profits of each firm are given by

$$E[\pi^C(\alpha)] = \left(\beta + \frac{c}{2}\right) \left(\frac{1}{2\,\beta\, + \gamma + c}\right)^2 (E[\alpha])^2. \tag{14}$$

Below, we show that the expected profits of each firm is always decreasing in the own-price sensitivity of the inverse demand curve of each firm (β), the degree of substitution between the firms' products (γ), and the marginal cost of producing a unit output for each firm (c).

Corollary 1. *The expected producer profits $E[\pi^C(\alpha)]$ are always decreasing in β, γ, and c.*

Proof. Differentiating (14) with respect to β we obtain

$$\frac{\partial E[\pi^C(\alpha)]}{\partial \beta} = \left(\frac{1}{2\,\beta\,+\gamma+c}\right)^2 \left[1-4\left(\beta+\frac{c}{2}\right)\left(\frac{1}{2\,\beta\,+\gamma+c}\right)\right](E[\alpha])^2$$

$$= \left(\frac{1}{2\,\beta\,+\gamma+c}\right)^2 \left(\frac{\gamma-2\beta-c}{2\,\beta\,+\gamma+c}\right)(E[\alpha])^2 < 0, \tag{15}$$

since $\gamma < \beta$ and $c > 0$ by assumption. Now, differentiating (14) with respect to γ we obtain

$$\frac{\partial E[\pi^C(\alpha)]}{\partial \gamma} = -2\left(\beta+\frac{c}{2}\right)\left(\frac{1}{2\,\beta\,+\gamma+c}\right)^3 (E[\alpha])^2 < 0. \tag{16}$$

Finally, differentiating (14) with respect to c yields

$$\frac{\partial E[\pi^C(\alpha)]}{\partial c} = \left(\frac{1}{2\,\beta\,+\gamma+c}\right)^2 \left[\frac{1}{2}-2\left(\beta+\frac{c}{2}\right)\left(\frac{1}{2\,\beta\,+\gamma+c}\right)\right](E[\alpha])^2$$

$$= \left(\frac{1}{2\,\beta\,+\gamma+c}\right)^2 \left(\frac{\frac{\gamma}{2}-\beta-\frac{c}{2}}{2\,\beta\,+\gamma+c}\right)(E[\alpha])^2 < 0, \tag{17}$$

completing the proof. □

Now, we will consider the welfare of consumers. Using (2), (3), (7), and (12) we can calculate at any α the equilibrium consumer surplus as

$$CS^C(\alpha) = \; = 2\,\alpha\, q^C(\alpha) - (\beta+\gamma)\,\left(q^C(\alpha)\right)^2 - 2\,p^C(\alpha)\,q^C$$

$$= (\beta+\gamma)\,(q^C)^2 = (\beta+\gamma)\left(\frac{1}{2\,\beta\,+\gamma+c}\right)^2 (E[\alpha])^2. \tag{18}$$

Since $CS^C(\alpha)$ is independent of the realization of α, the expected consumer surplus $E[CS^C(\alpha)]$ is equal to $CS^C(\alpha)$.

Corollary 2. *The expected consumer surplus $E[CS^C(\alpha)]$ is always decreasing in c.*

Proof. Since $E[CS^C(\alpha)] = CS^C(\alpha)$, we differentiate (18) with respect to c to obtain

$$\frac{\partial E[CS^C(\alpha)]}{\partial c} = -2\,(\beta+\gamma)\left(\frac{1}{2\,\beta\,+\gamma+c}\right)^3 (E[\alpha])^2 < 0, \tag{19}$$

completing the proof. □

The effects of β and γ on the expected consumer surplus, $E[CS^C(\alpha)]$, are more involving. As we show below, $E[CS^C(\alpha)]$ is decreasing (increasing) in β and γ if and only if the value of c, the marginal cost of producing a unit output, is sufficiently small (large).

Corollary 3. *The expected consumer surplus $E[CS^C(\alpha)]$ is decreasing in β if and only if $c < 2\beta + 3\gamma$ and decreasing in γ if and only if $c < \gamma$.*

Proof. Since $E[CS^C(\alpha)] = CS^C(\alpha)$, we differentiate (18) with respect to β to obtain

$$\begin{aligned}\frac{\partial E[CS^C(\alpha)]}{\partial \beta} &= \left(\frac{1}{2\,\beta\,+\gamma+c}\right)^2\left[1-4\,(\beta+\gamma)\left(\frac{1}{2\,\beta\,+\gamma+c}\right)\right](E[\alpha])^2\\ &= \left(\frac{1}{2\,\beta\,+\gamma+c}\right)^2\left(\frac{c-2\beta-3\gamma}{2\,\beta\,+\gamma+c}\right)(E[\alpha])^2. \qquad (20)\end{aligned}$$

Clearly, $\partial E[CS^C(\alpha)]/\partial\beta < 0$ if and only if $c < 2\beta + 3\gamma$. Now, differentiating (18) with respect to γ we obtain

$$\begin{aligned}\frac{\partial E[CS^C(\alpha)]}{\partial \gamma} &= \left(\frac{1}{2\,\beta\,+\gamma+c}\right)^2\left[1-2\,(\beta+\gamma)\left(\frac{1}{2\,\beta\,+\gamma+c}\right)\right](E[\alpha])^2\\ &= \left(\frac{1}{2\,\beta\,+\gamma+c}\right)^2\left(\frac{c-\gamma}{2\,\beta\,+\gamma+c}\right)(E[\alpha])^2. \qquad (21)\end{aligned}$$

Thus, $\partial E[CS^C(\alpha)]/\partial\gamma < 0$ if and only if $c < \gamma$. □

3.2 Supply Function Competition

In supply function competition, the firms set supply functions before production takes place, i.e., a strategy for firm $j = 1, 2$ specifies a function mapping a non-negative price for its product into a non-negative quantity of output, i.e., $S_j : [0, \infty) \to [0, \infty)$. The firms determine these strategies simultaneously and without observing the realization of the demand shock α. So, for each $i, j \in \{1, 2\}$ with $j \neq i$, if firm j chooses the supply function $S_j(p_j)$, then (ex-post) market clearing in firm j's product market implies

$$S_j(p_j) = a - b\,p_j + g\,p_i. \qquad (22)$$

We assume that if there exist some p_1 and p_2 that clear the product market of each firm, and if these prices are unique, then the actual outputs $S_1(p_1)$ and $S_2(p_2)$ are produced. Otherwise, each firm earns zero profits. Note that the demand intercept a appearing in (22) is a function of α. So, if equation (22) holds, we must have $p_j = \phi_j(p_i, \alpha)$ for some function ϕ_j. Implicitly differentiating $S_j(\phi_j(p_i, \alpha)) = a - b\,\phi_j(p_i, \alpha) + g\,p_i$ with respect to p_i yields

$$\frac{\partial \phi_j(p_i, \alpha)}{\partial p_i} = \frac{g}{S_j'(\phi_j(p_i, \alpha)) + b}. \qquad (23)$$

Substituting $\phi_j(p_i, \alpha)$ for p_j in the demand curve of firm i, we say that for a given α, firm i's profit maximizing price $p_i^{SF}(\alpha)$ solves

$$\max_{p_i} \; p_i \left[a - b\,p_i + g\,\phi_j(p_i, \alpha)\right] - \frac{c}{2}\left[a - b\,p_i + g\,\phi_j(p_i, \alpha)\right]^2. \tag{24}$$

The first-order necessary condition for the above maximization implies

$$[\alpha - b\,p_i + g\,\phi_j(p_i, \alpha)] + p_i \left[-b + g\,\frac{\partial \phi_j(p_i, \alpha)}{\partial p_i}\right] - c\,[a - b\,p_i + g\,\phi_j(p_i, \alpha)] \left[-b + g\,\frac{\partial \phi_j(p_i, \alpha)}{\partial p_i}\right] = 0. \tag{25}$$

A pair of supply functions $\hat{S}_1(p_1)$ and $\hat{S}_1(p_2)$ constitutes a Nash equilibrium if for each $i, j \in \{1, 2\}$ with $j \neq i$ the function $\hat{S}_i(p_i)$ maximizes the expected profits of firm i when firm j sticks to the supply function $\hat{S}_j(p_j)$. In a symmetric equilibrium, $\hat{S}_i(.) = \hat{S}_j(.) \equiv \hat{S}(.)$, and $p_i(\alpha) = p_j(\alpha) \equiv p(\alpha)$ for each α. Inserting these equalities and (23) into (25) yields

$$\hat{S}(p) + p\left[-b + \frac{g^2}{\hat{S}'(p) + b}\right] - c\hat{S}(p)\left[-b + \frac{g^2}{\hat{S}'(p) + b}\right] = 0. \tag{26}$$

Solving for $\hat{S}'(p)$ we can obtain

$$\hat{S}'(p) = -b - \frac{g^2(p - c\,\hat{S}(p))}{\hat{S}(p) - b\,(p - c\,\hat{S}(p))}. \tag{27}$$

Any solution to the above differential equation is a Nash equilibrium in supply functions.

Proposition 2 (Klemperer and Meyer, 1989). *In the studied duopolistic industry with differentiated products and demand uncertainty, there exists a unique and symmetric Nash equilibrium in supply functions where each firm chooses the supply function given by*

$$S^{SF}(p) = \xi p, \tag{28}$$

where

$$\xi = \frac{-\dfrac{(b^2 - g^2)}{b} + \sqrt{\left(\dfrac{b^2 - g^2}{b}\right)^2 + \dfrac{4}{c}\left(\dfrac{b^2 - g^2}{b}\right)\left(1 + \dfrac{1}{b\,c}\right)}}{2\left(1 + \dfrac{1}{b\,c}\right)}. \tag{29}$$

Proof. The proof is available in pages 1267-1269 of Klemperer and Meyer (1989). □

Note that given the equilibrium supply functions in (28)-(29) and given any realization of the demand shock α, we can calculate the market clearing price $p^{SF}(\alpha)$ using the market clearing condition in any market $i = 1, 2$, i.e., $S^{SF}(p^{SF}(\alpha)) = D_i(p^{SF}(\alpha), p^{SF}(\alpha))$, implying $\xi\, p^{SF}(\alpha)(\alpha) = a + (g - b)\, p^{SF}(\alpha)$, further implying

$$p^{SF}(\alpha) = \frac{a}{b - g + \xi} = \frac{\alpha}{1 + \xi(\beta + \gamma)}. \tag{30}$$

Given the equilibrium price in (30), it follows from (28) that under the supply function competition each firm must produce the equilibrium quantity

$$q^{SF}(\alpha) = \frac{\xi\alpha}{1 + \xi(\beta + \gamma)}. \tag{31}$$

Using (30) and (31), the equilibrium profits of each firm can be calculated as

$$\pi^{SF}(\alpha) = p^{SF}(\alpha)\, q^{SF}(\alpha) - \frac{c}{2}\left(q^{SF}(\alpha)\right)^2 = \left(\frac{1}{\xi} - \frac{c}{2}\right)\left(\frac{\xi\,\alpha}{1 + \xi(\beta + \gamma)}\right)^2. \tag{32}$$

Hence, the expected equilibrium profits of each firm become

$$E[\pi^{SF}(\alpha)] \quad = \quad \left(\frac{1}{\xi} - \frac{c}{2}\right)\left(\frac{\xi}{1 + \xi(\beta + \gamma)}\right)^2 E[\alpha^2]. \tag{33}$$

On the other hand, using (2), (3), (30), (31) we can calculate at any α the equilibrium consumer surplus as

$$\begin{aligned} CS^{SF}(\alpha) \quad &= \quad 2\,\alpha\, q^{SF}(\alpha) - (\beta + \gamma)\left(q^{SF}(\alpha)\right)^2 - \frac{2}{\xi}\left(q^{SF}(\alpha)\right)^2 \\ &= \quad (\beta + \gamma)\left(q^{SF}(\alpha)\right)^2 = (\beta + \gamma)\left(\frac{\xi\,\alpha}{1 + \xi\,(\beta + \gamma)}\right)^2 \end{aligned} \tag{34}$$

and the expected equilibrium consumer surplus as

$$E[CS^{SF}(\alpha)] \quad = \quad (\beta + \gamma)\left(\frac{\xi}{1 + \xi\,(\beta + \gamma)}\right)^2 E[\alpha^2]. \tag{35}$$

We observe that under the supply function competition both the expected producer profits and the expected consumer surplus depend on $E[\alpha^2]$. This term can be expressed as

$$E[\alpha^2] = \left(1 + \eta^2\right)(E[\alpha])^2, \tag{36}$$

where

$$\eta = \frac{\sigma(\alpha)}{E[\alpha]}. \tag{37}$$

Note that η is called the coefficient of variation, a unitless measure of the size of demand uncertainty. Also note that the higher the coefficient η, the higher the expected producer profits in (33) and the expected consumer surplus in (35). That is, the size of demand uncertainty positively affects the expected welfares of both producers and consumers under the supply function competition.

Now, we will investigate the welfare effects of the parameter c measuring the marginal cost of producing a unit output. For this, we have to first find how the slope of the equilibrium supply functions is affected by c. Note that using $(b^2 - g^2)/b = 1/\beta$ and $1/b = (\beta^2 - \gamma^2)/\beta$, equation (29) can be rewritten as

$$\xi = \frac{-1 + \sqrt{1 + \frac{4}{c}\left(\beta + \frac{\beta^2 - \gamma^2}{c}\right)}}{2\left(\beta + \frac{\beta^2 - \gamma^2}{c}\right)}. \tag{38}$$

Studying (38), we observe the following.

Lemma 1. *The slope of the equilibrium supply function, ξ, is always decreasing in c.*

Proof. Note that (38) implies

$$\frac{1}{\xi} = \frac{2A}{-1 + \sqrt{1 + \frac{4A}{c}}}, \tag{39}$$

where

$$A = \beta + \frac{\beta^2 - \gamma^2}{c}. \tag{40}$$

It follows that

$$\begin{aligned}
\frac{\partial}{\partial c}\left(\frac{1}{\xi}\right) &= \frac{2A_c\left(-1 + \sqrt{1 + \frac{4A}{c}}\right) - \frac{4A}{\sqrt{1 + \frac{4A}{c}}}\left(\frac{A_c}{c} - \frac{A}{c^2}\right)}{\left(-1 + \sqrt{1 + \frac{4A}{c}}\right)^2} \\
&= \frac{2A_c\left(-1 + \sqrt{1 + \frac{4A}{c}} - \frac{\frac{2A}{c}}{\sqrt{1 + \frac{4A}{c}}}\right) + \frac{\frac{4A^2}{c}}{\sqrt{1 + \frac{4A}{c}}}}{\left(-1 + \sqrt{1 + \frac{4A}{c}}\right)^2} \\
&= \frac{2A_c\left(1 + \frac{2A}{c} - \sqrt{1 + \frac{4A}{c}}\right) + \frac{4A^2}{c}}{\sqrt{1 + \frac{4A}{c}}\left(-1 + \sqrt{1 + \frac{4A}{c}}\right)^2},
\end{aligned} \tag{41}$$

which is always positive since $1+\dfrac{2A}{c} > \sqrt{1+\dfrac{4A}{c}}$. Thus, $1/\xi$ is always increasing in c, implying that ξ is always decreasing in c. $\square$

Using Lemma 1, we can show that under the supply function competition both the expected producer profits and the expected consumer surplus are always decreasing in c.

Corollary 4. *$E[\pi^{SF}(\alpha)]$ and $E[CS^{SF}(\alpha)]$ are always decreasing in c.*

Proof. Differentiating (33) with respect to c yields

$$\frac{\partial E[\pi^{SF}(\alpha)]}{\partial c} = -\frac{1}{2}\left(\frac{1}{\dfrac{1}{\xi}+(\beta+\gamma)}\right)^2 E[\alpha^2] - 2\left(\frac{1}{\xi}-\frac{c}{2}\right)\left(\frac{1}{\frac{1}{\xi}+(\beta+\gamma)}\right)^3 E[\alpha^2]\,\frac{\partial}{\partial c}\left(\frac{1}{\xi}\right). \quad (42)$$

Then, Lemma 1 implies that $\dfrac{\partial E[\pi^{SF}(\alpha)]}{\partial c} < 0$. On the other hand, differentiating (35) yields

$$\frac{\partial E[CS^{SF}(\alpha)]}{\partial c} = -2(\beta+\gamma)\left(\frac{1}{\dfrac{1}{\xi}+(\beta+\gamma)}\right)^3 E[\alpha^2]\,\frac{\partial}{\partial c}\left(\frac{1}{\xi}\right). \quad (43)$$

Now, Lemma 1 implies that $\dfrac{\partial E[CS^{SF}(\alpha)]}{\partial c} < 0$. $\square$

Since the functional forms of $E[\pi^{SF}(\alpha)]$ and $E[CS^{SF}(\alpha)]$ do not allow us to study their dependence on β and γ analytically, we illustrate this dependence in Figures 1 and 2 with the help of some numerical simulations. Note that the six graphs in each figure correspond to the six values of β in the set $\{1, 1.5, 2, 2.5, 3, 3.5\}$. In each of these graphs, we consider 15 sample points for c between 0.01 and 10.00 and 15 sample points for γ between 0.01 and 1.00. Figures 1 and 2 show that fixing all other parameters constant, the higher the value of the parameter β, i.e., the own-price sensitivity of the inverse demand in each product market, the smaller the expected producer profits and also the expected consumer surplus. On the other hand, the substitution parameter γ has asymmetric effects. Fixing all other parameters constant, an increase in γ decreases the expected producer profits (as shown in Figure 1), while increasing the expected consumer surplus (as shown in Figure 2).

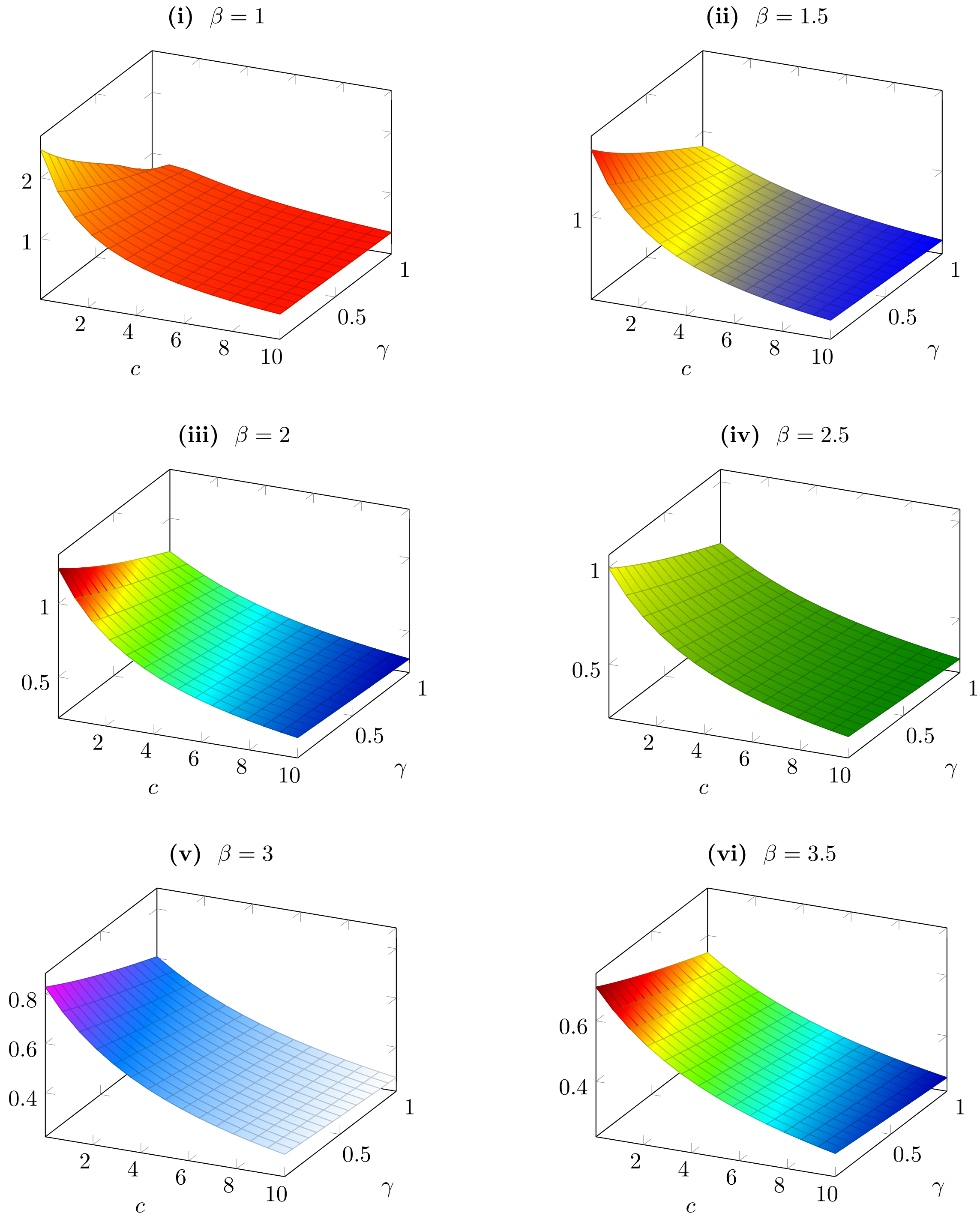

Figure 1. Plots of $E[\pi^{SF}(\alpha)]$ for various values of β, γ, and c when $E[\alpha^2] = 10$.

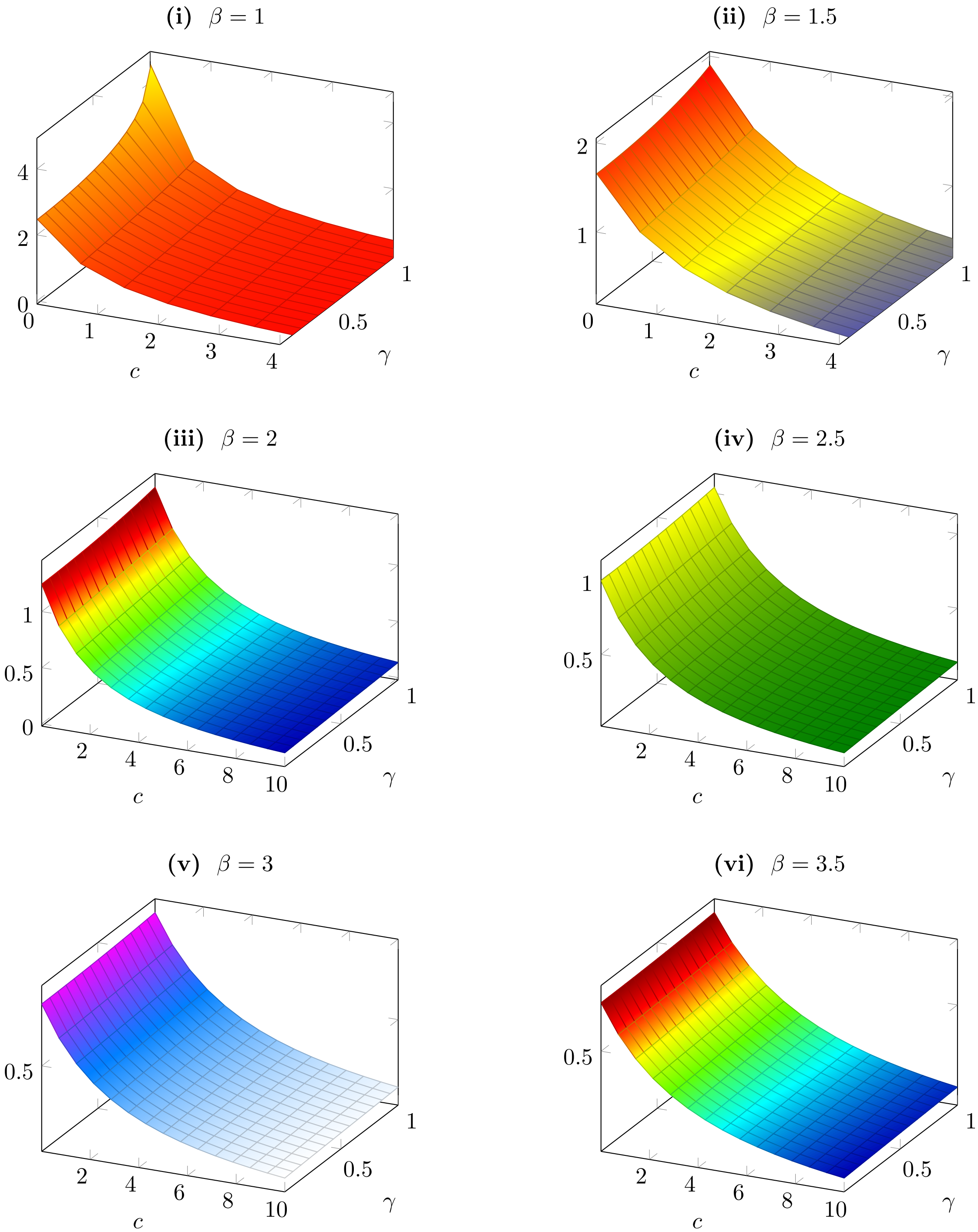

Figure 2. Plots of $E[CS^{SF}(\alpha)]$ for various values of β, γ, and c when $E[\alpha^2] = 10$.

3.3 Welfare Ranking

Now, we will investigate whether the supply function competition or the quantity competition can *always* yield a higher expected welfare to the duopolists or the consumers in the studied industry.

Proposition 3. *In the studied duopolistic industry with differentiated products and demand uncertainty, the expected consumer surplus under the supply function competition is always higher than under the quantity competition.*

Proof. Comparing (18) and (35) using (36) and (37), we observe that $E[CS^{SF}(\alpha)] > E[CS^{C}(\alpha)]$ if and only if

$$(\beta+\gamma)\left(\frac{\xi}{1+\xi(\beta+\gamma)}\right)^2\left(1+\eta^2\right)(E[\alpha])^2 > (\beta+\gamma)\left(\frac{1}{2\beta+\gamma+c}\right)^2(E[\alpha])^2, \tag{44}$$

implying

$$\frac{1}{(1/\xi)+(\beta+\gamma)}\mathcal{V} > \frac{1}{(\beta+c)+(\beta+\gamma)}, \tag{45}$$

where

$$\mathcal{V} = \sqrt{1+\eta^2}. \tag{46}$$

It follows from (45) and (46) that we have $E[CS^{SF}(\alpha)] > E[CS^{C}(\alpha)]$ if and only if

$$\xi > \kappa, \tag{47}$$

where

$$\kappa = \frac{1}{\mathcal{V}(\beta+c)+(\mathcal{V}-1)(\beta+\gamma)}. \tag{48}$$

Using (38) one can easily show that the inequality in (47) holds if and only if

$$\sqrt{1+\frac{4}{c}\left(\beta+\frac{\beta^2-\gamma^2}{c}\right)} > 2\kappa\left(\beta+\frac{\beta^2-\gamma^2}{c}\right)+1, \tag{49}$$

or

$$\frac{1}{\kappa^2}-\left(\beta c+\beta^2-\gamma^2\right)-\frac{c}{\kappa} > 0. \tag{50}$$

First assume that $\eta = 0$, implying $\mathcal{V} = 1$. Then, $\kappa = 1/(\beta+c)$. Inserting this into (50) yields

$$(\beta+c)^2-\left(\beta c+(\beta^2-\gamma^2)\right)-c(\beta+c) > 0, \tag{51}$$

reducing to

$$\gamma^2 > 0, \tag{52}$$

which always holds since $\gamma > 0$ by assumption. So, we have proved that (44) holds when $\eta = 0$. Since the left hand side of (44) is increasing in η while its right hand side is independent of it, (44) holds for $\eta > 0$, as well. So, it is always true that $E[CS^{SF}(\alpha)] > E[CS^{C}(\alpha)]$. $\square$

Now, we can consider a welfare comparison from the viewpoint of the producers. First note that using (36) and (37), the expected producer profits in (33) can be rewritten as

$$E[\pi^{SF}(\alpha)] = \left(\frac{1}{\xi} - \frac{c}{2}\right)\left(\frac{\xi}{1+\xi(\beta+\gamma)}\right)^2 \left(1+\eta^2\right)\left(E[\alpha]\right)^2. \tag{53}$$

As we can see, the demand uncertainty, measured by η, positively affects the expected profits in (53) obtained by the duopolists under the supply function competition, while it has no effect on the expected profits in (14) the duopolists obtain under the quantity competition. So, the profit difference $E[\pi^{SF}(\alpha)] - E[\pi^{C}(\alpha)]$ is increasing in η. Let $\bar{\eta}$ be the lowest value of demand uncertainty at which this profit difference is non-negative, i.e., $\bar{\eta} = \min\{\mu \geq 0 : E[\pi^{SF}(\alpha)] - E[\pi^{C}(\alpha)] \geq 0\}$. This value can be calculated using (14) and (53) as follows:

$$\bar{\eta} = \max\left\{0, \sqrt{-1 + \frac{\left(\beta + \frac{c}{2}\right)\left(\frac{1}{\xi} + \beta + \gamma\right)^2}{\left(\frac{1}{\xi} - \frac{c}{2}\right)(2\beta+\gamma+c)^2}}\right\} \tag{54}$$

Note that the coefficient of variation η is always non-negative and when it is above (below) the critical level $\bar{\eta}$, the expected producer profits under the supply function competition are higher (lower) than under the quantity competition.

Now, we will investigate how the critical level of demand uncertainty, $\bar{\eta}$, is affected by a change in any of the parameters β, γ, and c. However, because of the complexity of (38) and (54), we will be able to do this only by numerical simulations. We plot in Figure 3 the simulated graphs of $\bar{\eta}$. (The ranges of β, γ, and c are as in the previous two figures.) Comparing all six graphs in Figure 3 reveals that for extremely low values of γ, the value of $\bar{\eta}$ becomes zero, implying that at such values of γ the expected producer profits under the supply function competition always exceed the expected producer profits under the quantity competition.

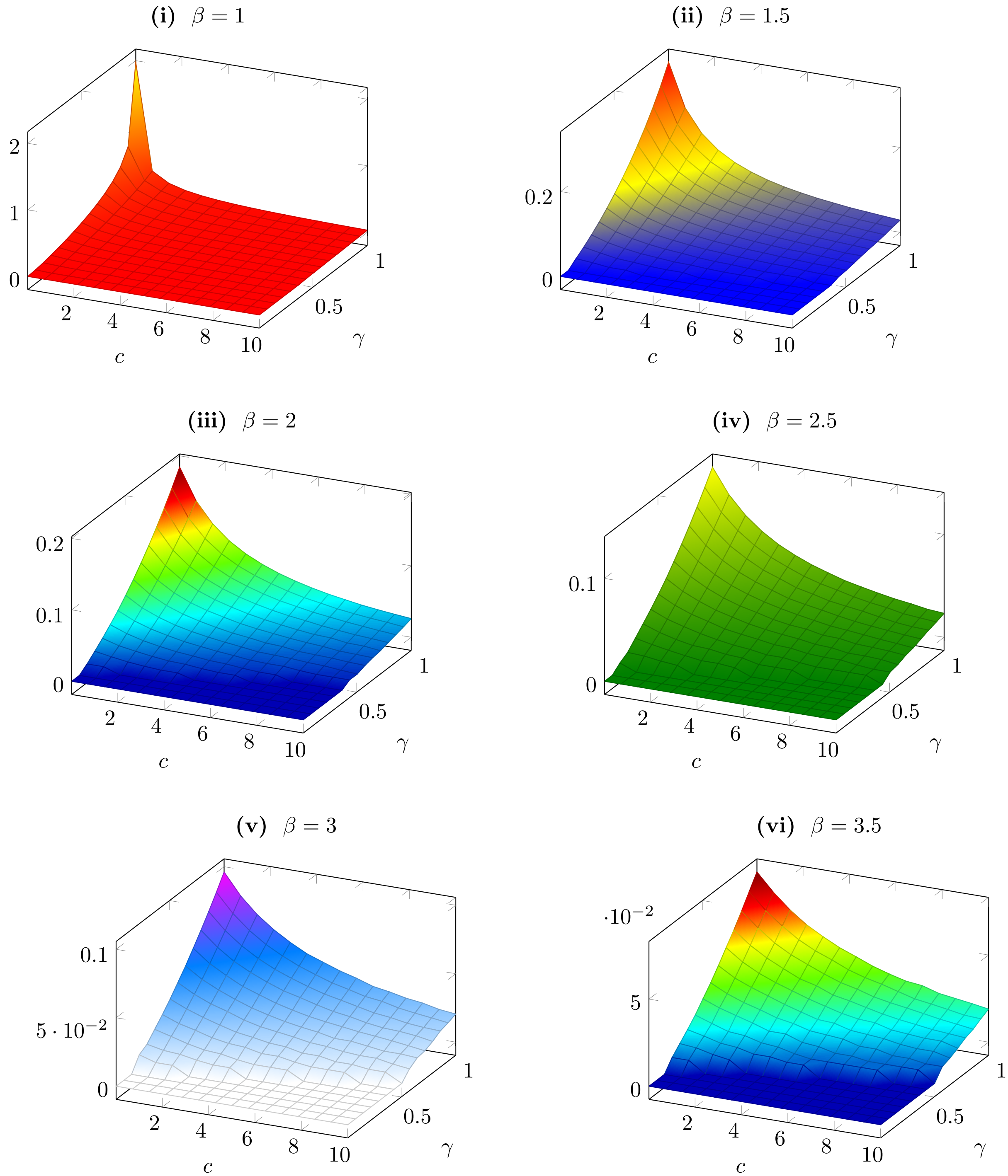

Figure 3. Plots of $\bar{\eta}$ for various values of β, γ, and c.

Figure 3 also shows that unless γ is extremely low, $\bar{\eta}$ is always positive, implying that for sufficiently low values of demand uncertainty the quantity competition can be a superior form of competition for the duopolistic firms, provided that their products are sufficiently close substitutes. Another finding in Figure 3 is that an increase in β decreases $\bar{\eta}$, at all values c in its domain unless γ is extremely low. This implies that if for each product the demand becomes smaller due to an increase in its own-price sensitivity, then the supply function competition would require a lower amount of uncertainty to dominate the quantity competition from the viewpoint of the duopolists. A similar result is also observed when the marginal cost of producing the unit output, c, becomes higher. On the other hand, the substitution parameter γ, when it is not extremely small, has a positive effect on $\bar{\eta}$. That is, as the products in the industry become closer substitutes, then the supply function competition would, in general, require a higher amount of uncertainty to dominate the quantity competition from the viewpoint of the duopolists.

4 Conclusion

In this paper we have made a welfare comparison between the supply function and quantity competitions in a duopolistic industry with differentiated products and demand uncertainty. We have presented in Propositions 1 and 2 the characterizations of the symmetric equilibrium obtained under each form of competition, and calculating the expected welfares of the producers and consumers at each of these equilibria, we have first studied how they would respond to changes in various model parameters. These parameters are the size of the demand uncertainty measured by the coefficient of variation (η), the own-price sensitivity of the demand faced by each duopolist (β), the degree of substitution between the products of the duopolists (γ), and the marginal cost faced by each duopolist to produce a unit output (c).

Under the quantity competition, both the expected consumer surplus and the expected producer profits are independent of η, while both of them are always decreasing in c. On the other hand, the parameters β and γ can have asymmetric effects on the welfares of the duopolists and the consumers. Whereas the expected profits of the duopolists are always decreasing in β and γ, the expected consumer surplus can be increasing in these two parameters unless the marginal cost of producing a unit output is sufficiently small for each duopolist. Under the supply function competition, we have found that both the expected consumer surplus and the expected profits of the duopolists are always increasing in the size of demand uncertainty, η, and always decreasing in both the cost parameter c and the own-price sensitivity parameter of the inverse demand, β. On the other hand, the substitution parameter γ has different effects on the expected welfares of producers and consumers. An increase in γ always decreases the expected producer profits, while always increasing the expected consumer surplus.

Next, we have studied how moving from one type of competition to the other one can affect the expected welfares of producers and consumers. In Proposition 3, we have showed that the expected consumer surplus under the supply function competition is always higher than under the quantity competition. By some numerical simulations, we have also found that the expected producer profits under the supply function competition can be lower than under the quantity competition if and only if the degree of product substitution is not extremely small and the size of the demand uncertainty is below a critical level, which is non-increasing in c and β and non-decreasing in γ.

The main results of this paper is that in a differentiated products duopoly with demand uncertainty (i) if the degree of product substitution is extremely low, then the supply function competition can be always Pareto superior to the quantity competition, and (ii) if the degree of product substitution is not extremely low, then the supply function competition can be Pareto superior to the quantity competition if and only if the size of demand uncertainty is sufficiently large. The second of these results is an extension of an earlier result of Saglam (2018) obtained for a homogeneous product duopoly. All in all, our results suggest that in electricity markets with differentiated products, the regulators should not intervene to impose the quantity competition in favor of the supply function competition unless the degree of product substitution is sufficiently high and the predicted demand fluctuations are sufficiently small.

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