Accounting for Busines Cycles in Canada: II. The Role of Money

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Abstract

I have explained business cycles in Canada focusing on the role of money. To do that, I have used both empirical and theoretical models. The empirical investigations include performing causality tests and computing impulse responses based on structural and co-integrated vector autoregressive models. The theoretical models consist of RBC and new-Keynesian models. Some of these theoretical models are: the inflation tax, the inflation and tax code, the sticky price, and the financial accelerator models.

The empirical models indicate monetary disturbances are instrumental in business cycle fluctuations but do not necessarily cause them. The theoretical models also point out that monetary disturbances contribute to business cycle fluctuations but not as much as technological change. Some channels through witch they propagate are: nominal capital gain tax, price stickiness, and deteriorating financial conditions. Price stickiness turns out to play the major role.

Keywords: Business Cycles, Macroeconomics, Monetary Policy, Sticky Prices, Vector Autoregression, Vector Error Correction.
JEL: E10, E31, E32,E37, E57

Contents

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Some Abbreviations and Acronyms

ADF	Augmented Dickey-Fuller
AIC	Akaike Information Criterion
BGP	Balanced Growth Path
CIA	Cash-in-advance
DSGE	Dynamic Stochastic General Equilibrium
FOC	First-Order Condition
GDP	Gross Domestic Product
GSS	General Social Survey
HP	Hodrick & Prescott
IRF	Impulse Response Function
LFS	Labor Force Survey
MIU	Money-in-the-utility
OLS	Ordinary Least Squares
RBC	Real Business Cycle
SC	Schwarz Criterion
TFP	Total Factor Productivity
US	United States
VAR	Vector Autoregression
VECM	Vector Error Correction Model

Some Unfamiliar Greek Letters

Letter	Name	Letter	Name
ε	varepsilon	η	eta
ϑ	vartheta	$\varkappa$	varkappa
ν	nu	ξ	xi
ϖ	varpi	υ Υ	upsilon
ϕ Φ	phi	φ	varphi
χ	chi	ψ	psi

The prefix var in some names stands for variant

Non-Technical Summary

Objectives The purpose of this research is to document business cycle facts in Canada and explain them emphasizing on the role of money. The issues addressed are: Do monetary disturbances cause aggregate fluctuations? If so how important are they and how do they propagate into the real economy?

Methodology First, I have undertaken some exploratory data analyses to document the business cycle facts to explain. I have then undertaken some time series analyses to find out how real variables actually respond to monetary disturbances. Finally, in turn, I have calibrated several theoretical growth models using actual data and simulated them. The cyclical behavior of the simulated series is compared to that of the actual ones.

Key Contributions I have explored the issue from multiple perspectives: empirically and theoretically using a variety of competing models. Besides, all the investigations are undertaken using Canadian data.

Findings The empirical investigations show that monetary disturbances are a determinant of business cycle fluctuations but the way they contribute to these fluctuations depend on the measure of money used. The theoretical models show the channels through which monetary disturbances propagate: nominal capital gain tax, price stickiness, deteriorating financial conditions.

Future Research To further the understanding of business cycle fluctuations in Canada, it is important to look at other determinants a part from technological change and monetary disturbances. Other determinants pointed to in the literature are expectations.

1 Introduction

Business cycles are a recurring sequence of expansion, recession, and recovery. Real business cycle (RBC) theory is largely premised on the assumptions that: markets are perfectly competitive, prices and real wages adjust instantly to clear simultaneously all markets, and technological change is the primary cause of the aggregate fluctuations in the economic activity. The importance given to technological change or, in general, to supply-side factors is empirically based on the negative correlation observed over the business cycle between gross domestic product (GDP) and its implicit price (Kydland and Prescott, 1990; Cooley and Ohanian, 1991; Apergis et al., 1996). [1] Money should, in principle, play no role a part from facilitating the exchange of goods and services. This idea referred to as the classical dichotomy does not hold empirically as money and real variables turn out to be correlated (Friedman and Schwartz, 1963; Barro, 1977; Blanchard, 1990). This empirical failure means prices and wages are not as flexible as posited to clear at once markets and money plays other roles than just facilitating the exchange of goods and services.

If money is not just a medium of exchange, in which ways does it influence real economic activity? According to theories purporting to explain the demand for money, it directly or indirectly yields utility to households (see Walsh, 2010, for a review). Money-in-the-utility (MIU) models treat money as a final good that directly yields utility. On the other hand, in shopping-time models, money is instead an intermediate good whose role is to make shopping less time consuming. Households could therefore allocate more time to leisure and consequently increase their utility by holding larger money balances. Cash-in-advance (CIA) models consider money as a medium of exchange that is absolutely required in certain transactions. It thus indirectly yields utility by allowing the purchase and consumption of goods.

Both the RBC and the new-Keynesian theories have incorporated these models of money demand into their frameworks to provide mechanisms through which monetary disturbances affect the real economic activity. But while the RBC theory keeps assuming markets are perfectly competitive, the new-Keynesian theory relies on imperfect competition and consequently on nominal price or wage rigidities as an additional and essential channel of influence of money on real macroeconomic variables. Empirical investigations using United States (US) micro data indicate that prices last about six (Bils and Klenow, 2004; Klenow and Kryvtsov, 2008) to twelve months (Blinder, Canetti, Lebow, and Rudd, 1998). The reason behind the rigidity of prices is that it costs firms to always change them. To illustrate that the concept of *menu cost* is put forward as restaurants, to change their prices,

[1] According to the law of demand and supply, when the supply of goods and services is high, prices decrease for markets to clear.

have to print new menus.

Another distinguishing feature between both schools of macroeconomic thought is the role of technological change in business cycle fluctuations. The new-Keynesian theory questions its importance. Mankiw (1989) argues that if business cycle were primarily driven by technological change, then recessions would be periods of technological regress, which is counterfactual. Gali (1999) deems the RBC paradigm empirically irrelevant because of its failure to replicate a central feature of business cycle: the strong positive correlation between output and hours worked. He shows that much of the fluctuations in these two variables are rather due to demand shocks. Demand shocks could be preference, money demand or investment efficiency shock. In addition, these shocks, unlike technological disturbances, generate a high positive correlation between the two variables. The evidence produced by Gali are mitigated by Ireland (2003) who finds estimating a sticky-price model that technological change accounts for the quasi-totality of the pre-1979 business cycle fluctuations in the US whereas the investment efficiency shock explains most of the post-1979 cyclical fluctuations. According to Ireland's evidence, both money demand and monetary policy shocks play a very small role.

To analyze monetary policy, both schools of thought use the same instruments: initially nominal money supply and later the short-term nominal interest rate. In the former case, the monetary authority directly controls the supply of money whereas in the latter case money supply adjusts to the target nominal interest rate. The monetary policy instruments can be exogenously set or allowed to adjust to the state of the economy, particularly to output or inflation fluctuations. In setting the money supply or the interest rate, the monetary authority can commit to a policy rule or, at will, change its plans. Two well-known simple policy rules are the Friedman k-percent rule (Friedman and Schwartz, 1963) and the Taylor (1993) rule. The Friedman rule suggests letting, each period, money supply grow at at a constant rate, regardless of the business cycles. The Taylor rule suggests raising nominal interest rate in response to the deviations of inflation and output from their targets. It stresses nominal interest rate must be raised more than one for one in response to a rise in inflation so that real interest rate also rises to stabilize output. An alternative to specifying directly a policy rule such as the one proposed by Friedman or Taylor is the targeting regime. In this framework, the policy rule is optimally derived from the monetary authority's objective function. The monetary authority's objective could be, *inter alia*, to minimize the volatility of inflation and output (Bernanke and Mishkin, 1997; Clarida, Gali, and Gertler, 1999, 2000) or to maximize households' welfare (Rotemberg and Woodford, 1997, 1999).

When the monetary authority is commuted to a policy rule, both RBC theorists (Gavin and Kydland, 1999; Flodén, 2000) and new-Keynesians (Ireland, 2003) agree that a change in that rule has a very little impact on the cyclical behavior of real variables; only nominal variables are affected. Even though the impact on real variables is small, Cooley and Hansen (1989) point out that it is significant As far as the effectiveness of monetary policy is concerned, most RBC theorists start sharing the new-Keynesian view that it does not just control inflation (Cooley and Hansen, 1989; Gavin, Kydland, and Pakko, 2007).

This paper is the second volume of an attempt at explaining business cycle in Canada using dynamic stochastic general equilibrium (DSGE) models. In the first volume, I emphasized on the role of technological change as a driving force (Accolley, 2016). The models I dealt with were only RBC models. This volume is about the role of monetary disturbances and demand-side factors in business cycle fluctuations in Canada.

Previous attempts at explaining the relation between nominal and real variables in Canada include Serletis and Molik (2000), Ambler, Dib, and Rebei (2004), Dib (2006), and Amano, Ambler, and Rebei (2007). [2] Serletis and Molik (2000) empirically investigate the relationship between the price level, nominal and real GDP, and fifteen alternative measures of money supply. They find none of the *broad measures of money* to be a leading indicator of the economic activity. Only some *narrow measures of money* appear to predict the economic activity. Ambler, Dib, and Rebei (2004) estimate, using Canadian and US data, a small open economy DSGE model that features rigidities in nominal wages and the prices of both domestic and imported goods. It turns out from their empirical investigations that the Bank of Canada has historically responded less strongly to fluctuations in output and paid too much attention to cyclical money growth in setting its key interest rate. They then produce estimates showing that, to maximize households' welfare, the Bank should not at all care about cyclical money growth but respond more strongly to fluctuations in output and inflation. Dib (2006) estimate a DSGE model that features costs of adjusting nominal price, physical capital, and employment. They find that combining nominal and real rigidities increase the persistence of monetary policy shocks. Amano, Ambler, and Rebei (2007) use a sticky-price general equilibrium model to investigate how trend inflation, *i.e.* the long-run value of inflation, affects macroeconomic variables. They model nominal price stickiness in three different ways. Simulating their model, they find that a rise in trend inflation lowers average output, consumption, and investment and raises their volatility and persistence. They also find that, at higher levels of inflation, monetary policy is inefficient in reducing inflation volatility as the Phillips curve becomes flatter.

To explain the relation between nominal and real variables, I have used empirical, RBC, and new-Keynesian models. The empirical investigations have consisted in some exploratory data analyses and fitting some dynamic econometric models to Canadian data. The exploratory data analyses show a co-movement between

[2] Cross and Bergevin (2012) dates the twelve recessions Canada experienced since 1926.

money and the business cycle measured as the cyclical real GDP. The sign of this association depends on the measure of money supply used. Whereas most narrow measures of money are positively correlated with business cycle, the broad measures are instead negatively correlated with it. Further econometric analyses have revealed a mutual dependence between the broad measures of money and business cycle. This evidence obtained using detrended time series contrasts with the findings of Serletis and Molik (2000) who rather used the raw data. The exploratory data analyses also reveal important breaks in the cyclical behavior of both real and nominal variables since the federal government and the Bank of Canada agreed in February 1991 to meet an inflation target through monetary policy.

The RBC models I have used are:

- the tax inflation model (Cooley and Hansen, 1989, 1991, 1992, 1995, 1998),
- the RBC model with endogenous money supply (Gavin and Kydland, 1999; Dittmar, Gavin, and Kydland, 2005),
- the inflation and tax code model (Gavin, Kydland, and Pakko, 2007).

The new-Keynesian models I have used are:

- the sticky price model (Yun, 1996; Ireland, 1997, 2001, 2003),
- the financial accelerator model (Bernanke, Gertler, and Gilchrist, 1999),

The inflation tax model introduces money into the neoclassical framework using the CIA constraint, *viz* households are required to pay for some consumption goods using cash. This model, compared to an equivalent cashless RBC model, explains a higher proportion of the fluctuations in consumption and wage. But it generates too much fluctuations in prices due to the assumption that this latter variable is flexible. A striking feature from methods of payment surveys is the fall in the use of cash in households' transactions. I have used this model to investigate the impacts of this fall. It has appeared that an economy using too little cash could experience the same level of welfare as one using too much cash since the relation between the two variables is U-shaped. On the other hand, inflation and welfare are inversely related.

In the RBC model with endogenous money supply, holding money reduces the time allocated to shopping for consumption goods. Money growth can be exogenous or depend on past output or current money supply. This model explains a lower share of the variability in output, consumption and the hours worked but a higher share of labor productivity's volatility. It has also emerged that alternative monetary rules, *i.e.* varying intensity in the monetary authority's reaction to fluctuations in money and output, have a very little impact on the cyclical behavior of real variables. However, they explain the break observed in the behavior of nominal variables. For instance, money could be pro-cyclical or countercyclical and prices could be highly or less volatile depending on the monetary authority's response to output.

The inflation and tax code model is an extension of the RBC model with endogenous money. It incorporates a government that sets the rules for taxing each type of income households earn. Monetary policy is implemented following an interest rate rule. It has turned out that taxing nominal capital gains discourages capital accumulation and worsens the welfare costs of inflation. Monetary disturbances generated by shocks to inflation target add to this model's ability to explain the cyclical behavior of real and nominal variables. Introducing nominal capital gain tax also adds to the volatility of real variables and lowers their correlation with output.

The sticky price model introduces two features: capital adjustment cost to avoid excessive investment volatility and nominal price rigidity as a channel of propagation of monetary disturbances. Simulating this model helps conclude that monetary policy shocks do not propagate in the absence of price stickiness. In addition to being able to account for the observed correlation between hours worked and productivity, this model can also explain the break observed after 1991 in the cyclical behavior of real and nominal variables.

According to the financial accelerator model, worsening credit market conditions fuels recessions. This model introduces into the sticky price model entrepreneurs that purchase their capital stocks out of equities, *i.e.* internal funds, and debts. Because of information asymmetry (financial friction), the creditors pays an auditing cost to observe entrepreneurs' cash flows. The external finance premium,, *i.e.* the difference in the interest rate on debts and equities, is a decreasing function of entrepreneurs' percentage equity holding. A change in the elasticity of the external finance premium with respect to the percentage equity holding significantly affects the cyclical behavior of the economy.

The rest of this paper is organized as follows. In Section 2, I have undertaken some exploratory data analyses using the cyclical components of some real and nominal macroeconomic variables. Further econometric analyses are carried out in Section 3. This includes testing for price stickiness and computing impulse responses to monetary policy shocks from vector autoregressive and vector error correction models. In Sections 4 through 8, the models listed above are sketched, calibrated using Canadian data, and then solved numerically. Section 9 concludes.

2 Business Cycle Facts

In this section, I have measured the fluctuations and co-movements in some nominal and real macroeconomic variables over the business cycle. The nominal variables include: the GDP deflator, the consumer price index (CPI) excluding the

Table 2.1: Definition of Monetary Aggregates

Aggregate	Component
Monetary Base	Coins and notes in circulation
	Commercial bank deposits held as reserve by the central bank
M1	Coins and notes in circulation
	Demand deposits
M1+	M1
	Checkable saving and notice deposits
M1++	M1+
	Non-checkable saving and notice deposits
M2	M1++
	Fixed-term saving deposits
M2+	M2
	Money market mutual funds
M2++	M2+
	Saving bonds
	Non-money market mutual funds

most volatile components, some short-term interest rates, and seven alternative measures of money supply (monetary base, M1, M1+, M1++, M2, M2+, M2++). The monetary base, M1, M1+ and M1++ are narrow measures of money and the other aggregates are broad measures. These monetary aggregates are defined in Table 2.1.

The real variables include the velocity of money that I have computed using the quantity theory of money, *i.e.* the relation $Mv = PY$, where M and v designate respectively the supply and velocity of money whereas Y and P are the real GDP and its deflator (also known as the implicit price of GDP).

I have used the Hodrick and Prescott (HP) filter, to isolate the cyclical components of the time series. I have then computed the standard deviation and first-order autocorrelation of the detrended series to show the magnitude and persistence of their fluctuations. To make the standard deviations scale invariant and enable comparisons, I have used not the detrended series in level but their percentage deviation from trend, except for rates. An alternative would be rather detrending the natural logarithm of the raw time series. [3]

I have also computed the correlation coefficient between cyclical real GDP and the macroeconomic variables to see how they relate to business cycle. Variables that are positively correlated with real GDP are said to be pro-cyclical. Those that are negatively correlated or not at all correlated with it are respectively said to be countercyclical and acyclical. In addition to the correlations between cyclical real

[3]Let Y_t, $t = 1, 2 \ldots T$ designates a real time series and Y_{ct}, its cyclical component. Its percentage deviation from trend, Y_{ct}/Y_t, is a first-order Taylor series approximation of the cyclical components of its logarithm.

GDP and the monetary aggregates, which indicate whether money is neutral or not, I computed the correlation between the growth in M1 and the macroeconomic variables, in order to check whether money is super-neutral.

The following stylized facts emerge from the summary statistics reported in Table 2.2.

1. The narrow measures of money supply (the monetary base, M1, and M1+) are pro-cyclical whereas its broad measures are countercyclical.

2. Growth in M1 is countercyclical.

3. M1 is the most volatile measure of money supply.

4. Money velocity is pro-cyclical and more volatile than output.

5. Interest rates are pro-cyclical and less volatile than output.

6. Interest rates are negatively correlated with growth in M1.

7. Prices are countercyclical while inflation is pro-cyclical.

8. International trade, *i.e.*exports and and imports, and exchange rate are pro-cyclical and more volatile than output.

9. Both prices and inflation are negatively correlated with growth in M1.

10. Both government consumption and investment are positively correlated with growth in M1.

11. Hours worked, which are pro-cyclical, are negatively correlated with growth in M1 while hourly earnings, which are countercyclical, are positively correlated with growth in M1.

In February 1991, the federal government and the Bank of Canada jointly set the 2% inflation-control target. Through monetary policy, the Bank committed to bring the year-over-year growth in the consumer price index (CPI) to 3% by the end of 1992 and finally to 2% by the end of 1995. This agreement has been renewed several times and is still effective. The average annual inflation rate, which was 4.7% between 1981 and 1990, went down to 1.96% thereafter. I have broken the sample at the end of 1990, to see how this change in the objective of monetary policy affected business cycles. Table 2.3 displays the statistics summarizing business cycles separately for the periods 1981:Q1-1990:Q4 and 1991:Q1-2015:Q4. Gavin and Kydland (1999) undertook a similar exercise using US data. They found no significant change in the cyclical behavior of real variables and observed substantial changes in the variability of monetary aggregates. Unlike the US, in

Table 2.2: Cyclical Behavior of the Canadian Economy, Percentage Deviation from Trend of Key Variables, 1981:Q1-2015:Q4

Variable	% Standard Deviation	Correlation with GDP	Correlation with M1 Growth	First-order Autocorrelation
Output (GDP)	1.56	1	-.12	.89
Household Consumption	1.14	.84	-.16	.85
Government Consumption	.94	-.06	.12	.69
Private Fixed Investment	4.9	.75	-.21	.88
Government Fixed Investment	3.42	-.04	.18	.85
Exports	3.64	.79	-.06	.79
Imports	4.87	.8	-.13	.85
Actual Hours Weekly Worked				
Average	.5	.77	-.07	.72
Total	1.41	.9	-.21	.9
Hourly Earnings	1.18	-.18	.06	.84
Productivity	.68	.42	.17	.62
Money Supply				
Monetary Base	1.59	.03	-.07	.78
M1	4.4	.4	.02	.92
M1+	3.2	.34	-.02	.9
M1++	2.16	-.19	-.11	.89
M2	1.73	-.26	.06	.93
M2+	1.73	-.37	.1	.94
M2++	1.18	-.1	-.04	.93
Money Velocity				
Monetary Base	2.66	.56	-.02	.83
M1	4.14	.11	-.18	.9
M1+	3.39	.27	-.12	.88
M1++	3.16	.64	.01	.89
M2	2.69	.71	-.1	.88
M2+	2.8	.74	-.1	.89
M2++	2.23	.75	-.05	.87
Interest Rates				
Overnight Rate	1.35	.56	-.12	.75
Bank Rate	1.32	.63	-.14	.8
Treasury Bills 1 Month	1.31	.61	-.11	.81
Treasury Bills 3 Month	1.3	.64	-.14	.8
Treasury Bills, 1 Year	1.23	.63	-.19	.77
Prices				
GDP Deflator	1.07	-.001	.01	.8
Consumer Price Index	.57	-.61	-.09	.85
Inflation	.28	.11	-.07	.12
Effective Exchange Rate	4.17	.06	.05	.81
Terms of Trade	2.5	.37	-.06	.77
Canadian $ in terms of US$	4.05	.13	-.12	.79

Table 2.3: Cyclical Behavior of the Canadian Economy, Percentage Deviation from Trend of Key Variables, 1981:Q1-1990:Q4 and 1991:Q1-2015:Q4

	1981:Q1-1990:Q4			1991:Q1-2015:Q4		
Variable	(1)	(2)	(3)	(1)	(2)	(3)
Output (GDP)	2.18	1	.9	1.21	1	.89
Household Consumption	1.74	.94	.9	.76	.68	.79
Government Consumption	.88	.21	.39	.95	-.22	.78
Private Fixed Investment	5.97	.88	.89	4.4	.65	.88
Government Fixed Investment	2.57	.19	.82	3.72	-.15	.86
Exports	3.96	.72	.72	3.5	.89	.82
Imports	6.74	.86	.85	3.91	.74	.85
Average Hours	.56	.85	.76	.47	.73	.69
Total Hours	2.1	.94	.93	.98	.85	.86
Hourly Earnings	1.4	-.46	.85	1.09	.08	.84
Productivity	.78	.23	.53	.64	.61	.66
Monetary Base	1.49	.38	.66	1.64	-.2	.81
M1	7.86	.56	.94	1.66	-.04	.77
M2	2.22	.06	.93	1.5	-.59	.92
Overnight Rate	1.95	.49	.7	1.01	.65	.83
Bank Rate	1.87	.58	.78	1.01	.68	.82
GDP Deflator	1.12	-.58	.9	1.05	.45	.76
Consumer Price Index	.86	-.77	.92	.39	-.34	.87
Inflation	.28	.28	.53	.29	.01	-.04
Terms of Trade	1.7	.32	.85	2.76	.45	.76
Canadian $ in terms of US$	3.05	.07	.91	4.41	.18	.77

Columns (1): Percentage Standard deviation, columns (2): Correlation with GDP, column (3): First-order autocorrelation

Canada, the target for inflation is explicitly set and this policy impacts on real business cycle fluctuations.

It thus emerges from Table 2.3 that after the Bank of Canada set the 2% inflation-control target:

1. Most nominal and real variables have become less volatile,

2. The volatility of CPI has dropped by 55% and that of M1 by about 80%,

3. Both government consumption and investment, monetary base, exchange rate and terms of trade have become more volatile,

4. Both government consumption and monetary base, M1, and M2 have become counter-cyclical,

5. Interest rate has become more pro-cyclical and less volatile and inflation is less persistent.

3 Some Empirical Models

The exploratory data analyses in the previous section indicates a co-movement between cyclical money, its growth rate, and the real economic activity. In this section, I have used some advanced econometric tools to find out whether money causes business cycle fluctuations and, if so, how monetary policy influences the real economic activity. Besides, according to the new-Keynesian theory, monetary disturbances affect business cycle because prices are sticky. I have finally fitted a price adjustment model to test for this assumption. The data used are from Statistics Canada and cover the sample period 1981:Q1-2015:Q4.

3.1 Money and Business Cycles

Does money cause business fluctuations ? To investigate the short-run dynamic relationship between money supply and real GDP, I have performed some bivariate Granger causality tests. This test, first, consists in estimating by ordinary least squares (OLS) unrestricted and restricted versions of the following reduced-form vector autoregressive (VAR) model of order p,

$$y_{ct} = \sum_{i=1}^{p} a_{1i} y_{c,t-i} + \sum_{i=1}^{p} b_{1i} m_{c,t-i} + c_1 + \varepsilon_{1t} \tag{3.1a}$$

$$m_{ct} = \sum_{i=1}^{p} a_{2i} y_{c,t-i} + \sum_{i=1}^{p} b_{2i} m_{c,t-i} + c_2 + \varepsilon_{2t}, \tag{3.1b}$$

where y_{ct} and m_{ct} respectively designate cyclical real GDP and money supply. The stochastic disturbances ε_{1t} and ε_{2t} are assumed to be uncorrelated. The parameters c_1 and c_2 are the intercepts. Then, one checks in turn the joint significance of the lagged values of m_{ct} in (3.1a) and that of the $y_{c,t-i}$s in (3.1b) performing F tests (for further details, see Hamilton, 1994, pp 302-9).

Before performing the F tests, I have selected p, the lag order, comparing information criteria computed after fitting several unrestricted VAR(p) models. Two information criteria often used are the Akaike information criterion (AIC) and the Schwarz criterion (SC)

$$AIC_j(p) = \ln\left(\frac{1}{T}\sum_{t=1}^{T} e_{jt}^2\right) + 2\frac{2p}{T}$$
$$SC_j(p) = \ln\left(\frac{1}{T}\sum_{t=1}^{T} e_{jt}^2\right) + \frac{2p}{T}\ln T,$$

where e_{jt}, $j = 1, 2$, are the residuals from regressing (3.1). I have used seven alternative measures of money supply: the monetary base, M1, M1+, M1++, M2, M2+,, and M2++. In all the cases, the VAR(2) models give the smallest SC.

The two null hypotheses of the Granger causality tests are:

$$\begin{aligned} \mathrm{H}_0 : &\quad b_{1,1} = b_{1,2} = \cdots = b_{1p} = 0 \\ \mathrm{H}_0' : &\quad a_{2,1} = a_{2,2} = \cdots = a_{2p} = 0. \end{aligned}$$

Rejecting H_0 would mean fluctuations in money supply cause business cycles and rejecting H_0' would mean the latter causes the former. The test statistic is

$$F = \frac{(RSS_R - RSS_U)/p}{RSS_U/(T-2p-1)} \sim F(p, T-2p-1),$$

where RSS_R and RSS_U are the residual sum of squares, $\sum_{t=1}^{T} e_{jt}^2$, from respectively the restricted and unrestricted models.

It appears in Table 3.1 that the conclusions of the Granger causality tests depend on the measure of money supply used. Fluctuations in the monetary base, M1, and M1++ do not help predict fluctuations in real GDP. On the other hand, M1+ and the broad measures of money supply Granger cause business cycles. Business cycles also turn out to Granger cause fluctuations in the broad measures of money, which points to a feedback relation, *viz* a mutual dependence, between the two variables.

Figure 3.1 plots the *cross-correlation* function between money and real GDP. The cross-correlation also known as dynamic correlation is the correlation between $m_{c,t+i}$ and y_{ct}, $i = -10\ldots 0\ldots 10$. I have used the aggregates M1 and M2 as measures of money supply.

Table 3.1: F Statistics from Granger Causality Tests

Measure of Money	H_0	H_0'
Monetary Base	.32	.38
M1	1.48	.57
M1+	2.03	.53
M1++	1.19	3.08
M2	6.93	5.51
M2+	10.27	5.06
M2++	5.71	6.5

$F_{5\%}(10, 119) = 1.91$

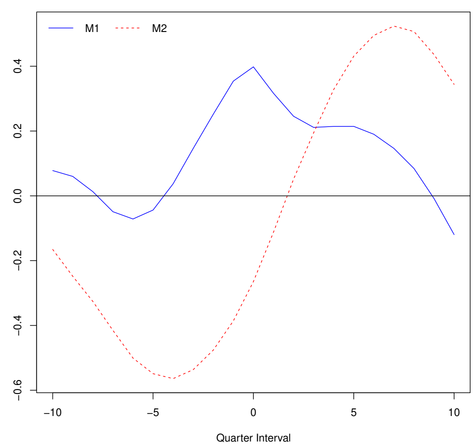

Figure 3.1: Cross-Correlation between Cyclical Money and GDP, Canada, 1981:Q1-2012:Q4

It appears in Figure 3.1 that the correlations between cyclical real GDP and both the lags and leads of cyclical M1 are not as strong as their contemporaneous correlation, which is consistent with the absence of causality suggested by the statistics in Table 3.1. Using M2, the correlations with the lags and leads are higher than the contemporaneous correlation and the feedback relationship suggested by the Granger causality tests transpires. The cross-correlations between business cycle and cyclical M2 suggest that economic booms are caused by contractions in M2 and followed by monetary expansions.

3.2 A Monetary Transmission Mechanism

Following Christiano, Eichenbaum, and Evans (2005), I have computed some empirical impulse responses to show how monetary policy influences the real economic activity. I have used two approaches to estimate the dynamic response of the economy to a monetary policy shock: the structural VAR and its vector error correction representation. The econometric model used is

$$\mathbf{A}_0\mathbf{y}_t = \mathbf{A}_1\mathbf{y}_{t-1} + \cdots + \mathbf{A}_p\mathbf{y}_{t-p} + \boldsymbol{\varepsilon}_t, \tag{3.2}$$

where $\mathbf{y}_t$ is a vector of nine macroeconomic policy and non-policy variables, $\mathbf{A}_i$, $i = 0 \ldots p$, are 9×9 matrices of coefficients, and $\boldsymbol{\varepsilon}_t$ is a vector of nine structural shocks. The structural shocks are normally distributed with mean $\mathbf{0}$ and variance-covariance matrix $\boldsymbol{\Sigma}_\varepsilon$.

The vector $\mathbf{y}_t$ is partitioned into two, $\mathbf{y}_t = [\mathbf{y}_{1t}, \mathbf{y}_{2t}]'$. The sub-vector $\mathbf{y}_{1t}$ consists of the natural logarithms of: real GDP, real consumption, GDP deflator (implicit price), real investment, real wage, and labor productivity. The sub-vector $\mathbf{y}_{2t}$ comprises the overnight rate, the natural logarithm of real profits proxied by the net operating surplus and the growth rate of M2. The overnight rate is the policy variable. [4] All the variables in $\mathbf{y}_t$ are integrated of order one, $I(1)$ in short, *viz* they are trended but their first differences are stationary. To test for the order of integration, I have performed some augmented Dickey-Fuller (ADF) unit root tests. [5] The test results are reported in Table 3.2.

I have selected the optimal lag length to enter in (3.2) performing some specification search tests. The SC suggests one lag while the AIC suggests two. It will be too restrictive choosing a lag length of one. [6] I have therefore followed the suggestion of the AIC.

[4] The overnight rate is the interest rate on one-day loans among financial institutions. It is the Bank of Canada's current key rate. Its original key rate was the bank rate.

[5] A non-stationary time series y_t following an autoregressive process of order p can be written as follows: $\Delta y_t = \lambda_0 + \pi y_{t-1} + \lambda_1 \Delta y_{t-1} + \cdots + \lambda_{p-1}\Delta y_{t-p+1} + \varepsilon_t$. If $\pi = 0$, y_t is said to be $I(1)$ or difference-stationary. The ADF test consists in checking the statistical significance of π. I have included a time trend in all the tests on the level variables except for the overnight rate.

[6] Setting the lag length to one would mainly restrict the short-run dynamics of all variables to error correction in long-run relations.

Table 3.2: Statistics from ADF Unit Root Tests

	Level		First-Difference	
Variable	Lags	τ-statistics	Lags	τ-statistics
GDP	1	-1.68	1	-5.751
Consumption	3	-2.179	2	-4.259
GDP Deflator	1	-2.369	1	-7.279
Investment	1	-2.158	1	-5.352
Wage	2	-2.327	1	-5.812
Productivity	1	-1.137	1	-9.337
Overnight Rate	4	-1.401	3	-5.747
Profit	1	-2.283	1	-7.078
M2	5	-2.873	4	-2.335
M2 Growth			3	-8.317
5%Critical Value		-3.43		-2.88

The Structural VAR Approach

The structural VAR(2) model can be reduced to the following one

$$\begin{aligned}\mathbf{y}_t &= \mathbf{A}_0^{-1}\mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_0^{-1}\mathbf{A}_2\mathbf{y}_{t-2} + \mathbf{A}_0^{-1}\boldsymbol{\varepsilon}_t \\ &= \boldsymbol{\Lambda}_1\mathbf{y}_{t-1} + \boldsymbol{\Lambda}_2\mathbf{y}_{t-2} + \mathbf{u}_t,\end{aligned}$$

where $\boldsymbol{\Lambda}_i = \mathbf{A}_0^{-1}\mathbf{A}_i$, $i = 1, 2$, and $\mathbf{u}_t = \mathbf{A}_0^{-1}\boldsymbol{\varepsilon}_t$. The vector of residuals $\mathbf{u}_t$, which is a linear combination of the structural shocks $\boldsymbol{\varepsilon}_t$, is normally distributed with mean $\mathbf{0}$ and variance-covariance matrix $\boldsymbol{\Sigma}_u$,

$$\boldsymbol{\Sigma}_u = \mathbf{A}_0^{-1}\boldsymbol{\Sigma}_\varepsilon\mathbf{A}_0^{-1\prime}. \tag{3.3}$$

One can directly estimate the matrices $\boldsymbol{\Lambda}_1$, $\boldsymbol{\Lambda}_2$, and $\boldsymbol{\Sigma}_u$ by OLS or maximum likelihood. But, given these estimates, it is not possible to solve for the structural matrices $\mathbf{A}_i, i = 0, 1, 2$, and $\boldsymbol{\Sigma}_\varepsilon$. To see why, let's consider relation (3.3). For 45 distinct estimates in $\boldsymbol{\Sigma}_u$, there are 117 distinct unknown parameters: the 72 off-diagonal elements of the 9×9 matrix $\mathbf{A}_0^{-1}$ in addition to the 9 variances and 36 distinct covariances in $\boldsymbol{\Sigma}_\varepsilon$. The matrices $\mathbf{A}_i$, $i = 0, 1, 2$ and $\boldsymbol{\Sigma}_\varepsilon$ are therefore said to be unidentified. Computing impulse responses becomes complicated because of this identification issue: a change in the residual $\mathbf{u}_t$ could stem from any combination of the structural shocks in ε_t.

A way to fix the identification issue is to make assumptions called identifying restrictions. The first identifying restrictions are that the structural shocks are orthogonal, which implies the 36 distinct covariances in $\boldsymbol{\Sigma}_\varepsilon$ are all nil and the 9 variances are equal to unity, $\boldsymbol{\Sigma}_\varepsilon = \mathbf{I}_9$. These restrictions reduce to 81 the number of unknown parameters in the 45 equations in (3.3). There is still a need for 36, *i.e.* 81-45, additional restrictions for the model to be just-identified. A strategy to complete the identification is the Choleski decomposition (Sims, 1980).

This decomposition consists in writing a positive definite matrix as the product of a lower triangular matrix and its transpose. After the orthogonality restrictions, (3.3) the variance-covariance matrix of $\mathbf{u}_t$ becomes

$$\mathbf{\Sigma}_u = \mathbf{A}_0^{-1}\mathbf{I}_9\mathbf{A}_0^{-1\prime}.$$

It then appears that Choleski decomposition is possible if one sets the 36 upper diagonal elements of $\mathbf{A}_0^{-1}$ to zero, which completely solves the remaining identification issue. This identification strategy requires ordering the variables in $\mathbf{y}_t$ from the least to the most endogenous. Christiano, Eichenbaum, and Evans (2005) used this strategy to identify the monetary policy shock. These restrictions imply that the variables in $\mathbf{y}_{1t}$ do not respond contemporaneously to monetary policy shock and that monetary policy does not respond contemporaneously to changes in the other two variables in $\mathbf{y}_{2t}$.

The Vector Error Correction Representation

The structural VAR(2) model can be written as follows

$$\begin{aligned}\mathbf{A}_0\mathbf{y}_t &= \mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_2\mathbf{y}_{t-1} - \mathbf{A}_2\mathbf{y}_{t-1} + \mathbf{A}_2\mathbf{y}_{t-2} + \boldsymbol{\varepsilon}_t\\ &= (\mathbf{A}_1 + \mathbf{A}_2)\mathbf{y}_{t-1} - \mathbf{A}_2\Delta\mathbf{y}_{t-1} + \boldsymbol{\varepsilon}_t.\end{aligned}$$

Subtracting $\mathbf{A}_0\mathbf{y}_{t-1}$ from both sides of the above relation, one ends up with the vector error correction representation of the VAR model.

$$\mathbf{A}_0\Delta\mathbf{y}_t = \mathbf{\Pi}\mathbf{y}_{t-1} - \mathbf{A}_2\Delta\mathbf{y}_{t-1} + \boldsymbol{\varepsilon}_t, \tag{3.4}$$

where $\mathbf{\Pi} = \mathbf{A}_1 + \mathbf{A}_2 - \mathbf{A}_0$.

The vector error correction model (VECM) can be used to investigate the existence of *co-integration*, *i.e.* a long-run equilibrium relationship, between money, the short-term interest rate, and the macroeconomic non-policy variables. Co-integration between the variables can be tested for following Johansen (1988, 1991, 1995). There are two possible tests: the eigen and the trace tests. Table 3.3 displays the statistics and critical values for these two tests.

The eigen test suggests three co-integrating relations while the trace test suggests five. If the number of co-integrating relations matches the number of variables in the sub-vector $\mathbf{y}_{2t}$, one can then treat the sub-vector $\mathbf{y}_{1t}$ as *weakly exogenous* and set additional identifying restrictions (Boswijk, 1995; Ericsson, 1995; Pagan and Pesaran, 2008).

Table 3.3: Statistics from Johansen Co-integration Tests

	Eigen Test		Trace Test	
	Test	5% Critical	Test	5% Critical
H_0	Statistic	Value	Statistic	Value
r = 0	117.55	57.42	379.96	202.92
r = 1	79.3	52	262.41	165.58
r = 2	65.78	46.45	183.11	131.7
r = 3	38.7	40.3	117.33	102.14
r = 4	28.22	34.4	78.63	76.07
r = 5	23.57	28.14	50.41	53.12

The existence of three co-integrating relations implies there are two 9×3 matrices, $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, such that the linear combination $\boldsymbol{\beta}'\mathbf{y}_{t-1}$ is stationary and (3.4) becomes

$$\mathbf{A}_0 \Delta \mathbf{y}_t = \boldsymbol{\alpha}\boldsymbol{\beta}'\mathbf{y}_{t-1} - \mathbf{A}_2 \Delta \mathbf{y}_{t-1} + \boldsymbol{\varepsilon}_t. \tag{3.5}$$

The matrices $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ contain respectively the adjustment (error correction) parameters and the co-integrating vectors. These two matrices are not identifiable because, for any nonsingular 3×3 matrix $\mathbf{M}$, it turns out $\boldsymbol{\alpha}\mathbf{M} \times \left(\boldsymbol{\beta}\mathbf{M}^{-1}\right)' = \boldsymbol{\alpha}\boldsymbol{\beta}'$.

After partitioning the matrices $\mathbf{A}_0$, $\mathbf{A}_2$, $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, and the vector $\boldsymbol{\varepsilon}_t$ in a conformable way with the vector $\mathbf{y}_t = [\mathbf{y}_{1t}, \mathbf{y}_{2t}]'$, I have set some identifying restrictions.

$$\begin{aligned}
\begin{bmatrix} \mathbf{A}_{0,11} & \mathbf{0}_{6\times 3} \\ \mathbf{A}_{0,21} & \mathbf{I}_3 \end{bmatrix} \begin{bmatrix} \Delta \mathbf{y}_{1t} \\ \Delta \mathbf{y}_{2t} \end{bmatrix} &= \begin{bmatrix} \boldsymbol{\alpha}_1 \\ \boldsymbol{\alpha}_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{\beta}_1' & \boldsymbol{\beta}_2' \end{bmatrix} \mathbf{y}_{t-1} \\
&\quad - \begin{bmatrix} \mathbf{A}_{2,11} & \mathbf{A}_{2,12} \\ \mathbf{A}_{2,21} & \mathbf{A}_{2,22} \end{bmatrix} \begin{bmatrix} \Delta \mathbf{y}_{1,t-1} \\ \Delta \mathbf{y}_{2,t-1} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\varepsilon}_{1t} \\ \boldsymbol{\varepsilon}_{2t} \end{bmatrix}
\end{aligned} \tag{3.6}$$

The sub-matrix $\mathbf{A}_{0,12}$ is set to $\mathbf{0}_{6\times 3}$ because the variables in $\mathbf{y}_{1t}$ are weakly exogenous, *viz* $\mathbf{y}_2$ does not contemporaneously explains $\mathbf{y}_1$. Since the monetary authority does not respond immediately to changes in the other two elements in $\mathbf{y}_{2t}$, which are the real profit and money growth, I have set the sub-matrix $\mathbf{A}_{0,22}$ to $\mathbf{I}_3$.

Given $\mathbf{y}_2$ does not contemporaneously explains $\mathbf{y}_1$, one can also set both the adjustment parameters $\boldsymbol{\alpha}_1$ and the sub-matrix $\mathbf{A}_{2,12}$ to $\mathbf{0}_{6\times 3}$. The vector of long-run parameters $\boldsymbol{\beta}_2$ is set to $\mathbf{I}_3$ so that each of the variables in $\mathbf{y}_2$ enters one and only co-integrating relation. Then pre-multiplying (3.6) by the inverse of the matrix $\mathbf{A}_0$ gives the following marginal and conditional models [7]

[7] To find the inverse of the matrix $\mathbf{A}_0$, solve the following equation $\begin{bmatrix} \mathbf{A}_{0,11} & \mathbf{0}_{6\times 3} \\ \mathbf{A}_{0,21} & \mathbf{I}_3 \end{bmatrix} \times \begin{bmatrix} \mathbf{x}_1 & \mathbf{0}_{6\times 3} \\ \mathbf{x}_2 & \mathbf{x}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{I}_6 & \mathbf{0}_{6\times 3} \\ \mathbf{0}_{3\times 6} & \mathbf{I}_3 \end{bmatrix}$, for unknown $\mathbf{x}_i$, $i = 1 \ldots 3$.

$$\begin{bmatrix}\Delta\mathbf{y}_{1t}\\ \Delta\mathbf{y}_{2t}\end{bmatrix} = \begin{bmatrix}\mathbf{0}_{6\times3} & \mathbf{0}_{3\times3}\\ \boldsymbol{\alpha}_2\boldsymbol{\beta}_1' & \boldsymbol{\alpha}_2\end{bmatrix}\mathbf{y}_{t-1} - \begin{bmatrix}\mathbf{A}_{0,11}^{-1}\mathbf{A}_{2,11} & \mathbf{0}_{6\times3}\\ \mathbf{A}_{2,21}-\mathbf{A}_{0,21}\mathbf{A}_{0,11}^{-1}\mathbf{A}_{2,11} & \mathbf{A}_{2,22}\end{bmatrix}\begin{bmatrix}\Delta\mathbf{y}_{1,t-1}\\ \Delta\mathbf{y}_{2,t-1}\end{bmatrix} + \begin{bmatrix}\mathbf{A}_{0,11}^{-1} & \mathbf{0}_{6\times3}\\ -\mathbf{A}_{0,21}\mathbf{A}_{0,11}^{-1} & \mathbf{I}_3\end{bmatrix}\begin{bmatrix}\boldsymbol{\varepsilon}_{1t}\\ \boldsymbol{\varepsilon}_{2t}\end{bmatrix}, \tag{3.7}$$

where the first block represents the marginal model for the weakly exogenous variables $\mathbf{y}_{1t}$ and the second block represents the conditional model of $\mathbf{y}_{2t}$ given $\mathbf{y}_{1t}$. The VECM (3.7) turns out to be a state-space model where the marginal model is the state equation and the conditional model represents the observation equation. [8] Further restrictions can be added, setting $\mathbf{A}_{2,11}$ to $\mathbf{0}_{6\times3}$ as Jacobs and Wallis (2010) among others did, which would mean the exogenous variables follow a random walk. The restrictions on the matrix $\mathbf{A}_2$ have consequences on the matrix of long-run multipliers, which is the cumulative impulse response function.

To get the matrix of long-run multipliers, rearrange the structural VAR to have

$$(\mathbf{I}_9 - \boldsymbol{\Lambda}\mathrm{L})\Delta\mathbf{y}_t = \mathbf{A}_0^{-1}\boldsymbol{\alpha}\boldsymbol{\beta}'\mathbf{y}_{t-1} + \mathbf{A}_0^{-1}\boldsymbol{\varepsilon}_t,$$

where L designates the lag operator and $\boldsymbol{\Lambda} = -\mathbf{A}_0^{-1}\mathbf{A}_2$. For $\mathrm{L} = 1$, it follows that

$$\Delta\mathbf{y}_t = (\mathbf{I}_9 - \boldsymbol{\Lambda})^{-1}\mathbf{A}_0^{-1}\boldsymbol{\alpha}\boldsymbol{\beta}'\mathbf{y}_{t-1} + (\mathbf{I}_9 - \boldsymbol{\Lambda})^{-1}\mathbf{A}_0^{-1}\boldsymbol{\varepsilon}_t.$$

Given the linear combination $\boldsymbol{\beta}'\mathbf{y}_{t-1}$ is stationary and consequently equals zero in the long-run, it follows from the above relation that the matrix of long-run multiplier is

$$\boldsymbol{\mu} = (\mathbf{I}_9 - \boldsymbol{\Lambda})^{-1}\mathbf{A}_0^{-1}. \tag{3.8}$$

To illustrate the matrix of long-run multipliers, let us consider a simple case where $\boldsymbol{\Lambda}$ is block diagonal, *i.e.* $\mathbf{A}_{2,21} = \mathbf{A}_{0,21}\mathbf{A}_{0,11}^{-1}\mathbf{A}_{2,11}$. This matrix equals

$$\boldsymbol{\mu} = \begin{bmatrix}(\mathbf{A}_{0,11}+\mathbf{A}_{2,11})^{-1} & \mathbf{0}_{6\times3}\\ -(\mathbf{I}_3+\mathbf{A}_{2,22})^{-1}\mathbf{A}_{0,21}\mathbf{A}_{0,11}^{-1} & (\mathbf{I}_3+\mathbf{A}_{2,22})^{-1}\end{bmatrix}$$

Setting $\mathbf{A}_{2,12}$ to $\mathbf{0}_{6\times3}$ implies that this matrix is block lower triangular. As Pagan and Pesaran (2008) pointed out, the six structural shocks $\boldsymbol{\varepsilon}_{1t}$ associated to the exogenous variables $\mathbf{y}_{1t}$ are permanent shocks and the three structural shocks associated to $\mathbf{y}_{2t}$ are transitory. Transitory shocks have no long-run effect on exogenous variables. Given $\boldsymbol{\varepsilon}_{2t}$ is made up of stationary shocks, one can set its long-run impact on $\mathbf{y}_{2t}$ to zero, which implies $(\mathbf{I}_3+\mathbf{A}_{2,22})^{-1} = \mathbf{0}_{3\times3}$.

To sum up, three identifying restrictions can be set doing co-integration analysis. First, the permanent-transitory shock decomposition suggests that the last

[8] For some notes on state-space models, see among others, (Hamilton, 1994, chap 13).

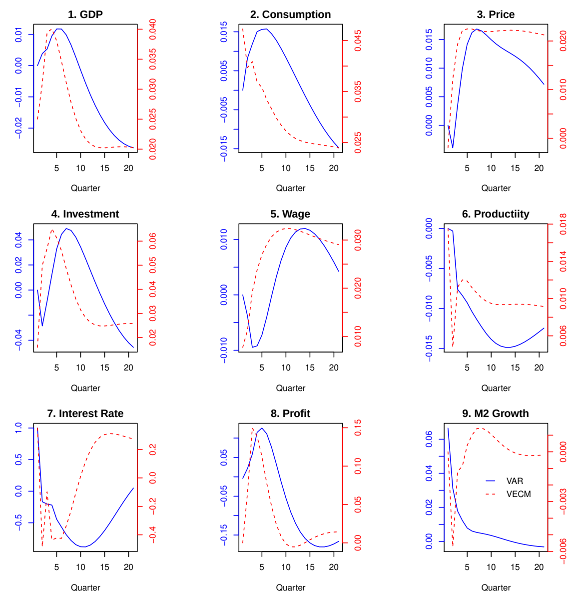

Figure 3.2: VAR- and VECM-Based Impulse Responses to an Expansionary Monetary Policy Shock (left scale: solid line, right scale: dashed line)

three columns in the matrix of long-run multipliers $\boldsymbol{\mu}$ are null, which means transitory shocks have no long-run effects. Second, the first sub-matrix in $\boldsymbol{\mu}$ is assumed to be block lower diagonal. Third, in the matrix of contemporaneous impacts, the partition $\mathbf{A}_{0,22}^{-1}$ is set to $\mathbf{I}_3$.

The Impulse Responses

I have computed impulse responses using in turn: the Choleski decomposition to identify the parameters in the structural VAR and the permanent-transitory shock decomposition to identify the parameters in the VECM. In the two cases, the impulse stems from an expansionary monetary policy, *i.e.* a negative shock of one standard deviation to interest rate. The R package *vars* has been used (Pfaff et al., 2008).

In Figure 3.2, what distinguishes the impulse responses from the structural VAR (the solid lines) from those from the VECM (the dotted lines) is that in the former case, monetary policy has no contemporaneous impact on the weakly exogenous variables whereas in the latter case, it has no long-run effect on these variables. This explains why the impacts from the VECMPfaff et al. (2008) are stronger and fade out faster than in the former case.

As Christiano, Eichenbaum, and Evans (2005) found using US data, it appears

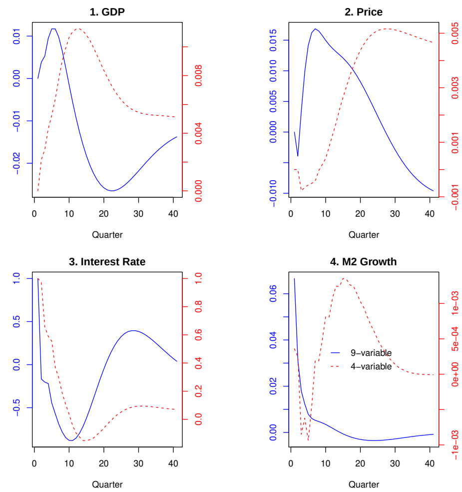

Figure 3.3: Impulse Responses to an Expansionary Monetary Policy Shock Based on a Nine- and a Four-Variable VAR (left scale: solid line, right scale: dashed line)

from the structural VAR model that, after an expansionary monetary policy shock,

1. the response of output, consumption, and investment is hump-shaped and peaks after about one and a half year,
2. the response of prices is also hump-shaped and peaks after about two years,
3. profits and money rise.

In Figure 3.3, I have compared the impulse responses of the nine-variable VAR to those of a four-variable VAR used by Sims (1992) and Serletis and Molik (2000). The four-variable VAR model is of order five. The responses of output and prices to the expansionary monetary shock are also hump-shaped but peaks later, respectively after about three and seven years. The information criteria are much in favor of the nine-variable VAR model.

It emerges from the variance decompositions that monetary policy only explains a very little share of the fluctuations in the variables.

3.3 Price Stickiness

According to the new-Keynesian theory, monetary disturbances affect the real economic activity because prices are sticky. There are four determinants of price adjustment: inflation inertia, level or Phillips-curve adjustment, rate-of-change adjustment, and inflation shock (Gordon, 1990). According to the inertia hypothesis,

Table 3.4: OLS Estimates of Price Adjustment Parameters, Canada, 1981:Q1-2015:Q4

	AR(2)		AR(3)		ARMA(2,1)	
Variable	Estimate	t-ratio	Estimate	t-ratio	Estimate	t-ratio
Intercept	.002	4.292	.002	3.185	.003	3.513
$\Delta \ln P_{t-1}$	.088	1.315	.092	1.375	.679	2.176
$\Delta \ln P_{t-2}$	-.007	-.121	-.064	-.982	.015	.092
$\Delta \ln P_{t-3}$			.118	2.028		
$\ln Y_t - \ln Y_{gt}$	.072	2.58	.066	2.348	- .09	-1.896
$\Delta (\ln P_t Y_t - \ln Y_{gt})$	.55	12.363	.556	12.571	.563	13.814
ε_{t-1}					-.356	-1.158
R^2	.621		.628		.65	
AIC	-1077		-1070		-1096	
SC	-1059		-1050		-1074	

$t_{2.5\%} = 1.978$, $t_{5\%} = 1.666$

past inflation helps predict current inflation if the proportion of firms adjusting their prices is low—see relation (7.14) on page 59. The level hypothesis relates inflation to output gap, *i.e.* the level of output relative to its trend. According to the rate-of-change hypothesis, fluctuations in prices are a constant fraction of the excess nominal GDP growth relative to natural real GDP. I have used the following econometric model to test for the significance of each of these determinants

$$\Delta \ln P_t = a_0 + \sum_{i=1}^{3} a_i \Delta \ln P_{t-i} + a_4 \ln \frac{Y_t}{Y_{gt}} + a_5 \Delta \ln \frac{P_t Y_t}{Y_{gt}} + a_6 \varepsilon_{t-1} + \varepsilon_t, \qquad (3.9)$$

where the variables P_t, Y_t, Y_{gt}, and ε_t respectively designate the GDP deflator, the real and natural real GDP, and the inflation shock. I have computed the natural real GDP using the HP filter. The parameters a_1, a_2, and a_3 measure the extent of inertia. The higher they are, the higher is inertia, and the stickier prices are. The level parameter a_4 is expected to be positive. If prices are sticky, they will be less sensitive to the cyclical fluctuations in real output. The rate-of-change parameter $0 \leq a_5 \leq 1$ measures the degree of nominal rigidity. A small a_5 indicates stickiness in prices and consequently a large fluctuations in real GDP.

OLS estimates of (3.9), the price adjustment equation, are displayed in Table 3.4. I have used three hypotheses to model inflation inertia. Inflation follows: (1) an autoregressive process of order 2, in short an $AR(2)$ process, (2) an autoregressive process of order 3, and (3) an autoregressive process of order 2 and a moving average process of order 1, in short $ARMA(2,1)$.

It appears in Table 3.4 that, in the absence of the lagged inflation shock ε_{t-1}, all the inertia parameters are very low. In the $ARMA(2,1)$ model, inflation response to its first lag has increased from .09 to .679 and has become statistically significant. The adjusted R^2 coefficients and the information criteria suggest that the ARMA process explains a higher proportion of the observed changes in the price level.

The level effect is low and statistically significant but its sign depends on the presence of the lagged inflation shock. It is positive, as expected, in the $AR(1)$ or $AR(2)$ models but negative in the $ARMA(2,1)$ model. Galı and Gertler (1999), in estimating an econometric model of the form $\Delta \ln P_t = \beta_1 \ln (Y_t/Y_{gt}) + \beta_2 \mathrm{E}_t \Delta \ln P_{t+1} + \varepsilon_t$, where E_t designates the expectation operator, also find a negative sign associated to the level effect. They use quarterly US data. They deem detrended real GDP is a poor proxy for output gap and recommends instead the use of marginal costs as the new-Keynesian Phillips curve suggests (see Appendix B.2).

Unlike the other parameters, the rate-of-change parameter does not change much across the models. It indicates that 56.3% of the growth in nominal demand is incorporated into the rate of inflation. The other 43.7% results in cyclical fluctuations in real GDP. The rate-of-change parameters is statistically significant.

4 The Inflation Tax Model

Cooley and Hansen (1989, 1991, 1992, 1995, 1998) introduced money into the indivisible labor RBC model using the cash-in-advance (CIA) constraint. Three types of agents populate their economy: households, firms, and the monetary authority. Households have their preferences defined over leisure and two types of consumption goods: *cash* and *credit goods*. Money is absolutely required to purchase cash goods whereas credit goods are financed out of the current period's income. Each time period, households have to keep money aside for next period's purchases of cash goods. Because inflation acts as a tax on money holdings, when households anticipate a higher inflation, to avoid losing purchasing power, they reduce their cash balances and consequently their future consumption of cash goods. Cash, credit, and investment goods are produced by firms using the same technology. Government conducts the monetary policy.

This model is used to investigate how anticipated inflation affects the long-run values of real variables and whether the way money is supplied plays a role in accounting for business cycles.

4.1 The Households

The representative household's preferences are represented by a quasi-linear utility function. This utility is logarithmic in the consumption of cash goods c_{1t} and credit goods c_{2t} and linear in leisure. [9]

$$\mathcal{U}\left(c_{1t}, c_{2t}, l_t\right) = a \ln c_{1t} + (1-a)\ln c_{2t} + \Upsilon(1-l_t) \tag{4.1}$$

The parameters $0 < a < 1$ and $\Upsilon > 0$ in the utility function are respectively the relative weight of the cash good and the relative weight of leisure. The variable l_t is the household's labor supply and $1 - l_t$ is consequently his leisure.

The representative household faces three resource constraints

$$p_t c_{1t} = m_t + \tau_t \tag{4.2a}$$

$$p_t\left(c_{1t} + c_{2t} + i_t\right) + m_{t+1} = p_t\left(w_t l_t + r_t k_t\right) + m_t + \tau_t \tag{4.2b}$$

$$k_{t+1} = (1-\delta)k_t + i_t \tag{4.2c}$$

The variables p_t, r_t, and w_t are respectively the price level, real interest rate and wage. The quantities i_t and k_t designate respectively the real investment and capital stock. The nominal variables m_t and τ_t designate respectively his cash holdings, and the lump-sum transfer of new cash issued by the monetary authority. The parameter δ is the depreciation rate of physical capital.

Relation (4.2a) is the CIA constraint. It says the household's spending on cash goods equals the currency he carried over from the previous period and new cash injected by the government. Relation (4.2b) is his nominal budget constraint. The third constraint is the law of motion of capital.

The following three relations are derived from his optimizing behavior.

$$\Upsilon c_{2t} = (1-a)w_t \tag{4.3a}$$

$$\beta \mathrm{E}_t\left[(1 + r_{t+1} - \delta)\frac{c_{2t}}{c_{2,t+1}}\right] = 1 \tag{4.3b}$$

$$\beta \mathrm{E}_t\left(\frac{c_{2t}}{c_{1,t+1}}\frac{p_t}{p_{t+1}}\right) = \frac{1-a}{a} \tag{4.3c}$$

Relation (4.3a) governs the intra-temporal trade-off between the consumption of the credit good and leisure. Relation (4.3b) is the Euler condition for the optimal consumption of the credit good. Constraint (4.2a) suggests that condition (4.3c) is the money demand equation. In the CIA model, money demand is not sensitive to interest rate but rather depends on inflation rate. The term p_{t+1}/p_t in (4.3c) is thr gross inflation rate. When the representative household anticipates a higher inflation, he immediately raises his consumption of credit goods and keeps aside less money for next period's consumption of cash goods.

[9] Linearity in leisure is based on the assumption that labor is indivisible, *viz* a household either works full-time or is unemployed. Indivisible labor was introduced by Hansen (1985) following the observation that most of the cyclical fluctuations in the aggregate hours worked are due to changes in the number of workers rather than the average hours worked. Much details on this can be found in the first volume of this paper.

4.2 The Firms

Firms produce the final good Y_t using physical capital K_t and labor L_t as inputs. The production technology is Cobb-Douglas, exhibits constant returns to scale, and is subject to stochastic technological changes.

$$Y_t = K_t^{\alpha}(z_t L_t)^{1-\alpha}, \quad 0 < \alpha < 1 \tag{4.4}$$

The exogenous technological change z_t, also known as total factor productivity (TFP), grows deterministically at the rate γ_z and randomly at the rate $\tilde{z}_t$. The random variable $\tilde{z}_t$ follows an $AR(1)$ process.

$$\begin{aligned} z_t &= \gamma_z^t \exp(\tilde{z}_t) \\ \tilde{z}_t &= \rho_z \tilde{z}_{t-1} + \epsilon_{zt}, \quad \epsilon_{zt} \sim \mathcal{N}\left(0, \sigma_z^2\right), \end{aligned} \tag{4.5}$$

where ρ is the persistence parameter and ϵ_{zt} is the technology shock. The technology shock is normally distributed with mean zero and standard deviation σ_z.

Firms are perfectly competitive and consequently maximize their profit taking the real interest rate and wage as given

$$\begin{aligned} &\max_{K_t,\ L_t} \ K_t^{\alpha}(z_t L_t)^{1-\alpha} - r_t K_t - w_t L_t \\ &K_t : \qquad r_t = \alpha \frac{Y_t}{K_t} \\ &L_t : \qquad w_t = (1-\alpha)\frac{Y_t}{L_t}. \end{aligned} \tag{4.6}$$

It has turned out from this optimization problem that the marginal product of each input equals its rental price.

4.3 The Monetary Authority

It supplies money M_t exogenously. Money supply can either follow the Friedman rule, *i.e.* grow at the constant rate γ_m, or a geometric random walk process. The general specification of the monetary rule is

$$M_{t+1} = \gamma_m \exp(\xi_t) M_t \tag{4.7a}$$

$$\xi_t = \rho_m \xi_{t-1} + \epsilon_{mt}, \quad \epsilon_{mt} \sim \mathcal{N}\left(0, \sigma_m^2\right). \tag{4.7b}$$

The money supply shock ϵ_{mt} occurs only when monetary policy is conducted erratically.

The government budget constraint is

$$T_t = M_{t+1} - M_t. \tag{4.8}$$

4.4 The General Equilibrium

The general equilibrium consists of a set of prices $\{(p_t, r_t, w_t)\}_{t=0}^{\infty}$, a state of the world $\{(z_t, \xi_t)\}_{t=0}^{\infty}$, an allocation $\{(c_{1t}, c_{2t}, i_t, l_t, k_t, m_t, \tau_t)\}_{t=0}^{\infty}$ for the representative household, an allocation $\{(K_t, L_t, Y_t)\}_{t=0}^{\infty}$ for firms, and an allocation $\{(M_t, T_t)\}_{t=0}^{\infty}$ for the monetary authority such that:

i. $\{(c_{1t}, c_{2t}, i_t, l_t, k_t, m_t, \tau_t)\}_{t=0}^{\infty}$ solves relations (4.2) and (4.3),

ii. $\{(K_t, L_t, Y_t)\}_{t=0}^{\infty}$ solves relations (4.4) and (4.6),

iii. $\{(M_t, T_t)\}_{t=0}^{\infty}$ solves relations (4.7a) and (4.8),

iv. $\{(z_t, \xi_t)\}_{t=0}^{\infty}$ is governed by relations (4.5) and (4.7b),

v. capital, labor, and money markets clear, *i.e.* $k_t = K_t$, $l_t = L_t$, $m_t = M_t$, and $\tau_t = T_t$.

The share of time allocated to labor and the real interest rate are stationary variables. Along the balanced growth path (BGP), consumption, investment, physical capital, output, and real wage are constrained to grow at the same rate γ_z as technological change. Price grows at the rate π. Money and government lump-sum transfer to households grow at the rate $\gamma_m = \pi\gamma_z$.

4.5 The Calibration

Following Cooley and Prescott (1995) and Gomme and Rupert (2007), I have consistently assigned values to the parameters using survey data, sample averages, and regression results.

The Capital Share It has been computed using GDP income-based estimates. Some of these estimates, such as the net operating surplus and the consumption of fixed capital, unambiguously remunerate the capital input and others, such as the compensation of employees, remunerate the labor input. The rest of the estimates are ambiguous in the sense they are both capital and labor incomes. An unknown proportion α of the ambiguous incomes is therefore apportioned to the capital input. At the same time, both the the unambiguous capital income and the share of capital in the ambiguous incomes represent a proportion α of GDP. This leads to solve the following equation for α

$$\alpha = \frac{\text{Unambiguous capital incomes } + \ \alpha\text{Ambiguous incomes}}{\text{GDP}},$$

which implies

$$\alpha = \frac{\text{Unambiguous capital incomes}}{\text{GDP} - \text{Ambiguous incomes}}.$$

Over the sample period 1981:Q1-2015:Q4, α averaged .329.

Table 4.1: OLS Estimates of Some Growth Rates, Canada, 1981:Q1-2015:Q4

Variable	Estimate	t-ratio
GDP Deflator	.007	10.146
CPI	.007	16.239
Real GDP	.006	8.729
M1	.022	14.547
Technological Change	.003	3.751

$t_{2.5\%}(138) = 1.977$

The Capital Depreciation Rate Its is the ratio of the current period's capital depreciation to the previous period's capital stock. The average annual depreciation rate is 5.9%, which implies a quarterly rate of 1.5%.

The Growth Rates Table 4.1 provides estimates of the quarterly average growth rate of prices, real GDP, M1 and technological change. Two measures of prices are used: GDP deflator and CPI excluding the most volatile components. The estimates of technological change are residuals from the growth accounting exercise

$$\begin{aligned}\Delta z_t &= \frac{1}{1-\alpha}\Delta \ln Y_t - \frac{\alpha}{1-\alpha}\Delta \ln K_t - \Delta \ln L_t \\ &= \ln \gamma_z + \Delta \tilde{z}_t.\end{aligned}$$

The persistence parameter ρ_z is set to .95 and it follows from the growth accounting exercise that σ_ϵ, the standard deviation of the technology shock, is .009.

Relation (4.9) presents OLS estimates of the stochastic process followed by M1.

$$\begin{aligned}\widehat{\Delta \ln m_t} &= .009 + .567\Delta \ln m_{t-1} \\ &\quad\ (4.81) \quad\ (8.04) \\ \bar{R}^2 &= .317 \qquad \sigma_m = .015\end{aligned} \tag{4.9}$$

The slope parameter in (4.9) is the estimate of ρ_m.

It appears in Table 4.1 that: (1) the growth rate of real GDP (output) differs from the rate of technological change as assumed, and (2) the relation $\gamma_m = \gamma_z \pi$ does not hold empirically. I have set π to 1.007, the gross growth rate of both the CPI and GDP deflator, and constrained γ_m to equal the product of π and γ_z.

The Discount Factor The average investment-output ratio is .162. Along the BGP, $i = (\gamma_z + \delta - 1)k$. This implies a capital-output ratio of 9.113. Evaluating then (4.3b) along the BGP, one has

$$\beta = \frac{\gamma_z}{1 + \frac{\alpha}{k/y} - \delta},$$

which equals .982.

The Share of Cash in Households' Consumption Following Cooley and Hansen (1995), I have defined money as M1. All the purchases made using cash, debit and stored-value cards, and personal check are consequently considered as cash goods. It emerges from the 2009 and 2013 methods-of-payment survey conducted by the Bank of Canada that these instruments represent respectively 59.2% and 51.9% of the value of households' transactions (Arango, Welte, et al., 2012; Henry, Huynh, Shen, et al., 2015). [10] This averages 50.6%. Calling this average v, one has

$$\begin{aligned} v_t &= \frac{p_t c_{1t}}{p_t c_{1t} + p_t c_{2t}} \\ &= \frac{m_{t+1}}{m_{t+1} + p_t c_{2t}}. \end{aligned}$$

Using then (4.3c) to eliminate $p_t c_{2t}$, one has

$$v_t = \frac{a\beta m_{t+1}}{a\beta m_{t+1} + (1-a)\mathrm{E}_t m_{t+2}}.$$

Given (4.7) the above relation becomes

$$v = \frac{a\beta}{a\beta + (1-a)\gamma_z \pi}, \tag{4.10}$$

where $\gamma_z \pi$ equals γ_m, the growth rate of money. Finally, solving (4.10) for a gives

$$a = \frac{\gamma_z \pi v}{\beta(1-v) + \gamma_z \pi v},$$

which equals .562.

Relation (4.10) sheds a light on the fall observed in the share of cash in the value of households transactions. This results from a decrease in β, an increase in either γ_z or π. The parameter β is the weight households put on their future consumption. Actually, an increase in time preference, *viz* impatience, will cause a decrease in β and by so doing lead households to consume more credit goods today and less cash goods tomorrow Second, households lose purchasing power when inflation π rises. They consequently hold less money and consume less cash goods. Third, technological progress raises aggregate output and consequently households' incomes, which rather increases their consumption of credit goods.

[10] These means of payments accounted for 80.7% and 49.9% of the volume of transactions respectively in 2009 and 2013.

Table 4.2: The Baseline Parameters of the Inflation Tax Model

Households	a	Relative Weight of Cash Good	.562
	β	Discount factor	.982
	π	Inflation rate	1.007
	Υ	Leisure weight	2.663
Firms	α	Capital share	.329
	γ_z	Growth rate of TFP	1.003
	δ	Depreciation rate	.015
	ρ_z	Persistence parameter of TFP	.95
	σ_z	Standard deviation of TFP shock	.009
Monetary	γ_m	Money growth rate	1.01
Authority	ρ_m	Persistence parameter of money supply	.567
	σ_m	Standard deviation of money supply shock	.015

The Leisure Weight According to the labor force survey of Statistics Canada, households allocate weekly, on average, 34.3 hours to labor. Censuses data indicate that the average weeks worked is 42.57. It also emerges from the general social survey that they daily allocate 10.45 hours to personal care. It follows that labor represents 29.6% of the discretionary time, *i.e.* the time not allocated to personal care. After computing the steady state value of all variables, the leisure weight turns out to be 2.663. The baseline values of all the parameters are reported in Table 4.2.

4.6 The Numerical Solution and Findings

Before solving numerically the model, all the non-stationary variables are normalized, *i.e.* divided by their gross growth factors. Relations (A.5) present the normalized DSGE model. I have then performed some Monte Carlo experiments and computed impulse responses using the package Dynare in Matlab (see Griffoli, 2007, for details on Dynare). Beforehand, Dynare linearized the model using a second-order approximation around the steady state.

The Monte Carlo experiments have consisted in simulating 100 times the model over 140 quarters, which corresponds to the length of the time series used to calibrate the model. The fluctuations in the variables are caused, in turn, by technology shocks and both technology and money growth shocks. For each of the 100 samples, I have reconstructed the non-stationary variables and detrended all variables using the HP filter. Some summary statistics have then been computed using the detrended series. These summary statistics, which are: the standard deviations, correlations with cyclical output and money growth, and first-order autocorrelations, are finally averaged.

In Table 4.3, one can compare the cyclical behavior of the Canadian economy to that of the inflation tax model. Under the Friedman rule, *i.e.* when money

Table 4.3: Cyclical Behavior of the Canadian Economy and the Inflation Tax Economy with Constant and Autoregressive Money Growth, Percentage Deviation from Trend of Key Variables, 140 Observations

	Canadian Economy				Inflation Tax Economy, Money Growth:						
					Constant			Autoregressive			
Variable	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(1)	(2)	(3)	(4)
Output (GDP)	1.56	1	.89	-.12	1.54	1	.7	1.49	1	.68	-.04
Consumption	1.14	.84	.85	-.16	.47	.88	.8	.64	.65	.6	-.7
Investment	4.9	.75	.88	-.21	7.51	.99	.69	7.57	.94	.65	.26
Hours	1.41	.9	.9	-.21	1.15	.98	.69	1.12	.98	.66	-.09
Productivity	68	.42	.62	.06	.47	.88	.8	.46	.87	.79	.1
M1	4.4	.4	.92	.02	.07	.03	.96	3.42	-.001	.89	.22
Price	1.07	-.001	.8	.01	.47	-.87	.8	3.7	-.12	.81	.44

Columns (1) Percentage standard deviations, columns (2) Correlation coefficient with output, columns (3) First-order autocorrelation coefficient, and columns (4) Correlation coefficient with money.

growth is constant, the model replicates most of the cyclical fluctuations observed in output, hours worked, and productivity. Its cyclical behavior is similar to that of the cashless indivisible labor model (see Accolley, 2016, for the comparison). However it explains a higher proportion of the fluctuations in consumption and productivity.
The cyclical behavior of the real variables does not change much when money growth becomes stochastic, except that consumption has become more volatile and less pro-cyclical. The money growth shock has improved the ability to replicate the volatility of money. But the predicted correlation between money and output is almost zero. Allowing technology and money growth shocks to be positively correlated makes money pro-cyclical but lowers the volatility of consumption, labor, and output. Because the price level is flexible, its volatility is too high.

The model does not well replicate the correlations with money growth. The correlation between money growth and investment is not correctly signed. The generated correlation between money growth and consumption is too high. So are the ones between money growth and the nominal variables. Moreover, the generated correlation between hours worked and productivity is .74 whereas the actual one is -.01.

I have performed other Monte Carlo simulations to study the impacts of a greater anticipated inflation or persistence of money growth. It has turned out that, when money growth is constant, a higher anticipated inflation has no impact on the cyclical behavior of real variables but increases the volatility of nominal variables. In Table 4.4, I have reported the cyclical behavior of the model following a 5% increase in the parameters π and ρ_m when money growth is autoregressive. A higher expected inflation lowers the volatility of consumption. A rise in the

Table 4.4: Cyclical Behavior of the Canadian Economy and the Inflation Tax Economy with Autoregressive Money Growth, Percentage Deviation from Trend of Key Variables, 140 Observations

	Canadian Economy			Inflation Tax Economy, 5% Increase in:					
				Inflation			Money Persistence		
Variable	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
Output (GDP)	1.56	1	.89	1.49	1	.68	1.49	1	.68
Consumption	1.14	.84	.85	.63	.65	.6	.66	.64	.6
Investment	4.9	.75	.88	7.56	.94	.65	7.58	.93	.65
Hours	1.41	.9	.9	1.12	.98	.66	1.12	.98	.66
Productivity	68	.42	.62	.46	.87	.79	.46	.86	.79
M1	4.4	.4	.92	5.61	.01	.94	3.57	-.003	.89
Price	1.07	-.001	.8	5.41	-.06	.89	3.85	-.12	.82

Columns (1) Percentage standard deviations, columns (2) Correlation coefficient with output, columns (3) First-order autocorrelation coefficient.

persistence of money growth raises the volatility of real variables and lowers their correlation with output. Figure 4.1 shows the responses of the inflation tax economy to money growth shocks for two different values of ρ_m.

4.7 The Welfare Analysis

A higher expected inflation only affects the short-run dynamics of consumption and nominal variables but it lowers the steady state and the average values of consumption, investment, hours worked, output, and money as Table 4.5 shows. When expected inflation rate is 5.7% instead of zero, consumption, investment, hours worked, and output all are, at steady state, 3% lower. The fall in the consumption of cash goods is 5.4%. Since all the other parameters have remained unchanged, the fall in consumption is only attributable to cash goods. Credit goods, real interest rate, and wage do not change. Even though hours worked fall and leisure consequently increases, households suffer an instantaneous welfare loss. This loss is calculated solving the following equation

$$\mathcal{U}\left(\tilde{c}_1, \tilde{c}_2, l\right)\big|_{\pi=1} = \mathcal{U}\left(\tilde{c}_1^* + \Delta\tilde{c}, \tilde{c}_2, l^*\right)\big|_{\pi=\{1.007,1.057\}},$$

with $(\tilde{c}_{1t}, \tilde{c}_{2t}) = (c_{1t}, c_{2t})/\gamma^t$. The left-hand side (lhs) element is the level of utility attained at steady steady when the inflation rate is zero and the variables $\tilde{c}_1^*$ and l^* on the right-hand side (rhs) are respectively the steady state consumption and hours worked when the inflation rate is either .7% or 5.7%. The compensating variation $\Delta\tilde{c}$ indicates, when trend inflation occurs, by how much the consumption of cash good has to increase to keep the utility unchanged. The welfare costs of inflation in Table 4.5 are the compensating variation expressed as a percentage of aggregate consumption or output. They are not that high but increase along with

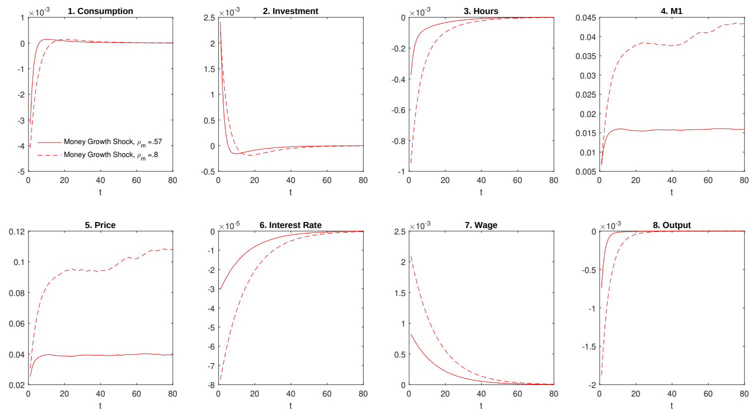

Figure 4.1: Impulse Responses to Money Growth Shock, Deviation form Steady State, the Inflation Tax Model

inflation. When expected inflation is 5.7%, the welfare cost of inflation represents .89% of steady state consumption.

I have finally investigated a new issue: the impacts of changes in the weight of cash good. This is motivated by the fall observed over time in the use of cash as a method of payment. A fall in a lowers the consumption of cash good and raises the consumption of credit good. Aggregate consumption increases because the rise in the consumption of credit good is higher. While investment, hours worked, and output increase, the demand of money decreases as the result of the fall in the consumption of the cash good. The relationship between welfare and the use of cash is U-shaped as it appears in Table 4.6 and Figure 4.2.

5 The RBC Model with Endogenous Money Supply

An alternative attempt to explain money demand is the shopping time model. It represents money as an intermediate good that households keep to reduce the time spent shopping for consumption goods. Gavin and Kydland (1999) introduced this model into the neoclassical growth framework to explain the relative instability in the cyclical behavior of US nominal variables due to a major change in the objectives of monetary policy that took place in 1979.

The model consists of households, firms, and a monetary authority. Households' behavior is described by the shopping time model. Firms behave the same way as in the inflation tax model— see Subsection 4.2. The monetary authority could supply money in various ways: either exogenously or endogenously taking

Table 4.5: Steady State and Average Values Associated with Different Values of Expected Inflation, the Inflation Tax Model

	π=1		π=1.007		π=1.057	
Variable	(1)	(2)	(1)	(2)	(1)	(2)
Output	.878	.879	.875	.876	.851	.853
Consumption	.736	.737	.733	.734	.714	.714
Investment	.142	.143	.142	.142	.138	.138
Hours	.297	.297	.296	.296	.288	.288
Wage	1.983	1.984	1.983	1.984	1.983	1.984
M1	.409	.414	.403	.408	.366	.37
Welfare	.879		.878		.872	
Welfare Costs: $100 \times \frac{\Delta \tilde{C}}{C}$	0		.08		.7	
$100 \times \frac{\Delta \tilde{C}}{Y}$	0		.067		.58	

Columns (1) Steady state value, columns (2) Average value of the simulated series

Table 4.6: Steady State and Average Values Associated with Different Values of the Weight of Cash Good, the Inflation Tax Model

	a=.8		a=.562		a=.4	
Variable	(1)	(2)	(1)	(2)	(1)	(2)
Output (GDP)	.869	.87	.875	.876	.879	.88
Consumption	.728	.729	.733	.734	.737	.737
Investment	.141	.141	.142	.142	.142	.143
Hours	.294	.294	.296	.296	.297	.298
Wage	1.983	1.984	1.983	1.984	1.983	1.984
M1	.574	.581	.403	.408	.29	.37
Welfare	1.062		.878		.892	

Columns (1) Steady state value, columns (2) Average value of the simulated series

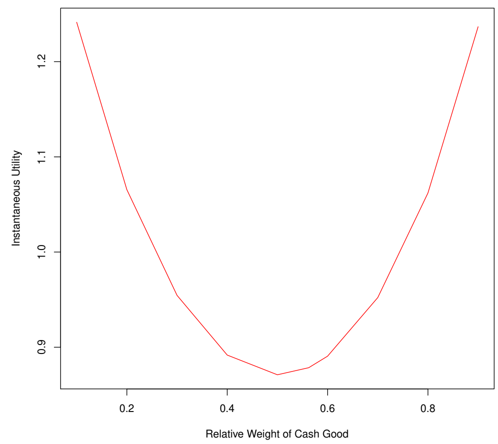

Figure 4.2: The Weight of Cash Good and Welfare, the Inflation Tax Model

into account the past level of output and the current stock of money. [11] Gavin and Kydland showed that money and prices could be either pro-cyclical or countercyclical depending on how intensely the monetary authority responds to last period's output level or current period money stock. My goal is to use this model to explain the changes in the cyclical behavior of variables that occurred after the Bank of Canada set the 2% inflation-control target in February 1991. These changes are reported in Table 2.3.

5.1 The Households

The representative household has preferences defined over consumption and leisure. Leisure is the share of time that has not been allocated to labor or shopping. These preferences are represented by a constant inter-temporal elasticity of substitution utility function.

$$\mathcal{U}(c_t, l_t, s_t) = \frac{1}{e} c_t^e \left(1 - l_t - s_t\right)^{ve}, \tag{5.1}$$

where $e < 1$ and $v > 0$ are respectively the substitution parameter and the relative leisure weight. The variable s_t designates the share of time allocated to shopping.

He faces three constraints, which are respectively the transaction or shopping

[11] In another version of the same model, the monetary authority follows an interest rate rule (Dittmar, Gavin, and Kydland, 2005).

technology, the budget constraint, and the law of motion of capital

$$s_t = \omega_0 - \omega_1 \left(\frac{m_t}{p_t c_t}\right)^{\omega_2} \tag{5.2a}$$

$$p_t (c_t + i_t) + m_{t+1} = p_t (w_t l_t + r_t k_t) + m_t + \tau_t \tag{5.2b}$$

$$k_{t+1} = (1-\delta) k_t + i._t \tag{5.2c}$$

Restricting in (5.2a) the curvature parameter, ω_2, to be negative and ω_1 to have the same sign ensure the shopping time positively depends on the consumption velocity of money $p_t c_t / m_t$, *viz* the higher the money holdings, the lower their velocity or how often transactions occur, which reduces shopping time. The following three conditions are derived from the representative household's utility maximization problem

$$1 - l_t - \omega_0 + (1 - \upsilon\omega_2)\omega_1 \left(\frac{m_t}{p_t c_t}\right)^{\omega_2} = \upsilon \frac{c_t}{w_t} \tag{5.3a}$$

$$\beta \mathrm{E}_t \left[(1 + r_{t+1} - \delta) \left(\frac{c_{t+1}}{c_t}\right)^{e} \left(\frac{1 - l_{t+1} - s_{t+1}}{1 - l_t - s_t}\right)^{\upsilon e - 1} \frac{w_t}{w_{t+1}} \right] = 1 \tag{5.3b}$$

$$\begin{aligned} \beta \mathrm{E}_t \left[\frac{p_t}{p_{t+1}} \frac{w_t}{w_{t+1}} + \omega_1 \omega_2 \frac{p_t w_t}{m_{t+1}} \left(\frac{m_{t+1}}{p_{t+1} c_{t+1}}\right)^{\omega_2} \right] \times \\ \left(\frac{c_{t+1}}{c_t}\right)^{e} \left(\frac{1 - l_{t+1} - s_{t+1}}{1 - l_t - s_t}\right)^{\upsilon e - 1} = 1. \end{aligned} \tag{5.3c}$$

Relations (5.3a) through (5.3c) respectively govern the optimal consumption-leisure, the consumption-investment, and the money-time trade-offs.

5.2 The Monetary Authority

The general specification of the monetary policy rule is

$$\Delta \ln M_{t+1} = \nu_y \ln \frac{Y_{t-1}}{Y} + \nu_m \ln \frac{M_t}{M} + \xi_t \tag{5.4}$$

where ν_y and ν_m are respectively the response to last period's output and current money stock, $\xi_t \sim \mathcal{N}\left(0, \sigma_\xi^2\right)$ is the money growth shock. The parameters M and Y are respectively the values of money and output along the BGP. For $\nu_y = 0$, money supply is exogenous and for $\nu_y = \nu_m = 0$, it follows a random walk.

5.3 The Calibration

The dynamic system consists of relations (5.2) through (5.4) and relations (4.4) through (4.6). As earlier, output, consumption, investment, capital, and wage are constrained to grow at the gross rate γ_z and prices grow at the gross rate π. The values of the parameters α, γ_z, δ, π, ρ_z, and σ_z are taken from Table 4.2. I have set the parameters ω_2 to -1.

The average consumption velocity of money, pc/m, is 3.04. The values of β and ω_1 are set evaluating along the BGP (5.3b) and (5.3c)

$$\beta = \frac{\gamma_z^{1-e}}{1 + \frac{\alpha}{k/y} - \delta}$$

$$\omega_1 = \frac{\pi\gamma_z^{1-e} - \beta}{\beta\omega_2(1-\alpha)\frac{y}{l}} \left(\frac{m}{p}\right)^{1-\omega_2} c^{\omega_2},$$

which gives, after setting e to .3, $\beta = .981$ and $\omega_1 = -.001$.

According to the general social survey of Statistics Canada, a households allocate, on average, .8 hour a day to shopping for consumption goods and services. This represents 5.9% of their discretionary time. Given the average shopping time, one can solve (5.2a) and (5.3a) for υ and ω_0, which respectively turn out to equal 1.728 and .0556.

The parameters ν_y and ν_m are chosen by the monetary authority. To have an idea of their historical values, I have estimated (5.4) abstracting from the trend regressors.

$$\begin{aligned} \Delta \widehat{\ln M_{t+1}} &= .004 \ln Y_{t-1} - .0024 \ln M_t \\ &\quad\;\; (1.787) \qquad\quad (-1.028) \\ \bar{R}^2 &= .602 \qquad\qquad \sigma_\xi = .018 \end{aligned} \tag{5.6}$$

The intercept term in (5.6), which represents $-(\nu_y \ln Y + \nu_m \ln M)$, is not statistically significant and has been dropped.

5.4 The Welfare Costs of Inflation

The welfare costs of inflation are calculated solving the following equation

$$\mathcal{U}\left(\tilde{c}, l, s\right)\big|_{\pi=1} = \mathcal{U}\left(\tilde{c}^* + \Delta\tilde{c}, l^*, s^*\right)\big|_{\pi=\{1.007,1.057\}}, \tag{5.7}$$

where $\tilde{c}$ denotes the normalized consumption at steady state when trend inflation is zero. The variables with asterisk are the consumption and use of time when trend inflation is non-zero. The compensating variation is denoted by $\Delta\tilde{c}$.

At steady state, a 5.7% trend inflation causes a .8% fall in consumption, investment, hours worked, and output. This is much lower than the 3% fall generated by the inflation tax model in Section 4. However, comparing Table 5.1 to Table 4.5, it turns out that the welfare cost of inflation is higher in this model. It represents

Table 5.1: Steady State Values Associated with Different Values of Expected Inflation, the RBC Model with Endogenous Money

Variable	π=1	π=1.007	π=1.057
Output	.876	.875	.869
Consumption	.734	.733	.728
Investment	.142	.142	.141
Hours	.296	.296	.294
Wage	1.983	1.983	1.983
M1	.277	.241	.148
Welfare	1.661	1.66	1.655
Welfare Costs: $100 \times \frac{\Delta\tilde{C}}{C}$	0	.144	.902
$100 \times \frac{\Delta\tilde{C}}{Y}$	0	.121	.756

.9% of steady state consumption versus .7% in the inflation tax model. The fact is that, in the inflation tax model, inflation only affects one type of consumption: the cash good.

When trend inflation is lower, precisely for $\pi = 1.007$, the welfare costs of inflation are nearly the double of the ones from the inflation tax model. The difference between the estimates from these two models decreases as the trend inflation rises.

5.5 The Impulse Responses

The responses of some variables to a positive money growth shock is plotted in Figure 5.1. The solid lines are the impulse responses that correspond to the historical value of ν_m, which is -.002, and the dotted lines are associated to a lower value of this parameter.

The parameter ν_m plays an important role in the magnitude of the responses of nominal and real variables to a monetary policy shock. Both nominal money supply and prices rise, after the expansionary money policy shock, and then gradually return to their steady state value. But their responses are less stronger and fade out more rapidly when ν_m is lower. Actually, ν_m determines the persistence in the growth of money supply. This parameter could well explain the break that appears in Table 2.3 in the cyclical behavior of nominal variables. As far as the real variables are concerned, consumption, interest rate, hours worked, and output peak higher when ν_m is lower, which is not the case for investment and wage.

The responses of output, hours worked, and wage turn out to be the opposite of those predicted by the inflation tax model (see Figure 4.1). This is due to the fact that money plays different role in the two models. In this model, money is an intermediate good used to reduce the time allocated to shopping, which explains why hours worked rise after the expansionary monetary policy shock.

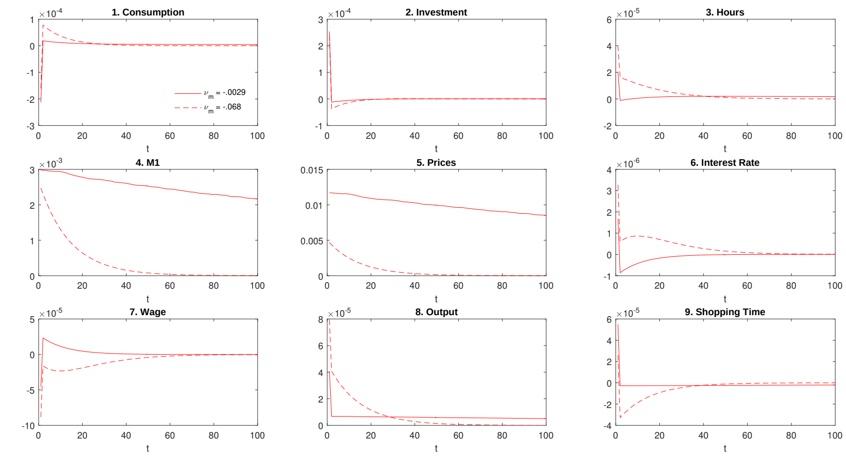

Figure 5.1: Impulse Responses to Money Growth Shock, Deviation form Steady State, the RBC model with Endogenous Money Supply ($\nu_y = .0682$ and $\sigma_\xi = .017$)

5.6 The Cyclical Properties

The summary statistics (standard deviations and correlation coefficients) in Table 5.2 are averages from 100 simulations of the model economy. Each simulation consists of 140 observations, which corresponds to the length of the sample used to calibrate the model. The second block of this table shows the cyclical behavior of the model economy when the stock of money is held fixed, *i.e.* when $\nu_y = \nu_m = \sigma_\xi = 0$ and only technology shocks drive business cycles. The third block displays the cyclical behavior when money is endogenous. In both cases, the elasticity of substitution parameter, e, is set to .3. A higher e will raise volatility but will lower the correlation between output and consumption.

When money supply is held fixed, the model explains 87% of output volatility. Compared to the inflation tax model (see Table 4.3), this model explains a lower share of the fluctuations in output, consumption, and hours worked. It explains 82% of the volatility in labor productivity, which is higher than in the inflation tax model. The cyclical behavior of the nominal variables (money and prices) is the same in the two models, when money growth is constant. The simulated correlation between hours worked and productivity, which is .57, is still high.

When money becomes endogenous and stochastic, business cycles are driven by both technology and money growth shocks. I have set ν_m to -.068 and ν_y to .0682 to have the model replicate 91% of the fluctuations in the price level and all its correlation with output. The share of the volatility in money rose to 48%.

Table 5.2: Cyclical Behavior of the Canadian Economy and the RBC Model with Fixed Money Supply, Percentage Deviation from Trend of Key Variables, 140 Observations

	Canadian Economy			RBC Economy					
				Fixed Money			Endogenous Money		
Variable	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
Output (GDP)	1.56	1	.89	1.35	1	.7	1.35	1	.69
Consumption	1.14	.84	.85	.35	.37	.95	.35	.36	.95
Investment	4.9	.75	.88	7.92	.97	.68	7.94	.97	.67
Hours	1.41	.9	.9	.87	.96	.68	.87	.96	.67
Productivity	68	.42	.56	.56	.92	.78	.56	.91	.78
M1	4.4	.4	.92	.07	.03	.96	2.13	.03	.68
Price	1.07	-.001	.8	.48	-.89	.8	.97	-.001	.68

Columns (1) Percentage standard deviations, columns (2) Correlation coefficient with output, and columns (3) First-order autocorrelation coefficient.

5.7 Sensitivity Analysis

Figure 5.2 shows the sensitivity of the cyclical behavior of money and the price level to the response coefficients ν_m and ν_y. Each data point is the average from 100 simulations of the model economy. In p,panels 1 and 2, money growth only depends on both the deviation of real output from trend and the monetary disturbances, *viz* ν_m is set to zero and ν_y varies between -.2 and .2. In panels 3 and 4, ν_y is held constant at .0682 and ν_m is varied between -.4 and 0.

It appears in Panel 1 that, when money supply growth is countercyclical, the volatility of the price level decreases as ν_y increases. After money supply growth has become pro-cyclical, precisely from $\nu_y = .04$, the volatility of the price level starts increasing along with this policy parameter. The volatility of money follows a similar pattern but is lower than that of the price level. The price level could be countercyclical or pro-cyclical depending on the magnitude of ν_y (Panel 2). It becomes procyclical at $\nu_y = .04$. where its volatility is the lowest.

When the policy parameter ν_y is held constant, Panel 3 shows that the standard deviation of money and prices behave differently as ν_m increases. The standard deviation of the price level is v-shaped and reaches its lowest value of $\nu_m = -.06$. For ν_m between -.4 and -.2, money is less volatile than the prices. Panel 4 shows that money is pro-cyclical and the price level is countercyclical as long as ν_m is less or equal to -.06.

Changes in the monetary policy rule affect the correlation between some real variables. Figure 5.3 shows the sensitivity of the correlation between hours worked and shopping time to each of the two policy parameters ν_m and ν_y. When ν_m is set to zero and ν_y is less than .02, both variables are negatively correlated (the lhs panel). This means when money supply growth is countercyclical or slightly

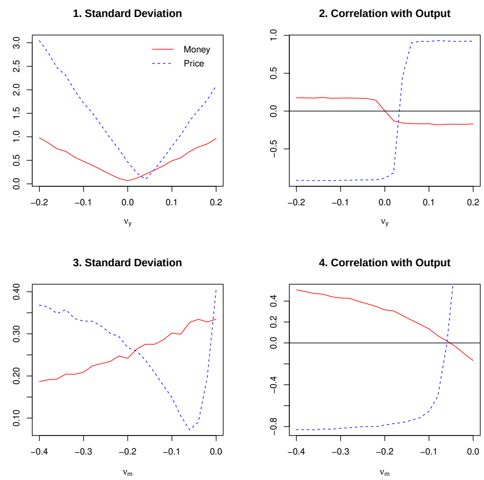

Figure 5.2: Cyclical Behavior of Nominal Variables as a Function of of the Responses of the Monetary Authority to Real Output (when $\nu_m = 0$) and Nominal Money Stock (when $\nu_y = .0682s$), the RBC Model with Endogenous Money (e=.3)

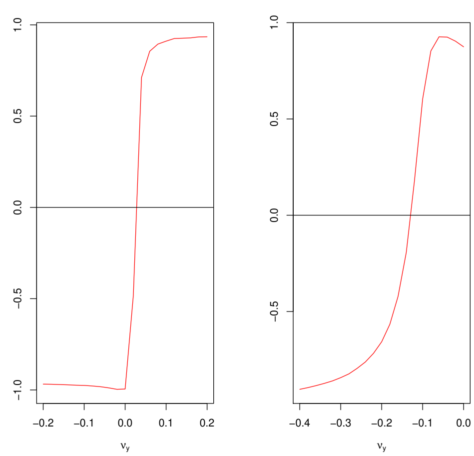

Figure 5.3: Correlation between Hours Worked and Shopping Time as a Function of the Policy Parameters ν_m smf ν_y

pro-cyclical and households save time on shopping, they do not increase the time they allocate to leisure but instead increase their hours worked. The rhs panel also shows that for ν_m lower than or equal to -.14 households substitute hours worked for shopping time.

6 The Inflation and Tax Code Model

This is an extension to a government with a nominal tax code of the RBC model with endogenous money supply sketched in Section 5. Each time period, for each type of income earned by households, the government exogenously sets the percentage that must be remitted as tax. Capital and labor incomes, coupon payments, *i.e.* interests on bonds, and capital gains are the incomes taxed. There are two types of capital gains: the accrued and the realized gains. The tax code specifies which one to tax. In the first scenario, the accumulated gains are taxed straight away whereas, in the second one, taxation is shifted into the future.

Instead of a money supply rule, the monetary authority uses the interest rate rule to achieve the inflation target. Gavin, Kydland, and Pakko (2007) use this model to study the interaction between inflation and the tax code. At steady state, they find that the welfare cost of inflation is higher when nominal capital gains is taxed. As a matter of fact, taxing capital gains discourages investment, which occasions an economic downturn. Suppressing the possibility to postpone the realization of capital gains, *i.e.* taxing instead the accrued gains, adds to the welfare cost. Over the business cycle, they find that the nominal capital gain tax is a propagation channel for inflation shocks. When shocks to trend inflation are highly persistent, inflation interacts with the nominal capital gain tax to generate high volatility in real variables and lower their correlation with output. Here, I only deal with the accrual-based taxation of capital gains.

6.1 The Households

The representative household's preferences are described by (5.1). Two of the constraints he faces remain unchanged: (5.2a) the shopping technology and (5.2c) the law of motion of physical capital. His nominal budget constraint is replaced with the following relation

$$\begin{aligned} P_t(C_t + I_t) + B_{t+1} + M_{t+1} + \tau_{gt} G_t = M_t &+ T_t + (1 - \tau_{lt}) P_t w_t L_t \\ &+ \left[(1 - \tau_{kt}) q_t + \tau_{kt}\delta\right] P_t K_t \\ &+ \left[1 + (1 - \tau_{bt})(R_t - 1)\right] B_t \qquad (6.1a) \end{aligned}$$

$$G_t = \left(P_t - P_{t-1}\right) K_t. \qquad (6.1b)$$

The rhs elements of (6.1a) indicate the sources of the representative household's income: money balances carried from the previous period, a tax refund and lump-sum transfer from the government T_t, the after-tax labor and capital incomes, the face value of the bonds he holds B_t, and the after-tax coupon payments. The variables q_t and R_t respectively denote the real rental price of capital and the gross nominal interest rate on bonds. The exogenous parameters τ_{bt}, τ_{kt}, and τ_{lt} are respectively the tax rates on bond yields, labor and capital incomes. The depreciated capital is not taxed.

The elements of (6.1a) show how he uses his after-tax incomes: buying consumption and investment goods, and bonds, keeping money aside for transactions, and paying tax on accrued capital gains. The capital gain G_t is specified in (6.1b) and the parameter τ_{gt} denotes the tax rate on this gain.

The representative household's optimal supply of labor, demand for money and investment, and the *no-arbitrage condition* are governed by the following four relations. The no-arbitrage condition indicates that holding bonds or physical capital yields the same after-tax real return.

$$1 - L_t - \omega_0 + (1 - \upsilon\omega_2)\omega_1 \left(\frac{M_t}{P_t C_t}\right)^{\omega_2} = \upsilon \frac{C_t}{(1-\tau_{lt})w_t} \tag{6.2a}$$

$$\left(\frac{M_t}{P_t}\right)^{\omega_2 - 1} = \frac{1-\tau_{bt}}{1-\tau_{lt}} \frac{R_t - 1}{w_t} \frac{C_t^{\omega_2}}{\omega_1\omega_2} \tag{6.2b}$$

$$\begin{aligned}\beta \mathrm{E}_t & \left[1 + (1-\tau_{k,t+1})(q_{t+1} - \delta) - \tau_{g,t+1}\left(1 - \frac{P_t}{P_{t+1}}\right)\right] \times \\ & \left(\frac{C_{t+1}}{C_t}\right)^e \left(\frac{1 - l_{t+1} - s_{t+1}}{1 - l_t - s_t}\right)^{\upsilon e - 1} \frac{1-\tau_{lt}}{1-\tau_{l,t+1}} \frac{w_t}{w_{t+1}} = 1\end{aligned} \tag{6.2c}$$

$$\begin{aligned}\mathrm{E}_t \left\{\left[1 + (1-\tau_{k,t+1})(q_{t+1} - \delta) - \tau_{g,t+1}\left(1 - \frac{P_t}{P_{t+1}}\right)\right] \frac{P_{t+1}}{P_t}\right\} &= 1 \\ + \mathrm{E}_t(1 - \tau_{b,t+1})(R_{t+1} - 1)&\end{aligned} \tag{6.2d}$$

The term $\tau_{gt+1}(1 - P_t/P_{t+1})$ in relations (6.2c) and (6.2d) shows the interaction of inflation with capital gain tax, which provides an additional mechanism that generates and propagates business cycle fluctuations.

6.2 The Government

The monetary authority, on behalf of the government, sets the nominal interest rate. It adjusts this key rate to deviations of inflation and its target from the BGP. The inflation target is stochastic but stationary, *viz* it fluctuates around the midpoint of a target range.

$$\ln \frac{R_{t+1}}{R} = (1 + \nu_\pi) \ln \frac{\pi_t}{\pi} - \nu_\pi \ln \frac{\gamma_{pt}}{\gamma_p} \tag{6.3a}$$

$$\ln \gamma_{pt} = \ln(1-\rho_\pi)\gamma_p + \rho_\pi \ln \gamma_{p,t-1} + \xi_t, \quad \xi_t \sim \mathcal{N}\left(0, \sigma_\xi^2\right) \tag{6.3b}$$

The response coefficient ν_π is positive and large enough for the model to have a solution.

The government collects taxes that are then entirely rebated to households. It prints new money, $M_{t+1}-M_t$, that is also transfered to households. The supply of money is endogenously determined from (6.2b), the money demand relationship. The government budget is therefore

$$T_t = \tau_{bt}(R_t-1)B_t + \tau_{gt}G_t + \tau_{kt}(q_t-\delta)P_tK_t + \tau_{lt}P_tw_tL_t + M_{t+1} - M_t. \tag{6.4}$$

6.3 Calibration

The following relations make up the DSGE model:

- relations (6.2a)-(6.2d) from the representative household's optimization problem and the constraints (5.2a), (5.2c), and (6.1a)-(6.1b).
- the aggregate production technology (4.4), the law of motion of TFP (4.5), and the two FOCs from firms' profit maximization problem: $q_t = \alpha Y_t/K_t$ and $w_t = (1-\alpha)Y_t/L_t$,
- the monetary policy rule (6.3a)-(6.3b),
- the government budget (6.4).

At equilibrium, the household specializes in investing in capital goods and will not acquire any bond, $B_t = 0$.

As previously, the share of time allocated to labor and shopping are stationary. So are the rental price of capital, the nominal interest rate, the inflation rate, and the tax rates. Consumption, investment, capital, output, and wage grow at the gross rate γ_z along the BGP. Prices grow at the gross rate $\pi = \gamma_p$. The nominal variables grow at the rate $\gamma_z\pi$.

The parameters α, γ_z, δ, π, ρ_z, σ_z, and ω_2 remain unchanged. The remaining parameters β, ν, τ_b, τ_g, τ_k, τ_l, ω_0, and ω_1 are set in a way that the model replicate along the BGP some sample averages.

The Tax Rates The average of some income tax rates released by the Fraser Institute is .305 (Palacios and Lammam, 2013; Palacios, Lammam, and Ren, 2015). This average is used to calculate the tax rate on labor income using income-based GDP estimates. Letting $\bar{\tau}$ denote the average income tax rate, one roughly has

$$\tau_l = \frac{\bar{\tau} \times \text{wages and salaries}}{\text{wages and salaries} + \text{employers' social contributions}},$$

which equals .2689.

I have related the tax rate on capital income to the two previous tax rates as follows: $\tau_k = [\bar{\tau} - (1-\alpha)\tau_l]/\alpha$. This gives $\tau_k = .3788$.

In Canada only half of capital gains is taxed. The effective tax rate on this income is therefore

$$\tau_g = \frac{\bar{\tau}\frac{G}{2}}{G} = \frac{\bar{\tau}}{2} = .1525.$$

To compute τ_b, the tax rate on bond incomes, I have evaluated (6.2d) along the BGP, which gives

$$\frac{I}{Y} = \frac{\alpha(\gamma_z + \delta - 1)(1-\tau_k)\pi}{(1-\tau_b)(R-1) + \delta(1-\tau_k)\pi - (1-\tau_g)(\pi - 1)}. \tag{6.5}$$

The average three-month treasury bill rate is 5.64% per annum over 1981-2015. This is equivalent to a quarterly rate of 1.4%. The average of the Bank of Canada's key rate, the overnight rate, is also almost the same. I have therefore set R to 1.014. Given the investment-output ratio, one can solve (6.5) for τ_b. The investment-output ratio used so far, .162, is the share of business investment in GDP. This ratio now turns out too low and yields a negative bond income tax rate. Using instead the sum of households' final consumption expenditure and business investment as the measure of output, the investment-output ratio $I/(C+I)$ becomes .237 and the solution of (6.5) becomes correctly signed, $\tau_b = .1725$.

The Discount Factor One get this parameter using either relation (6.2c) or the following relation, which is a combination of (6.2c) and (6.2d)

$$(1-\tau_b)(R-1) = \pi\frac{\gamma_z^{1-e}}{\beta} - 1.$$

The substitution parameter e is set to .395 as in Section 5.

The Leisure Weight and Shopping Technology Parameters Given the average consumption velocity of money, which is 3.04, and the share of time allocated to shopping, which is .0556, one can solve relations (5.2a), (6.2a), and (6.2b) to get the values of υ, ω_0, and ω_1. These values are reported in Table 6.2.

The Persistence of Inflation Stochastic Trend I have regressed the natural logarithm of trend inflation on its first lag to get estimates of ρ_π. Trend inflation is measured using the HP filter. It appears in Table 6.1 that its persistence was lower during the sub-period 1991:Q1-2015:Q4.

Table 6.1: OLS Estimates of Relation (6.3b), the Stochastic Process of Trend Inflation, Canada, 1981: Q1-2015:Q4

Parameter	1981:Q1-2015:Q4		1981:Q1-1990:Q4		1991:Q1-2015:Q4	
	Estimate	t-ratio	Estimate	t-ratio	Estimate	t-ratio
Intercept	-.0231	-12.81	-.0107	-4.68	-.061	-10.87
Persistence	.975	372.61	.941	124.19	.929	131.31
R^2	.9999		.9976		.9943	
σ_ξ	.015		.0106		.0123	
	$t_{2.5\%}(137) = 1.977$		$t_{2.5\%}(37) = 2.026$		$t_{2.5\%}(98) = 1.984$	

Table 6.2: The Baseline Parameters of the Inflation and Tax Code Model

Households	β	Discount factor	.997
	π	Inflation rate	1.007
	υ	Leisure weight	1.395
	ω_0	Shopping technology, intercept	.057
	ω_1	Shopping technology, slope	-.006
	ω_2	Shopping technology, curvature	-1
Firms	α	Capital share	.329
	γ_z	Growth rate of TFP	1.003
	δ	Depreciation rate	.015
	ρ_z	Persistence parameter of TFP	.95
	σ_z	Standard deviation of TFP shock	.009
Government	ρ_π	Persistence parameter of inflation	.975
	τ_b	Bond income tax rate	.173
	τ_g	Capital gain tax rate	.153
	τ_k	Capital income tax rate	.379
	τ_l	Labor income tax rate	.269

Table 6.3: Percentage Change in Steady State Values and Welfare Costs of Inflation, the Inflation and Tax Code Model

	No Taxes		No Capital Gain Tax		The Four Taxes	
	$\pi = 1.007$	$\pi = 1.057$	$\pi = 1.007$	$\pi = 1.057$	$\pi = 1.007$	$\pi = 1.057$
Output	-.145	-.694	-.155	-.742	-4.88	-26.751
Consumption	-.145	-.694	-.155	-.742	-2.75	-17.651
Investment	-.145	-.694	-.155	-.742	-11.136	-53.478
Hours	-.145	-.694	-.155	-.742	-1.653	-8.487
Wage	0	0	0	0	-3.281	-19.957
Welfare cost:						
$100 \times \frac{\Delta C}{C}$	.148	.714	.185	.889	1.807	12.934
$100 \times \frac{\Delta C}{Y}$	.105	.504	.138	.663	1.348	9.649

6.4 The Welfare Costs of Inflation

The welfare costs are computed using relation (5.7). Three scenarios are considered: (1) no income is taxed,, *i.e.* $\tau_b = \tau_g = \tau_k = \tau_l = 0$, (2) all incomes except capital gains are taxed, *i.e.* τ_b, τ_k, τ_l assume the values reported in Table 6.2 but $\tau_g = 0$, and (3) all incomes are taxed. All the other parameters have remained unchanged. The results are displayed in Table 6.3.

Comparing the results from the first scenario to those from the second one shows that introducing only bond, labor, and capital income taxes slightly raises the welfare costs of inflation. In the absence of any tax, the welfare cost of inflation represents .714% of the steady state consumption, when trend inflation is 5.7%. When the three aforementioned taxes are introduced, it goes up to .889%. These results are close to those in Table 5.1, which are obtained with different values assigned to the parameters β, υ, ω_0, and ω_1.

The welfare cost of inflation substantially increases when capital gains are taxed. It rises to 12.9% of steady state consumption when trend inflation is 5.7% instead of zero. This dramatic effect of inflation stems from the fact that taxing capital gain discourages investment, which falls by 53.5%.

6.5 The Impulse Responses

Figure 6.1 plots the response of some variables to a one-off negative shock of one standard deviation to trend inflation. This represents an expansionary monetary policy as it lowers the nominal interest rate. To see this clearly, consider relation (6.3a), the interest rate rule. Given, after normalizing prices, one can express the gross inflation rate as follows, $\pi_t = \gamma_{pt}\bar{P}_t//\bar{P}_{t-1}$, relation (6.3a) becomes

$$\ln \frac{R_{t+1}}{R} = \ln \frac{\gamma_{pt}}{\gamma_p} + (1+\nu_\pi) \ln \frac{\bar{P}_t}{\bar{P}_{t-1}}.$$

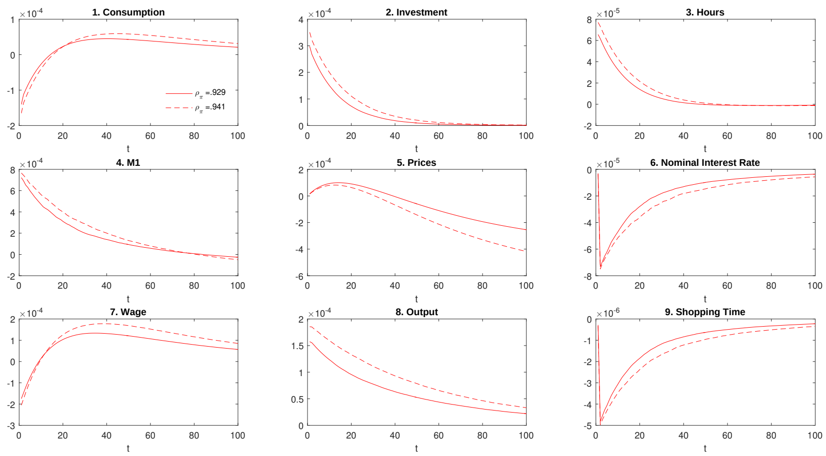

Figure 6.1: Impulse Responses to a Negative Inflation Shock, Deviation form Steady State, the Inflation and Tax Code Model

A negative shock to trend inflation thus lowers the nominal interest rate as it appears in panel 6 of Figure 6.1. The dotted and solid lines in Figure 6.1 are respectively the impulse responses associated to the periods 1981-1990' and 1991-2015's estimates of inflation persistence.

After the inflation shock, consumption and wage immediately fall whereas money, investment, hours worked, and output rise. These impacts are similar to those in Figure 5.1 where monetary policy is implemented by controlling directly the growth of money supply. Comparing these two figures also shows that the impact of a rise in inflation persistence on the impulse responses of real variables especially consumption, wage, and output do not fade out as quickly as those of a rise in the persistence of money supply.

As in Section 5, the responses of hours worked, wage, and output are the opposite of those from the inflation tax model plotted in Figure 4.1. This stems from the role money plays in this model: an intermediate good used to save time while shopping. When inflation rises, households immediately reduce their consumption and as a consequence the time they allocate to shopping. They could then allocate more time to labor, which immediately raises output. Real wage decreases as a result of the increase in labor supply.

6.6 The Cyclical Properties

I have here investigated two issues, which are the contribution to business cycle fluctuations of: (1) monetary disturbances, and (2) nominal capital gain tax. I

Table 6.4: Cyclical Behavior of the Canadian Economy and the Inflation and Tax Code Economy, Percentage Deviation from Trend of Key Variables, 140 Observations

	Canadian Economy			Inflation and Tax Code Economy					
				TFP Shock			TFP and Inflation Shocks		
Variable	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
Output (GDP)	1.56	1	.89	1.25	1	.7	1.3	1	.7
Consumption	1.14	.84	.85	.21	.9	.75	.5	.13	.69
Investment	4.9	.75	.88	4.72	1	.7	5.6	.95	.69
Hours	1.41	.9	.9	.72	1	.7	.91	.92	.69
Productivity	.68	.42	.62	.53	.99	.71	.57	.8	.71
M1	4.4	.4	.92	2.12	-.89	.72	4.05	-.17	.69
Inflation	.28	.11	.12	.05	.95	.71	.12	.15	.69

Columns (1) Percentage standard deviations, columns (2) Correlation coefficient with output, and columns (3) First-order autocorrelation coefficient.

have furthered the latter issue checking whether its importance depends on the persistence in the inflation target shock.

To simulate the model, I have set the monetary authority's response to inflation, ν_π, to .31. The estimated standard deviations of the shock to inflation target reported in Table 6.1 are too high and gives explosive solutions. To avoid this, I have lowered them to .001. The elasticity of substitution parameter e is set to .2 to make sure that consumption is always pro-cyclical and its volatility is as high as possible.

The contribution of monetary disturbances To investigate this issue, I have performed two Monte Carlo experiments: one with only technology shocks and the other with both technology and inflation shocks. The inflation shocks generate the monetary disturbances.

It appears in Table 6.4 that in the absence of monetary disturbances, the model explains about 81% of the fluctuations in output and only 24% of those in consumption. Introducing monetary disturbances has added to its ability to explain the fluctuations in consumption. The model now explains 42% of the fluctuations in consumption and 83% of the fluctuations in output. The share of the volatility in hours worked explained rises from 51% to 64%. The standard deviations of productivity and money also rise to get closer to observations. The explained standard deviation of inflation as well as its correlation with output have improved. It has also turned out that the correlation between output and the other variables has fallen. The data generated are less persistent than observations, except for inflation. The model does not replicate the absence of correlation between productivity and hours worked. Its prediction is .5.

Table 6.5: Cyclical Behavior of the Canadian Economy and the Inflation and Tax Code Economy, Percentage Deviation from Trend of Key Variables

	Canadian Economy			Inflation and Tax Code Economy					
				No Capital Gain Tax			All Taxes		
Variable	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
Panel 1 (1981-2015)									
Output (GDP)	1.56	1	.89	1.26	1	.7	1.3	1	.7
Consumption	1.14	.84	.85	.27	.75	.78	.5	.13	.69
Investment	4.9	.75	.88	4.43	.99	.69	5.6	.95	.69
Hours	1.41	.9	.9	.73	.99	.69	.91	.92	.69
Productivity	.68	.42	.62	.54	.99	.71	.57	.8	.71
M1	4.4	.4	.92	12.35	-.14	.68	4.05	-.17	.69
Inflation	.28	.11	.12	.29	.21	.69	.12	.15	.69
Panel 2 (1991-2015)									
Output (GDP)	1.21	1	.89	1.24	1	.69	1.23	1	.69
Consumption	.76	.68	.79	.23	.82	.79	.32	.58	.69
Investment	4.4	.65	.88	4.39	1	.68	4.71	.98	.68
Hours	.98	.85	.86	.72	.99	.68	.74	.97	.68
Productivity	.64	.61	.66	.54	.99	.71	.55	.94	.7
M1	1.66	-.04	.77	6.62	-.28	.67	3.76	-.25	.68
Inflation	.29	.01	.04	.17	.28	.67	.12	.19	.67

Columns (1) Percentage standard deviations, columns (2) Correlation coefficient with output, and columns (3) First-order autocorrelation coefficient.
Panel 1– Sample period:1981:Q1-2015:Q4 (140 observations), ρ_π =.975 and ν_π=.31.
Panel 2– Subsample period:1991:Q1-2015:Q4 (100 observations), ρ_π =.929 and ν_π=.5.

The Contribution of Nominal Capital Gain Tax I have performed another Monte Carlo experiment setting the capital gain tax to zero. Disturbances are generated by shocks to both the TFP and trend inflation. Some statistics summarizing this experiment are reported in the second block of the top panel of Table 6.5.

In the absence of capital gain tax, the model explains 81% of the fluctuations in output. The share of the volatility in consumption, hours worked, and productivity explained also dropped. The standard deviation of investment falls to get closer to its actual value. On the other hand the volatility of nominal variables has considerably increased: the volatility of inflation matches observations whereas that of money far exceeds them. The correlation between hours and productivity rose to .95.

The interaction of inflation and nominal capital gain tax has improved the model's ability to explain business cycle fluctuations, as it appears in the first panel of Table 6.5. My next interest is to find out whether capital gain tax still plays an important role when inflation becomes less persistent. In fact, over the period 1991:Q1-2015:Q4, the persistence in trend inflation has fallen to .929 as a result of the Bank of Canada's commitment to keep inflation low (see Table 6.1). I have simulated the model using this latter value and raising ν_π to .5 to indicate the more aggressive stance of the Bank of Canada against inflation. The results of the Monte Carlo simulations are displayed in the bottom panel of Table 6.5. One can observe that the introduction of capital gain tax has not much affected the cyclical behavior of output, hours worked, and labor productivity. Only the volatility of consumption and investment have increased but their percentage increases have shrunk compared to the case where inflation persistence is higher. When persistence in trend inflation drops from .975 to .929, the percentage increase in the volatility of consumption after adding capital gain tax drops from 85% to 39%.

6.7 Sensitivity Analysis

Two main findings from the previous section are: adding nominal capital gain tax raises the volatility of real variables and lowers their correlation with output. To check the robustness of these results, I have performed several other Monte Carlo experiments assigning different values to some key parameters, which are: the monetary authority's response to trend inflation, the persistence in trend inflation, and the elasticity of substitution parameter.

The Monetary Authority's Response to Trend Inflation Figure 6.2 shows the evolution of some statistics summarizing the cyclical behavior of the economy as the monetary authority's aggressive stance against inflation intensifies, *viz* as ν_π increases. The lowest value ν_π can assume in the absence of capital gain tax is .27, otherwise the model has no solution.

It appears in the first two panels of Figure 6.2 that the volatility in consumption and output decrease as ν_π increases. The standard deviation of consumption decreases faster in the absence of capital gain tax and that of output decreases faster in the model with capital gain tax. On the other hand, an increase in ν_π raises the correlation between consumption and output. The impacts of the changes in ν_π on the summary statistics are small and do not affect the qualitative results in the previous section.

The Persistence in Trend Inflation Figure 6.3 shows that the standard deviation of consumption and output increase along with ρ_π while the correlation between the two variables decreases. These impacts are the opposite of those occasioned by changes in ν_π.

This sensitivity analysis reveals that low values of ρ_π alter the qualitative results of the previous section. Adding capital gain tax does not raise the standard deviation of consumption when ρ_π is equal to or less than .825. It does not either raise the standard deviation of output when ρ_π is lower than .9.

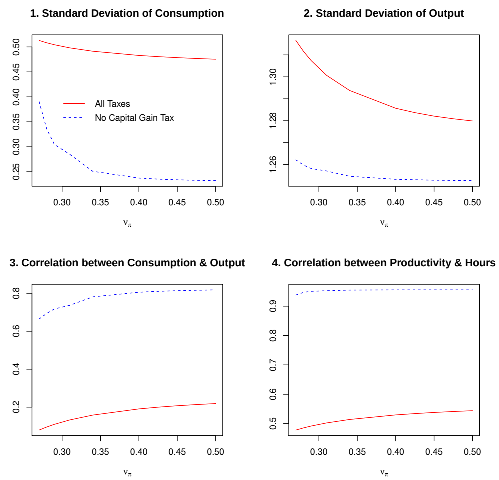

Figure 6.2: Cyclical Behavior of Some Real Variables as a Function of the Inflation Target Coefficient, the Inflation and Tax Code Model ($e = .2$ and $\rho_\pi = .975$)

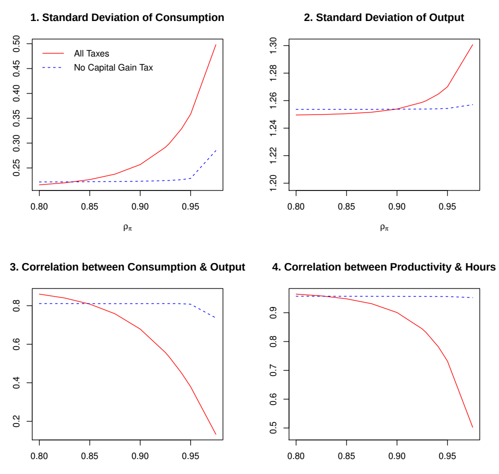

Figure 6.3: Cyclical Behavior of Some Real Variables as a Function of the Inflation Trend Persistence Coefficient, the Inflation and Tax Code Model ($e = .2$ and $\nu_\pi = .31$)

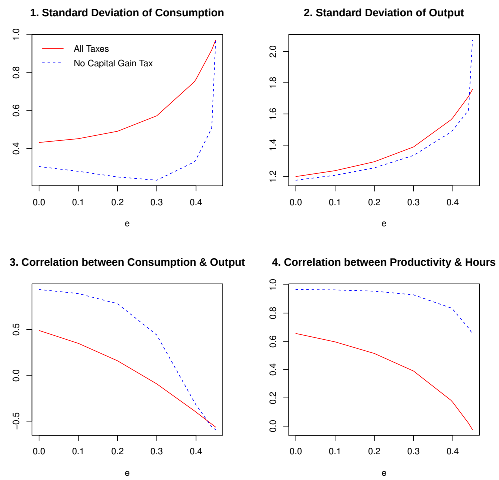

Figure 6.4: Cyclical Behavior of Some Real Variables as a Function of the Elasticity of Substitution Parameter, the Inflation and Tax Code Model ($\nu_\pi = .34$ and $\rho_\pi = .975$)

The Elasticity of Substitution Parameter It appears in the the third panel of Figure 6.4, that values this parameter can reasonably assume range from 0 to 2. For e greater than 2, consumption in the model with capital gain tax becomes countercyclical, which is counterfactual. It appears in the fourth panel of the same figure that the model is capable of replicating the absence of correlation between labor productivity and hours worked when e equals .44 or .45. For the desired values of e, adding capital gain tax raises the volatility of consumption and output and lowers their correlation. This prediction changes as e increases.

7 The Sticky Price Model

Four types of decision makers populate the economy: households, intermediate- and final-good-producing firms, and the monetary authority. Households derive utility from consuming the final good, holding money, and taking leisure time. The intermediate-good-producing firms (henceforth, intermediate firms) operate within a monopolistically competitive environment. Each of them supplies at a monopoly price to the final sector a differentiated good, *i.e.* a good that is an imperfect substitute for the others. The final good is a composite commodity produced by perfectly competitive firms out of the intermediate goods. The monetary authority sets the short-term nominal interest interest so as to minimize the volatility of inflation and output.

There are several variants of this model. What mostly distinguishes them is the way the behavior of households or the prices of intermediate goods are modeled. As far as the behavior of the prices of intermediate goods is concerned, following Rotemberg (1982), some assume that all firms face a quadratic cost of adjustment (Ireland, 1997, 2001, 2003; Dib, 2006). Others introduced nominal rigidity following Taylor (1980) or Calvo (1983) assuming only a constant fraction of firms charges a new price each period (Yun, 1996; Christiano, Eichenbaum, and Evans, 2005; Amano, Ambler, and Rebei, 2007). Herein, I have followed the latter approach.

7.1 The Households

Holding real money balances directly yields utility. The money-in-the-utility (MIU) function is represented as follows

$$\mathcal{U}\left(C_t, \frac{M_t}{P_t}, L_t\right) = a_t \left[\frac{C_t^e}{e} + \frac{\chi_t^{1-\eta}}{\eta}\left(\frac{M_t}{P_t}\right)^{\eta}\right] + \upsilon \ln(1 - L_t), \tag{7.1}$$

where the parameters $e < 1$ and $\eta < 1$ are the elasticity of substitution parameters. The stochastic parameters a_t and χ_t, which denote respectively the preference and money-demand shocks, follow an $AR(1)$ process

$$\ln a_t = \rho_a \ln a_{t-1} + \epsilon_{at}, \quad \epsilon_{at} \sim \mathcal{N}\left(0, \sigma_a^2\right) \tag{7.2a}$$
$$\ln \chi_t = \rho_\chi \ln \chi_{t-1} + (1-\rho_\chi)\ln\chi + \epsilon_{\chi t}, \quad \epsilon_{\chi t} \sim \mathcal{N}\left(0, \sigma_\chi^2\right). \tag{7.2b}$$

The representative household faces two constraints:

$$K_{t+1} = (1-\delta)K_t + I_t \tag{7.3a}$$
$$P_t(C_t + I_t) + \frac{\phi}{2}\left(\frac{K_{t+1}}{K_t} - \kappa\right)^2 P_t K_t + M_t + \frac{B_t}{R_t} = P_t(w_t L_t + q_t K_t) + M_{t-1} + B_{t-1} + D_t - T_t. \tag{7.3b}$$

The first constraint is the law of motion of capital and the second one is the nominal budget constraint. The household's financial resources are made up of labor and capital incomes, money balances carried from the previous period, one-period bonds B_{t-1}, and dividends D_t earned from holding shares in intermediate firms. Out of his incomes, he pays taxes T_t, purchases consumption and investment goods, buy new bonds, and keeps some money. Bonds are risk-free assets that pay the gross nominal interest rate R_t. Real rigidity occurs as it is costly to adjust capital stock inter-temporally. The third element on the lhs of (7.3b) is the capital adjustment cost, where ϕ and κ are respectively the adjustment cost parameter and the long-run gross growth rate of capital.

The four optimal conditions from maximizing (7.1) subject to (7.3) are:

$$\frac{\upsilon}{a_t} C_t^{1-e} = w_t(1 - L_t) \tag{7.4a}$$

$$\chi_t^{1-\eta} \left(\frac{M_t}{P_t}\right)^{\eta-1} = \frac{R_t - 1}{R_t} C_t^{e-1} \tag{7.4b}$$

$$\beta \mathrm{E}_t \left[\frac{a_{t+1}}{a_t} \left(\frac{C_{t+1}}{C_t}\right)^{e-1} \frac{P_t}{P_{t+1}}\right] R_t = 1 \tag{7.4c}$$

$$\beta \mathrm{E}_t \left\{\frac{a_{t+1}}{a_t} \left(\frac{C_{t+1}}{C_t}\right)^{e-1} \left[1 + q_{t+1} - \delta + \phi\left(\frac{K_{t+2}}{K_{t+1}} - \kappa\right)\left(\frac{K_{t+2}}{K_{t+1}} + \frac{\kappa}{2}\right)\right]\right\} = 1 + \phi\left(\frac{K_{t+1}}{K_t} - \kappa\right) \tag{7.4d}$$

Relation (7.4a) governs labor supply, (7.4b) is the demand for money, (7.4c) prevents arbitrage opportunities, and (7.4d) governs the household's investment decision.

7.2 The Final Sector

It aggregates a continuum of differentiated intermediate goods $Y_t(i)$ into a final good Y_t according to the Dixit-Stiglitz technology

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}, \ \theta > 1, \tag{7.5}$$

where θ is the elasticity of substitution between the intermediate goods.

The final firm rents its input at the monopoly price $P_t(i)$ and sells its output at the competitive price P_t. Its profit maximization problem is described as follows

$$\max_{Y_t(i)} P_t \left[\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}} - \int_0^1 P_t(i) Y_t(i) di.$$

It results from this maximization problem that the inverse demand for each type of intermediate good is

$$P_t Y_t(i)^{-\theta} \left[\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{1}{\theta-1}} = P_t(i),$$

hence the following demand

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\theta} Y_t. \tag{7.6}$$

Thus, in addition to being the elasticity of substitution between intermediate goods, the parameter θ also turns out to be the absolute value of the inverse of the price-elasticity of demand for the intermediate goods.

Eliminating $Y_t(i)$ in (7.5) using (7.6), one gets after rearrangement the price index expressed in terms of individual prices

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}}. \tag{7.7}$$

7.3 The Intermediate Firms

The production technology of each intermediate good is Cobb-Douglas and exhibits constant returns to scale.

$$Y_t(i) = K_t(i)^{\alpha} \left[z_t L_t(i)\right]^{1-\alpha},\ 0 < \alpha < 1 \tag{7.8}$$

Relation (4.5) describes the law of motion of z_t, the TFP.

The intermediate firms are monopolistically competitive. They pay their shareholders a dividend D_t after remunerating the capital and labor inputs. They therefore face a two-stage optimization problem, which consists of: (1) minimizing their total cost of producing a given level of output and (2) maximizing the dividend.

The total cost minimization problem

$$TC^*[Y_t(i)] = \min_{K_t(i), L_t(i)} q_t K_t(i) + w_t L_t(i) - \varphi_t \left\{K_t(i)^{\alpha} \left[z_t L_t(i)\right]^{1-\alpha} - Y_t(i)\right\},$$

has as FOCs

$$q_t = \varphi_t \alpha \frac{Y_t(i)}{K_t(i)} \tag{7.9a}$$

$$w_t = \varphi_t (1-\alpha) \frac{Y_t(i)}{L_t(i)}. \tag{7.9b}$$

Taking the derivative of TC^* with respect to $Y_t(i)$ shows that the Lagrange multiplier φ_t is also the marginal cost.

The nominal dividend firm i pays is

$$\begin{aligned} D_t(i) &= \left\{P_t(i) - P_t \frac{TC^*[Y_t(i)]}{Y_t(i)}\right\} Y_t(i) \\ &= \left[P_t(i) - P_t \varphi_t\right] Y_t(i), \end{aligned}$$

where the second line follows from the fact that average cost equals marginal cost when total cost is minimized. There is a probability ϖ_j that a firm keeps its price unchanged between time t and $t+j$, for $0 \leq j < \infty$, which will affect the discounted sum of its real dividends

$$\mathrm{E}_t \sum_{j=0}^{\infty} \varpi_j \beta^j \frac{\mu_{t+j}}{\mu_t} \left(\frac{P_t(i)}{P_{t+j}} - \varphi_{t+j} \right) Y_{t+j}(i)$$

where $\mu_t = a_t C_t^{e-1}$ is the marginal utility of consumption and the expression $\beta^j \mu_{t+j}/\mu_t$ is referred to as stochastic discount factor. Given (7.6) the dividend maximization problem of a firm reseting its price at time t can be written as follows

$$\min_{P_t(i)} \mathrm{E}_t \sum_{j=0}^{\infty} \varpi_j \beta^j \frac{\mu_{t+j}}{\mu_t} \left(\frac{P_t(i)}{P_{t+j}} - \varphi_{t+j} \right) \left[\frac{P_t(i)}{P_{t+j}} \right]^{-\theta} Y_{t+j}$$

with as as FOC for the optimal choice of $P_t(j)$

$$\mathrm{E}_t \sum_{j=0}^{\infty} \varpi_j \beta^j \frac{\mu_{t+j}}{\mu_t} \left[(1-\theta) \frac{P_t^*(i)}{P_{t+j}} + \theta \varphi_{t+j} \right] \left[\frac{P_t^*(i)}{P_{t+j}} \right]^{-\theta} \frac{Y_{t+j}}{P_t^*(i)} = 0,$$

which one solves to get

$$\frac{P_t^*(i)}{P_t} = \frac{\theta}{\theta-1} \frac{\mathrm{E}_t \sum_{j=0}^{\infty} \varpi_j \beta^j \frac{\mu_{t+j}}{\mu_t} \left(\frac{P_{t+j}}{P_t} \right)^{\theta} Y_{t+j} \varphi_{t+j}}{\mathrm{E}_t \sum_{j=0}^{\infty} \varpi_j \beta^j \frac{\mu_{t+j}}{\mu_t} \left(\frac{P_{t+j}}{P_t} \right)^{\theta-1} Y_{t+j}}. \tag{7.10}$$

The are several ways of modeling the probability ϖ_j, which include: (1) the Calvo (1983) pricing scheme where $\varpi_j = \varpi^j$ for $0 \leq j < \infty$, (2) the truncated Calvo pricing where $\varpi_j = \varpi^j$ for $0 \leq j < J$ and $\varpi_j = 0$ for $j \geq J$, and (3) the Taylor (1980) pricing where $\varpi_j = 1$ for $0 \leq j < J$ and $\varpi_j = 0$ for $j \geq J$ (Amano, Ambler, and Rebei, 2007). While under Calvo pricing, a firm can permanently keep its price unchanged, the truncated Calvo scheme restricts this possibility to J periods at most. Under Taylor pricing, each firm has to keep its price unchanged for exactly J periods.

To maintain tractability, I assume Calvo pricing. If $1-\varpi$ is the probability that a price change occur, it also designates the average number of adjusting firms per time unit; the total number of intermediate firms being normalized to unity. It follows that the average length of time between price changes is $1/(1-\varpi)$. [12]

[12]This result is the basis of the relationship between two statistical distributions: the Poisson distribution which describes the number of occurrences per time interval, and the exponential distribution,which describes the time interval between occurrences.

7.4 The Monetary Authority

Its policy rule is a generalization of the Taylor (1993) rule to allow for *interest rate smoothing*. Smoothing interest rate is to let it depend on its history. The monetary authority sets the short-term nominal interest rate not only in response to its past decision but also to the deviation of both inflation and output from their targets

$$\ln \frac{R_t}{R} = \nu_r \ln \frac{R_{t-1}}{R} + \nu_\pi \ln \frac{\pi_t}{\pi} + \nu_y \ln \frac{Y_t}{Y} + \xi_t, \quad \xi_t \sim \mathcal{N}\left(0, \sigma_\xi^2\right). \tag{7.11}$$

For nominal interest rate to be stationary, it is required $-1 < \nu_r < 1$. The Taylor principle requires the nominal interest rate respond more than one for one to changes in inflation ($\nu_\pi > 1$). The monetary authority chooses its response coefficients ν_r, ν_π, and ν_y to minimize a weighted sum of the variance of inflation and output

$$\min_{\nu_r, \nu_\pi, \nu_y} (\pi_t - \pi)^2 + \Lambda_y (Y_t - Y)^2. \tag{7.12}$$

Money demand then adjusts to the target nominal interest rate and lump-sum taxes are used to issue new money.

$$M_t = M_{t-1} + T_t \tag{7.13}$$

7.5 The General Equilibrium

All intermediate firms are endowed with the same production technology, face identical iso-elastic demand curves, and have the same marginal cost. As a consequence, the firms re-optimizing their dividends at time t will all choose the same price, *i.e.* $P_t^*(i) = P_t^*$. The average price of the other firms will be the one that prevailed at time $t-1$, *i.e.* P_{t-1}. Relation (7.7) can therefore be broken down as follows

$$P_t = \left[\int_0^\varpi P_{t-1}^{1-\theta} di + \int_\varpi^1 P_t^{*1-\theta} di \right]^{\frac{1}{1-\theta}},$$

which one rearranges to get

$$P_t^{1-\theta} = \varpi P_{t-1}^{1-\theta} + (1-\varpi) P_t^{*1-\theta}. \tag{7.14}$$

Relation (7.14) shows the effect of inertia in price adjustment. A high ϖ means a high inertia, *i.e.* many firms are not adjusting their prices. In that case, the past average price helps predicts the current one.

One gets the aggregate supply, Y_t^s, integrating over (7.6), the conditional demand function.

$$\begin{aligned}
Y_t^s &= \int_0^1 Y_t(i)di \\
&= Y_t S_t && (7.15a)\\
S_t &= \int_0^1 \left[\frac{P_t(i)}{P_t}\right]^{-\theta} di && (7.15b)\\
Y_t &= C_t + I_t + \frac{\phi}{2}\left(\frac{K_{t+1}}{K_t} - \kappa\right)^2 K_t && (7.15c)\\
Y_t^s &= K_t^\alpha \left(z_t L_t\right)^{1-\alpha}, && (7.15d)
\end{aligned}$$

where S_t measures price dispersion across firms. Because of the price dispersion, intermediate firms do not all produce the same amount of output. As a consequence, aggregate supply deviates from its long-run level Y_t. The price dispersion can be expressed recursively breaking down (7.15b) into two

$$\begin{aligned}
S_t &= \int_0^\varpi \left[\frac{P_t(i)}{P_{t-1}}\frac{P_{t-1}}{P_t}\right]^{-\theta} di + \int_\varpi^1 \left(\frac{P_t^*}{P_t}\right)^{-\theta} di \\
&= \pi_t^\theta \int_0^\varpi \left[\frac{P_{t-1}(i)}{P_{t-1}}\right]^{-\theta} di + (1-\varpi) p_t^{*-\theta} \\
&= \varpi \pi_t^\theta S_{t-1} + (1-\varpi) p_t^{*-\theta}, && (7.16)
\end{aligned}$$

where $p_t^* = P_t^*/P_t$. The lag of S_t in (7.16) follows from the fact the price dispersion across firms that are not re-optimizing their dividends at time t equals the price dispersion that prevailed across all firms at time $t-1$.

For convenience reasons, the price adjustment equation (7.10) is written as system of three equations using the artificial variables X_{1t} and X_{2t}

$$\begin{aligned}
1 &= \frac{\theta}{\theta-1}\frac{X_{1t}}{X_{2t}} && (7.17a)\\
X_{1t} &= \varpi\beta \mathrm{E}_t \pi_{t+1}^\theta \frac{\mu_{t+1}}{\mu_t}\left(\frac{p_t^*}{p_{t+1}^*}\right)^{-1-\theta} X_{1,t+1} + p_t^{*\ -1-\theta} Y_t \varphi_t && (7.17b)\\
X_{2t} &= \varpi\beta \mathrm{E}_t \pi_{t+1}^{\theta-1} \frac{\mu_{t+1}}{\mu_t}\left(\frac{p_t^*}{p_{t+1}^*}\right)^{-\theta} X_{2,t+1} + p_t^{*\ -\theta} Y_t, && (7.17c)
\end{aligned}$$

where $\pi_{t+1} = P_{t+1}/P_t$ (for more details, see Appendix B.1). An alternative to the above system of equations would be to approximate both (7.10) and (7.14) around the BGP to derive what is called the New-Keynesian Phillips curve. But this exercise is complicated by the fact that trend inflation occurs as prices grow at the gross rate $\pi > 1$ along the BGP. In Appendix B.2, I have derived one for the case $\pi = 1$.

The DSGE model consists of a set of prices $\{(P_t, P_t^*, q_t, R_t, w_t)\}_{t=0}^{\infty}$, a state of the world $\{(a_t, z_t, \chi_t)\}_{t=0}^{\infty}$, an allocation $\{(C_t, I_t, L_t^s, K_t^s, M_t^d)\}_{t=0}^{\infty}$ for the representative household, an allocation $\{(Y_t, S_t)\}_{t=0}^{\infty}$ for final firms, an allocation $\left\{(K_t^d, L_t^d, X_{1t}, X_{2t}, Y_t^s, \varphi_t)\right\}_{t=0}^{\infty}$ for intermediate firms, and an allocation $\{(M_t^s)\}_{t=0}^{\infty}$ for the monetary authority such that:

i. $\{(C_t, I_t, L_t^s, K_t^s, M_t^d)\}_{t=0}^{\infty}$ solves relations (7.3a) and (7.4a)-(7.4d),

ii. $\{(Y_t, S_t)\}_{t=0}^{\infty}$ solves relations (7.15a) and (7.16),

iii. $\left\{(K_t^d, L_t^d, X_{1t}, X_{2t}, Y_t^s, \varphi_t)\right\}_{t=0}^{\infty}$ solves relations (7.9a)-(7.9b), (7.15d), and (7.17a)-(7.17c),

iv. $\{(a_t, z_t, \chi_t)\}_{t=0}^{\infty}$ is governed by relations (4.5) and (7.2a)-(7.2b),

v. capital and labor markets clear, *i.e.* $K_t^d = K_t^s$ and $L_t^d = L_t^s$, equilibrium in the final good market is described by (7.15c), (7.11) ensures the money market clears, *i.e.* $M_t^d = M_t^s$, and (7.14) describes prices. .

The share of time allocated to labor is a stationary variable. So is the marginal cost. Relation (7.9b) thus constrains real output and wage to grow at the same rate. Relation (7.4a) constrains this rate to be zero, for $e \neq 0$. As a consequence, all the other real variables and the nominal interest rate are also stationary. It follows that the long-run gross growth rate of the TFP, γ_z, and the long-run gross growth rate of physical capital, κ, are equal to one. Money consequently grows at the same rate as prices.

7.6 Calibration

The capital share, α, and depreciation rate δ from the earlier calibration exercise are respectively .329 and .015. The gross rate of inflation π is 1.007 (Table 4.1).

The Probability of Price Inertia The OLS estimate of the degree of price stickiness (or rate-of-change parameter) in Table 3.4 is .56. This means 56% of the growth in nominal demand is incorporated into prices and 44% results in changes in real output. The probability that a firm keeps its price unchanged, ϖ is therefore set to .44.

The Stochastic Process of the Preference and Money Demand Shocks Log-linearizing households' labor supply and money demand equations, respectively relations (7.4a) and (7.4b), gives

$$\ln C_t = -\frac{\ln \gamma}{1-e} + \frac{\ln w_t}{1-e} + \frac{\ln a_t}{1-e} \tag{7.18a}$$

Table 7.1: OLS Estimates of the Stochastic Process Followed by the Preference and Money Demand Shocks

Dependent Variable	$\ln a_t$		$\ln \chi_t$	
	Estimate	t-ratio	Estimate	t-ratio
Intercept	-.001	-1.36	.0004	-.742 .23
Lag Dependent Variable	.908	48.28	.949	34.5
$\bar{R}^2$	.944		.896	
Residual Standard Error	.008		.021	

$t_{2.5\%}(137) = 1.977$

$$\ln \frac{M_t}{P_t} \approx \frac{\ln(R_t - 1)}{\eta - 1} + \frac{e-1}{\eta - 1} \ln C_t + \ln \chi_t, \tag{7.18b}$$

where $R_t - 1$, the first variable on the rhs of (7.18b), is a first-order approximation of $(R_t - 1)/R_t$ around 1. The natural logarithm of the preference shock, a_t, and the money demand shock, χ_t, are the residuals of the above two econometric models. The parameters of the $AR(1)$ process followed by the preference and money demand shocks estimated using the OLS residuals are reported in Table 7.1.

The OLS estimate of $1/(\eta - 1)$, the interest elasticity of money demand, in model (7.18b) is -.044. It follows that η equals -21.63.

The Other Parameters Evaluating the price adjustment equation along the BGP gives

$$p^* = \frac{\theta}{\theta - 1} \frac{1 - \varpi\beta\pi^{\theta-1}}{1 - \varpi\beta\pi^{\theta}} \varphi.$$

According Statistics Canada's annual retail trade survey, the gross profit margin averaged 26.8% over the period 2012-2015. The parameters β and θ are set so that $(p^\star - \varphi)/p^\star = .268$ and investment-output ratio equals .16 as observed.

The leisure weight, υ, depends on the value assigned to the elasticity of substitution parameter e. This latter parameter is set to .3, which ensures the simulated correlation between cyclical consumption and output is close to observations. The capital adjustment coefficient, ϕ, which is a free parameter, is set to 2, which ensures the simulated volatility of investment is also close to observations. The values of the parameters ν_r, ν_y, and ν_π that minimize the quadratic loss function (7.12) given the constraints $0 \leq \nu_r < 1$ and $1 < \nu_\pi < 2$ are returned by the Dynare command *osr*, which stands for optimal simple rule. They are reported in Table 7.2.

Table 7.2: The Baseline Parameters of the Sticky Price Model

Households	e	Inter-temporal elasticity of consumption parameter	.3
	β	Discount factor	.993
	η	Interest elasticity of money demand parameter	−21.629
	κ	Capital long-run gross growth rate	1
	ρ_a	Preference shock persistence parameter	.908
	ρ_χ	Money demand shock persistence parameter	.949
	σ_a	Preference shock standard deviation	.008
	σ_χ	Money demand shock standard deviation	.021
	υ	Leisure weight	1.312
	ϕ	Capital adjustment cost parameter	2
Firms	α	Capital share	.329
	γ_p	Inflation rate	1.007
	δ	Depreciation rate	.015
	θ	Elasticity of substitution between intermediate goods	3.787
	ϖ	Probability of price inertia	.44
	ρ_z	Persistence parameter of TFP	.95
	σ_z	Standard deviation of TFP shock	.009
Government	Λ_y	Weight of output gap in the loss function	1.5
	ν_r	Response to change in interest rate	.9
	ν_y	Response to change in output	.227
	ν_π	Response to change in inflation	1.9
	σ_ξ	Standard deviation of interest rate shock	.017

Table 7.3: Steady State and Average Values Associated with Different Values of Trend Inflation, the Sticky Price Model

	π=1		π=1.007		π=1.057	
Variable	(1)	(2)	(1)	(2)	(1)	(2)
Panel 1 ($\varpi = .44$)						
Output	.9591	.9573	.9590	.9572	.9398	..9376
Consumption	.8037	.8022	.8036	.8020	.7874	.7855
Investment	.1554	.1552	.1554	.1551	.1524	.1521
Hours	.2961	.2959	.2961	.2959	.2950	.2948
Wage	1.5995	1.5974	1.5993	1.5972	1.5744	1.5716
M1	.5508	.5729	.5353	.5553	.5010	.5234
Welfare	2.6606		2.6601		2.6403	
Welfare Costs: $100 \times \frac{\Delta \tilde{C}}{C}$	0		.0585		2.1651	
$100 \times \frac{\Delta \tilde{C}}{Y}$	0		.0490		1.8143	
Panel 2 ($\varpi = .66$)						
Output	.9591	.9568	.9587	.9561	.8480	
Consumption	.8037	.8016	.8033	.8010	.7096	
Investment	.1554	.1551	.1554	.1551	.1384	
Hours	.2961	.2958	.2961	.2958	.2901	
Wage	1.5995	1.5966	1.5990	1.5958	1.4537	
M1	.5508	.5759	.5353	.5587	.4994	
Welfare	2.6607		2.6597		2.5538	
Welfare Costs: $100 \times \frac{\Delta \tilde{C}}{C}$	0		.1053		10.9058	
$100 \times \frac{\Delta \tilde{C}}{Y}$	0		.0882		9.1387	

Columns (1) Steady state value, columns (2) Average value of the simulated series

7.7 The Welfare Cost of Inflation

Table 7.3 shows the impacts of trend inflation on both the model's steady state and stochastic means, *i.e.* the average value of the simulated series. To solve numerically the model and simulate it, its second-order approximation around the steady state is taken. Taking the second-order approximation instead of the first-order one enables capturing the impacts of the shocks on the stochastic means.

Trend inflation lowers the steady state value of output, consumption, investment, hours worked and wage (Panel 1 of Table 7.3). A .7% quarterly trend inflation lowers the steady state output and consumption by about .01%. As a result, households' welfare decreases. One computes the welfare cost of inflation solving (7.19): To compensate households for the welfare loss caused by a .7% trend inflation, one has to increase their steady state consumption by about .06%.

$$\mathcal{U}\left(C, \frac{M}{P}, L\right)\Big|_{\pi=1} = \mathcal{U}\left(C^* + \Delta C, \frac{M^*}{P^*}, L^*\right)\Big|_{\pi=\{1.007, 1.057\}}, \tag{7.19}$$

Trend inflation lowers intermediate firms' real marginal cost as real wage has fallen and capital rental price has remained unchanged. It also creates a price

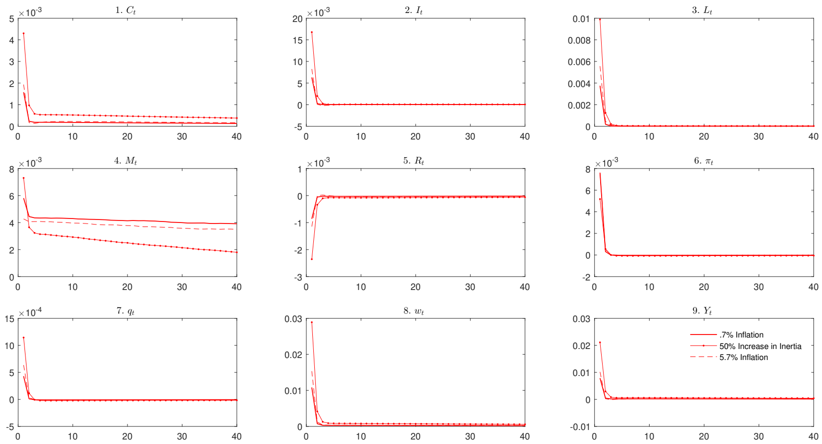

Figure 7.1: Impulse Responses to an Expansionary Monetary Policy Shock, Deviation form Steady State, the Sticky Price Model

dispersion across firms as those re-optimizing their dividends see an opportunity to charge a higher price.

It appears in Table 7.3 that the welfare cost is an increasing function of inflation rate. It also depends on other parameters such as: the inter-temporal elasticity of substitution of consumption, the elasticity of substitution between durables, and the degree of price stickiness. The second panel of Table 7.3 shows that when the probability of price inertia doubles, a .7% trend inflation lowers the steady state output and consumption by .05%, which is 4 times higher.

Table 7.3 also shows the impacts of trend inflation on the spread, *i.e.* the difference, between the steady state values and the stochastic means. In the absence of inflation, the steady state value of output and consumption were higher than their stochastic means by .19%. This spread monotonically increases along with trend inflation. Amano, Ambler, and Rebei (2007) attribute this to an increase in price dispersion across firms. The second panel of Table 7.3, I show that the spread between the steady state values and the stochastic means also increases along with the probability of price inertia. The fact is that, as it appears in relation (7.16), price dispersion becomes more persistent as ϖ increases.

7.8 The Impulse Responses

Figure 7.1 plots the response of the model to an expansionary monetary policy shock. The dotted and dashed lines are the responses respectively when the probability of price inertia is 50% higher and trend inflation is 5% higher.

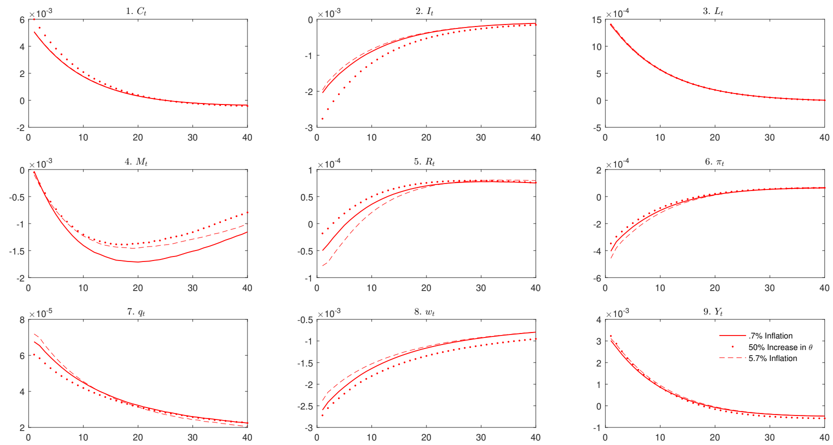

Figure 7.2: Impulse Responses to a Preference Shock, Deviation form Steady State, the Sticky Price Model

This shock that lowers the nominal interest rate (Panel 5) and raises money (Panel 4) stimulates the economy by encouraging consumption (Panel 1) and investment (Panel 2). Output, hours worked, and wage rise, as a consequence. These impacts contrast with those of the inflation and tax code model (Figure 6.1) where consumption and wage fall after an expansionary monetary policy shock. Besides, the impacts of the monetary policy shock in the sticky price model fade out very quickly compared to the inflation and tax code model. The impact on consumption lasts longer when the probability of price inertia is 50% higher. Besides, the immediate response of the real variables is stronger when price inertia or trend inflation is higher.

Figure 7.2 shows the responses to a preference shock. The dotted and dashed lines respectively correspond to the cases where the elasticity of substitution between durables is 50% higher and trend inflation is 5% higher. The response to this shock takes longer to fade out. A positive preference shock lowers the nominal interest rate, which causes households to prefer consumption over investment and leisure. Real wage rises as the result of the increase in hours worked. Consumption further increases and investment further decreases when the elasticity of substitution between durables is 50% higher.

7.9 The Cyclical Properties

I have performed some Monte Carlo simulations to check the model's ability to mimic the cyclical behavior of the Canadian economy. These experiments have

Table 7.4: Cyclical Behavior of the Canadian and the Sticky Price Economies, Percentage Deviation from Trend of Key Variables

	Canadian Economy			Sticky Price Economy					
				Only Technology Shock			All Shocks		
Variable	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
Panel 1 (Sticky Price: $\varpi = .44$)									
Output (GDP)	1.56	1	.89	.97	1	.73	1.3	1	.44
Consumption	1.14	.84	.85	.73	1	.71	1.1	.82	.66
Investment	4.9	.75	.88	2.23	1	.75	4.71	.72	.20
Hours	1.41	.9	.9	.31	.99	.80	1.36	.81	.13
Productivity	.68	.42	.62	.64	1	.73	.90	.86	.35
M1	4.4	.4	.92	.57	.60	.87	2.94	.23	.68
Interest Rate	1.35	.56	.75	.14	-1	.72	.16	-.96	.50
Inflation	.28	.11	.12	.15	-.96	.51	.74	.41	-.02
Panel 2 (Flexible Price: $\varpi = 0$)									
Output (GDP)	1.56	1	.89	.99	1	.70	1.07	1	.69
Consumption	1.14	.84	.85	.74	1	.78	1.08	.91	.68
Investment	4.9	.75	.88	2.33	1	.70	2.79	.54	.67
Hours	1.41	.9	.9	.34	.99	.70	.69	.79	.67
Productivity	.68	.42	.62	.66	1	.70	.68	.78	.68
Interest Rate	1.35	.56	.75	.14	-1	.70	.14	-.95	.69
M1	4.4	.4	.92	.57	-.61	.86	2.95	.12	.70
Inflation	.28	.11	.12	.15	-.95	.47	.88	-.18	-.66

Columns (1) Percentage standard deviations, columns (2) Correlation coefficient with output, and columns (3) First-order autocorrelation coefficient.

two purposes: (1) to check whether monetary disturbances are instrumental in business cycle fluctuations and (2) if so, to check whether the non-neutrality of money stems from nominal price rigidity. The summary statistics from these experiments are reported in the second and third blocks of Table 7.4.

It appears in Panel 1, that, with only technology shock, the sticky price model explains 62% of the fluctuations in output and less than half of those in investment. It explains respectively 22% and 13% of the volatility in hours worked and money. Adding monetary policy and the two other shocks, the share of the volatility of output explained rose from 62% to 83%. The model now explains almost all the fluctuations in consumption, investment, and hours worked. The simulated correlation of these variables with output become close to observations. While the standard deviation of money has risen to get close to the observed one, productivity and inflation has become too much volatile. Monetary policy shock contributed most to improving the volatility of output, investment, and hours worked while preference shocks contributed most to improving the volatility of consumption and money. When the monetary policy shock was added, the volatility of output rose by 28%, that of investment doubles, and that of hours worked quadruples

In Panel 2 of Table 7.4, the probability of price inertia, ϖ, is set to zero, which

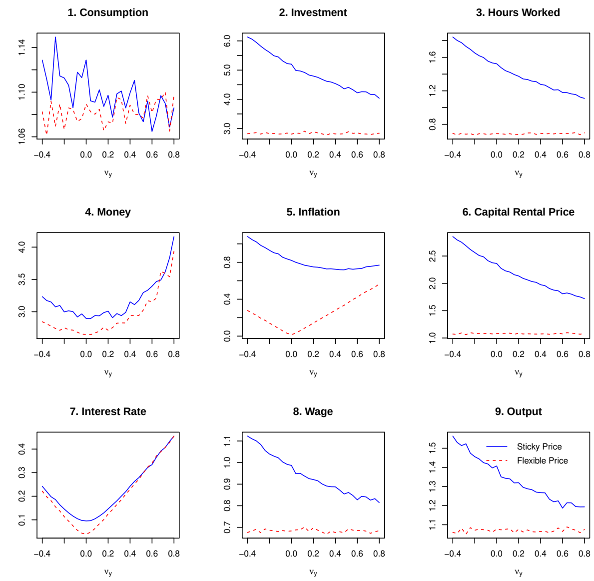

Figure 7.3: Standard Deviations as a Function of of the Monetary Authority's Response to Output Gap, the Sticky Price Model

means prices are flexible. When prices are flexible, monetary policy shock does not add much to the cyclical behavior of the model. Only the cyclical behavior of nominal variables and their correlation with output are affected. Most of the difference between the second and third blocks are brought by the preferences shocks.

These experiments show that the model needs the assumption of price stickiness to explain the cyclical behavior of the Canadian economy. Monetary policy shocks do not propagate into the real economy in the absence of price stickiness. Ireland (2003) came up up with the same conclusion after estimating by maximum likelihood a DSGE model in which price stickiness is modeled following Rotemberg.

The value of the monetary authority's response to output gap, ν_y depends on the weight of output, Λ_y in the objective function (7.12). The other parameters ν_r and ν_π are insensitive to Λ_y because of the constraints placed ($0 \leq \nu_r < 1$, $1 < \nu_\pi < 2$). I have therefore undertaken several other Monte Carlo simulations in order to check the sensitivity to ν_y of the role of price stickiness in the propagation of monetary policy shock. Figure 7.3 and 7.4 plot the simulated standard deviations and correlation coefficients with output of some key variables as a function of ν_y.

When prices are flexible, changes in ν_y do not much impact on the standard deviation of investment, hours worked, output, real wage, and the rental price of capital. On the other hand, when prices are sticky, the standard deviation of these

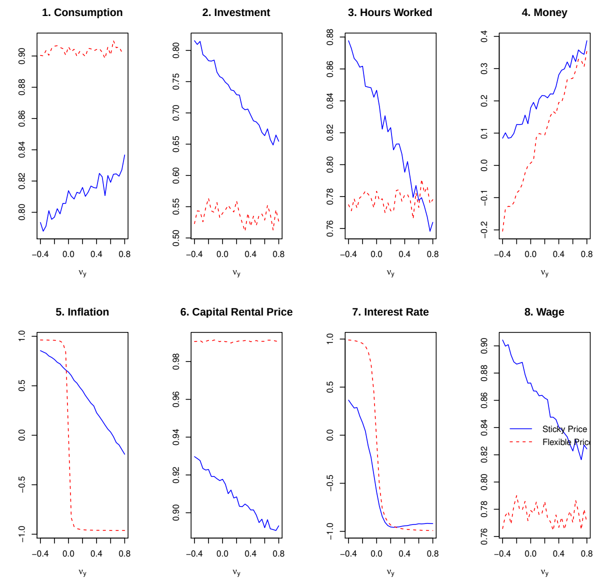

Figure 7.4: Correlation Coefficients with Output as a Function of of the Monetary Authority's Response to Output Gap, the Sticky Price Model

variables decreases as ν_y increases but the model keeps generating more volatility than when prices are flexible, except for consumption.

When prices are sticky, consumption and money become more pro-cyclical as ν_y increases. They other variables become less pro-cyclical. The nominal interest rate is pro-cyclical only when ν_y is negative.

Finally, Figures 7.3 and 7.4 reveal that changes in the monetary policy rule, especially the strength of the monetary authority's response to output deviation to its target, can explain the break in the cyclical behavior of real and nominal variables observed after 1991.

8 The Financial Accelerator Model

Credit market conditions play a central role in the propagation of business cycles. Their deterioration, *i.e.* sharp increases in insolvencies and bankruptcies, rising real debt burdens, collapsing asset prices, and bank failures, depress the economic activity. The Great Recession, for instance, was caused by the bursting of the global housing bubble and the subprime mortgage crisis that followed in 2007-08.

The fact that endogenous developments in the credit markets work to propagate and amplify real and monetary shocks to the economy is referred to as the financial accelerator effect. Bernanke, Gertler, and Gilchrist (1999) incorporate this effect into the new-Keynesian framework to illustrate how credit market imperfections (frictions) influence the transmission of the monetary policy.

The model is made up of five agents: households, retailers, entrepreneurs, financial intermediaries, and the monetary authority. The financial intermediaries lend funds received from households to entrepreneurs.

8.1 The Household

Their preferences are defined over consumption, real money balances, and leisure

$$\mathcal{U}\left(C_t, \frac{M_t}{P_t}, L_t\right) = \frac{a_t}{e}\ln\left[C_t^e + \chi_t^{1-e}\left(\frac{M_t}{P_t}\right)^e\right] + \upsilon\ln(1-L_t). \tag{8.1}$$

The laws of motion of a_t, the preference shock, and χ_t, the money demand shock, are described by (7.2). The budget constraint is similar to (7.3b) except the fact households do not hold any physical capital stock. They instead entrust their savings to financial intermediaries in return of a real risk-free gross interest rate R_t.

$$P_tC_t + P_tB_{t+1} + M_t = P_tw_tL_t + R_tP_tB_t + D_{t+1} + M_{t-1} - T_t, \tag{8.2}$$

where B_t denotes households' real savings.

The FOCs and Euler equation from maximizing (8.1) subject to (8.2) are:

$$\frac{\upsilon}{a_t}C_t^{1-e}\left[C_t^e + \chi_t^{1-e}\left(\frac{M_t}{P_t}\right)^e\right] = w_t(1-L_t) \tag{8.3a}$$

$$\chi_t^{1-e}\left(\frac{M_t}{P_t}\right)^{e-1} = \frac{R_{t+1}-1}{R_{t+1}}C_t^{e-1} \tag{8.3b}$$

$$\beta \mathrm{E}_t\left[\frac{a_{t+1}}{a_t}\left(\frac{C_{t+1}}{C_t}\right)^{e-1}\frac{C_t^e + \chi_t^{1-e}\left(\frac{M_t}{P_t}\right)^e}{C_{t+1}^e + \chi_{t+1}^{1-e}\left(\frac{M_{t+1}}{P_{t+1}}\right)^e}R_{t+1}\right] = 1. \tag{8.3c}$$

Comparing the optimal conditions in (8.3) to those in (7.4) shows that the logarithmic transformation of a MIU only affects the intra-temporal trade-off between consumption and leisure and the inter-temporal substitution of consumption for investment. The money demand equation remains unchanged provided the interest elasticity of money demand is still the same.

8.2 The Entrepreneurs

The entrepreneurs are finitely-lived. There is a probability ϖ_e that they survive to the next period. Their expected lifespan is therefore $1/(1-\varpi_e)$ as explained in

Footnote 12. They are risk-neutral and produce a wholesale good using labor and capital. They purchase the capital stock out of their internal funds and borrowings from financial intermediaries. They therefore face a profit maximization and a loan contracting problems.

The Profit Maximization Problem Entrepreneurs are perfectly competitive. They operate a constant returns to scale Cobb-Douglas technology using physical capital and labor as inputs. They produce themselves the capital stock. On the other hand, labor is a composite of households' labor L and entrepreneurs' labor L_e.

Entrepreneurs produce the capital goods using the final good purchased from retailers. An aggregate real investment expenditure of I_t on the final good yields $x_t I_t$ new capital goods. Capital therefore evolves as follows

$$K_{t+1} = (1-\delta)K_t + \exp(x_t)I_t, \tag{8.4}$$

where the stochastic parameter x_t denotes the investment-specific technological change (Greenwood, Hercowitz, and Huffman, 1988; Fisher, 2006). It follows a stationary autoregressive process of order 1

$$x_t = \rho_x x_{t-1} + \epsilon_{xt} \quad \epsilon_{xt} \sim \mathcal{N}\left(0, \sigma_x^2\right). \tag{8.5}$$

Entrepreneurs undergo adjustment costs while converting the final good into capital good. Following Christensen and Dib (2008), I assume these adjustment costs are quadratic. Given P_{kt}, the relative cost of a unit of capital, entrepreneurs choose I_t to maximize their gain

$$\max_{I_t} P_{kt}\exp(x_t)I_t - I_t - \frac{\phi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t,$$

which they achieve when

$$P_{kt} = \exp(-x_t)\left[1 + \phi\left(\frac{I_t}{K_t} - \delta\right)\right]. \tag{8.6}$$

The aggregate production of the wholesale good by entrepreneurs is

$$Y_t = K_t^\alpha \left(z_t L_t^\zeta L_{et}^{1-\zeta}\right)^{1-\alpha}, \quad 0 < \alpha, \zeta < 1, \tag{8.7}$$

where ζ denotes the share of households' labor in the wage bill. The wholesale good, is sold at the price P_{wt} to retailers. Household and entrepreneurial labor are remunerated respectively at the real rates w_t and w_{et}. As for the return on capital, it varies across entrepreneurs. Holding a unit of capital yields the entrepreneur j

the gross return $\nu_{jt}R_{kt}$, where ν_{jt} denotes its idiosyncratic or residual component. Assuming $\mathrm{E}\left(\nu_{jt}\right)=1$, the aggregate gross return on capital is R_{kt}.

The entrepreneurs choose K_{t+1}, L_t, and L_{et} to maximize their profits

$$\max_{K_{t+1},L_t,L_{et}} \frac{P_{wt}}{P_t}K_t^{\alpha}\left(z_t L_t^{\zeta} L_{et}^{1-\zeta}\right)^{1-\alpha} - (R_{kt}-1)P_{k,t-1}K_t - w_t L_t - w_{et}, L_{et}, \quad (8.8)$$

subject to (8.4).

This program has as FOCs

$$\mathrm{E}_t\left[\alpha\frac{\varphi_{t+1}}{P_{kt}}\frac{Y_{t+1}}{K_{t+1}} + (1-\delta)\frac{P_{k,t+1}}{P_{kt}}\right] = \mathrm{E}_t\left(R_{k,t+1}\right) \quad (8.9a)$$

$$\varphi_t(1-\alpha)\zeta\frac{Y_t}{L_t} = w_t \quad (8.9b)$$

$$\varphi_t(1-\alpha)(1-\zeta)\frac{Y_t}{L_{et}} = w_{et}, \quad (8.9c)$$

where $\varphi_t = P_{wt}/P_t$ is the relative price of the wholesale good. Relation (8.9a), the user cost of capital, is obtained solving inter-temporally the above maximization program. Comparing (8.9) to (7.9) shows the relative price of the wholesale good is also its real marginal cost.

The Contracting Problem To purchase new capital goods, the entrepreneurs use both their *net worth* and borrowing from financial intermediaries. The net worth of entrepreneurs or their internal funds is made up of their labor income and the unconsumed portion of the wealth they accumulated by operating firms.

Each entrepreneur's' agreement with his financial intermediary is to reimburse him out of the gross return on capital. Thus, entrepreneur j holding $K_{j,t+1}$ units of capital pays back $\bar{\nu}_t R_{k,t+1}P_{k,t}K_{j,t+1}$ out of his gross return $\nu_{jt}R_{k,t+1}P_{k,t}K_{j,t+1}$, provided $\bar{\nu}_t \leq \nu_{jt}$. He then keeps the difference as equity. In case of default, *i.e.* when $\bar{\nu}_t > \nu_{jt}$, the financial intermediary cashes all the gross return on capital. To be able to observe ν_{jt}, the financial intermediary pays a auditing cost $0 < \mathcal{A} < 1$.

The optimal contracting problem consists in maximizing the entrepreneurial equity subject to the expected return of the financial intermediary. The aggregate entrepreneurial equity equals

$$V_{t+1} = \mathrm{E}\left\{\int_{\bar{\nu}_t}^{\infty}\nu \mathrm{d}F(\nu) - \left[1-F(\bar{\nu}_t)\right]\bar{\nu}_t\right\} R_{k,t+1}P_{kt}K_{t+1} \quad (8.10)$$

where $F(\nu)$ is the probability distribution of the idiosyncratic component of the return on capital. Thus, $F(\bar{\nu})$ denotes the probability of default.

Entrepreneurs who give up on their business consume their equity

$$C_{et} = (1-\varpi_e)V_t. \quad (8.11)$$

Those who survive to the next period reinvest their equity. The net worth of the entrepreneurs is therefore

$$N_{t+1} = \varpi_e V_t + w_{et}, \tag{8.12}$$

which is the sum of the equity from investment made at period $t-1$ by surviving entrepreneurs and the income received from allocating inelastically one unit of labor to running business at time t. A share ϖ_e of the labor income w_{et} comes from entrepreneurs who survive from $t-1$ to t. The remaining share is a transfer from entrepreneurs who quit their business to startups.

The amount borrowed from the financial intermediaries is therefore $P_{kt}K_{t+1} - N_{t+1}$. Given the two states of nature, which are default or non-default, the repayment the financial intermediaries expect from the entrepreneurs is equal to the opportunity cost of lending, which is the real risk-free interest rate

$$\begin{aligned} \left\{[1-F(\bar{\nu}_t)]\bar{\nu}_t + (1-\mathcal{A})\int_0^{\bar{\nu}_t} \nu \mathrm{d}F(\nu)\right\} R_{k,t+1}P_{k,t}K_{t+1} = R_{t+1}\times \\ (P_{kt}K_{t+1} - N_{t+1})\,. \end{aligned} \tag{8.13}$$

It emerges from maximizing (8.10) subject to (8.13) that

$$\mathrm{E}(R_{k,t+1}) = s\left(\frac{N_{t+1}}{P_{kt}K_{t+1}}\right) R_{t+1}, \tag{8.14}$$

where the function s is decreasing in $N_{t+1}/P_{kt}K_{t+1}$, which measures the financial condition of entrepreneurs. Details on this optimization problem are provided in Appendix A.6. Relation (8.14), which is the supply curve for investment finance, shows that the premium on external funds, *i.e.* the ratio of the return on capital and the risk-free interest rate, is inversely related to the percentage equity holding of entrepreneurs. A low percentage equity holding worsens credit market frictions raising the premium on external funds. As for the demand for new capital, it is obtained by substituting the aggregate production and the price of capital relations, respectively (8.7) and (8.6), into the user cost of capital (8.9a).

Regarding the financial condition of entrepreneurs, changes over time in their net worth can be described endogenously by a difference equation. To get this, first, one substitutes (8.13), the financial intermediaries' expected income, into (8.10), the entrepreneurs' equity, which gives

$$\begin{aligned} V_t = &-\left(R_t + \frac{\mathcal{A}\int_0^{\bar{\nu}_t} \nu \mathrm{d}F(\nu) R_{kt}P_{k,t-1}K_t}{P_{t-1}K_t - N_t}\right)(P_{k,t-1}K_t - N_t) \\ &+ R_{kt}P_{k,t-1}K_t. \end{aligned} \tag{8.15}$$

Then, plugging (8.15) and (8.9c), the entrepreneurs' aggregate productivity, into (8.12), their net worth, one has after rearrangement

$$N_{t+1} = \varphi_t(1-\alpha)(1-\zeta)K_t^{\alpha}\left(z_t L_t^{\zeta}\right)^{1-\alpha} +$$
$$\varpi_e\left[R_{kt}P_{k,t-1}K_t - \left(R_t + \frac{\mathcal{A}\int_0^{\bar{\nu}_t}\nu \mathrm{d}F(\nu)R_{kt}P_{k,t-1}K_t}{P_{t-1}K_t - N_t}\right)(P_{k,t-1}K_t - N_t)\right]. \quad (8.16)$$

Note that in (8.16), both P_{wt}, the price of the wholesale good, and L_{et}, the entrepreneurial labor, have been set to unity. The financial accelerator operates through relations (8.14) and (8.16).

The natural logarithm of ν follows a normal distribution. This allows to write the expectation $\int_0^{\bar{\nu}_t}\nu \mathrm{d}F(\nu)$ in (8.16) as a cumulative distribution.

$$\int_0^{\bar{\nu}_t}\nu \mathrm{d}F(\nu) = \int_{-\infty}^{\ln(\bar{\nu}_t)}\frac{\nu}{\sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\ln(\nu)-\mu}{\sigma}\right)^2\right]\mathrm{d}\ln(\nu),$$

which becomes, after writing ν as $\exp[\ln(\nu)]$

$$\int_{-\infty}^{\ln(\bar{\nu}_t)}\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\ln(\nu)-\mu}{\sigma}\right)^2 + \ln(\nu)\right]\mathrm{d}\ln(\nu).$$

Performing a change of variable by defining $\mathcal{V} = \ln(\nu) - \mu$, one gets

$$\exp(\mu)\int_{-\infty}^{\bar{\mathcal{V}}_t+\mu}\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{1}{2}\frac{\mathcal{V}^2 - 2\mathcal{V}\sigma^2}{\sigma^2}\right)\mathrm{d}\mathcal{V},$$

which becomes after completing the square,

$$\exp(\mu+\frac{\sigma^2}{2})\int_{-\infty}^{\bar{\mathcal{V}}_t+\mu}\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\mathcal{V}-\sigma^2}{\sigma}\right)^2\right]\mathrm{d}\mathcal{V}.$$

This establishes that when ν is log-normally distributed,

$$\int_0^{\bar{\nu}_t}\nu \mathrm{d}F(\nu) = \exp(\mu+\frac{\sigma^2}{2})F\left[\ln(\bar{\nu}_t)\right].$$

Given the expected value of ν, *i.e.* $\int_{-\infty}^{\infty}\nu \mathrm{d}F(\nu)$, equals one, the following restriction should be placed on the first term on the rhs of the above relation: $\mu = -\sigma^2/2$. It follows that

$$\begin{aligned}\int_0^{\bar{\nu}_t}\nu \mathrm{d}F(\nu) &= F\left[\ln(\bar{\nu}_t)\right]\\ &= \Phi\left(\frac{\ln(\bar{\nu}_t)-\sigma^2/2}{\sigma}\right).\end{aligned}$$

The cutoff $\bar{\nu}_t$ is held constant over time, *i.e.* $\bar{\nu}_t = \bar{\nu}$.

8.3 The Retailers

They operate in a monopolistically competitive environment to produce each a differentiated good $Y_t(i)$ that is aggregated into a composite final good Y_t^s. The demand for the differentiated good to produce the final good is described by (7.6). The price of the final good, P_t, in terms of the differentiated goods is described by (7.7). Given there is a probability ϖ that retailers keep their prices, $P_t(i)$, unchanged, P_t can be written as a weighted average of the price of non-adjusting and adjusting firms as in (7.14). The price of non-adjusting retailers is the average price that prevailed last period, P_{t-1}, and the price of adjusting retailers is described by (7.10).

8.4 The Monetary authority

The monetary policy rule is

$$\ln \frac{R_{nt}}{R} = \nu_r \ln \frac{R_{n,t-1}}{R} + \nu_\pi \ln \frac{\pi_t}{\pi} + \nu_y \ln \frac{Y_t}{Y} + \xi_t, \quad \xi_t \sim \mathcal{N}\left(0, \sigma_\xi^2\right) \tag{8.17a}$$

$$R_{n,t+1} = \mathrm{E}_t \left(R_{t+1} \frac{P_{t+1}}{P_t} \right), \tag{8.17b}$$

where R_{nt} denotes the nominal interest rate. The monetary authority's objective is described by (7.12).

8.5 The General Equilibrium

Assuming prices ares are stationary, the first-order Taylor series approximation of (7.10) and (7.14) gives the new-Keynesian Phillips curve (see details in Appendix B.2)

$$\hat{\pi}_t = \frac{(1-\varpi)(1-\varpi\beta)}{\varpi} \hat{\varphi}_t + \beta \mathrm{E}_t \hat{\pi}_{t+1}, \tag{8.18}$$

where $\hat{\pi}_t$ and $\hat{\varphi}_t$ are the respective percentage deviations of π_t and φ_t from steady state.

According to the new-Keynesian Phillips curve, inflation is forward-looking and driven by firms' marginal cost. The traditional Phillips curve, as in model (3.9), describes inflation as a backward-looking variable that instead depends on output gap or unemployment rate.

The DSGE model consists of a set of prices $\{(P_t, P_t^*, P_{kt}, R_t, R_{kt}, w_t)\}_{t=0}^\infty$, a state of the world $\{(a_t, x_t, z_t, \chi_t)\}_{t=0}^\infty$, an allocation $\{(C_t, L_t^s, M_t^d)\}_{t=0}^\infty$ for the representative household, an allocation $\left\{(B_t, C_{et}, I_t, K_t, L_t^d, N_t, V_t, Y_t), \varphi_t)\right\}_{t=0}^\infty$ for entrepreneurs, an allocation $\{(S_t, Y_t^s)\}_{t=0}^\infty$ for retailers, and an allocation $\{(M_t^s)\}_{t=0}^\infty$ for the monetary authority such that:

i. $\{(C_t, L_t^s, M_t^d)\}_{t=0}^{\infty}$ solves relations (8.3a)-(8.3c),

ii. $\left\{(B_t, C_{et}, I_t, K_t, L_t^d, N_t, V_t, Y_t, \varphi_t)\right\}_{t=0}^{\infty}$ solves relations (8.4), (8.6), (8.7), (8.9a)-(8.9b), (8.11), (8.14), (8.15), and (8.16) ,

iii. $\{(S_t, Y_t^s)\}_{t=0}^{\infty}$ solves relations (7.15a) and (7.16),

iv. $\{(a_t, x_t, z_t, \chi_t)\}_{t=0}^{\infty}$ is governed by relations (4.5), (7.2a)-(7.2b), and (8.5),

v. capital and labor markets clear, *i.e.* $B_t = P_{k,t-1}K_t - N_t$ and $L_t^d = L_t^s$, equilibrium in the final good market is described by $Y_t = C_t + C_{et} + I_t + \mathcal{A}\Phi\left(\frac{\ln(\bar{\nu})-\sigma^2/2}{\sigma}\right) R_{kt}P_{k,t-1}K_t$, relations (8.17a) and (8.17b) ensure the money market clears, *i.e.* $M_t^d = M_t^s$, and relations (7.14) and (8.18) describe the evolution of prices.

8.6 Calibration

The values assigned to the parameters α, β, δ, ϖ, ρ_a, ρ_z, ρ_χ, σ_a, σ_z, and σ_χ are the same as those used in Section 7 (see Table 7.2). The value of the interest elasticity of money demand from the previous section, -21.629, is assigned to e. I assume ζ, households' share of labor income, is 99%.

Relation (8.14) is linearized to avoid deriving the functional form of s from a probability distribution, which gives

$$\mathrm{E}\left(\hat{R}_{k,t+1}\right) - \hat{R}_{t+1} = \nu_s\left(\hat{P}_{kt} + \hat{K}_{t+1} - \hat{N}_{t+1}\right), \tag{8.19}$$

where the hat over the variables K_{t+1}, N_{t+1}, P_{kt}, R_{t+1}, $R_{k,t+1}$ refers to their percentage deviations from steady state. The parameter $\nu_s \geq 0$, the elasticity of the external finance premium with respect to the percentage equity holding, is defined as follows

$$\begin{aligned}\nu_s &= -\frac{\mathrm{d}\ln s\left(\frac{N_{t+1}}{P_{kt}K_{t+1}}\right)}{\mathrm{d}\ln\frac{N_{t+1}}{P_{kt}K_{t+1}}}\\ &= \frac{\mathrm{d}\ln\frac{R_{k,t+1}}{R_{t+1}}}{\mathrm{d}\ln\psi\left(\frac{R_{k,t+1}}{R_{t+1}}\right)}.\end{aligned}$$

The parameter ν_s only appears in (8.19). It does not affect the steady state position of the economy given all the variables in this relation are expressed as percentage deviation. Its value will be set while simulating the model doing a sensitivity analysis.

According to Statistics Canada's data on firms' quarterly balance sheets, the ratio of the capital share and the capital assets averages .659 over the period 1988-2015. I have assigned this value to the percentage equity holding N/P_kK.

I have set $\int_0^{\bar{\nu}} \nu \mathrm{d}F(\nu)$, the expected value of the idiosyncratic risk among default firms, to .01. I have set the auditing cost, $\mathcal{A}$, to .12 as Bernanke, Gertler, and Gilchrist did. The values of ϖ_e and υ that result from the calibration exercise are respectively .994 and 1.468.

The investment-specific technological change parameter, x_t, is measured by the natural logarithm of the ratio of GDP and business gross fixed capital formation implicit prices. The values assigned to the parameters describing the investment-specific technological change stochastic process are then obtained from an OLS estimation of relation (8.5): $\rho_x = .968$ and $\sigma_x = .01$.

$$\begin{aligned} \hat{x}_t &= .0001 \quad + .968 x_{t-1} \\ &\quad (-.78) \quad\;\; (73.96) \\ \bar{R}^2 &= .975 \qquad \sigma_x = .0102 \end{aligned} \tag{8.20}$$

The t-ratios in parentheses help infer that the persistence parameter, ρ_x, is significantly positive and the intercept term of the regression is not different from zero.

8.7 The Cyclical Properties

To have the model produce summary statistics close to observations, I have set ν_s, the elasticity of the external finance premium with respect to the percentage equity holding, to .01 and ϕ, the capital adjustment cost parameter, to 10. Given $\Lambda_y = 1$, the values of ν_r, ν_π, and ν_y that minimize the monetary authority's objective function are respectively .9, 1.9, and .224.

Table 8.1 shows the cyclical behavior of the financial accelerator economy for two cases: (1) only TFP shock hits the economy and (2) all the four shocks considered occur simultaneously.

All the four shocks help explain 78% of the fluctuations in output, 86% of those in investment, and 95% of those in hours worked. The volatility in consumption, productivity, money, and inflation they generated exceed observations. TFP shock accounts for about half of the fluctuations in output and consumption and all of those in productivity.

As for the monetary policy shock, it adds an extra 18.6 percentage points (pp) to output volatility, 50 pp to the volatility of consumption, and 72.3 pp to that of hours worked. The additional fluctuations in consumption and investment are mainly due to the preference shocks and investment-specific technological change, respectively.

Adding the fours shocks has helped reproduced the correlation coefficient of output with consumption and improved those of output with money and inflation but deteriorated those with investment, hours worked, and nominal interest rate.

Besides, the model well reproduce the near-zero correlation between hours worked and productivity. Figures 8.1 and 8.2 show the sensitivity of the standard

Table 8.1: Cyclical Behavior of the Canadian and the Financial Accelerator Economies, Percentage Deviation from Trend of Key Variables

	Canadian Economy			Financial Accelerator Economy					
				Only Technology Shock			All Shocks		
Variable	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
Output (GDP)	1.56	1	.89	.85	1	.73	1.22	1	.40
Consumption	1.14	.84	.85	.64	1	.75	1.40	.83	.29
Investment	4.9	.75	.88	2.15	1	.70	4.22	.47	.70
Hours	1.41	.9	.9	.14	.94	.84	1.34	.78	.12
Productivity	.68	.42	.62	.69	1	.73	.93	.82	.43
M1	4.4	.4	.92	.64	.84	.87	8.17	.52	.02
Interest Rate	1.35	.56	.75	.12	-1	.72	.15	-.91	.47
Inflation	.28	.11	.12	.13	-.95	.50	.74	.48	-.03

Columns (1) Percentage standard deviations, columns (2) Correlation coefficient with output, and columns (3) First-order autocorrelation coefficient.

deviation of some variables and that of their correlation with output to the elasticity of the external finance premium with respect to entrepreneurs' percentage equity holding. An increase in this parameter raises fluctuations in consumption, entrepreneurs' net worth, the relative price of capital, and interest rates but reduces fluctuations in output, hours worked, and inflation. It makes consumption, inflation, and labor less procyclical while the price of capital and the net worth become more countercyclical.

8.8 The Impulse Responses

This subsection shows how the response of the economy to one-off shocks depends on ν_s, the elasticity of the external finance premium with respect to the percentage equity holding.

Figure 8.3 shows that an expansionary monetary policy, *i.e.* a fall in the nominal interest rate, on the one hand, raises consumption, hours worked, output, and inflation. On the other hand, investment, the relative price of capital, and the net worth of entrepreneurs fall as a result of the decrease in the return on capital. The magnitude and shape of the responses of these latter variables depend on the value of ν_s.

A positive money demand shock immediately lowers interest rate, consumption, hours worked, output, and inflation (Figure 8.4). It has the opposite effect on investment and the relative price of capital. These immediate impacts become less important as ν_s increases.

The preference shock, as it appears in Figure 8.5, immediately raises consumption. Output and hours worked also rise despite the fall in real wage. This shock has a negative impact on savings, which raises the risk-free interest rate. The relative price of capital goods falls because of the decline in investment. A higher

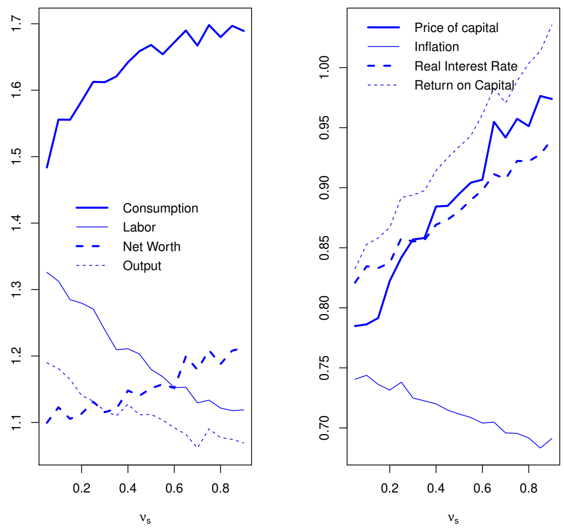

Figure 8.1: Standard Deviation as a Function of the Elasticity of External Finance Premium with Respect to Entrepreneurs' Percentage Equity Holding, the Financial Accelerator Model

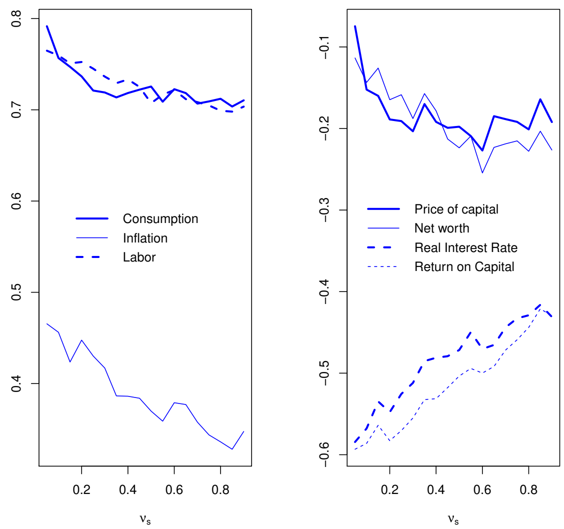

Figure 8.2: Correlation with Output as a Function of the Elasticity of External Finance Premium with Respect to Entrepreneurs' Percentage Equity Holding, the Financial Accelerator Model

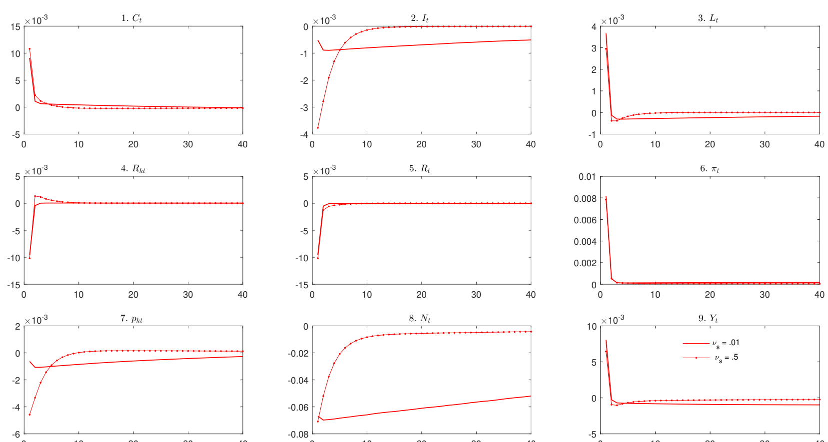

Figure 8.3: Impulse Responses to an Expansionary Monetary Policy Shock, Deviation form Steady State, the Financial Accelerator Model

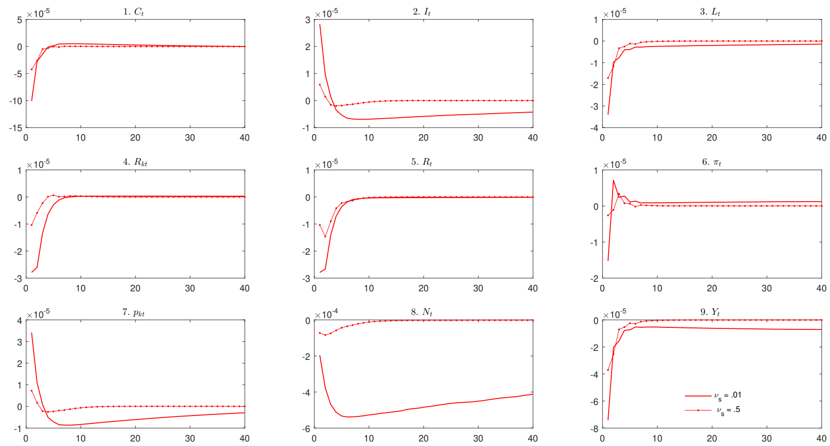

Figure 8.4: Impulse Responses to an Money Demand Shock, Deviation form Steady State, the Financial Accelerator Model

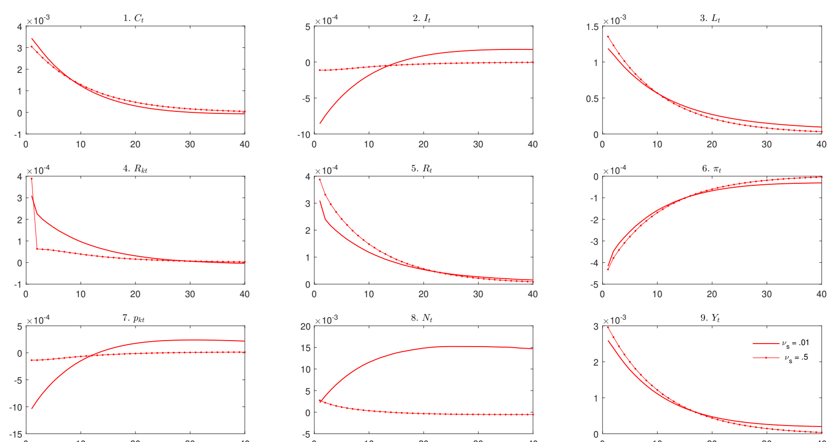

Figure 8.5: Impulse Responses to a Positive Preference Shock, Deviation form Steady State, the Financial Accelerator Model

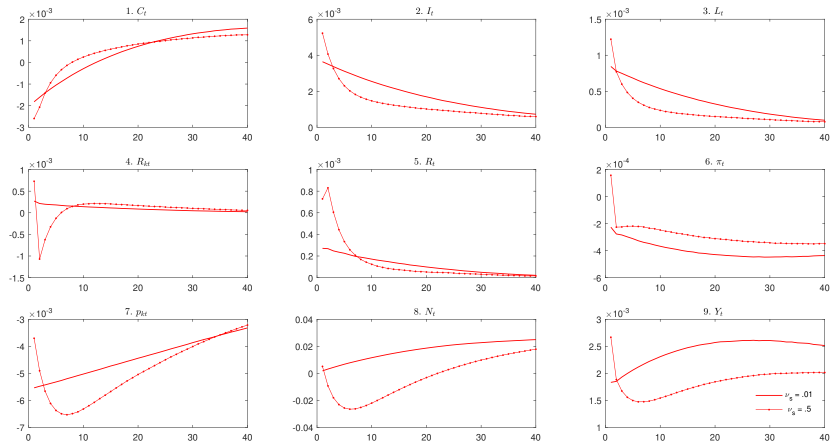

Figure 8.6: Impulse Responses to an Investment-Specific Technological Shock, Deviation form Steady State, the Financial Accelerator Model

ν_s limits the decline in investment and in the relative price of capital. For a higher ν_s, a smaller decline in savings and consequently in investment cause a higher increase in the risk-free interest rate, which significantly impacts on the response of entrepreneurs' net worth.

The investment-specific technological change lowers the relative price of capital and raises the demand for investment goods (Figure 8.6). Consumption declines as interest rate rises. Entrepreneurs' net worth also rise. These impacts are stronger as ν_s increases.

9 Conclusion

This research purports to explain business cycles in Canada and finding out whether monetary disturbances are instrumental in these fluctuations. I have used both empirical and theoretical models to investigate these issues. The empirical investigations have consisted in studying the relationship between detrended nominal and real time series and computing impulse responses from vector autoregressive models. The theoretical models comprised both RBC and new-Keynesian models that I have calibrated to Canadian data and simulated.

The empirical investigations have enabled to conclude that money is not neutral. The money measure M1 and its growth rate have turned out to be correlated with many real variables. So are other measures of money and short-term nominal interest rates. However, the sign and size of the correlation depend on the measure of money used. The monetary base and M1 are positively correlated with the business cycle whereas M2 is countercyclical. Furthermore, the monetary base and M1 explain business cycles without causing them whereas a mutual causality between M2 and business cycles emerged. The empirical investigations also reveal that a major change in the objective of monetary policy, which is the Bank of Canada's commitment at the beginning of 1991 to lower inflation to two percent, caused a break in the cyclical behavior of real and nominal variables. They have become less volatile. The three measures of money: the monetary base, M1, and M2, have all become countercyclical and nominal interest rates have become more pro-cyclical and persistent. This indicates monetary policy does not just curb inflation but also influences the real economic activity.

The theoretical models assign a role to money: a good required or used to save time in shopping, or a good that directly yields utility. They also point various channels whereby monetary disturbances propagate to the real economy: nominal capital gain tax, nominal price rigidity and deteriorating financial conditions. I have addressed three questions while simulating the theoretical models: Do monetary policy help account for the cyclical behavior of the Canadian economy? If so, could changes in the monetary policy rules help account for the break observed after 1991? Are the propagation channels: nominal capital gain tax, nominal price rigidity and deteriorating financial conditions essential features to account for business cycles?

Simulating the theoretical models reveals that TFP shock accounts for most of business cycle fluctuations. Monetary policy plays a limited role in RBC models and a more important role in new-Keynesian models. Sensitivity analyses help conclude that monetary policy shocks do not propagate in the absence of price stickiness or the interaction of inflation and the nominal capital gain tax. Moreover, changes in the way the monetary authority responds to output fluctuations, money growth, and inflation could cause breaks in the cyclical behavior of the economy.

To further the understanding of business cycle fluctuations in Canada, it is important to look at other determinants a part from technological change and monetary disturbances. Other determinants pointed to in the literature are expectations (Beaudry and Portier, 2004; Auray, Gomme, and Guo, 2013).

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Appendices

A The Optimization Problems

A.1 The Inflation Tax Model

$$
\begin{aligned}
\mathcal{V}(\mathcal{S}_t) = \max\ & a \ln c_{1t} + (1-a)\ln c_{2t} + \Upsilon(1-l_t) + \beta \mathrm{E}_t \mathcal{V}(\mathcal{S}_{t+1}) \\
& + \mu_{1t}(m_t + \tau_t - p_t c_{1t}) \\
& + \mu_{2t}\ \left[p_t(w_t l_t + r_t k_t) + m_t + \tau_t - p_t(c_{1t} + c_{2t} + i_t) - m_{t+1}\right] \\
& + \mu_{3t}\left[(1-\delta)k_t + i_t - k_{t+1}\right], \qquad \text{(A.1a)}
\end{aligned}
$$

with $\mathcal{S}_t = (k_t, m_t, z_t, \xi_t)$.

The First-Order Conditions (FOCs)

$$
\begin{aligned}
c_{1t}: &\quad \frac{a}{c_{1t}} = (\mu_{1t} + \mu_{2t})\, p_t && \text{(A.2a)} \\
c_{2t}: &\quad \frac{1-a}{c_{2t}} = \mu_{2t} p_t && \text{(A.2b)} \\
l_t: &\quad \Upsilon = \mu_{2t} p_t w_t && \text{(A.2c)} \\
i_t: &\quad \mu_{2t} p_t = \mu_{3t} && \text{(A.2d)} \\
k_{t+1}: &\quad \beta \frac{\partial \mathrm{E}_t \mathcal{V}(\mathcal{S}_{t+1})}{\partial k_{t+1}} = \mu_{3t} && \text{(A.2e)} \\
m_{t+1}: &\quad \beta \frac{\partial \mathrm{E}_t \mathcal{V}(\mathcal{S}_{t+1})}{\partial m_{t+1}} = \mu_{2t} && \text{(A.2f)}
\end{aligned}
$$

The Envelope Conditions

$$
\begin{aligned}
k_t: &\quad \frac{\partial \mathcal{V}(\mathcal{S}_t)}{\partial k_t} = \mu_{2t} p_t r_t + (1-\delta)\mu_{3t} && \text{(A.3a)} \\
m_t: &\quad \frac{\partial \mathcal{V}(\mathcal{S}_t)}{\partial m_t} = \mu_{1t} + \mu_{2t} && \text{(A.3b)}
\end{aligned}
$$

The following relations emerge from the FOCs and the envelope conditions

$$
\Upsilon c_{2t} = (1-a) w_t \qquad \text{(A.4a)}
$$

$$
\beta \mathrm{E}_t \left[(1 + r_{t+1} - \delta)\, \frac{c_{2t}}{c_{2,t+1}}\right] = 1 \qquad \text{(A.4b)}
$$

$$\beta \mathrm{E}_t \left(\frac{c_{2t}}{c_{1,t+1}} \frac{p_t}{p_{t+1}} \right) = \frac{1-a}{a} \tag{A.4c}$$

Relation (A.4a) results from a combination of (A.2b) and (A.2c). To get (A.4b), plug the envelope condition (A.3a) into the FOC (A.2e) and then use (A.2d), which gives

$$\beta \mathrm{E}_t \left[(1 + r_{t+1} - \delta) \frac{\mu_{2,t+1}}{\mu_{2t}} \frac{p_{t+1}}{p_t} \right] = 1.$$

Plugging then (A.2b) into the above relation gives (A.4b). To get (A.4c), first, plug the lead of the envelope condition (A.3b) into the FOC (A.2f). Then use (A.2a) and (A.2b)

The Normalized Equations

Let $\boldsymbol{v}_t$ and $\mathbf{v}_t$ respectively designate the vectors of non-stationary real and nominal variables, $\boldsymbol{v}_t = (c_{1t}, c_{2t}, i_t, k_t, w_t, y_t)$ and $\mathbf{v}_t = (m_t, \tau_t)$. Normalizing these vectors gives $\tilde{\boldsymbol{v}} = \boldsymbol{v}_t/\gamma_z^t$ and $\hat{\mathbf{v}} = \mathbf{v}_t/\gamma_m^t$, with $\gamma_m = \gamma_z \pi$. Normalizing the price level gives $\bar{p}_t = p_t/\pi^t$.

$$\bar{p}_t \tilde{c}_{1t} = \hat{m}_t + \hat{\tau}_t \tag{A.5a}$$

$$\bar{p}_t \left(\tilde{c}_{1t} + \tilde{c}_{2t} + \tilde{i}_t \right) + \gamma_z \pi m_{t+1} = \bar{p}_t \left(\tilde{w}_t l_t + r_t \tilde{k}_t \right) + \hat{m}_t + \hat{\tau}_t \tag{A.5b}$$

$$\tilde{c}_t = \tilde{c}_{1t} + \tilde{c}_{2t} \tag{A.5c}$$

$$\gamma_z \tilde{k}_{t+1} = (1-\delta)\tilde{k}_t + \tilde{i}_t \tag{A.5d}$$

$$\Upsilon \tilde{c}_{2t} = (1-a)\tilde{w}_t \tag{A.5e}$$

$$\beta \mathrm{E}_t \left[(1 + r_{t+1} - \delta) \frac{\tilde{c}_{2t}}{\tilde{c}_{2,t+1}} \right] = \gamma_z \tag{A.5f}$$

$$\beta \mathrm{E}_t \left(\frac{\tilde{c}_{2t}}{\tilde{c}_{1,t+1}} \frac{\bar{p}_t}{\bar{p}_{t+1}} \right) = \gamma_z \pi \frac{1-a}{a} \tag{A.5g}$$

$$\tilde{y}_t = \tilde{k}_t^{\alpha} [\exp(\tilde{z}_t) l_t]^{1-\alpha} \tag{A.5h}$$

$$\tilde{z}_t = \rho_z \tilde{z}_{t-1} + \epsilon_{zt} \tag{A.5i}$$

$$r_t = \alpha \frac{\tilde{y}_t}{\tilde{k}_t} \tag{A.5j}$$

$$\tilde{w}_t = (1-\alpha) \frac{\tilde{y}_t}{l_t} \tag{A.5k}$$

$$\hat{m}_{t+1} = \exp(\xi_t) \hat{m}_t \tag{A.5l}$$

$$\xi_t = \rho_m \xi_{t-1} + \epsilon_{mt} \tag{A.5m}$$

$$\hat{\tau}_t = \gamma_z \pi \hat{m}_{t+1} - \hat{m}_t \tag{A.5n}$$

A.2 The RBC Model with Endogenous Money

$$
\begin{aligned}
\mathcal{V}(\mathcal{S}_t) = \max\ & \frac{1}{e} c_t^e (1 - l_t - s_t)^{ve} + \beta \mathrm{E}_t \mathcal{V}(\mathcal{S}_{t+1}) \\
& + \mu_{1t} \left[\omega_0 - \omega_1 \left(\frac{m_t}{p_t c_t} \right)^{\omega_2} - s_t \right] \\
& + \mu_{2t} \ \left[p_t (w_t l_t + r_t k_t) + m_t + \tau_t - p_t (c_t + i_t) - m_{t+1} \right] \\
& + \mu_{3t} \left[(1-\delta) k_t + i_t - k_{t+1} \right],
\end{aligned} \tag{A.6a}
$$

with $\mathcal{S}_t = (k_t, m_t, z_t, \xi_t)$.

The FOCs

$$c_t : \quad c_t^{e-1} (1 - l_t - s_t)^{ve} + \mu_{1t} \frac{\omega_1 \omega_2}{c_t} \left(\frac{m_t}{p_t c_t} \right)^{\omega_2} = \mu_{2t} p_t \tag{A.7a}$$

$$l_t : \quad \upsilon c_t^e (1 - l_t - s_t)^{ve-1} = \mu_{2t} p_t w_t \tag{A.7b}$$

$$s_t : \quad -\upsilon c_t^e (1 - l_t - s_t)^{ve-1} = \mu_{1t} \tag{A.7c}$$

$$i_t : \quad \mu_{2t} p_t = \mu_{3t} \tag{A.7d}$$

$$k_{t+1} : \quad \beta \frac{\partial \mathrm{E}_t \mathcal{V}(\mathcal{S}_{t+1})}{\partial k_{t+1}} = \mu_{3t} \tag{A.7e}$$

$$m_{t+1} : \quad \beta \frac{\partial \mathrm{E}_t \mathcal{V}(\mathcal{S}_{t+1})}{\partial m_{t+1}} = \mu_{2t} \tag{A.7f}$$

The Envelope Conditions

$$k_t : \quad \frac{\partial \mathcal{V}(\mathcal{S}_t)}{\partial k_t} = \mu_{2t} p_t r_t + (1-\delta) \mu_{3t} \tag{A.8a}$$

$$m_t : \quad \frac{\partial \mathcal{V}(\mathcal{S}_t)}{\partial m_t} = -\mu_{1t} \frac{\omega_1 \omega_2}{p_t c_t} \left(\frac{m_t}{p_t c_t} \right)^{\omega_2 - 1} + \mu_{2t} \tag{A.8b}$$

The following relations emerge from the FOCs and the envelope conditions

$$1 - l_t - \omega_0 + (1 - \upsilon \omega_2) \omega_1 \left(\frac{m_t}{p_t c_t} \right)^{\omega_2} = \upsilon \frac{c_t}{w_t} \tag{A.9a}$$

$$\beta \mathrm{E}_t \left[(1 + r_{t+1} - \delta) \left(\frac{c_{t+1}}{c_t} \right)^e \left(\frac{1 - l_{t+1} - s_{t+1}}{1 - l_t - s_t} \right)^{ve-1} \frac{w_t}{w_{t+1}} \right] = 1 \tag{A.9b}$$

$$\beta \mathrm{E}_t \left[\frac{p_t}{p_{t+1}} \frac{w_t}{w_{t+1}} + \omega_1 \omega_2 \frac{p_t w_t}{m_{t+1}} \left(\frac{m_{t+1}}{p_{t+1} c_{t+1}} \right)^{\omega_2} \right] \times \left(\frac{c_{t+1}}{c_t} \right)^e \left(\frac{1 - l_{t+1} - s_{t+1}}{1 - l_t - s_t} \right)^{ve-1} = 1 \tag{A.9c}$$

A.3 The Inflation and Tax Code Model

$$\mathcal{V}(\mathcal{S}_t) = \max \ \frac{1}{e} C_t^e (1 - L_t - S_t)^{ve} + \beta \mathrm{E}_t \mathcal{V}(\mathcal{S}_{t+1})$$

subject to:

$$S_t = \omega_0 - \omega_1 \left(\frac{M_t}{P_t C_t} \right)^{\omega_2} \tag{A.10a}$$

$$\begin{aligned} P_t(C_t + I_t) + B_{t+1} + M_{t+1} + \tau_{gt} G_t = M_t + T_t + (1 - \tau_{lt}) P_t w_t L_t \\ + \left[(1 - \tau_{kt}) q_t + \tau_{kt} \delta \right] P_t K_t \\ + \left[1 + (1 - \tau_{bt})(R_t - 1) \right] B_t \end{aligned} \tag{A.10b}$$

$$K_{t+1} = (1 - \delta) K_t + I_t \tag{A.10c}$$

$$G_t = (P_t - P_{t-1}) K_t \tag{A.10d}$$

with $\mathcal{S}_t = (K_t, M_t, z_t)$.

The FOCs

$$C_t: \quad C_t^{e-1} (1 - L_t - S_t)^{ve} + \mu_{1t} \frac{\omega_1 \omega_2}{C_t} \left(\frac{M_t}{P_t C_t} \right)^{\omega_2} = \mu_{2t} P_t \tag{A.11a}$$

$$L_t: \quad v C_t^e (1 - L_t - S_t)^{ve-1} = \mu_{2t} (1 - \tau_{lt}) P_t w_t \tag{A.11b}$$

$$S_t: \quad -v C_t^e (1 - L_t - S_t)^{ve-1} = \mu_{1t} \tag{A.11c}$$

$$I_t: \quad \mu_{2t} P_t = \mu_{3t} \tag{A.11d}$$

$$G_t: \quad -\mu_{2t} \tau_{gt} = \mu_{4t} \tag{A.11e}$$

$$K_{t+1}: \quad \beta \frac{\partial \mathrm{E}_t \mathcal{V}(\mathcal{S}_{t+1})}{\partial K_{t+1}} = \mu_{3t} \tag{A.11f}$$

$$M_{t+1}: \quad \beta \frac{\partial \mathrm{E}_t \mathcal{V}(\mathcal{S}_{t+1})}{\partial M_{t+1}} = \mu_{2t} \tag{A.11g}$$

$$B_{t+1}: \quad \beta \frac{\partial \mathrm{E}_t \mathcal{V}(\mathcal{S}_{t+1})}{\partial B_{t+1}} = \mu_{2t} \tag{A.11h}$$

The Lagrange multipliers μ_{1t} through μ_{4t} are respectively associated to the constraints (A.10a) through (A.10d).

The Envelope Conditions

$$K_t: \quad \frac{\partial \mathcal{V}(\mathcal{S}_t)}{\partial K_t} = \mu_{2t}\left[(1-\tau_{kt})q_t + \tau_{kt}\delta\right]P_t + (1-\delta)\mu_{3t} + \mu_{4t}(P_t - P_{t-1}) \tag{A.12a}$$

$$M_t: \quad \frac{\partial \mathcal{V}(\mathcal{S}_t)}{\partial M_t} = -\mu_{1t}\frac{\omega_1\omega_2}{P_tC_t}\left(\frac{M_t}{P_tC_t}\right)^{\omega_2-1} + \mu_{2t} \tag{A.12b}$$

$$B_t: \quad \frac{\partial \mathcal{V}(\mathcal{S}_t)}{\partial B_t} = \mu_{2t}\left[1 + (1-\tau_{bt})(R_t - 1)\right] \tag{A.12c}$$

Combining the FOCs and the envelope conditions gives

$$1 - L_t - \omega_0 + (1-\upsilon\omega_2)\omega_1\left(\frac{M_t}{P_tC_t}\right)^{\omega_2} = \upsilon\frac{C_t}{(1-\tau_{lt})w_t} \tag{A.13a}$$

$$\left(\frac{M_t}{P_t}\right)^{\omega_2-1} = \frac{1-\tau_{bt}}{1-\tau_{lt}}\frac{R_t-1}{w_t}\frac{C_t^{\omega_2}}{\omega_1\omega_2} \tag{A.13b}$$

$$\beta \mathrm{E}_t\left[1 + (1-\tau_{k,t+1})(q_{t+1}-\delta) - \tau_{g,t+1}\left(1 - \frac{P_t}{P_{t+1}}\right)\right] \times \left(\frac{C_{t+1}}{C_t}\right)^e \left(\frac{1-l_{t+1}-s_{t+1}}{1-l_t-s_t}\right)^{\upsilon e-1} \frac{1-\tau_{lt}}{1-\tau_{l,t+1}}\frac{w_t}{w_{t+1}} = 1. \tag{A.13c}$$

$$\mathrm{E}_t\left\{\left[1 + (1-\tau_{k,t+1})(q_{t+1}-\delta) - \tau_{g,t+1}\left(1-\frac{P_t}{P_{t+1}}\right)\right]\frac{P_{t+1}}{P_t}\right\} = 1 + \mathrm{E}_t(1-\tau_{b,t+1})(R_{t+1}-1) \tag{A.13d}$$

A.4 The Sticky Price Model

$$\mathcal{V}(\mathcal{S}_t) = \max \ a_t\left[\frac{C_t^e}{e} + \frac{\chi_t^{1-\eta}}{\eta}\left(\frac{M_t}{P_t}\right)^\eta\right] + \upsilon\ln(1-L_t) + \beta\mathrm{E}_t\mathcal{V}(\mathcal{S}_{t+1})$$

subject to:

$$\begin{aligned} & w_tL_t + q_tK_t + \frac{M_{t-1}}{P_t} + \frac{B_{t-1}}{P_t} + \frac{D_t}{P_t} - \frac{T_t}{P_t} \\ &= C_t + K_{t+1} - (1-\delta)K_t + \frac{\phi}{2}\left(\frac{K_{t+1}}{K_t} - \kappa\right)^2 K_t \\ &\quad + \frac{M_t}{P_t} + \frac{B_t}{P_tR_t} \end{aligned} \tag{A.14a}$$

with $\mathcal{S}_t = (K_t, M_{t-1}, B_{t-1}, z_t)$.

The FOCs

$$C_t: \quad a_t C_t^{e-1} = \mu_t \tag{A.15a}$$

$$L_t: \quad \frac{\upsilon}{1-L_t} = \mu_t w_t \tag{A.15b}$$

$$K_{t+1}: \quad \beta \frac{\partial \mathrm{E}_t \mathcal{V}\left(\mathcal{S}_{t+1}\right)}{\partial K_{t+1}} = \mu_t \left[1 + \phi\left(\frac{K_{t+1}}{K_t} - \kappa\right)\right] \tag{A.15c}$$

$$M_t: \quad \frac{a_t \chi_t^{1-\eta}}{P_t}\left(\frac{M_t}{P_t}\right)^{\eta-1} + \beta \frac{\partial \mathrm{E}_t \mathcal{V}\left(\mathcal{S}_{t+1}\right)}{\partial M_t} = \frac{\mu_t}{P_t} \tag{A.15d}$$

$$B_t: \quad \beta \frac{\partial \mathrm{E}_t \mathcal{V}\left(\mathcal{S}_{t+1}\right)}{\partial B_t} = \frac{\mu_t}{P_t R_t} \tag{A.15e}$$

The Envelope Conditions

$$K_t: \quad \frac{\partial \mathcal{V}\left(\mathcal{S}_t\right)}{\partial K_t} = \mu_t \left[1 + q_t - \delta + \phi\left(\frac{K_{t+1}}{K_t} - \kappa\right)\left(\frac{K_{t+1}}{K_t} + \frac{\kappa}{2}\right)\right] \tag{A.16a}$$

$$M_{t-1}: \quad \frac{\partial \mathcal{V}\left(\mathcal{S}_t\right)}{\partial M_{t-1}} = \frac{\mu_t}{P_t} \tag{A.16b}$$

$$B_{t-1}: \quad \frac{\partial \mathcal{V}\left(\mathcal{S}_t\right)}{\partial B_{t-1}} = \frac{\mu_t}{P_t} \tag{A.16c}$$

Combining the FOCs and the envelope conditions gives

$$\frac{\upsilon}{a_t} C_t^{1-e} = w_t(1-L_t) \tag{A.17a}$$

$$\chi_t^{1-\eta}\left(\frac{M_t}{P_t}\right)^{\eta-1} = \frac{R_t - 1}{R_t} C_t^{e-1} \tag{A.17b}$$

$$\beta \mathrm{E}_t \left[\frac{a_{t+1}}{a_t}\left(\frac{C_{t+1}}{C_t}\right)^{e-1} \frac{P_t}{P_{t+1}}\right] R_t = 1 \tag{A.17c}$$

$$\beta \mathrm{E}_t \left\{\frac{a_{t+1}}{a_t}\left(\frac{C_{t+1}}{C_t}\right)^{e-1}\left[1 + q_{t+1} - \delta + \phi\left(\frac{K_{t+2}}{K_{t+1}} - \kappa\right)\left(\frac{K_{t+2}}{K_{t+1}} + \frac{\kappa}{2}\right)\right]\right\} = 1 + \phi\left(\frac{K_{t+1}}{K_t} - \kappa\right) \tag{A.17d}$$

A.5 The Financial Accelerator Model

$$\mathcal{V}\left(\mathcal{S}_t\right) = \max \ \frac{a_t}{e} \ln\left[C_t^e + \chi_t^{1-e}\left(\frac{M_t}{P_t}\right)^e\right] + \upsilon \ln(1-L_t) + \beta \mathrm{E}_t \mathcal{V}\left(\mathcal{S}_{t+1}\right)$$
$$+\mu_t P_t \left(w_t L_t + R_t B_t + \frac{D_{t+1} + M_{t-1} - M_t - T_t}{P_t} - C_t + B_{t+1}\right) \quad \text{(A.18a)}$$

with $\mathcal{S}_t = (B_t, M_{t-1}, z_t)$.

The FOCs

$$C_t: \quad a_t \frac{C_t^{e-1}}{C_t^e + \chi_t^{1-e}\left(\frac{M_t}{P_t}\right)^e} = \mu_t P_t \quad \text{(A.19a)}$$

$$L_t: \quad \frac{\upsilon}{1-L_t} = \mu_t P_t w_t \quad \text{(A.19b)}$$

$$B_{t+1}: \quad \beta \frac{\partial \mathrm{E}_t \mathcal{V}\left(\mathcal{S}_{t+1}\right)}{\partial B_{t+1}} = \mu_t P_t \quad \text{(A.19c)}$$

$$M_t: \quad \frac{a_t \chi_t^{1-e}}{P_t} \frac{\left(\frac{M_t}{P_t}\right)^{e-1}}{C_t^e + \chi_t^{1-e}\left(\frac{M_t}{P_t}\right)^e} + \beta \frac{\partial \mathrm{E}_t \mathcal{V}\left(\mathcal{S}_{t+1}\right)}{\partial M_t} = \mu_t \quad \text{(A.19d)}$$

The Envelope Conditions

$$B_t: \quad \frac{\partial \mathcal{V}\left(\mathcal{S}_t\right)}{\partial B_t} = \mu_t P_t R_t \quad \text{(A.20a)}$$

$$M_{t-1}: \quad \frac{\partial \mathcal{V}\left(\mathcal{S}_t\right)}{\partial M_{t-1}} = \mu_t \quad \text{(A.20b)}$$

Combining the FOCs and the envelope conditions gives

$$\frac{\upsilon}{a_t} C_t^{1-e}\left[C_t^e + \chi_t^{1-e}\left(\frac{M_t}{P_t}\right)^e\right] = w_t(1-L_t) \quad \text{(A.21a)}$$

$$\chi_t^{1-e}\left(\frac{M_t}{P_t}\right)^{e-1} = \frac{R_{t+1}-1}{R_{t+1}} C_t^{e-1} \quad \text{(A.21b)}$$

$$\beta \mathrm{E}_t\left[\frac{a_{t+1}}{a_t}\left(\frac{C_{t+1}}{C_t}\right)^{e-1} \frac{C_t^e + \chi_t^{1-e}\left(\frac{M_t}{P_t}\right)^e}{C_{t+1}^e + \chi_{t+1}^{1-e}\left(\frac{M_{t+1}}{P_{t+1}}\right)^e} R_{t+1}\right] = 1 \quad \text{(A.21c)}$$

A.6 The Optimal Contracting Problem

$$\max \mathrm{E}\left\{\int_{\bar{\nu}_t}^{\infty} \nu \mathrm{d}F(\nu) - [1-F(\bar{\nu}_t)]\,\bar{\nu}_t\right\} R_{k,t+1}P_{kt}K_{t+1}$$

subject to:

$$\left\{[1-F(\bar{\nu}_t)]\bar{\nu}_t + (1-\mathcal{A})\int_0^{\bar{\nu}} \nu \mathrm{d}F(\nu)\right\} R_{k,t+1}P_{k,t}K_{t+1}$$
$$= R_{t+1}\left(P_{kt}K_{t+1} - N_{t+1}\right) \tag{A.22}$$

The FOCs

Letting μ_t denote the Lagrange multiplier, one has

$$\bar{\nu}_t: \quad \mu_t\left\{1-F(\bar{\nu}_t) - \mathcal{A}\bar{\nu}_t f(\bar{\nu}_t)\right\} = 1 - F(\bar{\nu}_t) \tag{A.23a}$$

$$K_{t+1}: \quad \mathrm{E}\left\{\int_{\bar{\nu}_t}^{\infty} \nu \mathrm{d}F(\nu) - [1-F(\bar{\nu}_t)]\,\bar{\nu}_t\right\} R_{k,t+1} = \mu_t R_{t+1}$$
$$-\mu_t\left\{[1-F(\bar{\nu}_t)]\bar{\nu}_t + (1-\mathcal{A})\int_0^{\bar{\nu}_t} \nu \mathrm{d}F(\bar{\nu})\right\} R_{k,t+1} \tag{A.23b}$$

$$\mu_t: \quad \left\{[1-F(\bar{\nu}_t)]\bar{\nu}_t + (1-\mathcal{A})\int_0^{\bar{\nu}} \nu \mathrm{d}F(\nu)\right\} R_{k,t+1}P_{kt}K_{t+1} = R_{t+1}\times$$
$$\left(P_{kt}K_{t+1} - N_{t+1}\right). \tag{A.23c}$$

From j(A.23b), one has

$$\mu_t\frac{R_{t+1}}{R_{k,t+1}} = \mathrm{E}\left\{\int_{\bar{\nu}_t}^{\infty} \nu \mathrm{d}F(\nu) - [1-F(\bar{\nu}_t)]\,\bar{\nu}_t\right\}$$
$$+\,\mu_t\left\{[1-F(\bar{\nu}_t)]\bar{\nu}_t + (1-\mathcal{A})\int_0^{\bar{\nu}_t} \nu \mathrm{d}F(\bar{\nu})\right\},$$

and from (A.23c), one has

$$\frac{R_{t+1}}{R_{k,t+1}} = \left\{[1-F(\bar{\nu}_t)]\bar{\nu}_t + (1-\mathcal{A})\int_0^{\bar{\nu}} \nu \mathrm{d}F(\nu)\right\}\frac{P_{kt}K_{t+1}}{P_{k,t}K_{t+1} - N_{t+1}}.$$

Then plugging the latter relation into the former gives

$$\frac{P_{kt}K_{t+1}}{N_{t+1}} = 1 + \mu_t\frac{[1-F(\bar{\nu}_t)]\bar{\nu}_t + (1-\mathcal{A})\int_0^{\bar{\nu}} \nu \mathrm{d}F(\nu)}{\mathrm{E}\left\{\int_{\bar{\nu}_t}^{\infty} \nu \mathrm{d}F(\nu) - [1-F(\bar{\nu}_t)]\,\bar{\nu}_t\right\}}$$

$$= \Psi(\bar{\nu}_t), \tag{A.24}$$

with $\Psi'(\bar{\nu}_t) > 0$. The function Ψ is therefore invertible, *i.e.* one could write $\bar{v}_t = \Psi^{-1}(P_{kt}K_{t+1}/N_{t+1})$.

From (A.23b), one can express $R_{k,t+1}/R_{t+1}$, the premium on external funds, as a function of the cutoff value $\bar{\nu}_t$, *i.e.* $R_{k,t+1}/R_{t+1} = \rho(\bar{\nu}_t)$. Given $\rho'(\bar{\nu}_t) > 0$ for $\bar{\nu}_t > 0$ (see Bernanke, Gertler, and Gilchrist, 1999, p 1383), one can write $\bar{\nu}_t = \rho^{-1}(R_{k,t+1}/R_{t+1})$. Plugging this into (A.24) gives

$$\begin{aligned}\frac{P_{kt}K_{t+1}}{N_{t+1}} &= \Psi\left(\rho^{-1}\left(\frac{R_{k,t+1}}{R_{t+1}}\right)\right)\\ &= \psi\left(\frac{R_{k,t+1}}{R_{t+1}}\right),\end{aligned}$$

or

$$\frac{N_{t+1}}{P_{kt}K_{t+1}} = \left[\psi\left(\frac{R_{k,t+1}}{R_{t+1}}\right)\right]^{-1},$$

which one could inverse to get

$$\frac{R_{k,t+1}}{R_{t+1}} = s\left(\frac{N_{t+1}}{P_{kt}K_{t+1}}\right). \tag{A.25}$$

B The New-Keynesian Inflation Adjustment Equations

Twp cases are considered in turn: first, trend inflation is non-zero and second, no trend inflation.

B.1 Adjustment Equations with Trend Inflation

Let X_t be a variable following a first-order linear difference equation with a time-varying coefficient

$$X_t = \mathcal{A}\mathrm{E}_t\mathcal{B}_{t+1}X_{t+1} + F_t, \tag{B.1}$$

where E_t is the expectation operator, F_t is the forcing variable, and $\mathcal{A}$ and $\mathcal{B}_{t+1}$ are the coefficients. Iterating forward (B.1) gives the following general solution

$$X_t = \sum_{j=0}^{\infty}\mathcal{A}^j\mathrm{E}_t\left(\prod_{i=0}^{j}\mathcal{B}_{t+i}\right)F_{t+j} + \lim_{j\to\infty}\mathcal{A}^j\mathrm{E}_t\left(\prod_{i=0}^{j}\mathcal{B}_{t+i}\right)F_{t+j}.$$

For $0 < \mathcal{A} < 1$, the last element in the rhs of the above solution vanishes and one gets

$$X_t = \sum_{j=0}^{\infty}\mathcal{A}^j\mathrm{E}_t\left(\prod_{i=0}^{j}\mathcal{B}_{t+i}\right)F_{t+j}. \tag{B.2}$$

This result will be used to expressed recursively both the numerator and denominator of (7.10), the price adjustment equation. As explained earlier, at equilibrium, all adjusting firms choose the same price, $P_t^*(i) = P_t^*$. Besides, under Calvo pricing, $\varpi_j = \varpi^j$ for $0 \leq j < \infty$. Relation (7.10) thus becomes

$$p_t^* = \frac{\theta}{\theta-1}\frac{\mathrm{E}_t \sum_{j=0}^{\infty}(\varpi\beta)^j \frac{\mu_{t+j}}{\mu_t}\left(\frac{P_{t+j}}{P_t}\right)^{\theta} Y_{t+j}\varphi_{t+j}}{\mathrm{E}_t \sum_{j=0}^{\infty} \varpi_j \beta^j \frac{\mu_{t+j}}{\mu_t}\left(\frac{P_{t+j}}{P_t}\right)^{\theta-1} Y_{t+j}}$$

where $p_t^* = P_t^*/P_t$. By multiplying both sides of the above relation by p_t^{*-1}, one normalizes its lhs element to unity, which gives after rearrangement,

$$1 = \frac{\theta}{\theta-1}\frac{\mathrm{E}_t \sum_{j=0}^{\infty}(\varpi\beta)^j \frac{\mu_{t+j}}{\mu_t}\left(\frac{P_{t+j}}{P_t}\right)^{\theta} p_t^{*-1-\theta} Y_{t+j}\varphi_{t+j}}{\mathrm{E}_t \sum_{j=0}^{\infty} \varpi_j \beta^j \frac{\mu_{t+j}}{\mu_t}\left(\frac{P_{t+j}}{P_t}\right)^{\theta-1} p_t^{*-\theta} Y_{t+j}}.$$

A way to express the numerator and denominator of the above relation recursively as in (B.2) is to decompose the ratios into products of consecutive terms

$$1 = \frac{\theta}{\theta-1}\frac{\mathrm{E}_t \sum_{j=0}^{\infty}(\varpi\beta)^j \left[\prod_{i=0}^{j} \frac{\mu_{t+i+1}}{\mu_{t+i}} \pi_{t+i+1}^{\theta}\left(\frac{p_{t+i}^*}{p_{t+i+1}^*}\right)^{-1-\theta}\right] p_{t+j+1}^{*\ -1-\theta} Y_{t+j}\varphi_{t+j}}{\mathrm{E}_t \sum_{j=0}^{\infty}(\varpi\beta)^j \left[\prod_{i=0}^{j} \frac{\mu_{t+i+1}}{\mu_{t+i}} \pi_{t+i+1}^{\theta-1}\left(\frac{p_{t+i}^*}{p_{t+i+1}^*}\right)^{-\theta}\right] p_{t+j+1}^{*\ -\theta} Y_{t+j}}, \tag{B.3}$$

where $\pi_{t+j+1} = P_{t+j+1}/P_{t+j}$. Calling respectively X_{1t} and X_{2t} the numerator and the denominator of the second fraction on the rhs of (B.3), the New-Keynesian adjustment equations turns out to be

$$1 = \frac{\theta}{\theta-1}\frac{X_{1t}}{X_{2t}} \tag{B.4a}$$

$$X_{1t} = \varpi\beta \mathrm{E}_t \pi_{t+1}^{\theta}\frac{\mu_{t+1}}{\mu_t}\left(\frac{p_t^*}{p_{t+1}^*}\right)^{-1-\theta} X_{1,t+1} + p_t^{*\ -1-\theta} Y_t \varphi_t \tag{B.4b}$$

$$X_{2t} = \varpi\beta \mathrm{E}_t \pi_{t+1}^{\theta-1}\frac{\mu_{t+1}}{\mu_t}\left(\frac{p_t^*}{p_{t+1}^*}\right)^{-\theta} X_{2,t+1} + p_t^{*\ -\theta} Y_t. \tag{B.4c}$$

B.2 The New-Keynesian Phillips Curve

Let's consider relation (7.14), which is divided by $P_t^{1-\theta}$ to get

$$1 = \varpi \pi_t^{\theta-1} + (1-\varpi) p_t^{*1-\theta},$$

with $\pi_t = P_t/P_{t+1}$ and $p_t^* = P_t^*/P_t$. A first-order Taylor series approximation of this relation is

$$\hat{p}_t^* = \frac{\varpi}{1-\varpi}\hat{\pi}_t, \tag{B.5}$$

where the hat over the variables p_t^* and π_t denotes their percentage deviations from steady.

Let's now consider relation (7.10) at equilibrium, which can be rewritten as follows

$$p_t^* \mathrm{E}_t \sum_{j=0}^{\infty} (\varpi\beta)^j \mu_{t+j} \left(\frac{P_{t+j}}{P_t}\right)^{\theta-1} Y_{t+j} =$$
$$\frac{\theta}{\theta-1} \mathrm{E}_t \sum_{j=0}^{\infty} (\varpi\beta)^j \mu_{t+j} \left(\frac{P_{t+j}}{P_t}\right)^{\theta} Y_{t+j}\varphi_{t+j}.$$

A first-order Taylor series approximation of this relation gives

$$\frac{\mu Y}{1-\varpi\beta} + \frac{\mu Y}{1-\varpi\beta}\hat{p}_t^* + \mu Y \mathrm{E}_t \sum_{j=0}^{\infty} (\varpi\beta)^j \left[\hat{\mu}_{t+j} + \hat{Y}_{t+j} + (\theta-1)\left(\hat{p}_{t+j} - \hat{p}_t\right)\right] =$$
$$\frac{\mu Y}{1-\varpi\beta}\frac{\theta}{\theta-1}\varphi + \frac{\theta}{\theta-1}\varphi\mu Y \mathrm{E}_t \sum_{j=0}^{\infty} (\varpi\beta)^j \left[\hat{\mu}_{t+j} + \hat{Y}_{t+j} + \theta\left(\hat{p}_{t+j} - \hat{p}_t\right) + \hat{\varphi}_{t+j}\right]$$

When prices are flexible, *i.e.* for $\varpi = 0$, $p_t^* = \varphi_t\theta/(\theta-1)$, which implies that, at steady state, $\varphi\theta/(\theta-1) = 1$. Plugging this into the rhs of the above relation and then canceling the elements appearing on both sides gives

$$\frac{1}{1-\varpi\beta}\hat{p}_t^* = \mathrm{E}_t \sum_{j=0}^{\infty} (\varpi\beta)^j \left(\hat{p}_{t+j} - \hat{p}_t + \hat{\varphi}_{t+j}\right),$$

which one rearranges to get

$$\hat{p}_t^* + \hat{p}_t = (1-\varpi\beta)\mathrm{E}_t \sum_{j=0}^{\infty} (\varpi\beta)^j \left(\hat{p}_{t+j} + \hat{\varphi}_{t+j}\right).$$

As shown in Appendix B.1, this equation is the solution of a first-order difference equation of the form

$$\hat{p}_t^* + \hat{p}_t = (1-\varpi\beta)\left(\hat{p}_t + \hat{\varphi}_t\right) + \varpi\beta\mathrm{E}_t\left(\hat{p}_{t+1}^* + \hat{p}_{t+1}\right).$$

Rearranging this gives

$$\begin{aligned}\hat{p}_t^* &= (1-\varpi\beta)\hat{\varphi}_t + \varpi\beta\mathrm{E}_t\left(\hat{p}_{t+1}^* + \hat{p}_{t+1} - \hat{p}_t\right)\\ &= (1-\varpi\beta)\hat{\varphi}_t + \varpi\beta\mathrm{E}_t\left(\hat{p}_{t+1}^* + \hat{\pi}_{t+1}\right).\end{aligned}$$

Plugging (B.5) into the above relation gives,

$$\frac{\varpi}{1-\varpi}\hat{\pi}_t = (1-\varpi\beta)\hat{\varphi}_t + \frac{\varpi\beta}{1-\varpi}\mathrm{E}_t\hat{\pi}_{t+1}.$$

Finally, after rearrangement, one gets the new-Keynesian Phillips curve

$$\hat{\pi}_t = \frac{(1-\varpi)(1-\varpi\beta)}{\varpi}\hat{\varphi}_t + \beta\mathrm{E}_t\hat{\pi}_{t+1}. \tag{B.6}$$