# RD Growth and Business Cycles Measured with an Endogenous Growth DSGE Model

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# R&D Growth and Business Cycles Measured with an Endogenous Growth DSGE Model*

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**Abstract**

We consider how and the extent to which a pure technology shock driven by R&D activities impacts on business cycles as well as economic growth, using a medium-scale neo-classical dynamic stochastic general equilibrium (DSGE) model following Comin and Gertler (2006). We try to identify a pure technology shock by adopting "intellectual property product" first entered in 2008 SNA which can be regarded as R&D activity, and by assuming "time to build" by Kydland and Prescott (1982) in the process converting from innovations to products. Our empirical result based on a Bayesian analysis reports a common stochastic trend driven by the pure technology shock is likely to be procyclical, and it accounts for nearly half of variation of the real GDP whose remaining is explained by business cycle components. Meanwhile, a TFP shock, substituting for the R&D shocks, seems to move the common trend independently with business cycle.

*Keywords*: R&D shock, technology shock, dynamic stochastic general equilibrium model, common stochastic trend, endogenous growth model

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*The views expressed herein are of our own and do not represent those of the organizations the authors belongs to.

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# 1 Introduction

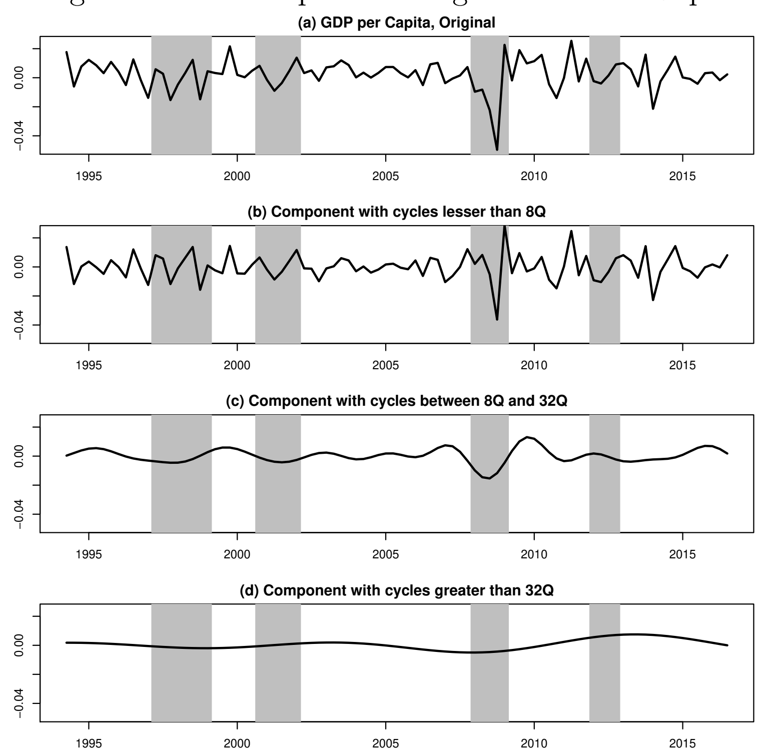
There is large volume of literature theoretically and empirically supports research and development (R&D) activities as an important channel for expanding economic growth through improving productivity of production process of private sectors. On the other hand, there are still controversial matters how a pure technology shock caused by R&D activities influence on business cycles; for example, Basu et al. (2006) theoretically and empirically advocated that the technology shock works as countercyclical using a two-sector model, while Alexopoulos (2011) empirically showed the shock is likely to be procyclical. Of course, conventional studies dealing with business cycles, especially concerning dynamic stochastic general equilibrium (DSGE) models, have regarded the technology shock as one of essential sources of fluctuations. In this way, effects of the technology shocks should be considered from the both sides of growth and business cycles.

At the same time, we can see another controversial matter concerning the relation between a technology shock and total factor productivity (TFP) in empirical studies of growth theories. According to neo-classical macroeconomic models such like real business cycle (RBC) models, the Solow residual is thought to be equivalent to the summation of the technology shock. It has, however, been usually a poor measure of technology progress in many empirical studies. It suggests that we need to split it from the Solow residual in order to classify the technology shock.

Commin and Getler (2006) coped with analyzing medium term business cycles specified as frequency interval between 8 quarters (Q) (or 2 years) and 200Q (or 50 years), by incorporating an endogenous growth model proposed by Romer (1990) into a RBC model, and compared the cycles between calibration derived from the model and data. Since earlier studies dealing with DSGE models have mainly focused on short span business cycles, an attempt by Commin and Getler (2006) is novel and influential for later related studies: for example, Ikeda and Kurozumi (2014), Guerron-Quintanna and Jinnai (2017) and Guerron-Quintanna et al (2017) considered a deep drop of growth rate in the Great recession triggered by the subprime loan crisis, using similar framework including an endogenous growth model. And Kung (2015) and Kung and Schmit (2015) studied how to fit data to term structure explained from a DSGE model in terms of this framework. Our paper is also along the lines with these studies. However, our study focuses on specifying technology shocks and investigating what relations exist between the shock and growth or business cycles.

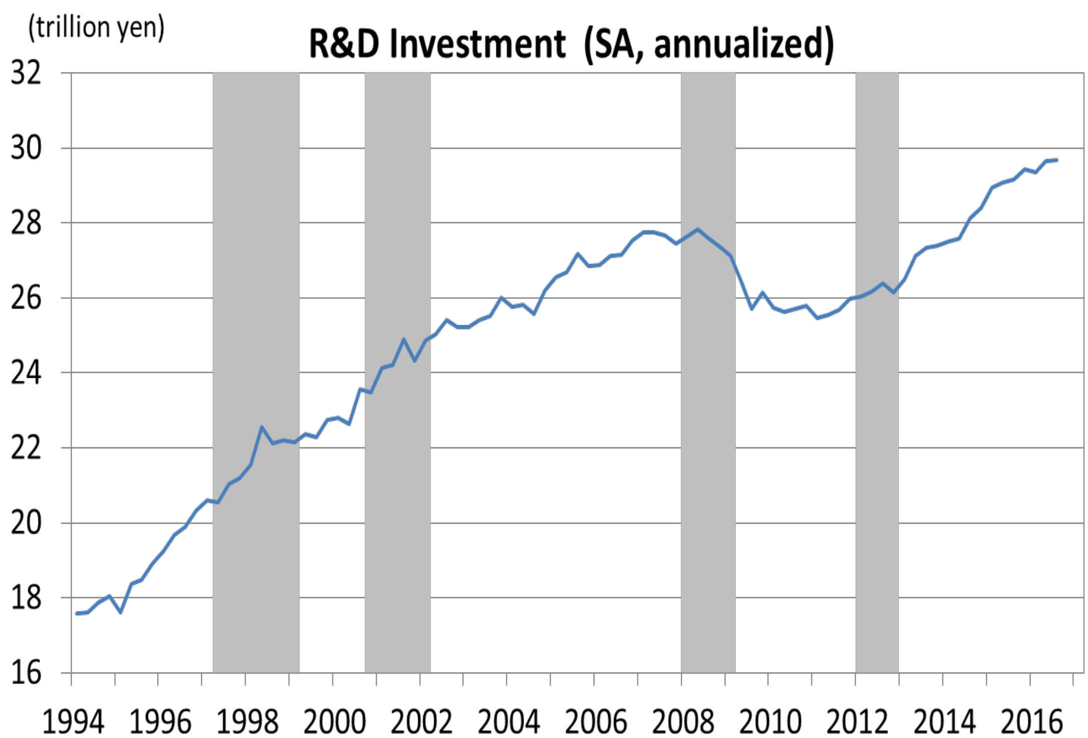
Figure 1 shows decompositions of the growth rate of Japan from 1994:Q2 to 2016:Q3, using the band-pass filter proposed by Christiano and Fitzgerald (2003) with recessions reported by the Cabinet Office (the gray shaded area). The black solid line of Panel (a) represents original data which is decomposed into three frequency regions. In Panel (b), high frequency interval (less than 8 Q) is drawn, while band-pass interval between 8Q and 32Q, and low-pass interval (greater than 32Q) are depicted in Panel (c) and (d), respectively. By definition, the solid line of Panel (c) is thought be represent business cycles, while that of Panel (d) is long ran growth. Although it is hard to extract growth component for such short period as much as nearly 90 quarters, the growth seems extremely smooth and flat even though the period includes the Lehman brother's collapse in September 2008. Instead, the high frequency region, usually regarded as noise component, in Panel (b) behaves the most volatile of all regions. Our interest is on which frequency region and the extent to which the pure technology driven by R&D activity impacts, from the viewpoint of an empirical model-based approach.

Figure 1: A decomposition of growth rate of Japan




The purpose of this paper is to identifying a technology shock by constructing a RBC-type DSGE model with endogenous growth following Commin and Gertler (2006), and to examine whether a technology shock influencing growth is procyclical or countercyclical. To this end, we conduct decomposition of cycles and growth extracted from data by using the technology shocks of a DSGE model, which especially contains two R&D related shocks regarded as the pure technology shocks, and incorporate the concept "time to build" proposed by Kydland and Prescott (1982) into the R&D sector. And we also include "intellectual property product" first entered in 2008 SNA which can be regarded as R&D activity, as an observed variable depicted as Figure 2. In addition, in order to check the roles of R&D data and R&D shocks, respectively, we construct two additional models by dropping R&D data from data set or by replacing the two R&D shocks with a TFP shock, and show how the R&D data and shocks can be helped to identify the common growth rate. Our empirical result based on a Bayesian analysis reports a common stochastic trend driven by the pure technology shock is likely to be procyclical, and it accounts for nearly half of variation of the real GDP whose remaining is explained by business cycle components. Meanwhile, a TFP shock, replaced from the R&D shocks, seems to move the common trend irrelevant to business cycle.

Figure 2: R&D investment



The rest of our paper is organized as follows. Section 2 describes our model. Estimation method and data are explained in Sections 3. Section 4 deals with the estimation results, while Section 5 describes the roles of R&D data and shocks. We conclude in Section 6. In the Appendix we show that the equilibrium conditions consist of the first order conditions (FOCs) and restrictions.

# 2 Model

Our model is basically an RBC-type dynamic model following Comin and Gertler (2006), and extended by adding the persistence of habit consumption and six structural shocks including two shocks involved to R&D activity, i.e., $z_t^{\lambda}$ and $z_t^{P}$, explained later. Our whole economy consists of three sectors, i.e., a R&D sector, producers and households. The R&D sector contributes to economic growth by inventing a blueprint of a new product which will be manufactured by the producers. Although Comin and Gertler (2006) set an innovation to contribute to increase quantity of final goods, an innovation increases varieties of intermediate goods in our framework .

## 2.1 R&D Sector

### Innovator

The innovator is assumed to be a representative agent who creates a new blueprint, $I_{d,t}$, by innovative activity which is adopted for increasing the new products of intermediate goods, $Y_{i,t}$. To draw the new blueprint, he needs to input final goods, $Y_t$, by obtaining loans from households. Then he sells the right to his blueprint to an adopter, who converts the blueprint into newly intermediate goods which contributes to increasing the quantities of final goods.

Let $Z_t$ be the total stock of blueprints drawn by the innovator. Then we obtain this dynamics as

$$Z_t = (1 - \delta_z) Z_{t-1} + \varPhi_t I_{dt}, \tag{1}$$

where $\delta_z$ denotes the obsolescence rate of the stock, and $\Phi_t$ is the R&D productive efficiency transforming from additional blueprint to the stock. And the R&D efficiency is defined as

$$\varPhi_t \equiv \chi_z \frac{Z_{t-1}}{\left(A_t^+\right)^\rho I_{dt}^{1-\rho}}, \tag{2}$$

with $0 < \rho \leq 1$ and where $\chi_z > 0$ is a scale parameter. And $A_t^+$ represents the level of composite technology of a production function, that is, $A_t^+ \equiv \Psi_t^{\frac{\alpha}{1-\alpha}} A_{t-1}$, where $\Psi_t^{\frac{\alpha}{1-\alpha}}$ is an investment-specific technology (IST) shock following Altig et al. (2011). This technology level is derived from the variety of intermediate goods, $V_t$ , that is $A_t^+ = \Psi_t^{\frac{\alpha}{1-\alpha}} V_t^{\frac{\lambda_I}{1-\alpha}}$ , follwoing Romer (1990). As Eq.(2), the R&D efficiency has the congestion effect in which larger $A_t^+$ than the steady state declines the value of $\Phi_t$.

Since the innovator faces perfect competition, he optimizes his profit and gets zero-profit satisfying the no-arbitrage condition such as

$$1 = \varPhi_t \left(1 - \delta_z\right) \mathbb{E}_t \Lambda_{t|t+1} J_{t+1}, \tag{3}$$

where $\boldsymbol{E}_{\mathbf{t}} \varLambda_{t|t+1}$ is stochastic discount factor (SDF) of households, and $J_t$ is value of the blueprint described in the following part. And Eq. (3) indicates equivalent exchange between his innovation and retail goods whose price is unity.

From Eq.(1) and Eq.(3), we obtain the dynamics of the innovator as

$$I_{dt} = \left(1 - \delta_z\right) \left\{Z_t - \left(1 - \delta_z\right) Z_{t-1}\right\} \mathbb{E}_t \Lambda_{t|t+1} J_{t+1}. \tag{4}$$

**Adopter**

The adopter is categorized as a representative agent who converts an available blueprint acquired from the innovator into a new product of intermediate goods. To buy the right to the innovation, he obtains loans from households, and he tries to manufacture a new product by inputing the final goods, $Y_t$. If he is successful, he sells it to an intermediate goods producer.

The value of a blueprint, which has not yet been adopted as productization, of the adopter is defined as

$$J_t = \max_{\{I_{at}\}} \left[-I_{at} + \left(1 - \delta_a\right) \left\{\lambda_t P_t^V + \left(1 - \lambda_t\right) \mathbb{E}_t \Lambda_{t|t+1} J_{t+1}\right\}\right], \tag{5}$$

where $I_{a,t}$ is the cost of investment for adoption as a new product, and $\delta_a$ is the obsolescence rate of the adopted blueprint. And $\lambda_t$ is the time-varying success probability of converting a blueprint into a new product in period $t$. $P_t^V$ denotes the value of the adopter successfully obtaining from a new product, and it indicates the present value of profit obtained by the adopter. The success probability is endogenously determined given as

$$\texttt{s.t. success rate}: \ \lambda_t \equiv \lambda_0 \, z_t^{\lambda} \left( \frac{V_{t-1} I_{at}}{Z_t^{+}} \right)^{\omega_a}, \tag{6}$$

where $\lambda_0 > 0$ and $\omega_a \in (0, 1)$. And $V_t$ denotes the stock of adopted blueprints, or the variety of newly realized intermediate goods. $z_t^{\lambda}$ is an auto-regressive (AR) process of a structural shock regarded as an *R&D success probability shock*. As Eq.(6), the success probability also has the congestion effect of adoption letting the speed of accumulation of $V_t$ slow down.

The first-order condition for investment, $I_{a,t}$, by maximizing Eq. (5) subject to Eq.(6) and Eq.(9) is written as

$$I_{at} = (1 - \delta_a) \, \omega_a \lambda_t \left( P_t^V - \mathbb{E}_t \Lambda_{t|t+1} J_{t+1} \right). \tag{7}$$

And using Eq.(5) and Eq.(7), we obtain the value of unadopted blueprints as

$$J_t = (1 - \delta_a) \left[ (1 - \omega_a) \, \lambda_t P_t^V + \{ 1 - (1 - \omega_a) \, \lambda_t \} \, \mathbb{E}_t \Lambda_{t|t+1} J_{t+1} \right], \tag{8}$$

where $J_t$ is also used as optimization of the innovator as Eq.(3). The increment, $\Delta_{a,t}$, of the stock $V_t$ depend on the size of success rate, $\lambda_t$, and the obsolescence rate, and it is obtained as

$$\mathit{\Delta}_{\mathrm{at}} = (1 - \delta_a) \; \lambda_t \; (Z_{t-1} - V_{t-1}), \tag{9}$$

where the term in bracket is the stock of blueprints that the adopter owns but does not yet adopt.

## 2.2 Households

Households are representative agents who face problem maximizing their intertemporal utility by attaining utility from consumption and leisure. The household's preference is given as

$$\max_{\left\{C_{jt},u^k_{jt},K_{jt-1},I_{jt-1},l_{jt}\ \mid\ j\in[0,1]\right\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t z^b_t \left\{ \ln\left(C_{jt} - hC_{jt-1}\right) - \gamma_l z^l_t \frac{l_{jt}^{1+\omega_l}}{1+\omega_l} \right\}, \tag{10}$$

where $C_t$ and $l_t$ denote aggregate consumption and labor supply, respectively. And we allow for habit persistence in preference by adding $h\,C_{t-1}$. $z^b_t$ and $z^l_t$ are preference shock and labor supply shock, respectively.

The budget constraint of the households is given as

$$\texttt{s.t. budget constraint} : C_{jt} + \frac{I_{jt}}{\Psi_t} = W_t l_{jt} + r^k_t u^k_t \frac{K_{jt-1}}{\Psi_t} + T_{jt}, \tag{11}$$

where $T_t$ is the lump-sum public transfer, and $W_t$ is real wage. And the dynamic of capital accumulation is

$$\text{s.t. fixed capital accumulation} : K_t = \left\{1 - \delta\left(u^k_t\right)\right\} K_{t-1} + \left\{1 - S\left(\frac{z^i_t}{\mu^+\mu^\psi}\frac{I_t}{I_{t-1}}\right)\right\} I_t. \tag{12}$$

Accordingly, by solving above problem, the FOC in terms of consumption is given as

$$\Lambda^c_t = \frac{z^b_t}{C_t - hC_{t-1}} - \beta h \mathbb{E}_t \frac{z^b_{t+1}}{C_{t+1} - hC_t}, \tag{13}$$

where $\Lambda^C_t$ is marginal utility with respect to consumption. Similarly, we obtain the FOCs of the problem as follows: The FOC w.r.t. utilization rate of capital, $\left(u^k_t\right)$ :

$$r^k_t = q^k_t \delta'\left(u^k_t\right). \tag{14}$$

The FOC w.r.t. capital, $(K_t)$ :

$$q^k_t = \mathbb{E}_t \frac{\Lambda_{t|t+1}}{\mu^\psi_{t+1}} \left[r^k_{t+1}u^k_{t+1} + q^k_{t+1}\left\{1 - \delta\left(u^k_{t+1}\right)\right\}\right]. \tag{15}$$

The FOC w.r.t. investment, $(I_t)$ :

$$\begin{aligned} 1 = q^k_t &\left\{1 - S\left(\tfrac{z^i_t}{\mu^+\mu^\psi}\tfrac{I_t}{I_{t-1}}\right) - S'\left(\tfrac{z^i_t}{\mu^+\mu^\psi}\tfrac{I_t}{I_{t-1}}\right)\tfrac{1}{\mu^+\mu^\psi}\tfrac{I_t}{I_{t-1}}\right\} \\ &+\mu^+\mu^\psi \mathbb{E}_t \tfrac{\Lambda_{t|t+1}}{\mu^\psi_{t+1}} q^k_{t+1} S\left(\tfrac{z^i_{t+1}}{\mu^+\mu^\psi}\tfrac{I_{t+1}}{I_t}\right)\left(\tfrac{1}{\mu^+\mu^\psi}\tfrac{I_{t+1}}{I_t}\right)^2. \end{aligned} \tag{16}$$

The FOC w.r.t. labor supply, $(l_t)$ :

$$W_t = \gamma_l \, z_t^l \, \frac{l_t^{\omega_l}}{\Lambda_t^c}. \tag{17}$$

## 2.3 Intermediate Goods Firms

There is a continuum of intermediate goods firms, indexed by $i \in [0, V_{t-1}]$, each of which are produced using labor, the physical capital stock $K_t$ and the variety of all intermediate goods $V_t$. We assume that they own $K_t$ and $V_t$, so the rental cost of physical capital does not exist in our model. Instead, the intermediate goods firms are assumed to pay their profits as dividends to the households which are regarded as owners of the intermediate goods firms.

The intermediate goods firms maximize the net present value of the profits by controlling the price of intermediate goods $p_{i,t}$, capital stock $K_t$, capital utilization rate $u_t^k$, labor demand $l_t$, and new product stock $V_t$, so their optimization problem is obtained as

$$\max_{\left\{p_{it}, l_{it}, u_{it}^k K_{it-1}, V_t \,|\, i\in[0,V_{t-1}]\right\}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^{t+s} \frac{\Lambda_{t+s}^c}{\Lambda_t^c} \left( \begin{array}{c} \int_0^{V_{t+s-1}} p_{it+s} Y_{it+s} di \\ -W_{t+s} \int_0^{V_{t+s-1}} l_{it+s} di - r_{t+s}^k \int_0^{V_{t+s-1}} \frac{u_{it+s}^k K_{it+s-1}}{\Psi_t} di \\ -P_{t+s}^V \Delta_{at+s} \end{array} \right). \tag{18}$$

We assume that there is a certain period to convert from blueprint invented by the innovator to a new intermediate product. To this end, let us apply the concept "time to build" of converting from investment to capital proposed by Kydland and Prescott (1982). and we incorporate delay of productization expressed as moving averages into the dynamic of variety of intermediate product given as

$$\texttt{s.t. variety accumulation}: \; V_t = (1-\delta_a) V_{t-1} + (1-\phi_{\mu 1}) \Delta_{at} + \phi_{\mu 1} \left\{ (1-\phi_{\mu 2}) \Delta_{at-1} + \phi_{\mu 2} \Delta_{at-2} \right\}, \tag{19}$$

where $1-\phi_{\mu 1}$, $\phi_{\mu 1}(1-\phi_{\mu 2})$ and $\phi_{\mu 1}\phi_{\mu 2}$ are the fractions of adopted blueprints to conversion of new goods over three periods, since it is assumed to take at least three quarters for implementing the project. Total fractions are set to unity. And also we have additional two constraints such as a demand function of the final goods and the

CES production of the intermediate goods, given as

$$\texttt{s.t. demand function of final goods firm}: \int_0^{V_{t-1}} Y_{it} di = \int_0^{V_{t-1}} p_{it}^{-\frac{1+\lambda_I}{\lambda_I}} Y_t di = V_{t-1}^{-\lambda_I} Y_t, \tag{20}$$

$$\texttt{s.t. production function}: \; Y_{it} = \left(u_{it}^k K_{it-1}\right)^{\alpha} l_{it}^{1-\alpha}. \tag{21}$$

Accordingly, the FOCs of the optimization problem consist of six equation as below. The FOC with respect to intermediate goods price is given as

$$p_{it} = (1 + \lambda_i)\, MC_t, \tag{22}$$

where the marginal cost of intermediate goods is

$$MC_t = \frac{V_{t-1}^{\lambda_I}}{1 + \lambda_I}. \tag{23}$$

The FOC with respect to labor demand $l_t$ is given as

$$W_t = (1 - \alpha)\, MC_t \left(\frac{u_t^k K_{t-1}}{l_t}\right)^{\alpha}. \tag{24}$$

The FOC with respect to capital utilization rate $u_t^k$ is obtained as

$$\frac{r_t^k}{\Psi_t} = \alpha MC_t \left(\frac{u_t^k K_{t-1}}{l_t}\right)^{\alpha - 1}. \tag{25}$$

The FOC w.r.t. $(\mathit{\Delta}_{at})$ is obtained as

$$P_t^V = (1 - \phi_{\mu 1})\, \mathit{\Gamma}_t + \phi_{\mu 1} (1 - \phi_{\mu 2})\, \mathbb{E}_t \mathit{\Lambda}_{t|t+1} \mathit{\Gamma}_{t+1} + \phi_{\mu 1} \phi_{\mu 2} \mathbb{E}_t \mathit{\Lambda}_{t|t+1} \mathit{\Lambda}_{t+1|t+2} \mathit{\Gamma}_{t+2}. \tag{26}$$

The FOC with respect to the stock of adopted innovation $V_t$ is written as

$$\mathit{\Gamma}_t = \mathbb{E}_t \mathit{\Lambda}_{t|t+1} \left\{ (1 - \delta_a)\, \mathit{\Gamma}_{t+1} + \lambda_I MC_{t+1} \frac{\left(u_{t+1}^k K_t\right)^{\alpha} l_{t+1}^{1-\alpha}}{V_t} \right\}, \tag{27}$$

where $\Gamma_t$ is the value of adopted innovation which is also used in the adopter's value function (5).

## 2.4 Final Goods Firms

A final goods firm is a representative agent that produces final goods $Y_t$ by bundling a set of intermediate goods $Y_{i,t}$ indexed by $i \in [0, V_{t-1}]$. Under the constraint of the production function (20), the final goods firm maximizes its profit, given as

$$\max_{\{Y_{it} \mid i \in [0, V_{t-1}]\}} \left( Y_t - \int_0^{V_{t-1}} p_{it} Y_{it} di \right), \tag{28}$$

where $p_{i,t}$ denote the price of the intermediate goods. The stock of adopted innovation $V_{t-1}$ is also assumed to be the variety of the intermediate goods accumulated from new products.

$$\texttt{s.t. producition function}: \; Y_t = \left( \int_0^{V_{t-1}} Y_{it}^{\frac{1}{1+\lambda_I}} di \right)^{1+\lambda_I}. \tag{29}$$

The FOC of final goods firm indicates the demand function of the intermediate goods given as

$$Y_{it} = p_{it}^{-\frac{1+\lambda_I}{\lambda_I}} Y_t. \tag{30}$$

Since the final goods are set as the numeral goods, i.e., $p_t = 1$, we obtain the price equation between the intermediate goods and the final goods as

$$p_{it} = V_{t-1}^{\lambda_I} \approx 1 = p_t, \tag{31}$$

where net markup rate of the intermediate good, $\lambda_t$, is set to a tiny value such as 0.1 shown in Table 2.

## 2.5 Market Clearing, Detrend and Structural Shocks

### Market Clearing

The aggregate output in the whole economy is composed of the sum of the demand for the final goods. The market clearing condition of the final goods is closed based on the SNA framework given as

$$Y_t = C_t + \frac{I_t}{\Psi_t} + I_t^{R\&D} z_t^P + \overline{g/y}\, y\, A_t^+ z_t^g, \tag{32}$$

where $I_t/\Psi_t$ is the dividend payment cost of the intermediate goods firms. $I_t^{R\&D}$ and $z_t^P$ denote the R&D investment and an AR (1) shock of R&D investment relative price, respectively. And $\overline{g/y}$ and $y$ are the average of government spending and net export share on GDP, and the steady state of real GDP par capita, respectively. $z_t^g$ denotes an exogenous expenditure shock such as the government sector. The R&D investment consist of two parts obtained as

$$I_t^{R\&D} = I_{at}\left(Z_{t-1} - V_{t-1}\right) + I_{dt} \tag{33}$$

where the first term is the R&D investment of the adopters, and $I_{dt}$ is the R&D investment of the innovators. However, each term in the equation does not strictly match the notion in the SNA framework.

### Detrend

The equilibrium conditions of the model are rewritten in terms of detrended variables around a steady state. To do so, firstly we set the investment specific progress rate, $\mu_t^\psi$, such as

$$\mu_t^\psi \; = \; \bar{\mu} \; \mu_t^V, \tag{34}$$

where $\mu_t^\psi \equiv \frac{\Psi_t}{\Psi_{t-1}}$ and $\mu_t^V \equiv \frac{V_{t+1}}{V_t}$. Then, using this notation, we set common stochastic growth rate as $\mu_t^+ \equiv \frac{A_t^+}{A_{t-1}^+} = \mu_{t-1}^A \left(\mu_t^\psi\right)^{\frac{\alpha}{1-\alpha}}$, since $A_t^+ \equiv \Psi_t^{\frac{\alpha}{1-\alpha}} A_{t-1}$, and $\mu_t^A \equiv \frac{A_t}{A_{t-1}}$.

Finally, the detrended variables expressed with small letters are given as $y_t \equiv \frac{Y_t}{A_t^+}$, $c_t \equiv \frac{C_t}{A_t^+}$, $i_t \equiv \frac{I_t}{A_t^+\Psi_t}$, $i_{at} \equiv \frac{I_{at}V_{t-1}}{A_t^+}$, $i_{dt} \equiv \frac{I_{dt}}{A_t^+}$, $v_t \equiv \frac{V_{t-1}}{Z_{t-1}}$, $\lambda_t^c \equiv \Lambda_t^c A_t^+$, $q_t^k \equiv \frac{\Lambda_t^k \Psi_t}{\Lambda_t^c}$, $k_t \equiv \frac{K_t}{A_t^+\Psi_t}$, $\widetilde{\Delta}_{at} \equiv \frac{\Delta_{at}}{V_{t-1}}$, $w_t \equiv \frac{W_t}{A_t^+}$, $j_t \equiv \frac{J_t A_{t-1}}{A_t^+}$, $p_t^V \equiv \frac{P_t^V Z_{t-1}}{A_t^+}$, $\phi_t \equiv \frac{\Phi_t A_t^+}{Z_{t-1}}$, $i_t^{R\&D} \equiv \frac{I_t^{R\&D}}{Z_t^+}$, $mc \equiv \frac{MC_t}{A_{t-1}^{\lambda_I}} = \frac{1}{1+\lambda_I}(= const.)$

### Equilibrium conditions and structural shocks

To acquire an equilibrium of the model and to estimate it, we use equations such as Eq.(1) through Eq.(4), Eq.(6) through Eq.(9), Eq.(11) through Eq.(17), and Eq.(19) through Eq.(34). There are six structural shocks including two R&D shocks such as $z_t^\lambda$ and $z_t^P$, of which all shocks follow the AR (1) process. Appendix A1 presents the conditions and the shocks.

# 3 Estimation Strategy

This section describes about estimation strategy including data and link between endogenous and observable variables.

## 3.1 Estimation Methods

In this paper the model is estimated following a Bayesian approach via Markov chain Monte Carlo (MCMC) simulation. We use a stylized solution method for estimation; specifically, we log-linearize the model shown above and convert it into a linear Gaussian state–space model after detrending the model variables around their steady states. Then we evaluate the posterior densities' combined likelihood derived from the Kalman filter with prior densities. To estimate the model, we adopt a climbing method to find the maximum a posteriori estimator (MAPE) as the posterior mode, and to find a hessian at the mode for using as a proper kernel in the randam walk Metropolis–Hastings (RWMH) algorithm. We then generate 1,500,000 draws as the posterior distribution of parameters with the RWMH algorithm and discard the first quarter of them (say, 375,000 draws) as burn-in. We estimate the parameters of sample period: 1994:Q2–2016:Q4.

## 3.2 Observable Variables and Data

As can be seen from Table 1, we adopt five observed variables – (1) output growth, (2) consumption, (3) investment, (4) labor supply and (5) R&D investment, from 1994:Q2 through 2016:Q3 in Japan. We collect real GDP figures, real private consumption: $C_t^{data}$, fixed capital formation: $I_t^{data}$, and intellectual property product: $I_t^{R\&D\ data}$, (this data is drawn as Figure 2 in Section 1) from the Cabinet Office's National Accounts, based on 2008 SNA, as the output growth, consumption, investment and R&D investment. Then we change them to par capta divided by working age population after age 15: $N_t^{data}$, reported by the Bureau of Statistics. We use the statistical release of the benchmark year 2011 that covers the period 1994:Q1–2016:Q3. Furthermore, the employment index for regular employees: $l_t^{obs}$, in the Monthly Labor Survey are used as the variables of labor supply $l_t$.

Table 1: Observable Variables

| Variable | Definition | Unit | Source |
|---|---|---|---|
| $Y_t^{data}$ | real GDP[1] | a billion yen | QE[2] |
| $C_t^{data}$ | real private consumption | a billion yen | QE |
| $I_t^{data}$ | real private investment | a billion yen | QE |
| $l_t^{data}$ | employment index for regular employees | 2010 average = 100 | MHLW[3] |
| $I_t^{R\&D\ data}$ | intellectual property products | 2010 average = 100 | QE |
| $N_t^{data}$ | working age population after age 15 | a thousand | Statistics Bureau, MIC[4] |

Notes:
1: Including net export and government spending.
2: Quarterly Estimates of GDP based on benchmark year revised into 2011.
3: Monthly Labor Survey, seasonally adjusted worked hour indices(2010 average = 100, S.A.), Ministry of Health, Labor and Welfare.
4: Labor Force Survey, Ministry of Internal Affairs and Communication.

## 3.3 Link between Observable and Endogenous Variables

We introduce measurement errors into five measurement equations to express them as noises included in the data. The noises in the equations are assumed to follow the normal distribution with independent and identical distribution (iid). The link between the observable and the endogenous variables are given as follows.

1. GDP growth rate

$$\Delta \ln(Y_t^{\text{data}}/N_t^{\text{data}}) = \ln(\mu_t^+) + \ln(y_t) - \ln(y_{t-1}) + \varepsilon_t^{\text{obs\_err\_y}}, \tag{35}$$

2. consumption growth rate

$$\Delta \ln(C_t^{\text{data}}/N_t^{\text{data}}) = \ln(\mu_t^+) + \ln(c_t) - \ln(c_{t-1}) + \varepsilon_t^{\text{obs\_err\_c}}, \tag{36}$$

3. fixed investment growth rate

$$\Delta \ln(I_t^{\text{data}}/N_t^{\text{data}}) = \ln(\mu_t^+) + \ln(\mu_t^\psi) + \ln(i_t) - \ln(i_{t-1}) + \varepsilon_t^{\text{obs\_err\_i}} \tag{37}$$

4. R&D investment growth rate

$$\Delta \ln(I_t^{R\&D\text{data}}/N_t^{\text{data}}) = \ln(\mu_t^+) + \ln(i_t^{R\&D}) - \ln(i_{t-1}^{R\&D}) + \varepsilon_t^{\text{obs\_err\_R\&D}}, \tag{38}$$

5. level of labor supply

$$\ln(l_t^{\mathrm{data}}) = \ln(l_t) + \varepsilon_t^{\mathrm{obs\_err\_l}}, \tag{39}$$

where the first four observable variables are taken the first differences. $\mu_t^+$ is logarithm of common growth rate., i.e., $\ln\left(A_t^+/A_{t-1}^+\right)$ . And each $\varepsilon_t^{obs}$ is a measurement error corresponding to a observable variable. Notice that the first four equations show that those variables have a long stable relation with stochastic common trend: $A_t^+$, since the size $\mu_t^+$are used in common by their growth rates. Besides, the growth of investment addtionally includes the investment specific technology growth, $\mu_t^\psi$ as Eq.(37).

## 3.4 Calibrated Parameters and Prior Distributions

In this model, we fix nine parameters in Table 2 for avoiding identification problems, instead of estimating them. The rest of model parameters are estimated and shown in the following section. The steady-state of ratio of government spending to output is set to the average of the data in the sample period, i.e., $\overline{g/y} = 0.24$. The depreciaton rate of physical capital stock $\delta$ and capital share $\alpha$ are calibrated to 2.5% and 40% based on averages of the statistics. Discount factor $\beta$ is cloeser to 1 (i.e., 0.9975) than several preceding studies, because of intending to focus on the recent Japanese economy, in which the monetary policy rate has been not templorarily but permanently near zero after 1999:Q1. The priors of estimating parameters are described in the third through the fifth columns of Table 3.

# 4 Empirical Results

## 4.1 Estimated Parameters

The posterior estimation of the parameters is described in Table 3. The posterior means, 90% credible intervals and 4% p-values of the convergence diagnostic are in the sixth through ninth columns of the table. To test the convergence of the MCMC simulation, we conduct Geweke's (1992) convergence diagnostic. We make several remarks on this results as follows.

First, the posterior mean of the habit formation of consumption, $h$, is around 0.922 indicating strong persistence of habit formation. Second, the parameter, $\omega_a$, representing the elasticity of success rate, $\lambda_t$, for investment of R&D, $I_{a,t}$, is estimated nearly

Table 2: Calibrated Parameters

| | Parameter | Value | Reference |
|---|---|---|---|
| $\beta$ | subjective discount factor | 0.9975 | - |
| $\delta$ | capital depreciation rate | 0.0250 | - |
| $\delta_z$ | obsoletion rate of ideas held by inventor[1] | 0.0250 | - |
| $\lambda_I$ | net markup rate of intemediate goods | 0.1000 | - |
| $\alpha$ | capital share | 0.4000 | - |
| $\overline{g/y}$ | government spending and net export share on GDP | 0.2587 | sample mean |
| $\mu^+$ | steady state composite technology progress rate[2] | 1.0023 | sample mean |
| $\mu^\psi$ | steady state investment specific technology progress rate | 1.0010 | sample mean[3] |
| $\overline{i^{R\&D}/y}$ | steady state R&D investment share on GDP | 0.0517 | sample mean |

Notes:
1: obsoletion rate of ideas held by adapter $\delta_a$is numerically calculated and decided according to estimated or calibrated parameter value in order to pin down R&D investment share on GDP.
2: steady state value of Harrod's neutral, or labor augmenting, technology progress rate $\mu^A$ is analytically solved and decided to keep $\mu^+$ get target value.
3: inverse of relative price inflation rate, or growth rate of private investment deflator compared with GDP deflator.

0.141 as the posterior mean, although it is set to as much as 0.5 as the priors. And it means thta the success rate is very inelastic for the investment. Third, the parameter $\rho$ of the R&D efficiency, $\mathit{\Phi}_t$ is very low such as 0.104 which suggests weakness of the congestion effect, since $\rho$ represents the degree of inelasticity of $\mathit{\Phi}_t$ which also indicates the congestion effect for the R&D investment.

## 4.2 Impulse Response Functions

Before considering relationship between the pure technology shocks and business cycles, we start from checking the impulse response of growth and cycles to the two R&D shocks which individually effects on them. Figure 3 shows the impulse response of four variables, the common stochastic growth rate, $\mu_t^+$, deviated from the steady state of rate, $\mu_{ss}^+$, and the deviations of output, consumption, and investment from their steady states (i.e., $\hat{y}_t$, $\hat{c}_t$, and $\hat{i}_t$) to both of the R&D investment's relative price shock, $z_t^p$, (the solid red lines) and the R&D success rate shock, $z_t^\lambda$, (the dashed blue lines). As the upper left panel, the success probability shocks increases common growth rate until nearly twelve quarters (or three years), but this shock does not seem to prolong the growth rate any more. Meanwhile, the R&D investment relative price shock is likely not to impact on the growth rate at all.

The upper right panel shows reactions of output which is regarded as fluctuation

Table 3: Prior and Posterior of Estimated Parameters

| | definitions | Prior | | | Posterior | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | mean | St. dev. | distributions | means | [ | 90% | HPD[2] | ] | C.D.(4% p-value)[3] |
| $h$ | habit formation | 0.4 | 0.075 | beta | 0.9223 | [ | 0.896 | 0.95 | ] | 0.407 |
| $\omega_a$ | success rate elasticity | 0.5 | 0.15 | gamma | 0.1406 | [ | 0.0765 | 0.2041 | ] | 0.551 |
| $\rho$ | R&D efficiency elasticity | 0.6 | 0.15 | beta | 0.1036 | [ | 0.0404 | 0.1637 | ] | 0.97 |
| $\zeta_k$ | capital depreciation elasticity | 1 | 0.1 | gamma | 0.8756 | [ | 0.7254 | 1.0174 | ] | 0.006 |
| $\overline{1/\zeta}$ | coefficients of investment adjustment cost | 7 | 1.5 | gamma | 2.5575 | [ | 1.1803 | 3.8744 | ] | 0.723 |
| $\omega_l$ | inverse of Frisch elasticity | 2 | 0.75 | gamma | 6.4061 | [ | 4.9161 | 7.9026 | ] | 0.388 |
| $\lambda_{ss}$ | steady state value of success rate | 0.15 | 0.05 | beta | 0.2844 | [ | 0.1836 | 0.3823 | ] | 0.162 |
| $\bar{l}$ | labor supply scale parameter | 0.964 | 0.005 | gamma | 0.9649 | [ | 0.9568 | 0.9728 | ] | 0.435 |
| $\phi_{\mu 1}$ | time to build coefficients 1 | 0.5 | 0.25 | beta | 0.5367 | [ | 0.1489 | 0.9413 | ] | 0.116 |
| $\phi_{\mu 2}$ | time to build coefficients 2 | 0.333 | 0.2 | beta | 0.2161 | [ | 0.0017 | 0.448 | ] | 0.669 |
| $\rho_b$ | persistence of preference shock | 0.5 | 0.2 | beta | 0.138 | [ | 0.0217 | 0.2437 | ] | 0.687 |
| $\rho_l$ | persistnece of labor disutility shock | 0.5 | 0.2 | beta | 0.6212 | [ | 0.5329 | 0.7146 | ] | 0.276 |
| $\rho_g$ | persisitence of exogenous spending shock | 0.5 | 0.2 | beta | 0.9858 | [ | 0.9741 | 0.9977 | ] | 0.199 |
| $\rho_i$ | persisitence of investment adjustment cost | 0.5 | 0.2 | beta | 0.8894 | [ | 0.8464 | 0.9314 | ] | 0.564 |
| $\rho_\lambda$ | persisitence of success rate shock | 0.8 | 0.1 | beta | 0.7606 | [ | 0.5775 | 0.9888 | ] | 0.101 |
| $\sigma_b$ | St.Dev. of preference shock | 0.5 | Inf | inv gamma | 0.1223 | [ | 0.0839 | 0.159 | ] | 0.694 |
| $\sigma_l$ | St.Dev. of labor disutility shock | 0.5 | Inf | inv gamma | 0.0564 | [ | 0.0491 | 0.0637 | ] | 0.284 |
| $\sigma_g$ | St.Dev. of exogenous spending shock | 0.5 | Inf | inv gamma | 0.0595 | [ | 0.0502 | 0.0683 | ] | 0.843 |
| $\sigma_i$ | St.Dev. of investment adjustment cost shock | 0.5 | Inf | inv gamma | 0.1391 | [ | 0.0878 | 0.1892 | ] | 0.393 |
| $\sigma_\lambda$ | St.Dev. of success rate shock | 0.5 | Inf | inv gamma | 0.1441 | [ | 0.0862 | 0.2007 | ] | 0.892 |
| $\sigma_y^{err}$ | St.Dev. of output growth rate obs.err. | 0.005 | 0.001 | inv gamma | 0.0096 | [ | 0.0081 | 0.011 | ] | 0.133 |
| $\sigma_c^{err}$ | St.Dev. of consumption growth rate obs.err. | 0.005 | 0.001 | inv gamma | 0.0051 | [ | 0.0035 | 0.0067 | ] | 0.248 |
| $\sigma_i^{err}$ | St.Dev. of l investment growth obs.err. | 0.005 | 0.001 | inv gamma | 0.0058 | [ | 0.0034 | 0.0081 | ] | 0.649 |
| $\sigma_{R\&D}^{err}$ | St.Dev. of R&D investment growth obs.err. | 0.005 | 0.001 | inv gamma | 0.0103 | [ | 0.0061 | 0.0143 | ] | 0.162 |
| $\sigma_l^{err}$ | St.Dev. of labor supply obs.err. | 0.005 | 0.001 | inv gamma | 0.0037 | [ | 0.0029 | 0.0044 | ] | 0.401 |

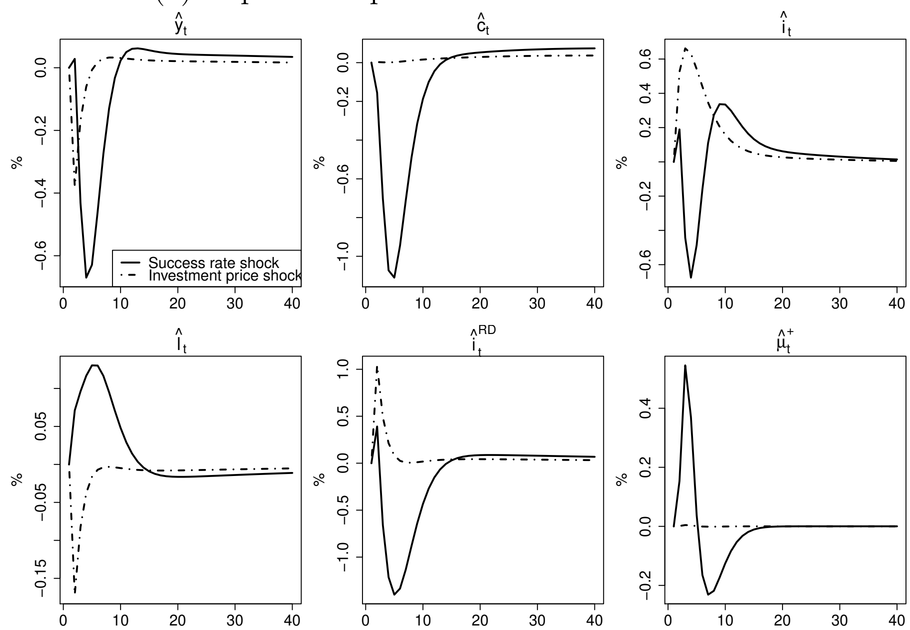
Notes:
1. We estimate the model during the sample period: 1994Q2-2016Q3, using MCMC simulation.And, we generate 600,000 draws from the posterior distribution of parameters with the Metropolis-Hastings algorithm and discard the first half of those(say 300,000 draws) as burn-in.
2. 90% Highest Posterior Density Interval.
3. Geweke(1992) Convergence Diagnostics.

driven by business cycles. The success rate shock might positively impact on $\hat{y}_t$ over 16 quarters (or 4 years) and longer than the case of common growth rate. A positive shock of relative price negatively impacts on $\hat{y}_t$ for quite short period such as nearly six quarters. The lower left chart shows responses of $\hat{c}_t$ to both shocks. As can be seen from the chart, the impacts of both shocks are tiny size although they are positively persistent for longer than 10 years. The lower right graph shows responses of $\hat{i}_t$ to both shocks. The success rate shock impact on investment as much as for 20 quarters (or five years), while a positive shock of R&D investment relative price also increases investment for relatively long such as 15 quarters.

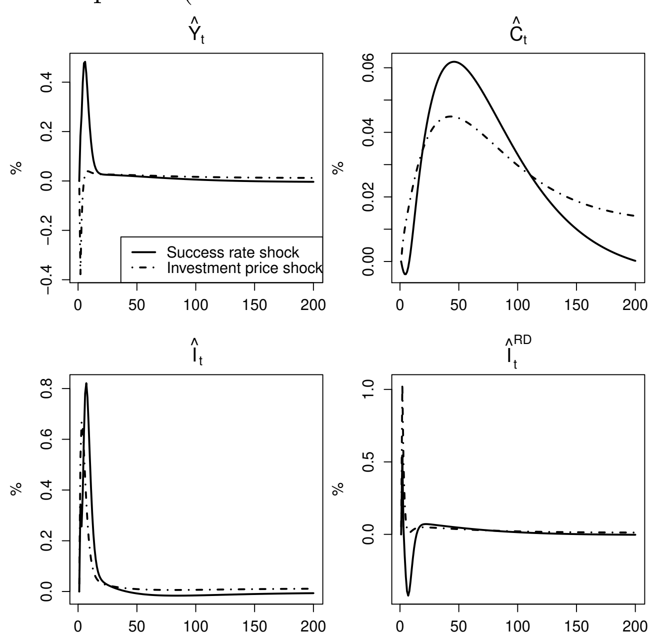
To sum up, a positive shock of the success rate, a sort of pure technology shocks, raises up common growth rate for short period, and also increase business cycle components of output and investment for similar period but not consumption. Meanwhile, a positive shock of R&D investment relative price hardly seem to influence the growth and the consumption at all, but it impacts on both of output and investment toward opposite directions by reducing the R&D investment itself.

Figure 3: Impulse Response Function to the R&D shocks

(a) Impulse Response of Detrended Variables




(b) Impulse Response (Deviation from the Balanced Growth Path)




Notes: The upper left panel plots IRF of the common stochastic growth rate, $\mu_t^+$, deviated from steady state, $\mu_{ss}^+$, to the R&D investment's relative price shock, $z_t^p$, (the solid red lines) and the R&D success rate shock, $z_t^\lambda$, (the dashed blue lines). The upper right chart plots IRF of deviation of output from steady state to the two R&D shocks. Similarly, the lower left shows that of comsumption, whereas the lower right shows that of investment.

## 4.3 Periodogram of Common Growth Rate

Next, we consider what relation there is between estimated common trend growth and business cycle components. The upper graph of Figure 4 (a) shows business cycle components of the real GDP per capita extracted from the band pass filter proposed by Christiano and Fitzgerald (2003) with specifying frequency band between 8 quarters and 32 quarters, while the lower graph shows estimated common growth rate, $\mu_t^+$, say, $\ln\left(A_t^+/A_{t-1}^+\right)$. As you see these graphs, both series seem to be completely coherent, although the upper chart represents extraction from the GDP growth rate alone. In contrast, the lower chart reflects a common growth among the observed variables.

To verify the coherence between them, we calculate periodgram of both series.[1] As Figure 4 (b), the periodgrams of these series show quite similar shape in which high frequency range such like upper 0.6 (which is corresponding to cycles with 6.67 quarters (=4/0.6). ) is quite silent, but low and medium ranges have high densities. The variance of $\mu_t^+$ is, however, about half as the GDP, i.e., 0.00267 vs. 0.00484. This indicates that half of fluctuations of the GDP is explained by the common growth induced by the two R&D shocks, and that the pure technology shocks characterized by the R&D shocks is likely to procyclically influence business cycles by shifting the common growth rate.

## 4.4 Historical Decomposition of Common Stochastic Trend

Now, we turn to consider what factors contribute to make the common trend growth rate. Figure 5 shows historical decomposition of common trend growth rate, $\mu_t^+$, which is obtained from the first difference of the composite technology level, i.e., $(\ln A_t^+) - (\ln A_{t-1}^+)$. The black line shows the estimated common growth, even in which the huge decline is observed in 2008:Q3 (i.e., at the time of Lehman brothers' collapse) as well

[1] These periodograms are obtained from

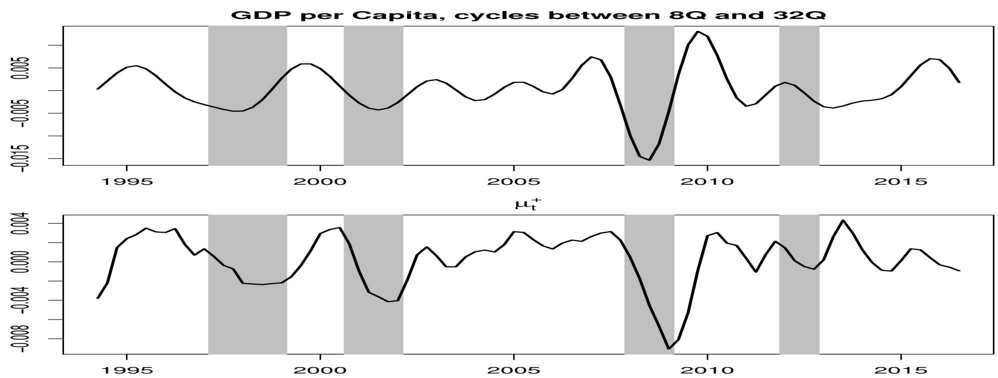
$$\hat{p}\left(\omega\right) = \frac{1}{2\pi}\sum_{k=-T+1}^{T-1}\hat{C}_k e^{-i\omega k},$$

where $\hat{C}_k$ is auto covariance function of data, $\omega$ is angular velocity, and $T$ is sample size. If the frequency $f$ is used instead of $\omega$, then we can rewrite it as

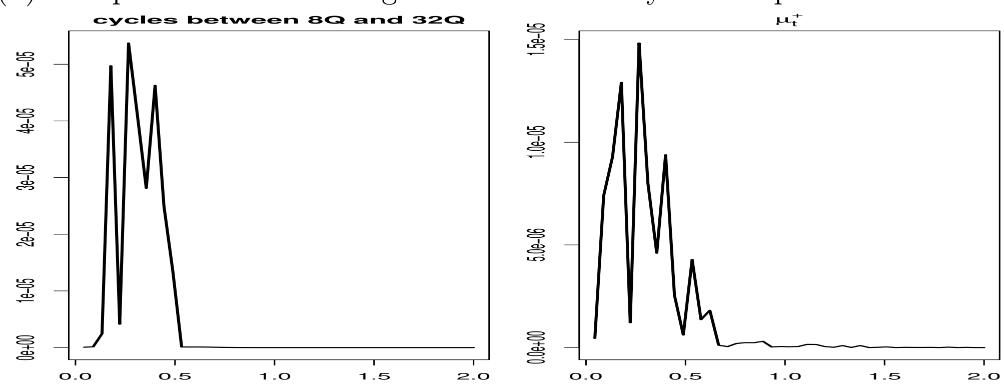
$$\hat{p}\left(f\right) = \frac{1}{2\pi}\sum_{j=-T+1}^{j=T-1}\hat{C}_k e^{-2\pi i\omega} = \frac{1}{2\pi}\left[\hat{C}_0 + \sum_{k=1}^{T-1}\hat{C}_k\cos\left(2\pi\omega k\right)\right].$$

Figure 4: Time Series and Periodgram of Common Growth

(a) Comparison with Time series of Business Cycle Component of real GDP




(b) Comparison with Periodgram of Business Cycle Component of real GDP



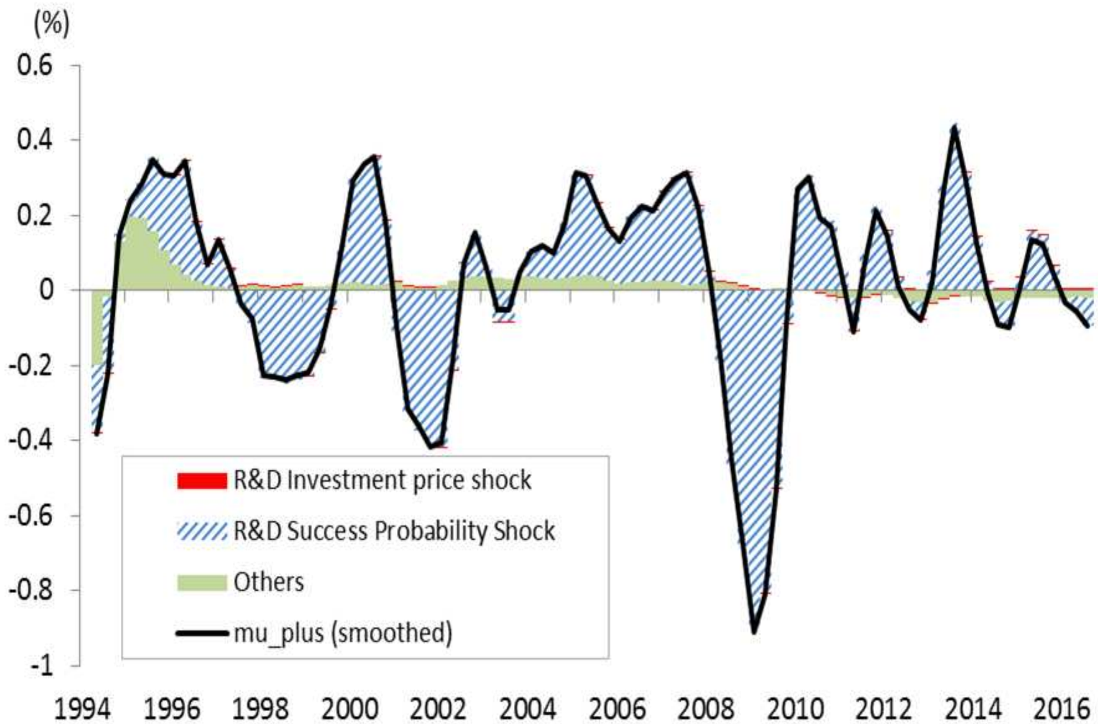

as business cycle components of output, consumption and investment. [2]

As Figure 5, the R&D success rate shock (the blue shade area) account for the most part of the common growth rate. On the other hand, the R&D investment relative price shock (the red shade area) and other four shocks such like preference, labor disutility, and investment specific shocks (the green shade area) hardly contribute to it. This result suggests that the R&D success probability shock is plausibly the main driving source of common growth, and that this shock also play an important role in amplifying

[2]When we see the historical decomposition of real GDP, we cannot observe that the first difference of GDP is sufficiently explained by the six structural shocks of our model. On the other hand, those of consumption and investment seem to be explained by the structural shocks very well. The fluctuations of the consumption are mainly driven by the preference and the exogenous spending shock including government spending and export. The fluctuations of the investment are mainly derived from the investment adjustment cost shock and the exogenous spending shock. However, the decline of investment in 2008 Lehman brothers collapse is contributed by R&D success rate shock with partly but non-negligible size.

magnitude of business cycles through increasing the R&D investment and growth rate. Again, such transition mechanism can explain nearly half of the variations of business cycle components of the real GDP excluding the noise components such as the frequency upper 8Q.

Figure 5: Historical Decomposition of Common Growth Rate, $\mu_t^+$




# 5 Roles of R&D shocks and Data

To verify significance of the presence of the two R&D shocks and also that of the R&D data, we introduce alternative models excluding those shocks or the data, and compare with the common stochastic trends of them.

We additionally construct two models to be compared with the original model considered so far. In order to consider a case excluding the R&D data from the original dataset, the first model, referred to as Model A1, is set to reduce to four measurement equations by cutting Eq.(38). The remaining settings is the same as the baseline model including the prior settings.

For considering another case of absence of the R&D shocks, the second model, referred to as Model A2, is constructed by replacing the two R&D related shocks with a temporary TFP shock. The five measurement equations are kept as the R&D data remains as observable data. In Model A2, we redefine the productive function, Eq.(20), as

$$Y_{it} = z_t^{\mathrm{TFP}} \left(u_{it}^k K_{it-1}\right)^{\alpha} l_{it}^{1-\alpha},$$

where the TFP shock follows an AR (1) process, i.e.,

$$\log(z_t^{\mathrm{TFP}}) = \rho^{\mathrm{TFP}} \log(z_{t-1}^{\mathrm{TFP}}) + \varepsilon_t^{\mathrm{TFP}}.$$

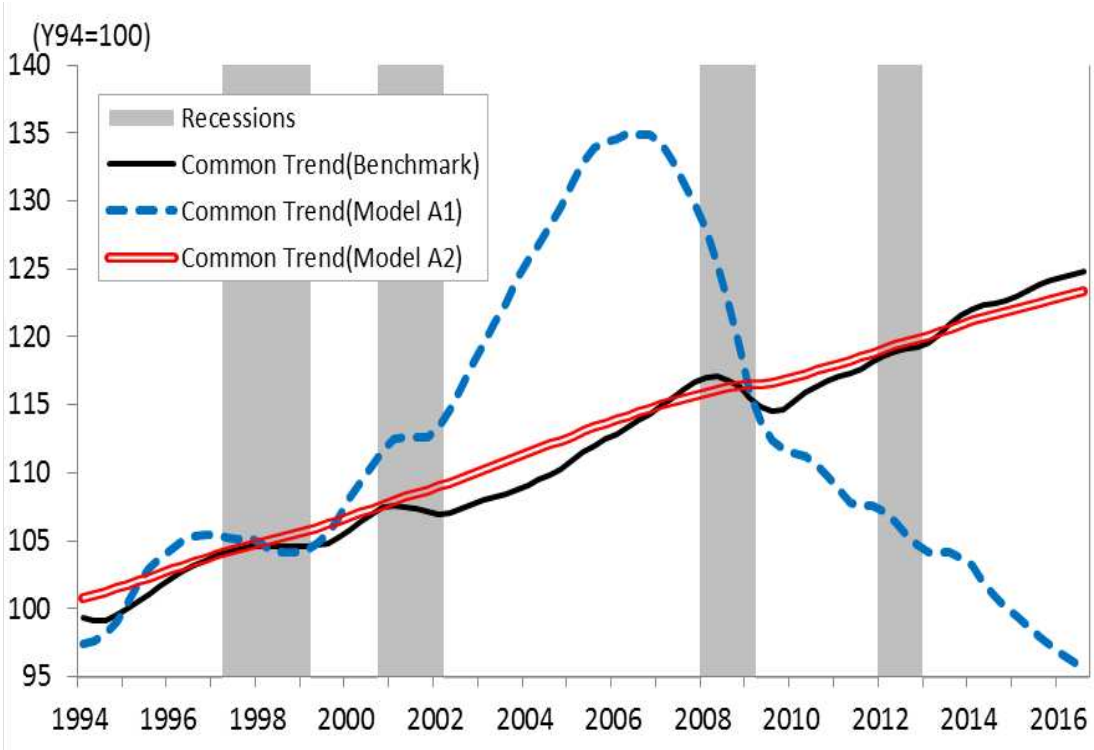
It is noteworthy that setting of theTFP structural shock involved in R&D sector is similar to that of Kung and Schmid (2015).

Figure 6 shows three lines of the estimated common growths from the three models including regions of recessions reported by the Cabinet Office. The red line, the green line and the purple represent the common trends of the original model, Model A1 without R&D data, Model A2 replaced with the TFP shock, respectively. As the green line denoting Model A1, the common trend (accumulation of the common component , $\mu_{t^+}$) has big swing such as going up steeply in the first half of sample periods and dropping down symmetrically in the last half, due to reducing one measurement equation (38) identifying the R&D activity. Omitting the R&D data seems to bring an identification problem of common growth, and as a result, induces inconsistency between model variables and observable variables through changing measurement equations.

On the other hand, the purple line of Model A2 shows that the common trend driven by the TFP shock, instead of the R&D shocks, grows with substantial linearity, even though we observe a change of R&D investment as data. And we see that the R&D investment plays an unimportant role in determining the growth rate, and that there is a small variance of the temporary TFP shocks compared with those of the R&D shocks.

As for comparison our baseline model with the alternative models, we summarize as follows. By adopting the R&D data for identifying the R&D investment, it is thought that we estimate successfully the common growth rate in terms of the same prior distributions of parameters, compared with the case of omitting the R&D data. Missing the data makes estimation of the variances of the R&D shocks much bigger than an envisioned size and brings the growth rate not plausible movement due to the identification

Figure 6: Trend Components




problem. On the other hand, embedding a TFP shock but not the R&D shocks makes the trend nearly linear, and the trend is rarely affected from the R&D data. And the TFP shock is also likely not to influence the economic growth because of tiny size of its variance.

Unlike above alternative models, our original model shows that the common growth rate characterized by a pure technology shock incorporated into endogenous growth model is time-varying plausible and persuadable thanks to adding both of the R&D data and shocks, especially the success probability shock. Accordingly, we conclude that the common trend identified by R&D activities is likely to be procyclical and that it accounts for nearly half of the variation of the real GDP, another half of which is explained by business cycle components. On the other hand, we also report that a TFP shock seems to shift the common trend independently with business cycle. Different structural shocks bring diffrence pictures even though we use same model and data except them.

# 6 Conclusion

We considered how and the extent to which a pure technology shock driven by R&D activities impacts on business cycles as well as economic growth, using a medium-scale neo-classical dynamic stochastic general equilibrium (DSGE) model with endogenous growth.

We tried to identify the pure technology shock by adopting intellectual property product first entered in 2008 SNA which can be regarded as the level of R&D activity, and by incorporating time to build assumption that it takes a certain period for R&D investment to be embodied in available technology.

Our empirical result based on a Bayesian analysis indicates that the common stochastic trend driven by the pure technology shock is likely to be procyclical, and it accounts for nearly half of the variation of the real GDP, while remaining is explained by the components more frequent than business cycles.

This result suggests that changes in R&D investment cause fluctuations at the business cycle frequency. The results of robustness check show the importance of R&D investment data and R&D related shocks. Without R&D data, the common trend seems to be not properly identified. The common trend extracted by an alternative model with a temporary TFP shock as a substitute for R&D related shocks is substantially linear.

# A Appendix

## A.1 Model Condition Equations

### A.1.1 Structural equations

1. marginal utility of consumption

$$\lambda_t^c = \frac{z_t^b \mu_t^+}{\mu_t^+ c_t - h c_{t-1}} - \beta h \mathbb{E}_t \frac{z_{t+1}^b}{\mu_{t+1}^+ c_{t+1} - h c_t} \tag{40}$$

2. capital rental cost

$$r_t^k = q_t^k \delta' \left( u_t^k \right) \tag{41}$$

3. stochastic discount factor

$$\mu_t^+ \Lambda_{t|t+1} = \beta \frac{\mathbb{E}_t \lambda_{t+1}^c}{\lambda_t^c} \tag{42}$$

4. Tobin's Q

$$q_t^k = \mathbb{E}_t \frac{\Lambda_{t|t+1}}{\mu_{t+1}^\psi} \left[ r_{t+1}^k u_{t+1}^k + q_{t+1}^k \left\{ 1 - \delta \left( u_{t+1}^k \right) \right\} \right] \tag{43}$$

5. FOC of fixed-capital investment

$$\begin{aligned} 1 = q_t^k & \left\{ 1 - S \left( z_t^i \tfrac{\mu_t^+ \mu_t^\psi}{\mu^+ \mu^\psi} \tfrac{i_t}{i_{t-1}} \right) - S' \left( z_t^i \tfrac{\mu_t^+ \mu_t^\psi}{\mu^+ \mu^\psi} \tfrac{i_t}{i_{t-1}} \right) \tfrac{\mu_t^+ \mu_t^\psi}{\mu^+ \mu^\psi} \tfrac{i_t}{i_{t-1}} \right\} \\ & + \mu^+ \mu^\psi \mathbb{E}_t \tfrac{\Lambda_{t|t+1}}{\mu_{t+1}^\psi} q_{t+1}^k S' \left( z_{t+1}^i \tfrac{\mu_{t+1}^+ \mu_{t+1}^\psi}{\mu^+ \mu^\psi} \tfrac{i_{t+1}}{i_t} \right) \left( \tfrac{\mu_{t+1}^+ \mu_{t+1}^\psi}{\mu^+ \mu^\psi} \tfrac{i_{t+1}}{i_t} \right)^2 \end{aligned} \tag{44}$$

6. goods variety accumulation with "time to build"

$$\mu_t^V = (1 - \delta_a) + (1 - \phi_{\mu 1}) \widetilde{\Delta}_{at} + \phi_{\mu 1} \left\{ (1 - \phi_{\mu 2}) \widetilde{\Delta}_{at-1} + \phi_{\mu 2} \widetilde{\Delta}_{at-2} \right\} \tag{45}$$

7. FOC of labor supply

$$w_t = \frac{1 - \alpha}{1 + \lambda_I} \frac{y_t}{l_t} \tag{46}$$

8. FOC of capital service supply

$$r_t^k = \frac{\alpha}{1 + \lambda_I} \frac{y_t}{u_t^k k_{t-1}} \mu_t^+ \mu_t^\psi \tag{47}$$

9. production function

$$y_t = \left( \frac{u_t^k k_{t-1}}{\mu_t^+ \mu_t^\psi} \right)^\alpha l_t^{1-\alpha} \tag{48}$$

10. success rate

$$\lambda_t = z_t^\lambda \lambda_{\mathbf{0}} i_{at}^{\omega_a} \tag{49}$$

11. gross increment of goods variety

$$\widetilde{\Delta}_{at} = (1 - \delta_a) \lambda_t \left( \frac{1}{v_{t-1}} - 1 \right) \tag{50}$$

12. FOC of adaptation invesment

$$i_{at} = (1 - \delta_a)\,\omega_a \lambda_t \left( p_t^V - \mathbb{E}_t \Lambda_{t|t+1} \frac{\mu_{t+1}^{+}}{\mu_t^V} \right) \tag{51}$$

13. Value function of adapter

$$j_t = (1 - \delta_a) \left\{ (1 - \omega_a)\,\lambda_t p_t^V + \frac{1 - (1 - \omega_a)\,\lambda_t}{(1 - \delta_z)\,\phi_t} \right\} \tag{52}$$

14. goods variety accumulation

$$\frac{1}{\upsilon_t} = \frac{1}{\mu_t^V} \left( \frac{1 - \delta_z}{\upsilon_{t-1}} + \phi_t i_{dt} \right) \tag{53}$$

15. R&D investment efficiency

$$\phi_t = \frac{\chi_z}{\upsilon_{t-1} i_{dt}^{1-\rho}} \tag{54}$$

16. FOC of R&D invention investment

$$i_{dt} = (1 - \delta_z) \left( \frac{1}{\upsilon_t} - \frac{1 - \delta_z}{\mu_t^V \upsilon_{t-1}} \right) \mathbb{E}_t \Lambda_{t|t+1} j_{t+1} \tag{55}$$

17. market clearing condition

$$y_t = c_t + i_t + i_t^{R\&D} z_t^{R\&D} + \overline{g/y} y z_t^g \tag{56}$$

18. composite technology progress rate

$$\mu_t^{+} = \left(\mu_t^V\right)^{\frac{\lambda_i}{1-\alpha}} \left(\mu_t^{\psi}\right)^{\frac{\alpha}{1-\alpha}} \tag{57}$$

19. capital depreciation rate function

$$\delta\left(u_t^k\right) = \delta_k + b_k \frac{\left(u_t^k\right)^{1+\zeta_k}}{1 + \zeta_k} \tag{58}$$

20. capital depreciation rate derived function

$$\delta'\left(u_t^k\right) = b_k \left(u_t^k\right)^{\zeta_k} \tag{59}$$

21. capital accumulation

$$k_t = \left(1 - \delta\left(u_t^k\right)\right) k_{t-1} + \left\{1 - S\left(z_t^i \frac{\mu_t^+ \mu_t^\psi}{\mu^+ \mu^\psi} \frac{i_t}{i_{t-1}}\right)\right\} i_t \tag{60}$$

22. FOC of newly added goods variety

$$p_t^V = (1 - \phi_{\mu 1}) \gamma_t + \phi_{\mu 1} (1 - \phi_{\mu 2}) \mathbb{E}_t \mathit{\Lambda}_{t|t+1} \gamma_{t+1} \frac{\mu_{t+1}^+}{\mu_t^V} + \phi_{\mu 1} \phi_{\mu 2} \mathbb{E}_t \mathit{\Lambda}_{t|t+1} \mathit{\Lambda}_{t+1|t+2} \gamma_{t+2} \tag{61}$$

23. FOC of goods variety stock

$$\gamma_t = \mathbb{E}_t \frac{\Lambda_{t|t+1} \mu_{t+1}^+}{\mu_t^V} \left\{(1 - \delta_a) \gamma_{t+1} + \frac{\lambda_I}{1 + \lambda_I} y_{t+1}\right\} \tag{62}$$

24. FOC of labor supply

$$w_t = \gamma_l \frac{z_t^l l_t^{\omega_l}}{\lambda_t^c} \tag{63}$$

25. investment specific technology progress rate

$$\mu_t^\psi = \bar{\mu} \mu_t^V \tag{64}$$

26. R&D investment

$$i_t^{R\&D} = i_{dt} + i_{at} \left(\frac{1}{v_{t-1}} - 1\right) \tag{65}$$

27. investment adustment cost function

$$S_t \left(\frac{\mu_t^+ \mu_t^\psi}{\mu^+ \mu^\psi} \frac{i_t}{i_{t-1}}\right) = \frac{1}{2} \overline{1/\zeta} \left(z_t^i \frac{\mu_t^+ \mu_t^\psi}{\mu^+ \mu^\psi} \frac{i_t}{i_{t-1}} - 1\right)^2 \tag{66}$$

28. derivative of invesment adjustment cost function

$$S_t' \left(\frac{\mu_t^+ \mu_t^\psi}{\mu^+ \mu^\psi} \frac{i_t}{i_{t-1}}\right) = \overline{1/\zeta} \left(z_t^i \frac{\mu_t^+ \mu_t^\psi}{\mu^+ \mu^\psi} \frac{i_t}{i_{t-1}} - 1\right) \tag{67}$$

### A.1.2 AR(1) processes of structural shocks

1. preference shock

$$\ln\left(z_t^b\right) = \rho_b \ln\left(z_{t-1}^b\right) + \varepsilon_t^b, \text{ for } \varepsilon_t^b \sim i.i.d.\ N\left(0, \sigma_b^2\right) \tag{68}$$

2. labor disutility shock

$$\ln\left(z_t^l\right) = \rho_l \ln\left(z_{t-1}^l\right) + \varepsilon_t^l, \text{ for } \varepsilon_t^l \sim i.i.d.\ N\left(0, \sigma_l^2\right) \tag{69}$$

3. exogenous spending shock

$$\ln\left(z_t^g\right) = \rho_g \ln\left(z_{t-1}^g\right) + \varepsilon_t^g, \text{ for } \varepsilon_t^g \sim i.i.d.\ N\left(0, \sigma_g^2\right) \tag{70}$$

4. R&D success probabilty shock

$$\ln\left(z_t^\lambda\right) = \rho_\lambda \ln\left(z_{t-1}^\lambda\right) + \varepsilon_t^\lambda, \quad \text{for } \varepsilon_t^\lambda \sim i.i.d.\ N\left(0, \sigma_\lambda^2\right) \tag{71}$$

5. R&D investment relative price shock

$$\ln\left(z_t^P\right) = \rho_\lambda \ln\left(z_{t-1}^P\right) + \varepsilon_t^P, \quad \text{for } \varepsilon_t^P \sim i.i.d.\ N\left(0, \sigma_P^2\right) \tag{72}$$

6. fixed investment adjustment cost shock

$$\ln\left(z_t^i\right) = \rho_i \ln\left(z_{t-1}^i\right) + \varepsilon_t^i, \quad \text{for } \varepsilon_t^i \sim i.i.d.\ N\left(0, \sigma_i^2\right) \tag{73}$$

### A.1.3 measurement equations

1. GDP growth rate

$$\Delta \ln(Y_t^{\text{data}}/N_t^{\text{data}}) = \ln(\mu_t^+) + \ln(y_t) - \ln(y_{t-1}) + \varepsilon_t^{\text{obs\_err\_y}}, \tag{74}$$

2. consumption growth rate

$$\Delta \ln(C_t^{\text{data}}/N_t^{\text{data}}) = \ln(\mu_t^+) + \ln(c_t) - \ln(c_{t-1}) + \varepsilon_t^{\text{obs\_err\_c}}, \tag{75}$$

3. fixed investment growth rate

$$\Delta \ln(I_t^{\text{data}}/N_t^{\text{data}}) = \ln(\mu_t^{+}) + \ln(\mu_t^{\psi}) + \ln(i_t) - \ln(i_{t-1}) + \varepsilon_t^{\text{obs\_err\_i}} \tag{76}$$

4. level of labor supply

$$\ln(l_t^{\text{data}}) = \ln(l_t) + \varepsilon_t^{\text{obs\_err\_l}}. \tag{77}$$

5. R&D investment growth rate

$$\Delta \ln(I_t^{R\&D\text{data}}/N_t^{\text{data}}) = \ln(\mu_t^{+}) + \ln(i_t^{R\&D}) - \ln(i_{t-1}^{R\&D}) + \varepsilon_t^{\text{obs\_err\_R\&D}}, \tag{78}$$

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