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ON THE TREATMENT OF SECONDARY PRODUCTS AND BY-PRODUCTS IN THE PREPARATION OF INPUT-OUTPUT TABLES

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Abstract

In a recent paper, Ten Raa, Chakraborty and Small (1984) proposed a method for treating secondary products and by-products in the preparation of commodity by commodity input-output matrices under the commodity technology assumption, when by-products result only from primary production. The presentation was limited to by-products that are *produced* at the margin by other activities. This paper extends that approach to inputs and by-products that are not produced at the margin.

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ON THE TREATMENT OF SECONDARY PRODUCTS AND BY-PRODUCTS IN THE PREPARATION OF INPUT-OUTPUT TABLES

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A recent paper by Ten Raa, Chakraborty and Small (1984), henceforth TRCS, proposed a method for treating secondary products and by-products in the preparation of commodity × commodity input-output matrices under the commodity technology assumption, when by-products result only from primary production. TRCS's presentation, itself an extension of Van Rijckeghem's (1967), was limited to by-products that are *produced* at the margin by other activities. This paper extends that approach to inputs and by-products that are not produced at the margin.

Following TRCS's notation, let $\mathbf{U} = (u_{ij})$ be the $n \times n$ matrix containing the use of produced commodities i by industries j, $\mathbf{W} = (w_{hj})$ be the $m \times n$ matrix containing the use of non-produced inputs h by industries j and let $\mathbf{V} = (v_{jk})$ be the $n \times (n + t)$ make matrix containing outputs of industries j consisting of produced commodities i and non-produced by-products. Note that **U** and **W** have been defined in terms of marginally produced and non-produced inputs, and not in terms of inputs other than factors, and value added. Consequently, inputs of one activity that are only non-produced by-products of others have been removed from **U** and assigned to **W**. By doing so we avoid the all too common error of leaving non-produced inputs in the **U** matrix as if they were produced. For example, oilseed cakes demanded by the animal feed industry will be recorded in the **W** matrix so that later, additional production of animal feed will not appear as demanding additional production of vegetable oils. Also note that in order to separate marginally produced from non-produced inputs, a considerable level of disaggregation in preparing the absorption and make matrices is required.

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In order to arrive at the commodity × commodity coefficient matrices, let us start by splitting the make matrix **V** into

$\mathbf{V}_1$ = the $n \times n$ matrix containing primary and secondary production of commodities i by industries j

$\mathbf{V}_2$ = the $n \times n$ matrix containing by-products (of primary products) i that are *produced* by other industries j as primary products

$\mathbf{V}_3$ = the $n \times t$ matrix of *non-produced* by-products of primary production j

Consequently, $\mathbf{V} = [(\mathbf{V}_1 + \mathbf{V}_2) \mid \mathbf{V}_3]$, where | indicates partitioning, and

$$[1]\,\mathbf{U} + [1]\,\mathbf{W} = [1]\,\mathbf{V}_1' + [1]\,\mathbf{V}_2' + [1]\,\mathbf{V}_3' \qquad (1)$$

Let us now define matrix $\mathbf{W}^+$ as matrix **W** augmented by as many zero rows as non-produced by-products that, while not specified in **W**, are present in $\mathbf{V}_3$. Similarly, $\mathbf{V}_3^+$ will be matrix $\mathbf{V}_3$ augmented by as many zero columns as non-produced inputs different from non-produced by-products present in **W**, so that $\mathbf{W}^+$ and $\mathbf{V}_3^{+\prime}$ have the same dimensions with rows and columns corresponding to the same non-produced goods. Consequently, equation (1) can also be written as

$$[1]\,\mathbf{U} + [1]\,\mathbf{W}^+ = [1]\,\mathbf{V}_1' + [1]\,\mathbf{V}_2' + [1]\,\mathbf{V}_3^{+\prime}$$

from which

$$\mathbf{A} = (\mathbf{U} - \mathbf{V}_2')\,(\mathbf{V}_1')^{-1}$$

$$\mathbf{F} = (\mathbf{W}^+ - \mathbf{V}_3^{+\prime})\,(\mathbf{V}_1')^{-1}$$

will be the commodity × commodity coefficient matrices for produced and non-produced inputs, respectively, obtained under the commodity technology assumption, when by-products only result from primary production.

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