A new irrelevance result in an endogenous timing with a consumer-friendly public firm

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Abstract

This study considers a mixed duopoly with a consumer-friendly public firm and analyzes an endogenous timing game in the presence of output subsidy and emission tax. We provide a new irrelevance result concerning the choice of government policy in which regardless of the policy mix, the equilibrium of endogenous market structure is determined by the public firm's concern on consumer surplus. We also show that the optimal policy mix can attain the first-best allocation for social welfare only when both firms have symmetric payoffs, which results in simultaneous-move outcome.

Keywords: irrelevance result; endogenous timing game; consumer-friendly public firm; emission tax; output subsidy

I. Introduction

Earlier studies for mixed markets where a public firm coexists with private ones have established a series of so-called "irrelevance results", which states that the first-best allocation for welfare can be attained if output is subsidized. It implies that privatization does not alter welfare, regardless of time structure, competition mode and the degree of privatization. Thus, it is also called "privatization neutrality theorem".[1] In their formulation, the objective function of the public firm is welfare-maximization, which considers not only consumer surplus but also industry profits. Thus, there exists the payoff interdependence between the public and private firms.[2]

Rather than focusing on the welfare effect of a public firm, this paper considers a variant of mixed duopoly with a consumer-friendly public firm, which cares for its own profits and some portion of consumer surplus, as a proxy of its concern on the society. This approach is a subset of corporate

[1] Since White (1996) and Pal and White (1998) reported, subsequent works supported these results. See Poyago-Theotoky (2001), Tomaru (2006), Kato and Tomaru (2007), Hashimzade et al. (2007), Zikos (2007), Tomaru and Saito (2010) and Matsumura and Okumura (2013) among others. However, Fjell and Heywood (2004), Cato and Matsumura (2013), Matsumura and Tomaru (2013, 2015) and Lee and Tomaru (2017) showed that it might fail under certain environments with foreign competitors, free entry, excess burden of taxation and R&D spillovers.

[2] As the payoff interdependent approach, Matsumura et al. (2013) and Matsumura and Okamura (2015) also formulated a mixed oligopoly market where the private firms consider their rival's profits as well.

social responsibility (CSR), which is increasingly popular not only in the real world but in academic research in both empirical and theoretical economics.[3] As a public firm may or may not share the same objectives as the government, we can also interpret this objective function as a situation that public firm takes its profit-maximizing decision under consumer surplus-constrained regulation in which consumer surplus in the market does not fall below a fixed level.[4]

In this study, we investigate an endogenous timing game in a mixed duopoly in which firms choose quantities simultaneously or sequentially. We also introduce a negative externality and abatement activities into the model where the production in both public and private firms leads to pollution. In the presence of emission tax and output subsidy, we show that a simultaneous movement is the only equilibrium when the concern on consumer surplus is small while a sequential movement can be an equilibrium when the concern on consumer surplus is large.

These results coincide with the previous findings which consider neither externality nor government intervention. Hamilton and Slutsky (1990) showed that firms decide simultaneously when both firms have symmetric payoffs while Pal (1998) showed that firms decide sequentially when a private firm competes with a public firm which maximizes not only consumer surplus but also industry profits. Similarly, as the consumer-friendly public firm more concerns consumer surplus in our formulation, it produces more aggressively than the simultaneous case and thus more industry outputs yields higher consumer surplus.

However, the literature in the standard mixed duopoly model with a welfare-maximizing public firm also showed that the outcomes of the endogenous timing game with externality depend critically on the significance of externality. In particular, Matsumura and Ogawa (2017) showed that a simultaneous-move (sequential-move) outcome is an equilibrium under a significant (insignificant) externality without emission tax. However, incorporating the optimal emission tax in each market structure, Lee and Xu (2017) showed that the results are reversed. They provided an important policy implications that the degree of internalization of externality may change the competition structure

[3] See CSR trend report by PricewaterhouseCoopers (2010) and KPMG (2015). Lyon and Maxwell (2004), Kitzmueller and Shimshack (2012) and Crifo and Forget (2015) provided fruitful discussions on the practical and academic issues on CSR.

[4] Note that the result of price regulation is equivalent to that of the rate-of-return regulation for public firm, in which the government obtains the market quantities and prices that maximize consumer surplus subject to permitting the firm to earn some fixed profit. For more detailed discussions on this point, see Brennan (1989), Lee (1998) and Xu et al. (2016).

and indirectly affect the resulting emission and welfare levels.

We re-examine the mixed duopoly with externality and government intervention through output subsidy and emission tax. In the presence of a consumer-friendly public firm, we investigate different timing of game structure that government chooses the policy mix of output subsidy and emission tax in the beginning of endogenous timing game. Interestingly, our analysis reveals that regardless of the policy mix, the equilibrium of endogenous market structure is determined solely by the consumer-friendly firm's concern on consumer surplus. Thus, the equilibrium mode of competition remains unaffected by government tax/subsidy policy. Thus, we provide a new irrelevance result in terms of the choice of government policy. This is surprisingly different to the standard mixed duopoly model where the government policy significantly affects endogenous timing outcomes that firms choose for production. Hence, our findings on the first-mover position of the government are in sharp contrast to the case that government takes second-mover position in mixed duopolies with a welfare-maximizing public firm.[5]

Our results also contain important implications on the objective functions of the public firm. If it is a welfare-maximizing firm, privatization does not alter welfare regardless of time structure, but an appropriately scheduled policy mix is important to obtain the best outcomes to the society. However, if it does not care for the industry profits, government subsidy/tax policies before the firms' endogenous choices of market roles will not change competition mode and market structure. This is a new finding that is neglected in the literature.

Finally, we show that the optimal policy mix can attain the first-best allocation for social welfare only when the concern on consumer surplus is abandoned and thus both firms have symmetric payoffs, which results in a simultaneous-move outcome.[6] Otherwise, two sequential-move outcomes might appear as equilibrium results.[7] We believe that our findings potentially provide new

[5] In Appendix, we also show that our findings do not hold under the standard mixed duopoly model where welfare-maximizing public firm exists. Tomaru and Saito (2010) examined the same timing of game structure with our formulation in a standard mixed duopoly model under output subsidization policy. They showed that simultaneous competition is an equilibrium outcome with a higher output subsidy rate only. Thus, the equilibrium also depends on the government subsidy policy.

[6] Under asymmetric information in a private oligopoly, Kim and Chang (1993), Kim and Lee (1995), and Lee (1997) proposed that optimal subsidy/tax policy can attain the first-best outcomes to the society.

[7] For example, Tomaru and Saito (2010) showed that simultaneous competition is an equilibrium outcome with a higher output subsidy while Lee and Xu (2017) showed that private leadership outcome can be an equilibrium outcome with an emission tax.

insights into the behavior of firms and contribute to the literature in endogenizing the strategic choice of government policy in mixed market settings.

The remainder of this paper is organized as follows. In section II, we formulate a mixed duopoly model with a consumer-friendly public firm. We analyze the simultaneous and sequential movements in section III, and compare the outcomes to find an endogenous timing equilibrium in section IV. In section V, we construct optimal policy mix and examine the first-best outcomes. Final section concludes the paper.

II. Model

We consider a mixed duopoly in a quantity-setting game. One of the firms is a consumer-friendly (CF) firm (hereafter referred to as firm 0) that cares for not only its profits but consumers surplus. The other is a for-profit (FP) firm (hereafter referred to as firm 1) that maximizes only its profits. Firms sell their outputs, $q_0 \geq 0$ and $q_1 \geq 0$, respectively, at the market clearing price $p(Q) = 1 - Q$ where $Q = q_0 + q_1$. We assume that both firms have identical technologies and the production cost function takes a quadratic form, $c(q_i) = \frac{1}{2}q_i^2,\ i \in \{0,1\}$.

The production in both CF and FP firms leads to pollution e_i, but each firm can prevent pollution by undertaking abatement activities. Suppose an end-of-pipe technology that the emission level of each firm is given by $e_i = q_i - a_i$ when firm i chooses pollution abatement level a_i, where firm i can reduce its emission by investing an amount of $\frac{a_i^2}{2}$ in abatement activities. The extent of environmental damage due to pollution by the industry is assumed to be given by $ED = \frac{(\sum_i e_i)^2}{2}$. The government imposes both an environmental tax on the emission level for which the tax rate is $t \geq 0$ and an output subsidy for which the subsidy rate is $s \geq 0$. The resulting total tax revenue is $T = t\sum_i e_i$ and the total subsidy expenditure is $S = sQ$

The profit of CF firm is given by $\pi_0 = (p + s) \cdot q_0 - \frac{1}{2}q_0^2 - t \cdot e_0 - \frac{a_0^2}{2}$. We assume that the CF firm maximizes profits plus a fraction of the consumer surplus (CS). Thus, the payoff that CF firm maximizes is as follows:

$$V_0 = \pi_0 + \theta CS \quad (1)$$

where $CS = \frac{Q^2}{2}$. The parameter $\theta \in [0,1]$ measures the degree of concern on consumer surplus that the CF firm has, which is exogenously given.

The FP firm seeks only for profit maximization:

$$\pi_1 = (p+s) \cdot q_1 - \frac{1}{2} q_1^2 - t \cdot e_1 - \frac{a_1^2}{2} \quad (2)$$

Then, the social welfare is the sum of consumer surplus CS, both firms' profits $\pi_0 + \pi_1$, and tax revenue T, minus environmental damage ED and subsidy expenditure S:

$$W = CS + \pi_0 + \pi_1 + T - ED - S \quad (3)$$

The game runs as follows: In the first stage, the government chooses the levels of emission tax and output subsidy to maximize social welfare. In the second stage, both firms commit market structure simultaneously. In the last stage, each firm decides output and abatement levels simultaneously according to its commitment move structure. The backward induction produces a sub-game perfect Nash equilibrium.

III. Fixed timing game

In this section, we consider fixed move structures: Simultaneous-move competition and Stackelberg competition with CF and FP leadership. We assume that the fixed move structures have interior solutions; $0 \leq t < 1 + s$.

a. Simultaneous-move game

In this game, the firms independently chooses its abatement effort level (a_i) and output (q_i), $i \in \{0,1\}$. Solving the first-order conditions for maximizing the payoffs of both firms in (1) and (2), respectively, we obtain the following equilibrium quantities and abatement level:

$$q_0^c = \frac{(1+s-t)(2+\theta)}{2(4-\theta)}, \quad q_1^c = \frac{(1+s-t)(2-\theta)}{2(4-\theta)}, \quad a_0^c = a_1^c = t \quad (4)$$

where superscript 'c' denotes the Cournot game. It may be useful to note the *ceteris paribus* effects of t, s and θ. We note that $\partial q_0^c / \partial t < \partial q_1^c / \partial t < 0$, $\partial q_0^c / \partial s > \partial q_1^c / \partial s > 0$, and $\partial q_1^c / \partial \theta < 0 <$

$\partial q_0^c/\partial\theta$, $\partial(q_0^c+q_1^c)/\partial\theta>0$. Thus, the CF firm is more sensitive to the tax and subsidy in a simultaneous-move game. Thus, if the concern on consumer surplus rises, its direct effect (ignoring any effect on the tax and subsidy) is predictably positive on the CF firm's output, negative on firm 1's output, but positive on the aggregate output.

The equilibrium payoffs and social welfare are, respectively:

$$V_0^c=\frac{t^2\left(76-20\theta-\theta^2\right)+\left(12(1+\theta)-5\theta^2\right)(1+s)(1+s-2t)}{8(4-\theta)^2},$$
$$\pi_1^c=\frac{t^2\left(76-44\theta+7\theta^2\right)+3(2-\theta)^2(1+s)(1+s-2t)}{8(4-\theta)^2},$$
$$\mathrm{W^c}=\frac{-\left(5+2s+s^2-2t-2st+13t^2\right)(4-\theta)^2+8(2+s+t)(1+s-t)(4-\theta)-36(1+s-t)^2}{4(4-\theta)^2} \qquad (5)$$

b. CF firm as a Stackelberg leader

In this case, firm 0 first chooses its output and abatement levels and then firm 1 makes its choice sequentially. Then, we have the following equilibrium quantities and abatement level:

$$q_0^{cf}=\frac{2(1+s-t)(3+\theta)}{21-4\theta},\quad q_1^{cf}=\frac{(1+s-t)(5-2\theta)}{21-4\theta},\quad a_0^{cf}=a_1^{cf}=t \qquad (6)$$

where superscript 'cf' denotes the equilibrium outcome in the Stackelberg game with CF firm leadership. It may be useful to note that $\partial q_0^{cf}/\partial t<\partial q_1^{cf}/\partial t<0$, $\partial q_0^{cf}/\partial s>\partial q_1^{cf}/\partial s>0$, and $\partial q_1^{cf}/\partial\theta<0<\partial q_0^{cf}/\partial\theta$, $\partial(q_0^{cf}+q_1^{cf})/\partial\theta>0$. These results are the same with those in a simultaneous-move game.

The equilibrium payoffs and social welfare are, respectively:

$$V_0^{cf}=\frac{t^2(25+\theta)+(4+5\theta)(1+s)(1+s-2t)}{2(21-4\theta)},$$
$$\pi_1^{cf}=\frac{4t^2\left(129-57\theta+7\theta^2\right)+3(5-2\theta)^2(1+s)(1+s-2t)}{2(21-4\theta)^2},$$
$$W^{cf}=\frac{-\left(1+2s+s^2-2t-2st+13t^2\right)(21-4\theta)^2+44(2+s+t)(1+s-t)(21-4\theta)-1089(1+s-t)^2}{4(21-4\theta)^2} \qquad (7)$$

c. FP firm as a Stackelberg leader

In this case, firm 1 first chooses its output and abatement levels and then firm 0 makes its choice sequentially. Then, we have the following equilibrium quantities and abatement level:

$$q_0^{fp} = \frac{(1+s-t)(5+(2-\theta)\theta)}{(7-\theta)(3-\theta)}, \quad q_1^{fp} = \frac{(1+s-t)(2-\theta)}{7-\theta}, \quad a_0^{fp} = a_1^{fp} = t \tag{8}$$

where superscript 'fp' denotes the equilibrium outcome in the Stackelberg game with FP firm leadership. We note that if $0 \leq \theta < \frac{7-\sqrt{41}}{4}$ then $\partial q_1^{fp}/\partial t < \partial q_0^{fp}/\partial t < 0$, and $\partial q_1^{fp}/\partial s > \partial q_0^{fp}/\partial s > 0$; if $\frac{7-\sqrt{41}}{4} < \theta \leq 1$, we have $\partial q_0^{fp}/\partial t < \partial q_1^{fp}/\partial t < 0$, and $\partial q_0^{fp}/\partial s > \partial q_1^{fp}/\partial s > 0$. In both cases, $\partial q_1^{fp}/\partial \theta < 0 < \partial q_0^{fp}/\partial \theta$, $\partial(q_0^{fp} + q_1^{fp})/\partial \theta > 0$. An increase in the environmental tax rate leads to output reduction in both firms. However, when the concern for consumer surplus is relatively low the reduction is more significant in the FP firm; otherwise, the CF firm' reduction is larger. On the other hand, an increase in the output subsidy leads to output expansion in both firms. When the concern for consumer surplus is relatively low the expansion is larger in the FP firm; otherwise, the CF firm' expansion is larger. Finally, if the concern on consumer surplus rises, its direct effect is predictably positive on the CF firm's output, negative on firm 1's output, but positive on the aggregate output.

The equilibrium payoffs and social welfare are, respectively:

$$V_0^{fp} = \frac{t^2(172-59\theta-5\theta^2+2\theta^3)+(25+32\theta-22\theta^2+3\theta^3)(1+s)(1+s-2t)}{2(7-\theta)^2(3-\theta)},$$

$$\pi_1^{fp} = \frac{t^2(25-14\theta+2\theta^2)+(2-\theta)^2(1+s)(1+s-2t)}{2(7-\theta)(3-\theta)},$$

$$W^{fp} = \frac{\begin{pmatrix} -16(1+s^2+2s(1-t)-2t+4t^2)(3-\theta)^2(7-\theta)^2 \\ +4(183+139s-51t-64\theta-52s\theta+28t\theta)(1+s-t)(7-\theta)(3-\theta) \\ -12(587-382\theta+63\theta^2)(1+s-t)^2 \end{pmatrix}}{16(7-\theta)^2(3-\theta)^2} \tag{9}$$

IV. Endogenous timing game

We now discuss the equilibrium choice in an endogenous timing game. Each firm i $(i = 0,1)$ simultaneously chooses whether to move early ($t_i = 1$) or late ($t_i = 2$). If both firms choose the same period, the equilibrium is a simultaneous-move game. Otherwise, the equilibrium is a sequential move game. Table I provides the payoff matrix of the observable delay game.

Table I: Payoff Matrix of the Observable Delay Game

Firm 0/1	$t_1 = 1$	$t_1 = 2$
$t_0 = 1$	(V_0^c, π_1^c)	(V_0^{cf}, π_1^{cf})
$t_0 = 2$	(V_0^{fp}, π_1^{fp})	(V_0^c, π_1^c)

Using the revealed fact that the payoff of a firm when it is the leader is never smaller than its payoff in the simultaneous-move game, $V_0^{cf} \geq V_0^c$ and $\pi_1^{fp} \geq \pi_1^c$, we have that $(t_0, t_1) = (2,2)$ never constitutes an equilibrium unless $V_0^{cf} = V_0^c$ and $\pi_1^{fp} = \pi_1^c$. We can show that $V_0^{cf} = V_0^c$ and $\pi_1^{fp} = \pi_1^c$ never hold simultaneously.

Comparing V_0^c and V_0^{fp}, we obtain the following lemma.

Lemma 1 $V_0^c \geq V_0^{fp}$ *if* $0 \leq \theta \leq \theta_1 \approx 0.658$ *or* $\theta = 1$. *Otherwise,* $V_0^c < V_0^{fp}$.

Proof. $V_0^c - V_0^{fp} = \frac{(1+s-t)^2(2-\theta)(1-\theta)(82-165\theta+66\theta^2-7\theta^3)}{8(7-\theta)^2(4-\theta)^2(3-\theta)}$. If $0 \leq \theta < 1$ and $0 \leq t < 1+s$, the sign of $V_0^c - V_0^{fp}$ depends on the sign of $82 - 165\theta + 66\theta^2 - 7\theta^3$. Then, if $0 \leq \theta \leq \theta_1 \approx 0.658$, we have non-negative sign of this equation. If $\theta = 1$, then $V_0^c = V_0^{fp}$.

Comparing π_1^c and π_1^{cf}, we obtain the following lemma.

Lemma 2 $\pi_1^c \geq \pi_1^{cf}$ *if* $0 \leq \theta \leq \frac{2}{3}$. *Otherwise,* $\pi_1^c < \pi_1^{cf}$.

Proof. $\pi_1^c - \pi_1^{cf} = \frac{3(1+s-t)^2(2-3\theta)(27+55(1-\theta)+8\theta^2)}{8(4-\theta)^2(21-4\theta)^2}$ If $0 \leq \theta \leq 1$ and $0 \leq t < 1+s$, the sign of $\pi_1^c - \pi_1^{cf}$ depends on the sign of $2 - 3\theta$.

From Lemmas 1-2, we obtain the following main result.

Proposition 1 *In a mixed duopoly with a consumer-friendly firm, the equilibrium of endogenous* *timing game is as follows:*

(a) If $0 \leq \theta < \theta_1$, the only equilibrium of the game is the simultaneous movement, that is, $(t_0, t_1) = (1{,}1)$;

(b) If $\theta = \theta_1$, either the simultaneous movement, $(t_0, t_1) = (1{,}1)$ or the sequential-move outcome in which the FP firm is the leader, $(t_0, t_1) = (2{,}1)$, are equilibrium outcomes;

(c) If $\theta_1 < \theta < \frac{2}{3}$, the sequential-move outcome in which the FP firm is the leader, $(t_0, t_1) = (2{,}1)$, is the unique equilibrium outcome;

(d) If $\frac{2}{3} \leq \theta \leq 1$, either the CF firm or the FP firm could be the Stackelberg leader of the game, that is, $(t_0, t_1) = (1{,}2)$ and $(t_0, t_1) = (2{,}1)$, are the equilibrium outcomes.

This proposition implies that regardless of the policy mix of emission tax and output subsidy, the equilibrium of endogenous market structure is determined solely by the consumer-friendly firm's concern on consumer surplus. It is a surprising result that the equilibrium mode of competition remains unaffected by government tax/subsidy policy. Thus, it provides a new irrelevance result regarding the choice of government policy.[8]

V. Optimal policy mix

The government will maximize the welfare function corresponding to the market structure, which is determined exclusively by θ accordingly to proposition 1 as follows:[9]

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[8] Since White (1996) and Pal and White (1998), the literature of mixed market where a public firm coexists with private ones have established a series of so-called "irrelevance results," which states that privatization does not alter welfare regardless of time structure.

[9] In the cases where there are multiple equilibria ($\theta = \theta_1$ or $\frac{2}{3} \leq \theta \leq 1$), we assume that the government will set the subsidy and tax rates such as $(s^*, t^*) = argmax_{s,t} W(s, t; \theta) = \{(s, t) | W(s, t; \theta) = \max\{W^{fp}, W^j\}, s \geq 0, 0 \leq t < 1 + s, 0 \leq \theta \leq 1\}$, where $j = c$ or cf, accordingly.

$$W(s,t;\theta)=\begin{cases} W^c & if \quad 0\leq\theta<\theta_1 \\ max\{W^c,W^{fp}\} & if \quad \theta=\theta_1 \\ W^{fp} & if \quad \theta_1<\theta<\frac{2}{3} \\ max\{W^{fp},W^{cf}\} & if \quad \frac{2}{3}\leq\theta\leq 1 \end{cases} \tag{10}$$

The corresponding value of s^* and t^*:

$$s^*(\theta)=\begin{cases} \frac{3(4-4\theta-\theta^2)}{44+3\theta^2} & if \quad 0\leq\theta<\theta_1 \\ \frac{3(49-67\theta+\theta^2+11\theta^3-2\theta^4)}{667-384\theta+129\theta^2-42\theta^3+6\theta^4} & if \quad \theta_1\leq\theta\leq\frac{1}{5}(9-\sqrt{26}) \\ \frac{3(49-48\theta-8\theta^2)}{667+12\theta+24\theta^2} & if \quad \frac{1}{5}(9-\sqrt{26})<\theta\leq\frac{1}{4}(-12+11\sqrt{2}) \\ 0, & \frac{1}{4}(-12+11\sqrt{2})\theta\leq 1 \end{cases} \tag{11}$$

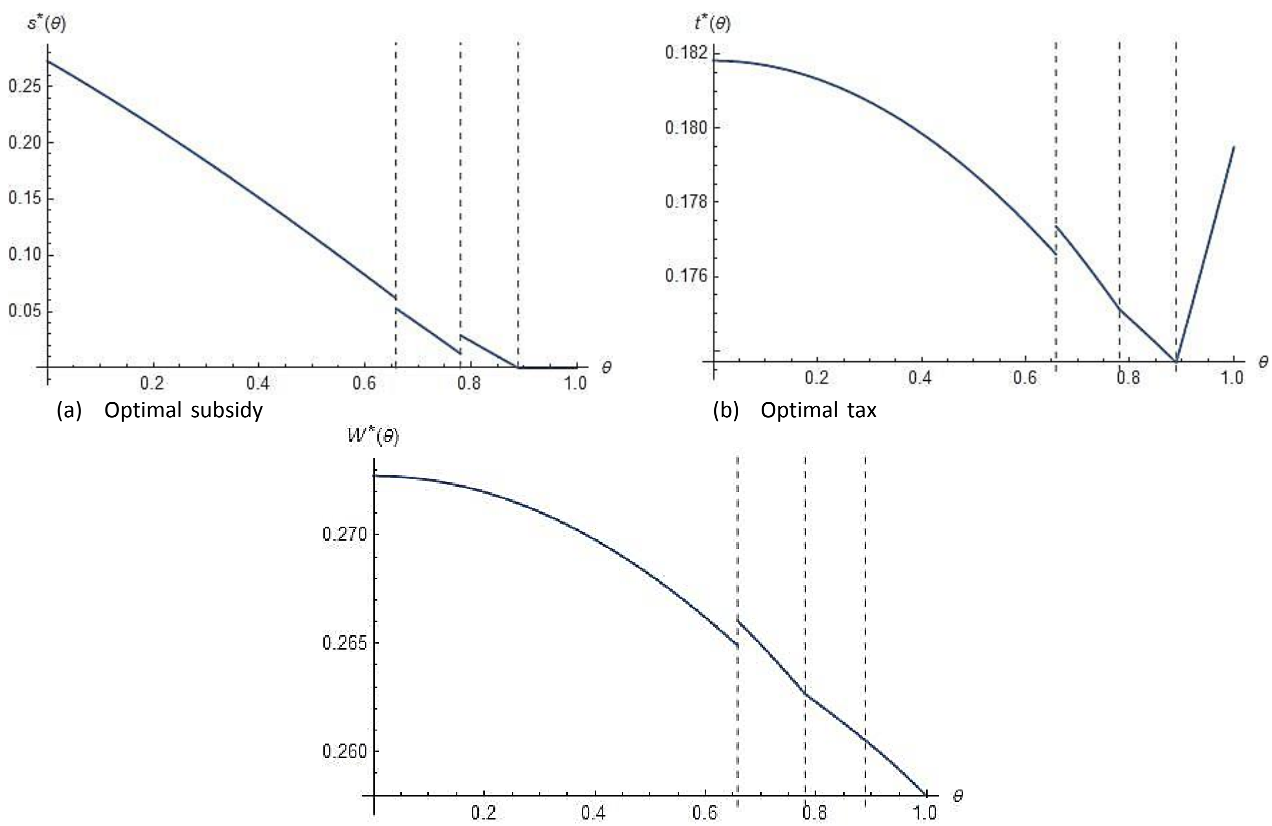
$$t^*(\theta)=\begin{cases} \frac{8}{44+3\theta^2} & if \quad 0\leq\theta<\theta_1 \\ \frac{(11-3\theta)^2}{667-384\theta+129\theta^2-42\theta^3+6\theta^4} & if \quad \theta_1\leq\theta\leq\frac{1}{5}(9-\sqrt{26}) \\ \frac{121}{667+12\theta+24\theta^2} & if \quad \frac{1}{5}(9-\sqrt{26})<\theta\leq\frac{1}{4}(-12+11\sqrt{2}) \\ \frac{534-40\theta+8\theta^2}{3873-1180\theta+104\theta^2}, & \frac{1}{4}(-12+11\sqrt{2})\theta\leq 1 \end{cases} \tag{12}$$

The CF firm produces more aggressively when the concern on consumer surplus θ, is sufficiently large, resulting in a reduction in the production of the FP firm. However the aggregate output increases. Thus, with a sufficiently large θ, the government has no incentive to stimulate the production of all firms, i.e., the optimal subsidy rate s becomes zero. Also, this expansion on the aggregate output implies larger pollution emissions, which makes the government raise the emission tax in order to mitigate the welfare loss due to environmental damage through the abatement efforts of the firms.

The resulting welfare after some simplifications:

$$W^{*}(\theta)=\begin{cases}\frac{12}{44+3\theta^{2}}, & 0\le\theta<\theta_{1}\\ \frac{3(11-3\theta)^{2}}{2(667-384\theta+129\theta^{2}-42\theta^{3}+6\theta^{4})}, & \theta_{1}\le\theta\le\frac{1}{5}\left(9-\sqrt{26}\right)\\ \frac{363}{2(667+12\theta+24\theta^{2})}, & \frac{1}{5}\left(9-\sqrt{26}\right)<\theta\le\frac{1}{4}\left(-12+11\sqrt{2}\right)\\ \frac{3\left(681-184\theta-16\theta^{2}\right)}{2(3873-1180\theta+104\theta^{2})}, & \frac{1}{4}\left(-12+11\sqrt{2}\right)\theta\le 1\end{cases}\tag{13}$$

Figure 1c shows that the corresponding welfare level is decreasing as θ increases. It is noteworthy that the first-best allocation can be achieved by the optimal policy mix with (11) and (12) only when the degree of concern on consumer surplus is zero and thus both firms have symmetric payoffs, which results in a simultaneous-move outcome.[10]



(a) Optimal subsidy

(b) Optimal tax

(c) Maximum welfare

Figure 1: Optimal subsidy, tax and social welfare

[10] The first-best allocations (denoted by 'B') where the output and abatement levels of the firms maximize the social welfare in (3) are as follows: $q_0^B = q_1^B = \frac{3}{11}$, $a_0^B = a_1^B = \frac{2}{11}$ and $W^B = \frac{3}{11}$

Proposition 2 *The first-best allocation can be attained by the optimal policy mix only when the concern on consumer surplus is abandoned, which results in a simultaneous-move outcome.*

This proposition implies that the emergence of a consumer-friendly public firm might not be socially beneficial even though the government uses an appropriate policy mix of emission tax and output subsidy before firms decide their roles in the endogenous timing game. Also, the sequential-move outcomes appear as equilibrium results when the concern on consumer surplus is not negligible, which causes welfare loss even under the optimal policy mix. These findings are consistent with the previous results in private and mixed duopolies. Under quantity setting game without government policy, firms decide simultaneously in a private duopoly with symmetric payoffs while firms decide sequentially in a mixed duopoly with asymmetric payoffs. (See Hamilton and Slutsky (1990), Pal (1998) and Tomaru and Kiyono (2010).) Further, under output subsidization Tomaru and Saito (2010) showed that only Cournot competition is an equilibrium outcome in a mixed duopoly with a higher subsidy rate.[11] Thus, we extend their analysis into the case with externality and show that the equilibrium does hold under the optimal emission tax.

VI. Concluding remarks

This paper aimed to provide a new irrelevance result. We considered a consumer-friendly public firm and analyzed an endogenous timing game when the government uses emission tax and output subsidy. We showed that regardless of the policy mix, the equilibrium of endogenous market structure is determined by the consumer-friendly public firm's concern on consumer surplus. Thus, the equilibrium mode of competition remains unaffected by government tax/subsidy policy. Finally, we showed that the first-best allocation for social welfare could be attained only when the concern on consumer surplus is abandoned, which results in a simultaneous-move outcome. Our results may have an important implication for economic models in mixed market settings. However, these

[11] Zikos (2007) examined an R&D subsidy while Lee and Xu (2017) considered an emission tax. Also, Lee and Tomaru (2017) analyzed the policy mix of output and R&D subsidies and showed that the first-best allocation could be attained irrespective of the asymmetric payoffs between the firms.

findings should further be treated in the context of differentiated products and different costs.

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Appendix I. The case where CF firm concerns about welfare

Consider the case where the payoff of the consumer friendly public firm is as follows:

$$V_0 = \pi_0 + \theta W \tag{I.1}$$

a. Fixed Timing Game

Simultaneous move game outputs and abatements:

$$q_0^c = \frac{s(2+2\theta-3\theta^2)+(1+\theta)(2+3\theta-2t(1-3\theta))}{8+23\theta+12\theta^2}$$

$$q_1^c = \frac{2+6\theta+3\theta^2+s(2+7\theta+5\theta^2)-t(2+9\theta+6\theta^2)}{8+23\theta+12\theta^2}$$

$$a_0^c = \frac{t(8-5\theta-6\theta^2)+\theta(4+3\theta+s(4+\theta))}{8+23\theta+12\theta^2}$$

$$a_1^c = t \tag{I.2}$$

CF firm as a Stackelberg leader outputs and abatements:

$$q_0^{cf} = \frac{6+15\theta+8\theta^2+s(6+9\theta-4\theta^2)-t(6-3\theta-10\theta^2)}{21+60\theta+32\theta^2}$$

$$q_1^{cf} = \frac{5+\theta(15+8\theta)+s(1+\theta)(5+12\theta)-t(5+21\theta+14\theta^2)}{21+4\theta(15+8\theta)}$$

$$a_0^{cf} = \frac{t(21-14\theta-18\theta^2)+\theta(11+8\theta+s(11+4\theta))}{21+60\theta+32\theta^2}$$

$$a_1^{cf} = t \tag{I.3}$$

FP firm as a Stackelberg leader outputs and abatements:

$$q_0^{fp} = \frac{\begin{array}{c}(1+\theta)\left(5+21\theta+25\theta^2+8\theta^3-t\left(5-3\theta-44\theta^2-28\theta^3\right)\right)\\ +s(5+18\theta+7\theta^2-25\theta^3-16\theta^4)\end{array}}{(1+2\theta)(21+78\theta+76\theta^2+22\theta^3)}$$

$$q_1^{fp} = \frac{(3+9\theta+5\theta^2)(2+6\theta+3\theta^2+s(2+7\theta+5\theta^2)-t(2+9\theta+6\theta^2))}{21+120\theta+232\theta^2+174\theta^3+44\theta^4}$$

$$a_0^{fp} = \frac{\begin{pmatrix}11(1+s)\theta+\theta^2\left(42+37s+43\theta+30s\theta+13\theta^2+7s\theta^2\right)\\ +t(21+46\theta-29\theta^2-73\theta^3-26\theta^4)\end{pmatrix}}{(1+2\theta)(21+78\theta+76\theta^2+22\theta^3)}$$

$$a_1^{fp} = \frac{t(21+122\theta+243\theta^2+189\theta^3+50\theta^4)-\theta(1+\theta)(2+6\theta+3\theta^2+s(2+7\theta+5\theta^2))}{21+120\theta+232\theta^2+174\theta^3+44\theta^4} \tag{I.4}$$

b. Endogenous timing game

Consider the payoff matrix in Table 1 to obtain the endogenous timing equilibrium. Note that the payoff of the firms when it is the leader is never smaller than its payoff in the simultaneous-move game: $V_0^c \leq V_0^{cf}$ and $\pi_1^c \leq \pi_1^f$.

Comparing V_0^c and V_0^{fp}, we obtain the following difference:

$$V_0^c - V_0^{fp} = \left(\frac{(1+\theta)(2(1+s-t)+(6+7s-9t)\theta+(3+5s-6t)\theta^2)}{2(1+2\theta)^2(8+23\theta+12\theta^2)^2(21+78\theta+76\theta^2+22\theta^3)^2}\right)\Gamma_0^V \quad \text{(I.5)}$$

where $\Gamma_0^V = 246(1+s-t) + 3(1258+1467s-1885t)\theta + (26339+35456s-59873t)\theta^2 + (107649 + 164146s - 337935t)\theta^3 + (279733 + 475485s - 1113250t)\theta^4 + 4(119095 + 223514s - 565567t)\theta^5 + 2(267467 + 552012s - 1455752t)\theta^6 + (391108 + 886777s - 2374489t)\theta^7 + (178871 + 445665s - 1188518t)\theta^8 + 8(5807 + 15907s - 41636t)\theta^9 + 16(327 + 985s - 2502t)\theta^{10}$.

Comparing π_1^c and π_1^{cf}, we obtain the following difference:

$$\pi_1^c - \pi_1^{cf} = \left(\frac{3(1+2\theta)(2(1+s-t)+(7+12s-22t)\theta+(4+18s-31t)\theta^2+4(2s-3t)\theta^3)}{2(8+23\theta+12\theta^2)^2(21+60\theta+32\theta^2)^2}\right)\Gamma_1^\pi \quad \text{(I.6)}$$

where $\Gamma_1^\pi = 82(1+s-t) + 37(13+14s-16t)\theta + (956+1136s-1385t)\theta^2 + 2(368+502s-611t)\theta^3 + 8(24+38s-45t)\theta^4$

The sign of both differences depends on s, t and θ, therefore the endogenous timing equilibrium also depends on s, t and θ.