Essential Interest-Bearing Money (2008)

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Abstract

I consider a model of intertemporal trade where agents lack commitment, agent types are private information, there is an absence of recordkeeping, and societal penalties are infeasible. Despite these frictions, I demonstrate that policy can be designed to implement the first-best allocation as a (stationary) competitive monetary equilibrium. The optimal policy requires a strictly positive interest rate with the aggregate interest expenditure financed in part by an inflation tax and in part by an incentive-compatible lump-sum fee. An illiquid bond is essential only in the event that paying interest on money is prohibitively costly.

1 Introduction

Kocherlakota (2003) describes an environment for which nominally risk-free bonds are essential in monetary economies. His argument is intriguing because it rationalizes the societal benefits of illiquid bonds. Put differently, and somewhat more provocatively, it demonstrates how a society can potentially be made better off by imposing, rather than removing a cash-in-advance constraint.

His environment contains a number of key frictions. First, individuals lack commitment and recordkeeping is absent; these properties are sufficient to ensure that money is essential; see Kocherlakota (1998). Second, agent types are private information; this has the effect of precluding type-contingent monetary transfers. And third, society cannot impose penalties on individuals (which precludes, among other things, the use of lump-sum taxes). The monetary equilibria that emerge in his environment are inefficient. The introduction of a liquid bond (a risk-free claim to money, usable for payments) can in no way improve efficiency; as liquid bonds are just another form of money. By restricting the liquidity of bonds, agents can engage in meaningful *ex post* money-bond

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trades; the effect of which is to direct liquidity to those who turn out to desire it most.

The initial motivation for my own paper was to generalize Kocherlakota's argument in the context of an environment where the societal benefits of an illiquid bond last for more than one period. Moreover, Kocherlakota only demonstrates how an illiquid bond can improve welfare; he does not characterize the optimal intervention. With this in mind, I chose to recast his model–maintaining all of the key frictions described above–in the context of a quasi-linear environment, similar to the one first introduced by Lagos and Wright (2005).

Proceeding in this manner, I discovered something interesting; namely, that I could not replicate his result. This was highly unexpected as our respective environments embed–as far as I can tell– the same critical frictions. Despite of these severe frictions, I demonstrate that a well-designed monetary policy can implement the first-best allocation making use of a single type of token; an illiquid bond is not essential.[1] What is essential in my model is that money earn interest and that at least some fraction of the aggregate interest expense be financed with a lump-sum "tax" that is paid voluntarily by agents. An illiquid bond is essential only if one rules out the possibility of paying interest on the economy's circulating medium.

To develop my argument, I proceed as follows. First, I describe a very simple quasi-linear environment along the lines of Andolfatto (2008) and characterize the first-best allocation.[2] I then employ the standard frictions that make money essential. I also assume that agent types are private information, which has the effect here of ruling out type-contingent transfers. I then characterize a (stationary) competitive monetary equilibrium under a given policy regime.

I begin by describing optimal policy under the assumption that lump-sum taxation is feasible. I also allow the government to pay interest on money. Not surprisingly, I find that the Friedman rule–paying zero interest on money and contracting the money supply via lump-sum taxation–is an optimal policy. Strictly speaking, however, the optimal policy is in a sense indeterminate. That is, the first-best allocation can also be implemented by setting a strictly positive interest rate in excess of the money growth rate; with the implied interest burden financed via lump-sum taxation. Hence, interest-bearing money can be part of an optimal policy; but it is not essential.

Next, I rule out lump-sum taxation. This restriction is one that is commonly employed in the literature. But it is important to understand exactly what is being ruled out here. The assumption is that *coercive* lump-sum taxation is infeasible; not that *voluntary* lump-sum payments (fees paid for some type of benefit) are infeasible. Implicitly, the literature assumes that lump-sum

[1] That the first-best is implementable is surely an artifact of my quasi-linear environment; but that constrained-efficient implementation requires no illiquid bond is likely robust.

[2] There is one critical difference that deserves mention. In Andolfatto (2008), I assume that society can punish individuals by banishing them to an autarkic state. In this paper, societal penalties of any form are entirely absent.

payments in any form are infeasible. If this is the case, I show that a constrained-efficient allocation can be implemented as an equilibrium with a constant money supply. Here, as before, it is possible to have implementation associated with a strictly positive interest rate; but once again, this is not essential.

The main point of my paper is then established by working off the insight described in the paragraph above. I demonstrate that if lump-sum taxation is ruled out, a policy can still be designed in a manner that implements the first-best allocation. The optimal policy requires a strictly positive interest rate with the aggregate interest expenditure financed in part by an inflation tax and in part by a lump-sum fee. Agents are free not to pay any fees; they must be compelled to do so voluntarily. I demonstrate that this can be accomplished by making the interest payment on money subject to a redemption charge (with the fee made contingent on the amount of money presented for redemption).

As for the societal benefit of introducing an illiquid bond, the conclusion appears to be valid in my environment only if one is willing to impose an additional extraneous restriction; namely, that paying interest on the economy's circulating medium is prohibitively costly. Even in this case, however, an optimal (incentive-feasible) policy requires the application of a lump-sum redemption fee (in this case on bonds, rather than money).

2 The Environment

The economy is populated by a continuum of *ex ante* identical agents, distributed uniformly on the unit interval. Each period $t = 0, 1, 2, ..., \infty$ is divided into two subperiods, labeled *day* and *night*. Agents meet at a central location in both subperiods; in particular, I abstract from the commonly employed assumption of random pairwise meetings in one of the subperiods.

All agents have common preferences and abilities during the day. Let $x_t(i) \in \mathbb{R}$ denote the consumption (production, if negative) of output in the day by agent i at date t. The key simplifying assumption is that preferences are linear in this term. The possibility of exchange then implies transferable utility. Output produced in the day is nonstorable, so an aggregate resource constraint implies:

$$\int x_t(i)di \leq 0; \tag{1}$$

for all $t \geq 0$.

At night, agents realize a shock that determines their type for the night. In particular, agents either have a desire to consume, an ability to produce, or neither. Refer to these types as *consumers*, *producers*, and *nonparticipants*, respectively. Types are determined randomly by an exogenous stochastic process. This process is *i.i.d.* across agents and time; there is no aggregate uncertainty. Let $\pi \in (0, 1/2)$ denote the measure of agents who become either consumers or producers; so that $(1 - 2\pi)$ denotes the measure of nonparticipants.

A consumer has utility $u(c)$ and a producer has utility $-g(y)$; where $c \in \mathbb{R}_+$ and $y \in \mathbb{R}_+$ denote consumption and production of the night good, respectively. Assume that $u'' < 0 < u'$, $\lim_{c \to 0} u'(c) = \infty$ and $g', g'' > 0$ with $\lim_{y \to 0} g'(y) = 0$. Nonparticipants neither value consume nor are they able to produce it; their utility is normalized to zero. As the night good is also nonstorable, there is another aggregate resource constraint given by:

$$\int c_t(i)di \leq \int y_t(i)di; \tag{2}$$

for all $t \geq 0$.

As agents are *ex ante* identical, their preferences can be represented as:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ x_t(i) + \pi \left[u(c_t(i)) - g(y_t(i)) \right] \right\}. \tag{3}$$

where $0 < \beta < 1$. Note that there is no discounting across subperiods.

Weighting all agents equally, a planner maximizes (3) subject to the resource constraints (1) and (2). As utility is linear in $x_t(i)$, agents are indifferent across any lottery over $\{x_t(i) : t \geq 0\}$ that delivers a given expected value. Without loss of generality, a planner may set $x_t(i) = 0$ for all i and all $t \geq 0$. Since g is strictly convex, all producers will be required to produce a common level of output $y \geq 0$. Given the strict concavity of u, all consumers will be allocated a common level of consumption $c \geq 0$. Given that the active population is divided equally among producers and consumers at night, the resource constraint (2) implies $c = y$. Hence, conditional on a given level of y (and invoking the fact that $Ex_t(i) = 0$), *ex ante* welfare is represented by:

$$W(y) = \left(\frac{\pi}{1-\beta} \right) \left[u(y) - g(y) \right]. \tag{4}$$

Clearly, there is a unique maximizer $0 < y^* < \infty$ satisfying:

$$u'(y^*) = g'(y^*). \tag{5}$$

In what follows, I refer to y^* as the *first-best* allocation. Associated with this allocation is any lottery over $x_t(i)$ that generates $E_t x_t(i) = 0$.

I impose the following additional restrictions on the environment. First, I assume that agents lack commitment and that private trading histories are unobservable. Together, these two frictions generate a role for fiat money. Second, I assume that agent types at night (whether consumer, producer, or neither) are unobservable. The role of this latter assumption is to rule out the possibility of type-dependent monetary transfers at night. Third, I assume that society cannot impose any penalties on individuals. This last assumption is critical; to highlight its role, I begin by first assuming that the contrary is true.

3 Individual Decision-Making

Let $z \geq 0$ denote the cash balances held by an agent at the end of the night. These balances are carried forward to the next day and transform into Rz units of money at the beginning of the day; where $R \geq 1$ denotes the (gross) nominal interest rate. For the moment, assume that all agents are obliged to pay a nominal lump-sum tax τ in the day (this represents a nominal transfer if τ is negative). Let (v_d, v_n) denote the values of money (the inverse of the price-levels) in the competitive spot markets prevailing in the day and night, respectively. I denote "next period" values with superscript "+" and "previous period" values with superscript "–".

3.1 The Day-Market

Let m denote money carried forward into the night-market. An agent's day-market budget constraint is given by:

$$x = v_d(Rz - \tau - m).$$

Let $N(m)$ denote the value of entering the night-market with m units of money. Assume, for the moment, that $N'' < 0 < N'$ (a conjecture that will prove to be true). Making use of the budget constraint above, the value of entering the day-market with money z can therefore be expressed as:

$$D(z) \equiv \max_m \left\{ v_d \left(Rz - \tau - m \right) + N(m) \right\}. \tag{6}$$

The demand for money during the day is characterized by:

$$v_d = N'(m). \tag{7}$$

As emphasized by Lagos and Wright (2005), the quasi-linear nature of preferences implies that money demand at this stage is independent of initial money balances and the lump-sum tax (transfer). Moreover, the value function $D(z)$ is linear in z; i.e.

$$D'(z) = Rv_d. \tag{8}$$

3.2 The Night-Market

3.2.1 Consumers

Let $C(m)$ denote the value associated with being a consumer, entering the night-market with money balance m. This money is used to make purchases of output y at the prevailing price-level v_n^{-1}; hence, future money balances must obey

$z^+ = m - v_n^{-1}y$, with the further restriction that $z^+ \geq 0$. Hence, the choice problem can be stated as:

$$C(m) \equiv \max_{y,,z^+} \left\{ u(y) + \beta D(z^+) : z^+ = \ m - v_n^{-1}y \geq 0 \right\}.$$

A well-known property of this solution is that for policy parameters away from those that implement the first-best allocation, the constraint $z^+ \geq 0$ will bind tightly.[3] In this case, the solution is given by:

$$y = v_n m; \tag{9}$$

so that the value function is given by:

$$C(m) \equiv u(v_n m) + \beta D(0). \tag{10}$$

By the envelope theorem:

$$C'(m) = v_n u'(y). \tag{11}$$

3.3 Nonparticipants

Let $I(m)$ denote the value associated with being "idle" (a nonparticipant), entering the night-market with money balance m. This type of agent faces no choice problem; i.e., $z^+ = m$. The associated value function is given by:

$$I(m) \equiv \beta D(m); \tag{12}$$

with

$$I'(m) = R\beta v_d^+; \tag{13}$$

where this latter result makes use of (8).

3.4 Producers

Let $P(m)$ denote the value associated with being a producer, entering the night-market with money balance m. If a producer makes sales of output y at the prevailing price-level v_n^{-1}, his future money balances are given by $z^+ = m + v_n^{-1}y$. Clearly, the constraint $z^+ \geq 0$ will not bind in this case and the choice problem may be formulated as:

$$P(m) \equiv \max_{y,z^+} \left\{ -g(y) + \beta D(z^+) : z^+ = \ m + v_n^{-1}y \right\}. \tag{14}$$

The supply of output is characterized by:

[3] The constraint will bind weakly when policy parameters are set to achieve the first-best.

$$v_n g'(y) = R\beta v_d^+; \tag{15}$$

where use has been made of (8). In addition, we have the envelope result:

$$P'(m) = R\beta v_d^+; \tag{16}$$

where again, use has been made of (8).

3.5 Gathering Restrictions

The *ex ante* value function associated with entering the night-market with money balances m is given by:

$$N(m) \equiv \pi C(m) + (1-2\pi)I(m) + \pi P(m). \tag{17}$$

Utilizing results (11), (16), and (13), we have:

$$N'(m) = \pi v_n u'(y) + (1-\pi)R\beta v_d^+. \tag{18}$$

Combining (18) with (7) and (15) implies:

$$v_d = \pi v_n u'(y) + (1-\pi)v_n g'(y).$$

Multiplying both sides of the expression above by $R\beta$ and updating one period results in the expression:

$$R\beta v_d^+ = R\beta v_n^+ \left[\pi u'(y^+) + (1-\pi)g'(y^+)\right].$$

Making reference once again to condition (15), the expression above can alternatively be stated as:

$$v_n g'(y) = R\beta v_n^+ \left[\pi u'(y^+) + (1-\pi)g'(y^+)\right]. \tag{19}$$

4 Equilibrium

In equilibrium, $m = M$, so that by condition (9), the night value of money satisfies $v_n = y/M$ and

$$\left(\frac{v_n^+}{v_n}\right) = \left(\frac{M}{M^+}\right)\left(\frac{y^+}{y}\right).$$

Assume that the money supply grows at a constant rate $\mu \geq \beta$, so that $M^+ = \mu M$. In what follows, I restrict attention to a steady-state in which $y = y^+$, so that $(v_n^+/v_n) = (1/\mu)$. Combining these restrictions with condition (19) allows us to characterize the (stationary) equilibrium level of output in the night-market $\hat{y}$ by:

$$g'(\hat{y}) = \left(\frac{R\beta}{\mu}\right)[\pi u'(\hat{y}) + (1-\pi)g'(\hat{y})] \tag{20}$$

The transactions that occur at night imply that, in equilibrium, consumers leave the night-market with $z^+ = 0$ money balances, producers leave with $z^+ = 2M$ money balances, and idle agents leave with $z^+ = M$ money balances. Hence, at the beginning of any arbitrary date, the distribution of money balances at the beginning of the day falls on a three-point set $z \in \{0, M^-, 2M^-\}$.

To complete the characterization of equilibrium, we need to ensure that the government policy parameters (μ, R) are consistent with government budget balance. At the beginning of the day, the money supply is given by $M^- = \pi(0) + (1-2\pi)M^- + \pi 2M^-$. The government creates new money $(\mu - 1)M^-$ and collects a lump-sum tax τ to finance interest payments $(R-1)M^-$. Hence, the government budget constraint is given by:

$$\tau = (R - \mu)M^-. \tag{21}$$

I am now in a position to characterize equilibrium activity in the day-market. Utilizing (15) and the fact that $\hat{y} = v_n^- M^-$, the equilibrium value of money in the day-market is given by:

$$v_d = g'(\hat{y})\left(\frac{\hat{y}}{R\beta M^-}\right). \tag{22}$$

The equilibrium level of output transacted in the day-market can now be deduced from (22), (21), and the budget constraint $x = v_d(Rz - \tau - m)$. In particular, it is easy to establish that producers use their excess money balances to purchase

$$\hat{x} = \beta^{-1} g'(\hat{y})\hat{y}$$

units of output in the day market. This output is supplied by consumers who desire to replenish their money balances (idle agents make no transactions in this market).[4] At the end of the day, the distribution of money holdings collapses over M.

4.1 Optimal Monetary Policy

Condition (20) reveals that the optimal monetary policy entails setting (μ, R) such that $R\beta = \mu$. From the government budget constraint (21), we see that an optimal policy requires lump-sum taxation; i.e., $\tau = (1-\beta)RM^- > 0$.

[4] Observe that from an *ex ante* perspective, agents are faced with a lottery over the day good. The expected value of this lottery satisfies:

$$\pi(-\hat{x}) + (1-2\pi)(0) + \pi(\hat{x}) = 0;$$

so that the risk induced by the sequence of spot markets entails no *ex ante* welfare loss.

It is frequently asserted that the optimal policy is characterized by the Friedman rule; i.e., $R = 1$ and $\mu = \beta$. But the analysis here demonstrates that this policy is not uniquely optimal. In particular, the first-best allocation can also be implemented by setting $R = \beta^{-1}$ and $\mu = 1$. In both cases, lump-sum taxation is necessary. For the Friedman rule, taxation is required to contract the money supply, which generates the requisite deflation. In the latter case, taxation is necessary to finance the interest paid on money. Both policy schemes work to implement the first-best allocation because both serve to align the marginal real return on money to the socially desirable rate.

Result 1 *When lump-sum taxation is feasible, a policy that sets $\tau^* = (1 - \beta)RM^-$ with any combination of $(\mu, R \geq 1)$ satisfying $R\beta = \mu$ implements the first-best allocation as a competitive monetary equilibrium. That is, interest-bearing money is not essential.*

One conclusion that follows here is that if lump-sum taxation is feasible, then paying interest on money is not essential–following the Friedman rule is sufficient. One way to bypass the Friedman rule as a prescription for optimal policy is to simply rule out lump-sum taxation; i.e., restrict policy so that $\tau \leq 0$, which from (21), implies $R \leq \mu$. When $R = 1$, the constrained-efficient monetary policy in this case is to set $\mu = 1$ (a constant money supply). By condition (20), the constrained-efficient level of output at night falls in the range $0 < y_C < y^*$.

Note that the analysis here suggests that the constrained-efficient level of output y_C can also be implemented with any policy in the form of $R = \mu \geq 1$. In other words, even when lump-sum taxation is ruled out, a constrained-efficient policy may involve a strictly positive interest rate. However, it appears that the original conclusion remains valid; i.e., paying interest on money is not essential (doing so does not alter the real marginal return on money).

In what follows, I wish to challenge the notion that interest-bearing money is not essential when taxation is infeasible. I claim that, not only is paying interest on money essential (whether directly, or indirectly via interest-bearing bonds), but that the first-best allocation is itself implementable.

5 Incentive Compatible Monetary Policy

Assume that society is unable to impose any direct penalties on individuals. That is, society cannot prevent people from trading; nor can it forcibly seize output or money. This restriction obviously rules out the use of coercive lump-sum taxation. However, it does not rule out the possibility that agents may voluntarily agree to pay a "tax" (better interpreted, in this case, as a fee for some type of service).

Monetary policy is designed as follows. First, the government agrees to pay interest on money $R \geq 1$. Second, the government stipulates that in order to receive interest on money, agents must display their money holdings to the government. Third, to be entitled to the interest payment, agents are asked to pay a lump-sum nominal fee f; this fee may be made conditional on the amount of money presented for redemption. Note that agents are free not to display their money or pay any fee; it is this sense in which "taxation" is voluntary.

I assume that the fee schedule takes the following form:

$$f(z) = \begin{cases} 0 & \text{if} \quad z > M^-; \\ \phi & \text{if} \quad z \in (0, M^-]; \\ 0 & \text{if} \quad z = 0; \end{cases} \tag{23}$$

where $\phi \geq 0$. Anticipating equilibrium behavior, this fee schedule implies that producers and consumers will be exempt from any redemption charge. Of course, as consumers return from the night-market with zero money balances, they would earn no interest and hence would not pay any fee even if it was applicable. If producers were subject to a redemption fee, they would have to evaluate the cost and benefit of incurring it. But as I have set the redemption charge on "large" money balances to zero, it follows that producers stand to gain from displaying their money.[5] Hence, the only "incentive-compatibility" condition that needs to be checked is whether it is indeed rational for idle agents (those with "small" money balances) to present their money for redemption.[6]

One benefit of the policy I have constructed here is that, if incentive-compatibility holds, then the relevant components of individual behavior are characterized exactly in the manner described earlier; in particular, condition (19) continues to hold.[7]

Let me now describe the way government finance works, anticipating some equilibrium behavior. First, a fraction π of the population will return to the day market with "large" money balances $2M^-$, which implies an interest expense of $(R-1)\pi 2M^-$. I assume that the government finances this interest expense entirely with new money creation $(\mu - 1)M^-$; so that

$$R = \left(\frac{\mu - 1}{2\pi} \right) + 1. \tag{24}$$

This clearly implies $R > \mu$ for all $\mu > 1$. In other words, this policy serves to increase the real marginal return to accumulating money balances for producers

[5] Note that the fee schedule (23) implies that the value function $D(z)$ is not differentiable at the point $z = M^-$. However, if the producer generates any positive output, the relevant value function $D(z^+)$ is differentiable over $z^+ \geq M$. We still need to check whether the producer might want to mimic the idle agent; i.e., he can do so by producing $y = 0$ so that $z^+ = M$. It is easy to verify that the producer will not want to do so.

[6] It has been pointed out to me that the fee schedule (23) may not be robust to a "coalitional deviation." In particular, idle agents may want to combine their money holdings and entrust one of the group to redeem a "large" pool of money so as to avoid the redemption penalty. Doing so, however, would require a degree of commitment; and would therefore be inconsistent with the properties of this environment.

[7] This would not be true if, for example, I considered a policy that conditioned the interest rate on individual money balances.

in the night-market (the resulting inflation tax is borne in part by other agents in the economy).

There is still the question of how the government is to finance the interest cost on "small" money balances. Here, I propose that the cost is met entirely by the lump-sum redemption fee ϕ; i.e.,

$$(1-2\pi)(R-1)M^{-} = (1-2\pi)\phi;$$

so that

$$\phi = (R-1)M^{-}. \tag{25}$$

With the fee designed in this manner, idle agents are just indifferent (in equilibrium) between displaying their money or not; so I assume that they do. In other words, the monetary policy described here is incentive-compatible.

For a given policy (μ, R), condition (20) describes an equilibrium $\hat{y}$. To implement the first-best allocation, we require $\mu = R\beta$. When lump-sum taxation is feasible, I noted above that the optimal interest rate policy is indeterminate (and that paying interest was inessential). In the present context, however, there is an additional restriction that needs to be satisfied; i.e., condition (24). Evidently, the optimal interest rate policy in this case is given by:

$$R^{*} = \left(\frac{1-2\pi}{\beta-2\pi}\right) > 1. \tag{26}$$

In what follows, I assume that $\beta > 2\pi$.

Result 2 *In the absence of societal penalties, the first-best allocation remains implementable under policy $(R^{*}, \mu^{*}, \phi^{*})$; where R^{*} given by (26), $R^{*} > \mu^{*} = \beta R^{*} > 1$, and $\phi^{*} = (R^{*}-1)M^{-} > 0$.*

One may be left wondering at this stage whether it is necessary to finance a component of the aggregate interest expense with a lump-sum charge. In particular, why not simply finance the entire interest expense by printing money? To address this question, assume that a fraction $0 \leq \lambda \leq 1$ of the interest expense in (25) is financed via new money creation; in which case,

$$(R-1)\left[2\pi + \lambda(1-2\pi)\right]M^{-} = (\mu-1)M^{-}.$$

Rearranging this expression and imposing $\mu = R\beta$ implies:

$$R = \left[\frac{1-2\pi-\lambda(1-2\pi)}{\beta-2\pi-\lambda(1-2\pi)}\right].$$

This expression corresponds to (26) when $\lambda = 0$. A look at the denominator reveals that this expression can only make sense under the parameter restriction:

$$\lambda < \left(\frac{\beta-2\pi}{1-2\pi}\right) < 1. \tag{27}$$

In other words, financing aggregate interest expenditures entirely by money creation ($\lambda = 1$) cannot implement the first-best allocation. That is, implementation requires a strictly positive lump-sum fee.

It is of some interest to point out, however, that financing some of the interest cost on "low" money balances is possible; i.e., by setting $\lambda > 0$ in a manner that does not violate the bound (27). The effect of such a policy would be to increase the rate of money creation and the nominal interest rate. At the same time, such a policy would ensure that $(R^* - 1)M^- > \phi^*$, so that individuals with "low" money balances would now strongly (rather than weakly) prefer to exercise redemption.

5.1 Discussion

When society is prevented from administering individual penalties, lump-sum taxation is precluded. When lump-sum taxation is precluded, the Friedman rule (setting the net nominal interest rate to zero and contracting the supply of money) is infeasible. It is widely presumed that when the Friedman rule is infeasible, the equilibrium allocation is constrained to be second-best. In the context of the environment laid out here, this second-best allocation can be achieved with a constant money supply.

There are two ways in which to view the nature of the inefficiency at the second-best. The first view highlights the fact that consumers are liquidity/debt constrained. Consumers (the high marginal utility types in Kocherlakota's model) would like to borrow if they could; but their anonymity precludes normal debt market operations. If types were observable, then policy might in principle rectify this situation by targeting money transfers to consumers in the night-market. But as types are private information, such a policy is infeasible; see also Levine (1991). In Kocherlakota (2003), an *ex post* asset market allows consumers to dispose of their (illiquid) bonds at discount for cash to meet their liquidity needs. The effect of this operation is to transfer purchasing power in the socially desirable direction; i.e., away from low marginal utility types and toward high marginal utility types. The former are rewarded in the future via the implied interest they earn on accumulated bond holdings.

The second view highlights the fact that the real rate of return on money is too low at the second-best. Hence producers (the low marginal utility types in Kocherlakota's model) are induced to undersupply output (undersave). One way to stimulate their saving is to increase the real return on money (the only vehicle for saving in this environment). In my setup, this is accomplished by paying interest on money and financing at least a part of the interest expense via lump-sum fees. In this manner, consumers do not need any additional liquidity. That is, the added production (saving) allows consumers to purchase more output with the money they already have.

These two views are, of course, related; they simply represent two sides of the same coin. The former view emphasizes the role of policy in helping consumers, while the latter view emphasizes the role of policy for inducing savers. In general equilibrium, this amounts to the same thing.

6 Money and Bonds

Consider an economy where society issues two types of fiat tokens; one of which is designed to circulate as money, and the other–called a bond–which constitutes a risk-free claim against the money token. Moreover, imagine that society purposely hampers the use of bonds as a means of payment; so that a cash-in-advance constraint is the product of policy, rather than any inherent property of the environment. Kocherlakota (2003) asks the following question: Under what circumstances does the policy of creating two fiat tokens and having one trade at a discount to the other constitute the optimal policy?

Such a policy is potentially beneficial only in circumstances where individuals have different intertemporal marginal rates of substitution in the equilibria of monetary economies. This condition is obviously not satisfied in the present environment if either: [1] lump-sum taxation is feasible; or [2] money can earn interest. Kocherlakota (2003) explicitly rules out [1] (there are no societal penalties) and implicitly rules out [2]. This latter restriction does not appear to follow from any property of his model's environment.[8]

If we rule out paying interest on the circulating medium (a policy that may be impractical in reality), then Kocherlakota's argument can be shown to hold in my environment as well. To see this, imagine that trade proceeds as before, with agents trading output for money in the sequence of spot markets. Money earns no interest. The only modification to the trading structure is the introduction of a "discount window," which is operated by the government just *subsequent* to night-market trading. At this window, money can be exchanged for bonds at the discount price $q \leq 1$. That is, a bond pays off q^{-1} units of money immediately the next day. Hence, bonds are purposely rendered illiquid in the sense that they can never be used to purchase output.[9]

Anticipating equilibrium behavior, it will be sufficient to issue bonds in two denominations; "large" and "small." The optimal policy will be to discount bonds at rate $q^* = (1/R^*)$; with $R^* > 1$ determined by (26). Moreover, an optimal policy will require expansion in the money supply at rate $\mu^* = \beta R^* > 1$ and a redemption fee $\phi^* = (R^* - 1)M^- > 0$ applied to small bonds only. At

[8] The same observation appears to apply to a related paper by Shi (2006).

[9] The reason that bonds must be made illiquid is as follows. First, an interest-bearing bond that circulates across time as a medium of exchange is ruled out by assumption (such a bond would constitute money, and I have ruled out paying interest on money).

However, it is possible to open the discount window just prior to night-market trading. If bonds can be used as payment in the night-market, then all agents would swap their money at this stage for interest-bearing bonds. The effect of this policy would simply be to expand the supply of money at the rate of interest; precluding implementation of the first-best allocation.

the conclusion of trades in the spot market at night, producers will swap their large money holdings for the large interest-bearing bond, and idle agents will swap their small money holdings for the small bond. Obviously, the role of the illiquid bond here is simply to replicate what could have been achieved by paying interest on money directly (if doing so was possible).

Result 3 *If the circulating medium cannot earn interest, then an optimal policy requires the introduction of an illiquid bond that trades at a strictly positive discount.*

7 Conclusion

My paper provides a rationale for why money should earn interest; or, for what (almost) amounts to the same thing, why risk-free claims to non-interest-bearing money should trade at discount. The rationale is as follows. In monetary economies, efficiency dictates that money earn a positive real rate of return. When lump-sum taxation is feasible, efficiency can be achieved by deflating at the Friedman rule. But when lump-sum taxation is infeasible, the Friedman rule is not incentive-compatible; i.e., agents are not willing to pay a "fee" if they derive no personal benefit (interest) from doing so. Hence, a strictly positive nominal interest rate is essential; with at least a part of the interest expense financed via a redemption fee designed to be incentive-compatible.

My results bear some relation to those of Berentsen, Camera, and Waller (2007). These authors examine a quasi-linear model similar to my own except that they introduce a financial intermediary with a limited recordkeeping technology. Fiat money is still valued in their model because the intermediary is prohibited from issuing its own notes (or it is forced to hold 100% cash reserves). In the context of my model, think of agents interacting with the intermediary prior to trading in the night-market, but just after realizing their shocks. The intermediary accepts cash deposits and issues cash loans at interest. Producers deposit their cash, which is redirected toward consumers in the form of cash loans. The interest-bearing deposits in their model play a role similar to the interest-bearing money in my model and the discounted illiquid bond in Kocherlakota (2003). However, what my analysis suggests is that the limited recordkeeping they introduce is redundant; implementation is possible without it.

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