An Estimated Dynamic Stochastic General Equilibrium Model of the Japanese Economy: A Bayesian Analysis

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Abstract

This paper estimates a dynamic stochastic general equilibrium (DSGE) model for the Japanese economy over 1970:Q1 through 1998:Q4, which is prior to the period of zero interest rate bound. More specifically, the New-Keynesian DSGE model with several frictions such as stickiness in price and wage, habit formation and adjustment cost in investment, developed by Christiano, Eichenbaum and Evans (CEE, 2005), is estimated using Bayesian inference via Markov-chain Monte Carlo (MCMC) simulation. The parameters and impulse response functions of nine shocks such as monetary policy shock and productivity shock are estimated to be quite consistent with those in the previous studies such as Onatski and Williams (2004) and Levin et al. (2005) for the U.S. and Smets and Wouters (2003) for the euro area. For example, the Japanese monetary authorities are found to have reacted very actively toward inflation. The only exception is investment, whose adjustment cost is estimated huge and whose shock is estimated to give long-lasting effects on output and consumption compared with those in the previous studies for the U.S. and euro area. Meanwhile, variance decomposition shows productivity shock and investment shock account for a large fraction of output fluctuation in long run in contrast to Smets and Wouters (2003).

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1 Introduction

After the publication of seminal work by Kydland and Prescott (1982), dynamic stochastic general equilibrium (DSGE) models have become popular in macroeconomics. One of the main advantages of DSGE models over macroeconometric models such as a vector autoregressive (VAR) model is that DSGE models can identify various shocks in a theoretically consistent way since they are structured from micro-foundation theories. Hence, DSGE models have attracted the attention of policy makers as well as macroeconomists.

DSGE models have been developed as New-Keynesian DSGE models with some market frictions. Among such New-Keynesian DSGE models, the most successful is the one proposed by Christiano, Eichenbaum and Evans (CEE, 2005). They introduce (1) Calvo-style nominal price and wage for expressing nominal rigidity, (2) habit formation in preference for consumption, (3) adjustment costs in investment and (4) variable capital utilization. As a result, their model has the ability to capture the time-series properties of macroeconomic data equivalent to a VAR model and can successfully explain inertia in inflation and persistence in output observed in the real world, which other macroeconomic models such as a real business cycle (RBC) model cannot explain.

This paper estimates the CEE (2005) model for the Japanese economy. The empirical applications of DSGE models have long been based on calibration without formal statistical methods. It is recent that formal statistical methods have become to be applied to DSGE models and such applications are still limited to the U.S. and euro economies. As far as we know, no studies have applied DSGE models to the Japanese economy using a formal statistical method. While CEE (2005) estimate their model using generalized methods of moments (GMM), we employ Bayesian inference via Markov chain Monte Carlo (MCMC) simulation following Smets and Wouters (2003), Onatski and Williams (2004) and Levin et al. (2005). This method samples the parameters from their posterior distribution and uses the sampled draws for parameter estimation. The method used for sampling from the posterior distribution is MCMC, where sampling is not random and depends on the draw obtained in the previous sampling. Specifically, the random walk Metropolis-Hastings (MH) algorithm, which is one of MCMC, is used for sampling the parameters in DSGE models. Bayesian inference via MCMC has some advantages over other methods such as GMM and maximum likelihood estimation. First, we can include prior information coming from microeconometric or macroeconometric studies into the prior distribution, which plays an important role for identifying shocks. Second, we can sample not only the parameters but also their functions such as impulse-response function from their posterior distribution. All we have to do is to substitute the sampled parameter values into those functions. It enables us to estimate impulse-response function taking the parameter uncertainty into consideration. Third, we may compare the DSGE models with non-nested models such as a VAR model using the posterior odds ratio, which is a usual tool for a Bayesian model comparison.

The data we use are the major seven macroeconomics quarterly series in Japan: real GDP, consumption, investment, labor input, real wage, inflation and nominal interest rate. As is well known, the zero interest rate bound started at 1999:Q1 in Japan. It is plausible that the macroeconomic behavior at the period of zero interest rate bound would be apart from the ordinary economic situation. Accordingly, the sample period is limited over 1970:Q1 through 1998:Q4, which is prior to the period of zero interest rate bound.

The parameter estimates and impulse response functions of nine real and nominal shocks such as monetary policy shock and productivity shock in the Japanese economy are estimated to be quite consistent with the previous studies such as Onatski and Williams (2004) and Levin et al. (2005) for the U.S. and Smets and Wouters (2003) for the euro area. For example, we find evidence that the Japanese monetary authorities reacted very actively toward inflation. Almost all shocks are estimated to give the reaction of all macroeconomic variables based on theoretical background. The only exception is investment, whose adjustment cost is estimated huge and whose shock is estimated to give long-lasting effects on output and consumption compared with those in the previous studies for the U.S. and euro area. On the other hand, variance decomposition shows that monetary policy shock do not influence the fluctuations of output and inflation in the long run, in contrast to Smets and Wouters (2003). Instead, productivity shock and investment shock account for a large fraction of all macroeconomic variables including output and inflation in the long run.

The remainder of this paper is organized as follows. Section 2 presents the CEE model and the derivation of the log-linearized model to be estimated. Section 3 explains the toolkit for estimating DSGE models such as the method for solving a linear rational expectations model proposed by Sims (2002), Kalman filter, Bayesian estimation and MCMC. Bayesian analysis of impulse response function and variance decomposition is also described. Section 4 describes the data used in the estimation. In Section 5, we present the estimation results of parameters, impulse response functions and variance decomposition. Section 6 concludes the paper.

2 The Model

2.1 The Household/Investor Sector

2.1.1 Preference and Budget Constraint

The household assumed in this paper follows that of Erceg, Henderson, and Levin (2000) (hereafter EHL) and CEE (2005). Each continuum of households are indexed by $h \in (0,1)$ and assumed to possess an identical preference toward consumption and leisure. In particular, each household seeks to maximize the following utility function;

$$E_0 \sum_{t=0}^{\infty} \beta^t \; u_t^c \; \left[\frac{(c_t(h) - H_t)^{1-\sigma_c}}{1-\sigma_c} - \frac{u_t^L \; l_t(h)^{1+\sigma_L}}{1+\sigma_L} \right], \tag{2.1}$$
$$\text{where} \quad H_t = \theta c_{t-1},$$

$c_t(h)$ stands for the real aggregate consumption of household h, H_t stands for the external habit formation which is exogenously given to household h, and $l_t(h)$ stands for the labor supplies of household h. Equation (2.1) also contains two persistent shocks: u_t^c denotes a preference shock and u_t^L denotes a shock to labor supply. An additional assumption that the external habit stock H_t is proportional to aggregate past consumption: $H_t = \theta c_{t-1}$, is introduced where θ denotes the habit persistence parameter. Each household h supplies a differentiated type of labor and, thus, decided to choose the amount of labor supply $l_t(h)$ monopolistically in the labor market. Parameter β stands for the discount rate, σ_c stands for the inverse of the long-run intertemporal elasticity of substitution, and σ_L stands for labor supply elasticity. Following EHL and CEE, we simply assume that $c_t(h)$ and $l_t(h)$ are additive-separable from each other. Now, the household h faces the following budget constraint for each period;

$$\begin{aligned} & B_t(h) + P_t \left[c_t(h) + inv_t(h) + a(u_t(h))K_t(h) \right] \\ & = R_{t-1}B_{t-1}(h) + W_t(h)l_t(h) + R_t^k \tilde{K}_t(h) + Div_t(h), \end{aligned} \tag{2.2}$$

where $B_t(h)$ stands for nominal bond holding by household h, P_t stands for the price index of real aggregate consumption goods which is common to all households, $inv_t(h)$ stands for physical investment by household h, R_{t-1} stands for the gross nominal interest rate from period $t-1$ to period t, $W_t(h)$ stands for the nominal wage rate uniquely associated to the household h's differentiated labor supply, and $Div_t(h)$ stands for the nominal dividend income from the firm that the household h owns. It should be noted that this dividend income is already maximized by the firm and therefore it is exogenous to the household's optimization problem.

Now, some detailed explanations are necessary for the variable related to the household's capital holdings. Here, in this model, the household not only act as a consumer/labor supplier, but also possess a characteristic of an investor. In other words, the household h lends out the capital, $K_t(h)$, to the firm and earns rental rate R_t^k from each effective capital, $\tilde{K}_t(h)$. Effective capital is defined as the product of actual capital holdings and capital utilization rate;

$$\tilde{K}_t(h) = u_t(h) K_t(h). \tag{2.3}$$

The household can increase the rental income by increasing the capital utilization rate. However, in so doing, the household need to pay the cost of capital utilization given by $a(u_t(h))K_t(h)$. Here, the utilization cost function $a(u_t(h))$ is assumed to be increasing and convex function – i.e., $a'(u) > 0$ and $a''(u) > 0$. Further, as in CEE, we assume the utilization cost to be zero when capital utilization is at the steady state – i.e., $a(u^{ss}) = 0$ where $u^{ss} = 1$.

In sum, LHS of the budget constraint (2.2) represents the total expenditure (bond investment, consumption expenditure, physical investment, and capital utilization cost) of household h at period t and RHS of the budget constraint represents the total revenue (bond carried over from period $t-1$, labor income, rental income, and dividend income) of the household.

Transforming the nominal budget constraint (2.2) into the real budget constraint, we obtain the following constraint,

$$\begin{aligned} & b_t(h) + c_t(h) + inv_t(h) + a(u_t(h))K_t(h) \\ & = \frac{R_{t-1}}{\Pi_t} b_{t-1}(h) + w_t(h) l_t(h) + r_t^k u_t(h) K_t(h) + div_t(h), \end{aligned} \tag{2.4}$$

where $b_t(h) = B_t(h)/P_t$ stands for real bond holdings, $\Pi_t = P_t/P_{t-1}$ stands for inflation rate from period $t-1$ to t, $w_t(h) = W_t(h)/P_t$ stands for real wage, $r_t^k = R_t^k/P_t$ stands for real rental rate, and $div_t(h) = Div_t(h)/P_t$ stands for real dividend.

2.1.2 Capital Accumulation and Capital Adjustment Cost

In addition to the budget constraint laid out above, following Smets and Wouters (2003) and Levin et al. (2005), the household accumulates the capital stock according to the following capital accumulation equation;

$$K_{t+1}(h) = (1-\delta)K_t(h) + \left[1 - S\Big(\frac{u_t^{inv}\ inv_t(h)}{inv_{t-1}(h)}\Big)\right] inv_t(h) \tag{2.5}$$

where δ stands for depreciation rate of capital and function $S(\cdot)$ stands for adjustment cost for capital defined as a quadratic function as follows.

$$S\left(\frac{inv_t(h)}{inv_{t-1}(h)}\right) = \frac{1}{\varphi}\frac{1}{2}\left(\frac{inv_t(h)}{inv_{t-1}(h)} - 1\right)^2. \tag{2.6}$$

As can be seen from the above specification, the bigger the deviation of the current physical investment from the previous period, the higher the adjustment cost. In other words, haphazard investment leads to a non-negligible "leakage" in capital installment and, therefore, it will be in the interest of the household to install the capital as smooth as possible to minimize the "leakage." The existence of this capital adjustment cost creates a motivation for the household to smooth out the physical investment over time. Also, it should be noted that adjustment cost to be zero in steady state – i.e., $S(1) = 0$ and $S'(1) = 0$. Further, notice that $\varphi = 1/S''(1)$ in steady state. Finally, a shock to the investment cost function $S(\cdot)$: u_t^{inv}, is contained in equation (2.5).

2.1.3 Euler Conditions of the Household/Investor Sector

From this point forward, assuming that each household h is facing the same initial condition and also assuming the complete state contingent commodity market, we omit the notation of h except for wage and labor supply. Given the budget constraint (2.4) and capital accumulation equation (2.5), the dynamic optimization problem for the household h can be formulated as follows.

$$\begin{aligned} L = E_0 \sum_{t=0}^{\infty} \beta^t u_t^c \Bigg\{ & \left[\frac{(c_t - H_t)^{1-\sigma_c}}{1-\sigma_c} - \frac{u_t^L \; l_t(h)^{1+\sigma_L}}{1+\sigma_L} \right] \\ & + \lambda_t \left[\frac{R_{t-1}}{\Pi_t} b_{t-1} + w_t(h) l_t(h) + r_t^k u_t K_t + div_t - b_t - c_t - inv_t - a(u_t) K_t \right] \\ & + q_t \left[(1-\tau) K_t + \left[1 - S\left(\frac{u_t^{inv} \; inv_t}{inv_{t-1}} \right) \right] inv_t - K_{t+1} \right] \Bigg\} \end{aligned} \tag{2.7}$$

where λ_t stands for the Lagrange multiplier attached to the budget constraint at period t and q_t[1] stands for the Lagrange multiplier attached to the capital accumulation equation. The household h seeks to maximize the utility over time by choosing the current consumption, bond holdings, magnitude of capital utilization, physical investment, and capital holdings. The decision regarding the amount of labor supply requires a special treatment due to the assumption of monopolistic competition in the labor market and will be analyzed separately.

The symmetric first order conditions associated with each control variable

[1] In CEE, SW, and LOWW, the Lagrange multiplier attached to capital accumulation constraint is defined as a product of shadow price of capital and shadow price of consumption goods (i.e., λ_t in our context). Thus, the Lagrange multiplier and shadow price of capital is strictly distinguished in their context. In our paper, for mechanical convenience, we continue to use Lagrange multiplier, q_t, without distinguishing it from shadow price of capital.

c_t, b_t, u_t, inv_t, and K_t will be as follows for any household h;

$$\text{consumption: } \lambda_t = (c_t - H_t)^{-\sigma_c} \tag{2.8}$$

$$\text{bond holdings: } \lambda_t = \beta E_t \left[\frac{R_t}{\Pi_{t+1}} \lambda_{t+1} \right] \tag{2.9}$$

$$\text{capital utilization: } r_t^k = a'(u_t) \tag{2.10}$$

$$\begin{aligned} \text{physical investment: } \lambda_t = {} & q_t \left[1 - S\left(\frac{u_t^{inv} \; inv_t}{inv_{t-1}} \right) - S'\left(\frac{u_t^{inv} \; inv_t}{inv_{t-1}} \right) \frac{u_t^{inv} \; inv_t}{inv_{t-1}} \right] \\ & + \beta E_t q_{t+1} \left[S'\left(\frac{u_{t+1}^{inv} \; inv_{t+1}}{inv_t} \right) \left(\frac{u_t^{inv} \; inv_{t+1}}{inv_t} \right) \left(\frac{inv_{t+1}}{inv_t} \right) \right] \end{aligned} \tag{2.11}$$

$$\text{capital holdings: } q_t = \beta E_t \left[q_{t+1}(1-\tau) + \lambda_{t+1} \left(r_{t+1}^k u_{t+1} - a(u_{t+1}) \right) \right] \tag{2.12}$$

The first order conditions regarding consumption and bond holdings are quite standard and, thus, we simply omit the explanation. Let us first turn to the first order condition associated with the capital utilization. As can be seen from equation (2.10), the optimality condition regarding capital utilization requires the household to equalize the marginal cost of capital utilization to the rental rate. By increasing capital utilization level marginally the household can increase the income by $r_t^k K_t$, which can be considered as a marginal benefit to the household. However, increase of the capital utilization comes with a cost. By increasing the capital utilization, on the margin, the household need to forgo $a'(u_t)K_t$ amount of consumption goods. Equalizing the marginal benefit associated to capital utilization with marginal cost yields the first order condition (2.10).

Next, turning to the first order condition associated with the physical investment, the LHS of equation (2.11) (i.e., λ_t) can be interpreted as the marginal cost of investment. By investing one additional consumption goods, the household need to forgo the same amount of consumption goods from his budget. Since the shadow price of consumption goods is λ_t, λ_t will be the marginal cost of investment. RHS of equation (2.11) basically represents the marginal benefit of investment. By additional one unit of investment the household can increase the amount of capital stock to some extent, but the magnitude of increase in capital stock is reduced due to the "leakage" in capital installment. This "leakage" of capital installment on the margin is represented by the term inside the first bracket in equation (2.11). In addition, due to the specification of the adjustment cost function (2.6), a marginal change of the current investment will also affect the next period's adjustment cost and this extra effect is represented by the term inside the second bracket in equation (2.11). Combining the "leakage" factor and "extra" effect and multiplying them by the shadow price of capital each will, roughly speaking, constitute the marginal benefit from additional investment. It should be noted that when the adjustment cost is constant, the first order condition associated with a physical investment will reduce to $\lambda_t = q_t(1 - \overline{S})$. Further, if there is no adjustment cost when forming the capital, the first order condition will trivially reduce to $\lambda_t = q_t$, which says that marginal cost of investment will be equal to the shadow price of capital.

Finally, equation (2.12) represents the first order condition associated with the capital holdings. Roughly speaking, shadow price of capital, q_t, on the LHS of equation (2.12) stands for the marginal cost of adding one unit of capital at period t. In return, the household can expect to increase $(1-\tau)$ unit of capital at next period and, thus, $\beta E_t q_{t+1}(1-\tau)$ can be considered as a present-valued marginal benefit of adding one unit of capital. However, it should be noted that there is also a side-effect from increasing capital. That is, by increasing the amount of capital, the household can expect to earn additional income via capital lending. This increase of income via capital lending is represented by $r^k_{t+1}u_{t+1}$ in equation (2.12). Of course, this additional capital lending comes with additional capital utilization cost and this is represented by $a(u_{t+1})$ in equation (2.12). Taking into these side-effect, the whole picture of the first order condition associated with the capital holding becomes to be equation (2.12).

2.1.4 Wage Setting and Labor Supply Behavior

In modeling household behavior in setting the wage, we basicaly follow EHL. Each continuum of household h is monopolistic supplier of differentiated labor, $l_t(h)$, and act as a wage setter in the labor market. Then, each differetiated labor supply is bundled into aggregate labor supply, l_t, accoriding to Dixit-Stiglitz type aggregator function.[2] By the same token, the wage levels that have been set by each household are also aggregated via Dixit-Stiglitz type aggregator function to yield the aggregate nominal wage, W_t.[3]

In contrast, the firm will act as a wage-taker in the labor market and the labor demand funtion for differentiated labor $l_t(h)$ is given as

$$l_t(h) = \left(\frac{w_t(h)}{W_t}\right)^{-(1+\lambda_w)/\lambda_w} L_t$$

where $w_t(h)$ is the wage of differentiated labor supplied by household h and

[2]Following EHL, the aggregate labor supply l_t is defined as below,

$$l_t = \left[\int_0^1 l_t(h)^{1/1+\lambda_w} dh\right]^{1+\lambda_w}.$$

[3]Again, following EHL, the aggregate nominal wage index W_t is defined as

$$W_t = \left[\int_0^1 w_t(h)^{-1/\lambda_w} dh\right]^{-\lambda_w}.$$

λ_w is the parameter governing the wage elasticity of labor demand. As it will be evident later, the parameter λ_w will be the wage markup over the marginal disutility of labor by household h. Taking this labor demand into account, the household h will monopolistically set the wage and also decide how much differentiated labor to supply.

In addition to the above monopolistic labor supply structure, following EHL, we introduce Calvo-Yun type sticky price environment for the wage setting problem. In particular, for any given period t, fraction of ξ_w of the entire households in the economy will not be able to revise their wage $w_t(h)$, while fraction of $(1-\xi_w)$ will have a chance to revise their wage. For any given chance to revise the wage $w_t(h)$, each household seeks to maximize the following objective funtion;

$$\max_{w_t(h)} E_t \sum_{i=0}^{\infty} \left[\beta^i \xi_w^i \left(\lambda_{t+i} \frac{w_t(h)}{p_{t+i}} l_{t+i}(w_t(h)) - \frac{l_{t+i}(w_t(h))^{1+\sigma_l}}{1+\sigma_l} \right) \right].$$

By supplying the labor, the household will expect to earn a real labor income of $w_t(h) \, l_{t+i}(h) / p_{t+i}$ for period $t+i$. Here, it should be noted that wage as of period t will stay at the same level during a spell of wage stickiness and cannot be revised until a household receive the next wage changing signal. A real labor income, whose unit is aggregate consumption goods, is converted to utility unit by multiplying marginal utility of real income – i.e., λ_{t+i}. Subtracting the labor disutility from "labor utility" yields the period-by-period net utility from labor. For any given chance to revise the wage, each household then maximize the expeted present value of the stream of net utility with respect to the nominal wage.

The FOC of the above problem will be as follows,[4]

$$E_t \sum_{i=o}^{\infty} \beta^i \xi_w^i \lambda_{t+i} L_{t+i} W_{t+i}^{(1+\lambda_w)/\lambda_w} \left[\frac{w_t^*}{p_{t+i}} - (1+\lambda_w) \frac{l_{t+i}(h)^{\sigma_l}}{\lambda_{t+i}} \right] = 0. \tag{2.13}$$

Further rearranging the above equation, we obtain the following relationship between the currrent real wage and the future stream of marginal rate of sub-

[4] As a special case of $\xi_w = 0$ where every household is able to revise their wage every period, it should be noted that the FOC (2.13) reduces to the standard intratemporal FOC without wage stickiness;

$$\frac{w_t^*}{p_t} = (1+\lambda_w) \frac{l_t(h)^{\sigma_l}}{\lambda_t}.$$

In other words, being able to set the wage each period, a household will set the real wage equal to markup over the marginal rate of substitution between labor supply and consumption.

stitution (MRS) between labor and consumption goods,

$$\frac{w_t^*}{p_t} = (1+\lambda_w) E_t \sum_{i=0}^{\infty} f_{t+i}^w \frac{p_{t+i}}{p_t} \frac{l_{t+i}(h)^{\sigma_l}}{\lambda_{t+i}} \tag{2.14}$$

$$\text{where } f_{t+i}^w \equiv \frac{\beta^i \xi_w^i L_{t+i} W_{t+i}^{(1+\lambda_w)/\lambda_w}}{E_t \sum_{i=0}^{\infty} \beta^i \xi_w^i L_{t+i} W_{t+i}^{(1+\lambda_w)/\lambda_w}}.$$

Thus, as can be seen from the above relationship, the optimal wage is set equal to the weighted average of future stream of MRS between labor and consumption goods marked up by the factor $(1+\lambda_w)$. Here, if the degree of wage stickiness is high (i.e., ξ_w is high), then the household will take into account the stream of MRS far into the future when setting the wage at period t. In contrast, if the degree of wage stickiness is low (i.e., ξ_w is low), then the household will be relatively shortsighted when setting the wage putting less emphasis on the future MRS. It is also useful to see the effect of inflation on the wage setting behavior. Suppose, in a partial equilibrium environment, a household is expecting a higher inflation in the future at period t. Then, as can be seen from equation (2.14), the future expected aggregate price index p_{t+i} will be higher that the household will put larger emphasis on the stream of MRS, ceteris paribus. Assuming that the effect of inflation do no affect the stream of MRS, aggregate labor demand, and aggregate wage index (which of course is not a plausible assumption in a general equilibrium setting), this will imply a higher wage at period t compared to the scenario where infaltion stays calm. Thus, higher expected inflation in the future will induce a household to set higher wage which, in turn, causes an inflation in the aggregate wage index.

For the sake of simplicity, we have so far, the above optimal wage setting rule was derived under the assumption that there is no wage indexation. Following CEE and SW and in order to model the persistence in nominal wage inflation, we introduce the partial indexation of nominal wage to price index. Under the scheme of partial wage indexation, the household who did not receive a "wage change signal" at period t will partially adjust their nominal wage taking into account the past inflation rate. Specifically, the partial indexation rule takes the following form,

$$\text{wage indexation to inflation: } \tilde{w}_t = \Pi_{t-1}^{\gamma_w} \tilde{w}_{t-1}, \tag{2.15}$$

where parameter γ_w controls the magnitude[5] of indexation to the past inflation. Under the wage indexation, the optimal wage setting rule will be modified as

[5]When $\gamma_w = 1$, the household who did not receive a "wage change signal" at period t will index their nominal wage to past inflation as in CEE. In contrast, when $\gamma_w = 0$, there is no wage indexation to past inflation and so the household who did not receive a signal will withold to the nominal wage set previously.

following,

$$E_t \sum_{i=o}^{\infty} \beta^i \xi_w^i \lambda_{t+i} L_{t+i} W_{t+i}^{(1+\lambda_w)/\lambda_w} \left[\frac{\tilde{w}_t}{p_{t+i}} \left(\frac{p_{t-1+i}}{p_{t-1}} \right)^{\gamma_w} - (1+\lambda_w) \frac{l_{t+i}(h)^{\sigma_l}}{\lambda_{t+i}} \right] = 0, \quad (2.16)$$

where $\tilde{w}_t$ stands for the optimal wage set by the household at period t and this wage will be automatically adjusted next period according to the wage indexation formula specified in equation (2.15).

Finally, from the definition of the aggregate wage index, the law of motion of the aggregate wage can be expressed as follows,

$$W_t^{-1/\lambda_w} = \xi_w \left[W_t \left(\frac{p_{t-1}}{p_{t-2}} \right)^{\gamma_w} \right]^{-1/\lambda_w} + (1-\xi_w) \tilde{w}_t^{-1/\lambda_w}.$$

2.2 The Firm Sector

2.2.1 Production Technology and Cost Minimization

In modeling the firms behavior, we basically follow Calvo (1983) and Yun (1996) type treatment. There are n monopolistically competitive firms each producing and selling disaggregated good, $y_{j,t}$, in the intermediate goods market. Following CEE, the production function for each monopolistic firms is identically defined as,

$$\text{production function: } y_{j,t} = u_t^a \, \tilde{K}_{j,t}^{\alpha} \, l_{j,t}^{1-\alpha} - \Phi \quad (2.17)$$

where u_t^a is the economy-wide technology shock affecting the productivity of all firms, $\tilde{K}_{j,t}$ stands for the borrowing of effective capital by the firm j, $l_{j,t}$ stands for the aggregate labor force employed by the firm j at period t, and Φ stands for the fixed cost. Notice that due to the existence of fixed cost inside the production function (2.17), a firm's production technology is no longer constant return-to-scale, but it will be increasing return-to-scale technology.[6]

Now by the assumption of perfectly competitive rental market for capital and since a firm behaves to be a price-taker in the labor market, the firm j takes the rental price r_t^k and real wage index w_t as given. Provided the target output level $y_{j,t}$, the cost minimization problem for the firm j can be expressed as follows.

$$\text{cost function: } \min_{\tilde{K}_{j,t}, l_{j,t}} \; w_t l_{j,t} + r_t^k \tilde{K}_{j,t} + mc_{j,t} \left(y_{j,t} - u_t^a \, \tilde{K}_{j,t}^{\alpha} \, l_{j,t}^{1-\alpha} + \Phi \right), \quad (2.18)$$

[6]As for another type of IRS specification in the DSGE literature, Tsuruga (2005) proposed to use the dynamic externality in production technology. In particular, without assuming the inflation indexation such as in CEE, Tsuruga (2005) showed that the impulse response function of inflation can be humped-shaped when there is a dynamic externality in the production technology.

where Lagrange multiplier, $mc_{j,t}$, has an interpretation of the marginal cost of producing $y_{j,t}$. Solving the above cost minimization, the first order condition becomes

$$\frac{w_t}{r_t^k} = \frac{1-\alpha}{\alpha}\frac{\tilde{K}_{j,t}}{l_{j,t}} \tag{2.19}$$

where LHS of equation (2.19) stands for the opportunity cost between the capital input and labor input and RHS stands for the marginal rate of technical substitution between two factors. Further, the firm j's marginal cost can be expressed as

$$\text{marginal cost: } mc_{j,t} = \frac{1}{u_t^a}\left(\alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}\right) w_t^{1-\alpha}(r_t^k)^{\alpha}. \tag{2.20}$$

As can be seen from equation (2.20), the specification of marginal cost does not depend on subscript j which implies that the marginal cost is symmetric across firms. This is because of the identical specification of the production function and price-taking behavior of firms in the capital market and aggregate labor market. Since marginal cost is symmetric across firms, we simply suppress subscript j from this point forward.

2.2.2 Optimal Pricing Rule under Sticky Price

Here, we investigate the optimal setting behavior of the firm j who behaves monopolistically in the intermediate goods market for $y_{j,t}$. Before investigating the optimal pricing rule, we need to specify the demand function for intermediate goods $y_{j,t}$. Following the literature, we specify the intermediate good demand function to be a standard one as follows.[7]

$$\text{demand function for } y_{j,t}\text{: } y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-(1+\lambda_p)/\lambda_p} y_t \tag{2.21}$$

where y_t stands for final goods,[8] P_t stands for aggregate price index of final goods y_t,[9] and λ_p is a parameter governing the price elasticity of demand and stands for the firm's markup over the marginal cost. Under Calvo (1983) - Yun (1996) type sticky price setting, for any given period t, fraction ξ_p of the

[7] For the derivation of the intermediate goods demand function, see for instance Woodford (2003).

[8] Following the literature, the final good y_t is produced using the intermediate goods $y_{j,t}$ and is defined as follows.

$$y_t = \left[\int_0^1 y_{j,t}^{1/(1+\lambda_p)} dj\right]^{1+\lambda_p}.$$

For more details, see Woodford (2003).

[9] Again, following the literature, the aggregate price index P_t is defined as

$$P_t = \left[\int_0^1 P_{j,t}^{-1/\lambda_p} dj\right]^{-\lambda_p}.$$

For more details, see Woodford (2003).

entire firms in the economy will not be able to revise their price $p_{j,t}$, whereas fraction $(1-\xi_p)$ will have a chance to revise their price. Now, for any given chance to revise the price $p_{j,t}$, the individual firm is faced with the following profit maximization problem.

$$\text{profit function: } \max_{P_{j,t}} E_t \sum_{i=0}^{\infty} \beta^i \, \xi_p^i \left[\left(\frac{P_{j,t}}{P_{t+i}} \right)^{-1/\lambda_p} - mc_{t+i} \left(\frac{P_{j,t}}{P_{t+i}} \right)^{-(1+\lambda_p)/\lambda_p} \right] y_{t+i}. \tag{2.22}$$

The first order condition for the above profit maximization problem yields

$$E_t \sum_{i=0}^{\infty} \beta^i \, \xi_p^i \, y_{j,t+i} \left[\frac{P_{j,t}}{P_{t+i}} - (1+\lambda_p) \, mc_{t+i} \right] = 0. \tag{2.23}$$

Rearranging further yields the following optimal pricing rule for firm j.

$$\frac{P_{j,t}}{P_t} = (1+\lambda_p) E_t \sum_{i=0}^{\infty} f_{t+i} mc_{t+i} \tag{2.24}$$

$$\text{where } f_{t+i} = \frac{\beta^i \xi_p^i (P_{t+i}/P_t)^{(1+\lambda_p)/\lambda_p} y_{j,t+i}}{\sum_{i=0}^{\infty} \beta^i \xi_p^i (P_{t+i}/P_t)^{1/\lambda_p} y_{j,t+i}}.$$

Thus, as can be seen from equation (2.24) the firm will set their price equal to the weighted average of the stream of future marginal costs marked up by the factor $(1+\lambda_p)$. Notice that, in the case of flexible price setting, the firm will set the price over the *current* marginal cost marked up by the factor $(1+\lambda_p)$, whereas, in the case of sticky price setting such as here, the firm who has a chance to revise their price at period t will set the price taking into account the *current and future* stream of expected marginal costs. If the degree of price stickiness is high (i.e., ξ_p is high), then the firm will take into account the future marginal costs far into the future when setting the price. On the other hand, if the degree of price stickiness is low (i.e., ξ_p is low), then the firm will be relatively shortsighted when considering the future marginal costs. As for the extreme case, when all the firms have a chance to revise their prices every period (i.e., $\xi_p = 0$), the pricing rule will reduce to flexible equilibrium pricing rule.

Now, in order to keep the exposition simple, the above pricing rule was derived under the assumption that firms that did not receive the "price-change signal" to keep their price unchanged from last period. In CEE and SW, in order to model the inflation persistence, they introduce, albeit in an ad-hoc way, a partial indexation to inflation. In other words, firms that did not have a chance to reoptimize their price will partially index their price to lagged inflation as follows.

$$\text{price indexation to inflation: } \tilde{p}_{j,t} = \left(\frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_p} \tilde{p}_{j,t-1}, \tag{2.25}$$

where parameter γ_p controls the magnitude of indexation[10] to the past inflation. Under this price indexation, the optimal pricing rule will be modified as follows.

$$E_t \sum_{i=0}^{\infty} \beta^i \, \xi_p^i \, y_{j,t+i} \left[\frac{\tilde{p}_{j,t}}{P_{t+i}} \left(\frac{P_{t-1+i}}{P_{t-1}} \right)^{\gamma_p} - (1+\lambda_p) \, mc_{t+i} \right] = 0, \tag{2.26}$$

where $\tilde{p}_{j,t}$ stands for the optimal price chosen by the optimizing firm at period t. It should be noted that this price $\tilde{p}_{j,t}$ will be automatically adjusted next period according to the indexation specified in equation (2.25), even if the firm does not receive a "price-changing signal". Taking a close look at equation (2.26) and comparing it with the pricing rule without indexation, we notice the presence of modifying term, $(P_{t-1+i}/P_{t-1})^{\gamma_p}$. Assuming the trend of positive inflation, the presence of this modifying term will render the optimal price $\tilde{p}_{j,t}$ to be lower compared to the case where there is no price indexation as in equation (2.23) – i.e., $\tilde{p}_{j,t} < P_{j,t}$. Thanks to the automatic price adjustment mechanism even for a period without a "price-changing signal," a firm is protected from a loss caused by an inflation and, thus, does not need to charge an "inflation premium" when setting a price at period t. In contrast, if there is no automatic price adjustment mechanism as in Yun (1996), then a firm need to take into account for the risk of future inflation and, therefore, need to charge "inflation premium" when setting the price at period t. This is the reason why the optimal price with inflation index will be lower than the case without inflation index.[11]

Finally, from the definition of the aggregate price index, the law of motion of the aggregate price index can be shown to be as follows.

$$P_t^{-1/\lambda_p} = \xi_p \left[P_{t-1} \left(\frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_p} \right]^{-1/\lambda_p} + (1-\xi_p) \tilde{p}_{j,t}^{-1/\lambda_p}. \tag{2.27}$$

2.3 Market Clearing Condition

We impose the market clearing condition for the final goods market. We require the supply of final goods to be equal to the demand of final goods for consumption, investment, capital utilization, and government expenditure. Thus, the market clearing condition can be expressed as follows.

$$y_t = c_t + inv_t + a(u_t) K_{t-1} + g_t. \tag{2.28}$$

[10]When $\gamma_p = 1$, firms that do not reoptimize will simply index their price to past inflation. This specification was adopted by CEE. Notice that when $\gamma_p = 0$, there is no indexation to past inflation and, therefore, firms that do not reoptimize will simply set the price equal to past price as in Yun (1996).

[11]Again, this argument assumes the trend of positive inflation. For the economy where deflation is prevailing, the argument needs to be reversed, discussing the issue of "deflation discount" – i.e., $\tilde{p}_{j,t} > P_{j,t}$.

2.4 Log-Linearization of the Model

For the sake of Bayesian estimation, which requires the model to be in the linear state-space form, we log-linearize the above model around the steady state. Without laboring on the derivation of the steady states and the log-linearization, we simply state the results from Smets and Wouters (2003) and Onatski and Williams (2004) here. The hat above a variable denotes log derivation from steady state: i.e. $\hat{x} = \ln x - \ln x^{ss}$ where x^{ss} is steady state.

2.4.1 Equilibrium Conditions from Housing/Investor Sector

(1) Consumption Euler equation:

$$\hat{c}_t = \frac{\theta}{1+\theta}\hat{c}_{t-1} + \frac{1}{1+\theta}E_t\hat{c}_{t+1} - \frac{1-\theta}{(1+\theta)\sigma_c}(\hat{R}_t - E_t\hat{\Pi}_{t+1}) + \frac{1-\theta}{(1+\theta)\sigma_c}(u_t^c - E_t u_{t+1}^c), \quad (2.29)$$

When $h = 0$, equation (2.29) reduces to the traditional forward-looking consumption equation. With external habit formation, consumption depends on a weighted average of past and expected future consumption. Note that in this case the interest elasticity of consumption depends not only on the intertemporal elasticity of substitution (σ_c), but also on habit persistence parameter. A high degree of habit persistence will tend to reduce the impact of the real rate on consumption for a given elasticity of substitution. A persistent shock u_t^c is AR(1) process with coefficient ρ^c, and, therefore, the expected value $E_t u_{t+1}^c$ can be rewritten as $\rho^c u_t^c$.

(2) Investment Euler equation

$$\widehat{inv}_t = \frac{1}{1+\beta}\widehat{inv}_{t-1} + \frac{\beta}{1+\beta}E_t\widehat{inv}_{t+1} + \frac{\varphi}{1+\beta}\hat{q}_t + \frac{\beta}{1+\beta}(E_t u_{t+1}^{inv} - u_t^{inv}) \quad (2.30)$$

where we set $\varphi = 1/\bar{S}''$ and the inverse, $1/\varphi$, implies the elasticity of investment on the price of capital. Modeling the capital adjustment cost as a function of the change in investment rather than its level introduces additional dynamics in the investment equation, which is useful in capturing the hump-shaped response of investment to various shocks including monetary policy shocks. A positive shock to the adjustment cost function, u_t^{inv}, temporarily reduces investment. The expected value $E_t u_{t+1}^{inv}$ is set as $\rho^{inv} u_t^{inv}$ using AR(1) coefficient ρ^{inv}.

(3) Asset Pricing Euler equation:

$$\hat{q}_t = -(\hat{R}_t - E_t\hat{\Pi}_{t+1}) + \frac{1-\tau}{1-\tau+\overline{r}^k}E_t\hat{q}_{t+1} + \frac{\overline{r}^k}{1-\tau+\overline{r}^k}E_t\hat{r}_{t+1}^k + \varepsilon_t^q \quad (2.31)$$

where $\beta = 1/(1-\tau-\bar{r}_k)$, τ is the depreciation rate, and $\bar{r}_k$ is steady-state rental rate. The current value of the capital stock, q_t, depends negatively on the ex ante real interest rate, and positively on its expected future value and the expected rental rate. The introduction of a white noise shock to the required rate of return on equity investment, ε_t^q, is meant as a shortcut to capture changes in the cost of capital that may be due to stochastic variations in the external finance premium.

(4) Real wage law of motion:

$$\begin{aligned}\hat{w}_t = & \frac{\beta}{1+\beta}E_t\hat{w}_{t+1} + \frac{1}{1+\beta}\hat{w}_{t-1} + \frac{\beta}{1+\beta}E_t\hat{\Pi}_{t+1} - \frac{1+\beta\gamma_w}{1+\beta}\hat{\Pi}_t + \frac{\gamma_w}{1+\beta}\hat{\Pi}_{t-1} \\ & - \frac{\lambda_w(1-\beta\xi_w)(1-\xi_w)}{(1+\beta)(\lambda_w+(1+\lambda_w)\sigma_L)\xi_w}\left[\hat{w}_t - \sigma_L\hat{l}_t - \frac{\sigma_c}{1-\theta}(\hat{c}_t - \theta\hat{c}_{t-1}) - u_t^L - \varepsilon_t^w\right]\end{aligned} \tag{2.32}$$

The real wage, $\hat{w}_t$, is a function of expected and past real wages and the expected, current, and past inflation rate where the relative weight depends on the degree of indexation of the nonoptimized wages, γ_w. When $\gamma_w = 0$, real wages do not depend on the lagged inflation rate. The last term implies a negative effect of the deviation of the actual real wage from the wage that would prevail in a flexible labor market. The size of this effect will be greater, the smaller the degree of wage rigidity, the lower the demand elasticity for labor and the lower the inverse elasticity of labor supply, σ_L. The shock to labor supply, u_t^L, follows the AR(1) process, while the shock to real wage, ε_t^w, is assumed to obey i.i.d-normal.

(5) Capital Accumulation equation:

$$\hat{K}_t = (1-\tau)\hat{K}_{t-1} + \tau\widehat{inv}_{t-1} \tag{2.33}$$

Capital, K_t, is decreased by the depreciation of capital and increased by investment. Note that τ is a double meaning: the depreciation rate of capital and the ratio of investment to capital so that the former are used in the first term of RHS and the latter is used in the second term.

2.4.2 Equilibrium Conditions from Firm Sector

(6) Cost minimization condition:

$$\hat{l}_t = -\hat{w}_t + (1+\psi)\hat{r}_t^k + \hat{K}_{t-1} \tag{2.34}$$

where $\psi = \psi'(1)/\psi''(1)$ is the inverse of elasticity of the capital utilization cost function. For a given installed capital stock, labor demand depends negatively on the real wage, $\hat{w}_t$, and positively on the rental rate of capital, $\hat{r}_t^k$.

(7) Production Function:

$$\hat{y}_t = \phi \hat{u}_t^a + \phi\alpha\hat{K}_{t-1} + \phi\alpha\psi\hat{r}_t^k + \phi(1-\alpha)\hat{l}_t \tag{2.35}$$

where ϕ is one plus the share of the fixed cost in production, and $\hat{u}_t^a$ is productivity shock. This equation is derived from the production function (2.17).

(8) Inflation law of motion:

$$\hat{\Pi}_t = \frac{\beta}{1+\beta\gamma_p}E_t\hat{\Pi}_{t+1} + \frac{\gamma_p}{1+\beta\gamma_p}\hat{\Pi}_{t-1} + \frac{(1-\beta\xi_p)(1-\xi_p)}{(1+\beta\gamma_p)\xi_p}\left[\alpha\hat{r}_t^k + (1-\alpha)\hat{w}_t - u_t^a + \varepsilon_t^p\right] \tag{2.36}$$

Inflation, $\hat{\Pi}_t$, depends on past and expected future inflation and the current marginal cost, which itself is a function of the rental rate on capital $\hat{r}_t$, the real wage $\hat{w}_t$, and productivity shock u_t^a. When $\gamma_p = 0$, this equation reduces to the standard purely forward-looking Phillips curve. In the other words, the degree of indexation, γ_p, determines how backward looking the inflation process is. The elasticity of inflation with respect to changes in the marginal cost, $\left[\alpha\hat{r}_t^k + (1-\alpha)\hat{w}_t - u_t^a + \varepsilon_t^p\right]$, depends mainly on the degree of price stickiness. When all prices are flexible ($\xi_p = 0$) and the i.i.d-normal price-markup shock, ε_t^p, is zero, this equation reduces to the normal condition that in a flexible price economy the real marginal cost should equal one.

(9) Employment equation:

$$\hat{e}_t = \beta\hat{e}_{t+1} + \frac{(1-\beta\xi_e)(1-\xi_e)}{\xi_e}(\hat{l}_t - \hat{e}_t) \tag{2.37}$$

where e_t is employment, l_t is labor input, and ξ_e is a constant probability at which firms are able to adjust employment to its desired total labor input. This equation reflects the fact that the employment is likely to respond more slowly than the labor input. Smets and Wouters (2003) and Onatski and Williams (2004) transformed labor input, l_t, from employment,e_t, using equation (2.37) since only the employment is available as data but not the labor input which is derived from total hours worked. However, we do not need to adopt equation (2.37), because we use instead labor input as data.

2.4.3 Miscellaneous Equilibrium Conditions

(10) Market clearing conditions:

$$\hat{y}_t = (1-\tau k_y - g_y)\hat{c}_t + \tau k_y\widehat{inv}_t + \overline{r}_t^k\psi k_y\hat{r}_t^k + g_y u_t^g \tag{2.38}$$

where k_y is the steady state capital-output ratio, and g_y is the steady state government spending-output ratio. We assume that the government spending shock follows a first-order autoregressive process with an i.i.d-normal error term: $u_t^g = \rho^g u_{t-1}^g + \varepsilon_t^g$.

(11) Monetary policy rule:

$$\hat{R}_t = \rho_m \hat{R}_{t-1} + (1-\rho_m)\left[\ \mu_\pi \hat{\Pi}_{t-1} + \mu_y \hat{y}_t\ \right] + \varepsilon_t^m \tag{2.39}$$

The monetary authorities follow a generalized Taylor rule by gradually responding to deviations of lagged inflation from a zero-percent inflation objective and the lagged output gap, $\hat{y}_t$. The parameter ρ_m captures the degree of interest rate smoothing. Also we assume monetary policy shock, ε_t^m, follows a white noise process. Smets and Wouters (2003) and Onatski and Williams (2004) adopted the more complicated rule as equation (2.39'). This rule, furthermore, considered a short-run feedback from the current changes in inflation and output gap and non-zero inflation target, π^*.

$$\hat{R}_t = \rho_m \hat{R}_{t-1} + (1-\rho_m)\left[\pi_t^* + \mu_\pi(\hat{\Pi}_{t-1} - \pi_t^*) + \mu_y \hat{y}_t\right] + \mu_{\Delta\pi}(\hat{\Pi}_t - \hat{\Pi}_{t-1}) + \mu_{\Delta y}(\hat{y}_t - \hat{y}_{t-1}) + \varepsilon_t^m \tag{2.39'}$$

2.4.4 Persistent Shocks and Forecast Errors

The five persistent shocks built in above equations are characterized by the first-order autoregressive process with an i.i.d-normal error term as follows.

(12) preference shock:

$$u_t^c = \rho^c u_{t-1}^c + \varepsilon_t^c, \tag{2.40}$$

(13) investment shock:

$$u_t^{inv} = \rho^{inv} u_{t-1}^{inv} + \varepsilon_t^{inv}, \tag{2.41}$$

(14) labor shock:

$$u_t^L = \rho^L u_{t-1}^L + \varepsilon_t^L, \tag{2.42}$$

(15) productivity shock:

$$u_t^a = \rho^z u_{t-1}^a + \varepsilon_t^a, \tag{2.43}$$

(16) government spending shock:

$$u_t^g = \rho^g u_{t-1}^g + \varepsilon_t^g. \tag{2.44}$$

And there are six forecast errors in the model as below.

(17) Inflation forecast error:

$$\eta_t^\pi = \hat{\pi}_t - E_{t-1}\hat{\pi}_t, \tag{2.45}$$

(18) Real wage forecast error:

$$\eta_t^w = \hat{w}_t - E_{t-1}\hat{w}_t, \tag{2.46}$$

(19) Equity premium forecast error:

$$\eta_t^q = \hat{q}_t - E_{t-1}\hat{q}_t, \tag{2.47}$$

(20) Investment forecast error:

$$\eta_t^{inv} = \widehat{inv}_t - E_{t-1}\widehat{inv}_t, \tag{2.48}$$

(21) Consumption forecast error:

$$\eta_t^c = \hat{c}_t - E_{t-1}\hat{c}_t, \tag{2.49}$$

(22) Rental Rate forecast error:

$$\eta_t^{rk} = \hat{r}_t^k - E_{t-1}\hat{r}_t^k. \tag{2.50}$$

2.4.5 System of the Log-Linearized Model

From equations (2.29) through (2.50) except (2.37), the system of the log-linearized model neighborhood the steady state are integrated as

$$\Gamma_0 \boldsymbol{s_t} = \Gamma_1 \boldsymbol{s_{t-1}} + \Psi \boldsymbol{\varepsilon_t} + \Pi \boldsymbol{\eta_t}, \tag{2.51}$$

where $\boldsymbol{s_t}$ is a vector of endogenous variables: $\boldsymbol{s_t} = [y_t, \pi_t, w_t, k_t, inv_t, c_t, R_t, r_t^k, L_t, E_t\pi_{t+1}E_t w_{t+1}, E_t w_{t+1}, E_t q_{t+1}, E_t inv_{t+1}, E_t c_{t+1}, E_t r_{t+1}^k, u_t^c, u_t^{inv}, u_t^L, u_t^a, u_t^g]'$, and $\boldsymbol{\varepsilon_t}$ is a vector of exogenous shocks: $\boldsymbol{\varepsilon_t} = [\varepsilon_t^c, \varepsilon_t^{inv}, \varepsilon_t^q, \varepsilon_t^L, \varepsilon_t^w, \varepsilon_t^a, \varepsilon_t^p, \varepsilon_t^g, \varepsilon_t^m]'$. $\boldsymbol{\eta_t}$ is a vector of forecast errors: $\boldsymbol{\eta_t} = [\eta_t^\pi, \eta_t^w, \eta_t^q, \eta_t^{inv}, \eta_t^c, \eta_t^{rk}]'$. $\boldsymbol{\Gamma_0}$, $\boldsymbol{\Gamma_1}$, $\boldsymbol{\Psi}$, and $\boldsymbol{\Pi}$ are the matrices of parameters. See Appendix A3 in which these matrices are described in detail. The next section describes how to solve and estimate the DSGE model using equation (2.51).

3 Bayesian Estimation of DSGE Models

In this section, we explain how to solve the DSGE model and the MCMC based Bayesian method for the analysis of the DSGE model. For readers interested in the developing field, we explain these methods in much more detail compared with the previous literature.

3.1 Solving DSGE model

3.1.1 General Form of Linear Rational Expectation Model

A linear rational expectations model (hereafter, LRE model) proposed by Blanchard and Kahn (1980) has been the representative of LRE models [12]. Nowadays, Sims (2002), however, generalized their linear rational expectations model[13]. Blanchard and Kahn (1980) do not explicitly build one-step-ahead prediction errors of endogenous variables in LRE models by setting these errors as zero (i.e. these endogenous variables are treated as predetermined ones.), whereas Sims (2002) do explicitly build the one-step-ahead prediction errors in the models [14]. The solving methods are characterized by whether the errors are built in the model or not. [15] The method proposed by Klein (2000) based on Blanchard and Kahn (1980) is adopted by Otrok(2001) and DeJong et al (2000a,b) etc, the Sims (2002) method is adopted by Schorfheide (2000), and Lubik and Schorfheide (2004).

The LRE model used in Sims (2002) can be represented as

$$\Gamma_0 \boldsymbol{s_t} = \Gamma_1 \boldsymbol{s_{t-1}} + \Psi \boldsymbol{\varepsilon_t} + \Pi \boldsymbol{\eta_t}, \tag{3.1}$$

where $\boldsymbol{s_t}$ is a vector of endogenous variables, $\boldsymbol{\varepsilon_t}$ is a vector of exogenous shock variables, and $\boldsymbol{\eta_t}$ is a vector of one-step-ahead prediction errors (or rational expectations errors), satisfying $E(\eta_{t+1}) = 0$.

The vector $\boldsymbol{s_t}$ denotes the variables in the model with the more advanced subindices, as well as the conditional expectations in the model. All of them

[12]Klein (2000) takes over from the form which builds no endogenous prediction error in the DSGE model used by Blanchard and Kahn (1980).

[13]This section follows the work by Novales et al. (1999)

[14]According to Sims (2002, pp.1-2), there are four advantages of the method as follows. (1) It covers all of the linear models with endogenous prediction error. (2) The approach handles automatically situations where linear combinations of variables are predetermined, while Blanchard and Kahn (1980) require that the analyst specifies which elements of endogenous variables are predetermined. (3) This approach makes an extension to continuous time possible. (4) Blanchard and Kahn (1980) assume that boundary conditions at infinity are given in the form of maximal rate of growth for any element of the endogenous variables. Meanwhile, this approach recognizes that in general only certain linear combinations of variables are required to grow at bounded rates and that different linear combinations may have different growth rate restrictions.

[15]Following Klein (2000, p1407), Sims (2002) transforms the LRE model into a triangular one using the Schur decomposition described above and isolates the unstable block of equations. This block is solved forward, and the endogenous prediction error process is solved for by imposing the informational restriction that the solution must be adapted to the given filtration. At this stage, no extraneous assumption (e.g. what variables are predetermined.) are invoked. all information about the solution is given in the coefficient matrices of the difference equation itself. Meanwhile, following Blanchard and Kahn (1980), Klein (2000) solves the unstable block of the triangular system forward without having to solve for prediction error separately. Instead, the endogenous prediction error process is solved for when solving the stable block of equations

are determined in the model. The vector ε_t denotes variables which are determined outside the model such as demand shocks, supply shocks, or errors in controlling government policy variables. The vector η_t denotes prediction errors, which will be solved for endogenously, together with state and decision variables s_t in the model.

As mentioned above, the features of Sims' (2002) model are that conditional expectation is defined as the endogenous variables s_t and that the prediction errors η_t are built in the LRE model. And if a stability condition does not hold in equation (3.1), the vector of endogenous variables s_t always traces unstable path which will violate the transversality conditions under arbitrary initial conditions s_0 and sample realizations for ε_t. However, s_t converges to equilibrium by necessity, if the stability condition holds in the model (3.1), although the structure of the stability conditions are generally model-specific. The linear combinations of prediction errors, η_t, which are endogenously determined in the models as explained later, contribute to the setup of the stability conditions.

Note that Sims (2002) proposed two approaches to find the solution and the stable condition depending on the property of the matrix Γ_0. In general, the second method, however, is more commonly used regardless of this property. In the case that the matrix Γ_0 is invertible, the first method is applied. In the method, we can find the eigenvalues $\mathbf{\Lambda}$ of $\mathbf{\Gamma}_0^{-1}\mathbf{\Gamma}_1 (= \boldsymbol{P}\mathbf{\Lambda}\boldsymbol{P}^{-1})$ using Jordan decomposition. Then we get the recursive equilibrium law of motion which will thread out stable path consisting of the stable eigenvalues and their corresponding eigenvectors. Meanwhile, in the case that the matrix Γ_0 is not invertible, i.e, it is singular, the second method is applied. In the method, we need to compute the generalized eigenvalues of the pair (Γ_0, Γ_1) using Schur decomposition (or QZ decomposition) as explained in the next subsection.

3.1.2 Solving DSGE model by Schur decomposition

In this section, we deal with the solving method of DSGE model by Schur decomposition (or QZ decomposition) [16]. When sampling parameters in the underlying DSGE model as explained in the next section, whether the models specified by sampled parameter set traces on stable path or on unstable path, is judged by this method. Only parameter sets in which the model traces on stable path are saved and otherwise are removed from the sample.

In the LRE model, equation (3.1), explained in the last subsection such as

$$\Gamma_0 \boldsymbol{s_t} = \Gamma_1 \boldsymbol{s_{t-1}} + \Psi \boldsymbol{\varepsilon_t} + \Pi \boldsymbol{\eta_t},$$

[16] This subsection follows Sims (2002).

the matrix Γ_0 and Γ_1 are decomposed by QZ decomposition as below.

$$Q'\Lambda Z' = \Gamma_0,$$
$$Q'\Omega Z' = \Gamma_1,$$

where $Q'Q = Z'Z = I$, and Q and Z are both possibly complex. Also Ω and Λ are possibly complex and upper triangular. Note that the above QZ decomposition always exists. Letting $\omega_t = Z's_{t,}$, and premultiplying the both side of equation (3.1) by Q, then we get

$$\Lambda\boldsymbol{\omega_t} = \Omega\boldsymbol{\omega_{t-1}} + Q\Psi\boldsymbol{\varepsilon_t} + Q\Pi\boldsymbol{\eta_t}. \tag{3.2}$$

Although QZ decomposition is not unique, the ratio of diagonal elements of Ω and Λ, $\{\omega_{ii}/\lambda_{ii}\}$, which is referred to generalized eigenvalue, is generally unique. The matrix Ω and Λ are ordered with respect to the absolute value of the ratio $\{\omega_{ii}/\lambda_{ii}\}$ (or generalized eigenvalue) by ascending order. By partitioning equation (3.2) in two blocks so that the stable generalized eigenvalues corresponding to $|\omega_{ii}/\lambda_{ii}| < \bar{\xi}$ and the unstable generalized eigenvalue corresponding to $|\omega_{ii}/\lambda_{ii}| \geq \bar{\xi}$, it is rewritten as equation (3.3). The upper and the lower in equation (3.3) are the stable block and the unstable block, respectively. Here, $\bar{\xi}$ is the bound of maximal growth rate of endogenous variables $\boldsymbol{s_t}$, that holds the transversality condition.

$$\begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ 0 & \Lambda_{22} \end{bmatrix} \begin{bmatrix} \omega_S(t) \\ \omega_U(t) \end{bmatrix} = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ 0 & \Omega_{22} \end{bmatrix} \begin{bmatrix} \omega_S(t-1) \\ \omega_U(t-1) \end{bmatrix} + \begin{bmatrix} Q_{1\cdot} \\ Q_{2\cdot} \end{bmatrix} \Big(\Psi\varepsilon(t) + \Pi\eta(t)\Big). \tag{3.3}$$

where $Q_{1\cdot}$ and $Q_{2\cdot}$ denote the first and the second rows of the matrix Q. For canceling out the term of expectation errors $\boldsymbol{\eta(t)}$ from equation (3.3), we premultiply equation (3.3) by $[I \quad -\Phi]$ and translate its stable block into the upper of equation (3.4). Note that Φ is set to satisfy a linear combination, $Q_{1\cdot}\Pi = \Phi Q_{2\cdot}\Pi$, and this linear combination of expectation errors $\boldsymbol{\eta(t)}$ is the stability condition of the DSGE model.

Meanwhile, on the unstable block (i.e. the lower) in equation (3.3), the last term, $Q_{2\cdot}\Pi\eta_{t+1}$, is solved forward[17], and then it becomes $Q_{2\cdot}\Pi\eta_{t+1} = \sum_{s=1}^{\infty} M^{s-1}\Omega_{22}^{-1}Q_{2\cdot}\Psi\epsilon_{t+s}$. Here, we set $M = \Omega_{22}^{-1}\Lambda_{22}$. Substituting it into equation (3.3), we get

$$\begin{bmatrix} \Lambda_{11} & \Lambda_{12} - \Phi\Lambda_{22} \\ 0 & I \end{bmatrix} \begin{bmatrix} \omega_S(t) \\ \omega_U(t) \end{bmatrix} = \begin{bmatrix} \Omega_{11} & \Omega_{12} - \Phi\Omega_{22} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \omega_S(t-1) \\ \omega_U(t-1) \end{bmatrix}$$
$$+ \begin{bmatrix} Q_{1\cdot} - \Phi Q_{2\cdot} \\ 0 \end{bmatrix} \Psi\varepsilon(t) + E_t \begin{bmatrix} 0 \\ \sum_{s=1}^{\infty} M^{s-1}\Omega_{22}^{-1}Q_{2\cdot}\Psi\varepsilon_{t+s} \end{bmatrix} \tag{3.4}$$

[17]This derivation is described in Sims (2002). Here, we omit it since this calculation is not used in the later part of our study.

Here, we set $E_t(\varepsilon_{t+s}) = 0$ for $s = 1 \cdots T$ in the last term of equation (3.4) and remind that $\omega_t = Z' s_t$, then we get the recursive equilibrium law of motion such as equation (3.5).

$$\boldsymbol{s_t} = \boldsymbol{\Theta_1 s_{t-1}} + \boldsymbol{\Theta_0 \varepsilon_t}, \tag{3.5}$$

where

$$\boldsymbol{\Theta}_1 = Z_{\cdot 1}\Lambda_{11}^{-1}[\Omega_{11} \ \ (\Omega_{12} - \Phi\Omega_{22})]Z,$$

$$\boldsymbol{\Theta}_0 = H \begin{bmatrix} Q_{1\cdot} - \Phi Q_{2\cdot} \\ 0 \end{bmatrix} \Psi,$$

$$H = Z \begin{bmatrix} \Lambda_{11}^{-1} & -\Lambda_{11}^{-1}(\Lambda_{12} - \Phi\Lambda_{22}) \\ 0 & I \end{bmatrix} Z,$$

where $Z_{\cdot 1}$ denotes the first column of matrix Z. Equation (3.5) traces the stable path converging to the equilibrium and corresponds to our target, say, the solution of the DSGE models.

From equation (3.5), we set a state space model which consists of a transition equation and a measurement equation using $\boldsymbol{\Theta_1}$ and $\boldsymbol{\Theta_0}$ as below. The transition equation (or recursive equilibrium law of motion) is given by

$$\boldsymbol{s_t} = \boldsymbol{\Theta_1 s_{t-1}} + \boldsymbol{\Theta_0 \varepsilon_t}, \tag{3.6a}$$

And the measurement equation is given by

$$\boldsymbol{y_t} = \boldsymbol{A s_t}, \tag{3.6b}$$

where, $\boldsymbol{y_t}$ is a vector of observed variables, $\boldsymbol{s_t}$ is a vector of endogenous variables. $\boldsymbol{A}$ is a $n \times k$ matrix expressing relations between observed variables $\boldsymbol{y_t}$ and unobserved variables $\boldsymbol{s_t}$.

For this state space model with Gaussian error terms, unobservable variables $\boldsymbol{s_t}$ and the likelihood of the model are obtained using Kalman filter. In the next subsection, the Bayesian estimation for the state space model with the recursive equilibrium law of motion is explained.

3.2 Bayesian Inference via MCMC Simulation

3.2.1 Likelihood of DSGE models

Equations (3.6a) and (3.6b) constitute a linear Gaussian state space model, whose likelihood can be evaluated using the Kalman filter. The Kalman filter is the algorithm that provides the mean and the covariance matrix of the state vector $\boldsymbol{s_t}$ $(t = 1, \ldots, T)$ conditional on the observations up to t, i.e., $(\boldsymbol{y_1}, \cdots, \boldsymbol{y_t})$ in a linear Gaussian state space model (see Harvey 1989 and Durbin and Koopman 2001 for a detailed discussion of the Kalman filter).

In the model that consists of equations (3.6a) and (3.6b), the equations of the Kalman filter are:
One-Step-Ahead Prediction:

$$\boldsymbol{s}_{t|t-1} = \boldsymbol{\Theta}_1 \boldsymbol{s}_{t-1|t-1}, \tag{3.7}$$

$$\boldsymbol{P}_{t|t-1} = \Theta_1 \boldsymbol{P}_{t-1|t-1} \Theta_1' + \Theta_0 \Theta_0', \tag{3.8}$$

Updating:

$$\boldsymbol{s}_{t|t} = \boldsymbol{s}_{t|t-1} + \boldsymbol{P}_{t|t-1} \boldsymbol{A}' \boldsymbol{F_t}^{-1} \boldsymbol{\nu}_t, \tag{3.9}$$

$$\boldsymbol{P}_{t|t} = \boldsymbol{P}_{t|t-1} - \boldsymbol{P}_{t|t-1} \boldsymbol{A}' \boldsymbol{F_t}^{-1} \boldsymbol{A} \boldsymbol{P}_{t|t-1}, \tag{3.10}$$

where $\boldsymbol{s}_{t|t-1}$ is the mean of $\boldsymbol{s}_t$ conditional on $(\boldsymbol{y}_1, \cdots, \boldsymbol{y}_{t-1})$, $\boldsymbol{s}_{t|t}$ is the mean of $\boldsymbol{s}_t$ conditional on $(\boldsymbol{y}_1, \cdots, \boldsymbol{y}_t)$, $\boldsymbol{P}_{t|t-1}$ is the covariance matrix of $(\boldsymbol{s}_t - \boldsymbol{s}_{t|t-1})$, and $\boldsymbol{P}_{t|t}$ is the covariance matrix of $(\boldsymbol{s}_t - \boldsymbol{s}_{t|t})$. $\boldsymbol{\nu}_t$ is the prediction error vector defined by

$$\boldsymbol{\nu}_t = \boldsymbol{y}_t - \boldsymbol{A} \boldsymbol{s}_{t|t-1}, \tag{3.11}$$

and $\boldsymbol{F}_t$ is its covariance matrix, given by

$$\boldsymbol{F}_t = \boldsymbol{A} \boldsymbol{P}_{t|t-1} \boldsymbol{A}'. \tag{3.12}$$

Once the initial values of $\boldsymbol{s}_{t|t-1}$ and $\boldsymbol{P}_{t|t-1}$ are given, equations (3.7)–(3.12) can be solved recursively. Those initial values are usually set equal to the unconditional mean and covariance matrix of the state vector, i.e.,

$$\boldsymbol{s}_{1|0} = \mathrm{E}\left(\boldsymbol{s}_t\right) = \mathbf{0}, \tag{3.13}$$

$$\mathrm{vec}(\boldsymbol{P}_{1|0}) = \mathrm{vec}\left(\mathrm{Var}(\boldsymbol{s}_t)\right) = \left[\boldsymbol{I} - \boldsymbol{\Theta}_1 \otimes \boldsymbol{\Theta}_1\right]^{-1} \mathrm{vec}\left(\boldsymbol{\Theta}_0 \boldsymbol{\Theta}_0{}'\right), \tag{3.14}$$

where $\boldsymbol{I}$ is the $n \times n$ identity matrix, $\otimes$ is the Kronecker product, and the vec$(\cdot)$ operator indicates that the columns of the matrix are being stacked one upon the other.

Since the error term $\boldsymbol{\varepsilon}_t$ is normally distributed for all t, the prediction error $\boldsymbol{\nu}_t$ given by equation (3.11) is normally distributed with mean $\mathbf{0}$ and covariance matrix $\boldsymbol{F}_t$. Therefore, the log likelihood is given by

$$\ln L = -\frac{nT}{2} \ln 2\pi - \frac{1}{2} \sum_{t=1}^{T} \ln |\boldsymbol{F}_t| - \frac{1}{2} \sum_{t=1}^{T} \boldsymbol{\nu}_t{}' \boldsymbol{F_t}^{-1} \boldsymbol{\nu}_t, \tag{3.15}$$

where T is the number of observations.

Maximizing this function with respect to the unknown parameters will produce their maximum likelihood estimates. Several authors use this maximum likelihood estimation (MLE) to estimate the parameters in DSGE models (Altug 1989, McGrantten, Rogerson and Wright 1997, Ireland 2001, 2004, Kim 2000).

3.2.2 MCMC Bayesian Estimation

Recent years have seen a surge in the application of the Markov-chain Monte Carlo (MCMC) based Bayesian estimation instead of the MLE to DSGE models (De Jong et al 2000a,b, Schorfheide 2000, Otrok 2001, Smets and Wouters 2003 and Lubik and Schorfheide 2004). Let $\boldsymbol{\theta}$ denote the set of the unknown parameters. The conventional Bayesian method proceeds as follows.

1. Set the prior distribution $f(\boldsymbol{\theta})$, which is the distribution the researcher has in mind before observing the data.
2. Convert the prior distribution to the posterior distribution $f(\boldsymbol{\theta}|\text{data})$, which is the distribution conditional on the data, using the Bayes theorem

$$f(\boldsymbol{\theta}|\text{data}) = \frac{f(\text{data}|\boldsymbol{\theta})f(\boldsymbol{\theta})}{\displaystyle\int f(\text{data}|\boldsymbol{\theta})f(\boldsymbol{\theta})d\boldsymbol{\theta}}. \tag{3.16}$$

3. Estimate the parameters $\boldsymbol{\theta}$ using the posterior distribution.

One of the most widely used prior distributions is the normal-gamma, which leads to the same normal-gamma posterior distribution for a simple linear model with normal errors. Notice that the unknown parameter vectors $\boldsymbol{\Theta}_1$, $\boldsymbol{\Theta}_2$ and $\boldsymbol{Z}$ in equations (3.6a) and (3.6b) are non-linear functions of the original parameters in the DSGE model, so that the posterior distribution is non-standard even if we use the normal-gamma prior distribution. When the posterior distribution is non-standard, it may be difficult to obtain the denominator of the right-hand-side of Bayes theorem (3.16) and to conduct the above Step 3 analytically.

In a MCMC based Bayesian estimation, $\boldsymbol{\theta}$ is sampled from the posterior distribution and the sampled draws are used for parameter estimation. The method used for sampling from the posterior distribution is MCMC, where sampling is not random and depends on the draw obtained in the previous sampling. Since the likelihood of DSGE models can be evaluated by executing the Kalman filter, it is straightforward to evaluate the numerator of the right-hand-side of Bayes theorem (3.16) analytically. In such a case, we may use the Metropolis-Hastings (MH) algorithm (see Chib and Greenberg 1995), which is one of MCMC methods.

To use the MH algorithm, we must choose a proposal density $g(\cdot|\cdot)$ from which it is possible to sample and an initial value $\boldsymbol{\theta}_0$. Then, we can sample $(\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_N)$ from $f(\boldsymbol{\theta}|\text{data})$ by executing the following algorithm.

(1) Set $n = 1$.

(2) Sample from $g(\boldsymbol{\theta}|\boldsymbol{\theta}_{n-1})$ and, using the sampled draw $\boldsymbol{\theta}_n^{(\text{proposal})}$, calculate the acceptance probability q as follows.

$$q = \min\left[\frac{f(\boldsymbol{\theta}_n^{(\text{proposal})}|\text{data})g(\boldsymbol{\theta}_{n-1}|\boldsymbol{\theta}_n^{(\text{proposal})})}{f(\boldsymbol{\theta}_{n-1}|\text{data})g(\boldsymbol{\theta}_n^{(\text{proposal})}|\boldsymbol{\theta}_{n-1})}, 1\right].$$

(3) Accept $\boldsymbol{\theta}_n^{(\text{proposal})}$ with probability q and reject it with probability $1-q$. Set $\boldsymbol{\theta}_n = \boldsymbol{\theta}_n^{(\text{proposal})}$ when accepted and $\boldsymbol{\theta}_n = \boldsymbol{\theta}_{n-1}$ when rejected.

(4) If $n < N$, set $n = n+1$ and return to (2). Otherwise, set $n = N$ and end.

All previous literature that applies the MH algorithm to DSGE models uses a method called the random-walk MH algorithm, where the proposal $\boldsymbol{\theta}_n^{(\text{proposal})}$ is sampled from the random-walk model:

$$\boldsymbol{\theta}_n^{(\text{proposal})} = \boldsymbol{\theta}_{n-1} + \boldsymbol{\nu_t}, \quad \boldsymbol{\nu_t} \sim \text{i.i.d.N}(0, c\boldsymbol{H}),$$

where c is a scalar called the adjustment coefficient, whose choice will be explained below, and $\boldsymbol{H}$ is usually set arbitrarily or equal to $-l''^{-1}(\hat{\boldsymbol{\theta}})$, where $l(\boldsymbol{\theta}) = \ln f(\boldsymbol{\theta}|\text{data})$ and $l''^{-1}(\hat{\boldsymbol{\theta}})$ is the inverse of the second derivative of $l(\boldsymbol{\theta})$ at $\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}$.

The merit of using this random-walk proposal is that $g(\boldsymbol{\theta}_{n-1}|\boldsymbol{\theta}_n^{(\text{proposal})}) = g(\boldsymbol{\theta}_n^{(\text{proposal})}|\boldsymbol{\theta}_{n-1})$, so that the acceptance probability q collapses to:

$$q = \min\left[\frac{f(\boldsymbol{\theta}_n^{(\text{proposal})}|\text{data})}{f(\boldsymbol{\theta}_{n-1}|\text{data})}, 1\right],$$

which does not depend on the proposal density $g(\cdot|\cdot)$. Hence, we need not find a proposal density that mimics the posterior density. We must, however, be careful for $\boldsymbol{\theta}_n^{(\text{proposal})}$ not to deviate from $\boldsymbol{\theta}_{n-1}$ so much because the acceptance probability q may be low when those deviate far from each other. This may be achieved by making c low, but $\boldsymbol{\theta}_n^{(\text{proposal})}$ may be sampled only from the narrow range if c is too low. It is a common practice to choose c such that the acceptance probability is around 25%. Following the previous literature, we simply use this random-walk MH algorithm with $\boldsymbol{H} = -cl''^{-1}(\hat{\boldsymbol{\theta}})$.[18]

3.2.3 Sampling from Prior Distribution.

The form of a prior density of each parameter is given in advance by an investigator in the Bayesian inference. In general, the prior densities in the DSGE models are set up as follows.

[18] We also tried a different algorithm called the independence M-H algorithm. In this algorithm, it is important to make the acceptance probability q as close to one as possible especially around the mode of the posterior density $f(\boldsymbol{\theta}|\text{data})$ because the same values are sampled consecutively if q is low. To achieve this purpose, we should choose the proposal density $g(\cdot|\cdot)$ that mimics the posterior density $f(\boldsymbol{\theta}|\text{data})$ especially around its mode, which may be possible by approximating the log of the true density $l(\boldsymbol{\theta})(= f(\boldsymbol{\theta}|\text{data}))$ using the second order Taylor expansion around its

It is assumed that the exogenous shocks ε_t such as technology shock, preference shock and monetary shock are persistent for their past shocks and these motions follow an AR (1) process such that $u_t = \rho u_{t-1} + \varepsilon_t$ where error term ε_t is *i.i.d.* Since the coefficient ρ must be between zero and one in the AR(1) process with the stationary property, their prior densities obey *beta distributions*. The variances of the error term ε_t are set up to be based on *inverted gamma distributions*. For the other parameters of the DSGE models *normal distributions* are adopted as their prior densities.

The distinction of the prior in the DSGE models is not to use normal-gamma distributions directly like other Bayesian estimations but to build up their own prior distributions by sampling the draws of the prior distributions given above. The aim that the priors are built up by sampling is to exclude the drawn parameters which are on *unstable* path in the DSGE model or are *not* the equilibrium solution from the sampling of the priors, and to include only the draws which are on *stable* path in the DSGE model or are the equilibrium solution into the sampling of the priors

The procedure to build up the priors is as follows. Firstly, draw around 2-3000 candidates of the parameters randomly from the given prior distributions, and save them. Next, using the candidates, solve the DSGE model for each candidate and obtain the recursive equilibrium law of motion (or stable path to the equilibrium) as equation (3.5). If one candidate derives the indeterminacy or no-existence of the equilibrium solution which indicates the DSGE cannot be solved, then this candidate is removed from the sample of the prior distribution. If the DSGE model can be solved using one candidate, then this candidate is saved in the sample of the prior distribution. Finally, depict a histogram from the saved sample, where all draws of the parameters form the recursive equilibrium law of motion, as the prior density.

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mode as follows (Watanabe 2001).

$$\begin{aligned} l(\boldsymbol{\theta}) &\approx l(\hat{\boldsymbol{\theta}}) + (\boldsymbol{\theta} - \hat{\boldsymbol{\theta}})' l'(\hat{\boldsymbol{\theta}}) + \frac{1}{2}(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}})' l''(\hat{\boldsymbol{\theta}})(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}), \\ &= \text{constant} + \frac{1}{2}[\boldsymbol{\theta} - \hat{\boldsymbol{\theta}} + l''^{-1}(\hat{\boldsymbol{\theta}}) l'(\hat{\boldsymbol{\theta}})]' l''(\hat{\boldsymbol{\theta}})[\boldsymbol{\theta} - \hat{\boldsymbol{\theta}} + l''^{-1}(\hat{\boldsymbol{\theta}}) l'(\hat{\boldsymbol{\theta}})], \\ &= \ln c g(\boldsymbol{\theta}), \end{aligned}$$

where $g(\boldsymbol{\theta})$ is the normal density with mean $\hat{\boldsymbol{\theta}} - l''^{-1}(\hat{\boldsymbol{\theta}}) l'(\hat{\boldsymbol{\theta}})$ and covariance $-l''^{-1}(\hat{\boldsymbol{\theta}})$, which may be used for a proposal density. This proposal density does not depend on $\boldsymbol{\theta}_{n-1}$.

This algorithm is much more efficient than the random-walk MH algorithm if the number of parameters is small (Kasuya, Nakajima and Watanabe 2005). However, the number of parameters estimated in this paper is 27 and they are transformed nonlinearly for state space representation. As a result, we find that the acceptance probability is very low such as 1–2% and cannot be improved by using the accept-reject (AR) MH algorithm proposed by Tierney (1994).

3.3 Impulse Response Functions

Here, how to derive the impulse response function by the Bayesian approach is described. The feature of the function by the Bayesian approach is that each value of the impulse response function is calculated using each draw of the parameter set, and that the sample of these values are saved as the posterior densities of the impulse response function, and used for calculating the moments (e.g. mean or median) and credible interval of the posterior densities.[19]

Let M denote the sampling size of the MCMC simulation. The impulse response function by the Bayesian approach might be assembled using M draws of parameters set sampled from the posterior densities in Section 3.2.2. and the procedure is presented as below.

First of all, the matrix $\mathbf{\Theta_1}$, and $\mathbf{\Theta_0}$ in equation (3.6a) are derived for every draw of parameters set sampled. Using $\mathbf{\Theta_1}$ and $\mathbf{\Theta_0}$, calculate the impulse response function $IR(k)^i$ from the first horizon via k horizon up to T horizon for i-th draws as below.

$$IR(k)^i = \boldsymbol{A} \times \mathbf{\Theta}_{1,i}^{k-1} \times \mathbf{\Theta}_{0,i} \times \boldsymbol{\varepsilon}, \qquad for \;\; \boldsymbol{\varepsilon} = \begin{bmatrix} 0 \\ \vdots \\ \varepsilon_j \\ \vdots \\ 0 \end{bmatrix},$$

where $\boldsymbol{A}$ denotes the matrix in equation (3.6b), and $\mathbf{\Theta}_{1,i}$ and $\mathbf{\Theta}_{0,i}$ are derived from the i-th draws of parameters set, and k is the number of horizon. ε_j is the j-th exogenous shock whose size is one standard deviation estimated in Section 3.2. Then, this calculation is implemented for all of M draws and all of M impulse responses are saved as the posterior densities. And calculate moments such as mean and confidence intervals (e.g., 90% interval) from the M samples. Finally, plot the moments and the confidence intervals of the impulse response functions for each observed variables y_t in equation (3.6b).

3.4 Variance Decomposition

As well as the impulse response functions, forecast error variance decomposition is derived using the parameters of equations (3.6a and b). The mean squared error (MSE) of h-period-ahead forecast of endogenous variables $\boldsymbol{y_t}$ can

[19] The method explained here is based on Schorfheide (2000) and Lubik and Schorfheide (2004).

be written as

$$MSE[\boldsymbol{y_t}(h)] = \sum_{j=1}^{N}\sum_{k=1}^{h}[\boldsymbol{A} \times \boldsymbol{\Theta_1^{k-1}} \times \boldsymbol{\Theta_0} \times \boldsymbol{\varepsilon_j} \times \boldsymbol{\varepsilon_j'} \times \boldsymbol{\Theta_0'} \times \boldsymbol{\Theta_1^{k-1\prime}} \times \boldsymbol{A'}\ ], \qquad for\ \ \boldsymbol{\varepsilon_j} = \begin{bmatrix} 0 \\ \vdots \\ \varepsilon_j \\ \vdots \\ 0 \end{bmatrix},$$

where ε_j is the standard diviation of jth structural shock, N is the number of shocks ε_j, and k is the number of horizon. This equation indicates that the MSE consists of the sum of h-period-ahead forecast error variances accounted for by the sum of N shocks. Notice that $MSE[\boldsymbol{y_t}(h)]$ is expressed by the matrix in which ith diagonal element is the MSE of variable y_i. With this expression, we can calculate the contribution of jth structural shock to the MSE of the h-period-ahead forecast of variables $\boldsymbol{y_t}$ as below.

$$Var[\boldsymbol{y_{t,j}}(h)] = \sum_{k=1}^{h}[\boldsymbol{A} \times \boldsymbol{\Theta_1^{k-1}} \times \boldsymbol{\Theta_0} \times \boldsymbol{\varepsilon_j} \times \boldsymbol{\varepsilon_j'} \times \boldsymbol{\Theta_0'} \times \boldsymbol{\Theta_1^{k-1\prime}} \times \boldsymbol{A'}\ ], \qquad for\ \ \boldsymbol{\varepsilon_j} = \begin{bmatrix} 0 \\ \vdots \\ \varepsilon_j \\ \vdots \\ 0 \end{bmatrix}.$$

Denoting the ith diagonal element of the matrix $MSE[\boldsymbol{y_{t,j}}(h)]$ and $Var[\boldsymbol{y_t}(h)]$ by $MSE[y_i(h)]$ and $Var[y_{i,j}(h)]$, respectively, we obtain the proportion of the h-period-ahead error variance of variable y_i accounted for by structural shock ε_j,

$$\omega_{i,j}(h) = Var[y_{i,j}(h)]/MSE[y_i(h)].$$

This value $\omega_{i,j}(h)$ indicates the forecast error variance decomposed into components accounted for by jth shock in the variable y_i at h horizon. In this way, we get a sample of variance decomposition using each sample of parameters, and save it as the posterior distribution.

4 Data

In estimating the model, following Smets and Wouters (2003), we chose seven quarterly macroeconomic series: output, consumption, investment, labor input, real wage, inflation, nominal rate as data. The sample period is from 1970:Q1 to 1998:Q4. The raw data is picked up from FERIS that is the database system of the Bank of Japan.

The seven series are as follows. Output series is real GDP per labor force, seasonally adjusted (unit is 1 million yen at 1990). Consumption series is real consumption per labor force, s.a., (unit same as above). Investment series is real investment per labor force, s.a., (unit same as above). Labor input series are derived from Work hour index times total employment divided by labor population. Real wage series is real wage index calculated from nominal wage index divided by GDP deflator. Inflation series is GDP deflator inflation rate (quarterly, annual rate, decadal demeaned). Nominal rate series is the uncollateralized call rate (annual rate, decadal demeaned). The four real series, output, consumption, investment, and labor, and real wage, are transformed to their logarithms and then detrended using the Hodrick and Prescott (HP) filter. Nominal rate and inflation are detrended using HP filter without log-transformation. After those procedures, all above data for estimating the model are obtained by being demeaned.

There is an issue of how to filter inflation rate. For the Japanese case, we simply assumed that 70's inflation target, 80's inflation target and 90's inflation target were different. Thus, by constructing decadal dummies for 70's and 80's, we demeaned the inflation rate accordingly. Also, we have demeaned the call rate using same decadal dummy coefficients. We know this is a controversial treatment, but we didn't know any better way to deal with it.

5 Estimation Results

5.1 Preliminary Setting

In order to estimate the parameters of the DSGE model described in Section 2, the data is limited over the period 1970:Q2 - 1998:Q4, because of excluding the period of zero interest rate bound from 1999:Q1, in which the law of equilibrium motions of macro-economies is plausible to be apart away from the ordinary economic dynamics. The seven key observed variables used as the data are real GDP, inflation, real wage, real investment, real consumption, nominal interest rate and labor input as can be seen from Figure 1, whereas the capital stock and the rental rate on capital are dealt with as unobserved variables based on the manner of Smets and Wouters (2003).

[Insert Figure 1.]

The fact that the model contains nine structural shocks and there are only seven observable variables raises a general identification issue. That is, for instance, it is difficult to separately identify the labor supply shock and the wage markup shock in equation (2.28). Identification is conducted by assuming that the each of the structural shocks is uncorrelated and that the three "cost-push" shocks (equity premium shock, price markup shock and wage markup shock) and monetary policy shock follow a white noise process. The remain five shocks (preference shock, productivity shock, investment shock, labor supply shock and government spending shock) are assumed to follow an AR(1) process where the autoregressive parameter has a relatively tight prior distribution with a mean of 0.85 and a standard error of 0.10, clearly distinguishing them from the white noise shocks.

In our DSGE model following the earlier studies such as Smets and Wouters (2003) who studied the euro area, Onatski and Williams (2004) and Levin, Onatski, Williams and Williams (2005) who studies the U.S. area, some parameters need to be calibrated. We chose most of the calibrated parameters following Hayashi and Prescott (2002). We set the discount factor, β, equal to 0.98, and the depreciation rate, τ, equal to 0.08, and the share of capital, α, equal to 0.35. The ratio of steady-state government spending to total output, g_y, is assumed to be 0.15, while the steady-state capital output ratio, k_y, is assumed to be 2.2. In addition, we also need to fix the parameter capturing the markup in wage setting, λ_w, as this parameter is not identified. We set λ_w equal to 0.20. The steady-state rental rate on capital (or the value of capital) is derived such as $\bar{r}^k = \frac{1}{\beta} - (1 - \tau)$. Finally, our study sets zero-percent rate as inflation target rate, π^*, and assumes the monetary policy response on the current change in inflation and the output gap, $\mu_{\Delta\pi}$ and $\mu_{\delta y}$, are zero as can be seen from equation (2.39).

The prior distributions of the other 27 estimated parameters following the manner of Smets and Wouters (2003) are given in Table 1. All the variances of the shocks are assumed to be distributed as an inverted Gamma distribution with a degree of freedom equal two. This distribution guarantees a positive standard deviation with a rather large domain. The distribution of the autoregressive parameters in the six persistent shocks is assumed to follow a beta distribution with mean 0.85 and standard error 0.1. The beta distribution covers the range between zero and one, but a rather tight standard error was used in order to have a clear separation between the persistent shocks and temporary shocks. The technology, utility, and price-setting parameters were assumed to be either Normal distributed or Beta distributed (for the parameters were restricted to the $0 - 1$ range).

[Insert Table 1.]

The mean of the prior was typically set at values that correspond to those in Smets and Wouters (2003)'s work. The standard deviations were set so that the domain covers a reasonable range of parameter values. For example, we set the mean of the Calvo parameters in price and wage setting equations, ξ_p, ξ_w so that average length of the contract is about one year following Smets and Wouters (2003) and in the estimates of Gali, Gertler, and Lopez-Salido (2001) for the European economy, and the standard deviation equal to 0.15, which is larger than 0.5 assumed by Smets and Wouters (2003). Similarly, the mean of the intertemporal elasticity of substitution σ_c is set equal to one. The elasticity of the capital utilization cost function has a mean of 0.2, and include in its domain the value of 0.1 suggested by King and Rebelo (2000) for the U.S. economy. The share of fixed cost (or the elasticity of the cost of adjusting investment) in total production, ϕ, has a mean of 1.45 which is close to those CEE (2005) for the United States. A wide range of calibrations has been used for the inverse elasticity of labor supply. We took as a starting point a value of two, which falls in between the relatively low elasticities that are typically estimated in the microlabor literature and the higher elasticities typically used in DSGE models. Finally, the priors on the means of the coefficients in the monetary policy reaction, i.e. 1.7, helps to guarantee a unique solution path when solving the model; the prior on the lagged interest rate is set at 0.8, and the prior on the output gap reaction coefficient corresponds to the Taylor coefficient of 0.5.

5.2 Parameters Estimates

For parameter estimation, we conduct the MCMC simulation with 350,000 iterations. The first 250,000 draws are discarded and then the next 100,000 are recorded. Using these 100,000 draws for each of the parameters, we calculate the posterior means, the standard errors of the posterior means, the standard deviations, the 90% intervals and the convergence diagnostic (CD) statistics proposed by Geweke (1992). The posterior means are computed by averaging the simulated draws. The standard errors of the posterior means are computed using a Parzen window with a bandwidth of 10,000 (see Shephard and Pitt 1997, p.665). The standard deviations are computed as the sample standard deviation of the simulated draws. The 90% intervals are calculated using the 5th and 95th percentiles of the simulated draws. Geweke (1992) suggests assessing the convergence of the MCMC by comparing values early in the sequence with those late in the sequence. Let $X^{(i)}$ be the ith draw of a parameter in the recorded 100,000 draws, and let $\bar{X}_A = \frac{1}{n_A}\sum_{i=1}^{n_A} X^{(i)}$ and $\bar{X}_B = \frac{1}{n_B}\sum_{i=100{,}001-n_B}^{100{,}000} X^{(i)}$. Using these values, Geweke (1992) proposes the following statistics called convergence diagnostics (CD).

$$\mathrm{CD} = \frac{\bar{X}_A - \bar{X}_B}{\sqrt{\hat{\sigma}_A^2/n_A + \hat{\sigma}_B^2/n_B}}, \tag{1}$$

where $\sqrt{\hat{\sigma}_A^2/n_A}$ and $\sqrt{\hat{\sigma}_B^2/n_B}$ are standard errors of $\bar{X}_A$ and $\bar{X}_B$. If the sequence of $X^{(i)}$ is stationary, it converges in distribution to the standard normal. We set $n_A = 10,000$ and $n_B = 50,000$ and compute $\hat{\sigma}_A^2$ and $\hat{\sigma}_B^2$ using Parzen windows with bandwidth of 1,000 and 5,000 respectively.

Table 2 reports our estimation results for the Japanese economy prior to the period of zero interest rate bound together with the posterior means of the parameters for the euro area by Smets and Wouters (SW, 2003) and the United States by Onatski and Williams (OW, 2004) and Levin et al. (LOWW, 2005). According to the CD values, the null hypothesis that the sequence of 100,000 draws is stationary is accepted at the 1% significance level for all parameters. Figures 2A through 2C depict the prior and posterior distribution of each parameter. The latter is obtained from the recorded 100,000 draws. The persistence in shocks is estimated as an autoregressive parameter ρ, whose posterior mean lies between 0.37 (for preference shock) and 0.87 (for investment shock).

[Insert Table 2 and Figure 2.]

Here we focus on the four parameters that represent the degree of price and wage stickiness. For instance, the posterior mean of price indexation parameter γ_p is 0.61, indicating that the weight on lagged inflation in the inflation equation (2.36), $\frac{\gamma_p}{1+\beta\gamma_p}$, is only 0.38. There is, however, a considerable degree of Calvo wage and price stickiness because the posterior means of ξ_w and ξ_p are 0.37 and 0.65 respectively and $1-\xi_w$ and $1-\xi_p$ indicate the probabilities that a given price and wage can be optimized in a quarterly period. The average duration of wage contracts is estimated to be about 1.6 quarter, whereas that of price contract is about 2.9 quarters. Both of the durations are quite short compared with the earlier studies such as SW, OW and LOWW, and the greater stickiness in prices relative to wages is somewhat counterintuitive.

The posterior mean and the 90% interval of the inverse intertemporal elasticity of substitution σ_c are greater than one, which is consistent with most of the RBC literature that assumes an elasticity of substitution between 0.5 and 1 and with the results of SW, OW and LOWW. On the other hand, the posterior mean of the external habit formation θ is 0.64, which is much higher relative to the U.S. and the euro area. The posterior mean of the weight of the present consumption on the past consumption, $\theta/(1+\theta)$, is about 0.39, while that on the future consumption, $1/(1+\theta)$, is about 0.61.

The posterior mean of the adjustment cost parameter $1/\varphi$ is about 8.34, which is quite low compared with the U.S. and the euro area. It implies that investment increases only by 0.06 percent in the short-run and 0.12 percent in the long run following a one percent increase in the current value of capital stock, q_t. The posterior means of the fixed cost share ϕ and the capital utilization cost ψ are 1.58 and 0.18 respectively. The posterior mean of inverse labor supply elasticity with respect to real wage, σ_L, is about 2.43, indicating that labor supply increases by 0.41 percent following a one percent increase in real wage.

Finally, we obtain plausible estimates for the long-run reaction function of the monetary authorities. The estimates state that the response of interest rate to inflation, μ_π, is much greater than one, indicating that the Japanese monetary authorities reacted very actively toward inflation.[20] On the contrary, the response to output, μ_y, is small. The posterior mean of the response to lagged interest rate, ρ_m, is 0.68, indicating relatively low persistence compared with the U.S. and the euro area.

We also extend the sample period to 1970:Q1 – 2004:Q4 such that the period of zero-percent interest rate bound is included. However, the parameter estimates are similar to those excluding the period of zero interest rate bound. Accordingly, we omit to explain these results here.

5.3 Impulse Response Analysis

The impulse responses to each of the nine structural shocks are estimated using a selection of 10,000 parameters from the posterior sample of 100,000 which were described in the last subsection. Figures 3A through 3I plot the median response together with the 5th and the 95 percentiles. The estimated impulse responses to all shocks except investment shock and equity premium shock, are consistent with those in SW (2003) for the euro economy.

(1) Positive Productivity Shock (Figure 3A)

A positive productivity shock leads to a hump-shaped rise in output, consumption and investment and a hump-shaped fall in inflation, nominal interest rate, rental rate of capital and labor input. No significant effect on real wage is observed because the 90% interval includes 0. All these results are consistent with those in SW (2003).

[Insert Figure 3A.]

(2) Negative Labor Supply Shock (Figure 3B)

A negative labor shock leads to a hump-shaped rise in inflation, nominal interest rate, real wage, rental rate of capital and a hump-shaped fall in output, consumption, investment and labor input. These responses would be similar to those of "negative" productivity shock except for real wage and labor input. All these results are consistent with those in SW (2003) except for rental rate, which falls in SW (2003).

[Insert Figure 3B.]

[20] If $\mu_\pi < 1$, it leads to the indeterminacy of solution to the LRE model. We preclude such an indeterminacy case by assuming beta distribution for the prior of μ_π. See Lubik and Schorfheide (2004) for the analysis that does not preclude the indeterminacy case.

(3) Positive Wage Markup Shock (Figure 3C)

A positive wage markup shock leads to a hump-shaped rise in inflation, nominal interest rate, real wage and rental rate and a hump-shaped fall in output, consumption, investment and labor input. All these results are consistent with those in SW (2003).

[Insert Figure 3C.]

(4) Positive Price Markup Shock (Figure 3D)

The effects of a positive price markup shock on output, consumption, inflation, nominal interest rate, investment and labor input are similar to those of the above positive wage markup shock. The effects on real wage and the rental rate are opposite to those of the wage markup shock. These results are consistent with those in SW (2003).

[Insert Figure 3D.]

(5) Positive Preference Shock (Figure 3E)

A positive preference shock leads to a hump-shaped rise in all variables except investment (and capital stock). These results are consistent with those in SW (2003).

[Insert Figure 3E.]

(6) Positive Investment Shock (Figure 3F)

The effects of a positive investment shock are different from those in SW (2003) except for real wage, rental rate and investment. Output and consumption rise monotonically over time in our estimation while they rise in a hump-shaped manner in SW (2003). The effects on inflation and labor input are not significant in our estimation while they rise in a hump-shaped manner in SW (2003).

The reason to our counterintuitive results might be our tiny estimate of φ. As can be seen from equation (2.26), a tiny value of φ makes investment almost independent of the current value of the capital stock, q_t, and hence the convergence of investment shock very slow. Investment equation (2.26), which is based on a financial market with complete information, might be misspecified. It is worthwhile to extend to a more advanced model, for example, based on incomplete information.

[Insert Figure 3F.]

(7) Positive Equity Premium Shock (Figure 3G)

The effects of a positive equity premium shock on output and nominal interest rate are also counterintuitive and not consistent with those in SW (2003).

Output decreases gradually in SW (2003), but it is not true for our estimation. The effect on nominal interest rate is not significant while it rises in SW (2003). The reason might to be the same as that for the above investment shock.

[Insert Figure 3G.]

(8) Positive Government Spending Shock (Figure 3H)

A positive government spending shock leads to a hump-shaped rise in output, inflation, nominal interest rate, rental rate and labor input and a hump-shaped fall in consumption and investment. The effect on real wage is not significant. These results are consistent with those in SW (2003).

[Insert Figure 3H.]

(9) Positive Monetary Policy Shock (Figure 3I)

A rise in nominal interest rate leads to a hump-shaped fall in all variables except nominal interest rate, which is also consistent with the result in SW (2003).

[Insert Figure 3I.]

5.4 Variance decomposition

The forecast error variance decompositions of the seven observable variables to each of the nine structural shocks are calculated using a selection of 5,000 parameters from the posterior sample 100,000. Table 3 reports the mean of the variance decompositions at four horizons from contemporary horizon ($t = 0$) to long run ($t = 100$, 25 years) via short run ($t = 4$, 1 year) and medium run ($t = 10$, 2.5 years).

[Insert Table 3]

Smets and Wouters (2003) shows that labor supply shock accounts for a large friction of the variance of almost macroeconimic variables in the long run in euro area. Meanwhile, Table 3 shows that productivity shock and investment shock accounts for a substantial portion of the fluctuation of the seven macroeconomic variables in the long run in the Japanese economy.

The variance of output is driven mainly by preference shock, government spending shock and monetary policy shock in the contemporary horizon. The preference shock accounts for around 53 %, the government spending shock for 27%, and the monetary policy shock for 11%. However, these effects weaken as horizon is longer. Instead, the effects of the productivity shock and the price markup shock enlarge in the medium run: the ratio of the productivity shock is 41 %, and that of the price markup shock is 9 %. In the long run (25 years), the investment shock and the productivity shock play the main role in the output variation: the former account for about 60 %, the latter for 26 %. In contrast to Smets and Wouters (2003), labor supply shock and monetary policy shock do not influence the fluctuation of output in the long run.

The preference shock accounts for 78 % in the variance of consumption in the contemporary horizon. But the effect of the preference shock is short-lived. And the other factor contributing to the variance is the monetary policy shock which accounts for around 14 %. The contribution of the productivity shock plays the important role from the short run to the long run. This effect in the long run is 25 % of the variance in consumption, whereas the investment shock becomes the primary factor which account for 60 % in the long run.

Similar to Smets and Wouters (2003), the price markup shock plays the main role in the variance of inflation at all horizon from the short run to the long run. It accounts for 87 % in the contemporary horizon, for 62 % in the medium run, and for 48 % in the long run. And the productivity shock influences inflation. The effect in the short run is about 9% and it grows up to 20 % in the medium and long runs. The investment shock is not a negligible factor in the long run. In contrast to Smets and Wouters (2003), monetary policy shock does not occupy a large fraction of the fluactuation of inflation. And also Table 3 shows that the variance of nominal wage is very similar to that of inflation.

The variance of the nominal rate is dominated by the monetary policy shock in the very short run. As the horizon is long, the contribution of the shock gradually reduces such as 57 % in the short run, 50 % in the medium run, and 40% in the long run. After the short run, price markup shock and investment shock influence the nominal interest rate at a certain level.

6 Conclusion

This paper estimates the CEE (2005) model, which is the most successful among New-Keynesian DSGE models in explaining the behavior of macroeconomic variables in the U.S. and euro area, for the Japanese economy over 1970:Q1 through 1998:Q4, which is prior to the period of zero interest rate bound. Using Bayesian inference via MCMC simulation, we find that the parameters and impulse response functions in the Japanese economy are estimated to be quite consistent with the earlier studies such as SW (2003) for the euro area and OW (2004) and LOWW (2005) for the U.S. area. For example, we find evidence that the Japanese monetary authorities reacted very actively toward inflation. The only exception is investment, whose adjustment cost is estimated huge and whose shock is estimated to give long-lasting effects on output and consumption compared with those in the previous studies for the U.S. and euro area. On the other hand, variance decomposition shows that monetary policy shock do not influence the fluctuations of output and inflation in the long run, in contrast to Smets and Wouters (2003). Instead, productivity shock and investment shock account for a substantial portion of all macroeconomic variables including output and inflation in the long run.

This paper is a starting point of the Bayesian analysis of DSGE models for the Japanese economy, and there remain some issues that should be pursued. First, we should calculate the marginal likelihood to compare with other DSGE models and reference models such as VAR and VAR-DSGE models (Smets and Wouters 2003, Del Negro and Schorfheide 2004, Del Negro, Schorfheide, Smets and Wouters 2004 and An and Schorfheide 2005). These issues are now under study. Second, since we find that the CEE (2005) model leads to counterintuitive results on investment, it is important to develop an alternative model taking Japanese companies' investment behavior into account.

Appendix

A Simplified Smets and Wouters (2003) Model Skelton

A.1 Model Description (Log-linearized version)

A.1.1 Consumer/Investor's Equilibrium Conditions

1. Consumption Euler equation:

$$\hat{c}_t = \frac{\theta}{1+\theta}\hat{c}_{t-1} + \frac{1}{1+\theta}E_t\hat{c}_{t+1} - \frac{1-\theta}{(1+\theta)\sigma_c}(\hat{R}_t - E_t\hat{\pi}_{t+1}) + \frac{1-\theta}{(1+\theta)\sigma_c}(1-\rho^c)u_t^c \quad (2.29)$$

where we set $E_t u_{t+1}^c = \rho^c u_t^c$.

2. Investment Euler equation:

$$\widehat{inv}_t = \frac{1}{1+\beta}\widehat{inv}_{t-1} + \frac{\beta}{1+\beta}E_t\widehat{inv}_{t+1} + \frac{\varphi}{1+\beta}\hat{q}_t + \frac{\beta}{1+\beta}(1-\rho^{inv})u_t^{inv} \quad (2.30)$$

where we set $E_t u_{t+1}^{inv} = \rho^{inv} u_t^{inv}$.

3. Asset pricing Euler equation:

$$\hat{q}_t = -(\hat{R}_t - E_t\hat{\pi}_{t+1}) + \frac{1-\tau}{1-\tau+\bar{r}^k}E_t\hat{q}_{t+1} + \frac{\bar{r}^k}{1-\tau+\bar{r}^k}E_t\hat{r}_{t+1}^k + \varepsilon_t^q \quad (2.31)$$

4. Wage setting equation.:

$$\begin{aligned}\hat{w}_t = &\frac{\beta}{1+\beta}E_t\hat{w}_{t+1} + \frac{1}{1+\beta}\hat{w}_{t-1} + \frac{\beta}{1+\beta}E_t\hat{\pi}_{t+1} - \frac{1+\beta\gamma_w}{1+\beta}\hat{\pi}_t + \frac{\gamma_w}{1+\beta}\hat{\pi}_{t-1} \\ &- \frac{1}{1+\beta}\Psi_w\left[\hat{w}_t - \sigma_L\hat{L}_t - \frac{\sigma_c}{1-\theta}(\hat{c}_t - \theta\hat{c}_{t-1}) - u_t^L - \varepsilon_t^w\right]\end{aligned} \quad (2.32)$$

where $\Psi_w = \frac{(1-\beta\xi_w)(1-\xi_w)}{\left(1+\frac{(1+\lambda_w)\sigma_L}{\lambda_w}\right)\xi_w}$

A.1.2 Firm's Equilibrium Conditions

1. Production function:

$$\hat{y}_t = \phi u_t^a + \phi\alpha\hat{k}_{t-1} + \phi\alpha\psi\hat{r}_t^k + \phi(1-\alpha)\hat{L}_t \quad (2.35)$$

2. Labor demand:

$$\hat{L}_t = -\hat{w}_t + (1+\psi)\hat{r}_t^k + \hat{k}_{t-1} \quad (2.34)$$

3. Price setting equation.:

$$\hat{\pi}_t = \frac{\beta}{1+\beta\gamma_p}E_t\hat{\pi}_{t+1} + \frac{\gamma_p}{1+\beta\gamma_p}\hat{\pi}_{t-1} + \frac{1}{1+\beta\gamma_p}\Psi_p\left[\alpha\hat{r}_t^k + (1-\alpha)\hat{w}_t - u_t^a + \varepsilon_t^p\right] \quad (2.36)$$

where $\Psi_p = \frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p}$

A.1.3 Miscellaneous Equilibrium Conditions

1. Resource constraint:

$$\hat{y}_t = (1 - \tau k_y - g_y)\hat{c}_t + \tau k_y \widehat{inv}_t + \bar{r}^k \psi k_y r_t^k + g_y u_t^g \tag{2.38}$$

2. Capital accumulation equation:

$$\hat{k}_t = (1 - \tau)\hat{k}_{t-1} + \tau \widehat{inv}_{t-1} \tag{2.33}$$

3. Monetary policy rule:

$$\hat{R}_t = \rho_m \hat{R}_{t-1} + (1 - \rho_m)\left[\mu_\pi \hat{\pi}_{t-1} + \mu_y \hat{y}_t\right] + \varepsilon_t^m \tag{2.39}$$

Persistent Shocks

1. : preference shock: $u_t^c = \rho^c u_{t-1}^c + \varepsilon_t^c$
2. : investment shock: $u_t^{inv} = \rho^{inv} u_{t-1}^{inv} + \varepsilon_t^{inv}$
3. : labor shock: $u_t^L = \rho^L u_{t-1}^L + \varepsilon_t^L$
4. : productivity shock: $u_t^a = \rho^z u_{t-1}^a + \varepsilon_t^a$
5. : government spending shock: $u_t^g = \rho^g u_{t-1}^g + \varepsilon_t^g$

Forecast Errors

1. Inflation forecast error: $\hat{\pi}_t = E_{t-1}\hat{\pi}_t + \eta_t^\pi$
2. Wage forecast error: $\hat{w}_t = E_{t-1}\hat{w}_t + \eta_t^w$
3. Q forecast error: $\hat{q}_t = E_{t-1}\hat{q}_t + \eta_t^q$
4. Investment forecast error: $\widehat{inv}_t = E_{t-1}\widehat{inv}_t + \eta_t^{inv}$
5. Consumption forecast error: $\hat{c}_t = E_{t-1}\hat{c}_t + \eta_t^c$
6. Capital cost forecast error: $\hat{r}_t^k = E_{t-1}\hat{r}_t^k + \eta_t^{rk}$

A.1.4 Endogenous Variables

y_t : output
π_t : inflation rate
w_t : nominal wage
k_t : capital stock
q_t : shadow price of capital stock
inv_t : physical investment

c_t : consumption
R_t : nominal interest rate
r_t^k : rental rate on capital (cost of capital)
L_t : labor input
$u_t^c, u_t^{inv}, u_t^L, u_t^a, u_t^g$: persistent shocks to consumption, investment, labor, productivity, and government spending, respectively.

A.1.5 Exogenous Shock Variables, (i.i.d. Normal distribution)

ε_t^c : preference shock
ε_t^{inv} : investment shock
ε_t^q : equity premium shock
ε_t^L : labor shock
ε_t^w : wage mark-up shock
ε_t^a : productivity shock
ε_t^p : price mark-up shock
ε_t^g : government spending shock
ε_t^m : monetary policy shock

A.1.6 Forecast Errors

η_t^π : forecast error of inflation
η_t^w : forecast error of real wage
η_t^q : forecast error of equity premium
η_t^{inv} : forecast error of investment
η_t^c : forecast error of consumption
η_t^{rk} : forecast error of rental rate

A.2 Preliminary Settings

A.2.1 Estimated Parameters

θ: habit formation, σ_c: inverse long-run IES, σ_L: inverse labor supply elasticity, φ: inverse adj.cost, ϕ: fixed cost share, ψ: capital utilization cost, γ_p: price indexation, γ_w: wage indexation, ξ_p: Calvo price no-revise prob., ξ_w: Calvo wage no-revise prob., ρ_m: lagged interest rate, μ_π: reaction on inflation, μ_y: reaction on output, ρ_c: persitence, preference , ρ_{inv}: persistence, investment, ρ_L: persistence, labor supply, ρ_a: persistence, productivity, ρ_g: persistence, government spending, ε_c: S.D., preference shock, ε_{inv}: S.D., investment shock, ε_q: S.D., equity premium shock, ε_L: S.D, labor supply shock, ε_w: S.D., wage markup shock, ε_z:

S.D., productivity shock, ε_p: S.D., price markup shock, ε_g: S.D., gov. spending shock, ε_m: S.D., monetary policy shock.

A.2.2 Values of Calibrated Parameters

discount factor: $\beta = 0.99$,
depriciation rate of capital: $\tau = 0.025$,
share of capital: $\alpha = 0.3$,
capital-output ratio: $k_y = 2.2$,
government spending-output ratio: $g_y = 0.2$,
wage markup: $\lambda_w = 0.05$,
steady-state rental rate: $\bar{r}^k = \frac{1}{\beta} - 1 + \tau$, (Smets and Wouters 2003, p1135)

A.3 Canonical LRE Form

$$\Gamma_0 \begin{bmatrix} y_t \\ \pi_t \\ w_t \\ k_t \\ q_t \\ inv_t \\ c_t \\ R_t \\ r_t^k \\ L_t \\ E_t\pi_{t+1} \\ E_t w_{t+1} \\ E_t q_{t+1} \\ E_t inv_{t+1} \\ E_t c_{t+1} \\ E_t r_{t+1}^k \\ u_t^c \\ u_t^{inv} \\ u_t^L \\ u_t^a \\ u_t^g \end{bmatrix} = \Gamma_1 \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ w_{t-1} \\ k_{t-1} \\ q_{t-1} \\ inv_{t-1} \\ c_{t-1} \\ R_{t-1} \\ r_{t-1}^k \\ L_{t-1} \\ E_{t-1}\pi_t \\ E_{t-1} w_t \\ E_{t-1} q_t \\ E_{t-1} inv_t \\ E_{t-1} c_t \\ E_{t-1} r_t^k \\ u_{t-1}^c \\ u_{t-1}^{inv} \\ u_{t-1}^L \\ u_{t-1}^a \\ u_{t-1}^g \end{bmatrix} + \Psi \begin{bmatrix} \varepsilon_t^c \\ \varepsilon_t^{inv} \\ \varepsilon_t^q \\ \varepsilon_t^L \\ \varepsilon_t^w \\ \varepsilon_t^a \\ \varepsilon_t^p \\ \varepsilon_t^g \\ \varepsilon_t^m \end{bmatrix} + \Pi \begin{bmatrix} \eta_t^\pi \\ \eta_t^w \\ \eta_t^q \\ \eta_t^{inv} \\ \eta_t^c \\ \eta_t^{rk} \end{bmatrix}$$

where coefficient matrices Γ_0, Γ_1, Ψ, and Π are set as follows.

$$
\Gamma_0 = \begin{array}{c}
\begin{array}{cccccccc}
y_t & \pi_t & w_t & k_t & q_t & inv_t & c_t & R_t
\end{array} \\
\left[\begin{array}{cccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{1-\theta}{(1+\theta)\sigma_c} \\
0 & 0 & 0 & 0 & -\frac{\varphi}{1+\beta} & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & \frac{1+\beta\gamma_w}{1+\beta} & 1+\frac{\Psi_w}{1+\beta} & 0 & 0 & 0 & -\frac{\sigma_c\Psi_w}{(1+\beta)(1-\theta)} & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -\frac{\Psi_p(1-\alpha)}{1+\beta\gamma_p} & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & -\tau k_y & -(1-\tau k_y-\tau g_y) & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
-(1-\rho_m)\mu_y & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right.
\end{array}
$$

$$
\begin{array}{ccccccccccccc}
r_t^k & L_t & E_t\pi_{t+1} & E_t w_{t+1} & E_t q_{t+1} & E_t inv_{t+1} & E_t c_{t+1} & E_t r_{t+1}^k & u_t^c & u_t^{inv} & u_t^L & u_t^a & u_t^g
\end{array}
$$

$$
\left.\begin{matrix}
0 & 0 & -\frac{1-\theta}{(1+\theta)\sigma_c} & 0 & 0 & 0 & -\frac{1}{1+\theta} & 0 & -\frac{(1-\theta)(1-\rho^c)}{(1+\theta)\sigma_c} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{\beta}{1+\beta} & 0 & 0 & 0 & -\frac{\beta(1-\rho^{inv})}{1+\beta} & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & -\frac{1-\tau}{1-\tau+Rk^*} & 0 & 0 & -\frac{Rk^*}{1-\tau+Rk^*} & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{\sigma_L\Psi_w}{1+\beta} & -\frac{\beta}{1+\beta} & -\frac{\beta}{1+\beta} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\Psi_w}{1+\beta} & 0 & 0 \\
-\phi\alpha\psi & -\phi(1-\alpha) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\phi & 0 \\
-(1+\psi) & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{\Psi_p\alpha}{1+\beta\gamma_p} & 0 & -\frac{\beta}{1+\beta\gamma_p} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\Psi_p}{1+\beta\gamma_p} & 0 \\
-Rk^*\psi k_y & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -g_y \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{matrix}\right]
$$

$$
\Gamma_1 = \begin{array}{c}
\begin{array}{cccccccc} y_{t-1} & \pi_{t-1} & w_{t-1} & k_{t-1} & q_{t-1} & inv_{t-1} & c_{t-1} & R_{t-1} \end{array} \\
\left[\begin{array}{cccccccc}
0 & 0 & 0 & 0 & 0 & 0 & \frac{\theta}{1+\theta} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{1+\beta} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\gamma_w}{1+\beta} & \frac{1}{1+\beta} & 0 & 0 & 0 & -\frac{\sigma_c \Psi_w \theta}{(1+\beta)(1-\theta)} & 0 \\
0 & 0 & 0 & \phi\alpha & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & \frac{\gamma_p}{1+\beta\gamma_p} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1-\tau & 0 & \tau & 0 & 0 \\
0 & (1-\rho_m)\mu_\pi & 0 & 0 & 0 & 0 & 0 & \rho_m \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
\end{array}
$$

$$
\begin{array}{ccccccccccccc}
r^k_{t-1} & L_{t-1} & E_{t-1}\pi_t & E_{t-1}w_t & E_{t-1}q_t & E_{t-1}inv_t & E_{t-1}c_t & E_{t-1}r^k_t & u^c_{t-1} & u^{inv}_{t-1} & u^L_{t-1} & u^a_{t-1} & u^g_{t-1}
\end{array}
$$

$$
\left.
\begin{array}{ccccccccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho^c & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho^{inv} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho^L & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho^z & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho^g
\end{array}
\right]
$$

$$
\Psi = \begin{array}{c}
\begin{matrix} \varepsilon_t^{c} & \varepsilon_t^{inv} & \varepsilon_t^{q} & \varepsilon_t^{L} & \varepsilon_t^{w} & \varepsilon_t^{a} & \varepsilon_t^{p} & \varepsilon_t^{g} & \varepsilon_t^{m} \end{matrix} \\
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{\Psi_w}{1+\beta} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{\Psi_p}{1+\beta\gamma_p} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\end{array},
\qquad
\Pi = \begin{array}{c}
\begin{matrix} \eta_t^{\pi} & \eta_t^{w} & \eta_t^{q} & \eta_t^{inv} & \eta_t^{c} & \eta_t^{Rk} \end{matrix} \\
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{array}
$$

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Table 1. Prior Distributions of the Parameters

parameters	meanings	type	mean	S. D.
Structural Parameters				
θ	habit formation	beta	0.7	0.1
σ_c	inverse long-run IES	normal	1	0.375
σ_L	inverse labor supply elasticity	normal	2	0.75
$1/\varphi$	inverse adj. cost	normal	4	1.5
ϕ	fixed cost share	normal	1.45	0.25
ψ	capital utilization cost	normal	0.2	0.075
γ_p	price indexation	beta	0.75	0.15
γ_w	wage indexation	beta	0.75	0.15
ξ_p	Calvo price no-revise prob.	beta	0.75	0.15
ξ_w	Calvo wage no-revise prob.	beta	0.75	0.15
ξ_e	Calvo employment	beta	0.50	0.15
Policy Parameters				
ρ_m	policy, lag interest	beta	0.8	0.1
μ_π	policy, inflation	normal	1.7	0.1
μ_y	policy, output	normal	0.125	0.05
$\mu_{\Delta\pi}$	policy, delta inflation	normal	0.3	0.1
$\mu_{\Delta y}$	policy, delta output	normal	0.0625	0.05
Shock Persistence				
ρ_{target}	persist, target	beta	0.85	0.1
ρ_a	persist, productivity	beta	0.85	0.1
ρ_c	persist, preference	beta	0.85	0.1
ρ_g	persist, gov. expenditure	beta	0.85	0.1
ρ_L	persist, labor supply	beta	0.85	0.1
ρ_{inv}	persist, investment	beta	0.85	0.1
S. D. of Shocks				
ε_c	preference shock	inv. gamma	0.2	2
ε_{inv}	investment shock	inv. gamma	0.1	2
ε_q	equity premium shock	inv. gamma	0.4	2
ε_a	productivity shock	inv. gamma	0.4	2
ε_p	price markup shock	inv. gamma	0.15	2
ε_L	labor supply shock	inv. gamma	1.0	2
ε_w	wage markup shock	inv. gamma	0.25	2
ε_g	gov. expenditure shock	inv. gamma	0.3	2
ε_m	monetary policy shock	inv. gamma	0.1	2
ε_{target}	inflation target shock	inv. gamma	0.02	2

Table 2. Posterior Distributions of the Parameters
(Before the Period of Zero Interest Rate Bound: 1970:Q1 — 1998:Q4)

Parameters	SW (2003) mean	OW (2004) median	LOWW (2005) median	This Paper mean	S. E.	S. D.	90-percent interval		CD
Structural Parameters									
θ	0.592	0.4	0.293	0.641	0.016	0.102	[0.451	0.780]	0.345
σ_c	1.391	2.178	2.167	2.041	0.028	0.296	[1.565	2.530]	-0.202
σ_L	2.503	3.0	1.359	2.427	0.081	0.718	[1.241	3.589]	-0.032
$1/\varphi$	6.962	0.152*	0.541*	8.338	0.036	0.914	[6.870	9.890]	0.185
ϕ	1.417	1.8	1.084	1.581	0.019	0.239	[1.186	1.969]	0.417
ψ	0.201	2.8	0.212	0.182	0.005	0.076	[0.056	0.308]	-0.490
γ_p	0.477	0.323	0.078	0.613	0.006	0.109	[0.439	0.804]	1.412
γ_w	0.728	0.0	0.82	0.578	0.005	0.135	[0.353	0.803]	-2.566
ξ_p	0.905	0.930	0.834	0.650	0.005	0.042	[0.579	0.726]	-0.985
ξ_w	0.742	0.704	0.764	0.367	0.012	0.077	[0.236	0.500]	0.023
ξ_e	0.597	0.400	N.A.	N.A.	N.A.	N.A.	[N.A	N.A]	N.A.
Policy Parameters									
ρ_m	0.956	0.962	0.83	0.682	0.003	0.034	[0.625	0.736]	0.145
μ_π	1.688	4.0	2.749	1.589	0.005	0.095	[1.436	1.747]	0.262
μ_y	0.098	0.099	0.055	0.053	0.003	0.042	[-0.013	0.126]	1.208
$\mu_{\Delta\pi}$	0.151	0.14	0.295	N.A.	N.A.	N.A.	[N.A.	N.A.]	N.A.
$\mu_{\Delta y}$	0.158	0.159	0.505	N.A.	N.A.	N.A.	[N.A.	N.A.]	N.A.
Shock Persistence									
ρ_{target}	0.855	0.582	0.995	N.A.	N.A.	N.A.	[N.A.	N.A.]	N.A.
ρ_a	0.811	0.957	0.962	0.851	0.006	0.077	[0.691	0.937]	2.116
ρ_c	0.838	0.876	0.946	0.368	0.020	0.145	[0.170	0.642]	-0.668
ρ_g	0.943	0.972	0.945	0.792	0.004	0.075	[0.664	0.911]	1.421
ρ_L	0.881	0.974	0.983	0.462	0.007	0.106	[0.284	0.636]	0.582
ρ_{inv}	0.913	0.943	0.734	0.871	0.004	0.048	[0.782	0.935]	1.481
S. D. of Shocks									
ε_c	0.407	0.24	0.125	0.077	0.002	0.015	[0.056	0.106]	-0.087
ε_{inv}	0.113	0.059	1.045	0.046	0.001	0.014	[0.029	0.074]	0.573
ε_q	0.613	7.0	4.047	0.114	0.001	0.017	[0.088	0.144]	0.888
ε_a	0.639	0.343	0.595	0.110	0.002	0.025	[0.078	0.157]	0.444
ε_p	0.165	0.172	0.205	0.245	0.018	0.104	[0.092	0.428]	0.477
ε_L	3.818	2.351	2.352	0.074	0.000	0.005	[0.066	0.082]	0.941
ε_w	0.297	0.246	0.294	0.079	0.003	0.022	[0.052	0.127]	-0.829
ε_g	0.335	0.354	0.287	0.043	0.000	0.003	[0.038	0.048]	1.007
ε_m	0.089	0.0	0.0002	0.011	0.000	0.001	[0.010	0.012]	0.709
ε_{target}	0.033	1.0	0.117	N.A.	N.A.	N.A.	[N.A.	N.A]	N.A

Note:

(a) SW denotes Smets and Wouters' (2003) model. OW denotes Onatski and Willams' (2004) model. LOWW denotes Levin, Onatski, Williams and Williams' (2005) model.

(b) * indicates that it is the mean of φ instead of $1/\varphi$.

(c) The first 250,000 draws of MH algorithm are discarded to guarantee convergence and then the next 100,000 draws are used for calculating the posterior means, the standard errors of the posterior means (S.E.), the standard deviations (S.D.), the 90% intervals and the convergence diagnostic (CD) statistics proposed by Geweke (1992).

(d) The posterior mean is computed by averaging the simulated draws.

(e) S.E. is computed using a Parzen window with a bandwidth of 10,000.

(f) S.D. is computed as the sample standard deviation of the simulated draws.

(g) The 90% intervals refer to 90 % posterior probability bands. These bands are calculated using the 5th and 95th percentiles of the simulated draws.

(h) CD is computed using equation (5.1), where we set $n_A = 10,000$ and $n_B = 50,000$ and compute $\hat{\sigma}_A^2$ and $\hat{\sigma}_B^2$ using a Parzen window with bandwidths of 1,000 and 5,000 respectively.

Table 3. Variance Decomposition
(Before the Period of Zero Interest Rate Bound: 1970:Q1 — 1998:Q4)

		C	I	Y	L	π	W	R
t = 0	Productivity shock	0.041	0.110	0.038	0.286	0.086	0.097	0.000
	Labor supply shock	0.001	0.003	0.001	0.001	0.002	0.043	0.000
	Wage markup shock	0.001	0.001	0.000	0.001	0.002	0.079	0.000
	Price markup shock	0.042	0.050	0.040	0.002	0.872	0.759	0.000
	Preference shock	0.772	0.024	0.527	0.394	0.018	0.015	0.001
	Investment shock	0.001	0.459	0.008	0.005	0.006	0.001	0.000
	Equity premium shock	0.000	0.295	0.003	0.002	0.000	0.000	0.000
	Government spending shock	0.005	0.007	0.273	0.208	0.004	0.001	0.000
	Monetary policy shock	0.138	0.051	0.111	0.085	0.011	0.006	0.999
t = 4	Productivity shock	0.259	0.206	0.275	0.175	0.209	0.422	0.112
	Labor supply shock	0.008	0.006	0.008	0.013	0.008	0.043	0.004
	Wage markup shock	0.003	0.002	0.003	0.005	0.004	0.036	0.002
	Price markup shock	0.133	0.035	0.125	0.121	0.640	0.462	0.241
	Preference shock	0.432	0.040	0.305	0.365	0.046	0.011	0.040
	Investment shock	0.002	0.622	0.029	0.023	0.039	0.012	0.020
	Equity premium shock	0.000	0.056	0.003	0.002	0.000	0.001	0.000
	Government spending shock	0.023	0.012	0.125	0.155	0.008	0.004	0.008
	Monetary policy shock	0.138	0.021	0.123	0.142	0.046	0.009	0.572
t = 10	Productivity shock	0.398	0.216	0.412	0.185	0.210	0.558	0.150
	Labor supply shock	0.013	0.006	0.013	0.021	0.008	0.025	0.006
	Wage markup shock	0.005	0.002	0.005	0.008	0.004	0.019	0.003
	Price markup shock	0.106	0.014	0.094	0.120	0.624	0.274	0.218
	Preference shock	0.321	0.034	0.212	0.343	0.044	0.012	0.050
	Investment shock	0.014	0.682	0.086	0.034	0.056	0.093	0.063
	Equity premium shock	0.002	0.025	0.005	0.003	0.001	0.005	0.001
	Government spending shock	0.034	0.012	0.085	0.149	0.008	0.007	0.010
	Monetary policy shock	0.106	0.089	0.087	0.137	0.046	0.007	0.500
t = 100	Productivity shock	0.256	0.203	0.264	0.184	0.204	0.262	0.159
	Labor supply shock	0.005	0.004	0.006	0.020	0.007	0.005	0.006
	Wage markup shock	0.002	0.001	0.002	0.008	0.003	0.002	0.002
	Price markup shock	0.022	0.006	0.021	0.111	0.479	0.026	0.176
	Preference shock	0.075	0.022	0.058	0.309	0.039	0.017	0.042
	Investment shock	0.593	0.733	0.598	0.108	0.219	0.664	0.198
	Equity premium shock	0.007	0.014	0.008	0.003	0.002	0.008	0.002
	Government spending shock	0.018	0.012	0.025	0.135	0.009	0.011	0.010
	Monetary policy shock	0.022	0.004	0.020	0.122	0.037	0.003	0.403

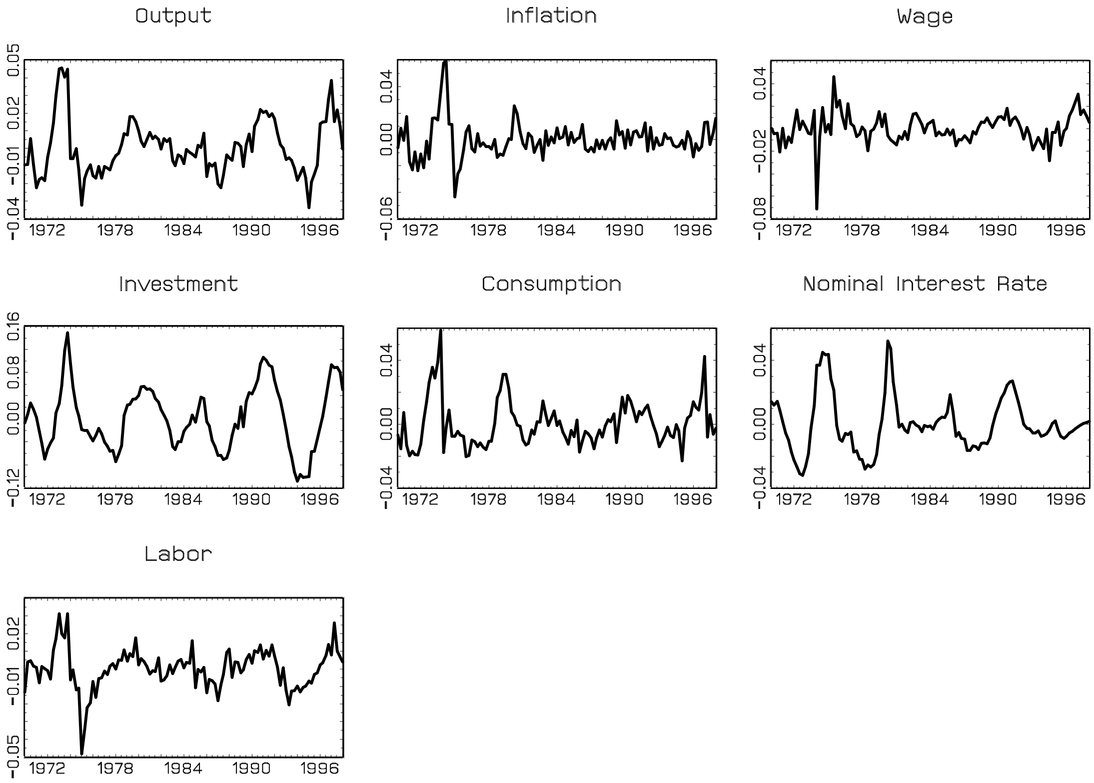


Figure 1. Data

Note: Series output: Real GDP per labour force, seasonally adjusted (unit is 1 million yen at 1990). Series consumption: Real consumption per labour force, s.a., (unit same as above). Series investment: Real investment per labour force, s.a., (unit same as above). Series Labor: Labour input index = (Work hour index*Total Employment) / Labour Population. Series wage: Real wage index = Nominal wage index / GDP deflator. Series inflation: GDP deflator inflation rate (quarterly, annual rate, decadal demeaned). Series Nominal Interest Rate: Uncollateralized call rate (annual rate, decadal demeaned). Output, consumption, investment, labor and wage are transformed to their logarithms and then detrended using the Hodrick and Prescott (HP) filter. Nominal rate and inflation are detrended using HP filter without log-transformation. After those procedure, all above data are obtained by being demeaned.

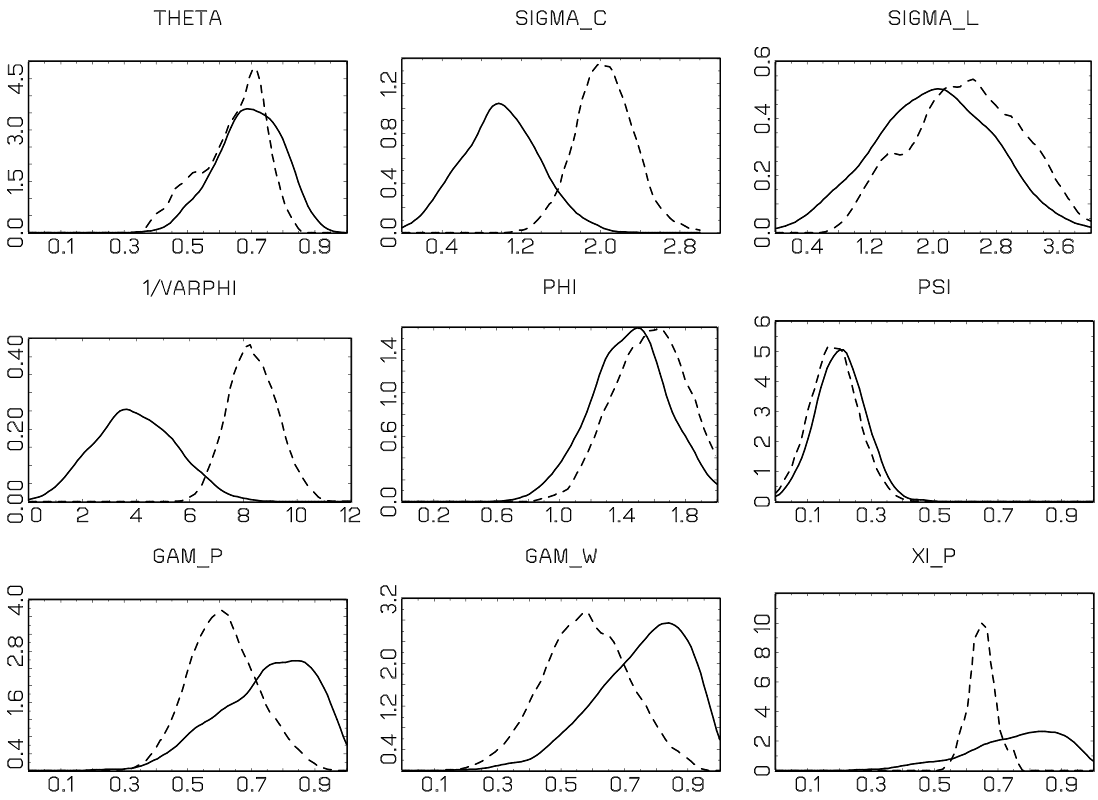


Figure 2A. Estimated Parameter Distribution
(Before the Period of 0% Interest Rate Bound: 1970:Q1 – 1998:Q4)

Note: The straight line plots prior distribution. The dush line plots posterior distribution. THETA is habit persistent parameter. SIGMA C is the inverse intertemporal elasticity of substitution. SIGMA L is the inverse elasiticy of labor supply. 1/VARPHI is the inverse adjustment cost of investment. PHI is the fixed cost share. PSI is capital utilization cost. GAM P is price indexation. GAM W is wage indexation. XI P is Calvo price.

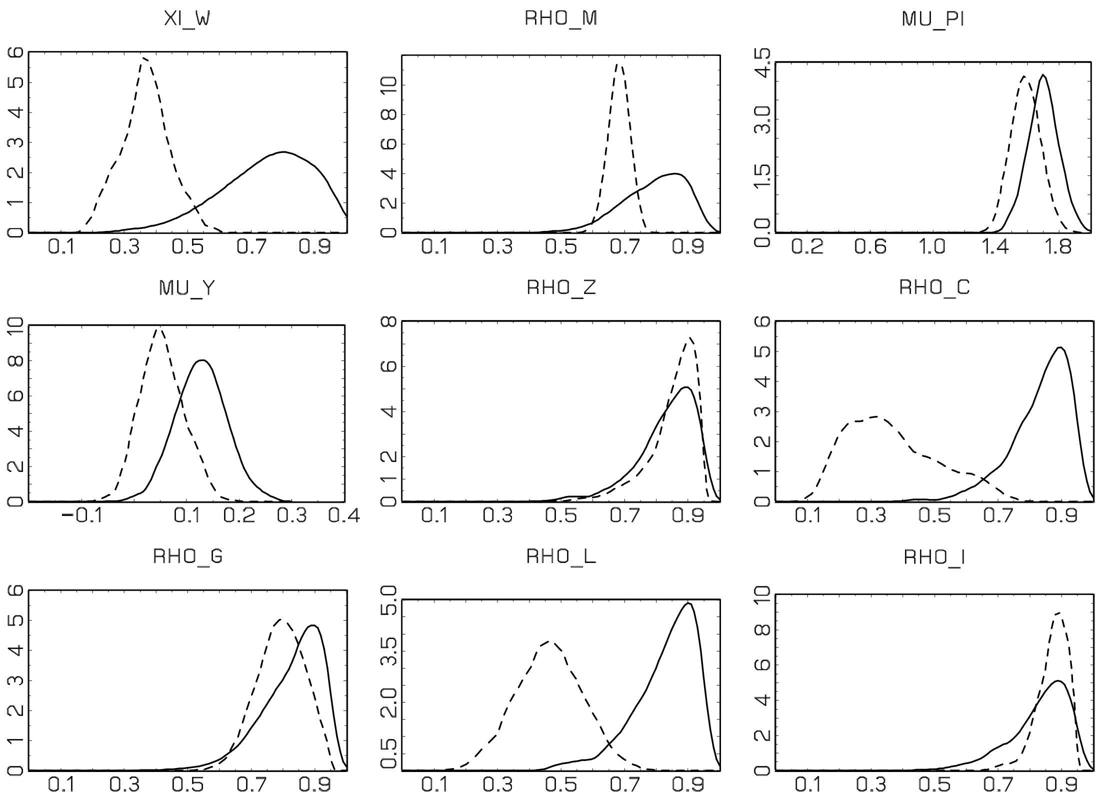


Figure 2B Estimated Parameter Distribution
(Before the Period of 0% Interest Rate Bound: 1970:Q1 – 1998:Q4)

Note: The straight line plots prior distribution. The dush line plots posterior distribution. XI W is Calvo wage. RHO M is lagged interest rate. MU PI is the monetary policy response on inflation. MU Y is the monetary response on output. RHO Z is persistent of productivity shock. RHO C is persistent of preference shock. RHO G is persistent of government spending shock. RHO L is persistent of labor supply shock. RHO I is persistent of investment shock.

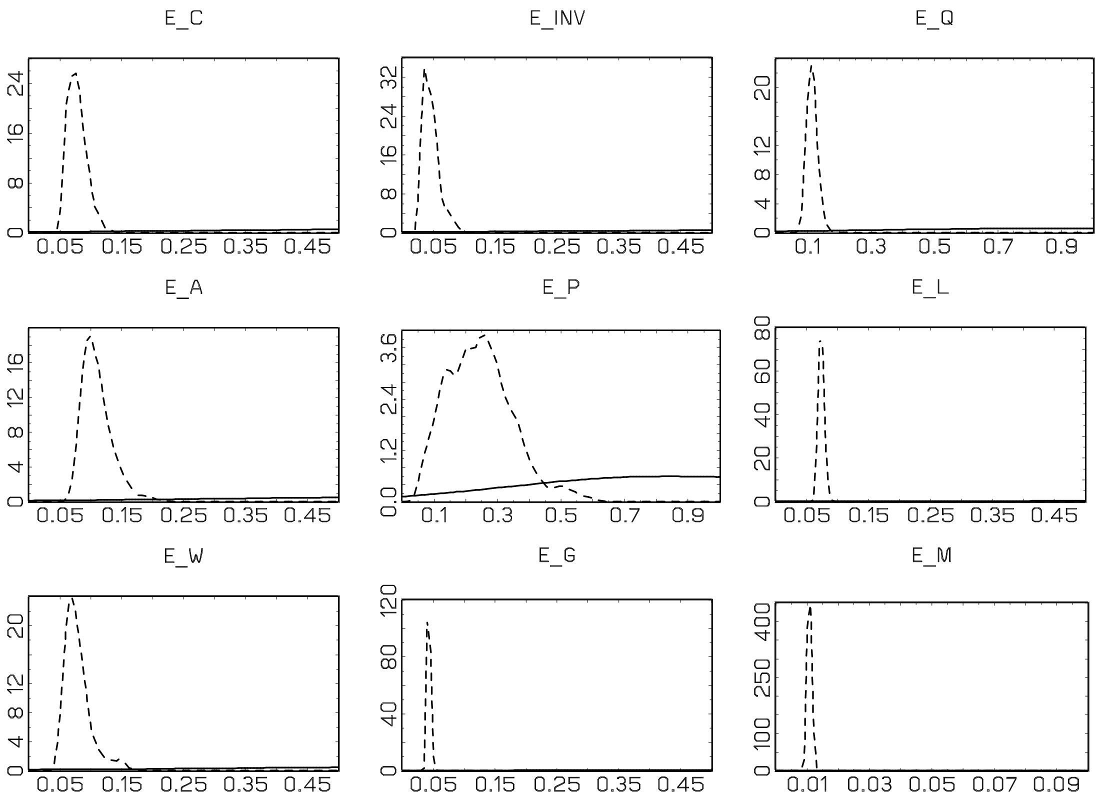


Figure 2C   Estimated Parameter Distribution
(Before the Period of 0% Interest Rate Bound: 1970:Q1 – 1998:Q4)

Note: The straight line plots prior distribution. The dush line plots posterior distribution. E C is the standard error of preference shock. E INV is the standard error of investment shock.. E Q is the standard error of Equity Premium shock. E A is the standard error of productivity shock. E P is the standard error of price-markup shock. E L is the standard error of labor supply shock. E W is the standard error of wage-markup shock. E G is the standard error of government spending shock. E M is the standard error of monetary policy shock.

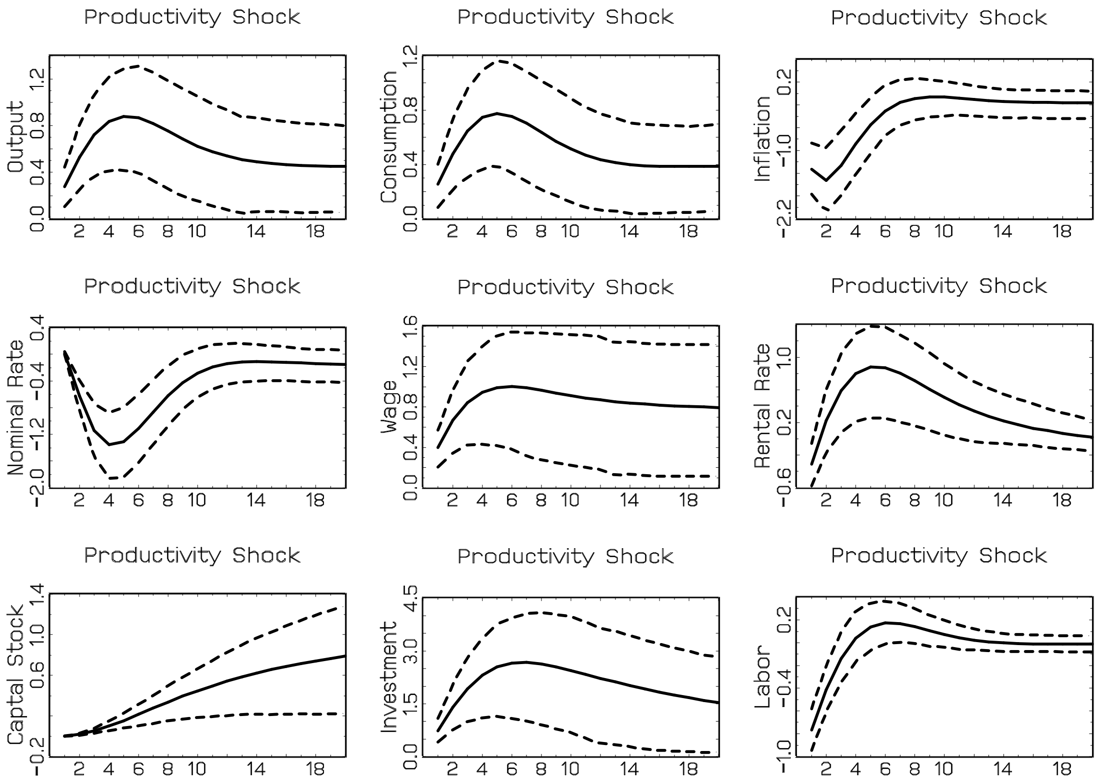


Figure 3A. Productivity Shock

Note: The straight line plot the median response. The dush lines plot the 5th and the 95 percentiles of the response.

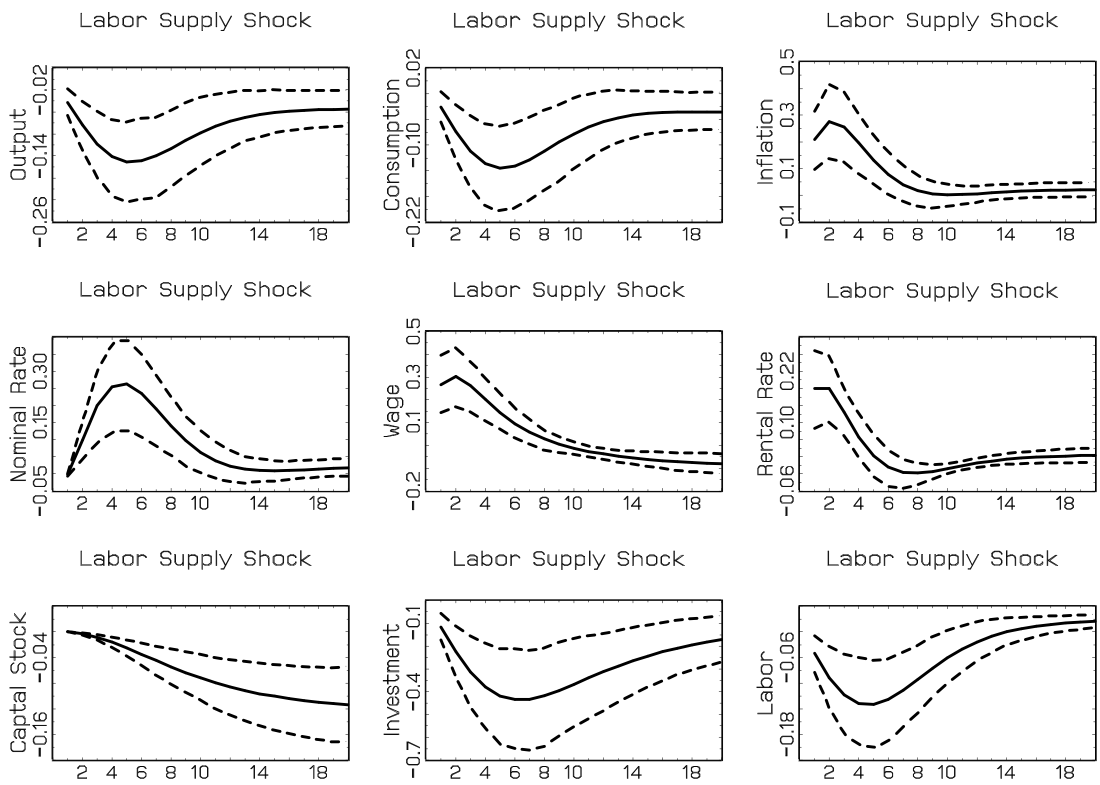


Figure 3B. Labor Supply Shock

Note: The straight line plot the median response. The dush lines plot the 5th and the 95 percentiles of the response.

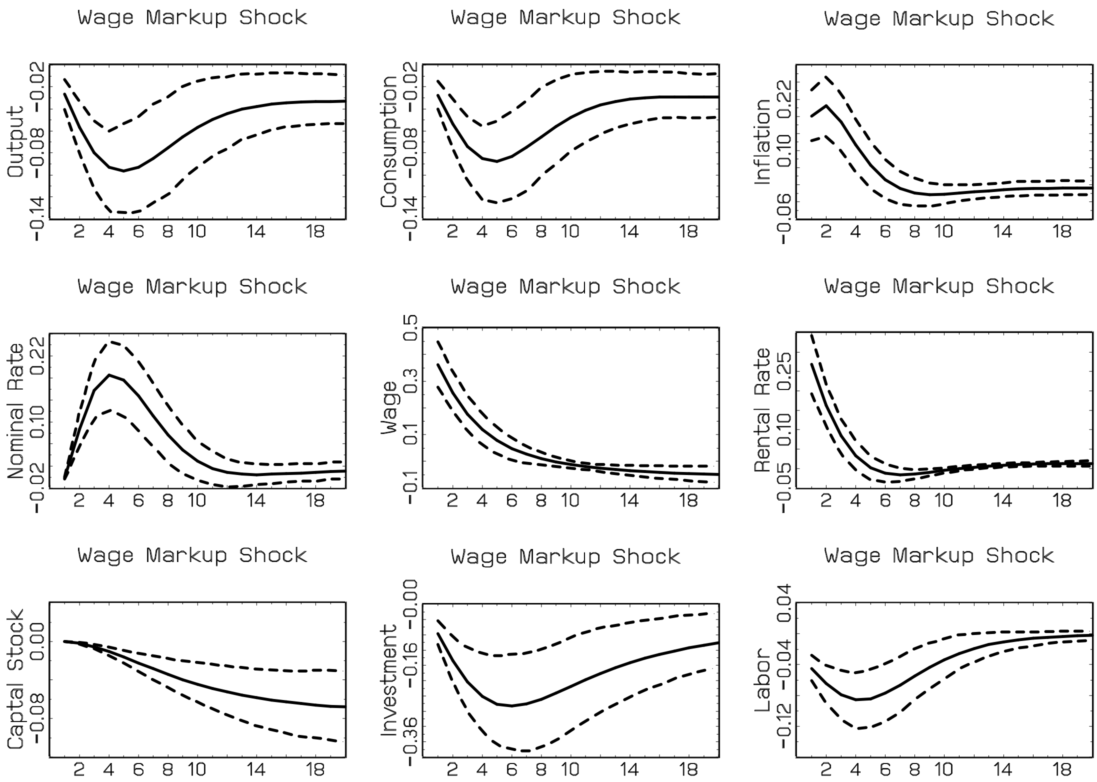


Figure 3C. Wage Markup Shock

Note: The straight line plot the median response. The dush lines plot the 5th and the 95 percentiles of the response.

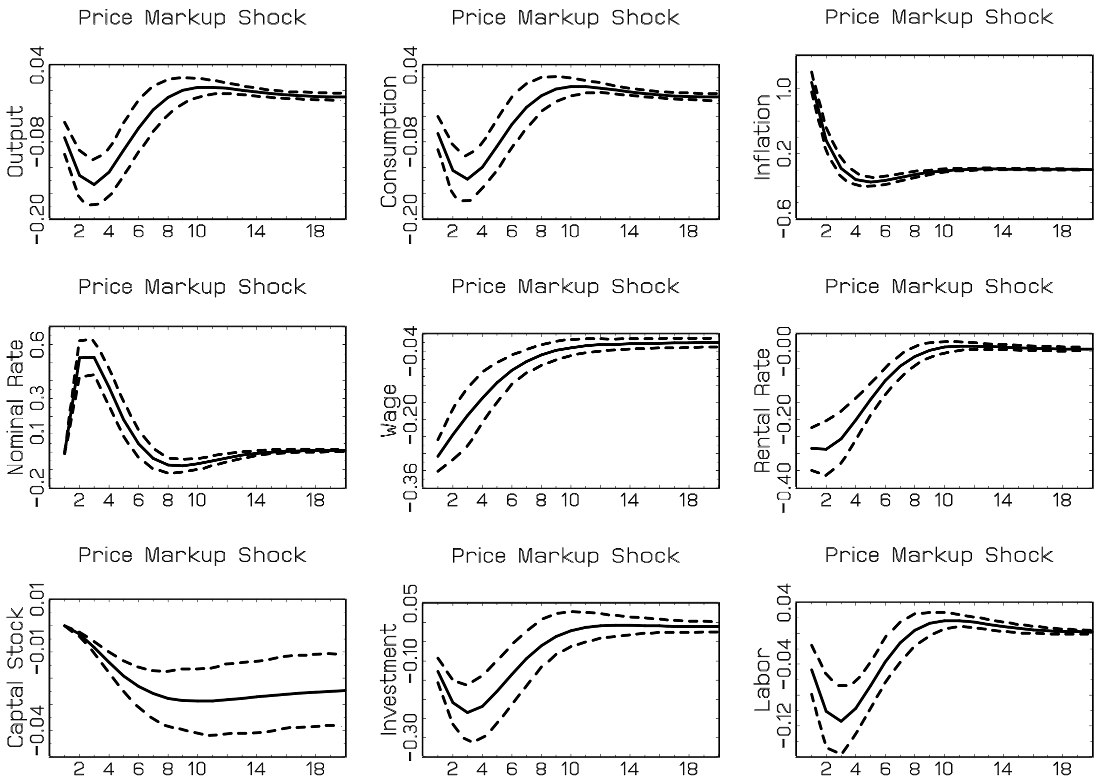


Figure 3D. Price Markup Shock

Note: The straight line plot the median response. The dush lines plot the 5th and the 95 percentiles of the response.

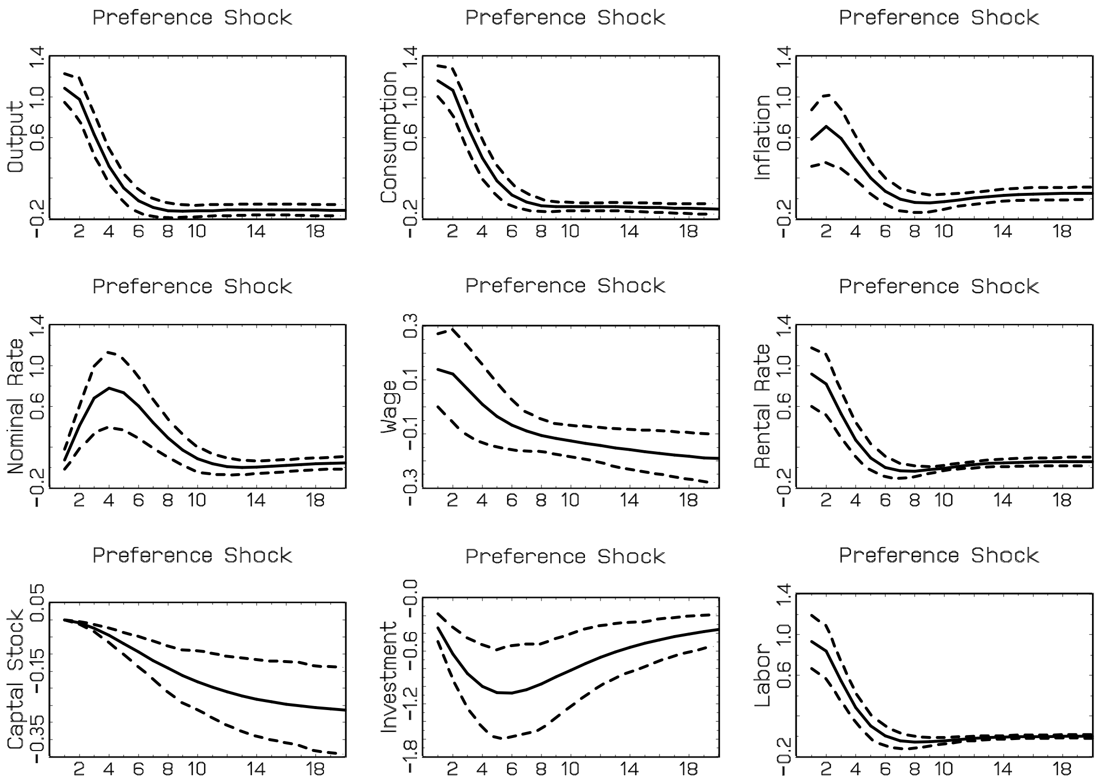


Figure 3E. Preference Shock

Note: The straight line plot the median response. The dush lines plot the 5th and the 95 percentiles of the response.

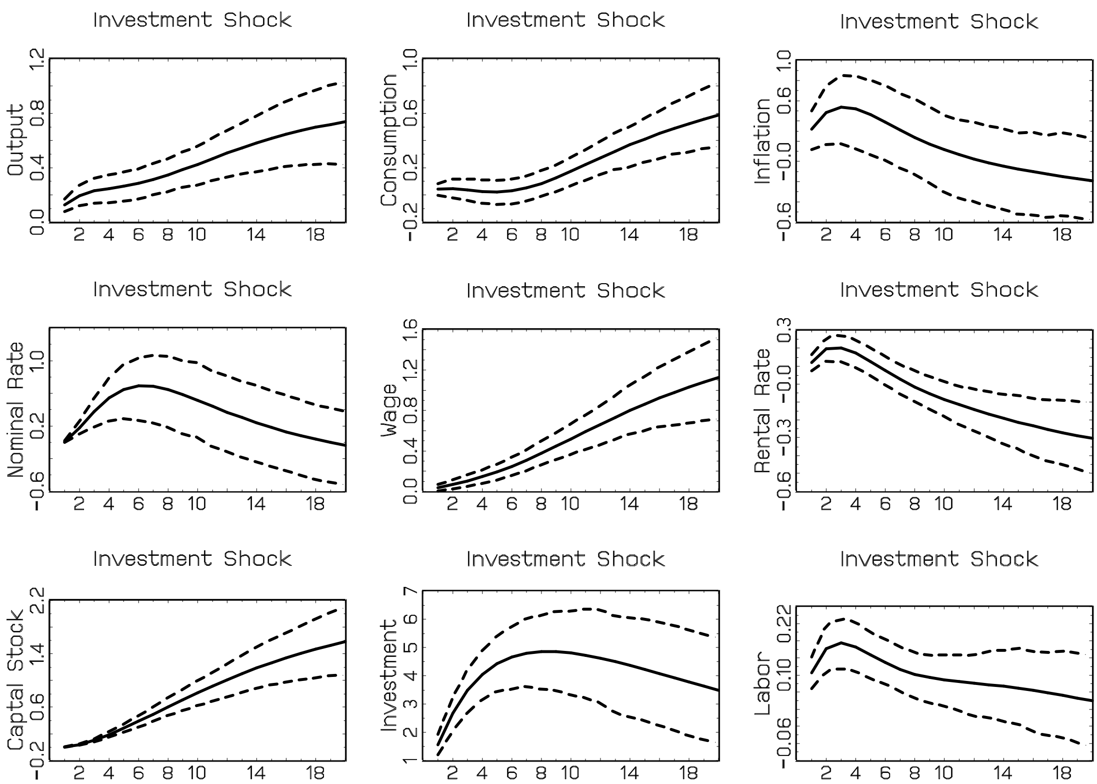


Figure 3F. Investment Shock

Note: The straight line plot the median response. The dush lines plot the 5th and the 95 percentiles of the response.

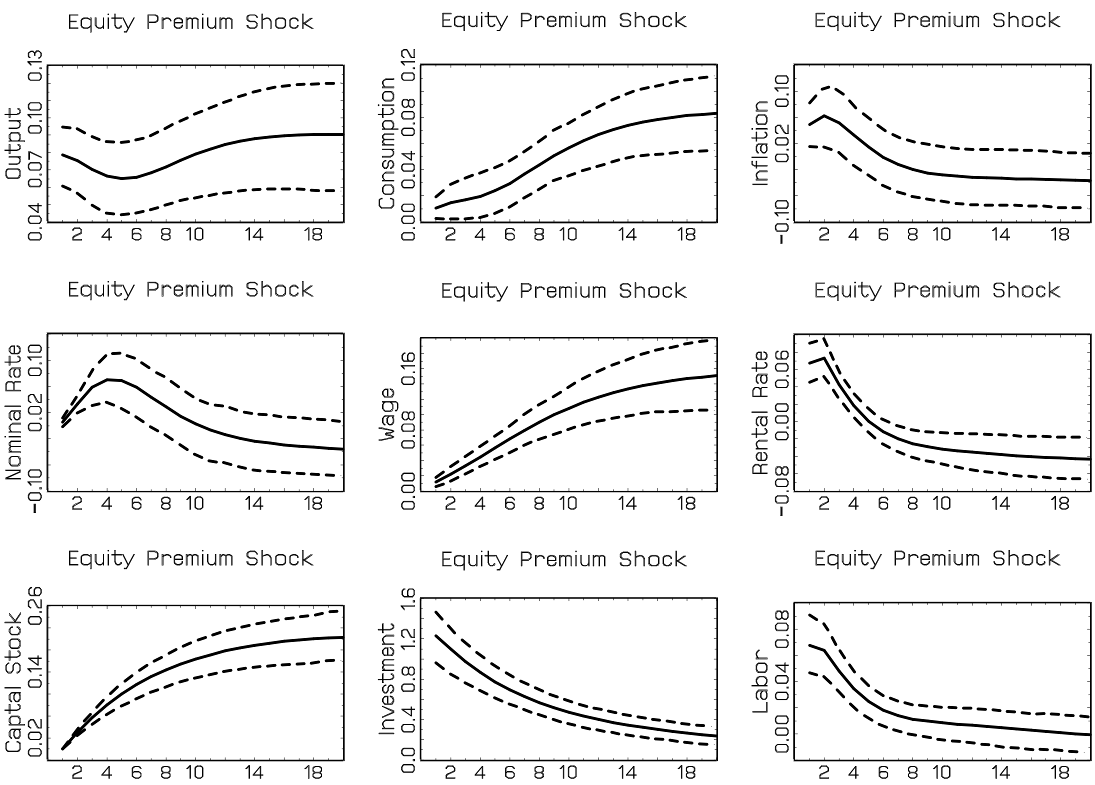


Figure 3G. Equity Premium Shock

Note: The straight line plot the median response. The dush lines plot the 5th and the 95 percentiles of the response.

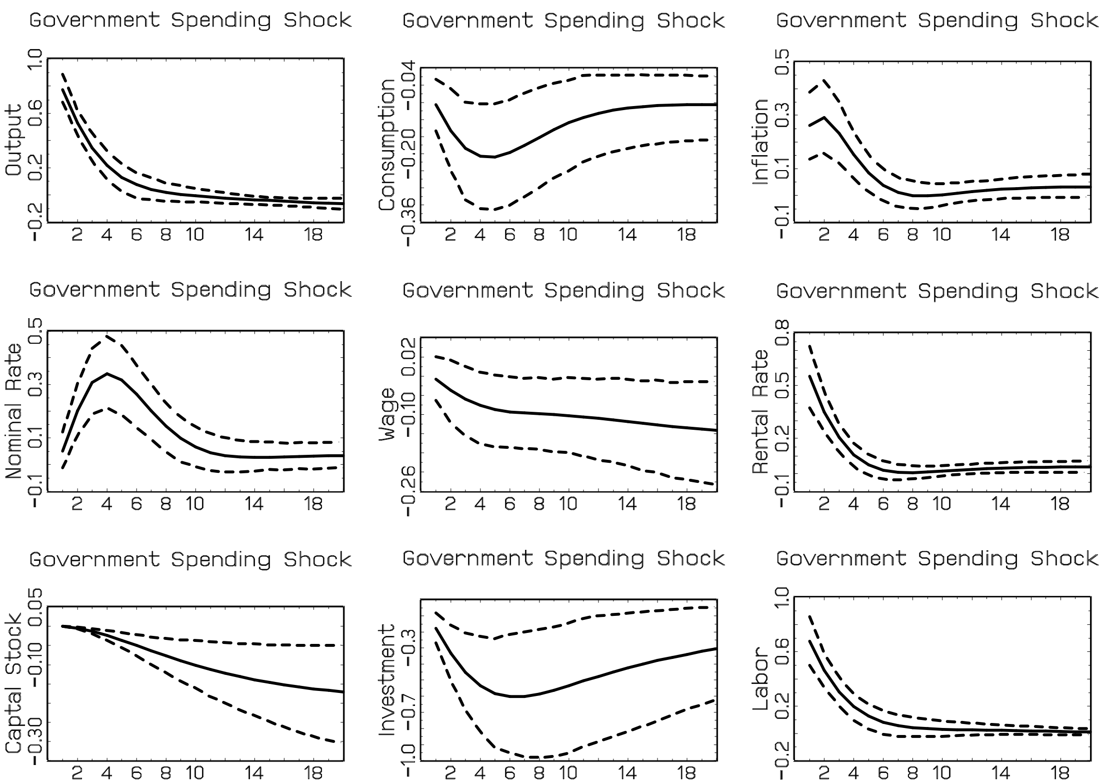


Figure 3H. Government Spending Shock

Note: The straight line plot the median response. The dush lines plot the 5th and the 95 percentiles of the response.

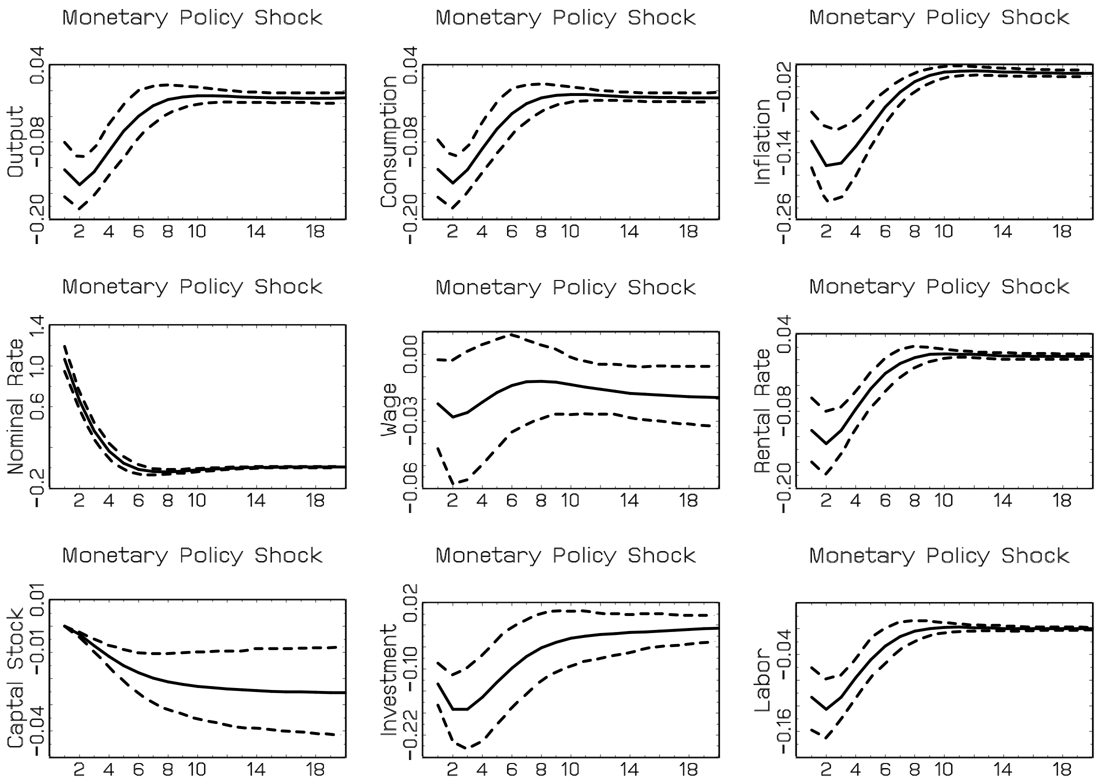
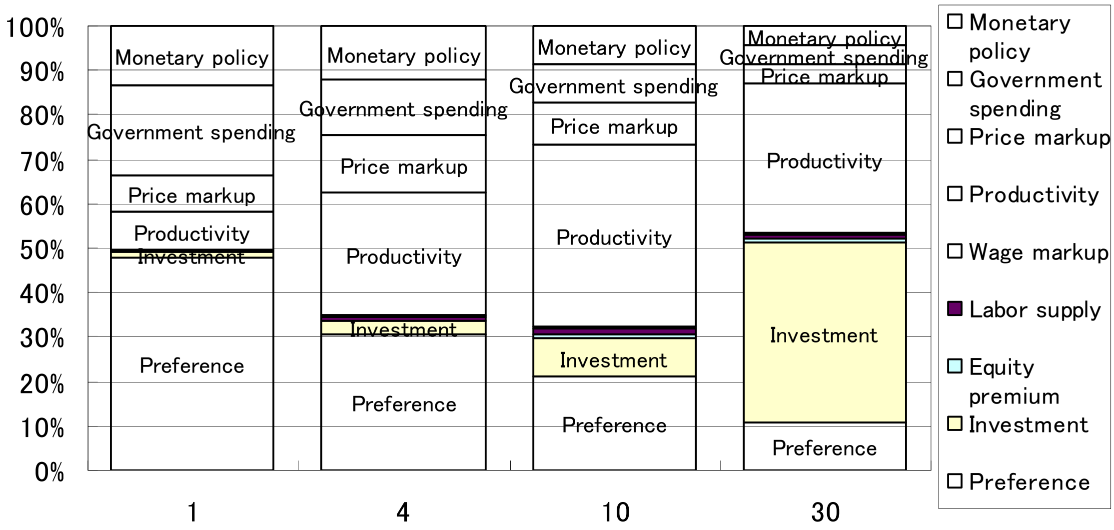


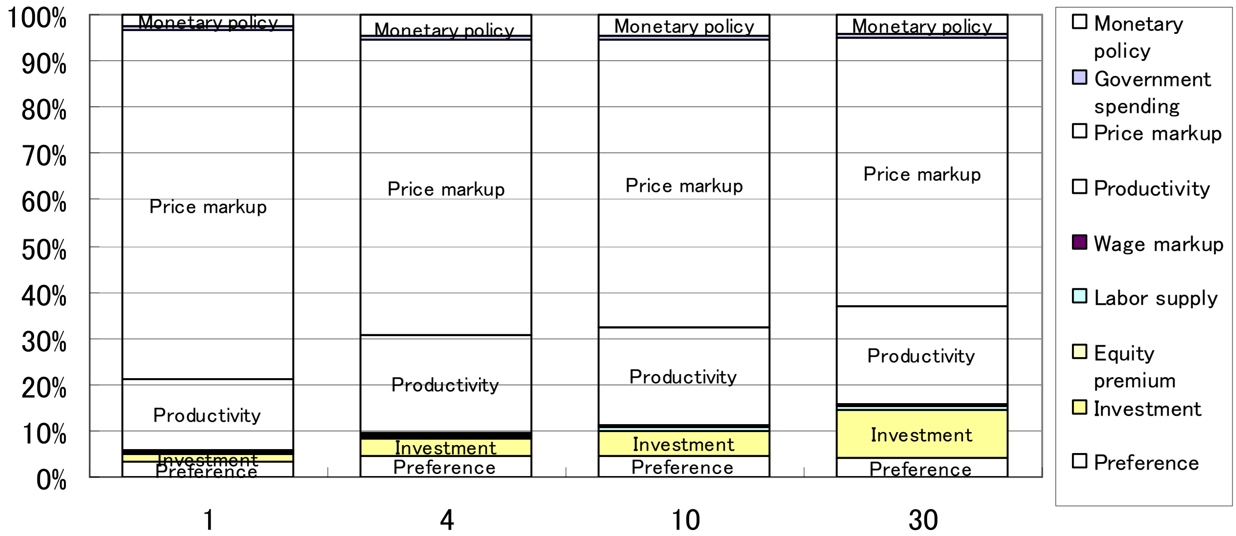
Figure 3I. Monetary Policy Shock

Note: The straight line plot the median response. The dush lines plot the 5th and the 95 percentiles of the response.



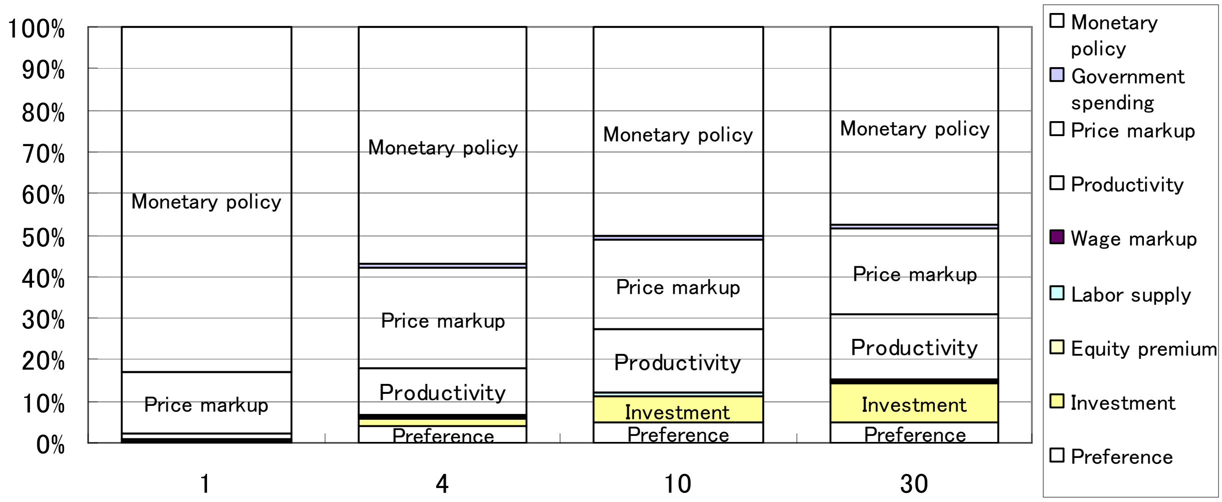
	1		4		10		30	
	5%	95%	5%	95%	5%	95%	5%	95%
Preference	0.39	0.58	0.20	0.41	0.10	0.31	0.04	0.18
Investment	0.00	0.02	0.01	0.05	0.03	0.15	0.17	0.65
Equity premium	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.01
Labor supply	0.00	0.00	0.00	0.02	0.00	0.02	0.00	0.02
Wage markup	0.00	0.00	0.00	0.01	0.00	0.01	0.00	0.01
Productivity	0.01	0.15	0.05	0.44	0.14	0.67	0.02	0.58
Price markup	0.04	0.12	0.08	0.18	0.04	0.14	0.01	0.07
Government spending	0.13	0.27	0.08	0.18	0.05	0.13	0.02	0.07
Monetary policy	0.08	0.19	0.07	0.17	0.04	0.13	0.01	0.07

Figure 4A. Variance Decomposition of Output.



	1		4		10		30	
	5%	95%	5%	95%	5%	95%	5%	95%
Preference	0.01	0.06	0.01	0.08	0.01	0.08	0.01	0.08
Investment	0.00	0.03	0.00	0.08	0.00	0.12	0.02	0.18
Equity premium	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Labor supply	0.00	0.01	0.00	0.02	0.00	0.02	0.00	0.01
Wage markup	0.00	0.00	0.00	0.01	0.00	0.01	0.00	0.01
Productivity	0.06	0.24	0.08	0.33	0.07	0.34	0.07	0.34
Price markup	0.66	0.87	0.52	0.77	0.50	0.75	0.43	0.72
Government spending	0.00	0.01	0.00	0.01	0.00	0.01	0.00	0.01
Monetary policy	0.01	0.04	0.02	0.07	0.01	0.07	0.01	0.07

Figure 4B. Variance Decomposition of Inflation.



	1		4		10		30	
	5%	95%	5%	95%	5%	95%	5%	95%
Preference	0.00	0.01	0.01	0.07	0.01	0.09	0.01	0.09
Investment	0.00	0.00	0.00	0.04	0.00	0.12	0.02	0.17
Equity premium	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Labor supply	0.00	0.00	0.00	0.01	0.00	0.01	0.00	0.01
Wage markup	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Productivity	0.00	0.02	0.04	0.18	0.05	0.24	0.04	0.26
Price markup	0.10	0.20	0.17	0.31	0.15	0.29	0.14	0.28
Government spending	0.00	0.00	0.00	0.01	0.00	0.02	0.00	0.02
Monetary policy	0.77	0.89	0.48	0.67	0.40	0.60	0.35	0.57

Figure 4C. Variance Decomposition of Interest Rate.