Tracing value-added and double counting in sales of foreign affiliates and domestic-owned companies

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TRACING VALUE-ADDED AND DOUBLE COUNTING IN SALES OF FOREIGN AFFILIATES AND DOMESTIC-OWNED COMPANIES

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ABSTRACT: The literature on trade in value-added has emphasised that gross trade flows do not adequately measure the income generated by trade when many intermediate inputs are imported. While this literature has deepened our understanding of global value chains, it is still missing an important element when analysing income generation along the value chain: the fact that domestic value-added often results from activities of foreign-owned companies. Studies that look at activities of multinational enterprises (MNEs) still rely on the concept of sales of foreign affiliates, which is a gross concept also subject to double counting when it comes to the use of intermediate inputs.

In this paper, we propose a new accounting framework for the decomposition of value-added into domestic, foreign and double counting terms in domestic sales. In this framework, we show where the value-added double counting is derived from and give an explicit expression of domestic and foreign double counting terms based on the Inter-Country Input-Output (ICIO) tables' Ghosh insight. We can distinguish domestic sales from exports and trace the value added and double counting in sales of foreign affiliates and domestic-owned enterprises. Based on this framework, we then calculate the value-added by foreign-owned and domestic-owned firms in exports and in domestic sales by using an Inter-Country Input-Output table split according to ownership. Preliminary results suggest that there is much more double counting in sales of foreign affiliates than in exports and that more value-added is created through exports than through sales of foreign affiliates in world GDP.

Keywords: inter-country input-output, value-added decomposition, global value chains, foreign affiliates

JEL Codes: E01, E16, F23, L14

1. Introduction

The literature on trade in value-added (Johnson and Noguera, 2012; Koopman et al., 2014; Los et al., 2016; Nagengast and Stehrer, 2016) as well as empirical datasets, such as the Trade in Value-Added (TiVA) indicators released by OECD and WTO in 2013, have emphasised that gross trade flows do not adequately measure the income generated by trade in a world characterised by global supply chains where intermediate products are traded across countries. New accounting frameworks have been developed to identify the domestic value-added in gross exports and in final demand and to remove the double counting of intermediate inputs that cross international borders more than once.

But trade is only one dimension in the activities of firms involved in global production. Some of these firms are multinational enterprises (MNEs) that rely on foreign affiliates to source inputs or produce abroad. According to UNCTAD (2013), 80% of global trade is co-ordinated by these MNEs (when including their arm's length trade transactions as well as trade flows related to franchising, contract manufacturing and strategic alliances).

The empirical literature analysing activities of MNEs relies on the concept of sales of foreign affiliates (Dunning, 1980; Brainard, 1997; Bergstrand and Egger, 2007), which is also a gross concept and includes some double counting with respect to foreign and domestic inputs. Somehow this concept has not yet been through the kind of 'value-added revolution' that has significantly changed the analysis of trade. Also, the concept of sales of foreign affiliates covers both their domestic sales and their exports. When exporting, the output of foreign affiliates can be decomposed with the same tools created for TiVA analysis. But it is different in the context of domestic sales.

In this paper, we are interested in decomposing not only trade but also domestic sales in a consistent framework that can allow us to identify the activities of foreign-owned firms. Such a decomposition can shed light on the reasons why firms engage in FDI. The literature suggests that foreign affiliates can be involved: (1) in the production of (final) goods for domestic consumers in the case of 'horizontal FDI' (Markusen, 1984); (2) in the production of (final) goods for foreign consumers in the case of 'export platform FDI' (Ekholm et al., 2007); or (3) in the production of inputs for other affiliates in the host economy or abroad in the case of 'vertical FDI' (Helpman, 1984).[1] More recent work indicates that in many instances firms engage in 'complex FDI' combining horizontal and vertical motives (Alfaro and Charlton, 2009), or set up affiliates for other purposes than contributing to the production process such as 'conglomerate FDI' or FDI for financial purposes (Herger and

[1] This literature tends to not distinguish final from intermediate goods. While vertical FDI is more about investment in order to supply inputs, the case of final assembly by the affiliate is still ambiguous.

McCorriston, 2016; Ray, 2016). There is therefore a need for more empirical work on value creation in relation to activities of foreign affiliates.

Moreover, by applying to sales of foreign affiliates the same kind of treatment applied to trade flows in the context of value-added analyses, we would like to provide a more accurate measurement of the importance of MNEs in global production. Figures such as the one proposed by UNCTAD seem to overstate the true importance of MNEs in trade and output as they are based on gross figures and not on a value-added decomposition.

The paper is organised as follows. Section 2 details the methodology, first presenting an alternative mathematical framework to derive the domestic and foreign value-added multiplier coefficients in an inter-country input-output framework. Based on this, we provide a full decomposition of the value-added in domestic sales following the Ghosh insight and identifying double counting terms in addition to domestic and foreign value-added. In Section 3, we use this conceptual framework to decompose GDP in the context of ICIO tables that have an ownership dimension (splitting data for domestic-owned and foreign-owned firms). Section 4 concludes.

2. Methodology

This section introduces a new type of value-added decomposition in the inter-country input-output (ICIO) framework. The starting point is that gross output consists of domestic sales (i.e. domestic shipments) and exports (i.e. shipments to foreign countries). Some important efforts have been devoted to tracing domestic and foreign value-added, as well as double counting, in gross exports (following Koopman et al., 2014), but not for domestic shipments. Domestic sales are also interesting in terms of their domestic and foreign value-added content, especially when these domestic sales result from activities of foreign-owned companies. Our objective is to provide a full decomposition of GDP in a given economy that would allow us to identify the domestic and foreign value-added in domestic sales and in exports, and ultimately to compare the foreign value-added coming from exports with the value added by foreign-owned firms in the domestic economy (which is part of the "domestic" value-added in current decompositions).

2.1 Clarifying the domestic sales in IO tables and value-added multiplier coefficients

Leontief (1936) established that the amount and type of intermediate inputs needed in the production of one unit of output can be estimated based on the input-output (IO) structure across industries. Using the linkages across industries, one can trace output in all stages of production needed to produce one unit of final goods. When the gross output flows associated with a specific level of final demand are

known, value-added production and trade can be simply derived by multiplying these flows with the value added to gross output ratio in each industry.

In the IO framework, all gross output must be used either as an intermediate good or as a final good,

$$X = AX + Y \tag{1}$$

where, X is the $N \times 1$ gross output vector, Y is the $N \times 1$ final demand vector, and A is the $N \times N$ I-O coefficients matrix.

The accounting relationship between domestic sales H and final demand in an Inter-Country Input-Output (ICIO) model can be expressed as:

$$H = \overline{A}H + \overline{Y} \tag{2}$$

Here, $\overline{A} = A^D(I - A^F)^{-1}$, with A^D the domestic coefficients in the global ICIO table (i.e. the block-diagonal matrix of the *A* matrix in the ICIO table). A^F is the export matrix of A indicating the use of intermediate inputs from one country into another country. In addition, $\overline{Y} = Y^D + \overline{A}Y^F$, Y^D denoting domestic final demand and Y^F final demand in foreign countries. The full derivation is provided in Appendix I (Lemma 1).

Each element of the $\overline{A}$ matrix describes how domestic intermediate goods are sent abroad (or transported domestically) to produce one unit of domestic sales in foreign countries (or in the domestic economy). For example, the element $\overline{A}_{ij}$ means that in order to produce one unit of domestic sales in country *j*, country *i* needs to produce $\overline{A}_{ij}$ units of intermediate inputs that are then embodied in domestic sales in country *j*. $\overline{A}_{ij}H_j$ means that country *i* needs to produce $\overline{A}_{ij}H_j$ intermediate inputs for domestic sales H_j in country *j*, so we can call $\overline{A}$ as the 'direct domestic sales requirements matrix'. Re-arranging equation (2) above, we obtain $H = \overline{B}\overline{Y}$, and $\overline{B} = (I - \overline{A})^{-1}$, similar to $B = (I - A)^{-1}$ in the IO model. We can define matrix $\overline{B}$ as the 'total domestic sales requirements matrix'.

With respect to $\overline{B}$, we have

$$\begin{aligned}
&\overline{B} = (I - \overline{A})^{-1} = [I - A^D(I - A^F)^{-1}]^{-1} = [(I - A^F)(I - A^F)^{-1} - A^D(I - A^F)^{-1}]^{-1} \\
&= [(I - A^F - A^D)(I - A^F)^{-1}]^{-1} \\
&= (I - A^F)B = (I - A + A^D)B = I + A^D B
\end{aligned}$$

So, for any element in matrix $\overline{B}$, $\overline{B}_{ij} = \begin{cases} I + A_{ii}B_{ij} & i = j \\ A_{ii}B_{ij} & i \neq j \end{cases}$.

For H_i, the domestic sales in country i, all the intermediate inputs needed are $\sum_{j}^{G} \overline{A}_{ji} H_i$. We can thus calculate the value-added in domestic sales in country i as $VaH(i)^T = H_i - \sum_{j}^{G} \overline{A}_{ji} H_i$. This value-added does not only include country i's value-added but also other countries' value-added. We can then express the value-added multiplier coefficients in domestic sales in the form of a matrix $\overline{V}$, defined as:

$$\overline{V} = u(I - \overline{A}) = u(I - A)(I - A^F)^{-1} = V(I - A^F)^{-1} \tag{3}$$

where V is a 1×N direct value-added coefficients vector. Each element of V_i gives the share of direct domestic value-added in total output. It is equal to one minus the intermediate input share from all countries (including domestically produced intermediates): $V_i = u[I - \sum_{j}^{G} A_{ji}]$, where u is a 1×N unit vector. If we use the notation $B^F = (I - A^F)^{-1}$, we obtain the expression for value-added coefficients in domestic sales in country i: $\overline{V}_i = V_i B_{ii}^F + \sum_{j \neq i}^{G} V_j B_{ji}^F$. They can be divided into two parts: the value-added from country i (domestic part) $V_i B_{ii}^F$ and the value-added from other countries (foreign part) $\sum_{j \neq i}^{G} V_j B_{ji}^F$.

2.2 Measurement of value-added in domestic sales

Moreover, we also can derive a consistent measure of the domestic and foreign value-added (or GDP) in domestic sales from the initial ICIO model. In the ICIO model, gross exports and gross output can be written as:

$$E = (I - A^F)^{-1} A^F H + (I - A^F)^{-1} Y^F \tag{4}$$

$$X = H + E = (I - A^F)^{-1} H + (I - A^F)^{-1} Y^F = B^F H + B^F Y^F \tag{5}$$

Based on the expression above, for country i's output X_i:

$$X_i = B_{ii}^F H_i + \sum_{j \neq i}^{G} B_{ij}^F H_j + \sum_{j}^{G} B_{ij}^F Y_j^F \tag{6}$$

Therefore, country *i*'s GDP can be measured as:

$$GDP_i = V_i X_i = V_i B_{ii}^F H_i + V_i \sum_{j \neq i}^{G} B_{ij}^F H_j + V_i \sum_{j}^{G} B_{ij}^F Y_j^F \tag{7}$$

According to equation (7), country *i*'s GDP can be divided into 3 parts. $V_i \sum_{j}^{G} B_{ij}^F Y_j^F$ is the share of GDP in exports of final products, $V_i B_{ii}^F H_i$ is GDP in country *i*'s domestic sales and $V_i \sum_{j \neq i}^{G} B_{ij}^F H_j$ is GDP in foreign countries' domestic sales via exports of intermediates from country *i* to other countries and measured as foreign value-added for other countries. Similarly, we can also express domestic value-added in domestic sales in a consistent way and regard coefficients $V_i B_{ii}^F$ as the domestic value-added multiplier coefficients for a country's domestic sales.

Symmetrically, we can obtain the expression of country *j*'s GDP as $GDP_j = V_j X_j = V_j B_{ji}^F H_i + V_j \sum_{k \neq i}^{G} B_{jk}^F H_k + V_j \sum_{k}^{G} B_{jk}^F Y_k^F$. From the point of view of country *j*'s GDP, the part $V_j B_{ji}^F H_i$ is included in country *i*'s domestic sales. If we sum up the value-added from all countries (except country *i*) in country *i*'s domestic sales, we also obtain an expression for the foreign value-added part of country *i*'s domestic sales, measured as $\sum_{j \neq i}^{G} V_j B_{ji}^F H_i$. Therefore, the value-added in country *i*'s domestic sales should be equal to the domestic part plus the foreign part: $V_i B_{ii}^F H_i + \sum_{j \neq i}^{G} V_j B_{ji}^F H_i = \bar{V}_i H_i$.

2.3 Tracing value-added in domestic sales: the Ghosh insight

The Ghosh model (Ghosh, 1958), in turn, is also known as the 'supply–driven' input-output model, since value-added is the exogenously specified driving force of the model. Although the Ghosh model is generally interpreted as a price model (Dietzenbacher, 1997), it can be applied to the analysis of the structure of value-added flows as an alternative to Leontief's 'demand-driven' model. The 'supply–driven' accounting identity states that country *i*'s total input (which should be equal to domestic sales here) is equal to the value of its initial inputs (domestic and foreign value-added) plus its intermediate input flows from all other countries (which can be interpreted as value-added double counting terms).

In the IO table, the output coefficient is defined as $l_{ij} = x_{ij} / x_i$. Output coefficients give the output percentage of industry *i* that is sold to industry *j*. The accounting equation can be rewritten as:

$$X^T = VA^T + X^T L = VA \cdot G \tag{8}$$

where $G = (I - L)^{-1}$ denotes the Ghosh inverse; Meanwhile, $G = \hat{X}^{-1} B \hat{X}$, where $\hat{X}$ is a $N \times N$ diagonal matrix with output on the diagonal.

Similarly, in the domestic sales input-output table, domestic sales can be written as $H^T = VaH^T + H^T \bar{L} = VaH^T \cdot \bar{G}$. Here $\bar{G} = \hat{H}^{-1} \bar{B} \hat{H}$, $\bar{L} = \hat{H}^{-1} \bar{A} \hat{H}$ and $\bar{L}_{ij} = \hat{H}_i^{-1} \bar{A}_{ij} \hat{H}_j$. $\bar{L}_{ij}$ gives the share of country *i*'s goods in country *j*'s domestic sales.

To illustrate the relationship between domestic sales and value-added, we can refer to the Taylor expansion:

$$H^T = VaH^T (I + \bar{L} + \bar{L}^2 + \bar{L}^3 + \cdots) \tag{9}$$

In the value-added input VaH^T , the domestic sales value is H^T , which is decomposed into three value-added terms: an initial input VaH^T , a direct input $VaH^T \cdot \bar{L}$ in the first round and indirect input in subsequent rounds amounting to $VaH^T (\bar{L}^2 + \bar{L}^3 + \cdots)$.

Following the Ghosh insight, we can give the full decomposition for country *i*'s domestic sales:

$$\begin{aligned} H_i^T = {} & VaH(i)^T + VaH(i)^T \bar{L}_{ii} + \sum_{j \neq i}^{G} VaH(j)^T \bar{L}_{ji} \\ & + VaH(i)^T [\bar{L}]^2_{ii} + VaH(i)^T [\bar{L}]^3_{ii} + \cdots \\ & + \sum_{j \neq i}^{G} VaH(j)^T [\bar{L}]^2_{ji} + \sum_{j \neq i}^{G} VaH(j)^T [\bar{L}]^3_{ji} + \cdots \end{aligned} \tag{10}$$

The above expression provides an explicit interpretation of the decomposition of domestic sales according to the Ghosh insight. Every sub-term has an economic interpretation.

The initial effect is the value-added in country *i*'s domestic sales. According to the equation above, this term is equal to $VaH(i)^T = \bar{V}_i \hat{H}_i = (V_i B_{ii}^F + \sum_{j \neq i}^{G} V_j B_{ji}^F) \hat{H}_i$, which is the initial value-added input in the Ghosh insight. As value-added is fully measured in this round, the value-added found in the later rounds is part of double counting terms (i.e. value-added that has already been measured and further

goes to sectors and/or countries as intermediate input). If we expand this term, it includes domestic value-added initial inputs $V_i B_{ii}^F \hat{H}_i$ and foreign value-added initial inputs $\sum_{j \neq i}^{G} V_j B_{ji}^F \hat{H}_i$ which are contained in goods imported from country *j*.

In the first round, it means that the value-added term which is already counted in the initial round propagates through the matrix $\overline{L}_{ii} = \hat{H}_i^{-1} \overline{A}_{ii} \hat{H}_i$ (having in mind that $\overline{A}_{ii} = A_{ii} B_{ii}^F = A_{ii} + A_{ii}[A^F]_{ii}^2 + A_{ii}[A^F]_{ii}^3 + \cdots$, this value-added propagation route includes not only what has gone across the country border but also across domestic sectors as intermediate inputs). Because this value-added was already measured in the initial round, it should be part of the value-added double counting terms in later rounds. The direct effect can be divided into two parts, the effect from country *i*'s inputs (which is not domestic value-added here) and from other country *j*'s inputs (which is not foreign value-added here). Country *i*'s input is equal to:

$$VaH(i)^T \overline{L}_{ii} = \overline{V}_i \hat{H}_i \cdot \hat{H}_i^{-1} \overline{A}_{ii} \hat{H}_i = \overline{V}_i \overline{A}_{ii} \hat{H}_i \tag{11}$$

The other countries' value-added within intermediate inputs is imported from country *j*. These terms are equal to:

$$\sum_{j \neq i}^{G} VaH(j)^T \cdot \overline{L}_{ji} = \sum_{j \neq i}^{G} \overline{V}_j \hat{H}_j \cdot \hat{H}_j^{-1} \overline{A}_{ji} \hat{H}_i = \sum_{j \neq i}^{G} \overline{V}_j \overline{A}_{ji} \hat{H}_i \tag{12}$$

In the second round, the additional value-added has a similar interpretation and can also be divided into the domestic input and foreign input parts. It still accounts for value-added double counting terms passed from country *i*'s domestic propagation to the other countries and returned back home. This implies that for the domestic input part, the value-added coming from country *i* is $VaH(i)^T \sum_{k}^{G} \overline{L}_{ik} \overline{L}_{ki}$, reflecting value-added from country *i* $VaH(i)^T \overline{L}_{ik}$ propagated to country *k*. The $\overline{L}_{ki}$ part in country *k* returned back home. This part of value-added has already been measured in the initial round, so it should still be counted as value-added double counting term (domestic). We have

$$\begin{aligned} VaH(i)^T [\overline{L}]_{ii}^2 &= VaH(i)^T \sum_{k}^{G} \overline{L}_{ik} \overline{L}_{ki} \\ &= \overline{V}_i \hat{H}_i \sum_{k}^{G} \hat{H}_i^{-1} \overline{A}_{ik} \hat{H}_k \cdot \hat{H}_k^{-1} \overline{A}_{ki} \hat{H}_i = \overline{V}_i \sum_{k}^{G} \overline{A}_{ik} \overline{A}_{ki} \hat{H}_i \end{aligned} \tag{13}$$

For the value-added contributed by country *j*, we have $VaH(j)^T \sum_k^G \overline{L}_{jk}\overline{L}_{ki}$, reflecting the value-added from country *j* $VaH(j)^T \overline{L}_{jk}$ propagated to country *k*. $\overline{L}_{ki}$ is the part in country *k* that has returned back to country *i*. This part has also already been measured in the initial round as foreign value-added input, so it should be counted as value-added double counting term (foreign). Also, we have

$$\begin{aligned}&\sum_{j\neq i}^G VaH(j)^T[\overline{L}]^2_{ji} = VaH(j)^T \sum_k^G \overline{L}_{jk}\overline{L}_{ki} \\ &= \overline{V}_j \hat{H}_j \sum_k^G \hat{H}_j^{-1}\overline{A}_{jk}\hat{H}_k \cdot \hat{H}_k^{-1}\overline{A}_{ki}\hat{H}_i = \overline{V}_j \sum_k^G \overline{A}_{jk}\overline{A}_{ki}\hat{H}_i\end{aligned} \tag{14}$$

Therefore, in round 2, the whole double counted value-added in the foreign part is $\sum_{j\neq i}^G \overline{V}_j \sum_k^G \overline{A}_{jk}\overline{A}_{ki}\hat{H}_i$.

And for the domestic part, the domestic double counted value-added is:

$$\begin{aligned}&VaH(i)^T \overline{L}_{ii} + VaH(i)^T[\overline{L}]^2_{ii} + VaH(i)^T[\overline{L}]^3_{ii} + \cdots = \\ &\overline{V}_i(\overline{A}_{ii} + \sum_j^G \overline{A}_{ij}\overline{A}_{ji} + \sum_k^G\sum_j^G \overline{A}_{ij}\overline{A}_{jk}\overline{A}_{ki} + \cdots)\hat{H}_i = \overline{V}_i(\overline{B}_{ii} - I)\hat{H}_i = \overline{V}_i A_{ii}B_{ii}\hat{H}_i\end{aligned} \tag{15}$$

While the foreign double counted value-added is:

$$\begin{aligned}&\sum_{j\neq i}^G VaH(j)^T \cdot \overline{L}_{ji} + \sum_{j\neq i}^G VaH(j)^T[\overline{L}]^2_{ji} + \sum_{j\neq i}^G VaH(j)^T[\overline{L}]^3_{ji} + \cdots = \\ &\sum_{j\neq i}^G \overline{V}_j(\overline{A}_{ji} + \sum_k^G \overline{A}_{jk}\overline{A}_{ki} + \sum_t^G\sum_k^G \overline{A}_{jk}\overline{A}_{kt}\overline{A}_{ti} + \cdots)\hat{H}_i = \sum_{j\neq i}^G \overline{V}_j\overline{B}_{ji}\hat{H}_i\end{aligned} \tag{16}$$

Merging the expression of value-added coefficients $\overline{V}_i = V_i B_{ii}^F + \sum_{j\neq i}^G V_j B_{ji}^F$ in the domestic sales ICIO framework, the domestic value-added in country *i*'s domestic sales should be equal to country *i*'s value-added portion in the initial round: $V_i B_{ii}^F \hat{H}_i$. Moreover, country *i*'s value-added double counting term in domestic sales should be equal to the sum of the country *i*'s value-added portion in the double counting content (including the domestic input term and foreign input term): $V_i B_{ii}^F A_{ii}B_{ii}\hat{H}_i + \sum_{j\neq i}^G V_i B_{ij}^F \overline{B}_{ji}\hat{H}_i$.

Theorem 1: In the value-added decomposition of domestic sales, the sum of the domestic value-added and the double counting term is equal to the domestic content in domestic sales.

$$V_i B_{ii}^F \hat{H}_i + V_i B_{ii}^F A_{ii} B_{ii} \hat{H}_i + \sum_{j \neq i}^{G} V_i B_{ij}^F \overline{B}_{ji} \hat{H}_i = V_i B_{ii} \hat{H}_i$$

Similarly, the foreign value-added for country *i*'s domestic sales should be equal to the sum of foreign countries' value-added in the initial round's foreign input: $\sum_{j \neq i}^{G} V_j B_{ji}^F \hat{H}_i$. Foreign value-added double counting term in country *i*'s domestic sales should be equal to the foreign value-added portion in the double counting content: $\sum_{j \neq i}^{G} V_j B_{ji}^F A_{ii} B_{ii} \hat{H}_i + \sum_{s \neq i}^{G} \sum_{j \neq i}^{G} V_s B_{sj}^F \overline{B}_{ji} \hat{H}_i$.

Theorem 2: In the value-added decomposition of domestic sales, the sum of the foreign value-added and the double counting term is equal to the foreign content in domestic sales.

$$\sum_{j \neq i}^{G} V_j B_{ji}^F \hat{H}_i + \sum_{j \neq i}^{G} V_j B_{ji}^F A_{ii} B_{ii} \hat{H}_i + \sum_{s \neq i}^{G} \sum_{j \neq i}^{G} V_s B_{sj}^F \overline{B}_{ji} \hat{H}_i = \sum_{j \neq i}^{G} V_j B_{ji} \hat{H}_i$$

See Appendix I for the proof of these theorems.

2.4 The meaning of double counting terms: further clarifications

In the input-output framework, double counting comes from the concept of intermediate input. Output is equal to (domestic) value-added plus intermediate inputs (domestic value-added double counting, foreign value-added and foreign double counting), so the double counting term in output decomposition is a subset of intermediate inputs. This translates in the Ghosh model into an initial round where all (domestic) value-added is captured and subsequent rounds that correspond to double counting in output. Foreign value-added, when relevant, first appears in the first round.

When it comes to value-added in exports, there is no consensus yet on the definition of double counting. Some authors, such as Koopman et al. (2014), Nagengast and Stehrer (2016) and Borin and Mancini (2017) propose to base the definition on the number of international border crossings. Also, Los and Timmer (2018) point out that the double counted domestic value-added is the sum of the bilateral domestic value-added across all partners minus the unilateral one (i.e. with partner world).

Alternatively, Miroudot and Ye (2017) rely on the Ghosh insight. In their framework, domestic value-added is the initial round input and foreign value-added is the first round, which is consistent with the analysis of output. In addition, every round in this framework is conceptually value-added crossing the

'supply side's country border' (i.e. not considering the destination where it is absorbed)[2]. This definition based on the Ghosh framework is consistent with Los and Timmer (2018), as the double counting can also be expressed as the sum of the value-added across all bilateral relationships minus value-added in unilateral trade.

The decomposition for domestic sales presented in the previous section is similar with the exports decomposition in Miroudot and Ye (2017). However, exports and domestic sales are different concepts. If we compare the double counting in domestic sales with the one in exports, we find that double counting terms in domestic sales are significantly higher. It comes from the fact that rather than the country border for international transactions, the double counting for domestic sales starts with the 'sector border'. When value-added crosses the sector border as intermediate input more than once, it becomes part of double counting.

Let assume that a car is exported with all inputs produced domestically except one which is imported. Only the imported input can create some double counting in the case of exports (for example if it includes domestic value-added coming back). If the car is sold on the domestic market, all the inputs required (and the inputs used to produce these inputs) are also domestic sales and the double counting will be significantly higher. Section 3 includes empirical results that illustrate this difference in double counting. From a conceptual point of view, it seems however important to have a consistent definition of double counting in exports and in domestic sales, particularly if we start to decompose GDP into both exports and domestic sales as in the next sub-section.

2.5 GDP decomposition into exports and domestic sales (with an overlap)

As previously highlighted, the accounting relationship between domestic sales H and final demand in destination in the ICIO model can be written as $H = \overline{A}H + \overline{Y}$. In a similar way, we can also obtain the accounting relationship between gross exports E and final demand in different destinations in the Inter-Country Input-Output (ICIO) model (see Appendix I):

[2] The country border in the Ghosh framework can be seen as conceptually different from the border crossed by products in the real world. As the model is supply-driven, value-added crosses the border from the supply side rather than the demand side and cannot be merged with value-added finally absorbed in the destination country. Concretely, it means that in the initial round of the Ghosh model applied to the decomposition of exports, value-added crosses the domestic border only once while from the demand side it would have crossed countries' borders for indefinite times. We believe this difference explains why the decomposition in Miroudot and Ye (2017) cannot be described as 'arbitratry' as Los and Timmer (2018) suggest for decompositions that focus on border crossings from the demand side.

$$E = \tilde{A}E + \tilde{Y} \tag{17}$$

with $\tilde{A} = A^F(I - A^D)^{-1}and\tilde{Y} = Y^F + \tilde{A}Y^D$.

Re-arranging equations (2) and (17), we can express gross exports and domestic sales as:

$$E = [I - A^F(I - A^D)^{-1}]^{-1}[A^F(I - A^D)^{-1}Y^D + Y^F] \tag{18}$$

$$H = [I - A^D(I - A^F)^{-1}]^{-1}[A^D(I - A^F)^{-1}Y^F + Y^D] \tag{19}$$

Therefore, in the ICIO model, gross output can be written as:

$$X = A^D X + Y + A^F X + Y^F = A^D X + Y + E \tag{20}$$

$$\text{Or } X = A^D X + Y + A^F X + Y^F = H + A^F X + Y^F \tag{21}$$

Rearranging equations (20) and (21), we get:

$$X = (I - A^D)^{-1}Y^D + (I - A^D)^{-1}E \tag{22}$$

$$\text{And } X = (I - A^F)^{-1}Y^F + (I - A^F)^{-1}H \tag{23}$$

The expression $(I - A^D)^{-1}$ is sometimes described as the local Leontief inverse in the ICIO.

GDP can then be calculated as follows:

$$GDP = VX = V(I - A^D)^{-1}Y^D + V(I - A^D)^{-1}E \tag{24}$$

$$\text{Or } GDP = VX = V(I - A^F)^{-1}Y^F + V(I - A^F)^{-1}H \tag{25}$$

According to equation (24), GDP can be divided into two parts. The first part is the share of GDP that does not participate in international trade and is just for domestic final demand. The second part, $V(I - A^D)^{-1}E$, is the share of GDP in exports. GDP in exports includes some value-added that can return home. This is why the split is not based on whether final consumption takes place in the domestic economy or abroad. Exports include both intermediate and final products.

From equation (25), GDP can also be decomposed into two parts along another dimension: $V(I - A^F)^{-1}H$ reflects the value-added in domestic sales while $V(I - A^F)^{-1}Y^F$ corresponds to value-added for the foreign final demand. Again, it does not indicate where value-added is ultimately going as the concept of domestic sales is still a mix of intermediate and final products.

Merging equations (18), (19), (24) and (25), we obtain the following GDP decomposition:

$$GDP = VBA^F (I - A^D)^{-1} Y^D + V(I - A^D)^{-1} Y^D + VBA^D (I - A^F)^{-1} Y^F + V(I - A^F)^{-1} Y^F \quad (26)$$

For a specific country *i,* the equation can be written as follows:

$$GDP_i = V_i \sum_j^G \sum_{s \neq j}^G B_{is} A_{sj} (I - A_{jj})^{-1} Y_j^D + V_i (I - A_{ii})^{-1} Y_i^D + V_i \sum_j^G \sum_s^G B_{is} A_{ss} B_{sj}^F Y_j^F + V_i \sum_j^G B_{ij}^F Y_j^F \quad (27)$$

Equation (27) allows us to divide GDP into 4 terms that we can interpret in the following way. The first term, $V_i \sum_j^G \sum_{s \neq j}^G B_{is} A_{sj} (I - A_{jj})^{-1} Y_j^D$, measures the value-added that has propagated in domestic and global value chains[3] and is absorbed by destination countries as an intermediate product. The second term, $V_i (I - A_{ii})^{-1} Y_i^D$, measures the purely domestic value-added which has not been part of international trade and ends up in domestic final demand. The third term, $V_i \sum_j^G \sum_s^G B_{is} A_{ss} B_{sj}^F Y_j^F$, measures the value-added that has participated in domestic and global value chains and is ultimately absorbed by destination countries as a final product. The last term, $V_i \sum_j^G B_{ij}^F Y_j^F$, measures the value-added that has not participated in the domestic propagation and is absorbed by foreign countries.

Terms 1, 3 and 4 are equal to the domestic value-added in exports, as measured by Koopman et al. (2014) or by Los et al. (2016), which includes the value-added in exports coming back to the domestic economy. The second term corresponds to value-added going into domestic final demand without having transited through other countries.

Theorem 3: The GDP decomposition in equation (27) is consistent with GDP decomposition according to final demand. We have:

$$VBA^F (I - A^D)^{-1} Y^D + V(I - A^D)^{-1} Y^D = VBY^D$$
$$VBA^D (I - A^F)^{-1} Y^F + V(I - A^F)^{-1} Y^F = VBY^F$$

[3] In this sub-section, we use the concept of 'global value chain' the way it is used in the trade literature to describe the production of goods and services across different countries that provide inputs at different stages of production before final production takes place. The Leontief inverse in a global ICIO can be understood as a matrix describing the fragmentation of production across countries and industries.

From the above decomposition, we can also provide expressions for the value-added in exports and in domestic sales as follows:

$$\begin{aligned} V(I-A^D)^{-1}E &= VBA^F(I-A^D)^{-1}Y^D + VBA^D(I-A^F)^{-1}Y^F + V(I-A^F)^{-1}Y^F \\ V(I-A^F)^{-1}H &= VBA^F(I-A^D)^{-1}Y^D + VBA^D(I-A^F)^{-1}Y^F + V(I-A^D)^{-1}Y^D \end{aligned} \quad (28)$$

These equations highlight an important feature of this value-added decomposition. There is an overlap between the value-added in exports and in domestic sales (as some domestic sales are intermediates that are then incorporated into exports). The overlap can be seen in $VBA^F(I-A^D)^{-1}Y^D$ and $VBA^D(I-A^F)^{-1}Y^F$, as these two terms not only participate in the domestic propagation but also in international trade.

Coming back to equation (27), it means that while terms 1, 3 and 4 are equal to domestic value-added in exports, value-added in domestic sales is equal to terms 1, 2 and 3. The overlapping terms are 1 and 3. They correspond to value-added that has transited through the domestic value chain before being exported and absorbed abroad (or back to the domestic economy).

In gross terms, we can distinguish gross exports from domestic sales but in value-added terms there is by definition on overlap. There is no reason for the value-added reflected in terms 1 and 3 to be attributed to exports or domestic sales only. And it highlights that decompositions of gross exports include value-added from domestic sales. This finding has implications for authors comparing exports and sales of foreign affiliates or in the case of trade in services the comparison between cross-border trade (Mode 1, 2 and 4 in the General Agreement on Trade in Services) and trade through commercial presence (Mode 3).

3. Empirical results

In order to illustrate how our framework can be used to analyse domestic sales and compare sales of foreign affiliates with exports, we rely on the World Input-Output Database (WIOD) inter-country input-output tables. Some information on WIOD can be found in Timmer et al. (2015). We use the 2016 update of the database (Timmer et al., 2016). The World Input-Output Tables (WIOTs) do not have information on domestic and foreign ownership of firms but the OECD has split these tables according to ownership as part of its analytical AMNE database (Cadestin et al., 2018a). In this project, WIOD data have been merged with statistics on the output, value-added and exports of domestic-owned and foreign-owned firms in each country and industry. Full input-output tables are available where in each industry the information is split between domestic-owned and foreign-owned firms across columns and rows.

3.1 Value-added in domestic sales by foreign-owned and domestic-owned firms

We first decompose domestic sales according to the equations presented in Section 2.3 (Table 1). This decomposition allows us to identify the domestic value-added, domestic double counting, foreign value-added and foreign double counting in domestic sales. The information is further split between domestic-owned and foreign-owned companies.

As previously highlighted, domestic sales feature a high level of double counting due to the fact that most of the intermediate inputs used in the production process are also contributing to domestic sales. It is an important difference with the decomposition of gross exports where only inputs crossing international borders can create double counting. The foreign double counting is also important in domestic sales as imported inputs also go through processing in the domestic economy.

Table 1. Decomposition of domestic sales for selected economies, 2014

Country	Ownership	Domestic sales (mio USD)	Domestic VA (%)	Domestic double counting (%)	Foreign VA (%)	Foreign double counting (%)
China	Domestic-owned	27,192,850	34.28	54.51	2.57	8.64
	Foreign-owned	2,126,788	22.89	59.05	6.01	12.06
Germany	Domestic-owned	4,383,927	57.45	30.93	4.84	6.78
	Foreign-owned	1,000,560	43.89	33.02	13.36	9.74
France	Domestic-owned	3,750,263	57.42	30.31	5.02	7.25
	Foreign-owned	510,217	41.91	35.53	12.56	10.00
Japan	Domestic-owned	7,563,495	53.69	33.16	4.70	8.46
	Foreign-owned	287,727	48.34	37.14	6.00	8.52
United States	Domestic-owned	26,677,432	58.09	35.57	2.38	3.97
	Foreign-owned	2,366,500	44.48	39.75	8.60	7.17

Source: Authors' calculations based on WIOD and the OECD analytical AMNE database.

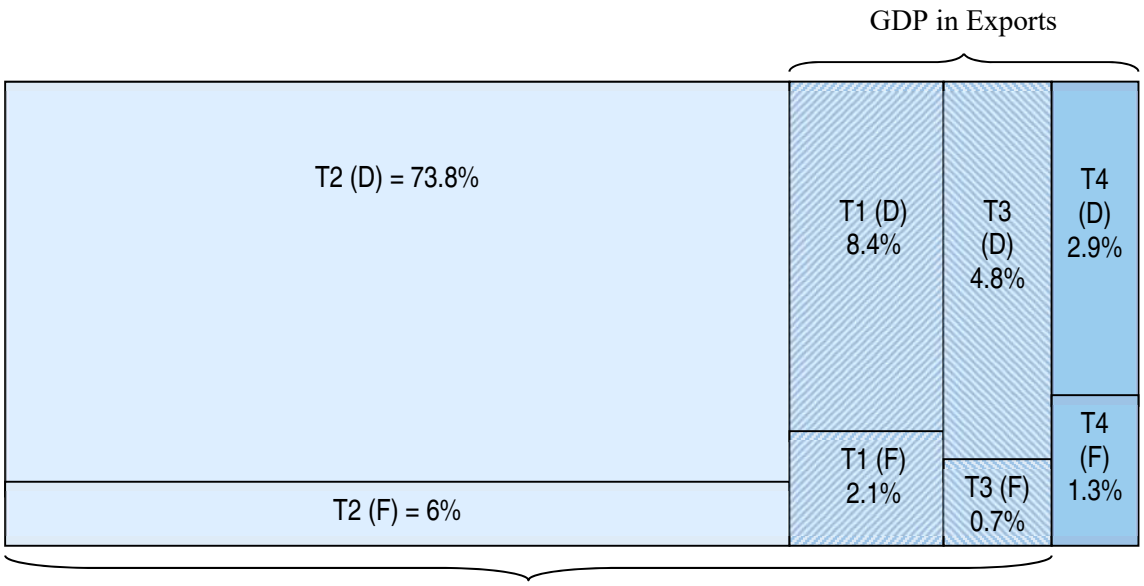
Another result illustrated in Table 1 is that affiliates of foreign firms also rely to a large extent on domestic inputs to produce in the host economy. We find that foreign affiliates rely relatively more on foreign VA as compared to domestic-owned firms, but the difference is quite small and domestic VA is in all countries higher than foreign VA for foreign-owned firms.

3.2 Decomposition of GDP: Value-added in exports and in domestic sales by domestic-owned and foreign-owned firms

Using equation (27) in the ICIO split according to ownership, we can provide a full decomposition of world GDP indicating whether value-added is derived from domestic sales or from exports and then whether this value-added is generated by domestic-owned firms or foreign-owned firms. The results

are summarised in Figure 1 through a chart that indicates the relative importance of the value-added generated in each case, decomposed between domestic-owned firms (D) and foreign-owned firms (F). Table 2 (in Appendix II) reports detailed results by country for the four terms of equation (27). Figure 1 illustrates the overlap between value-added in exports and in domestic sales, through T1 and T3 (i.e. the value-added that propagates through domestic and global value chains and is finally absorbed in the destination country as an intermediate or as a final good). T1 and T3 belong both to GDP in exports and in domestic sales. This overlap amounts to 16% of world GDP.

Figure 1. Decomposition of world GDP according to equation (27), domestic-owned and foreign-owned firms, 2014



Source: Authors' calculations based on WIOD and the OECD analytical AMNE database. D = Domestic-owned firms; F = Foreign-owned firms. T1 = value-added that has propagated in domestic and global value chains and is absorbed by destination countries as an intermediate product; T2 = purely domestic value-added which has not been part of international trade and ends up in domestic final demand, T3 = value-added that has participated in domestic and global value chains and is ultimately absorbed by destination countries as a final product, and T4 = value-added that has not participated in the domestic propagation and is absorbed by foreign countries.

Figure 1 highlights that more value-added is created through trade (exports of domestic-owned and foreign-owned firms) than through the sales of foreign affiliates (domestic sales and exports of foreign-owned firms), a result maybe in contradiction with the literature based on gross flows that overstate the importance of sales of foreign affiliates when ignoring the double counting. The value-added in exports by domestic-owned firms and foreign-owned firms (8.4%+4.8%+2.9%+2.1%+0.7%+1.3%=20.2%) is double the size of the value-added in domestic sales and exports by foreign-owned firms (6%+2.1%+0.7%+1.3%=10.1%). The overlap between trade and

investment is relatively small – i.e. 4.1% of world GDP - which corresponds to the value-added in exports by foreign-owned firms (2.1%+0.7%+1.3%). These 4.1% seem to suggest that at the world level foreign affiliates account for a rather small share of trade in value-added terms. It is an indication that global value chains operate with many arm's length trade transactions and maybe less within pure MNE networks (e.g. inputs transferred between affiliates).

Figure 1 also points out that 73.8% of world GDP is value-added created by domestic-owned firms in domestic sales without transiting through global value chains (T2). For most products and particularly for services, a high share of value-added is purely domestic. However, MNEs can still play a role in this domestic value-added as they are involved in domestic sales in the parent country and their operations can in some cases not rely on international trade.[4]

4. Concluding remarks

This paper has introduced a new type of GDP decomposition that allows us to trace value-added and double counting not only in exports but also in domestic sales. The motivation is that traditional I-O analysis looking at value-added in final demand is not sufficient to discuss trade and investment in global value chains. Looking at trade in value-added terms and decomposing gross exports has brought many interesting analytical results and led to new policy implications. We believe the same will happen when looking at activities of multinational enterprises in value-added terms and decomposing sales of foreign affiliates. This is why we need for domestic sales tools similar to what was developed for gross exports.

By using our methodology to compare the value-added in exports and in sales of domestic-owned and foreign-owned firms, there are already interesting findings. In particular, it seems that the double counting in sales of foreign affiliates is much more pronounced than in exports, as affiliates of foreign firms rely even more on inputs from the host economy and inputs transactions in the domestic economy lead to double counting in sales. The whole literature on the benefits of FDI and the impact of activities of MNEs will have to be revisited in light of this value-added analysis which can provide a better indication of how income is generated and who really benefits from production by foreign-owned firms.

[4] When adding the contribution of MNEs in the parent economy, the share of MNEs in world GDP is 31% according to Cadestin et al. (2018b).

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Appendix I

Lemma 1：The accounting relationship between domestic shipments H and final demand in destination in an Inter-Country Input-Output (ICIO) model can be expressed as:

$$H = \bar{A}H + \bar{Y}$$

Proof: Gross output X is the sum of gross exports E and gross domestic shipments H. From the accounting identity in equation (1), $X = AX + Y$, we can express E the vector of exports and H the vector of gross domestic shipments as:

$$E = A^F(E+H) + Y^F$$
$$H = A^D(E+H) + Y^D$$

Solving for E, we obtain:

$$E = (I - A^F)^{-1}A^F H + (I - A^F)^{-1}Y^F$$

Merging the expression for H and for E, we obtain:

$$\begin{aligned} H &= A^D(E+H) + Y^D \\ &= A^D[H + (I - A^F)^{-1}A^F H + (I - A^F)^{-1}Y^F] + Y^D \\ &= A^D[I + (I - A^F)^{-1}A^F]H + A^D(I - A^F)^{-1}Y^F + Y^D \\ &= A^D(I - A^F)^{-1}H + A^D(I - A^F)^{-1}Y^F + Y^D \\ &= \bar{A}H + \bar{Y} \end{aligned}$$

with $\bar{A} = A^D(I - A^F)^{-1}$ and $\bar{Y} = \bar{A}Y^F + Y^D$.

Lemma 2：The accounting relationship between gross exports E and final demand in destination in an Inter-Country Input-Output (ICIO) model can be expressed as:

$$E = \tilde{A}E + \tilde{Y}$$

Proof: Similar to Lemma1.

Lemma 3：In the domestic sales accounting framework, we have

$$B^F \overline{B} = B$$

Here, $B^F = (I - A^F)^{-1}$ and $\overline{B} = (I - \overline{A})^{-1}$, B is the 'total requirements matrix' in the ICIO table which is $B = (I - A)^{-1}$.

Proof: Expanding the expression of B^F and $\overline{B}$, we obtain:

$$\begin{aligned}
&B^F \overline{B} = (I - A^F)^{-1}(I - \overline{A})^{-1} = [(I - \overline{A})(I - A^F)]^{-1} = \{[I - A^D(I - A^F)^{-1}](I - A^F)\}^{-1} \\
&= \{[(I - A^F)(I - A^F)^{-1} - A^D(I - A^F)^{-1}](I - A^F)\}^{-1} \\
&= [(I - A^F - A^D)(I - A^F)^{-1}(I - A^F)]^{-1} \\
&= (I - A)^{-1} = B
\end{aligned}$$

Theorem 1: In the value-added decomposition of domestic sales, the sum of the domestic value-added and the double counting term is equal to the domestic content in domestic sales.

$$V_i B_{ii}^F \hat{H}_i + V_i B_{ii}^F A_{ii} B_{ii} \hat{H}_i + \sum_{j \neq i}^{G} V_i B_{ij}^F \overline{B}_{ji} \hat{H}_i = V_i B_{ii} \hat{H}_i$$

Proof: According to lemma 3, we can obtain the submatrix *i*'s expression as

$$\sum_{j}^{G} B_{ij}^F \overline{B}_{ji} = B_{ii}^F \overline{B}_{ii} + \sum_{j \neq i}^{G} B_{ij}^F \overline{B}_{ji} = B_{ii}$$

merging the expression of matrix $\overline{B}_{ii} = I + A_{ii} B_{ii}$, we have

$$B_{ii}^F + B_{ii}^F A_{ii} B_{ii} + \sum_{j \neq i}^{G} B_{ij}^F \overline{B}_{ji} = B_{ii}$$

Theorem 2: In the value-added decomposition of domestic sales, the sum of the foreign value-added and the double counting term is equal to the foreign content in domestic sales.

$$\sum_{j \neq i}^{G} V_j B_{ji}^F \hat{H}_i + \sum_{j \neq i}^{G} V_j B_{ji}^F A_{ii} B_{ii} \hat{H}_i + \sum_{s \neq i}^{G} \sum_{j \neq i}^{G} V_s B_{sj}^F \overline{B}_{ji} \hat{H}_i = \sum_{j \neq i}^{G} V_j B_{ji} \hat{H}_i$$

Proof: Similar to theorem 1.

Theorem 3: The four terms GDP decomposition is consistent with GDP decomposition in the final demand. We have:

$$VBA^F(I-A^D)^{-1}Y^D + V(I-A^D)^{-1}Y^D = VBY^D$$
$$VBA^D(I-A^F)^{-1}Y^F + V(I-A^F)^{-1}Y^F = VBY^F$$

Proof: For the first equation, we have

$$BA^F(I-A^D)^{-1} + (I-A^D)^{-1} = (BA^F + I)(I-A^D)^{-1}$$
$$= [BA^F + B(I-A)](I-A^D)^{-1} = [B(I-A+A^F)](I-A^D)^{-1}$$
$$= B$$

The same can be done with the second equation.

Appendix II

Table 2. Four terms GDP decomposition, domestic-owned and foreign-owned firms, 2014

Country	Ownership	GDP (million USD)	T1 (%)	T2 (%)	T3 (%)	T4 (%)
AUS	Domestic-owned	1,214,354	12.3	83.4	3.0	1.4
	Foreign-owned	170,022	16.9	75.1	3.8	4.2
AUT	Domestic-owned	327,350	13.9	70.7	7.5	7.8
	Foreign-owned	77,984	22.9	49.8	9.6	17.7
BEL	Domestic-owned	396,349	13.7	72.1	8.3	5.9
	Foreign-owned	105,777	40.8	22.3	8.4	28.5
BGR	Domestic-owned	38,364	16.6	69.1	8.3	6.1
	Foreign-owned	14,280	28.6	48.0	9.1	14.3
BRA	Domestic-owned	2,040,815	5.7	90.8	2.3	1.2
	Foreign-owned	222,888	12.9	80.2	3.3	3.7
CAN	Domestic-owned	1,408,567	11.6	81.4	4.2	2.8
	Foreign-owned	289,454	36.7	44.7	5.7	12.9
CHE	Domestic-owned	520,723	13.9	70.5	8.6	7.1
	Foreign-owned	177,939	28.6	40.4	10.0	21.0
CHN	Domestic-owned	9,722,440	7.2	82.4	8.2	2.1
	Foreign-owned	676,281	16.3	55.6	11.2	16.9
CYP	Domestic-owned	20,808	15.0	72.4	6.3	6.3
	Foreign-owned	1,084	41.1	21.6	7.3	30.0
CZE	Domestic-owned	127,956	16.5	64.4	10.8	8.2
	Foreign-owned	70,374	24.1	40.7	13.9	21.2
DEU	Domestic-owned	3,003,899	13.0	69.3	9.1	8.6
	Foreign-owned	616,411	20.6	54.0	11.0	14.4
DNK	Domestic-owned	272,384	12.3	71.3	7.7	8.7
	Foreign-owned	44,256	29.3	39.0	9.5	22.1
ESP	Domestic-owned	1,112,447	7.5	82.1	6.1	4.4
	Foreign-owned	191,803	15.0	64.9	8.4	11.7
EST	Domestic-owned	17,860	20.8	64.5	8.0	6.7
	Foreign-owned	6,917	31.4	43.3	8.9	16.5
FIN	Domestic-owned	215,757	13.0	76.5	5.9	4.7
	Foreign-owned	32,603	22.0	56.5	7.9	13.7
FRA	Domestic-owned	2,342,619	7.8	82.4	5.7	4.1
	Foreign-owned	306,513	20.1	57.6	8.2	14.1
GBR	Domestic-owned	2,216,930	8.2	83.8	4.6	3.4
	Foreign-owned	589,523	21.3	59.1	6.7	12.9
GRC	Domestic-owned	200,986	9.3	83.1	3.8	3.8
	Foreign-owned	15,636	18.2	69.0	5.9	6.9
HRV	Domestic-owned	41,934	15.3	69.0	7.7	8.0
	Foreign-owned	9,274	19.8	59.9	8.3	11.9
HUN	Domestic-owned	78,673	14.2	69.6	9.6	6.6
	Foreign-owned	47,953	27.5	33.7	9.4	29.4
IDN	Domestic-owned	773,937	11.7	81.8	3.9	2.6
	Foreign-owned	106,166	18.5	70.2	5.3	6.0
IND	Domestic-owned	2,050,622	5.9	86.8	3.6	3.6
	Foreign-owned	55,870	23.2	60.6	5.1	11.1
IRL	Domestic-owned	130,844	19.4	59.9	7.1	13.6

	Foreign-owned	105,680	38.0	22.9	9.2	29.9
ITA	Domestic-owned	1,783,155	7.8	80.2	7.5	4.4
	Foreign-owned	214,811	15.9	63.3	9.6	11.2
JPN	Domestic-owned	4,317,665	6.3	86.6	4.2	2.9
	Foreign-owned	171,540	13.0	70.4	6.4	10.1
KOR	Domestic-owned	1,301,738	15.3	67.9	10.4	6.5
	Foreign-owned	64,859	27.5	47.2	10.9	14.4
LTU	Domestic-owned	35,987	21.2	57.2	8.1	13.5
	Foreign-owned	10,147	27.5	45.2	9.3	17.9
LUX	Domestic-owned	44,273	32.9	41.2	12.5	13.3
	Foreign-owned	16,815	46.4	19.5	11.1	23.0
LVA	Domestic-owned	22,177	16.7	69.7	7.3	6.3
	Foreign-owned	6,710	27.4	50.6	8.7	13.3
MEX	Domestic-owned	1,069,198	7.9	83.2	4.6	4.4
	Foreign-owned	157,166	20.9	62.0	5.4	11.7
MLT	Domestic-owned	9,083	17.1	54.6	10.5	17.8
	Foreign-owned	720	31.8	30.3	10.8	27.0
NLD	Domestic-owned	661,801	20.4	64.1	8.3	7.2
	Foreign-owned	167,688	40.0	30.3	9.3	20.4
NOR	Domestic-owned	372,748	18.2	73.4	4.9	3.5
	Foreign-owned	103,663	38.8	46.6	7.2	7.4
POL	Domestic-owned	372,271	12.9	70.6	9.4	7.1
	Foreign-owned	139,757	18.9	55.3	10.6	15.2
PRT	Domestic-owned	179,199	10.9	78.6	5.5	5.0
	Foreign-owned	36,314	19.6	61.5	7.3	11.6
ROU	Domestic-owned	120,926	12.9	75.1	6.9	5.0
	Foreign-owned	65,941	19.7	59.9	8.6	11.8
RUS	Domestic-owned	1,661,111	20.0	73.9	4.2	2.0
	Foreign-owned	63,243	24.7	66.4	4.5	4.4
SVK	Domestic-owned	67,895	14.9	68.2	10.6	6.3
	Foreign-owned	29,571	27.7	30.7	12.0	29.7
SVN	Domestic-owned	37,279	17.3	63.1	9.8	9.8
	Foreign-owned	7,943	30.4	33.3	10.2	26.1
SWE	Domestic-owned	415,489	13.2	73.8	6.6	6.4
	Foreign-owned	117,726	23.1	51.0	8.8	17.2
TUR	Domestic-owned	677,637	9.2	77.8	7.5	5.5
	Foreign-owned	72,919	15.8	64.0	8.0	12.2
TWN	Domestic-owned	467,366	22.1	61.6	8.6	7.7
	Foreign-owned	66,026	31.2	48.1	7.7	13.1
USA	Domestic-owned	16,149,852	4.4	91.5	2.3	1.8
	Foreign-owned	1,266,994	12.1	76.6	3.7	7.6
ROW	Domestic-owned	9,801,750	15.8	75.2	5.9	3.1
	Foreign-owned	886,774	24.0	61.1	6.4	8.6

Source: Authors' calculations based on WIOD and the OECD analytical AMNE database. The decomposition is based on equation (27). T1 = value-added that has propagated in domestic and global value chains and is absorbed by destination countries as an intermediate product; T2 = purely domestic value-added which has not been part of international trade and ends up in domestic final demand, T3 = value-added that has participated in domestic and global value chains and is ultimately absorbed by destination countries as a final product, and T4 = value-added that has not participated in the domestic propagation and is absorbed by foreign countries.