Environmental policies with excess burden of taxation in free-entry mixed markets

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The timing of environmental policies with excess burden of taxation in free-entry mixed markets

Abstract

This study investigates the timing of environmental policies in free-entry mixed markets with excess burden of taxation. We consider two scenarios in which the government chooses the optimal tax before or after firms enter the market, and compare these ex-ante and ex-post taxations. When the excess burden of taxation is small (large), we find that ex-post taxation imposes a lower (higher) tax level than ex-ante taxation, which induces a larger (smaller) number of firms and a higher (lower) environmental damage. Finally, the effect of excess burden of taxation can increase the welfare, but ex-ante taxation always yields higher welfare than ex-post taxation.

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1. Introduction

Since the 1980s, the waves of market liberalization and environmental protection have been salient features of economic policies in polluting industries around the world. On the one hand, government has continuously conducted market-based environmental regulation by using taxes, subsidies and cap-and-trades. For example, various studies have explored the effect of pollution taxation in imperfect competition markets and concerned about the optimal tax rate. The traditional tax rule suggests that the optimal rate is equal to the marginal environmental damage in perfect competition markets while it falls short of the marginal damage in imperfect competition markets.[1]

On the other hand, technological and managerial improvement in public domains have motivated the entry of private firms having profit-maximizing incentives into the markets where public firms already exist and government regulates their activities. In the literature, many works have pointed out welfare implications of public policies in free entry markets are in sharp contrast with those in entry-regulated markets.[2] However, relatively few economic analyses on mixed markets where public firm competes with newly entering private firms have concerned on how an environmental policy interacts with a competition policy. The welfare effect of environmental policies in free entry mixed markets has

[1] See, for example, Buchanan (1969) and Barnett (1980) for monopoly and Levin (1985) and Shaffer (1995) for oligopoly.
[2] For example, Matsumura and Okumura (2014) compared specific taxation and volume quotas in a free entry oligopoly while Wang (2016) examined import tariff and output subsidy rates under open economy.

become an important concern of policy decision-making in the polluting industries such as oil, steel, electricity and so on.

Since Mankiw and Whinston (1986) indicated the welfare loss of business-stealing effect in free-entry markets, which causes not only less production but excessive entry under imperfect competition, many research in different fields have supported the robustness of these inefficiencies. In the literature of environmental economics, for example, Katsoulacos and Xepapadeas (1995) and Lee (1999) examined the welfare effect of environmental policies in free-entry private industries. They found that the optimal tax should be set at the marginal environmental damage because the tax effect on the output is fully offset by the effect on the number of entering firms and thus the inefficiency of less production is invariant while the optimal tax can work for reducing excessive entry.[3]

In the literature of mixed markets, Matsumura and Kanda (2005), Brandao and Castro (2007) and Ino and Matsumura (2010) examined the possible benefits of public ownership in free-entry mixed industries. They found that the existence of a welfare-maximizing public firm can induce the expansion of its output and less number of entering firms, which will fully offset the output substitution effect but the inefficiency of less production is invariant while the public firm can reduce excessive entry.[4]

However, the government's public policy causes the shadow cost of the public funds, which carries an excess burden of taxation.[5] In the context of mixed oligopolies, for example, Capuano and De Feo (2010) examined the effect of the shadow cost of public funds on privatization in a mixed duopoly. Wang and Chen (2011) and Matsumura and Tomaru (2013) showed that the excess burden of taxation sharply affects the comparisons of optimal subsidy, total output, and social welfare in mixed oligopolies. Matsumura and Tomaru (2015) also examined the relationship between the equilibrium level and the efficient level of product differentiation with the shadow cost of the public funds.

This paper investigates the environmental policies in free-entry mixed markets and examines the effect of excess burden of taxation, which comes from the tax revenue and the profit of the public firm. In particular, we examine the impact of the timing of environmental taxation and compare two models:

[3] In the subsequent research, it is proved that the optimal emission tax rate in free-entry markets depends on the curvature of market demand (Requate, 2007), the degree of product differentiation (Fujiwara, 2009), the output elasticity of emissions (Sugeta, 2017) and consumers awareness (Hsu *et al.*, 2017). However, all these analysis still support that the optimal tax can reduce excessive entry.

[4] In the subsequent research, Cato (2008) considered environmental externalities and showed that the optimal degree of privatization is invariant in free-entry mixed polluting markets. However, Cato and Matsumura (2012, 2015) showed that the optimal ownership of the public firm decreases as foreign penetration increases or the import tariff rate increases.

[5] The profits of public firms can be used for public finance while, as pointed out by Laffont and Tirole (1986), Lin *et al.* (1998) and Lin and Tan (1999), the government's public policy through the public firm might cause the welfare loss. Further, the emission tax revenue can provide double-dividend effect, which can be also used for public finance. Also, as pointed out by Bovenberg and Mooij (1994), Bovenberg and Goulder (1996) and Fullerton (1997), an environmental tax has its own distorting effects on labor supply, which can have the same excess burden as a tax on labor income.

ex-ante taxation and ex-post taxation. In the former, the government decides environmental tax and then liberalizes the market, whereas the order of the policies is reversed in the latter. The ex-ante case might correspond to the situation that the government is able to make a commitment on the optimal tax rate before it opens the market to the newly entering private firms. It also represents the economic policy-neutral case where the industrial and environmental policies do not incur any political cost. However, this neutral case is not always realistic. There might be strategic relationship between economic decision and political decision because of lobbying activities of firms and industrial associations for tax policies or the government's budget constraint and fiscal concerns. Further, a change in economic environment such as technological improvement or ownership structure in the industry under open economy can affect the ex-post optimal decisions on the environmental policies. Thus, the ex-post case corresponds to this situation in which the adjustment of tax rate is easy and flexible.

The main findings are summarized as follows. First, as the shadow cost increases, public firm becomes more aggressive and thus produces more outputs, which results in that the equilibrium number of private firms always decreases under both ex-ante and ex-post taxation cases. Second, under ex-ante taxation the optimal tax should be lower than the marginal damage and the efficient departure between them increases as the shadow cost increases. However, under ex-post taxation the efficient departure depends on the entry cost and the shadow cost, and the optimal tax might exceed the marginal damage when both entry cost and the shadow cost are large. Third, ex-post taxation yields a lower (higher) environmental tax level than the ex-ante taxation when the shadow cost is small (large). It induces that ex-post taxation yields a larger (smaller) number of private firms and a higher (lower) environmental damage. Finally, the effect of excess burden of taxation can increase the welfare, but the ex-ante taxation always yields higher welfare than ex-post taxation.

The remainder of this paper is organized as follows. Section 2 presents the basic model. Section 3 analyzes the *ex-ante* and *ex-post* taxation models and presents a comparative analysis of the two cases. Section 4 concludes the study.

2. Model

We consider a mixed market with Cournot oligopoly in which $n+1$ operating firms produce homogeneous products. Firm 0 is a public firm and firm i (i=1,2,..., n) is a private firm. The inverse demand function is $P=1-Q$, where $\mathrm{Q}=\sum_{j=0}^{n} q^{j}$ is the market output and q_j is firm j's output, j=0,1,..., n , respectively. Then, consumer surplus can be measured as $CS=Q^2/2$.

All firms have the same cost function, $\mathrm{C}(q_j)=(q^j)^2/2+F^2$, where F^2 is the entry cost of each private firm. In the analysis, following Matsumura and Kanda (2005), we assume that the public firm is already established in the initial market before the private firms enter the market and thus the fixed cost of public firm is a sunk cost to the government when it decides its policy decisions.

We assume that production leads to the emission of pollution e^j and the emission level is proportional to the production level, that is, $e^j = q^j$. The environmental damage due to emission is given by $ED = \sum_{j=0}^{n} (e^j)^2/2$. The government imposes an emission tax on the emission level with the tax rate of t. The resulting total tax revenue is $T = t\sum_{j=0}^{n} e^j$. The profits of each private firm is

$$\pi^i = Pq^i - \left(q^i\right)^2/2 - te^i - F^2 \qquad \text{for } i=1,\ldots, n\,, \tag{1}$$

We consider a free-entry mixed market in which private firms can enter the market where the public firm already exists. Then, the number of private firms is determined at the point where the profit of the private firm is zero. We allow the profit of public firm to be either negative or positive in the mixed market. The government revenue, which includes not only the government revenue from emission tax but the profit of public firm, can be used to reduce tax rates in other markets, which results in the reduction of dead weight loss due to distortionary taxation. That is, we consider the excess burden of taxation, i.e., $EBT = \lambda(\pi^0 + T)$.

The welfare is defined as the sum of consumer surplus, producer surplus, tax revenue, and ETB minus industrial pollution:

$$W = CS + \pi^0 + \sum_{i=1}^{n} \pi^i + T + \lambda(\pi^0 + T) - ED, \tag{2}$$

where $\lambda \geq 0$ represents the shadow cost of the government revenue, which can be used to reduce the EBT as a public finance.

Finally, the firms' objective functions are subject to their ownership structures. We suppose that the private firm, which is completely characterized by private property rights, maximizes its profits, while the public firm, which is fully owned by the government, maximizes the social welfare.

3. Analysis and comparison

3.1. Ex-ante taxation

We consider ex-ante taxation model in which the government chooses the level of environmental tax before private firms enter the market. The timing of this game is as follows. In the first stage, the government chooses the level of tax to maximize social welfare. In the second stage, with exogenously given tax rate each private firm decides whether to enter the market where the number of private firms is endogenously determined by a free-entry market condition, i.e., zero-profit condition. In the third stage, the public and private firms compete in quantities in a Cournot fashion. The subgame perfect Nash equilibrium is solved by backward induction.

In the third stage, the public firm chooses q^0 to maximize the social welfare, *W*, and the private firm chooses q^i to maximize its own profits, π^i simultaneously. Assuming interior solutions, the first-order conditions provide the following equilibrium outputs:[6]

$$q^o = \frac{2(1+\lambda)+n(t(2+\lambda)-1)}{6+n+2(3+n)\lambda}, \tag{3}$$

$$q^i = \frac{(2-3t)(1+\lambda)}{6+n+2(3+n)\lambda}. \tag{4}$$

The profits of the public and private firms are respectively

$$\pi^0 = \frac{(2(1+\lambda)+n(t(2+\lambda)-1))(6(2t-1)(1+\lambda)+n(t(2+\lambda)-1))}{2(6+n+2(3+n)\lambda)^2}, \tag{5}$$

$$\pi^i = \frac{3(2-3t)^2(1+\lambda)^2}{2(6+n+2(3+n)\lambda)^2} - F^2. \tag{6}$$

The welfare is given by

$$W = \frac{(1+\lambda)\left(12(1+\lambda)^2+n(1+\lambda)(8-9t(t+4(t-1)\lambda))-n^2\left(1+3t^2(2+\lambda(4+3\lambda))-2t\left(3+\lambda(5+4\lambda)\right)\right)\right)}{2(6+n+2(3+n)\lambda)^2} - nF^2. \tag{7}$$

In the second stage, each private firm earns zero profit in the equilibrium. Setting the profit of the private firms in equation (6) to zero yields the following equilibrium number of private firms:

$$n = \frac{(1+\lambda)(\sqrt{6}(2-3t)-12F)}{2F(1+2\lambda)}. \tag{8}$$

Note that the number of private firms decreases as the tax increases, i.e., $\frac{\partial n}{\partial t} < 0$, or as the shadow cost increases, i.e., $\frac{\partial n}{\partial \lambda} < 0$. This fact implies that the effect of tax on the number of firms can be complementary to the effect of the shadow cost under the ex-ante taxation, in which both effects can reduce the number of entering firms.

The welfare is given by

$$W = \frac{(1+\lambda)\{24F^2\lambda(2+\lambda)+4\sqrt{6}F\left(1-\lambda-3t(1+\lambda+\lambda^2)\right)+3\left(2t\left(3+\lambda(5+4\lambda)\right)-1+3t^2\left(2+\lambda(4+3\lambda)\right)\right)\}}{6(1+2\lambda)^2}. \tag{9}$$

In the first stage, the government chooses the optimal tax level from the differentiation of *W* in equation (9) with respect to *t*:

$$t_B = \frac{3+\lambda(5+4\lambda)-2\sqrt{6}F(1+\lambda+\lambda^2)}{6+3\lambda(4+3\lambda)}, \tag{10}$$

where subscript "B" denotes the equilibrium outcome of this game (taxation **before** entry). Note that

[6] In this study, all of the second-order conditions are satisfied.

$t_B > 0$ in the region of $0 < F < F_B = \frac{\sqrt{6}}{12}$, which ensure a positive equilibrium number of private firm under ex-ante taxation. The tax decreases as the shadow cost increases, i.e., $\frac{\partial t_B}{\partial \lambda} < 0$ when $0 < F < F_B$, and thus a higher effect of EBT brings about a lower tax rate.

By substituting equation (10) into equation (8), we obtain the following equilibrium number of private firms:

$$n_B = \frac{(\sqrt{6}-12F)(1+\lambda)^2}{2F(2+\lambda(4+3\lambda))}. \tag{11}$$

The number of private firms decreases as the shadow cost increases, i.e., $\frac{\partial n_B}{\partial \lambda} < 0$ when $0 < F < F_B$. It implies that under ex-ante taxation a higher shadow cost induces not only lower optimal tax rate but lower equilibrium number of private firms in a free-entry mixed market. Notice that the shadow cost directly reduce the number of private firms while it also indirectly increases the number of firms through taxation effect. However, this indirect effect is outweighed by the direct effect and thus, the number of private firms decreases as the shadow cost increases.

The outputs of public and private firms are:

$$q^o{}_B = \frac{12F(1+\lambda)(2+\lambda)+\sqrt{6}\lambda(1+2\lambda)}{3\sqrt{6}(2+\lambda(4+3\lambda))} \text{ and } q^i{}_B = \frac{\sqrt{6}}{3}F \tag{12}$$

where $q^o{}_B > q^i{}_B$. The output of public firm increases as the shadow cost increases, i.e., $\frac{\partial q^o{}_B}{\partial \lambda} > 0$, while the output of private firm is independent of the shadow cost, i.e., $\frac{\partial q^i{}_B}{\partial \lambda} = 0$, which is determined by the zero-profit condition.[7]

The public firm's profit is:

$$\pi^o{}_B = \frac{-\lambda^2(1+2\lambda)^2+4\sqrt{6}F(\lambda+2\lambda^2)^2+24F^2(1+\lambda)(2+\lambda)(2+5\lambda(1+\lambda))}{18(2+\lambda(4+3\lambda))^2} \tag{13}$$

The public firm's profit decreases as the shadow cost increases, i.e., $\frac{\partial \pi^o{}_B}{\partial \lambda} < 0$. Thus, as the EBT is more significant, the public firm is more aggressive to produce more outputs while the total outputs of private firms decreases because the number of private firms decreases.

Total market output become:

[7] Proof: Direct calculations from the zero-profit condition yields that the equilibrium price is equal to the average cost (AC) of each private firm. Also, the profit-maximization condition yields that the marginal revenue is equal to the marginal cost (MC) of each private firm. Thus, we have AC = MC + q at free-entry equilibrium, which yields $q^i{}_B = \frac{\sqrt{6}}{3}F$ in (12).

$$Q_B = \frac{\sqrt{6}(3+\lambda(7+5\lambda))-12F(1+\lambda)(1+2\lambda)}{3\sqrt{6}(2+\lambda(4+3\lambda))}, \tag{14}$$

where $\frac{\partial Q_B}{\partial \lambda} > 0$. Under ex-ante taxation, the increasing output effect of the public firm outweighs the decreasing total outputs effect of the private firms.

Total and marginal environmental damages are

$$ED_B = \frac{\{\sqrt{6}(3+\lambda(7+5\lambda))-12F(1+\lambda)(1+2\lambda)\}^2}{108(2+\lambda(4+3\lambda))^2} \text{ and } MD_B = \frac{\sqrt{6}(3+\lambda(7+5\lambda))-12F(1+\lambda)(1+2\lambda)}{3\sqrt{6}(2+\lambda(4+3\lambda))} \tag{15}$$

Note that these environmental damage are increasing in the shadow cost, i.e., $\frac{\partial ED_B}{\partial \lambda} > 0$ and $\frac{\partial MD_B}{\partial \lambda} > 0$ when $0 < F < F_B$. Note also that $MD_B = t_B = \frac{3-2\sqrt{6}F}{6}$ if $\lambda = 0$. This indicates that under ex-ante taxation, Pigouvian tax rate is optimal in the absence of the EBT.[8] However, as the shadow cost increases, the tax decreases while the marginal damage increases, i.e., $\frac{\partial t_B}{\partial \lambda} < 0$ and $\frac{\partial MD_B}{\partial \lambda} > 0$. It implies that the efficient departure between the optimal tax and marginal damage increases as the shadow cost increases. In particular, we have the following efficient tax gap:

$$t_B - MD_B = \frac{(-1+2\sqrt{6}F)\lambda(2+\lambda)}{6+3\lambda(4+3\lambda)} \leq 0 \text{ when } \lambda \geq 0.$$

Then, EBT is

$$EBT_B = \frac{\lambda(3+6\lambda+4\lambda^2-4\sqrt{6}F(1+\lambda)^2+24F^2(1+\lambda)^2)}{6(2+\lambda(4+3\lambda))} \tag{16}$$

Note that EBT_B is positive and increasing in the shadow cost, i.e., $\frac{\partial EBT_B}{\partial \lambda} > 0$ when $0 < F < F_B$.

Finally, the welfare is

$$W_B = \frac{(1+\lambda)\{3-4\sqrt{6}F(1+\lambda)^2+24F^2(1+\lambda)^2+2\lambda(3+2\lambda)\}}{6(2+\lambda(4+3\lambda))}. \tag{17}$$

Then, the shadow cost affects both EBT and welfare positively, i.e., $\frac{\partial W_B}{\partial \lambda} > 0$ when $0 < F < F_B$.

3.2. Ex-post taxation

We consider ex-post taxation in which the government chooses the level of taxation after the entry of private firms. In other words, each private firm decides whether to enter the market in the first stage

[8] Katsoulacos and Xepapadeas (1995) and Lee (1999) examined the welfare effect of environmental tax policy in free-entry private industries and found that the optimal tax should be set at the marginal environmental damage. See footnote 3 for further discussion.

before the government decides the level of taxation.[9]

In the third stage, both public and private firms simultaneously choose their output levels. The equilibrium outputs of the firms are derived in (3) and (4). In the second stage, the government chooses the optimal level of taxation. The differentiation of W in (7) with respect to t yields the following result:

$$t = \frac{18\lambda(1+\lambda)+n(3+\lambda(5+4\lambda))}{3(3+2n+(15+4n)\lambda+3(4+n)\lambda^2)}. \tag{18}$$

Note that the tax increases as the number of private firms increases, i.e., $\frac{\partial t}{\partial n} > 0$, or as the shadow cost increases, i.e., $\frac{\partial t}{\partial \lambda} > 0$. Contrary to the ex-ante taxation, the effect of the shadow cost on tax can be changed under the ex-post taxation.

By substituting equation (18) into equation (6), we obtain the following equilibrium profits of private firms:

$$\pi_i = \frac{3(1+\lambda)^4}{2(3+2n+(15+4n)\lambda+3(4+n)\lambda^2)^2} - F^2. \tag{19}$$

In the first stage, each private firm enters the market until it earns zero profit. Setting equation (19) to zero yields the following number of private firms in the equilibrium:

$$n_A = \frac{(1+\lambda)(\sqrt{6}(1+\lambda)-6F(1+4\lambda))}{2F(2+\lambda(4+3\lambda))}, \tag{20}$$

where subscript "A" denotes the equilibrium outcome of this game (taxation **after** entry). The number of private firms decreases as the shadow cost increases, i.e., $\frac{\partial n_A}{\partial \lambda} < 0$ in the region of $0 < F < F_A = \frac{\sqrt{6}(1+\lambda)}{6(1+4\lambda)}$, which ensures a positive equilibrium number of private firm under ex-post taxation. Therefore, as EBT is more significant, the excessive number of entering private firms under both ex-ante and ex-post taxation cases will be reduced.

By substituting equation (20) into equation (18), we obtain the following optimal tax:

$$t_A = \frac{3+\lambda(5+4\lambda)-\sqrt{6}F(3+2(1-\lambda)\lambda)}{6+3\lambda(4+3\lambda)}. \tag{21}$$

Note that $t_A > 0$ in the region of $0 < F < F_A$. But, as the shadow cost increases, the optimal tax rate takes U-shape in which it decreases first and then increases, i.e., $\frac{\partial t_A}{\partial \lambda} \leq 0$ when $0 < F \leq F_M = \frac{2+(2-\lambda)\lambda}{2\sqrt{6}(4+\lambda(13+7\lambda))}$ and $\frac{\partial t_A}{\partial \lambda} > 0$ when $F_M < F < F_A$. This is because the shadow cost directly increases

[9] Note that this case provides the same results as the case of the simultaneous choice between taxation and free entry because the equilibrium number of entering firms is given when the government decides the optimal environmental tax.

tax while it also indirectly decreases tax through decreasing the number of firms. Thus, these two separating effects work for deciding the optimal tax rate.

The outputs of public and private firms are:

$$q^{o}{}_{A} = \frac{\sqrt{6}\lambda(1+2\lambda)+6F(2+\lambda)(1+4\lambda)}{3\sqrt{6}(2+\lambda(4+3\lambda))} \text{ and } q^{i}{}_{A} = \sqrt{\frac{2}{3}}F \tag{22}$$

where $q^{o}{}_{A} > q^{i}{}_{A}$. The output of public firm increases as the shadow cost increases, i.e., $\frac{\partial q^{o}{}_{A}}{\partial \lambda} > 0$, while the output of private firm is still independent of the shadow cost, i.e., $\frac{\partial q^{i}{}_{A}}{\partial \lambda} = 0$.

The public firm's profit is:

$$\pi^{o}{}_{A} = \frac{-\lambda^2(1+2\lambda)^2+2\sqrt{6}F\lambda(2+3\lambda+4\lambda^3)+6F^2(2+\lambda)(1+4\lambda)(6+\lambda(7+8\lambda))}{18(2+\lambda(4+3\lambda))^2}. \tag{23}$$

The public firm's profit decreases as the shadow cost increases either (i) when $\lambda \geq \frac{1}{2}$ or (ii) when $\lambda < \frac{1}{2}$ and $F < \frac{\lambda(1+2\lambda)}{\sqrt{6}(2+\lambda(-1+2\lambda))}$. Otherwise, the public firm's profit increases as the shadow cost increases. In either case, as the EBT is more significant, the public firm becomes more aggressive and produces more outputs. However, the total equilibrium output of private firms depends on the fixed cost and the shadow cost because the optimal tax also depends on the shadow cost.

Total market output become:

$$Q_A = \frac{\sqrt{6}(3+\lambda(7+5\lambda))-6F(1+6\lambda+8\lambda^2)}{3\sqrt{6}(2+\lambda(4+3\lambda))}. \tag{24}$$

Under ex-post taxation the market output decreases first and then increases as the shadow cost increases, i.e., $\frac{\partial Q_A}{\partial \lambda} \geq 0$ when $0 < F \leq F_M$ and $\frac{\partial Q_A}{\partial \lambda} < 0$ when $F_M < F < F_A$.

Total and marginal environmental damages are

$$ED_A = \frac{\{\sqrt{6}\left(3+\lambda(7+5\lambda)\right)-6F\left(1+6\lambda+8\lambda^2\right)\}^2}{108(2+\lambda(4+3\lambda))^2} \text{ and } MD_A = \frac{\sqrt{6}(3+\lambda(7+5\lambda))-6F(1+6\lambda+8\lambda^2)}{3\sqrt{6}(2+\lambda(4+3\lambda))}. \tag{25}$$

Note that these environmental damage depend on F_M. In specific, $\frac{\partial ED_A}{\partial \lambda} \geq 0$ but $\frac{\partial MD_A}{\partial \lambda} \leq 0$ when $0 < F \leq F_M$ while the reverse is true otherwise. Note also that $MD_A = \frac{3-\sqrt{6}F}{6} > t_A = \frac{3-3\sqrt{6}F}{6}$ if $\lambda = 0$. It indicates that under ex-post taxation, Pigouvian tax rate is not optimal even in the absence of the EBT. It is well-known result under the situations that the number of firms is given and each firm behaves under imperfect competition.[10] Thus, the optimal tax is always less than marginal damage in the

[10] The optimal tax rate depends on the relative size of the externality and market power distortions under the

absence of EBT, but the efficient departure between optimal tax and marginal damage depends on the shadow cost. In particular, we have:

$$t_A - MD_A = \frac{-\sqrt{6}\lambda(2+\lambda)+12F(\lambda(2+5\lambda)-1)}{3\sqrt{6}(2+\lambda(4+3\lambda))}.$$

Then, we have $t_A < MD_A$ either (i) when $\lambda \leq \frac{1}{2}$ or (ii) when $\lambda > \frac{1}{2}$ and $0 < F < \frac{\lambda(2+\lambda)}{2\sqrt{6}(-1+2\lambda+5\lambda^2)}$. However, we have $t_A > MD_A$ when $\lambda > \frac{1}{2}$ and $\frac{\lambda(2+\lambda)}{2\sqrt{6}(-1+2\lambda+5\lambda^2)} < F < F_A$. It is noteworthy that the optimal tax exceeds the marginal damage under ex-post taxation when the effect of EBT is more significant and the entry cost is high.

Then, EBT is

$$EBT_A = \frac{\lambda(3+6\lambda+4\lambda^2-4\sqrt{6}F(1+\lambda)^2+18F^2(1+4\lambda))}{6(2+\lambda(4+3\lambda))} \quad (26)$$

Note that EBT_A is positive and increasing in the shadow cost, i.e., $\frac{\partial ETB_A}{\partial \lambda} > 0$ when $0 < F < F_A$.

Finally, the welfare is

$$W_A = \frac{(1+\lambda)\{\sqrt{6}(3+6\lambda+4\lambda^2)+18\sqrt{6}F^2(1+4\lambda)-24F(1+\lambda)^2\}}{6\sqrt{6}(2+\lambda(4+3\lambda))}. \quad (27)$$

Then, the shadow cost affects both ETB and welfare positively, i.e., $\frac{\partial W_A}{\partial \lambda} > 0$ when $0 < F < F_A$.

3.3. Comparisons

We now compare the ex-ante and ex-post taxation cases.

Lemma: $F_B - F_A = \frac{\sqrt{6}(2\lambda-1)}{12(1+4\lambda)} \lesseqgtr 0$ when $\lambda \lesseqgtr \frac{1}{2}$.

Let $F^* \equiv \min\{F_B, F_A\}$. Then, we have the following proposition in the range of $0 < F < F^*$.

Proposition: *When the shadow cost is small (large), the ex-post taxation yields a lower (higher) emission tax, a larger (smaller) number of private firms and a higher (lower) environmental damage. However, the ex-ante taxation yields always higher social welfare than the ex-post taxation.*

Proof: When $\lambda \lesseqgtr \frac{1}{2}$, we can show that (i) $t_B - t_A = \frac{\sqrt{6}F(1-2\lambda)(1+2\lambda)}{3(2+4\lambda+3\lambda^2)} \gtreqless 0$, (ii) $n_B - n_A = \frac{3(1+\lambda)(2\lambda-1)}{2+4\lambda+3\lambda^2} \lesseqgtr 0$ and (iii) $ED_B - ED_A = \frac{F(1-2\lambda)(1+2\lambda)(9F(1+2\lambda)^2-\sqrt{6}(3+\lambda(7+5\lambda)))}{9(2+\lambda(4+3\lambda))^2} \lesseqgtr 0$ in the range of $0 < F < F^*$.

given number of firms in oligopoly. See, for example, Shaffer (1995), Lee (1999) and Requate (2007).

Finally, we have that $W_B - W_A = \frac{F^2(2\lambda-1)^2(1+\lambda)}{2+4\lambda+3\lambda^2} \geq 0$ and the equality holds only when $\lambda = \frac{1}{2}$.

The economic explanations behind these results are as follows. Under the fixed number of firms in imperfect competition markets without EBT, it is well-know that the ex-post optimal tax rate should be lower than the marginal damage because oligopolistic firms produce less outputs due to the market power distortion. On the other hand, under the free-entry equilibrium, the ex-ante optimal tax rate should be equal to the marginal damage because the tax effect on the outputs can be offset by the effect on the endogenously chosen number of firms. Thus, ex-post taxation yields a lower tax level than the ex-ante taxation when the shadow cost is small. It also induces that ex-post taxation yields a larger number of private firms and a higher environmental damage

As the shadow cost increases, however, the effect of EBT is more significant and thus, the government induces public firm become more aggressive and thus produces more outputs. Since the output of private firm is determined by the zero-profit condition in a free-entry market equilibrium, total number of entering firms and tax rate determine the total market outputs. Further, a higher concern on the EBT induces a lower number of private firms under both ex-ante and ex-post taxation cases, but it works on the tax rates differently under ex-ante and ex-post taxation cases. Under ex-ante taxation the total outputs of private firms decreases as the shadow cost increases and thus, in order to remedy the distortion of less production, the efficient departure between the optimal tax and the marginal damage increases. On the contrary, under ex-post taxation the total outputs of private firms depend on the shadow cost. In particular, if the shadow cost is very high, the optimal tax should be higher than the marginal damage in order to reduce welfare loss from the other markets. Hence, ex-post taxation yields higher tax level than the ex-ante taxation when the shadow cost is large. It induces that ex-post taxation yields a smaller number of private firms and a lower environmental damage.

Finally, regardless of the shadow cost, the effect of EBT can increase the welfare, and the ex-ante taxation always yields higher welfare than ex-post taxation. This is because the government is able to commit to set the tax rate at the optimal ex-post tax rate under ex-ante taxation before the firms enter the market, but the reverse cannot be achievable. These results are summarized in Table 1.

4. Concluding remarks

This paper investigates the environmental policies in free-entry mixed markets and examines the effect of excess burden of taxation and the impact of the timing of environmental taxation. We compare two models in free-entry mixed markets. We find that ex-post taxation yields a lower (higher) environmental tax level than the ex-ante taxation case when the excess burden of taxation is small (large). It induces that ex-post taxation yields a larger (smaller) number of private firms and a higher (lower) environmental damage. We also find that the effect of excess burden of taxation can increase the welfare, but the ex-ante taxation always yields higher welfare than ex-post taxation.

Regarding the effect of EBT, the number of private firms is always decreasing in the shadow cost, which is independent of the timing of implementation of environmental policy. However, the optimal tax is decreasing in the shadow cost if it is implemented before entry, while it is decreasing (increasing) when the excess burden of taxation is small (large) if it is implemented after entry. These results suggest that market liberalization policy and environmental policy can be either complements or substitutes depending on the timing of taxation and the size of excess burden of taxation. The endogenous choice on the relationship between these two policies remains for future research.

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Table 1: The effects of EBT on the equilibrium results of the two models

	Ex-ante taxation	Ex-post taxation
n	$\frac{(\sqrt{6}-12F)(1+\lambda)^2}{2F(2+\lambda(4+3\lambda))}$ where $\frac{\partial n_B}{\partial\lambda}<0.$	$\frac{(1+\lambda)(\sqrt{6}(1+\lambda)-6F(1+4\lambda))}{2F(2+\lambda(4+3\lambda))}$ where $\frac{\partial n_A}{\partial\lambda}<0.$
t	$\frac{3+\lambda(5+4\lambda)-2\sqrt{6}F(1+\lambda+\lambda^2)}{6+3\lambda(4+3\lambda)}$ where $\frac{\partial t_B}{\partial\lambda}<0.$	$\frac{3+\lambda(5+4\lambda)-\sqrt{6}F(3+2(1-\lambda)\lambda)}{6+3\lambda(4+3\lambda)}$ where $\frac{\partial t_A}{\partial\lambda}\lesseqgtr 0$ if $F\lesseqgtr\frac{2+(2-\lambda)\lambda}{2\sqrt{6}(4+\lambda(13+7\lambda))}.$
Q	$\frac{\sqrt{6}(3+\lambda(7+5\lambda))-12F(1+3\lambda+2\lambda^2)}{3\sqrt{6}(2+\lambda(4+3\lambda))}$ where $\frac{\partial Q_B}{\partial\lambda}>0.$	$\frac{\sqrt{6}(3+\lambda(7+5\lambda))-6F(1+6\lambda+8\lambda^2)}{3\sqrt{6}(2+\lambda(4+3\lambda))}$ where $\frac{\partial Q_A}{\partial\lambda}\gtreqless 0$ if $F\lesseqgtr\frac{2+(2-\lambda)\lambda}{2\sqrt{6}(4+\lambda(13+7\lambda))}.$
ED	$\frac{\{\sqrt{6}(3+\lambda(7+5\lambda))-12F(1+3\lambda+2\lambda^2)\}^2}{108(2+\lambda(4+3\lambda))^2}$ where $\frac{\partial ED_B}{\partial\lambda}>0.$	$\frac{\{\sqrt{6}(3+\lambda(7+5\lambda))-6F(1+6\lambda+8\lambda^2)\}^2}{108(2+\lambda(4+3\lambda))^2}$ where $\frac{\partial ED_A}{\partial\lambda}\gtreqless 0$ if $F\lesseqgtr\frac{2+(2-\lambda)\lambda}{2\sqrt{6}(4+\lambda(13+7\lambda))}.$
MD	$\frac{\sqrt{6}(3+\lambda(7+5\lambda))-12F(1+3\lambda+2\lambda^2)}{3\sqrt{6}(2+\lambda(4+3\lambda))}$ where $\frac{\partial MD_B}{\partial\lambda}>0.$	$\frac{\sqrt{6}(3+\lambda(7+5\lambda))-6F(1+6\lambda+8\lambda^2)}{3\sqrt{6}(2+\lambda(4+3\lambda))}$ where $\frac{\partial MD_A}{\partial\lambda}\gtreqless 0$ if $F\lesseqgtr\frac{2+(2-\lambda)\lambda}{2\sqrt{6}(4+\lambda(13+7\lambda))}.$
W	$\frac{(1+\lambda)\{3-4\sqrt{6}F(1+\lambda)^2+24F^2(1+\lambda)^2+2\lambda(3+2\lambda)\}}{6(2+\lambda(4+3\lambda))}$ where $\frac{\partial W_B}{\partial\lambda}>0.$	$\frac{(1+\lambda)\{\sqrt{6}(3+6\lambda+4\lambda^2)+18\sqrt{6}F^2(1+4\lambda)-24F(1+\lambda)^2\}}{6\sqrt{6}(2+\lambda(4+3\lambda))}$ where $\frac{\partial W_A}{\partial\lambda}>0.$
	$t_B\leq MD_B$ when $\lambda\geq 0$	$t_A<MD_A$ when $0\leq\lambda\leq\frac{1}{2}$ $t_A\gtreqless MD_A$ when $\lambda>\frac{1}{2}$ and $\frac{\lambda(2+\lambda)}{2\sqrt{6}(-1+2\lambda+5\lambda^2)}\lesseqgtr F.$