General Equilibrium Model of Arbitrage Trade and Real Exchange Rate Persistence

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Abstract

Heterogeneity of marginal shipping costs leads to persistent and volatile deviations in real exchange rate. In a two-country, three-good endowment general equilibrium model, arbitrage firms use a transportation technology which depends positively on distance and physical mass of goods. The model exhibits endogenous tradability, non-linearity of law of one price deviations and trade-inducing and suppressing substitution effects due to heterogeneity in trade costs. When endowments follow an AR(1) process that matches quarterly HP-filtered US and EU GDPs, and the aggregate trade costs consume 1.7% of GDP, persistence of real exchange rate matches the data. A model with quadratic adjustment costs also induces sufficient real exchange rate volatility.

Keywords: Arbitrage trade, heterogeneity, real exchange rate, persistence, volatility

JEL Classification: F3, F41

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1 Introduction

[1]

This paper explains persistence and volatility of real exchange rate deviations as a result of heterogeneous shipping costs in a dynamic general equilibrium framework with arbitrage trade. In a two-country three-good endowment model with identical households, arbitrage trading firms chose trade volumes in response to profitable arbitrage opportunities. Because the marginal shipping costs are heterogeneous (motivated by the heterogeneity of physical characteristics important in shipment) a country-specific shock may lead to trade in some goods but not in others. Moreover, the heterogeneity leads to substitution effects between traded and non-traded goods within each country. This substitution can induce or suppress trade and has a measurable influence on the dynamic properties of the real exchange rate. A careful calibration of the model matches persistence of the real exchange rate in the data and, when adjustment costs are added, also generates volatility in real exchange rate deviations.

The concept of purchasing power parity (PPP) maintains that national price levels should be equal when expressed in the units of a common currency (Cassel 1918). Translated into observables, it states that the real exchange rate should be constant. The central puzzle in the international business cycle literature is that fluctuations in the real exchange rate are very large and persistent. Traditional attempts to address this puzzle based on the Harrod-Balassa-Samuelson objection to PPP (Balassa 1961) are empirically unwarranted for developed countries (e.g., Engel 1999)[2]. In particular, many empirical studies document large, volatile and persistent deviations in the prices of traded goods across countries. Several avenues have been explored to motivate the deviations in prices of traded goods from parity. Betts and Devereux (2000) and Bergin and Feenstra (2001) find that pricing to market with segmented markets and nominal rigidities creates volatile deviations in the real exchange rate. A year-long price stickiness combined with a low degree of intertemporal elasticity of substitution and consumption - leisure separable preferences generates sufficient volatility

[1] I thank Michael B. Devereux, John F. Helliwell and James M. Nason for their encouragement and support. I have also benefited from discussions with Jenny Xu and Henry Siu. All errors and omissions are mine.

[2] Harrod-Balassa-Samuelson (HBS) objection is based on the relative price of traded and non-traded goods. Engel (1999) shows that in the U.S. data, no more than 2% of the variation in the real exchange rate can be attributed to the fluctuations in the relative price of non-traded to traded goods. HBS proposition holds holds better for emerging and developing economies, and at lower frequencies. See, i.a., Choudhri & Khan (2004)

but not sufficient persistence in the real exchange rate (Devereux 1997, Chari, Kehoe and McGrattan 2002). A distribution costs approach (e.g., Corsetti and Dedola 2005, Burstein, Neves and Rebello 2003) justifies wedges between the prices of tradable goods by very large costs to product distribution (up to 60% of product price) in order to match the volatility of the real exchange rate. Differences in preferences across countries have also been used to rationalize deviations from the law of one price (e.g., Lapham and Vigneault 2001) but rely on volatile and highly persistent shocks to preference substitution parameters to match the observed fluctuations in the prices of traded goods. Finally, models of the costs of arbitrage trade were so far unsuccessful in generating sufficiently persistent law of one price deviations (e.g., Obstfeld and Rogoff 2000, Dumas 1992, Ohanian and Stockman 1997, Canjels, Prakash-Canjels and Taylor 2004, Sercu, Uppal and van Hulle 1995)[3].

Recent evidence (e.g., O'Connel and Wei 2002, Crucini, Telmer and Zachariadis 2005) shows that law of one price deviations behave in a threshold non-linear and heterogeneous way. Obstfeld and Taylor (1997) find that threshold estimates for sectoral RERs are significantly related to exchange rate volatility and city distances, a result which holds also at an international level and at various frequencies (Zussman 2002). Imbs et. al. (2003) confirm this at a sectoral level. Berka (2009) finds that, at the level of individual goods, heterogeneity of marginal transport costs, proxied by price-to-weight ratios, explains a large part of the variation in thresholds and conditional half-lives of price differences. Prices of heavier or more voluminous goods deviate further before becoming mean reverting, suggesting that shipping costs are important in explaining heterogeneous behaviour of law of one price deviations[4].

The two general equilibrium models presented in this paper show how heterogeneity of shipping costs can explain persistence and volatility in deviations of good prices – and the real exchange rate – from parity. Three goods which only differ by their marginal shipping costs (physical weight) are traded for arbitrage purposes[5]. Arbitrage trading firms decide

[3]The border effect literature tries to understand the vastly higher density of trade flows when two equidistant locations are separated by a border. This phenomenon also includes a very high cross-border price volatility of identical products, and is therefore closely related to literature on real exchange rates. See, i.a., Engel and Rogers 1996, and Jenkins and Rogers (1995).

[4] Hummels (1999) documents that shipping costs depend on weight or volume of the transported goods.

[5]Because the purpose of this paper is to explain price differences and not trade volume, the modelling approach does not require existence of a large amount of arbitrage trade to be justified. A threat of arbitrage is sufficient in keeping a check on price deviations. Arbitrage trade can also be thought of as a limiting case of specialized production and trade of substitutable goods and offers a simple way of introducing shipping costs

on the timing and magnitude of trade to maximize their profits by comparing marginal revenues (proportional to the size of the price difference and trade volume) with arbitrage costs (proportional to shipping distance and the heterogeneous good friction). In the second model, arbitrage costs also include quadratic adjustment costs in the change of trade volume. This makes large changes in the volume of trade more than proportionately costly due to adjustments in legal contracts, infrastructure, such as costs of establishing new (or changing existing) business relationships and distribution networks. Firms then optimally smooth the trade volume leading to more volatile price differentials.

Equilibrium in both models has three notable characteristics. First, the tradability of goods is determined endogenously by the endowment shock and the physical characteristics the product. Second, price differences exhibit threshold non-linearity. Size of the symmetric threshold in the linear model equals the marginal trade cost. Third, size of the law of one price deviation depends on physical characteristics of *all* products and their endowments. General equilibrium effects due to substitution among traded and non-traded goods in each country can induce or suppress trade and affect the real exchange rate distribution. Logarithm of the real exchange rate exhibits a string-type nonlinearity[6]. For large deviations from parity, thresholds of all RER components are crossed, yielding a stronger mean-reverting tendency and a larger arbitrage trade volume. Real exchange rate persistence declines in the volatility of the endowment shock process and increases in the persistence of the endowment shocks and in the trade friction. Volatility of the real exchange rate increases in all three of the above factors (it is concave in shock volatility).

A careful calibration of the first model matches the persistence of real exchange rate found in the data, while producing meaningful persistence and co-movements of various price- and quantity- constructs. However, due to small size of transportation friction and instantaneous adjustment, volatility of RER is low. The quadratic adjustment cost model yields a dynamic and highly non-linear model which retains its core features but improves results in a dynamic environment. It goes a long way towards matching both RER persistence and volatility while giving qualitatively meaningful results along other dimensions.

into the model.

[6]This is an empirical regularity, documented by Taylor, Peel and Sarno 2001, Kilian and Taylor 2003 who show that smooth-threshold AR models provide a better empirical description of the data.

The rest of the paper is structured as follows. Sections 2 and 3 discuss models with linear heterogeneous shipping costs and quadratic adjustment costs, respectively. Section 4 analyzes stochastic properties of the real exchange rate and section 5 discusses parameter calibration. Section 6 analyzes persistence, comovement and volatility of the real exchange rate and other variables. Section 7 concludes.

2 General equilibrium model of arbitrage trade

The two-country world consists of households and arbitrage trading firms. Each country is endowed with positive amounts of three tradable goods. Goods differ in their physical characteristics, proxied here by their weight.

2.1 Households

A representative household at Home chooses its consumption path to maximize instantaneous CES utility function subject to a resource budget constraint:

$$\max_{C_{1t},C_{2t},C_{3t}} \sum_{t=1}^{\infty} \beta^t \left\{ \frac{1}{1-\theta} \left[\gamma_1^{\frac{1}{\theta}} C_{1t}^{1-\frac{1}{\theta}} + \gamma_2^{\frac{1}{\theta}} C_{2t}^{1-\frac{1}{\theta}} + \gamma_3^{\frac{1}{\theta}} C_{3t}^{1-\frac{1}{\theta}} \right]^{1-\theta} \right\}$$

$$\text{s.t.} \quad p_{1t}C_{1t} + p_{2t}C_{2t} + C_{3t} = p_{1t}Y_{1t} + p_{2t}Y_{2t} + Y_{3t} + \frac{1}{2}AP_t \tag{1}$$

given AP_t and $Y_{it},\ i = \{1, 2\}$, where Y_{it} is an endowment of good i at time t, $\sum_{i=1}^{3} \gamma_i = 1$, $\theta > 1$, p_{it} $(i = 1, 2)$ is the relative price of goods i to good 3 and AP_t is the amount of current-period arbitrage profits transferred to the household from a firm, assuming an equal splitting rule between households at home and abroad. The first order conditions for this problem imply the usual demand functions:

$$C_{1t} = \gamma_1 p_{1t}^{-\theta} \frac{Y_t}{P_t^{1-\theta}} \tag{2}$$

$$C_{2t} = \gamma_2 p_{2t}^{-\theta} \frac{Y_t}{P_t^{1-\theta}} \tag{3}$$

$$C_{3t} = \gamma_3 \frac{Y_t}{P_t^{1-\theta}} \tag{4}$$

where Y_t is home country's real GDP expressed in the units of good 3 ($Y_t = p_{1t}Y_{1t} + p_{2t}Y_{2t} + Y_{3t} + \frac{1}{2}AP_t$) and P_t is a composite price index $P_t = (\gamma_1 p_{1t}^{1-\theta} + \gamma_2 p_{2t}^{1-\theta} + \gamma_3)^{\frac{1}{1-\theta}})$. Preferences of households at Home and Abroad are identical, with prices abroad denoted by an asterix.

2.2 Arbitrage trading firms

There is a representative arbitrage trading firm in each country. It chooses the time and amount traded of each good, taking into account the transportation costs.

$$\begin{aligned} \max_{N_1,N_2} A\Pi_t &= \max_{N_1,N_2} \sum_{t=1}^{\infty} \beta^t AP_t \\ &= \max_{N_1,N_2} \sum_{t=1}^{\infty} \beta^t \left[\sum_{i=1}^{3} (p_{it}^* - p_{it})N_{it} - T(N_{1t}, N_{2t}) \right] \end{aligned} \tag{5}$$

where N_{it} is the amount of trade in good i ($N > 0$ implies exports from Home to Abroad) and $T(N_{1t}, N_{2t})$ is the cost function of the arbitrage trading firm. An arbitrage firm has to purchase $T(N_1, N_2)$ units of good 3 to trade $\{N_1, N_2\}$. It is assumed that good 3 has zero trade friction, implying that the law of one price always holds for this good[7]. The cost function is linear in the heterogeneous trade friction t_i:

$$T(N_{1t}, N_{2t}) = (t_1|N_{1t}| + t_2|N_{2t}|) = (aw_1|N_{1t}| + aw_2|N_{2t}|)$$

where t_i is assumed to be a linear function of the weight of a good i w_i and a positive constant homogeneous component of the shipping cost a[8]. The first order conditions for the arbitrage trading firm approximately yield:

$$\begin{aligned} &\mathrm{I(N)}(p_i^* - p_i) = aw_i \qquad \text{iff } |N_i| > 0 \\ &\mathrm{I(N)}(p_i^* - p_i) < aw_i \quad \text{iff } N_i = 0 \text{ for } i = 1, 2 \end{aligned} \tag{6}$$

[7]The assumption of zero trade friction is innocuous. A positive friction for each good would make the computation more complicated but would not change the results qualitatively. Parameters t_1 and t_2 can be thought of as trade frictions of goods 1 and 2 *relative* to the trade friction of good 3.

[8]a can be thought of as a per-kilogram fraction of good 3 which is used when a good is transported between Home and Abroad. For the sake of simplicity and expositional clarity, insurance costs, costs of setting up distribution networks, and other costs are ignored in this specification.

where I(N) is an indicator function, such that I(N)$= 1$ when $N \geq 0$, I(N)$= -1$ otherwise. Trade occurs when the marginal revenue of arbitrage (left-hand side of (6)) exceeds the marginal cost (right-hand side (6)). Trade leads to price convergence, and stops when all profit opportunities are eliminated and absolute value of price difference equals marginal trade cost. FOCs hold with inequality only in autarky. It is intuitive to rewrite (6) as:

$$-a \leq \quad \frac{\overbrace{p_i^* - p_i}^{\text{MRA per kg}}}{w_i} \quad \leq \quad \overbrace{a}^{\text{MCA per kg}} \qquad i = 1, 2$$

The middle part of this inequality captures the marginal revenue of arbitrage per kilogram of good i (MRA) and the outside parts represent the marginal arbitrage cost per kilogram of good i (MCA). While MCA is identical across goods, MRA is not. Goods that are relatively heavier (or for another reason have a larger marginal shipping cost) need a larger price difference in order for MRA to exceed MCA. Thus, maximum law of one price deviation for each good proportional to its weight:

$$\overbrace{|p_1^* - p_1|}^{\text{LOPD}} \quad \leq \quad t_1 \tag{7}$$

$$|p_2^* - p_2| \quad \leq \quad t_2 \tag{8}$$

This leads to *heterogeneous filtering*. Consider an endowment shock x which leads to an identical law of one price deviation for goods 1 and 2. The value of x can be divided into three subsets in terms of its effect on the price deviations. $x \in [0, x_1^*)$ results in autarky because the law of one price deviations for goods 1 and 2 are in a no-trade region ($|LOPD_i| < t_i \iff MR_i < MC_i \quad i = 1, 2$). For $x \in [x_1^*, x_2^*)$, only the lighter good (thereafter good 1) is traded because autarky price difference exceeds t_1 but not t_2: $|LOPD_1| > t_1 \iff MR_1 > MC_1$, $|LOPD_2| < t_2 \iff MR_2 < MC_2$. For $x \in [x_2^*, \infty)$, all goods are traded as respective autarky price differences exceed t_i ($|LOPD_i| > t_i \iff MR_i > MC_i \quad i = 1, 2$).

2.3 Market clearing

Three goods markets clear at home as well as abroad. The direction of trade in goods 1 and 2 depends on the size and sign of the initial deviation from a law of one price, as determined by the endowments. With two countries, $N_i \equiv EXP_i = IMP_i^* = -EXP_i^* \equiv -N_i^*$. The market clearing conditions can then be written as:

$$C_1 + N_1 = Y_1, \qquad C_1^* - N_1 = Y_1^* \tag{9}$$

$$C_2 + N_2 = Y_2, \qquad C_2^* - N_2 = Y_2^* \tag{10}$$

$$C_3 + N_3 + \tfrac{1}{2}T(N_1, N_2) = Y_3, \quad C_3^* - N_3 + \frac{1}{2}T(N_1, N_2) = Y_3^* \tag{11}$$

2.4 Equilibrium

The equilibrium is a set of prices and quantities $\{p_1, p_1^*, p_2, p_2^*, C_1, C_1^*, C_2, C_2^*, C_3, C_3^*, N_1, N_2, N_3\}$ such that the households maximize their utility (equations (1)-(4)), arbitrage trading firms maximize their profits (eqs. (7) to (8)) and markets clear (eqs. (9) - (11)).

2.4.1 Frictionless trade

Without transportation costs ($t_i = 0$), profit maximization problem faced by the firm implies that law of one price holds for all goods ($p_i^* = p_i,\ i \in \{1, 2\}$). The equilibrium relative prices then depend on the world endowments and the preference parameters:

$$\frac{p_i}{p_j} = \frac{p_i^*}{p_j^*} = \left[\frac{Y_j^W}{Y_i^W}\frac{\gamma_i}{\gamma_j}\right]^{\frac{1}{\theta}} \quad \forall\ i \tag{12}$$

where $Y_i^W \equiv Y_i + Y_i^*$. The equilibrium consumption levels are

$$C_1 = Y_1 \frac{\left(\frac{\gamma_1}{\gamma_2}\right)^{\frac{1}{\theta}} \left(\frac{Y_2^W}{Y_1^W}\right)^{\frac{1}{\theta}} + \frac{Y_2}{Y_1}}{\left(\frac{\gamma_1}{\gamma_2}\right)^{\frac{1}{\theta}} \left(\frac{Y_2^W}{Y_1^W}\right)^{\frac{1}{\theta}} + \frac{Y_2^W}{Y_1^W}}, \qquad C_2 = Y_1 \frac{Y_2^W}{Y_1^W} \frac{\left(\frac{\gamma_1}{\gamma_2}\right)^{\frac{1}{\theta}} \left(\frac{Y_2^W}{Y_1^W}\right)^{\frac{1}{\theta}} + \frac{Y_2}{Y_1}}{\left(\frac{\gamma_1}{\gamma_2}\right)^{\frac{1}{\theta}} \left(\frac{Y_2^W}{Y_1^W}\right)^{\frac{1}{\theta}} + \frac{Y_2^W}{Y_1^W}}$$

and similarly for C_1^* and C_2^*. $C_i = Y_i,\ C_i^* = Y_i^*$ iff $\frac{Y_1}{Y_2} = \frac{Y_1^*}{Y_2^*}$. Country which is endowed with a *relatively* larger amount of good i will export good i and import good j.

2.4.2 Equilibrium with positive trade frictions $t_2 > t_1 > 0$

With positive trade frictions and $Y_1 = Y_2$, three cases can arise. In Case 1, endowments are such that $LOPD_i < MC_i$ in autarky (i.e., (7) and (8) hold with inequality) and no goods are traded. In Case 2, the endowments imply autarky prices which exceed the marginal cost of arbitrage for one good but not the other. Consequently, trade occurs in one good but not the other (one of (7) and (8) holds with equality, the other with inequality). Finally, in Case 3 the endowments imply autarky prices such that the law of one price exceeds MC$_i$ $\forall i \in \{1, 2\}$, both goods are traded. I summarize the equilibrium in all three cases.

Case 1: No trade in goods 1 & 2 The equilibrium conditions are:

$$
\begin{aligned}
\gamma_1 p_1^{-\theta} \frac{Y}{P^{1-\theta}} = Y_1, \qquad & \gamma_1 p_1^{*-\theta} \frac{Y^*}{P^{*1-\theta}} = Y_1^* \\
\gamma_2 p_2^{-\theta} \frac{Y}{P^{1-\theta}} = Y_2, \qquad & \gamma_2 p_2^{*-\theta} \frac{Y^*}{P^{*1-\theta}} = Y_2^* \\
\gamma_3 \left(\frac{Y}{P^{1-\theta}} + \frac{Y^*}{P^{*1-\theta}} \right) \quad & = \quad Y_3 + Y_3^*
\end{aligned}
$$

where $Y = p_1 Y_1 + p_2 Y_2 + Y_3$, $Y^* = p_1^* Y_1^* + p_2 Y_2^* + Y_3^*$, $P = (\gamma_1 p_1^{1-\theta} + \gamma_2 p_2^{1-\theta} + \gamma_3)^{1/(1-\theta)}$ and $P^* = (\gamma_1 p_1^{*1-\theta} + \gamma_2 p_2^{*1-\theta} + \gamma_3)^{1/(1-\theta)}$. Walras' law implies that the system can be uniquely solved for prices $\{p_1, p_2, p_1^*, p_2^*\}$, which recursively define other equilibrium values.

Case 2: No trade in good j In this case, $N_j = 0$ and the equilibrium is characterized by:

$$
\begin{aligned}
\gamma_i \left(p_i^* - I(N_i) t_i\right)^{-\theta} \frac{Y}{P^{1-\theta}} + \gamma_i p_i^{*-\theta} \frac{Y^*}{P^{*1-\theta}} \quad & = \quad Y_i + Y_i^* \\
\gamma_j p_j^{-\theta} \frac{Y}{P^{1-\theta}} = Y_j \ \text{ and } \ \gamma_j p_j^{*-\theta} \frac{Y^*}{P^{*1-\theta}} = Y_j^* \quad & \\
\gamma_3 \frac{Y}{P^{1-\theta}} + \gamma_3 \frac{Y^*}{P^{*1-\theta}} + t_i \left[Y_i - \gamma_i \left(p_i^* - I(N_i) t_i\right)^{-\theta} \frac{Y}{P^{1-\theta}} \right] \quad & = \quad Y_3 + Y_3^*
\end{aligned}
$$

where $Y = (p_1^* - I(N_1) t_1) Y_1 + p_2 Y_2 + Y_3$, $P = (\gamma_1 (p_1^* - I(N_1) t_1)^{1-\theta} + \gamma_2 p_2^{1-\theta} + \gamma_3)^{1/(1-\theta)}$ and $I(.)$ is the indicator function defined in (6). Walras' law implies that this system uniquely determines $\{p_i^*, p_j^*, p_j\}$ and consequently all other equilibrium values as functions of preferences, endowments, and the trade friction t_i.

Case 3: All goods traded Here, equilibrium prices solve the following reduced system:

$$\begin{aligned}
(p_1^* - I(N)t_1)^{-\theta}\frac{Y}{P^{1-\theta}} + p_1^{*-\theta}\frac{Y^*}{P^{*1-\theta}} &= \frac{1}{\gamma_1}(Y_1 + Y_1^*) \\
(p_2^* - I(N)t_2)^{-\theta}\frac{Y}{P^{1-\theta}} + p_2^{*-\theta}\frac{Y^*}{P^{*1-\theta}} &= \frac{1}{\gamma_2}(Y_2 + Y_2^*) \\
\gamma_3\frac{Y}{P^{1-\theta}} + \gamma_3\frac{Y^*}{P^{*1-\theta}} + t_1\left[Y_1 - \gamma_1(p_1^* - I(N_1)t_1)^{-\theta}\frac{Y}{P^{1-\theta}}\right] &= Y_3 + Y_3^*
\end{aligned}$$

where $Y = (p_1^* - I(N_1)t_1)Y_1 + (p_2^* - I(N_2)t_2)Y_2 + Y_3)$ and $P = (\gamma_1(p_1^* - I(N_1)t_1)^{1-\theta} + \gamma_2(p_2 - I(N_2)t_2)^{1-\theta} + \gamma_3)^{1/(1-\theta)}$. Walras' law reduces the above system into two equations that solve uniquely for $\{p_1^*, p_2^*\}$ and implicitly all other variables as functions of endowments, preferences, and the trade frictions.

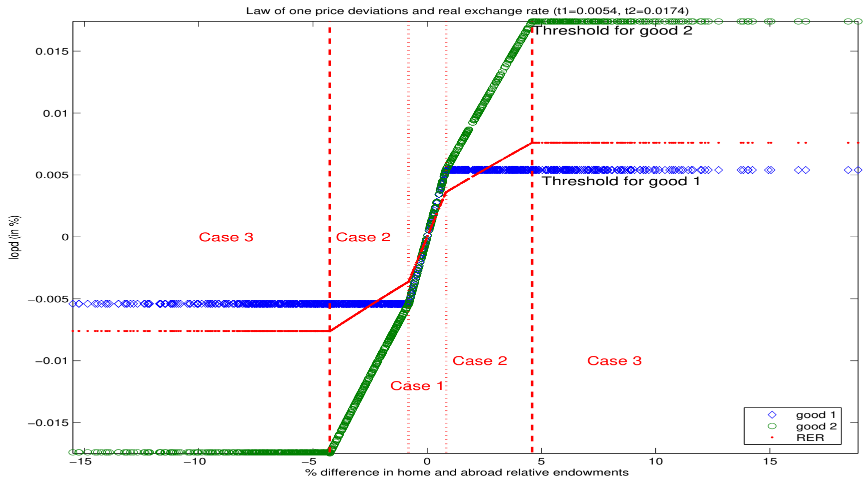
2.4.3 Properties of the equilibrium

Trade frictions affect equilibrium prices and allocations in all three cases: directly in cases 2 and 3 and indirectly in cases 1 and 2 by defining endowments for which the autarky solutions apply. Furthermore, *prices of non-traded good in Case 2 are affected by the price convergence in traded good*. This general equilibrium effect is caused by consumers in exporting country substituting away from traded (whose price rises due to shrinking domestic supply) into non-traded good and consumers in the importing country moving away from non-traded into traded good. Consequently, law of one price deviation for the non-traded good is smaller when the other good is traded than it would have been if both good were not traded. When endowment shocks are country- or sector-specific, this substitution effect can induce or suppress trade and affects the dynamic properties of the real exchange rate.

Figure 1 plots the equilibrium law of one price deviations against the endowment difference. Keeping the endowments Abroad fixed, Home endowments of goods 1 and 2 vary by the same amount, leading to changes in p_1 and p_2[9]. In case 1, price differences are smaller than marginal costs of trade. In case 2, trade occurs for good 1 but good 2 remains non-traded. When Home exports good 1, p_1 rises and p_1^* declines until $p_1^* - p_1 = t_1$. Therefore, graph of $LOPD_1$ has a threshold in case 2. As the demand for non-traded good rises in exporting

[9]This is the simplest way to perturb the model to illustrate the three aforementioned cases.

Figure 1: Model solution: thresholds of price deviations in linear model



and declines in importing country (due to the aforementioned substitution effect), $LOPD_2$ increases in the endowment difference at a lower rate than when good 1 is not traded. Consequently, slope of $LOPD_2$ is lower in case 2 than in case 1. To the extent that goods are substitutable, trade in one sector lowers the law of one price deviations in non-traded sectors[10]. Finally, when the endowment differences induce trade in the second goods, equilibrium $LOPD_2$ reaches a threshold.

This implies that when shocks to endowments are identical across sectors, goods with larger trade friction have, on average, larger and more volatile LOPDs. Larger shocks increase the size of the LOPD but only to the point where arbitrage takes place; excess shock volatility does not affect the mean nor standard deviation of LOPDs.

3 Arbitrage trade model with adjustment costs to trade

The second model has an identical endowment and preference setting. However, trade costs also include quadratic adjustment costs in the change of trade volume. Changes in the trade volume require hiring of labour resources, adjustment in the distribution system and possibly investment in new (or a changes of the existing) trade infrastructure. Larger swings in trade

[10]For example, trade in shaving machines would reduce law of one price deviation in barber services.

volume are therefore more-than-proportionately costly. The arbitrage firms' problem is now:

$$\begin{aligned}
\max_{N_{1t},N_{2t}} A\Pi_t &= \max_{N_{1t},N_{2t}} \sum_{j=t}^{\infty} \beta^{(j-1)} AP_j \\
&= \max_{N_{1t},N_{2t}} \sum_{j=t}^{\infty} \beta^{(j-1)} \left[\sum_{i=1}^{2} (p^*_{ij} - p_{ij}) N_{ij} - T(N_{1j}, N_{2j}) \right] \qquad (13) \\
\text{s.t. } T(N_{1t}, N_{2t}) &= t_1|N_{1t}| + t_2|N_{2t}| + c_1 \Delta N_{1t}^2 + c_2 \Delta N_{2t}^2 \qquad (14)
\end{aligned}$$

where N_{it} is the amount of trade in good i at time t from Home to Abroad, $T(N_{1t}, N_{2t})$ the total cost function of the arbitrage trading firm and p_{it} the price of good i relative to good 3. The firm has to purchase $T(N_{1t}, N_{2t})$ units of good 3 to trade $\{N_{1t}, N_{2t}\}$[11]. The total cost consists of a shipping cost and an adjustment cost. Shipping cost is identical to that in the first model: $t_i = aw_i,\ i = 1, 2$ where w_i is the weight of good i and a is a constant. Adjustment cost is quadratic in the change of volume of trade from the previous period to the current period[12]. Parameters c_i are not related to the physical characteristics of goods.

The difficulty of summarizing the behaviour of the firm with its first order conditions lies in the non-differentiability of the absolute value function at 0. A smooth approximation $G(.)$ to the absolute value function is used to allow a continuous mapping between the first order conditions and the objective function. Let $\mathrm{I}(N_{i,t}) \equiv \mathrm{d}G(.)$ denote the first order derivative of a "smooth" absolute value function. $\mathrm{I}(N_{i,t})$ can be thought of as an approximation to the indicator function: $\mathrm{I}(N_{i,t}) = 1$ when $N_{i,t} > 0$, $\mathrm{I}(N_{i,t}) = -1$ when $N_{i,t} < 0$ and $\mathrm{I}(N_{i,t}) = 0$ when $N_{i,t} = 0$ (see Appendix A). The first order optimality conditions then yield:

$$\begin{aligned}
0 &= \left\{ (p^*_{i,t} - p_{i,t}) - \frac{\partial T(._t)}{\partial N_{i,t}} - \beta \mathrm{E}_t \frac{\partial T(._{t+1})}{\partial N_{i,t}} \right\} \\
0 &= p^*_{i,t} - p_{i,t} - [t_i \mathrm{I}(N_{i,t}) + 2c_i(N_{i,t} - N_{i,t-1})] - \beta \mathrm{E}_t[-2c_i(N_{i,t+1} - N_{i,t})]
\end{aligned}$$

Rearranging, we get

$$\frac{1}{2c_i} \left[p^*_{i,t} - p_{i,t} - t_i \mathrm{I}(N_{i,t}) \right] = -\beta \mathrm{E}_t N_{i,t+1} + (1+\beta) N_{i,t} - N_{i,t-1}$$

[11] The simplifying assumption that trade in good 3 is costless remains.

[12] A quadratic adjustment cost function provides a reduced form which captures firm's gradual response in a smoother way than the assumption of a pre-determined volume of shipment (i.a., Ravn & Mazzenga 2004).

$$= \quad [-\beta + (1+\beta)\mathrm{B} - \mathrm{B}^2]\mathrm{E}_t N_{i,t+1} \ \forall i$$

where B is a backshift operator. The quadratic form on the right hand side non-stationary root allowing us to rewrite the equation as: $-(\mathrm{B}-1)(\mathrm{B}-\beta)$. The first order conditions for the firm can then be written as

$$-\frac{1}{2c_i}\left[p^*_{i,t} - p_{i,t} - t_i \mathrm{I}(N_{i,t})\right] = (1-\mathrm{B}^{-1})(1-\beta\mathrm{B}^{-1})\mathrm{E}_t N_{i,t-1} \ \forall i$$

Expanding the stable eigenvalue forward and the unstable backward, the first order condition for the arbitrage trading firm can be re-written in the forward-looking form:

$$N_{i,t} = N_{i,t-1} + \frac{1}{2c_i}\mathrm{E}_t \sum_{j=0}^{\infty} \beta^j \left(p^*_{i,t+j} - p_{i,t+j} - t_i \mathrm{I}(N_{i,t+j})\right) \ \forall i \tag{15}$$

The optimal amount of trade in good i in period t depends positively on the volume of trade in the last period and on the expected future path of price differences in excess of the trade friction. Firms care about the future path of LOPDs because they prefer to smooth their trade pattern over time. The size of expected price difference in excess of trade friction in period $t+j$ increase trade in all periods after t. The expected future direction of trade $\mathrm{E}_t \mathrm{I}(N_{i,t+j})$ is also important: if the firm expects a future price process in which periods with expected export regime are followed by periods with expected import regime, it optimally lowers today's trade volume relative to a scenario in which only one regime prevails.

3.1 Equilibrium

The equilibrium is a set of prices and quantities $\{p_{1,t}, p^*_{1,t}, p_{2,t}, p^*_{2,t},\ C_{1,t}, C^*_{1,t}, C_{2,t},\ C^*_{2,t}, C_{3,t}, C^*_{3,t}, N_{1,t}, N_{2,t},\ N_{3,t}\}_{t=0}^{\infty}$ such that the representative household maximizes its utility (equations (1)-(4)), arbitrage trading firms maximize their profits (equation (15) for both goods) and all markets clear (equations (9) - (11)). It can be simplified into a 4-by-4 system in $\{p_{1,t}, p_{2,t}, p^*_{1,t}, p^*_{2,t}\}$:

$$\Delta Y_{i,t} - \gamma_i \left[p_{i,t}^{-\theta}\frac{Y_t}{P_t^{1-\theta}} - p_{i,t-1}^{-\theta}\frac{Y_{t-1}}{P_{t-1}^{1-\theta}}\right] = \frac{1}{2c_i}\mathrm{E}_t \sum_{j=0}^{\infty} \beta^j \left[p^*_{i,t+j} - p_{i,t+j} - t_i \mathrm{I}(N_{i,t+j})\right], i \in \{1,2\} \tag{16}$$

$$\gamma_i p_{i,t}^{-\theta} \frac{Y_t}{P_t^{1-\theta}} + \gamma_i p_{i,t}^{*-\theta} \frac{Y_t^*}{P_t^{*1-\theta}} = Y_{i,t} + Y_{i,t}^*, i \in \{1,2\} \quad (17)$$

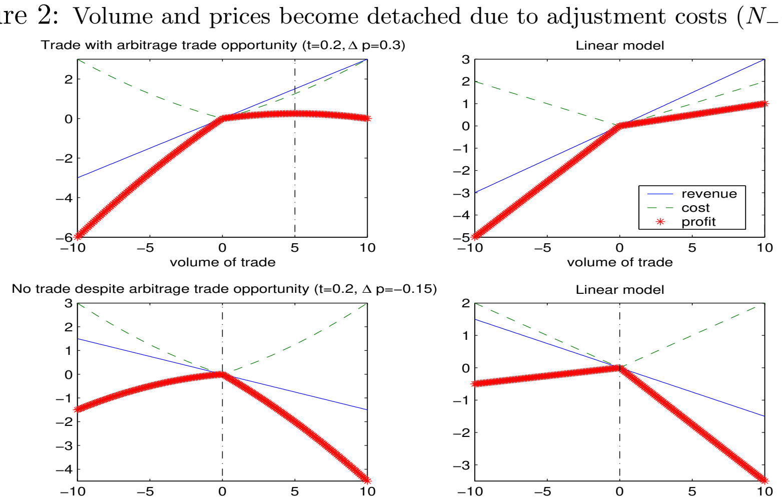
where $Y_t = p_{1,t}Y_{1,t} + p_{2,t}Y_{2,t} + Y_{3,t} + \frac{1}{2}AP_t$, $P_t = (\gamma_1 p_{1,t}^{1-\theta} + \gamma_2 p_{2,t}^{1-\theta} + \gamma_3)^{\frac{1}{1-\theta}}$ and AP_t are the contemporaneous arbitrage profits. For goods 1 and 2, equations (16) and 17) represent the intertemporal and intratemporal equilibrium conditions, respectively.

3.1.1 Intuition

Two pieces of intuition about the influence of adjustment costs can be built by considering a one-period partial equilibrium version of the model. First, firm chooses a finite trade volume with adjustment costs while it would chose an infinite trade volume in their absence. Second, price deviations can exceed shipping costs in equilibrium. Conversely, trade may occur when price difference does not exceed shipping costs[13].

In a one-period version of the model with one good and a positive trade friction t, first order condition implies: $p^* - p - \mathrm{I}(N)t = 2c(N - N_{-1})$ where N_{-1} is the last period's trade volume. With $c > 0$ and $|p^* - p| > t$, firm chooses a finite volume of trade that depends positively on $p^* - p$ and last period's trade volume N_{-1}, and negatively on the cost parameters t and c. Figure 2 compares the profit functions between linear and simplified QAC models

Figure 2: Volume and prices become detached due to adjustment costs ($N_{-1} = 0$)



when $t = 0.2$ and $N_{-1} = 0$. The upper segment illustrates situations when price Abroad is

[13]Note that the quadratic adjustment cost model nests the linear shipping cost model. When $c = 0$, (16)is identical to (6).

30% higher than at Home in autarky and trade takes place. The trade volume in a simple QAC model is finite because the profit function is parabolic (with a kink). In the lower segment, p^* is 15% below p, and no trade takes place.

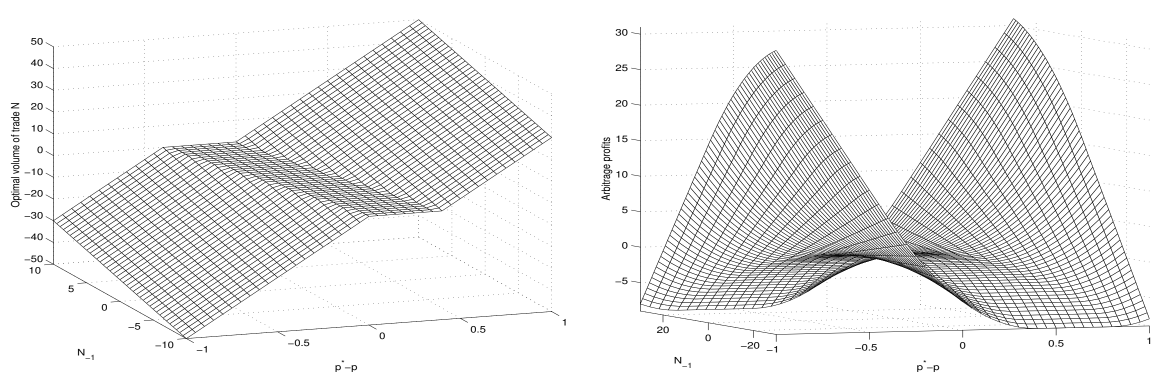
Because trade is the only source of price adjustment, a smaller trade volume requires a smaller price adjustment. By lowering trade volume, adjustment costs can sustain law of one price deviations which exceed threshold t in equilibrium. *Although quadratic adjustment cost model creates the same no-trade region (in terms of price differences) as the linear model, law of one price deviation can exceed the trade frictions in equilibrium.* It can be shown that, in the one-period model, for any N_{-1}, an increase in home endowment will decrease home price:

$$\frac{\partial p_i}{\partial Y_i} = \mathrm{D}\left[-2c - t\frac{\partial \mathrm{I}(N)}{\partial Y_i}\left(1 + \frac{1}{A}\right)\right]$$

where D>0 and $A < -1$. Further, LOPD increases in c because a larger adjustment cost leads to a smaller adjustment in volume.

The intuition changes slightly when $N_{-1} \neq 0$ because $N = N_{-1} + \frac{1}{2c}(p^* - p - \mathrm{I}(N)t)$. The relationship between N and LOPD is qualitatively unchanged as long as the good remains traded and $\mathrm{I}(N)$ does not change. But the range of autarkic values of LOPD decreases in N_{-1} (left-hand panel of Figure 3). When $N_{-1} \neq 0$, costly trade deceleration can imply positive trade volume even though $|p^* - p| < t$ as the firm strikes balance between contemporaneously loss-making trade and costs of trade deceleration. Therefore, profits can be negative in equilibrium when $c > 0$ (the right-hand panel of figure 3). Larger values of $|N_{-1}|$ require smaller $|p^* - p|$ to optimally induce trade.

Figure 3: Trade and profits in partial equilibrium in QAC model for various LOPDs and N_{-1}. c=0.01, t=0.2



The tendency for price differences to exceed marginal shipping cost is visible in the full version of the model: the initial law of one price deviations increase in the endowment difference even when both goods are traded (the "increasing thresholds" in figure 9). As in the linear shipping cost model, three cases exist, and the influence of the substitution effect is visible in the change of the slope of $LOPD_2$ after good 1 becomes traded. Trade volume depends negatively on frictions c and t. The adjustment costs force firms to spread trade in more steps of smaller magnitude: length of adjustment time depends positively on c and $\mathrm{d}Y_i$.

3.2 Solution method

Due to a high degree of non-linearity, the model is solved numerically. First, to limit the time span for adjustment, I assume a steady state equilibrium to which countries converge following a shock, and a number of time periods T available for the adjustment. Conditional on T, the model is solved using method of relaxation by Boucekkine (1995) in which a finite-period approximation $\widehat{f(.)}_{t=1:T} = 0$ to the system $f(.)_{t=1:\infty} = 0$ is solved by stacking all equations for all time periods into one large system $F(.) \equiv [\widehat{f(.)}_{t=1} \ ... \ \widehat{f(.)}_{t=T}]\prime = 0$ which is then solved numerically. Second, in order to compute the Jacobian of the stacked system $F(.)$ in one step, it is necessary to select a functional form for $I(N_{i,t})$. The selection is described in detail in Appendix A. Third, to facilitate the numerical solver in finding an equilibrium, (16) is replaced with their simpler forms (18) and (19) which do not include an infinite forward-looking sum. This step facilitates convergence because an error in p_{it} by the numerical solver only affects the $4(t-1) : 4(t+1)$ partition of the Jacobian, not all $(4T)^2$ values it would otherwise[14].

$$\begin{aligned}
\frac{1}{2c_1}(p^*_{1,t} - p_{1,t} - t_1 I(N_{1,t})) \quad &= \quad (1+\beta)Y_{1,t} - \beta Y_{1,t+1} - Y_{1,t-1} + \gamma_1 p_{1,t-1}^{-\theta}\frac{Y_{t-1}}{P_{t-1}^{1-\theta}} \\
&\quad -(1+\beta)\gamma_1 p_{1,t}^{-\theta}\frac{Y_t}{P_t^{1-\theta}} + \beta\gamma_1 p_{1,t+1}^{-\theta}\frac{Y_{t+1}}{P_{t+1}^{1-\theta}} \qquad (18) \\
\frac{1}{2c_2}(p^*_{2,t} - p_{2,t} - t_2 I(N_{2,t})) \quad &= \quad (1+\beta)Y_{2,t} - \beta Y_{2,t+1} - Y_{2,t-1} + \gamma_2 p_{2,t-1}^{-\theta}\frac{Y_{t-1}}{P_{t-1}^{1-\theta}} \\
&\quad -(1+\beta)\gamma_2 p_{2,t}^{-\theta}\frac{Y_t}{P_t^{1-\theta}} + \gamma_2\beta p_{2,t+1}^{-\theta}\frac{Y_{t+1}}{P_{t+1}^{1-\theta}} \qquad (19)
\end{aligned}$$

[14] When $T = 30$, this translates into 144 rather than 14400 values. The latter prevents convergence even for relatively small errors.

A system $\widehat{f(.)}_{t=i}$, part of the large stacked system $F(.)$, consists of equations (18), (19) and (17). Period $T+1$ values found in the inter-temporal Euler equations of $\widehat{f(.)}_T$ are set to steady-state equilibrium values associated with a full adjustment to the shock. Finally, values of $I(N_{i,t})$ in the approximate solution obtained above are replaced with $1, -1$, or 0 and system $F(.)$ is solved again to ensure that the approximation is valid.

4 Real exchange rate

This section explains the behaviour of the real exchange rate in the model for a range of parameter values when endowments are stochastic. Logarithm of the real exchange rate from the model is a weighted average of the three law of one price deviations[15]: $\log(RER) = \gamma_1 \log(LOPD_1) + \gamma_2 \log(LOPD_2) + \gamma_3 \log(LOPD_3)$. At first, endowments Abroad are fixed while at Home they follow an AR(1) process: $Y_{i,t} = \alpha Y_{i,t-1} + (1-\alpha)\bar{Y} + u_t \; i = 1, 2$ ($u_t \sim$ N$(0, \sigma^2)$). The assumption that only one country is subject to the shocks and that both sectors receive the same shock is relaxed in sections 4.2 and 4.3, respectively.

4.1 Real exchange rate in a linear shipping cost model

Persistence of real exchange rate in the linear model *increases in shipping costs* which determine the size of a no-arbitrage threshold. This relationship gets stronger as t_2/t_1 increases, implying that the *heterogeneity* of shipping costs increases RER persistence. Finally, persistence *decreases in the volatility of endowment shocks* because smaller (persistent) shocks tend to remain longer below the no-arbitrage threshold.

Conditional on the trade friction, persistence of the real exchange rate is positively related to the persistence of the endowment shock process as measured by α (Table 1). For $\alpha \leq 0.9$, half lives of convergence do not exceed 6 time periods. Half life increases sharply in α for values near 1, to about 11 when $\alpha = 0.95$, and up to 933 time periods when $\alpha = 0.99$. Variance of shocks decreases half life because it increases the likelihood of triggering arbitrage and consequently price convergence.

[15]This is the method of constructing of RER in the empirical literature. Each country j's CPI is a geometric average of goods and services with weights corresponding to the consumption shares. Hence, $\log(CPI_t^j) = \gamma_1^j p_{1,t}^j + \gamma_2^j p_{2,t}^j + \gamma_{3,t}^j$. When γ_i is the same in both countries, the RER result follows.

Volatility of the real exchange rate increases both in shock persistence α and shock volatility σ (Table 1). Endowment shocks increase LOPD volatility as long as at least one good is not traded. When both goods are traded, additional shock volatility is neutral because the additional price differences are arbitraged away. Higher α leads to longer-lived LOPDs, thus increasing their volatility, ceteris paribus. This is especially visible when σ is small so that most shocks leave LOPDs below their thresholds. RER volatility then exceeds σ. Conversely, high σs only have a marginal effect on std(RER).

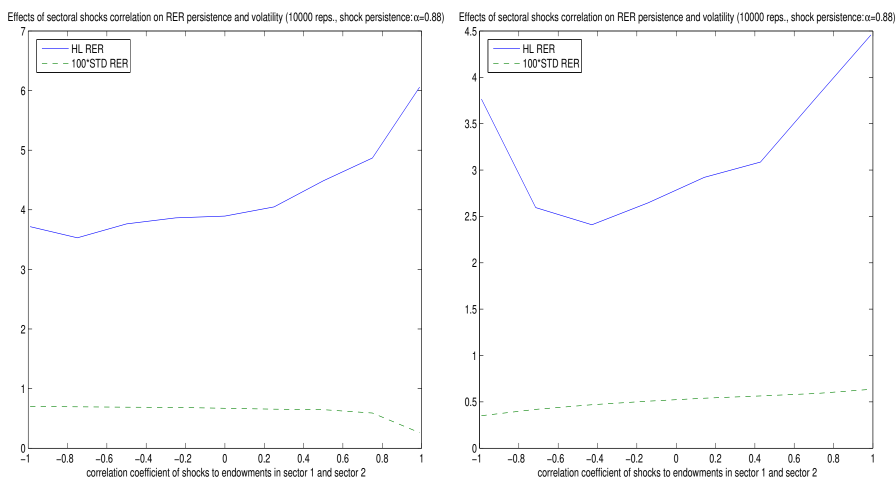
Shipping cost increases persistence of real exchange rate for any given α and σ because it requires a larger endowment shock in order for arbitrage trade to occur. Moreover, *heterogeneity* of the shipping costs increases persistence and volatility of RER because of a substitution from traded into non-traded good in the exporting country (see the following sub-section for a more detailed explanation). This yields the increasing loci of persistence and volatility in t_2/t_1 (Table 2). The effect is stronger at higher values of α.

4.2 Country-specific shocks

Now let the endowments vary in both countries, assuming they follow a similar AR(1) process: $\hat{Y}_{i,t} = \alpha\hat{Y}_{i,t-1} + (1-\alpha)\bar{Y} + \hat{u}_t$ for $i = 1, 2$ where $\hat{Y}_{i,t} = [Y_{i,t}\ Y^*_{i,t}]'$, $\hat{u}_t = [u_t\ u^*_t]'$ and $\hat{u}_t \sim \mathrm{N}(0, \hat{\Omega})$ where $\hat{\Omega} = \begin{pmatrix} \sigma^2 & \gamma \\ \gamma & \sigma^2 \end{pmatrix}$. The left panel of Figure 4 shows that RER *persistence* increases in the correlation coefficient η ($\eta \equiv \frac{\gamma}{\sigma^2}$) while *volatility* decreases in η. Negatively correlated shocks lead to relatively larger LOPDs and larger average RER while positively correlated shocks lead to relatively smaller LOPDs and smaller average RER. With more mass of the RER distribution near the mean when $\eta > 0$, RER deviations do not change much from one period to another, leading to a more persistent and less volatile RER. When $\eta < 0$, RER distribution has a relatively larger proportion of the mass in tails (near the thresholds). Repeated draws from this distribution lead to a process with less persistence (deviations differ from mean more often) and a higher volatility. The monotonicity in average LOPDs as η increases leads to monotonicity in persistence as well as volatility when shocks are country-specific.

The substitution effect from a traded into a non-traded good (case 2[16]) affects the size of the RER and therefore its persistence and volatility. As the traded good is exported, its domestic price increases, prompting a substitution to the non-traded good, and increasing p_{NT} (vice versa in the importing country). Changes in LOPD$_T$ and LOPD$_{NT}$ are positively correlated: as trade lowers $|LOPD_T|$ to arbitrage threshold, $|LOPD_{NT}|$ also declines. Thus, the substitution effect lowers the average $|RER|$[17]. Because the proportion of case 2 trades in all trades increases in η when shocks are country-specific, influence of the substitution effect on RER is also increasing in η. RER persistence is up to 8% higher and volatily up to 9% lower as a result of the substitution from traded into non-traded goods.

Figure 4: RER properties with country- and sector- specific endowment shocks in linear model (shock volatility as a proportion of GDP: $\sigma = 0.034$)



4.3 Sector-specific shocks

Now assume that the endowments differ across sectors. For simplicity, endowments Abroad are kept constant and Home endowments follow an AR(1) process: $\tilde{Y}_t = \alpha\tilde{Y}_{t-1} + (1-\alpha)\bar{Y} + \tilde{u}_t$ for $i = 1, 2$ where $\tilde{Y}_{i,t} = [Y_{1,t}\ Y_{2,t}]'$, $\tilde{u}_t = [u_{1,t}\ u_{2,t}]'$ and $\tilde{u}_t \sim \mathrm{N}(0, \tilde{\Omega})$ where $\tilde{\Omega} = \begin{pmatrix} \sigma^2 & \gamma \\ \gamma & \sigma^2 \end{pmatrix}$. The right panel of Figure 4 shows the asymmetric U-shaped relationship between RER persistence and $\tilde{\eta}$, the correlation coefficient of shocks across sectors. Volatility

[16] In case 2 when both goods are traded, substitution effect does not have a measureable effect on ex-post price deviations, only on the volume of trade.

[17] When $\alpha = 0.88$, difference in the average $|RER|$ and $|RER_{noS.E.}|$ increases in η from 3 to 9%.

of RER increases monotonically in $\tilde{\eta}$.

When $\tilde{\eta} < 0$, shocks to sectoral endowments at Home tend to be of opposite signs, leading to opposite signs of LOPDs (and the direction of trade flows) of goods 1 and 2. Such LOPDs partly cancel each other. Therefore, $|RER|$ tends to be closer to zero. When $\tilde{\eta} > 0$, shocks to sectoral endowments at Home tend to have the same sign, leading to LOPDs of identical signs and a larger average $|RER|$. From the definition of the shock process, frequency of $\{\text{sign}(LOPD_1)=\text{sign}(LOPD_2)\}$ increases in $\tilde{\eta}$, which causes $|RER|$ to be increasing in $\tilde{\eta}$ also. This drives the increasing tendency in RER volatility: repetitive draws from a distribution which is more compressed around its mean ($\tilde{\eta} < 0$) lead to a less volatile RER process. As $\tilde{\eta}$ increases, frequency of situations when both goods are traded in the same direction increases, and with it the frequency of RER reaching its threshold ($\gamma_1 t_1 + \gamma_2 t_2$). As the mass of the distribution of $|RER|$ increases around the threshold, RER persistence increases[18].

The influence of the substitution effect on RER also depends on the signs of sectoral shocks. If the shocks are of opposite signs, the positive correlation between changes in $LOPD_T$ and $LOPD_{NT}$ due to substitution effect leads to *trade induction*. $LOPD_{NT}$ can be brought to its no-arbitrage threshold and, consequently, become traded. Conversely, if the sectoral shocks are of the same sign, non-traded good is less likely to become traded. This *trade suppression* effect is also a result of the substitution from a traded to non-traded good in the exporting country[19]. Trade induction can either increase or decrease $|RER|$, depending on which good is not traded. Trade suppression always decreases $|RER|$. As the proportion of trade suppression increases in $\tilde{\eta}$, so does the downward influence of the substitution effect on $|RER|$ (mean $|RER|$ decreases by 2 to 8% and the standard deviation of RER by 3 to 5%). The shift of the RER distribution away from the RER threshold due to substitution effect leads to a decline in RER half-life between 1 and 22% (depending on $\tilde{\eta}$).

Persistence of RER does not imply equal persistence in its components. Moreover, small RER deviations may be mean reverting because they originate from larger deviations for individual goods of opposite magnitude - an effect which has been empirically documented

[18]RER persistence is high when $\tilde{\eta}$ is close to -1 because $\{\text{sign}(LOPD_1)=-\text{sign}(LOPD_2)\}$ *at all times*, thus preventing $|RER|$ to exceed $\gamma_2 t_2 - \gamma_1 t_1$.

[19]Equivalently, we can think of the effective degree of substitutability between goods depending on trade: when sectors receive endowment shocks of opposite signs, the ability to subsitute between the products is limited by the (induced) trade, and conversely with shocks of identical signs.

by Crucini, Telmer and Zachariadis (2005)[20].

4.4 Real exchange rate in a model with quadratic adjustment costs

Real exchange rate in the quadratic adjustment cost model is *more volatile and more persistent* than in a linear shipping cost model. The additional friction of quadratic adjustment costs in changes in trade volume reduces profitability of arbitrage, allowing price differences in excess of the shipping cost in equilibrium (see section 3.1.1 above) and increasing volatility. Contemporaneous arbitrage profits may be negative because firms balance costs of trade deceleration with a potentially negative marginal revenue of arbitrage. Persistence increases because firms adjust to profitable arbitrage opportunity over a longer period of time.

Let the autoregressive endowment process follow $Y_{i,t} = \alpha Y_{i,t-1} + (1-\alpha)\bar{Y} + u_t \;\; i = 1, 2$ where $u_t \sim \mathrm{N}(0, \sigma^2)$, assuming that $t \in [1, T]$ and $u_t = 0$ for $t > 1$. This T-period simulation is repeated M times[21]. Table 3 reports the means of RER half-life and volatility estimates. As in the linear cost model, half life of convergence decreases in σ and increases in α. The convergence speed declines in σ at a much slower rate than in the linear shipping cost model. Adjustment costs increase the half life estimates for any α and σ. Volatility of RER is higher for any α and σ than in the linear shipping cost model (Table 4). Compared to the linear shipping cost model, RER volatility is less sensitive to σ. Higher σ leads to higher LOPD volatility, keeping their ratio unchanged. The standard deviation estimates decline in α because of a smoother adjustment in prices imposed by the quadratic adjustment costs.

5 Calibration

Preference parameters are calibrated to the usual values in the literature: weights of the utility function are symmetric ($\gamma_i = \frac{1}{3} \; \forall i$) and the inverse of the elasticity of substitution θ assumes the standard value 1.5 (see Chari, Christiano and Kehoe 1994, and McGrattan 1994). Shipping costs are calibrated directly as a tax (a heterogeneous iceberg cost) that disappears in the course of shipment, assuming they exhibit constant returns to scale. In

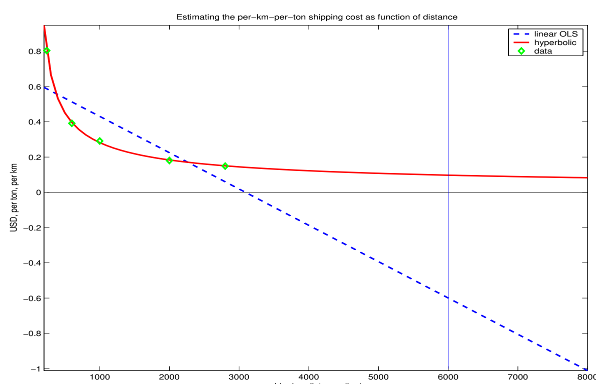
[20] This effect works in the opposite direction to the "aggregation bias" effect introduced by Imbs et al. (2003).

[21] Because of the limit on the number of periods needed for adjustment, results are not perfectly comparable between the two models. They are less precise in the QAC model.

particular, they depend multiplicatively on the distance and the weight of a good[22]. The US and EU are chosen as locations because of their similar size. Distance between their two major ports New York and Hamburg (6000km) is used as the shipping distance (most goods are shipped by sea between Europe and the US).

Marginal shipping costs are calibrated from two sources. In a survey of transportation modes, Runhaar et. al (2001) quote an average price in 2001 for a standard 40' container on a route Rotterdam – Singapore of NLG 3060 (USD 1220), including a fuel surcharge. They estimate the average load of a 40' container is 16.25 ton, yielding an average rate of USD 0.0077 per ton per km. Perishable goods such as most of foodstuffs are shipped in chilled containers. In a survey of shipping costs for fish (chilled) containers Brox et. al. (1984) survey costs across a range of distances. The implied per ton per km shipping costs is well approximated by a hyperbolic function (Figure 5). At the 6000km, it implies a unit cost of USD 0.11 per ton per km between US and Europe. A dataset of physical weights and average

Figure 5: Calibration of per-kg-per-km shipping cost for cooled sea transport



prices in Berka (2009) implies that 24% of the goods require refrigerating for transport. This yields an average shipping cost per ton per km of USD 0.033. An average weight of a good in the dataset is 43kg, and the average price USD 745 (2001 prices). When two weights (20kg and 66kg) are picked to match the average weight of a good in that dataset, per-kg-per-km shipping frictions are $t_1 = 0.0054$ and $t_2 = 0.0174$, respectively. That is, about 0.54% of good 1 and 1.74% of good 2 get used in transportation. These cost estimates are conservative

[22] The CRS assumption is inconsequential, as there is only 1 distance (2 countries). Many authors calibrate the transportation costs using indirect estimates. For an example, see Ravn & Mazzegna (2004). The dependance of shipping costs on weight has been established by many (e.g., see Table 7 in Hummels 1999).

compared to the literature[23]. Calibration of the quadratic adjustment cost parameter c does not appear in the literature. Therefore, c is calibrated indirectly by matching the co-movement of consumption vectors between the two countries, implying $c = 0.2$.

The stochastic endowment process at Home is calibrated to match the logged and H-P-filtered quarterly U.S. GDP series from 1973:1 to 1994:4, implying AR(1) coefficient $\alpha = 0.88$ and the standard deviation of the residuals equal to 0.8% of GDP. Correlation between output processes is equal to US-EU output correlation: $corr(Y, Y^*) \equiv \eta = \frac{\gamma}{\sigma^2} = 0.6$ (see Chari, Kehoe and McGrattan 2002).

6 Simulation results

6.1 Simulation results in a linear shipping cost model

A bivariate vector of 10,000 normally distributed shocks is used to generate the stochastic endowment vectors at Home and Abroad: $\hat{Y}_{i,t} = \alpha \hat{Y}_{i,t-1} + (1-\alpha)\bar{Y} + \hat{u}_t$ for $i = 1, 2$ where $\hat{Y}_{i,t} = [Y_{i,t}\ Y^*_{i,t}]'$, $\hat{u}_t = [u_t\ u^*_t]'$ and $\hat{u}_t \sim \mathrm{N}(0, \hat{\Omega})$ where $\hat{\Omega} = \begin{pmatrix} \sigma^2 & \gamma \\ \gamma & \sigma^2 \end{pmatrix}$. Qualitative properties of the solution are described in section 2.4[24]. Table 5 summarizes statistics of interest.

6.1.1 Persistence

Persistence of logarithm of the real exchange rate in linear model matches the persistence in the data, as in Chari, Kehoe and McGrattan (2002) ("CKM" hereafter). Model's AR(1) estimate $\hat{\alpha} = 0.8286$ with a standard error 0.0056 implies a half-life of convergence of about 3.7 quarters. Deviations from parity for good 1 are more persistent than for good 2 (AR(1) slope estimates of 0.7379 vs. 0.847, respectively). This is the core result of the linear model: heterogeneity in marginal shipping costs leads to persistent RER deviations. The model also generates consumption and net export correlations that are very close to the data.

[23] Harrigan (1993) finds transportation barriers of 20%. Hummels (1999) uses 2-digit SITC data to estimate a transportation costs of 9%. Using 4-digit SITC data, Ravn & Mazzegna (2004) find that the weighted average of transportation costs declined from 6.31% in 1974 to 3.49% in 1994. IMF frequently uses 11% as a rule of thumb for transportation costs. All these are greater than the 1.14% average in my calibration.

[24] Good 1 is traded more frequently (86% of the time periods) than good 2 (32%) (see Figure 7). Distribution of law of one price deviations in Figure 7 is clearly bimodal, with peaks corresponding to thresholds.

6.1.2 Comovements

The correlation between NX and GDP is negative, as in the data, but close to zero. This is caused by the aggregate constraint which requires that the good with the smallest friction flows in the opposite direction to the flow of the other two goods[25]. Real exchange rate is partially disconnected from the real economy. A sufficiently large endowment difference lowers prices at Home relative to Abroad – a depreciation (increase) in the real exchange rate that leads to a positive $corr(RER, Y)$. Real exchange rate is positively correlated with relative consumption vectors (0.96 compared to -0.35 in data and 1 in CKM) as the expenditure-switching motive is not sufficiently strong to decouple the two. Correlation of consumptions between countries is strong (0.62, vs. 0.38 in the data and 0.49 in CKM) because trade instantaneously eliminates endowments differences that lead to arbitrage opportunities.

6.1.3 Volatility

The risk-sharing role of international trade lowers consumption volatility relative to the endowments to 0.75, bringing it very close to 0.83 in the data. Volatility of trade is then necessarily slightly higher than in the data (0.19 vs. 0.11). This is in part the result of modeling only the arbitrage motive to trade which can lead to frequent changes and an on/off trade pattern. Linear shipping cost model does not generate sufficient RER volatility because it assumes instantaneous adjustment to an endowment shock. Consequently, $|RER|$ deviation has a well-defined maximum, equal to a weighted average of the trade frictions $(= (t_1 + t_2)/3)$[26].

6.2 Simulation results in a quadratic adjustment cost model

The additional friction brings countries' consumption sets closer to their endowments (see Section 3.1.1) and therefore leaves equilibrium prices longer and further away from the parity. Hence, persistence and volatility of RER increase while trade volume declines and becomes

[25] When goods 1 and 2 do not flow in the same directions, net flow of good 3 depends on the prices of goods 1 and 2 and the volume of trade.

[26] Nominal rigidities could increase volatility of the price aggregates over that of the endowment shocks. However, unlike quadratic adjustment costs, nominal rigidities are orthogonal to the mechanism present in the model.

smoother.

6.2.1 Persistence and comovements

On average, real exchange rate is marginally more persistent in the quadratic adjustment cost model than in the data, with an AR(1) coefficient estimate of 0.87. Persistence of consumptions and net exports is very close to the data, as the consumption risk-sharing role of trade is reduced. Also the comovement of variables in QAC model is very close to the data. Correlation of consumptions at Home and Abroad matches the data (this is used to calibrate c), and $corr(RER, Y)$ and $corr(RER, NX)$ are both significantly closer to the data than in the linear model or in CKM.

6.2.2 Volatility

Quadratic adjustment cost model is successful in creating volatility of prices relative to GDP. The average standard deviation estimate of 7.2 is higher than the 4.4 in the data. Histogram of all standard deviation estimates (Figure 10) shows that $LOPD_2$ is more volatile than $LOPD_1$, and that the distribution of LOPDs is bimodal with a larger mass near the thresholds. The bimodality is not as pronounced as in the linear model because thresholds increase in endowment differences (Figure 9). Aggregate consumption is more volatile because countries are more disconnected. On the other hand, small trade volume leads to an insufficient volatility of net exports. Co-movement of relative consumptions is at 0.28 closer to the data than it was in the linear model.

7 Conclusions and extensions for future research

This paper studies two general equilibrium models in which persistence and volatility of real exchange rate in equilibrium is a result of heterogeneity in shipping costs due to importance of goods' physical characteristics in shipment. In both models, tradability of a good is endogenously determined by the endowment differences and trade frictions of all goods. Goods with larger trade frictions need a larger deviation from parity to become traded and are therefore traded relatively less frequently. Calibration exercise shows that half life of real exchange rate deviation can match the estimates observed in the data.

Firms in the second model also pay a quadratic adjustment cost if they change their volume of trade from one period to the next. Adjustment costs arise from additional legal and infrastructure expenses or search costs, and are aimed to capture the time dimension of shipping to eliminate the unrealistic assumption of instantaneous adjustment. In the dynamic non-linear environment, firms' aversion to react to endowment shocks by large adjustments in trade volume creates larger and longer-lasting real exchange rate deviations. Adjustment costs limit trade between countries and the co-movement of their consumption levels. Simulation results of the second model generates great RER volatility, nearly matches the RER persistence, and performs very well in bringing comovements of other relevant variables close to the data. In this sense, heterogeneity in shipping costs is a plausible and an empirically relevant candidate explanation for the observed persistence and volatility in real exchange rates.

The importance of modeling heterogeneity of shipping costs is highlighted by the effects of substitution between traded and non-traded goods in each country on trade volume and price differences. Consumers substituting away from rising price of the export good increase the price of a non-traded product. This brings the price difference of the non-traded good further from the marginal shipping cost and lowers the probability that the good becomes traded. To the extent that trade adjusts countries economies to shocks, this effect causes a larger degree of insulation of the economy (a converse result is possible depending on the exact type of the endowment shock). Equivalently, we can think of the effective degree of substitution between products as being endogenous to the size of heterogeneous trade frictions.

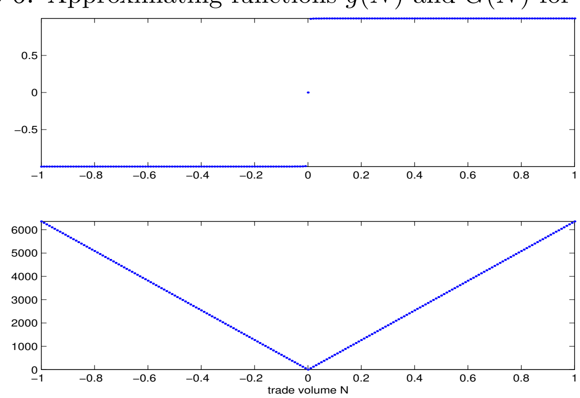
A Appendix: Approximating the absolute value function

A suitable choice is $I(N_{i,t}) \equiv \mathrm{d}G = \frac{2}{\pi}\arctan(\lambda N_{i,t})$ where λ is a choice parameter which governs the approximation error. An inverse of a trigonometric function $\tan(x)$, $\arctan(x)$ has a range of $[-\pi/2, \pi/2]$ for $x \in \mathbf{R}$ and is monotonically increasing, continuously differentiable, and has a convenient property that $\arctan(x) < 0$ when $x < 0$, $\arctan(x) > 0$ when $x > 0$ and $\arctan(0) = 0$. Further, $\arctan(\lambda x)$ can reach the bounds arbitrarily fast. Premultiplying it by $2/\pi$ changes its range to [-1,1], creating a "continuous step function". High λ lowers the approximation error, as can be seen in Figure 8. A choice of $\lambda = 10^{40}$ makes the approximation error indistinguishable from zero for any feasible stopping criterion of the numerical solver. However, it is misleading to use this approximation to describe the first order conditions of a system with $|N_{i,t}|$ because the absolute value function is not differentiable at 0. Therefore, a smooth approximation $G(N_{i,t})$ to $|N_{i,t}|$ needs to be constructed first, and then differentiated. Conveniently, function

$$G(N_{i,t}) \equiv \int g(N_{it}) = \frac{2}{\pi}\left[\lambda N_{i,t}\left(\frac{2}{\pi}\arctan(\lambda N_{i,t}) - 0.5\log(1+(\lambda N_{i,t})^2)\right)\right]$$

can be used to arbitrarily closely approximate $|N_{i,t}|$ by a choice of λ (see figure (6)).

Figure 6: Approximating functions $g(N)$ and $G(N)$ for $\lambda = 10^5$.



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Figures and Tables

Figure 7: Distribution of trade and price differences. US-EU simulation of the linear model

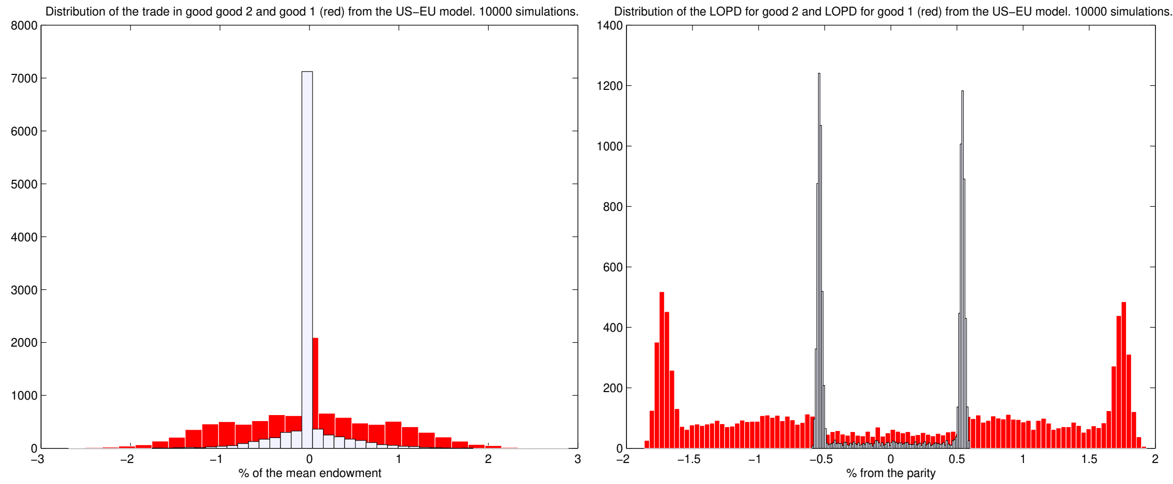

Figure 8: Approximating the indicator function in QAC model

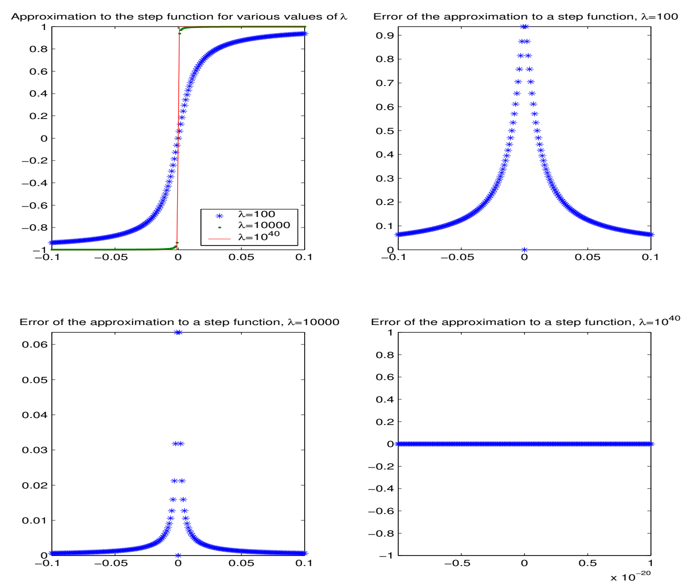

Figure 9: Thresholds in QAC model when c=0.001 and c=0.1

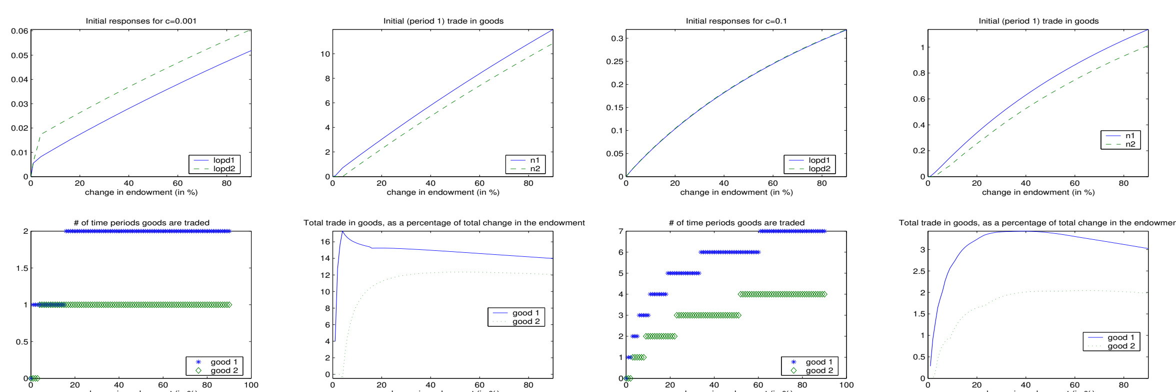

Figure 10: Distribution of standard deviation and price estimates in a QAC model, c=0.1

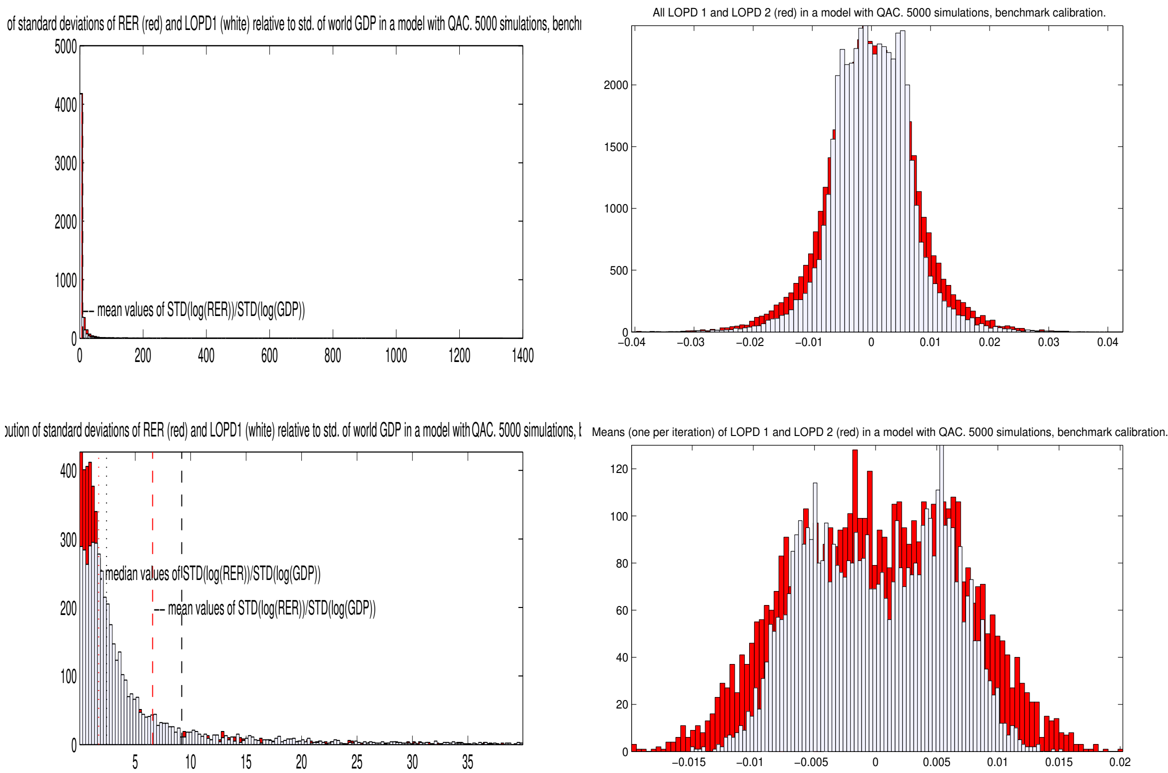

Table 1: log(RER) Half-lives, standard deviation, and the shock process in a linear model

	$\alpha = 0.65$	$\alpha = 0.7$	$\alpha = 0.75$	$\alpha = 0.8$	$\alpha = 0.85$	$\alpha = 0.9$	$\alpha = 0.95$	$\alpha = 0.99$
				log(RER) half-life				
$\sigma = 0.008$	1.65	1.99	2.44	3.12	4.20	6.16	11.14	933.5
$\sigma = 0.019$	1.45	1.72	2.09	2.61	3.36	4.64	7.5	576.9
$\sigma = 0.034$	1.26	1.48	1.77	2.15	2.71	3.58	5.53	172.2
$\sigma = 0.068$	1.06	1.23	1.46	1.75	2.17	2.8	4.35	60.8
				log(RER) standard deviation				
$\sigma = 0.008$	0.008	0.009	0.009	0.010	0.011	0.012	0.014	0.015
$\sigma = 0.019$	0.013	0.013	0.014	0.014	0.015	0.016	0.017	0.017
$\sigma = 0.034$	0.015	0.015	0.016	0.016	0.017	0.017	0.018	0.020
$\sigma = 0.068$	0.016	0.017	0.017	0.018	0.018	0.019	0.020	0.022

Each result is based on 10000 simulations of the linear shipping cost model when t_1=0.02 and t_2=0.04. α is the AR(1) coefficient of the shock process, σ is the standard deviation as a proportion of mean GDP.

Table 2: Half-lives of log (RER) and the relative trade friction in a linear model

	$\alpha = 0.6$	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 0.99$
$t_2/t_1 = 2$	1.4	1.9	2.9	5.0	263.5
$t_2/t_1 = 4$	1.4	2.0	3.1	6.1	653.4
$t_2/t_1 = 6$	1.4	2.0	3.1	6.2	1060.2
$t_2/t_1 = 8$	1.4	2.0	3.1	6.2	1641.0

Each result is based on 2000 simulations of the linear shipping cost model starting from t_1=0.02 and t_2=0.04. α is the AR(1) coefficient of the shock process, σ is the standard deviation as a proportion of the mean GDP.

Table 3: Mean half-lives of log(RER) in a quadratic model

	c=0.01				**c=0.1**			
	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 0.99$	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 0.99$
$\sigma = 0.8\%$	1.9444	3.12	7	–	1.943	3.105	6.567	68
$\sigma = 1.9\%$	1.9438	3.11	6.8	–	1.9429	3.1048	6.567	66
$\sigma = 3.4\%$	1.9436	3.11	6.7	–	1.9428	3.1047	6.515	65
$\sigma = 6.8\%$	1.9433	3.1	6.6	–	1.9428	3.09	6.522	63

Each result is based on 1000 simulations of the model when $T = 20$, $t_1 = 0.0054$ and $t_2 = 0.0174$ (see section 5). α is the AR(1) coefficient of the shock process.

Table 4: Volatility of log(RER) in a quadratic model

	[Mean std(lRER)]/[Mean std(lGDP)]				[Median std(lRER)]/[Med. std(lGDP)]			
	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 0.99$	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 0.99$
$\sigma = 0.8\%$	27.62	14.39	12.59	8.49	1.951	1.952	1.953	2.03
$\sigma = 1.9\%$	17.83	14.88	12.35	8.42	1.950	1.951	1.952	2.08
$\sigma = 3.4\%$	17.68	14.52	12.24	8.54	1.950	1.950	1.951	2.36
$\sigma = 6.8\%$	17.07	14.25	12.14	8.75	1.949	1.949	1.950	2.71

Each result is based on 1000 simulations of the model when $T = 20$, $t_1 = 0.0054$ and $t_2 = 0.0174$ (see section 5). α is the AR(1) shock coefficient.

Table 5: Properties of the US-EU model simulation

	data	linear model[1]	QAC $c = 0.05$	model[2] $c = 0.2$	CKMcG[3]
Autocorrelations					
Ex. rates & prices					
RER	0.83	0.8286	0.868	0.87	0.62
Business cycle stat					
GDP	0.88	0.88*	0.88*	0.88*	0.62
Consumption	0.89	0.88	0.854	0.877	0.61
Net Exports	0.82	0.87	0.700	0.78	0.72
STD rel. to GDP					
Ex. rates & prices					
RER	4.36	0.002	6.41 (1.65)	7.2 (1.82)	4.27
Business cycle stat					
Consumption	0.83	0.75	1	1	0.83
Net Exports	0.11	0.19	0.001	0.0004	0.09
Cross-Correlat.					
GDPs	0.6	0.6*	0.6*	0.6*	0.49
Consumptions	0.38	0.62	0.28	0.38	0.49
NX & GDP	-0.41	-0.03	0.05	0.05	0.04
RER & GDP	0.08	0.69	-0.02	-0.002	0.51
RER & NX	0.14	0.88 (-0.02)	0.027	0.032	-0.04
RER & Relat. C	-0.35	0.96	0.956	0.97	1.00

[1] Based on 10,000 simulations of the linear shipping model with parameter calibration described in section 5.
[2] Based on 5,000 simulations ($T = 30$) of the quadratic adjustment cost model.
[3] Results of the model simulation in Chari, Kehoe and McGrattan (2002).
* Denotes a calibrated value.