An axiomatic approach to sustainable development

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Abstract. The paper proposes two axioms that capture the idea of sustainable development and derives the welfare criterion that they imply. The axioms require that neither the present nor the future should play a dictatorial role.

Theorem 1 shows there exist *sustainable preferences*, which satisfy these axioms. They exhibit sensitivity to the present and to the long-run future, and specify trade-offs between them. It examines other welfare criteria which are generally utilized: discounted utility, lim inf. long run averages, overtaking and catching-up criteria, Ramsey's criterion, Rawlsian rules, and the criterion of satisfaction of basic needs, and finds that none satisfies the axioms for sustainability.

Theorem 2 gives a characterization of all continuous independent *sustainable preferences*. Theorem 3 shows that in general sustainable growth paths cannot be approximated by paths which approximate discounted optima. Proposition 1 shows that paths which maximize the present value under a standard price system may fail to reach optimal sustainable welfare levels, and Example 4 that the two criteria can give rise to different value systems.

1. Introduction

Economics is at a crossroads created by two major trends. One is advances in information technology, which are changing the way we work, live and think. The other is the environmental agenda, which leads us to reconsider the foundations of economics, the central question of resource allocation. Economics traditionally

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considers resources as passive, so to say "there for the taking". But many resources are alive, like forests or biodiversity, and even those which are not alive have their own rational and dynamics, such as the planet's atmosphere or its water bodies. The interaction of resources with human organizations such as markets requires attention now because for the first time in history, human activity has reached levels at which it can alter the global environment, such as the planet's climate and its biological mix.

Understanding the interaction between resources and human organizations is not easy. While economic motives are the driving force behind the exhaustion of resources, the destruction of biodiversity and the changes to the atmosphere, economics does not measure and therefore cannot perceive properly the consequences, which are biological or geophysical cause and effect are separated. Each science sees only one side of the equation.

The need for reformulating the economics of resource allocation is plain to outsiders who demand it, but takes an abstract and rarefied form for the economist. It appears for example as a call for "sustainable development" strategies within international organizations[1], a term which has not been well defined until now and which many still consider undefinable. Nonetheless, the useful survival of economics as a discipline lies in its ability to adjust its principles and its analysis and to offer needed insights and policies. We must ask what is sustainable development, and we must find an answer on which economics can be built reliably and usefully.

In the broadest sense, sustainable development means development which gives an equal opportunity to future generations. Current patterns of development conjure up images of "slash and burn" industrialization. The industrial countries with the highest levels of income consume most of the earth's resources, and generate most of the carbon emissions from their burning of fossil fuels[2]. Global carbon emissions generated since the second world war are generally believed to have the ability to alter seriously the earth's climate, although there is as yet no scientific agreement on the precise magnitude of the effects[5]. Biologists see the loss of biodiversity during the last fifty years as one of the four or five largest incidents of destruction of life in the planet[5].

The evidence of the last fifty years shows that carbon emissions and biodiversity loss are inversely related to population growth [43]. Carbon emissions are positively related to a country's income, because they are closely connected with energy use. Industrial countries which have $\frac{1}{5}$ of the world's population, the highest income levels and the lowest population growth, use most of the world's produced energy and correspondingly emit about 70% of the world's carbon, while developing countries which contain about $\frac{4}{5}$ of the world's population and the highest rates of population growth, account only for about 30% of the world's emissions[3]. These observations indicate that current patterns of growth in the industrial countries may not be sustainable and should not be imitated by developing countries. They could be incompatible with the sustainable use of one of the most valuable resources: the planet's atmosphere. Several years ago in the *Bariloche Model*[4] we

[1] E.g. the World Bank, the United Nations and the OECD.
[2] Chichilnisky [18, 19, 43].
[3] Coppel [22], Chichilnisky [19], WRI-UNEP-UNDP [43].
[4] [6, 10].
[5] [43].

pointed out that the patterns of development followed by industrial societies were probably not sustainable, and should not be imitated by developing countries. We introduced and formalized the concept of development based on the satisfaction of basic needs [10, 11], and carried out an empirical analysis of world development based on this criterion, which we showed to be more consistent with the planet's resources and its ecological dynamics [6], [10].

Further impetus to the same theme was given in 1987 by the Brundtland Commission which updated the term "sustainability" and anchored it also to the concept of needs: development should "satisfy the needs of the present without compromising the needs of the future[6]". The 1992 UN Earth Summit in Rio de Janeiro saw sustainable development as one of the most urgent issues of international policy: sustainable development is the official charge of a UN Commission which was made responsible for advancing environmental policy in the international arena as embodied in UN Agenda 21. Other UN conventions cover Climate Change and Biodiversity, but it has been said that sustainable development comprises them all. The topic is of widespread interest, and is compelling to many. But *what is sustainable development*?

Solow [39] pointed out that recent discussion of sustainability has been mainly an occasion for the expression of emotions and attitudes, with very little analysis of sustainable paths for a modern industrial economy. He suggests that sustainable development means that we are not to consume humanity's capital, in a general sense which Hicks anticipated [30]. But the replacement of one form of capital with another is, in his view, acceptable. As a thought experiment, consider the replacement of all trees on the planet by equally valuable capital stock. Are we to deem this as sustainable development? A large number of people would disagree strongly with such a view, although many economists would not. There is an ambiguity here in how value is defined, and in this lies the crux of the matter. How do we define economic value? What shall we measure, and how?

Value is usually defined in terms of utility. This is almost a tautology in the case of an individual, but leads to notorious difficulties when attempting to compare the utilities of different people and of different generations. The definitions given above suggest that the issue is how to describe value so that is does not underestimate the futures' interests, so that the future is given an equal treatment. Implicit in this statement is the idea of fair treatment for both the present and the long-run future. The challenge is to develop economic theory which formalizes this aim, the equal treatment for the present and for the future, with the level of clarity and substance achieved by neoclassical growth theory. The formalization should suggest a program of empirical research with clear implications for public policy.

To follow this plan of action, we need to start from a clear notion of how to rank alternatives and how to evaluate trade-offs. From such rankings practical criteria for optimality are derived, which allow us to evaluate alternatives and trade-offs, the essential elements of an economist's task.

This paper proposes simple axioms which capture the concept of sustainability. They require intergenerational equity in the sense that neither the "present" nor the "future" are favored over the other. I neither accept the romantic view which relishes the future without regards for the present, nor the consumerist view which ranks the present above all. But is there anything left?

[6] [9, Ch. 2, para 1].

The answer is, perhaps surprisingly, yes. What is left is a characterization of a family of *sustainable preferences*[7] which has points in common with Hick's notion of income as a "sustainable" level of consumption [30]. These preferences are sensitive to the welfare of all generations, and offer an equal opportunity to the present and to the future. Trade-offs between present and future consumption are allowed. The examples and a characterization of the family of sustainable preferences appears in Theorems 1 and 2. These show that sustainable preferences are different from all other criteria used so far in the analysis of optimal growth and of markets.

But how different is this criterion in practice from other standard measures? Does it lead to different optimal paths from those which emerge from using discounted utility? Can we always approach sustainable optima by paths which in the foreseeable future resemble those of discounted optima, albeit with small discount rates?

An answer to these questions is in Theorem 3 and Example 4. These show that sustainable optima can be quite different from discounted optima, no matter how small is the discount factor. The difference is most striking in decision problems with irreversibilities, which impose constraints today leading to substantial differences in the long run, as in Example 1. But the general issue is captured by Hicks' idea of income, and Solow's concern for not consuming humanity's capital: the long run has a substantial weight.

It remains to show how practical is my criterion for ranking development paths and for evaluating projects. This is the subject of companion papers, Beltratti et al. [7] and Heal [29], which study "green golden rules" and sustainable optimal growth paths. They compute optimal solutions and derive shadow prices which can be used to decentralize optimal solutions, although possibly not by the maximization of the present value of profits under standard price systems.[8]

The axioms and the characterizations I propose have a compelling minimal logic, but they are by no means unique. They are meant as an aid to formalizing our thinking on sustainability and as a way of providing a level of analytical clarity which could improve our understanding. Analytical underpinnings are as necessary for the practical evaluation of sustainable policies as they are for our thinking of the available options.

2. The state of the art

The theory of economic growth developed about 50 years ago, and anticipated the patterns of development which were followed throughout this period. Exponential growth of population and resource use are viewed as steady states. Most studies of

[7] An alternative name, suggested by Robert Solow is "intertemporally equitable preferences".

[8] The "present value under a standard price system" is a value which is defined by a sequence of dated prices, which defines a (finite) value for all bounded sequences of commodities, see definition in Sect. 9. Heal [29] proves that in the standard "cake eating" or exhaustible resource problem, the solution according to the sustainability criterion proposed in this paper is the same as that involving a discounting criterion, because the maximum sustainable optimal level of consumption in such a problem is always zero. A different solution is obtained when the problem is modified so that utility is derived not only from the flow but also from the remaining stock of the resource; in this case an optimal path uses less than all the stock, and the exact amount remaining depends on the actual weight that the sustainable preference gives to the long run.

optimal growth and most project evaluations rank alternatives using a discounted utility criterion. However, discounting future utility is generally inconsistent with sustainable development. It can produce outcomes which seem patently unjust to later generations. Indeed, under any positive discount rate, the long-run future is deemed irrelevant.

For example, at a standard 5% discount rate, the present value of the earth's aggregate output discounted 200 hundred years from now, is a few hundred thousand dollars. A simple computation shows that if one tried to decide how much it is worth investing in preventing the destruction of the earth 200 years from now on the basis of measuring the value of foregone output, the answer would be no more than one is willing to invest in an apartment.

This appears to contradict the observation that people and their governments seem seriously concerned with the long-run future and willing to invest substantial sums of money to control the disposal of nuclear waste or to prevent global climate change. Both involve potentially disastrous risks and very long time horizons. For example, a recent OECD study considers a global carbon tax yielding a revenue of US$150 billion annually[9]. Practitioners have argued that discounted utility is inappropriate for evaluating the costs of global warming, given the very long time horizons involved, and have suggested that the use of a zero discount rate should be considered[10].

Other criteria for project evaluation and optimal growth take into consideration the long-run future, but share a drawback: their insensitivity to present generations[11]. They include undiscounted methods such as long-run averages and lim-infimum rules. In fact, these rules ignore the welfare of any finite number of generations, no matter how large this number may be.

It is also possible to select development paths using a Rawlsian criterion, Rawls [37], or alternatively the criterion of satisfaction of basic needs, introduced in Chichilnisky [10, 11]. In addition, welfare criteria other than discounted utility have been proposed for evaluating optimal growth paths by Ramsey [36], von Weizacker [42], Gale [26], and Koppmans [31], Hammond [27], Heal [28] and McFadden [34].

It seems fair to say that no criterion has achieved the analytical clarity of the discounted sum of utilities. This criterion has led to simple conditions for the existence of optimal paths; to a characterization of optimal paths, and to prices which can decentralize a social optimum[12]. In an analytical sense, discounted utility compares favorably with Rawlsian rules and basic needs, which take into consideration solely the welfare of the neediest. The latter are rather insensitive, giving the same welfare weight to any two utility streams which coincide in the utility of the neediest, even though in one of them infinitely many generations achieve strictly higher welfare. The discounted utility criterion is, by contrast, sensitive to increases in utility by any one generation. In fact, the sustainable welfare criteria introduced here also satisfies such sensitivity: if two utility streams give identical welfare to all generations but one, and one stream gives strictly larger welfare to the remaining generation, then the latter ranks higher.

The discounted utility criterion shows its analytical strength when compared with Ramsey's criterion and with von-Weizacker's overtaking criterion. The latter

[9] Coppel [22] and Chichilnisky [19].
[10] Broome [8] and Cline [21].
[11] Beltratti et al. [7].
[12] See Arrow and Kurz [2], Dasgupta and Heal [24], and Chichilnisky and Kalman [12].

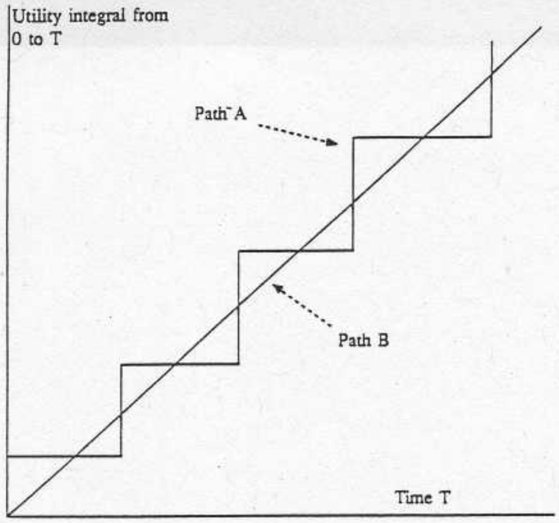

Fig. 1. The overtaking criterion cannot rank paths A and B

criteria are incomplete orders which fail to rank many reasonable alternatives. The discounted utility criterion is, instead, a complete order. The problem is that the overtaking criterion fails to rank any two paths which switch between overtaking and being overtaken by the other, as shown in Fig. 1 above. It leads to a seriously incomplete or indecisive ranking of alternative paths. A similar difficulty emerges with Ramsey's criterion, which ranks paths according to their distance from a blissful level of consumption, a distance which is measured according to the area between the path to be evaluated and the bliss path. Such distances are often ill-defined, because in an infinite world the measure of the area between two bounded paths can be unboundedly large. Therefore the ranking is indecisive. Instead, discounted utility provides a complete ranking of all feasible paths. My sustainable welfare criterion is also a complete ranking. The usefulness of an indecisive criterion is limited: In many cases it leads to no choice. As has already been argued, the task for economists is to develop a criterion for sustainable development with a level of analytical clarity and completeness similar to that which has been achieved by the parts of neoclassical growth theory which rest on discounted utility. Being decisive is an important part of this task.

These concerns motivate the axioms for sustainability, the results on existence and characterization of the welfare criteria which satisfy these axioms, which I call *sustainable preferences*, and the study of the connection between sustainable preferences and other welfare criteria presented below.

3. Social choice and sustainable development

I propose two axioms which reflect a concern for sustainable development. These axioms are non-dictatorship properties, and as such have a connection with social choice theory[13].

[13] See Arrow [1], Chichilnisky [20]. In this case we are concerned with fairness across generations, see also Solow [38], Lauwers [32], Lauwers and van Liederkierke [33].

A welfare criterion used to evaluate sustainable development must be a complete order over utility paths which does not assign a dictatorial role either to the present or to the future, and which increases with increases in the welfare of any generation. Axiom I requires that the present should not dictate the outcome in disregard for the future: it requires sensitivity to the welfare of generations in the distant future. This, it turns out, eliminates the discounted utilitarian approach. Axiom 2 requires that the welfare criterion should not be dictated by the long-run future, and thus requires sensitivity to the present. This latter axiom eliminates lim inf and long run averages which only consider the very long run. In this sense, the axioms require a form of symmetry or "equal treatment" of the present and of the future, one which is reminiscent of the anonymity condition in social choice[14].

Theorem 1 shows that Axioms 1 and 2 eliminate all known criteria for evaluating infinite utility streams used so far. It also establishes that there exists a well-defined class of welfare criteria which do satisfy the two axioms: I call them *sustainable preferences.*

Theorem 2 identifies completely all the *continuous* independent welfare criteria which satisfy the axioms[15]. Continuity has the role of ensuring the existence of sufficient statistics. When the welfare criterion has the same rate of substitution between the utilities of any two generations whatever their levels of utility, sustainable preferences are shown to have a simple and general characterization.

Because it is important that the axioms should be practical in nature, and usable for solving sustainable optimal growth problems and evaluating sustainable projects, companion papers[16] study optimal growth models with renewable resources[17], in which the welfare criterion are sustainable preferences. These I call *sustainable growth models.* The results in [7] and [29] exhibit the extent to which these axioms lead to a well defined optimal growth theory with the analytical clarity and substance of neoclassical growth theory. In [7] we characterize a "green golden rule" which is the natural extension of the "golden rule" of neoclassical growth theory, and analyze sustainable optimal growth paths. The aim is to offer a *sustainable optimal growth* analysis at the level of applicability of neoclassical growth theory.

Sustainable preferences can of course be used in a general equilibrium context. When the market economy is infinitely lived, an hypothesis which is often made in order to eliminate the potential inconsistencies arising from unknown terminal dates, it may be appropriate and indeed desirable that the traders and the producers should have sustainable preferences. Indeed, current German national accounting practices include a concept of sustainability in the determination of a firm's value. It has been suggested that Universities' budgetary decisions, which typically involve a requirement of keeping the long run value of the institution intact, have sustainable characteristics as well[18].

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[14] Chichilnisky [13, 20].

[15] Continuity has been used profitably in social choice theory for the last ten years, see e.g. Chichilnisky [13, 14].

[16] Beltratti et al. [7] and Heal [29].

[17] Chichilnisky [14].

[18] I owe this comment to Donald Kennedy, Chairman of the Global Environmental Program at the Institute for International Studies of Stanford University, and ex-president of Stanford University.

4. Sustainability and value

In addition to introducing axioms for sustainability and characterizing the welfare functions which satisfy them, I show that paths that maximize the present value of profits may be quite different from a sustainable optimum. In particular, it may not be possible to decentralize a solution by means of "market prices" as in neoclassical growth theory[19]. Theorem 3 and Example 4 show that even if an optimal sustainable solution exists and is unique, in general it will not be approximated by paths which are optimal under discounted criteria, no matter how small the discount rate.

Environmental assets may therefore have a well defined and substantial value according to a sustainable preference, but an arbitrarily small market or profit maximizing value, see Example 4. The concept of present value of profits may differ substantially from the value implicit in our axioms for sustainability. In general, however, there will be some overlap between the two, and some trade-offs are possible.

5. Axioms for sustainability

Consider an infinitely lived world, an assumption that obviates the need to make decisions contingent on an unknown terminal date. Each generation is represented by an integer g, $g = 1, \ldots, \infty$. Generations could overlap or not; indeed one can consider a world in which some agents are infinitely long lived. In this latter case, one is concerned about the manner in which infinitely long lived agents evaluate development paths for their own futures.

In order to compare the axioms and results to those of growth theory I shall adopt a formulation which is as close as possible to that of the neoclassical model. Each generation g has a utility function u_g for consumption of n goods, some of which could be environmental goods such as water or soil, so that consumption vectors are in R^n, and $u_g: R^n \to R$. The availability of goods in the economy could be constrained in a number of ways, for example by a differential equation which represents the growth of the stock of a renewable resource[20], and/or the accumulation and depreciation of capital. Ignore for the moment population growth; this issue can be incorporated with little change in the results[21]. The space of all feasible consumption paths is indicated F.

$$F = \{x : x = \{x_g\}_{g=1,2,\ldots}, \; x_g \in R^n\}. \tag{1}$$

In common with the neoclassical growth literature, utility across generations is assumed to be comparable. Each generation's utility functions is bounded below and above and we assume $u_g: R^n \to R^+$, and $\sup_{x \in R^n}(u_g(x)) < \infty$. This is not

[19] This often occurs because the "cone condition" which is necessary and sufficient for supporting optimal growth programs with non-zero price sequences, defined Chichilnisky and Kalman [12] and Chichilnisky [15] is typically not satisfied in sustainable growth models.

[20] See [17, 7].

[21] Population growth and utilitarian analysis are known to make an explosive mix, which is however outside the scope of this paper.

a restrictive assumption: one cannot have utilities which grow indefinitely in either the positive or the negative direction when there are an infinite number of generations[22]. In order to eliminate some of the most obvious problems of comparability I normalize the utility functions u_g so that they all share a common bound, which I assume without loss of generality to be 1:

$$\sup_g (u_g(x_g))_{x_g \in R^n} \leq 1. \tag{2}$$

The space of feasible utility streams Ω is

$$\Omega = \{\alpha: \alpha = \{\alpha_g\}_{g=1,2,\dots}, \alpha_g = u_g(x_g)\}_{g=1,2,\dots} \text{ and } x = \{x_g\}_{g=1,2,\dots} \subset F\}. \tag{3}$$

Because I normalized utilities, each utility stream is a sequence of positive real numbers, all of which are bounded by the number 1. The space of all utility streams is therefore contained in the space of all infinite bounded sequences of real numbers, denoted ℓ_∞[23]. The welfare criterion W should rank elements of Ω, for all possible $\Omega \subset \ell_\infty$.

5.1. *Sensitivity and completeness*

The welfare criterion W must be represented by an increasing real valued function on the space of all bounded utility streams[24] $W: \ell_\infty \to R^+$. The word increasing means here that if a utility stream α is obtained from another β by increasing the welfare of some generation, then W must rank α strictly higher than β[25]. This eliminates the Rawlsian criterion and the basic needs criterion, both of which, are insensitive to the welfare of all generations but those with the lowest welfare. Completeness and sensitivity eliminate the Ramsey criterion as well as the overtaking criterion.

5.2. *The present*

How to represent the present? Intuitively, when regarding utility streams across generations, the present is the part of those streams that pertains to finitely many

[22] This would lead to paradoxical behavior. The argument parallels interestingly that given by Arrow [3] on the problem that originally gave rise to Daniel Bernouilli's famous paper on the "St. Petersburg paradox", see *Utility Boundedness Theorem*, page 27. If utilities are not bounded, one can find a utility stream for all generations with as large a welfare value as we wish, and this violates standard continuity axioms.

[23] Formally, $\Omega \subset \ell_\infty$, where $\ell_\infty = \{y: y = \{y_g\}_{g=1,\dots}: y_g \in R^+, \sup_g |y_g| < \infty\}$. Here $|\cdot|$ denotes the absolute value of $y \in R$, which is used to endow ℓ_∞ with a standard Banach space structure, defined by the norm $\|\cdot\|$ in ℓ_∞

$$\|y\| = \sup_{g=1,2\dots} |y_g|. \tag{4}$$

The space of sequences ℓ_∞ was first used in economics by Debreu [23].

[24] The representability of the order W by a real valued function can be obtained from more primitive assumptions, such as e.g. transitivity, completeness and continuity conditions on W.

[25] Formally, if $\alpha > \beta$ then $W(\alpha) > W(\beta)$.

generations. The present will therefore be represented by all the parts of feasible utility streams which have no future: for any given utility stream α, its "present" is represented by all finite utility streams which are obtained by cutting α off after any number of generations. Formally,

Definition 1. For any utility stream $\alpha \in \ell_\infty$, and any integer K, let α^K be the "K-cutoff" of the sequence α, the sequence whose coordinates up to and including the K-th are equal to those of α, and zero after the K-th[26].

Definition 2. The present consists of all feasible utility streams which have no future, i.e. it consists of the cutoffs of all utility streams.

5.3. No dictatorial role for the present

Definition 3. We shall say that a welfare function $W: \ell_\infty \to R$ gives a dictatorial role to the present, or that W is a dictatorship of the present, if W is insensitive to the utility levels of all but a finite number of generations, i.e. W is only sensitive to the "cutoffs" of utility streams, and it disregards the utility levels of all generations from some generation on.

Definition 4. The "K-th tail" of a stream $\alpha \in \ell_\infty$, denoted α_K, is the sequence with all coordinates equal to zero up to and including the K-th, and with coordinates equal to those of α after the K-th[29].

Formally, for any two $\alpha, \gamma \in \ell_\infty$, let (α^K, γ_K) be the sequence defined by summing up or "pasting together" the K-th cutoff of α with the K-th tail of γ. W is a dictatorship of the present if for any two utility streams α, β

$W(\alpha) > W(\beta) \Leftrightarrow$

$\exists N = N(\alpha, \beta)$ s.t. if $K > N$, $W(\alpha^K, \gamma_K) > W(\beta^K, \sigma_K)$ for any utility streams $\gamma, \sigma \in \ell_\infty$[27].

The following axiom eliminates dictatorships of the present:

Axiom 1. No dictatorship of the present.

This axiom can be seen to eliminate all forms of discounted sums of utilities, as shown in Theorem 1[28].

5.4. The Future

For any given utility stream α, its "future" is represented by all infinite utility streams which are obtained as the "tail" resulting from modifying α to assign zero utility to any initial finite number of generations.

[26] In symbols: $\alpha^K = \{\sigma_g\}_{g=1,2\ldots}$ such that $\sigma_g = \alpha_g$ if $g \le K$, and $\sigma_g = 0$ if $g > K$.
[27] Recall that all utility streams are in ℓ_∞ and they are normalized so that $\sup_{g=1,2\ldots} (\alpha(g)) = \|\alpha\| < 1$ and $\sup_{g=1,2\ldots} (\beta(g)) = \|\beta\| < 1$.
[28] Boundedness of the utilities is important here, although as shown above, it is not a strong assumption, see Arrow's [3] Utility Boundedness Theorem.
[29] In Symbols: $\sigma_K = \{\sigma_g\}_{g=1,2\ldots}$ such that $\sigma_g = 0$ if $g \le K$, and $\sigma_g = \alpha_g$ if $g > K$.

5.5. No dictatorial role for the future

Definition 5. Welfare function $W: \ell_\infty \to R$ gives a dictatorial role to the future, or equivalently W is a dictatorship of the future, if W is insensitive to the utility levels of any finite number of generations, or equivalently it is only sensitive to the utility levels of the "tails" of utility streams.

Formally, for every two utility streams α, β

$$W(\alpha) > W(\beta) \Leftrightarrow$$
$$\exists N = N(\alpha, \beta)\text{: if } K > N,\ W(\gamma^K, \alpha_K) > W(\sigma^K, \beta_K),\ \forall \gamma, \sigma \in \ell_\infty.$$

The welfare criterion W is therefore only sensitive to the utilities of "tails" of streams, and in this sense the future always dictates the outcome independently of the present. The following axiom eliminates dictatorships of the future:

Axiom 2. No dictatorship of the future.

This axiom excludes all welfare functions which are defined solely as a function of the limiting behavior of the utility streams. For example, it eliminates the lim-inf and the long run averages.

Definition 6. A sustainable preference is a complete sensitive preference satisfying Axioms 1 and 2.

6. Previous welfare criteria

While the axioms proposed in the previous section appear quite reasonable, and they could be said to have a compelling minimal logic, they suffice to exclude most welfare criteria used in the literature. In this section I define some of the more widely used welfare criteria, and also provide examples which will be used in Theorem 1 of the next section.

A function $W: \ell_\infty \to R$ is called a *discounted sum of utilities* if it is of the form

$$W(\alpha) = \sum_{g=1}^{\infty} \lambda_g \alpha_g, \quad \forall \alpha \in \ell_\infty, \tag{5}$$

where $\forall_g, \lambda_g \geq 0$ and $\sum_{g=1}^{\infty} \lambda_g < \infty$; λ_g is called a discount factor. *Ramsey's welfare criterion* [36] ranks a utility stream $\alpha = \{\alpha_g\}_{g=1,2,\ldots} \in \ell_\infty$ above another $\beta = \{\beta_g\}_{g=1,2,\ldots} \in \ell_\infty$ if the utility stream α is "closer" to the bliss path, namely to the sequence $\zeta = \{1, 1, \ldots, 1, \ldots\}$, than is the sequence β. Formally,

$$\sum_{g=1}^{\infty} (1 - \alpha_g) \leq \sum_{g=1}^{\infty} (1 - \beta_g). \tag{6}$$

A Rawlsian rule [37] ranks two utility streams according to which has a higher infimum value of utility for all generations. This is a natural extension of the criterion proposed initially by Rawls [37]. Formally, a utility stream α is preferred to another β if

$$\inf\{\alpha_g\}_{g=1,2\ldots} > \inf\{\beta_g\}_{g=1,2\ldots} \tag{7}$$

The criterion of *satisfaction of basic needs* [11] ranks a utility stream α over another β if the time required to meet basic needs is shorter in α than in β. Formally,

$$T(\alpha) \leq T(\beta), \tag{8}$$

where $T(\alpha) = \min\{t\colon \alpha_g \geq b \;\forall g \geq t\}$, for a given b which represents basic needs. The *overtaking criterion* [42] ranks a utility stream α over another β if α eventually leads to a permanently higher level of aggregate utility than does β. Formally, α is preferred to β if $\exists N$:

$$\forall M > N, \sum_{g=1}^{M} \alpha_g \geq \sum_{g=1}^{M} \beta_g. \tag{9}$$

The *long run average* criterion can be defined in our context as follows: a utility stream α is preferred to another β if in average terms, the long run aggregate utility[30] achieved by α is larger that achieved by β. Formally, $\exists N, K > 0$:

$$\frac{1}{T}\left(\sum_{g=M}^{T+M} \alpha_g\right) \geq \frac{1}{T}\left(\sum_{g=M}^{T+M} \beta_g\right), \quad \forall T > N \quad M > K. \tag{10}$$

7. Existence and characterization of sustainable preferences

Why is it difficult to rank infinite utility streams? Ideally one wishes to give equal weight to every generation. For example, with finitely many N generations, each generation can be assigned weight $1/N$. But when trying to extend this criterion to infinitely many generations one encounters a problem: one usual in the limit, every generation is given zero weight.

To solve this problem attaches more weight to the utility of near generations, and less weight to future ones[31]. An example is the sum of discounted utilities. Discounted utilities give a bounded welfare level to every utility stream which assigns each generation the same utility. Since two numbers can always be compared, the criterion so defined is complete. However, the sum of discounted utilities is not even-handed: it disregards the long run future. I show below that as soon as the total welfare assigned to finite utility streams is well defined, the welfare criterion thus obtained is a dictatorship of the present. Therefore the sum of discounted utilities is unacceptable under my axioms, for any discount factor no matter how small.

Another solution is offered in the theory of repeated games: here, instead, more weight is given to the future and less to the present. An example is the criterion defined by the long-run average of a utility stream, a criterion used frequently in

[30] This is only one of the possible definitions of long run averages. For other related definitions with similar properties see Dutta [25].

[31] With time Robert Solow has suggested making the discounting factor smaller than one and decreasing: it is possible to show that under certain conditions this is what must be done to ensure the existence of solutions.

repeated games. However, this criterion is not even-handed either, this time because it is biased in favour of the future and against the present. The decision of ranking one utility stream over another is made on the basis of the long run behavior of the utility stream, and is quite independent of what any finite number of generations is assigned. We have jumped from one unacceptable extreme to the other.

Here matters stood for some time. Asking for the two axioms together, the no-dictatorship of the present and the no-dictatorship of the future, as I do here appears almost to imply an impossibility theorem. But not quite.

It turns out the utilities just described are each unacceptable on their own, but not when taken together. Lets' reason again by analogy with the case of finite generations. To any finite number of generations one can assign weights which decline into the future, and then assign some extra weight to the last generation. This procedure, when extended naturally to infinitely many generations, is neither dictatorial for the present nor for the future. It is similar to adding to a sum of discounted utilities, the long run average of the whole utility stream. Neither part of the sum is acceptable on its own, but together they are. In other words: two partial answers make a complete answer. This is Theorem 1 below. Furthermore, this is the only way to get things right, under regularity assumptions independent. This is Theorem 2 below, which gives a complete characterization of all continuous sustainable preferences.

Formally, this section establishes that there exist sustainable preferences which satisfy the two axioms: they neither give a dictatorial role to the present nor to the future. This is achieved by taking the sum of two welfare criteria. The first gives a dictatorial role to the present, but is sensitive to the welfare of each and every generation, for example a sum of discounted utilities. The second part of the sum gives a dictatorial role to the future, for example, the long run average of the sequence of utilities. The sum of a dictatorship of the present plus a dictatorship of the future is neither. This is because the first part of the sum is sensitive to the present, and the second is sensitive to the future. Furthermore such a sum admits trade-offs between the welfare of the present and that of the future. Theorem 1 below shows that sustainable preferences do exist; they are represented diagrammatically in Fig. 2, which shows the trade-offs between the present and the future's utilities. The three axis represent the utility levels of generations 1, 2, and, figuratively, ∞. The two triangular planes represent two indifference surfaces. One gives more utility to generations 1 and 2, and under a dictatorship of the present these choices would prevail; however the second surface gives more utility to the long run, so that under certain conditions the second surface is chosen over the first. Theorem 1 makes this reasoning rigorous.

The second part of Theorem 1 shows that all known criteria of optimality used until now fail to satisfy the axioms postulated here: some, such as the sum of discounted utilities, are dictatorship of the present. Others, such as the long run averages, are dictatorship of the future. Yet others are incomplete, such as overtaking and the Ramsey's criterion, and others are insensitive, such as the Rawlsian criteria and basic needs. Therefore the sustainable preferences defined here perform a role that no previously used criteria did.

What is somewhat surprising is that the sustainable welfare criteria constructed here, namely the sum of a dictatorship of the present and on of the future, exhaust all the continuous independent preferences which satisfy my two axioms. This means that all such sustainable preferences must be of the form just indicated. This

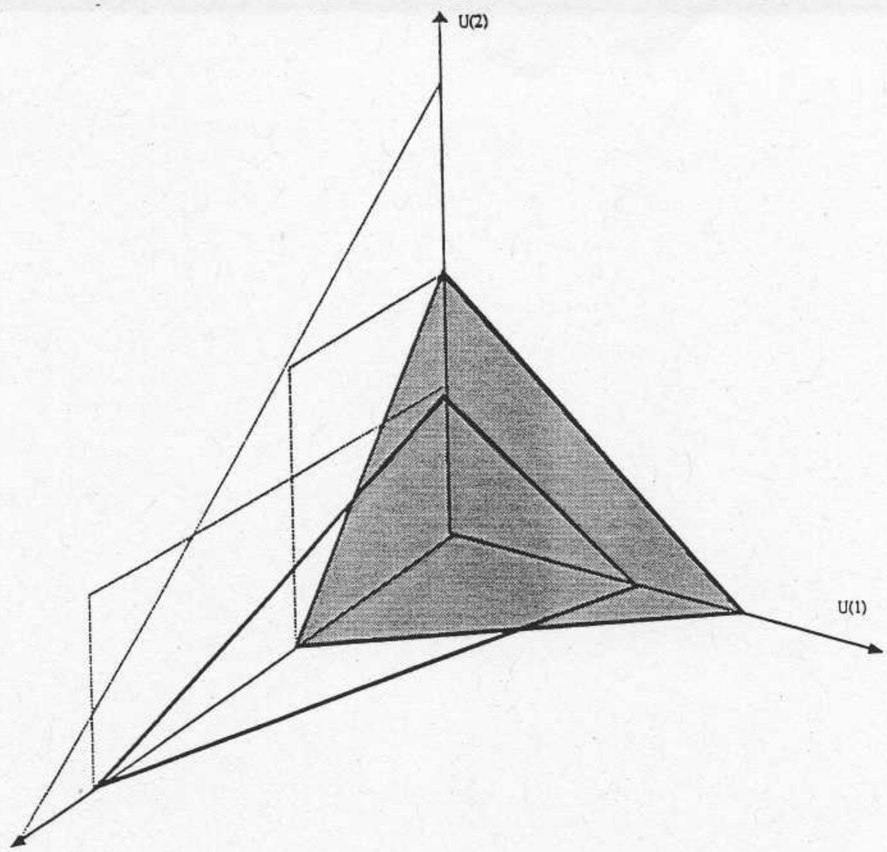

Fig. 2. The 3 axes represent the utilities of finite generations ($U(1)$ and $U(2)$) and the limiting utility value Lim inf U. Two level sets of the ranking are shown. One dominates over finite generations but has a lower lim inf. As the weights on finite generations are fixed, the ranking in these dimensions can be represented by the intersection of the level set restricted to the $U(1)$–($U(2)$, plane with the vertical axis. The overall ranking is then shown as the sum of this ranking (the countably additive measure) with the ranking in the lim inf dimension (the purely additive measure)

is Theorem 2, proved in the Appendix. The mathematics needed to make all this work is not trivial, but the intuition is clear.

7.1. The existence of sustainable preferences

Theorem 1. *There exists a sustainable preference $W : \ell_\infty \to R$, i.e. a preference which is sensitive and does not assign a dictatorial role to either the present or the future*:

$$W(\alpha) = \sum_{g=1}^{\infty} \lambda_g \alpha_g + \phi(\alpha), \tag{11}$$

where $\forall g, \lambda_g > 0, \sum_{g=1}^{\infty} \lambda_g < \infty$, *and* $\phi(\alpha) = \lim_g(\alpha_g)$ *extended to all of* ℓ_∞[32].

The following welfare criteria are not sustainable preferences: (*a*) *the sum of discounted utilities, for any discount factor*, (*b*) *Ramsey's criterion*, (*c*) *the overtaking criterion*, (*d*) lim inf, (*e*) *long run averages*, (*f*) *Rawlsian rules, and* (*g*) *basic needs.*

Proof. In the Appendix. □

An intuitive explanation of this result follows. The preference defined in (11) is sustainable because it is complete, its first term is sensitive to the present, in fact it increases with increases in the welfare of every generation, and its second term is sensitive to the long run future.

(*a*) The sum of discounted utilities is a dictatorship of the present because for every $\varepsilon > 0$, there exist a generation N so that the sum of discounted utilities of all other generations beyond N is lower than ε for all utility streams since all utilities are bounded by the number 1. Now, given any two utility streams α, β, if $W(\alpha) > W(\beta)$ then $W(\alpha) > W(\beta) - \varepsilon$ for some $\varepsilon > 0$; therefore there exists a generation N beyond which the utilities achieved by any generation beyond N do not count in the criterion W. This is true for any discount factor.

(*b*) The Appendix establishes that the Ramsey's criterion is incomplete; this derives from the fact that the distance to Ramsey's bliss path is ill-defined for many paths.

(*c*) The Appendix establishes that the overtaking criterion is incomplete: see also Fig. 1[33].

(*d*) and (*e*) Lim inf and long run averages are dictatorships of the future: furthermore the long run averages is also incomplete[34].

Both (*f*) and (*g*), Rawlsian and basic needs criteria, are insensitive because they rank equally any two paths which have the same infimum even if one assigns much higher utility to the other generations.

Figure 2 represents two indifferences surfaces for a sustainable preference as defined in (11).

7.2. A complete classification of sustainable preferences

The following result characterizes sustainable preferences. Additional conditions on the welfare criterion W are now introduced: W is *continuous* when it is defined by a continuous function $W: \ell_\infty \rightarrow R$[35]. Continuity has played a useful role in social choice theory in the last ten years, in effect replacing the axiom of independence of irrelevant alternatives and allowing a complete characterization of domains in which social choice exists, Chichilnisky [13, 14] and Chichilnisky and Heal [16]. A similar role is found here for continuity: the following theorem gives a full characterization to sustainable criteria which are continuous.

---

[32] The linear map $\lim_N(\alpha_N)$ is defined only on a closed subset of ℓ_∞ consisting of those sequences which have a limit; it can be extended continuously to all of ℓ_∞ preserving its norm by Hahn-Banach's theorem. This extension is ϕ.

[33] Other interesting incomplete intergenerational criteria which have otherwise points in common with sustainable preferences are found in Asheim [4, 5].

[34] Take two sequences: (1, 0, 0, 1, 1, 1, 0, 0, 0, 0, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 1, … } and (0, 1, 1, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 0, …): they are not comparable according to the long run averages criterion.

[35] A function W which is continuous with respect to the standard norm of the space of sequences ℓ_∞. The norm is $\|\alpha\| = \sup_{g=1,2\ldots} |\alpha_g|$, and was defined above. Different forms give rise to different notions of continuity but in the context of equitable treatment of generations the sup norm is a natural candidate.

A standard property of neoclassical analysis, one which we wish to consider here as well, is that the rate of substitution between two generations, while possibly dependent on their level of consumption, be nevertheless independent from their levels of utility. A welfare criterion W satisfying this property is called *independent*[36]; for example, a standard sum of discounted concave utilities has this property. The characterization of a sustainable criterion W in the following theorem is given when W is continuous and independent.

The following theorem decomposes a sustainable criterion into the sum of two functions. The first is a discounted utility with variable discount factors and the second is a generalization of long run averages. Called a *purely finitely additive* measure, this second function ϕ assigns all welfare weight to the very long run. In particular, ϕ assigns the value zero to any sequence with has finitely many non zero terms[37].

Theorem 2. *Let $W : \ell_\infty \to R^+$ be a continuous independent sustainable preference. Then W is of the form $\forall \alpha \in \ell_\infty$:*

$$W(\alpha) = \sum_{g=1}^{\infty} \lambda_g \alpha_g + \phi(\alpha), \tag{12}$$

where $\forall g$, $\lambda_g > 0$, $\sum_{g=1}^{\infty} \lambda_g < \infty$, and ϕ is a purely finitely additive measure.

Proof. In the Appendix. □

8. Sustainable optima can be far from discounted optima

The previous results showed that sustainable preferences are substantially different from all other welfare criteria. It remains however to study how different they are in practice, for example whether the optimal solutions of problems which maximize sustainable preferences are substantially different from the optimal solutions to discounted problems.

To answer this question I shall compare problems which are defined over the same constraint set, but each of which maximizes different welfare criteria. The purpose is to explore what difference this makes in practice. An optimal problem which maximizes a sustainable preference will be called a *sustainable problem.* If the welfare criterion is a discounted sum of utilities as defined in (5), I call this a *discounted problem.* The corresponding solutions are called sustainable optima and discounted optima[38].

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[36] See the Appendix. This simply means that the indifference surfaces of the welfare criterion W are hyperplanes so that it is possible to represent W by a linear function on utility streams: $W(\alpha + \beta) = W(\alpha) + W(\beta)$. Note that this does not restrict the utilities of the generations, u_g, in any way; in particular the u_g's need not be linear.

[37] A finitely additive measure on the integers Z is a function μ defined on subsets of the integers, satisfying $\mu(A \cup B) = \mu(A) + \mu(B)$ when $A \cap B = \emptyset$; it is called purely finitely additive when it assigns measure zero to any finite subset of integers. See also the Appendix.

[38] Formally, let F be a convex and closed subset of feasible paths in a linear space X Such as for example $X = \ell_\infty$, or $X = R^N$. A vector $\beta \in F$ is called optimal in F if it maximizes the value of a function $U : X \to R$. The vector β is called a *discounted optimum* if $U : \ell_\infty \to R$ is a discounted sum of utilities as defined in (5): β is called a *sustainable optimum* if $U : X \to R$ is a sustainable preference.

Can one always approximate a sustainable optimum by paths which optimize discounted problems? Or even better: can one always approximate a sustainable optimum by a sequence of paths which approximate the solutions of a discounted problem? The following result gives a negative answer to these questions. It is not always possible to approximate sustainable optima by paths which approach discounted optima. Sustainable optima and discounted optima can be far apart.[39]

Theorem 3. *Consider a sustainable problem:*

$$\underset{\{\alpha:\alpha\in\Omega\}}{\text{Max}}\ W(\alpha_g), \tag{13}$$

where $W(\alpha) = \sum_{g=1}^{\infty} \lambda_g \alpha_g + \Phi(\alpha)$ is a sustainable preference and $\Omega \subset \ell_\infty$ is the set of all feasible utility streams. Let α^* be a unique sustainable optimum of (13), and $W^* = W(\alpha^*)$. Assume also that there exist a unique discounted optimum β^*, for the problem of maximizing over the same set Ω the discounted utility $\sum_{g=1}^{\infty} \lambda_g \alpha_g$[40]. Then in general the sustainable optimum α^* cannot be approximated by a sequence of feasible utility streams $\{\beta^n\}_{n=1,2...}$ which approximates the discounted optimum β^*. This is true for any sequence of discount factors $\{\lambda_g\}$.

Proof. In the Appendix. □

The intuition for this result is in the following example:

Example 1. Consider an economy which uses trees as a necessary input to production or consumption; without this input the economy's utility is zero. The dynamics of tree reproduction requires that unless the first N periods the economy refrains from cutting more than a certain number of trees, the species becomes extinct after $K + N$ periods, in which case there is zero utility at every period from there on. The economy's feasible set of utility streams Ω is described then as follows: a minimum investment denoted $\varepsilon > 0$ is required during each of the first N periods to ensure that the utility levels in all periods after the $(N + K)$-th is above zero. Once this threshold is reached, then the utility levels in each period after the $(N + K)$-th exceed ε. Then for every discount factor, there is an N, K for which the sum of discounted utilities is maximized at a path which leads to the eventual elimination of the forest. Instead, for a sustainable preference which gives sufficient weight to the long run the optimum will keep the forest alive and yielding a minimum utility level ε forever. Therefore the two optima are apart by at least ε; any sequence of paths which approaches the discounted optimum will not approach the sustainable solution.

[39] This result has also implications for the support and decentralization of sustainable optima (Corollary 1 and Example 4). It may not be possible to approximate sustainable optima by paths which approximately maximize the present value of profits under any standard sequence of prices. However, level independent sustainable preferences can be used in certain cases to define shadow prices at which the optimal paths maximize value. These are "generalized" prices, but they can be given a precise mathematical formulation and a precise economic interpretation. They value both the short and the long run, and can be called "sustainable prices".

[40] This result shows that optimizing a sustainable preference may be quite different from optimizing its "discounted" part. This is why we compare the optimum of W with that of $\sum \lambda_g \alpha_g$. One could allow the case where the discount factor varies over all possible discount factors; this is considered in Example 4 below.

9. Sustainability and value

The following corollary and example show that the notion of value derived from sustainable preferences is rather distinctive. Paths which are optimal under sustainable preferences may not maximize present discounted value according to any standard price system. Therefore, environmental resources with a large value in the long run, may not appear valuable under a standard notion of present value profit maximization.

The following example explores the connection between sustainable optima and the maximization of present value[41]. A *standard price p* is a sequence of prices, $p = (p_1, \dots, p_g, \dots)$ which assigns a well-defined present value to every stream, $p(\gamma) = \sum_{g=1}^{\infty} p_g \gamma_g, < \infty$ for all $\gamma \in \ell_\infty$[42]. There exist continuous linear functions on ℓ_∞ which cannot be represented in this manner. An example is the long run average of a sequence α: this assigns a present value zero to every sequence with finitely many terms. If the long run average was representable as a sequence, in the limit, it would assign zero value to every sequence, which is obviously not true[43]. Example 4 constructs a specific economic example.

9.1. Example 4: A sustainable optimum which does not maximize expected value at any standard price system

Consider a feasible path $\beta \in \ell_\infty$ which maximizes a continuous concave utility function U within a convex set $F \subset \ell_\infty$. I will show that at no standard price system p does β maximize present value[45]. For $c \in [0, \infty)$ let

$$u_t(c) = 2^t c \quad \text{for } c \leq 1/2^{2t} \quad \text{and} \quad u_t(c) = 1/2^t \quad \text{for } c > 1/2^{2t}.$$

Now, for any sequence $c \in \ell_\infty^+$ let $U(c) = \sum_{t=1}^{\infty} u_t(c_t)$, which is well defined, continuous, concave, and increasing on ℓ_∞^+. Let $\beta \in \ell_\infty^+$ be defined by

$$\beta_t = 1/2^{2t+1}$$

[41] Here I consider a special case where the utilities u_g are linear, the problem can then be formulated readily without introducing any further notation. The non linear case can be anlayzed along similar lines, at the cost of more notation. General formulations of the problem of optima and intertemporal profit maximization can be found in the literature (Debreu [23]); a simple formulation in infinite dimensional space that fits well our purposes is in Chichilnisky and Kalman [12].

[42] In some cases, the present value coincides with intertemporal utility maximization, see Chichilnisky and Kalman [12]. Note that if p is a standard price system, represented by the sequence $(p_1, p_2, \dots)$, then by definition this sequence satisfies $\sum_{g=1}^{\infty} p_g < \infty$.

[43] All purely finitely additive measures on Z define real valued functions on sequences which cannot be represented themselves by sequences.

[44] Footnote deleted.

[45] This is from Example 1 in Chichilnisky and Heal [16, p. 369], which is reproduced here for the reader's convenience. This example deals with the minimization rather than the maximization of a function over a set, but the results are equivalent. The example constructs a feasible set $F \subset \ell_\infty$ which is non-empty, closed and concave, and a continuous concave function $U: \ell_\infty^+ \to R$ which attains an infimum $U(\beta)$ at β in F, such that the only sequence of prices $p = \{p_n\}_{n=1,2\dots}$ which can support β in F is identically zero.

and let[48]

$$F = U^{\beta} = \{\gamma \in \ell_{\infty} : U(\gamma) \geq U(\beta)\};^{46}$$

F is a closed convex subset of ℓ_{∞}. Now assume that $p = \{p_t\}_{t=1,2\ldots}$ is a standard supporting price system for the set U^{β}, $p_t \geq 0$, i.e. $p \cdot \gamma \geq p \cdot \beta \; \forall \gamma \in U^{\beta}$. By the usual marginal rate of substitution arguments,

$$p_t = p_1 2^{t-1}. \tag{15}$$

I shall show that p_1 must be zero, so that the whole sequence $\{p_t\}_{t=1,2\ldots}$ must be zero. Assume to the contrary that $p_1 \neq 0$. Define $z \in \ell_{\infty}^{+}$ by

$$z_t = 1/p_t$$

and $z^n \in \ell_{\infty}^{+}$ by

$$z_t^n = z_t \quad \text{if } t \leq n \quad \text{and} \quad 0 \text{ otherwise.}$$

Then $\forall n,\, z \geq z^n$ so that

$$p(z) \geq p(z^n), \tag{16}$$

but $\sum_{t=1}^{\infty} p_t z_t^n = n > p(z)$ for some n sufficiently large, contradicting (16). The contradiction arises from the assumption that p_1 is not zero. Therefore $p_1 = 0$ and by (15) the entire price sequence $p = \{p_t\}_{t=1,2\ldots}$ is identically zero. It is therefore not possible to support the concave set U^{β} with a non-zero standard price system[47]. □

10. Conclusions

I have defined a set of axioms which capture the idea of sustainability, and characterized the sustainable preferences that they imply (Theorems 1 and 2). I also analyzed other criteria used in the literature, and found that they do not satisfy my axioms (Theorem 1). Discounted utility fails to satisfy the non-dictatorship of the present, Axiom 2, and in this sense it is not appropriate for the study of sustainable development. This agrees with the viewpoint of many practitioners, who have pointed out the inadequacy of discounted utility for analyzing sustainable growth[48]. Rawlsian and basic needs criteria are insensitive, since they only regard the welfare of the generation which is less well-off. The *overtaking* criterion and its relative the *catching up* criterion are incomplete orders. They fail to compare many reasonable alternatives. This decreases their value in decision making. Ramsey's criterion has a similar drawback: it is defined as the integral of the distance to a "bliss" utility level, but this integral can be ill-defined. Even when paths converge to the bliss point, the criterion may fail to rank these paths if the convergence of the path to the bliss level of utility is slow. The Ramsey criterion is therefore incomplete since no finite value can be attached to those paths with ill defined integrals.

[46] We call this set U^{β} in sympathy with the notation of Chichilnisky and Heal [16].
[47] Further examples of phenomena related to the results in Theorem 2 and Corollary 1 can be found in Dutta [25].
[48] E.g. Dasgupta and Heal [24], Broome [8], Cline [21].

Another way of looking at the problem is that in many cases the welfare level could be infinite. In such cases, it is impractical to use this criterion as a foundation for policy, since this would involve calculus with infinite magnitudes[49].

The sustainable preferences proposed in Theorem 1 and characterized in Theorem 2 circumvent all of these problems. From the practical point of view, they give rise to optimal solutions which are different from those obtained by discounted optimization criteria. Theorem 3 establishes that a path which is optimal under a sustainable preference may not be approximated by paths which approximate discounted optima.

The notion of value derived from sustainable preferences is distinctive. Paths which are optimal under sustainable preferences may not maximize value according to any standard price system (Example 4). Therefore, environmental resources with a large value in the long run, may not appear valuable under a standard notion of profit maximization.

This may help to disentangle the apparent contradictions in values which were discussed in the beginning of this paper. I noted that governments and international organizations seem prepared to invest sums of money which exceed by several orders of magnitude the discounted value of the planet's economic product in order to prevent global climate change. If we accept my axioms for sustainable preferences, the contradiction is resolved. Discounted profit maximization and sustainability lead to different value systems. Some trade-offs are possible, but the two values are not the same.

As Solow has proposed, sustainability should allow intergenerational trade-offs, but no generation should be favored over any other. This standard is met by sustainable preferences when applied to the "present" and to "future" generations. The long run does matter and so does the short run. Indeed, independent sustainable preferences can define *shadow prices* for sustainable optima, which can be used for project evaluation and for the characterization of optimal solutions. Several of the aims of this paper have therefore been reached, and several of the questions that we posed have been answered. But perhaps the results open up at least as many new questions.

It remains to understand the concern for the long run future which is observed in practice, and which appears formalized in the axioms proposed here and their implied preferences. Nobody alive today, not even their heirs, has a stake on the welfare of 50 generations into the future. Yet many humans care about the long run future of the planet, and the results of this paper indicate that axioms which formalize this concern are not altogether unacceptable. One may then ask: whose welfare do sustainable preferences represent?

Perhaps an answer for this riddle may be found in a wider understanding of humankind as an organism who seeks its overall welfare over time. Such proposals have been advanced in the concepts of a "selfish gene", or more practically, in Eastern religions such as Buddhism which view the unity of humankind as a natural phenomenon. If such unity existed, humankind would make up an

[49] Hammond (see [35]) has defined agreeable paths as those which are approximately optimal for any sufficiently long horizon, in the sense that the welfare losses inflicted by considering only finite horizons go to zero as the length of the finite horizon goes to infinity. The criterion is incomplete: it is not designed as a complete order but rather as a way of indentifying acceptable paths. A similar issue arises with the overtaking criterion, which is ill-defined in many cases.

unusual organism, one whose parts are widely distributed in space and time and who is lacking a nervous system on which the consciousness of its existence can be based. Perhaps the advances in information technology described at the beginning of this article, with their global communications reach, are a glimmer of the emergence of a nervous system from which a global consciousness for humankind could emerge.

11. Appendix

11.1. Continuity

In practical terms the continuity of W is the requirement that there should exist a sufficient statistic for inferring the welfare criterion from actual data. This is an expression of the condition that it should be possible to approximate as closely as desired the welfare criterion W by sampling over large enough finite samples of utility streams. Continuity of a sustainable criterion function $W : \ell_\infty \to R$ is not needed in Theorem 1; it is used solely for the characterization in Theorem 2. Continuity is defined in terms of the standard topology of ℓ_∞: the norm defined by $\|\alpha\| = \sup_{g=1,2\ldots}|\alpha(g)|$.

11.2. Independence

The welfare criterion $W : \ell_\infty \to R$ is said to give independent trade-offs between generations, and called *independent*, when the marginal rate of substitution between the utilities of two generations g_1 and g_2 depends only on the identities of the generations, i.e. on the numbers g_1 and g_2, and not on the actual utility levels of the two generations. Independence of the welfare criterion is not needed in Theorem 1. It is used solely in the characterization of Theorem 2, to obtain a simple representation of sustainable preferences. Formally, let ℓ_∞^* be the space of all continuous real valued linear functions on ℓ_∞.

Definition 7. The welfare criterion $W : \ell_\infty \to R$ is independent if $\forall \alpha \in \ell_\infty$,

$$W(\alpha) = W(\beta) \Leftrightarrow \exists \lambda \in \ell_\infty^*, \lambda = \lambda(W), \text{ such that } \lambda(\alpha) = \lambda(\beta)$$

This property has a simple geometric interpretation, which is perhaps easier to visualize in finite dimensions. For example: consider an economy with n goods and 2 periods. Let $\alpha = (\alpha_1, \alpha_2), \beta = (\beta_1, \beta_2) \in R^2$ denote two feasible utility streams. Then α and β are equivalent according to the welfare criterion $W : R^2 \to R$, i.e. $W(\alpha) = W(\beta)$, if and only if there exists a number $\mu = \mu(W)$, such that

$$\frac{\alpha_2 - \beta_2}{\alpha_1 - \beta_1} = \mu. \tag{17}$$

The geometric interpretation of (17) is that the indifference surfaces of W are affine linear subspaces of R^2. Independence implies that the indifference surfaces of the welfare function W are affine hyperplanes in ℓ_∞. In particular, W can be represented by a linear function on utility streams. Examples of welfare criteria which satisfy this axiom are all time-separable discounted utility functions, any linear real valued non-negative function on ℓ_∞, and the welfare criteria in Theorem 2. As already mentioned, this axiom is used to provide a tight representation of sustainable preferences, but is not necessary for the results in Theorem 1.

Definition 8. A continuous independent sustainable preference is a complete, sensitive preference satisfying Axioms 1 and 2 which is continuous and independent.

11.3. Countable and finitely additive measures

Definition 9. Let $(S, \sum)$ denote the field of all subsets of a set S with the operations of unions and intersections of sets. A real valued, bounded additive set function on $(S, \sum)$ is one which assigns a real value to each element of $(S, \sum)$, and assigns the sum of the values to the union of two disjoint sets.

Definition 10. A real valued bounded additive set function is called countably additive if it assigns the countable sum of the values to a countable union of disjoint sets.

Example 2. Probability measures on the real numbers, R, or on the integers Z, are typical examples of countably additive functions. Any sequence of positive real numbers $\{\lambda_g\}_{g=1,2...}$ such that $\sum_{g=1}^{\infty} \lambda_g < \infty$ defines a countably additive measure μ on the integers Z, by the rule

$$\mu(A) = \sum_{g \in A} \lambda_g, \forall A \subset Z.$$

Definition 11. A real valued bounded additive set function φ on $(S, \sum)$ is called purely finitely additive (see Yosida and Hewitt [41]) if whenever a countably additive function v satisfies: $\forall A \in (S, \sum), v(A) \leq \varphi(A)$, then $v(A) = 0 \ \forall A \in (S, \sum)$.

This means that the only countably additive measure which is absolutely continuous with respect to a purely finitely additive measure is identically zero.

Example 3. Any real valued linear function $V: \ell_\infty \to R$ defines a bounded additive function $\hat{V}$ on the field $(Z, \sum)$ of subsets of the integers Z as follows:

$$\forall A \subset Z, \quad \hat{V}(A) = V(\alpha^A), \tag{18}$$

where α^A is the "characteristic function" of the set A, namely the sequence defined by

$$\alpha^A = \{\alpha_g^A\}_{g=1,2...}$$

$$\alpha_g^A = 1 \quad \text{if } g \in A \quad \text{and} \quad \alpha_g^A = 0 \text{ otherwise.} \tag{19}$$

Example 4. Typical purely finitely additive set functions on the field of all subsets of the integers, $(S, \sum)$, are the lim inf function on ℓ_∞, defined for each $\alpha \in \ell_\infty$ by

$$\liminf(\alpha) = \lim \inf_{g=1,2...} \{\alpha_g\}. \tag{20}$$

Recall that the lim inf of a sequence is the infimum of the set of points of accumulation of the sequence. The "long run averages" function is another example: it is defined for each $\alpha \in \ell_\infty$ by

$$\lim_{K,N\to\infty} \left(\frac{1}{K}\sum_{g=N}^{K+N} \alpha_g\right). \tag{21}$$

It is worth noting that a purely finitely additive set function ϕ on the field of subsets of the integers $(S, \sum)$ cannot be represented by a sequence of real numbers in the sense that there exists no sequence of positive real numbers, $\lambda = \{\lambda_n\}$ which defines ϕ, i.e. there is no λ such that

$$\forall A \subset Z, \quad \phi(A) = \sum_{n\in A} \lambda_n.$$

For example the lim inf: $\ell_\infty \to R$, defines a purely finitely additive set function on the integers which is not representable by a sequence of real numbers.

11.4. Proof of Theorem 1

To establish the existence of a sustainable preference $W: \ell_\infty \to R$, it suffices to exhibit a function $W: \ell_\infty \to R$ satisfying the two axioms. For any $\alpha \in \ell_\infty$ consider

$$W(\alpha) = \textstyle\sum_{g=1}^{\infty} \delta^g \alpha_g + \phi(\alpha).$$

with $0 < \delta < 1$, and ϕ purely finitely additive and increasing. W satisfies the axioms because it is a well defined, non-negative, increasing function on ℓ_∞; it is not a dictatorship of the present (Axiom 1) because its second term makes it sensitive to changes in the "tails" of sequences; it is not a dictatorship of the future (Axiom 2) because its first term makes it sensitive to changes in "cutoffs" of sequences.

The next task is to show that the following welfare criteria do not define sustainable preferences: (*a*) Ramsey's criterion, (*b*) the overtaking criterion, (*c*) the sum of discounted utilities, (*d*) lim inf, and (*e*) long run averages (*f*) Rawlsian criteria and (*g*) basic needs.

The Ramsey's criterion defined in (6) fails because it is not a well defined real valued function on all of ℓ_∞, and cannot therefore define a complete order on ℓ_∞. To see this it suffices to consider any sequence $\alpha \in \ell_\infty$ for which the sum in (6) does not converge. For example, let $\alpha = \{\alpha_g\}_{g=1,2,\dots,}$ where $\forall g, \alpha_g = (g-1)/g$. Then $\alpha_g \to 1$ so that the sequence approaches the "bliss" consumption path $\beta = (1, 1, \dots, 1, \dots)$. The ranking of α is obtained by the sum of the distance between α and the bliss path β. Since $\lim_{N\to\infty} \sum_{g=1}^{N} (1-\alpha_g) = \lim_{N\to\infty} \sum_{g=1}^{N} 1/g$ does not converge, Ramsey's welfare criterion does not define a sustainable preference.

The overtaking criterion defined in (9) is not a well defined function of ℓ_∞, since it cannot rank those pair of utility streams $\alpha, \beta \in \ell_\infty$ in which neither α overtakes β, nor β overtakes α. Fig. 1 above exhibits a typical pair of utility streams which the overtaking criterion fails to rank.

The long run averages criterion defined in (10) and the lim inf criterion defined in (20) fail on the grounds that neither satisfies Axiom 2; both are dictatorships of the future.

Finally any discounted utility criterion of the form

$$W(\alpha) = \sum_{g=1}^{\infty} \alpha_g \lambda_g, \quad \text{where } \forall g, \lambda_g > 0 \text{ and } \sum_{g=1}^{\infty} \lambda_g < \infty$$

is a dictatorship of the present, and therefore fails to satisfy Axiom 1. This is because

$$\forall \gamma \in \ell_\infty \text{ s.t. } \sup_{g=1,2\ldots} (\gamma_g) \leq 1, \text{ and } \forall \varepsilon > 0,$$

$$\exists N > 0, N = N(\varepsilon): \sum_{g=N}^{\infty} \gamma_g \lambda_g < \varepsilon. \tag{22}$$

Therefore, since

$$W(\alpha) > W(\beta) \;\Rightarrow\; \exists \varepsilon > 0: W(\alpha) - W(\beta) > 3\varepsilon,$$

then by (22) $\exists N > 0$ such that $\forall \sigma, \gamma \in \Omega$, $W(\alpha^K, \sigma_K) > W(\alpha^K, \gamma_K)$, $\forall K > N$. The function W thus satisfies the first part of the definition of a dictatorship of the present, i.e.

$$W(\alpha) > W(\beta) \;\Rightarrow$$

$$\exists N, N = N(\alpha, \beta): \forall \gamma, \sigma \in \ell_\infty \text{ with } \|\gamma\| \leq 1$$

$$\text{and } \|\sigma\| \leq 1, \; W(\alpha^K, \gamma_K) > W(\beta^K, \sigma_K), \quad \forall K > N.$$

The reciprocal part of the definition of dictatorship of the present is immediately satisfied: if $\forall \sigma, \gamma \in \ell_\infty$ such that $\|\alpha\| \leq 1$, $\|\beta\| \leq 1$, $W(\alpha^K, \sigma_K) > W(\alpha^K, \gamma K)$, this implies $W(\alpha) > W(\beta)$. Therefore W is a dictatorship of the present and violates Axiom 1.

Finally the Rawlsian welfare criterion and the criterion of satisfaction of basic needs do not define independent sustainable preferences: the Rawlsian criterion defined in (7) fails because it is not sensitive to the welfare of many generations: only to that of the less favoured generation. Basic needs has the same drawback. □

11.5. Proof of Theorem 2

Consider a continuous independent sustainable preference. It must satisfy Axioms 1 and 2. There exists a utility representation for the welfare criterion $W: \ell_\infty \to R$, defining a non-negative, continuous linear functional on ℓ_∞. As seen above in Example 3, (18) and (19), such a function defines a non-negative, bounded, additive set function denoted $\hat{W}$ on the field of subsets of the integers Z, $(Z, \sum)$.

Now the representation theorem of Yosida and Hewitt ([41, 40]), establishes that every non-negative, bounded, additive set function on $(S, \sum)$, the field of subsets $\sum$ of a set S, can be decomposed into the sum of a non-negative measure μ_1 and a purely finitely additive, non-negative set function μ_2 on $(S, \sum)$. It follows from this theorem that $\hat{W}$ can be represented as the sum of a countably additive measure μ_1, and a purely finitely additive measure on the integers Z. It is immediate to verify that this is the representation in (11). To complete the characterization of an independent sustainable preference it suffices now to show that neither λ nor ϕ are identically zero in (11). This follows from Axioms 1 and 2: we saw above that discounted utility is a dictatorship of the present, so that if $\phi \equiv 0$, then W would be a dictatorship of the present, contradicting Axiom 1. If on the other hand $\lambda \equiv 0$, then W would be a dictatorship of the future because all purely finitely additive measures are, by definition, dictatorships of the future, contradicting Axiom 2. Therefore neither λ nor ϕ can be identically zero.

11.6. Proof of Theorem 3

The statement of Theorem 3 is

Consider a sustainable optimum growth problem

$$\max_{\alpha \in \Omega} W(\alpha_g), \quad \text{where} \quad \alpha_g = \{u_g(x_g)\}_{g=1,2\ldots} \in \Omega \subset \ell_\infty. \tag{23}$$

where Ω is the set of all feasible utility streams and W is an independent sustainable preference. By Theorem 2, W must be of the form

$$W(\alpha) = \sum_{g=1}^{\infty} \lambda_g \alpha_g + (1-\theta)\phi(\alpha), \quad \forall \alpha \in \ell_\infty, \tag{24}$$

where $\forall g, \lambda_g > 0, \sum_{g=1}^{\infty} \lambda_g < \infty$, $\phi \neq 0$ is a purely finitely additive independent measure on Z and $0 < \theta < 1$. Assume that there exists a unique solution to problem (23), denoted α^* and called a sustainable optimum, with welfare value $W^* = W(\alpha^*)$. Assume also that there exist a unique solution, denoted β^* and called a discounted optimum, for the problem of maximizing over the same set Ω the discounted utility

$$\max_{\alpha \in \Omega} (U(\alpha)), \quad \text{where } U(\alpha) = \theta_g \sum_{g=1}^{\infty} \lambda_g \alpha_g, \tag{25}$$

which is the first term defining the preference W in (24). Then in general the sustainable optima α^* cannot be approximated by a sequence of feasible utility streams $\{\beta^n\}_{n=1,2\ldots}$ which approximates the discounted optimum β^*, i.e. for all such sequences

$$\lim_{n\to\infty} \left(\sum_{g=1}^{\infty} \lambda_g \beta_g^n \right) \neq \max_{\gamma \in \Omega} \left(\sum_{g=1}^{\infty} \lambda_g \gamma_g \right).$$

This is true for any sequence of "discount factors" $\{\lambda_g\}_{g=1,2\ldots}$ satisfying $\forall g, \lambda_g > 0$ and $\sum_{g=1}^{\infty} \lambda_g < \infty$.

Proof. I define a family of optimal growth problems, each with a welfare function of the form (24), and each having a feasible set $\Omega \subset \ell_\infty$, all satisfying the conditions of the Theorem. I will show that for each problem in this family, the optimum α^* cannot be approximated by a sequence which approximates the optima β^* of discounted utility functions of the form (25). This is true for any discount factors $\lambda: Z \to R$ which satisfy $\forall g, \lambda_g > 0, \sum_{g=1}^{\infty} \lambda_g < \infty$.

Define the set of feasible utility streams Ω as follows: $\Omega = \{\alpha \in \ell_\infty^+ : \alpha = \{\alpha_g\}_{g=1,2\ldots}, \text{Sup}_g (\alpha_g\} \leq 1$ and $\exists \varepsilon > 0$, and integers N and K such that if $\alpha_g < \varepsilon$ $\forall g \leq N$ then $\alpha_g = 0 \forall g > K + N$, while if $\alpha_g > \varepsilon \forall g \leq N$, then $\alpha_g \geq \varepsilon$, $\forall g > K + N$. Each set of parameters ε, N and K define a different feasible set of utility streams Ω.

If the welfare function W is a discounted utility of the form (25), then there exists ε, N and K such that the discounted optimum $\beta^* = \{\beta_g^*\}_{g=1,2\ldots} \in \Omega$ satisfies

$$\beta_g^* = 1 \quad \text{for } g \leq N + K \quad \text{and} \quad \beta_g^* = 0 \quad \text{for } g > K + N.$$

The sustainable optima α^* is quite different when in the definition of W, (24), the purely finitely level independent measure ϕ has most of the "weight", i.e. when $\theta \sim 0$. Indeed, for $\theta \sim 0$, the sustainable optimum α^* satisfies

$$\alpha_g^* \geq \varepsilon \quad \text{for } g > K + N.$$

Since both α^* and β^* are unique, and $\|\alpha^* - \beta^*\| \geq \varepsilon > 0$, it is clear that a sequence $\{\beta^n\}$ which approaches β^* cannot approach also α^*. This completes the proof of the theorem. □

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