Endogenous Fertility and Pension System

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11 April 2018

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April 11, 2018

Abstract

This paper presents consideration of two public pension systems having a Defined Contribution (DC) or a Defined Benefit (DB) structure. The differences between these two pension structures are considerably important. In fact, DC benefits for older people are changed according to a budget under a constant contribution rate by younger people, but DB entails a contribution rate that changes based on maintenance of a balanced budget, providing constant benefits for older people. In addition, this paper presents consideration of the child care of two types: one for the child care service and the other for the child care time. The noteworthy result shows that the DB pension system derives the multiple fertility if the child care is given by the time because of the contribution rate affects both the household disposable income and opportunity cost to have children.

Keywords:Defined benefit, Defined contribution, Endogenous fertility

JEL Classifications:J13, H55

*I am grateful to Makoto Hirazawa, Ryo Horii, Takeshi Koba, Koji Kitaura, Takashi Kurosaki, Kazutoshi Miyazawa, Tamotsu Nakamura, Takashi Oshio, Yurika Shiozu, and seminar participants for their useful comments. Any remaining errors are mine. The research for this paper was financially supported by Grants-in-Aid for Scientific Research (No.21370159, 17K03746, 17K03791).

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1 Introduction

In some OECD countries, an aging society with fewer children poses severe difficulties because of the need for social security payments. A decrease in the number of younger people is expected to lead to a shortage of revenues for social security transfers such as pensions and medical insurance. Governments must consider Defined Contribution (DC) and the Defined Benefit (DB) pension systems. For instance, in Japan, the pension budget is managed as a DC system. A fixed-benefit pension system is regarded as a DB system. Such pension systems fix a benefit and set payments to maintain a balanced budget. By contrast, a fixed payment pension system is regarded as a DC system. Such pension systems fix the payment and set the benefit to maintain a balanced budget. It is crucially important to consider how a pension system should be designed for an aging society with fewer children. Many studies have explored this question.

Some related studies have examined endogenous fertility models, which derive fewer children endogenously and which analyze pension systems such as DC. For instance, van Groezen, Leers, and Meijdam (2003), van Groezen and Meijdam (2008), Yasuoka and Goto (2011), and Yasuoka and Goto (2015) consider endogenous fertility models that fix pension payments (that is, DC for which the benefit depends on the intergenerational population ratio) and insist on the necessity of a child allowance to raise fertility. Wigger (1999) described a relation between the contribution rate and the growth of income per capita in an endogenous fertility model that can be regarded as DC.

By contrast, some researchers have examined fixed benefit pensions such as DB. For example, in Oshio and Yasuoka (2009), if younger people become fewer, then the burden of payment on each of them becomes heavier. People will therefore be unable to gain sufficient income to have children. For that reason, the pension system is not sustainable in the long run because younger people decrease over time. Without the heavy burden of payments for pensions, two steady states occur: fertility converges to a steady state or zero. Moreover, Lin and Tian (2003), considering children as investment goods, examined the relation between an increase in the pension benefit financed by a consumption tax and income per capita in a fixed benefit pension system with an endogenous fertility model. Other studies have treated children as investment goods (e.g., Nishimura and Zhang, 1992; Zhang and Zhang, 1998). Still others have considered them as consumption goods (e.g., van Groezen, Leers, and Meijdam, 2003; Fanti and Gori, 2009). The decision-making related to fertility differs between these models. The pension effects on fertility also differ. Therefore, we should consider the basis of fertility even if DB is examined in endogenous fertility model. Lin and Tian (2003) examined DB. However, Lin and Tian (2003) used a fertility model with children cast not as consumption goods but as investment goods. The result derived by Lin and Tian (2003) is inapplicable to a fertility model incorporating children as consumption goods. Therefore, it is important to examine DB in a fertility model that includes children as consumption goods.

Ono (2003) analyzed the pension system with public debt and a mortality rate applied during the old period in an exogenous fertility model. Moreover, that study incorporated a pension system that was balanced by public debt under both a fixed contribution rate and a fixed benefit rate. The results demonstrated how income per capita and the public debt stock are determined under such a pension system.

The earlier studies described above posit no uncertainty for pension benefits, but are instead perfect foresight models. We think that it is desirable that a DB fixes the benefit level to allay uncertainty about pension benefits. Thereby, people can know their pension benefit in advance. Therefore, we must analyze how the pension system is desirable under uncertainty. In fact, some earlier studies of this question persist. Borgmann (2005) considered the uncertainty generated by economic and population growth and demonstrated which pension system (DC or DB) is desirable in terms of social welfare. Thøgerson (1998) demonstrated that a public pension system is better than the private pension system under wage uncertainty. However, these models did not consider endogenous fertility.

The aims of this paper are to derive how fertility is determined. This paper presents consideration of the pension system of two types: DC and DB. In addition, this paper presents consideration of the child care of two types: one for child care services and the other for the child care time. The pension system affects fertility through the household disposable income in the model of child care services. In addition to this effect, the pension system affects the opportunity cost of rearing children, which in turn affects fertility in the model of child care time. Because of this effect, DB in a model of child care time leads to multiple fertility.

The remainder of this paper is as follows. Section 2 of this paper establishes the model. Section 3 derives the equilibrium in the case of child care services. Section 4 derives the equilibrium in the case of child care time. The final section presents results.

2 The Model

This model economy consists of a two-period (young and old) overlapping generations model. Three agents exist in this model: households, firms, and a government. In the following subsection, we explain each agent.

2.1 Households

Each household lives in two periods: young and old, and supplies labor to gain an income during the young period. This model economy assumes that some child-care service or time is necessary to rear children. In this model economy, we consider equilibrium of two types: one is the equilibrium with child care time; the other is the equilibrium with child care service. In the model of child care time, the younger people divide their time (unity) into child care time and labor supply. Then, the budget constraint are given as

$$c_{1t} + \frac{c_{2t+1}}{1+r_{t+1}} = (1-\tau_t)(1-\phi n_t)w_t + \frac{p_{t+1}}{1+r_{t+1}}. \tag{1}$$

Therein, n_t represents the number of children. The ϕ unit time is necessary to rear a child ($0 < \phi < 1$); then the labor time is reduced to $1 - \phi n_t$ if the household has n_t children. In addition, c_{1t} and c_{2t+1} respectively denote consumption in young and old periods. Here, w_t shows the unit time labor supply. Interest rate $1 + r_{t+1}$ represents the return to savings. Younger people face contribution rate τ_t for the pension system. Older people receive pension benefit p_{t+1}.

In the model of child care service, younger people can provide full time for the labor supply, but they must buy the child care service from the market. Assuming z_t as the price for caring for a child, the budget constraint is given as

$$z_t n_t + c_{1t} + \frac{c_{2t+1}}{1+r_{t+1}} = (1-\tau_t)w_t + \frac{p_{t+1}}{1+r_{t+1}}. \tag{2}$$

A household's utility function is assumed as

$$u_t = \alpha \ln n_t + \beta \ln c_{1t} + (1-\alpha-\beta) \ln c_{2t+1}, \quad 0 < \alpha, \beta < 1, \ \alpha + \beta < 1. \tag{3}$$

This function form is generally used in the endogenous fertility model.[1]

Under the budget constraint (1) in the model of child care time, households decide the allocations of c_{1t}, c_{2t+1} and n_t to maximize their utility as

$$c_{1t} = \beta \left((1-\tau_t)w_t + \frac{p_{t+1}}{1+r_{t+1}} \right), \tag{4}$$

$$c_{2t+1} = (1-\alpha-\beta)(1+r_{t+1}) \left((1-\tau_t)w_t + \frac{p_{t+1}}{1+r_{t+1}} \right), \tag{5}$$

$$n_t = \frac{\alpha \left((1-\tau_t)w_t + \frac{p_{t+1}}{1+r_{t+1}} \right)}{(1-\tau_t)\phi w_t}. \tag{6}$$

Under budget constraint (2) in the model of child care service, households decide the allocations of

[1] This utility form is used by van Groezen, Leers, and Meijdam (2003) and others. This is the conventional form in an endogenous fertility model with consumption goods.

c_{1t}, c_{2t+1} and n_t to maximize their utility as

$$c_{1t} = \beta \left((1-\tau_t)w_t + \frac{p_{t+1}}{1+r_{t+1}} \right), \tag{7}$$

$$c_{2t+1} = (1-\alpha-\beta)(1+r_{t+1}) \left((1-\tau_t)w_t + \frac{p_{t+1}}{1+r_{t+1}} \right), \tag{8}$$

$$n_t = \frac{\alpha \left((1-\tau_t)w_t + \frac{p_{t+1}}{1+r_{t+1}} \right)}{z_t}. \tag{9}$$

2.2 Firms

A representative firm produces final good Y_t with constant returns to scale or a neoclassical product function, as shown by

$$Y_t = F(K_t, L_t),\ F_K > 0,\ F_L > 0,\ F_{KK} < 0,\ F_{LL} < 0. \tag{10}$$

The firm inputs capital stock K_t and labor L_t. With a perfectly competitive market, wage rate w_t and interest rate r_t are shown as

$$w_t = f(k_t) - f'(k_t)k_t, \tag{11}$$

$$1 + r_t = f'(k_t). \tag{12}$$

In those equations, $k_t \equiv \frac{K_t}{L_t}$ and $f(k_t) \equiv \frac{Y_t}{L_t}$. The capital stock depreciates fully in one period.

In this model, there exists the child care service sector. Based on Yasuoka and Miyake (2010), we assume $Y_t^c = \rho L_t^c$ as the child care service production function ($0 < \rho$). Here, Y_t^c and L_t^c respectively denote the output of child care service and the labor input for child care service sector. Assuming free labor mobility between the final goods sector and child care service sector, the profit function π_t is

$$\pi_t = z_t \rho L_t^c - w_t L_t^c. \tag{13}$$

Then, the profit maximization derives the price of child care service as

$$z_t = \frac{w_t}{\rho}. \tag{14}$$

2.3 Government

The government supplies two policies: one for pay-as-you-go pensions and the other for child allowances. Moreover, we consider pension systems of two types: DC and DB.

Defined Contribution: DC This pension system fixes the contribution rate for younger people ($\tau_t = \bar{\tau}$) and determines the benefit level for older people, which depends on the intergenerational population ratio, to hold a balanced budget. Considering the balanced budget, the budget constraint in the model of child care time is shown as

$$N_t p_{t+1} = \bar{\tau} N_{t+1}(1-\phi n_{t+1}) w_{t+1} \;\leftrightarrow\; p_{t+1} = \bar{\tau} n_t (1-\phi n_{t+1}) w_{t+1}. \tag{15}$$

In that equation, N_t and N_{t+1} respectively denote the population size of older people in $t+1$ period (younger people in t period) and that of the younger people in $t+1$ period. The intergenerational population ratio is given as $n_t = \frac{N_{t+1}}{N_t}$. Large n_t, which represents the intergenerational population ratio, increases the benefit for older people. However, if households have many children, then the labor supply decreases.

The government budget constraint in the model of child care service is

$$N_t p_{t+1} = \bar{\tau} N_{t+1} w_{t+1} \;\leftrightarrow\; p_{t+1} = \bar{\tau} n_t w_{t+1}. \tag{16}$$

As shown by this equation, the child care time does not affect the pension benefit for older people.

Defined Benefit: DB This pension system fixes the benefit level for older people ($p_{t+1} = \bar{p} w_{t+1}$), and determines the contribution rate for the younger people. Considering a balanced budget, the budget constraint in the model of child care time is

$$N_t \bar{p} w_{t+1} = \tau_{t+1} N_{t+1}(1-\phi n_{t+1}) w_{t+1} \;\leftrightarrow\; \tau_{t+1} = \frac{\bar{p}}{n_t(1-\phi n_{t+1})}. \tag{17}$$

The greater number of younger people in $t+1$ period, which means large n_t, decreases the contribution rate. However, if the younger people have many children, then the contribution rate increases because of the small labor supply.

The government budget constraint in the model of child care service is

$$N_t \bar{p} w_{t+1} = \tau_{t+1} N_{t+1} w_{t+1} \;\leftrightarrow\; \tau_{t+1} = \frac{\bar{p}}{n_t}. \tag{18}$$

As shown by this equation, the child care time does not affect the pension benefit received by older people. With a large population of younger people, the contribution rate is low.

3 Equilibrium in Child Care Service

This paper presents derivation of the equilibrium for a small open economy. Therefore, the interest rate is given by an exogenous interest rate r; the wage rate w is also decided exogenously. This section presents derivation of the fertility dynamics as the equilibrium in the model of child care service. However, there exist pension systems of two types. We show the equilibrium of DC and DB.

3.1 DC Case

Considering (9), (14), and (16), we can obtain fertility n_t as

$$n_t = \frac{\alpha(1-\bar{\tau})}{\frac{1}{\rho} - \frac{\alpha\bar{\tau}}{1+r}}. \tag{19}$$

Here, $\frac{1}{\rho} > \frac{\alpha\bar{\tau}}{1+r}$ should be held to be positive n_t. There is no dynamics of n_t.

3.2 DB Case

Considering (9), (14) and (18), we can obtain fertility n_t as

$$n_t = \alpha\rho\left(1 + \frac{\bar{p}}{1+r} - \frac{\bar{p}}{n_{t-1}}\right). \tag{20}$$

For given n_{t-1}, the fertility in t period n_t is derived. The fertility dynamics of (20) can be portrayed as the figure below.

[Insert Fig.1 around here.]

The solid line has two steady state equilibria: one for the stable steady state equilibrium and the other for the unstable one. The dashed line has no steady state. With $\alpha\rho\left(1 + \frac{\bar{p}}{1+r}\right)^2 - 4\bar{p} \geq 0$, we can obtain the steady state equilibrium.[2]

The results presented in this section are obtained using many related studies of the literature such as those by van Groezen, Leers and Meijdam (2003), and by Fanti and Gori (2009).

4 Equilibrium in Child Care Time

This section presents derivation of the fertility dynamics as the equilibrium in the model of child care time in the cases of DC and DB.

4.1 DC Case

Considering (6) and (15), we can obtain $n_t = \frac{\alpha\left((1-\bar{\tau})w + \frac{\bar{\tau}n_t(1-\phi n_{t+1})w}{1+r}\right)}{(1-\bar{\tau})\phi w}$, i.e.,

$$n_{t+1} = \frac{1+r}{\alpha\phi\bar{\tau}}\left(\frac{\alpha(1-\bar{\tau})}{n_t} + \frac{\alpha\bar{\tau}}{1+r} - (1-\bar{\tau})\phi\right). \tag{21}$$

The dynamics of (21) shows an up sloping curve as portrayed in Fig. 2.

[Insert Fig.2 around here.]

[2] Assuming the steady state $n = n_t = n_{t+1}$, we can obtain $n^2 - \alpha\rho\left(1 + \frac{\bar{p}}{1+r}\right)n + \alpha\rho\bar{p} = 0$. Then, $n = \frac{\alpha\rho\left(1+\frac{\bar{p}}{1+r}\right) \pm \sqrt{\alpha^2\rho^2\left(1+\frac{\bar{p}}{1+r}\right)^2 - 4\alpha\rho\bar{p}}}{2}$ is obtainable if $\alpha^2\rho^2\left(1 + \frac{\bar{p}}{1+r}\right)^2 - 4\alpha\rho\bar{p} > 0$.

As shown in Fig. 2, the dynamics of (21) has the positive steady state equilibrium. However, this steady state is not always locally stable. With $\frac{dn_{t+1}}{dn_t} = -\frac{(1+r)(1-\bar{\tau})}{\bar{\tau}\phi n^2} > -1$, the steady state is the locally stable one.

4.2 DB Case

Considering (6) and (17), we can obtain $n_{t+1} = \frac{\alpha\left(\left(1-\frac{\bar{p}}{n_t(1-\phi n_{t+1})}\right)w+\frac{\bar{p}w}{1+r}\right)}{\left(1-\frac{\bar{p}}{n_t(1-\phi n_{t+1})}\right)\phi w}$, i.e.,

$$\phi^2 n_t n_{t+1}^2 - \phi\left(\left(1+\alpha+\frac{\alpha\bar{p}}{1+r}\right)n_t - \bar{p}\right)n_{t+1} - \alpha\left(\bar{p} - \left(1+\frac{\bar{p}}{1+r}\right)n_t\right) = 0. \tag{22}$$

Then, the fertility n_{t+1} can be derived as

$$n_{t+1} = \frac{\phi\left(\left(1+\alpha+\frac{\alpha\bar{p}}{1+r}\right)n_t - \bar{p}\right) \pm \sqrt{\phi^2\left(\left(1+\alpha+\frac{\alpha\bar{p}}{1+r}\right)n_t - \bar{p}\right)^2 + 4\phi^2 n_t\bar{p}\left(\bar{p}-\left(1+\frac{\bar{p}}{1+r}\right)n_t\right)}}{2\phi^2 n_t}. \tag{23}$$

Obtaining two positive n_{t+1}

$$\phi^2\left(\left(1+\alpha+\frac{\alpha\bar{p}}{1+r}\right)n_t - \bar{p}\right)^2 + 4\phi^2 n_t\bar{p}\left(\bar{p}-\left(1+\frac{\bar{p}}{1+r}\right)n_t\right) > 0 \tag{24}$$

requires positive n_t. With

$$4\phi^2 n_t\bar{p}\left(\bar{p}-\left(1+\frac{\bar{p}}{1+r}\right)n_t\right) < 0, \tag{25}$$

we can obtain two multiple n_{t+1} for n_t, i.e., the multiple equilibrium. Then, the following proposition can be established.

Proposition 1 In the model of child care time with a DB pension, if (24) and (25) are held, then two positive fertility n_{t+1} can be derived for n_t.

Why can multiple equilibria be derived? The reason is given by the child care time with DB pension. If n_{t+1} is large, then the contribution rate $\tau = \frac{\bar{p}}{n_t(1-\phi n_{t+1})}$ remains high because of a small $1-\phi n_{t+1}$. This effect reduces the household disposable income, which reduces fertility. However, $\tau = \frac{\bar{p}}{n_t(1-\phi n_{t+1})}$ affects the opportunity cost of rearing children, and the cost of child care time is small. Consequently, the household wants to have more children. Then, high fertility n_{t+1} can be achieved.

However, if n_{t+1} is small, then the contribution rate $\tau = \frac{\bar{p}}{n_t(1-\phi n_{t+1})}$ is low. This effect raises fertility. However, a low contribution rate means high child care costs because of the high opportunity cost of rearing children. Then, low fertility n_{t+1} can be achieved.

If the steady state exists, then the locally stable condition is given as $-1 < \frac{dn_{t+1}}{dn_t} < 1$. $\frac{dn_{t+1}}{dn_t}$ is given by

$$\frac{dn_{t+1}}{dn_t} = \frac{\phi n \left(1 + \alpha + \frac{\alpha \bar{p}}{1+r}\right) - \phi^2 n^2 - \alpha \left(1 + \frac{\bar{p}}{1+r}\right)}{2\phi^2 n^2 - \phi \left(\left(1 + \alpha + \frac{\alpha \bar{p}}{1+r}\right) n - \epsilon\right)}. \tag{26}$$

5 Conclusions

This paper sets the endogenous fertility model with pay-as-you-go pension and examines how the pension system affects fertility. If the household can use child care services, then the DC pension gives no dynamics of fertility. With the DB pension, the fertility fluctuates over time. However, if the household cannot use child care services, then the household should use the time for rearing children. Therefore, because of the change of the labor supply, the fertility dynamics is complicated. Especially in the DB case, there exist multiple equilibria: Multiple fertility can be derived because the contribution rate affects not only the household disposable income but also the opportunity cost of rearing children.

References

[1] Borgmann C. (2005), "Social Security, Demographics, and Risk," Springer.

[2] de la Croix D. and Doepke M. (2003), "Inequality and Growth: Why Differential Fertility Matters," *American Economic Review* Vol.93-4, pp.1091-1113.

[3] Fanti L. and Gori L. (2009), "Population and Neoclassical Economic Growth: a New Child Policy Perspective," *Economics Letters* Vol.104, pp.27-30.

[4] Galor O. and Weil N. (1996), "The Gender Gap, Fertility, and Growth," *American Economic Review*, Vol.86-3, pp.374-387.

[5] Lin S. and Tian X. (2003), "Population Growth and Social Security Financing," *Journal of Population Economics*, Vol.16, pp.91-110.

[6] Nishimura K. and Zhang J. (1992), "Pay-As-You-Go Public Pension with Endogenous Fertility," *Journal of Public Economics*, Vol.48, pp.239-258.

[7] Ono T. (2003),"Social Security Policy with Public Debt in an Aging Economy," *Journal of Population Economics*, Vol.16, pp.363-387.

[8] Oshio T. and Yasuoka M. (2009), "Maximum size of social security in a model of endogenous fertility," *Economics Bulletin*, Vol.29-2, pp.656-666.

[9] Thøgersen O. (2003), "A Note on Intergenerational Risk Sharing and the Design of Pay-As-You-Go Pension Programs," *Journal of Population Economics*, Vol.11, pp.373-378.

[10] van Groezen B., Leers T. and Meijdam L. (2003),"Social Security and Endogenous Fertility: Pension and Child Allowances as Siamese Twins," *Journal of Public Economics*, Vol.87, pp.233-251.

[11] van Groezen B. and Meijdam L. (2008), "Growing Old and Staying Young: Population Policy in an Ageing Closed Economy," *Journaol of Population Economics*, Vol.21, pp.573-588.

[12] Wigger B.U. (1999), "Pay-As-You-Go Financed Public Pensions in a Model of Endogenous Growth and Fertility," *Journal of Population Economics*, Vol.12, pp.625-640.

[13] Yasuoka M. and Goto N. (2011), "Pension and Child Care Policies with Endogenous Fertility," *Economic Modelling*, Vol.28(6), pp.2478-2482.

[14] Yasuoka M. and Goto N. (2015), "How is the Child Allowance to be Financed? By Income Tax or Consumption Tax?," *International Review of Economics*, Vol.62(3), pp.249-269.

[15] Yasuoka M. and Miyake A. (2010), "Change in the Transition of the Fertility Rate," *Economics Letters*, Vol.106(2), pp.78-80.

[16] Zhang J. and Zhang J. (1998), "Social Security, Intergenerational Transfers, and Endogenous Growth," *Canadian Journal of Economics*, Vol.31-5, pp.1225-1241.

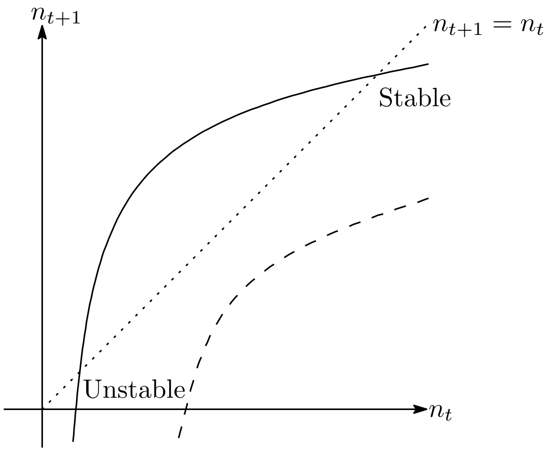

Fig. 1: Dynamics of n_t.

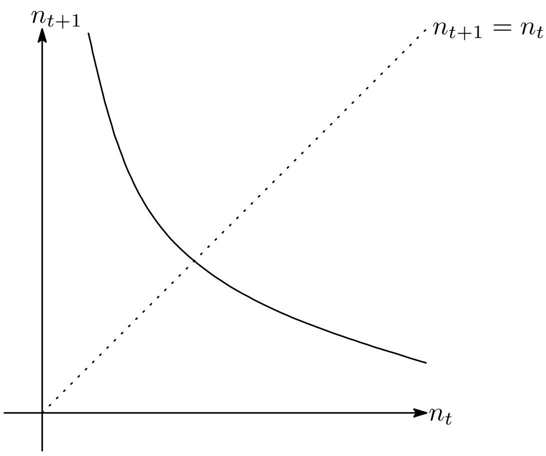

Fig. 2: Dynamics of n_t.