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Abstract

The theory of strategic managerial delegation has recently been extended by incorporating bargaining over managerial contracts (van Witteloostuijn et al. 2007, etc). Assuming that bargaining involves only the incentive rates of managers, this line of research has shown that market outcomes (profits and social welfare) depend crucially on the intra-firm allocation of bargaining powers. In the current paper we revisit the bargaining framework assuming that negotiations involve all contractual terms (incentive rates and transfers). We show that contrary to the earlier results, the market equilibrium is independent of bargaining powers, the latter determining only the transfers. Hence the outcome of our model is identical to the outcome of the delegation model with no bargaining.

Keywords: Strategic delegation; oligopoly; Nash bargaining; equivalence

JEL Classification: L13, L21.

1 Introduction

Strategic managerial delegation is an important branch of modern industrial economics. The relevant literature was launched with the seminal papers of Vickers (1985), Fershtman & Judd (1987) and Sklivas (1987). These works postulated that the production and pricing decisions of firms are taken by their managers, the incentives of which are strategically distorted by the owners of firms. The distortion materializes via the terms of the contracts that the owners offer to managers. The signing of contracts is a credible device for inducing the managers to behave "aggressively" in the market and to enhance the profits of their firms.

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The analysis of strategic delegation in oligopoly is based on two-stage games of the following form: in the first stage, firms’ owners choose the terms of the managerial contracts. These terms essentially determine the objective function of each firm. In the second stage, managers select their market strategies (quantities or prices), based on the outcome of the first stage.

The literature has examined various types of managerial contracts. The most frequently encountered are the sales-based contracts, where the manager’s reward is a function of his firm’s profit and output (Vickers 1985) or profit and revenue (Fershtman & Judd 1987, Sklivas 1987), the relative performance contracts, where the reward depends on the difference between own firm’s profit and the opponent firm’s profit (Salas Fumas 1992) and the market share contracts, where the reward depends on own firm’s profit and market share (Jansen et. al 2007).

These works spurred a large number of extensions. The basic delegation model was enriched via the incorporation of R&D (Zhang and Zhang 1997, Kopel and Riegler 2009), quality competition (Ishibashi 2001), collusion (Lambertini and Trombetta 2002, Pal 2010), mixed oligopoly (White 2001), mergers (Krakel and Sliwka 2006, Ziss 2001), patent licensing (Saracho 2002), wage bargaining (Szymanski 1994), endogenous mode of market competition (Miller and Pazgal 2001), two-period models (Mujumdar and Pal 2007), Stackelberg competition (Kopel and Loffler 2008), etc.

The literature described above was built on the assumption that the owners of firms have the power to impose their terms on the contracts they sign with their managers. Recently, van Witteloostuijn et al. (2007) pioneered a line of research where (some of) the terms of the managerial contracts are bargained over by owners and managers. Negotiations are modeled via a Nash bargaining game within each firm. Agents are characterized by their bargaining powers, which they are assumed to be exogenous. Bargaining deals with the incentive rate of managers, i.e., the parameter that determines their market behavior. The other terms of the contract, i.e., transfers to the managers, are not included in the bargaining agenda.

Using the above framework, van Witteloostuijn et al. (2007) shows that bargaining plays a role: incentive rates, and thus quantities, prices and profits, depend crucially on the allocation of bargaining powers. In particular, if the bargaining power of managers increases, profits fall (increase) provided that sales-based (relative performance) contracts are used. The opposite is true for social welfare. Van Witteloostuijn et al. (2007) focused on a homogeneous Cournot market. Their work was subsequently extended to include markets with differentiated goods and Bertrand competition (Nakamura 2008a), non-constant returns to scale (Nakamura 2008b), sequential competition (Kamaga & Nakamura 2008) and international oligopolies (Nakamura 2011). These works show, again, that bargaining affects the market outcome.

The current note revisits the bargaining framework by assuming that all contractual terms (i.e., incentive rates and transfers) are bargained over by owners and managers. We show that this framework produces a completely different result compared to the earlier works on bargaining: incentive rates, and hence market equilibrium outcomes, are completely independent of the allocation of bargaining powers. Bargaining affects only the intra-firm transfers (which have no impact on market equilibrium). Since the market outcome does not depend on bargaining powers, it is identical to the outcome of the seminal
delegation model where the owners fully impose their terms on the contracts (Vickers 1985, Fershtman & Judd 1987, Sklivas 1987). This equivalence holds for all known types of managerial contracts. It holds also irrespective of the demand and cost structure of firms, the number of competitors, the mode (Cournot or Bertrand) and timing of market competition (simultaneous or sequential choice of strategies).

The reason why the early works on bargaining deliver results under which bargaining does play role is the fact that their bargaining games neglect intra-firm transfers. The role of transfers is to align the interests of owners and managers, in the sense that both parties inside each firm are interested in pure profit maximization (as in the seminal delegation model). The justification for not taking into account the full contract is that the managerial reward is simply set equal to the opportunity cost of the manager.\(^1\) Although such an assumption is plausible when the manager has no bargaining power, it is less plausible when dealing with a bargaining framework where both parties have positive bargaining power.

In what follows, we present the model and the analysis in section 2. Section 3 discusses extensions of the main results and section 4 provides brief concluding remarks.

## 2 Results

Consider a duopolistic market with firms 1 and 2. The price and quantity of firm \(i\) are \(p_i\) and \(q_i\), respectively; its cost function is given by \(C_i(q_i)\); its profit function is \(\pi_i = p_i q_i - C_i(q_i)\), \(i = 1, 2\). Our framework is general enough so that we do not need to specify the mode of market competition (i.e., whether firms compete by selecting prices or quantities). The mode of competition is thus assumed to be fixed without being specified. Our formulation allows also for either homogeneous or differentiated goods.

Firms are characterized by ownership-management separation. The manager of each firm is assigned the task of choosing his firm’s market price or quantity by maximizing an objective (or incentive) function. Some well known examples of this function are based on the profit-quantity or profit-revenue schemes (Vickers 1985, Fershtman & Judd 1987, Sklivas 1987), the relative performance scheme (Salas Fumas 1992) or the market share scheme (Jansen et.al 2007).

Under the first two schemes, the manager of firm \(i\) is delegated the objective function \(v_i = \pi_i + a_i q_i\) or \(v_i = a_i \pi_i + (1 - a_i) p_i q_i\); under the third, the objective function is \(v_i = \pi_i - a_i \pi_j\); and under the last scheme, the objective function is \(v_i = \pi_i + a_i \frac{q_i}{Q}\), where \(a_i \geq 0\) is manager \(i\)'s incentive rate and \(Q = q_1 + q_2\). The total compensation that the manager receives is \(y_i = \lambda_i v_i + t_i\), where \(\lambda_i\) and \(t_i\) are constants. The manager’s utility function and reservation utility respectively are \(w_i(y_i)\) and \(\bar{w}_i\). We assume that \(w_i\) is increasing in \(y_i\), \(i = 1, 2\).

The seminal approach in the literature has assumed that the owners of firms impose their own terms in the managerial contracts. I.e., \(a_i\) is chosen via the maximization of firm \(i\)'s profit function.\(^2\) On the other hand, the values of \(\lambda_i\) and \(t_i\) are chosen so as to equate

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\(^1\)See for example Nakamura (2011).

\(^2\)Witteeloostuiju et al. (2007), Nakamura (2008a, 2008b, 2011) and Kamaga & Nakamura (2008) are exceptions. We elaborate more on these works later on.
the manager's utility to his reservation utility (the values of $\lambda_i$ and $t_i$ do not affect the market decisions of firm $i$'s manager).

In the current note we assume that the incentive rate $a_i$ and the terms $\lambda_i$ and $t_i$ are determined via a Nash bargaining game inside firm $i$, $i = 1, 2$. The timing of the interaction is as follows: in stage 1, the owners and manager of firm $i$ bargain over the triplet $(a_i, \lambda_i, t_i)$, $i = 1, 2$. Bargaining is simultaneous across firms. The bargaining outcomes become commonly known. In stage 2, the two managers choose quantities (if market competition is of the Cournot type) or prices (if market competition is of the Bertrand type). We call this game $G$ (irrespective of the mode of competition); and we call $G_-$ the game with the same time structure but without bargaining (i.e., the game where the terms of the managerial contracts are chosen by the firms’ owners).

Assume that the manager of firm $i$ has been delegated the objective function $v_i$, $i = 1, 2$. We do not need to specify the exact form of $v_i$: the result that we derive below does not depend on the exact type of the contract. We only assume that $v_i$ is such that the game of quantity or price competition has a unique interior solution. By backwards induction, let us first turn to stage 2 of $G$ where the manager of firm $i$ solves the problem:\[\max_{a_i} v_i, \quad i = 1, 2,\] where $s_i$ denotes either $q_i$ (if market competition is of the Cournot type) or $p_i$ (if market competition is of the Bertrand type). Let $s_i(a)$ denote his choice, where $a = (a_1, a_2)$. Let $v_i(a)$ and $\pi_i(a)$ denote the corresponding values of the objective and profit functions. Denote by $\beta_i \in (0, 1)$ the bargaining power of firm $i$’s owners. The Nash product within firm $i$ in stage 1 of $G$ is:\[B_i = [\pi_i(a) - y_i]^{\beta_i} [w_i(y_i) - \bar{w}_i]^{1-\beta_i}, \quad i = 1, 2\] (1)

The optimization problems in stage 1 are
\[\max_{t_i, \lambda_i, a_i} B_i, \quad i = 1, 2\]

We have the following result.

**Proposition 1** The incentive rates $a_i$, $i = 1, 2$, and the market equilibrium outcomes in $G$ and $G_-$ are identical.

**Proof** The first-order condition for the optimal $t_i$, $i = 1, 2$, is given by
\[\frac{\partial B_i}{\partial t_i} = 0 \iff t_i = \pi_i(a) - \lambda_i v_i(a) - \frac{\beta_i}{(1 - \beta_i)} \frac{\partial w_i(y_i)}{\partial y_i} (w_i(y_i) - \bar{w}_i)\] (2)
where we used the relation $\frac{\partial w_i(y_i)}{\partial t_i} = \frac{\partial w_i(y_i)}{\partial y_i} \cdot \frac{\partial y_i}{\partial t_i} = \frac{\partial w_i(y_i)}{\partial y_i}$.

The optimality condition for $\lambda_i$ reads as
\[\frac{\partial B_i}{\partial \lambda_i} = 0 \iff \lambda_i = \frac{\pi_i(a) - t_i}{v_i(a)} - \frac{\beta_i (w_i(y_i) - \bar{w}_i)}{(1 - \beta_i) \frac{\partial w_i(y_i)}{\partial y_i} v_i(a)}\] (3)

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3The manager faces the same maximization problem in $G_-$.

4For notational simplicity we drop the symbol $a$ from $y_i = \lambda_i v_i(a) + t_i$. 

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where we used \[ \frac{\partial w_i(y_i)}{\partial \lambda_i} = \frac{\partial w_i(y_i)}{\partial y_i} \cdot \frac{\partial y_i}{\partial \lambda_i} = \frac{\partial w_i(y_i)}{\partial y_i} v_i(a). \]

By substituting (2) in (3), we can see that the latter holds for all \( \lambda_i \). Hence there is a continuum of optimal values of \( \lambda_i \) and \( t_i \). This does not pose a restriction, though, as market prices and quantities do not depend on \( \lambda_i \) or \( t_i \).

On the other hand, we have

\[
\frac{\partial B_i}{\partial a_i} \geq 0 \Leftrightarrow \beta_i(w_i(y_i) - \bar{w}_i)(\frac{\partial \pi_i(a)}{\partial a_i} - \lambda_i \frac{\partial v_i(a)}{\partial a_i}) + \\
(1 - \beta_i)(\pi_i(a) - \lambda_i v_i(a) - t_i) \frac{\partial w_i(y_i)}{\partial y_i} \frac{\partial y_i}{\partial a_i} \geq 0
\] (4)

Notice by (2) that

\[
\pi_i(a) - \lambda_i v_i(a) - t_i = \frac{\beta_i}{(1 - \beta_i) \frac{\partial w_i(y_i)}{\partial y_i}} (w_i(y_i) - \bar{w}_i)
\] (5)

Using (5) into (4), we get

\[
\frac{\partial B_i}{\partial a_i} \geq 0 \Leftrightarrow (w_i(y_i) - \bar{w}_i)\beta_i(\frac{\partial \pi_i(a)}{\partial a_i} - \lambda_i \frac{\partial v_i(a)}{\partial a_i} + \frac{\partial w_i(y_i)}{\partial y_i} \frac{\partial y_i}{\partial a_i}) \geq 0
\] (6)

However,

\[
\frac{\partial y_i}{\partial a_i} = \lambda_i \frac{\partial v_i(a)}{\partial a_i}
\] (7)

Combining (6) and (7), we have that \( \frac{\partial B_i}{\partial a_i} \geq 0 \) iff \( (w_i(y_i) - \bar{w}_i)\beta_i(\frac{\partial \pi_i(a)}{\partial a_i} - \lambda_i \frac{\partial v_i(a)}{\partial a_i} + \frac{\partial w_i(y_i)}{\partial y_i} \frac{\partial y_i}{\partial a_i}) \geq 0 \). Clearly, in any solution we must have \( w_i(y_i) - \bar{w}_i > 0 \). Therefore, \( \frac{\partial B_i}{\partial a_i} \geq 0 \) iff \( \frac{\partial \pi_i(a)}{\partial a_i} \leq 0 \).

Hence the optimal value of \( a_i \) in \( G \) is derived by solving the problem as in \( G_- \). Since \( a_i \) \((i = 1, 2)\) is the same in \( G \) and \( G_- \), so are the corresponding market quantities and prices.

The result of Proposition 1 is driven by the existence of the intra-firm transfers \( t_i \) and \( \lambda_i \). Under the Nash bargaining solution, the transfers guarantee that what actually matters is the value that the firm creates, i.e., its profit. In other words, the transfers align the interests of the two parties towards pure profit maximization. Hence both are interested in choosing an incentive rate so as \( \pi_i(a) \) is maximized, similarly to the no-bargaining case.

The only differences between \( G \) and \( G_- \) are the values of \( t_i \) and \( \lambda_i \) : in \( G \) they are no longer set at the level that equates \( w_i(.) \) to \( \bar{w}_i \), \( i = 1, 2 \).

The bargaining agenda in the works of Witteloostuijn et al. (2007), Nakamura (2008a, 2008b, 2011) and Kamaga & Nakamura (2008) includes only the incentive rates \( a_i, i = 1, 2 \). The terms \( t_i \) and \( \lambda_i, i = 1, 2 \), are not included in the agenda or even in the Nash product.
As a result, the coordination of the interests of the two parties inside each firm is not fully achieved. Hence, there is room for bargaining to play a role: the allocation of bargaining power affects the incentive rates and subsequently affects the market equilibrium outcome (unlike what happens in a full-fledged bargaining model).

3 Discussion

Our analysis has assumed a duopoly market. However, our point holds also for markets with any number of firms (the proof of Proposition 1 would go through.) It also holds for markets where managers select their strategies (prices or quantities) sequentially or for the case where bargaining takes places sequentially. In what follows, we provide a short proof for the latter game [with the understanding that a similar procedure will work for the game of sequential market (price or quantity) competition].

Consider the following three-stage game: in the first stage, the owners and manager of (say) firm 1 bargain over the terms of the managerial contract. The outcome becomes commonly known. In the second stage, the owners and manager of firm 2 bargain too over their contractual relation. Then, in the last stage the managers of the two firms select simultaneously the market strategies of their firms (either quantities or prices). We denote this game by $G^*$, whereas the corresponding game where no bargaining takes place is denoted by $G_{-}^*$.

**Corollary 1** The incentive rates $a_i, i = 1, 2,$ and the market equilibrium outcomes in $G^*$ and $G_{-}^*$ are identical.

**Proof** Clearly, the outcome of the last stage of $G^*$ is as in $G$. Let us next move back to the second stage. We can apply Proposition 1 and directly conclude that the bargained incentive rate of firm 2’s manager is identical to the incentive rate that maximizes firm 2’s profit. Denote this choice by $a_2(a_1)$. Furthermore, a continuum of solutions in $\lambda_2$ and $t_2$ will arise again. Consider next the first stage of $G^*$. Let $v_1(a_1, a_2(a_1))$ and $\pi_1(a_1, a_2(a_1))$ denote the corresponding objective and profit functions for firm 1; and with a slight abuse of notation, let $y_1$ denote again the compensation of its manager. The Nash product within firm 1 is

$$B_1 = [\pi_1(a_1, a_2(a_1)) - y_1]^\beta_1 [w_1(y_1) - \bar{w}_1]^{1-\beta_1}$$

By straightforward calculations, the optimal $t_1$ and $\lambda_1$ are given by expressions similar to (2) and (3), i.e.,

$$t_1 = \frac{\beta_1}{(1 - \beta_1) \frac{\partial w_1(y_1)}{\partial y_1}} (w_1(y_1) - \bar{w}_1)$$

and

$$\lambda_1 = \frac{\pi_1(a_1, a_2(a_1)) - t_1}{v_1(a_1, a_2(a_1))} - \frac{\beta_1 (w_1(y_1) - \bar{w}_1)}{(1 - \beta_1) \frac{\partial w_1(y_1)}{\partial y_1} v_1(a_1, a_2(a_1))}$$

Hence again we reach to a continuum of solutions in $\lambda_1$ and $t_1$. Moreover,
\[
\frac{\partial \mathcal{B}_1}{\partial a_1} \geq 0 \Leftrightarrow \beta_1 (w_1(y_1) - \bar{w}_1) \left( \frac{d\pi_1(a_1, a_2(a_1))}{da_1} - \lambda_1 \frac{dv_1(a_1, a_2(a_1))}{da_1} \right) + (1 - \beta_1)(\pi_1(a_1, a_2(a_1)) - \lambda_1 v_1(a_1, a_2(a_1)) - t_1) \frac{\partial w_1(y_1)}{\partial y_1} \frac{dy_1}{da_1} \geq 0 \quad (11)
\]

where
\[
\frac{d\pi_1(a_1, a_2(a_1))}{da_1} = \frac{\partial \pi_1(a_1, a_2(a_1))}{\partial a_1} + \frac{\partial \pi_1(a_1, a_2(a_1))}{\partial a_2} \cdot \frac{\partial a_2(a_1)}{\partial a_1}
\]

and likewise for \( \frac{dv_1(a_1, a_2(a_1))}{da_1} \) and \( \frac{dy_1}{da_1} \). Using (9) in (11) and taking into account the relation
\[
\frac{dy_1}{da_1} = \lambda_1 \frac{dv_1(a_1, a_2(a_1))}{da_1}
\]

we can easily reach to the conclusion that \( \frac{\partial \mathcal{B}_1}{\partial a_1} \geq 0 \) iff \( \frac{d\pi_1(a_1, a_2(a_1))}{da_1} \geq 0 \). Hence, \( a_1 \) is chosen by maximizing firm 1’s profit, as in \( G^*_1 \).

**Examples**

1. Consider a market where firms produce homogeneous goods and compete in quantities. The price in the market is given by \( p = a - q_1 - q_2 \); firms produce with 0 per-unit cost. The manager of firm \( i \) is assigned the objective function \( v_i = \pi_i + a_i q_i \). His compensation is \( y_i = \lambda_i v_i \), i.e., we assume that \( t_i = 0 \); his utility and reservation utility are \( w_i(y_i) = y_i \) and \( \bar{w}_i = 0 \), \( i = 1, 2 \). Bargaining across firms is simultaneous and so is quantity competition.

By straightforward calculations, we derive the profit and objective functions \( \pi_i(a) = (1 - a_i - a_j)(1 + 2a_i - a_j)/9 \) and \( v_i(a) = (1 + 2a_i - a_j)^2/9 \). For \( i = 1, 2 \), the Nash product within firm \( i \) is
\[
\mathcal{B}_i = \frac{1}{9} [(1 + 2a_i - a_j)[(1 - \lambda_i)(1 - a_j) - a_i(1 + 2\lambda_i)]^{\beta_i} [\lambda_i(1 + 2a_i - a_j)^2]^{1 - \beta_i}
\]

The optimal value of \( \lambda_i \) as function of the incentive rates is
\[
\lambda_i = \frac{(1 - \beta_i)(1 - a_i - a_j)}{1 + 2a_i - a_j}
\quad (12)
\]

Furthermore,
\[
\frac{\partial \mathcal{B}_i}{\partial a_i} \geq 0 \Leftrightarrow -4(2\lambda_i + 1)a_i + 4(1 - \lambda_i) + (-4 + 3\beta_i + 4\lambda_i)a_j - 3\beta_i \geq 0
\]

Substituting \( \lambda_i \) from (12) we get
\[
\frac{\partial \mathcal{B}_i}{\partial a_i} \geq 0 \Leftrightarrow a_i \geq (1 - a_j)/4
\quad (13)
\]

Hence at the solution we have \( a_1 = a_2 = 1/5 \).
We will reach the same solution if we derive the incentive rate by maximizing $\pi_i(a)$ w.r.t. $a_i$, $i = 1, 2$:

$$\frac{\partial \pi_i(a)}{\partial a_i} \gtrless 0 \iff -4a_i + 1 - a_j \gtrless 0$$

i.e., iff $a_i \lesssim \frac{1}{56} (1 - a_j)/4$, as in (13).

2. Consider a market where firms produce differentiated goods and compete in prices. The demand of firm $i$ is $q_i = \alpha - p_i + p_j/2$. Firms produce with 0 cost. The manager of firm $i$ is assigned the objective function $v_i = \pi_i + a_i q_i$. His compensation again is $y_i = \lambda_i v_i$; his utility and reservation utility are $w_i(y_i) = y_i$ and 0. Again, bargaining is simultaneous and so is price competition.

We have $\pi_i(a) = \frac{2}{225} (10 + 7a_i - 2a_j)(5\alpha - 4a_i - a_j)$; $v_i(a) = \frac{1}{225} (10 + 7a_i - 2a_j)^2$.

Hence the Nash product is

$$B_i = \frac{1}{225} [(10\alpha + 7a_i - 2a_j)[10\alpha(1 + \lambda_i) + a_i(8 + 7\lambda_i) + a_j(2 - \lambda_i)]^{\beta_i}[\lambda_i(10\alpha + 7a_i - 2a_j)^2]^{1-\beta_i}$$

The optimal value of $\lambda_i$ is

$$\lambda_i = \frac{2(1 - \beta_i)(5\alpha - 4a_i - a_j)}{10\alpha + 7a_i - 2a_j}$$

(14)

Furthermore,

$$\frac{\partial B_i}{\partial a_i} \gtrless 0 \iff -(69\lambda_i + 56)a_i + (70 - 75\beta_i - 70\lambda_i)\alpha + (15\beta_i - 14)a_j \gtrless 0$$

Substituting $\lambda_i$ from (14), we get that

$$\frac{\partial B_i}{\partial a_i} \gtrless 0 \iff a_i \lesssim -5\alpha/56 + a_j/56$$

(15)

Hence at the optimum, $a_1 = a_2 = 0$.

We would reach the same conclusion if we had instead maximized directly $\pi_i(a)$ w.r.t. $a_i$:

$$\frac{\partial \pi_i(a)}{\partial a_i} \gtrless 0 \iff -10\alpha + 2a_j - 112a_i \gtrless 0$$

or iff $a_i \lesssim -5\alpha/56 + a_j/56$, i.e., as in (15).

4 Conclusions

In this note we revisited the model of bargaining over managerial contracts in oligopoly. We showed that whenever firms and managers negotiate over all terms of the contracts, the incentive rate of each manager is equal to the rate that he would be offered in a market without bargaining. As a result, equilibrium market prices, quantities and profits under our
bargaining framework are identical to those under no bargaining. This is in sharp contrast with the conclusions of recent works which are based on the assumption that bargaining involves only a subset of the contractual terms, hence showing that bargaining crucially affects market outcomes.

References


