On cross-system interactions and the sustainability of (economic) development

Stijepic, Denis and Wagner, Helmut

Fernuniversität in Hagen

4 December 2017
On cross-system interactions and the sustainability of (economic) development

Denis Stijepic and Helmut Wagner
University of Hagen*

11th April 2018

Abstract. We propose a system-theoretical model for analyzing the sustainability of (economic) growth and development. In particular, we set up a general dynamic system describing the dynamics of the economic and non-economic system (where the latter encompasses, e.g., the ecological, socio-cultural, and political subsystem), their interactions, the dynamics of development indicators, and the sustainability concepts. Then, we discuss the major aspects of sustainability in this framework, in particular, drivers of sustainable development and their direct and indirect/cross-system impacts on development indicators, dynamic equilibria in relation to sustainability, cross-system feedbacks, intra-system interactions, critique of non-interdisciplinary sustainability studies, and sustainability policy design.

Keywords. Economic growth, economic development, long-run dynamics, sustainability, systems, cross-system interactions, economic system, socio-cultural system, political system, ecological system, dynamic systems theory, dynamic equilibrium.

JEL Codes. O40, A12.

* Address: Lehrstuhl für Makroökonomik, Fernuniversität in Hagen, Universitätsstrasse 41, D-58084 Hagen, Germany. Phone: +49 2331 987 2640. Fax: +49 2331 987 391. Email: denis.stijepic@fernuni-hagen.de and helmut.wagner@fernuni-hagen.de. The authors thank Daniel Stähr for valuable research assistance.
1. Introduction

Sustainability is one of the fundamental concepts in economic growth and development theory referring to the properties of (a) the growth and development process or (b) development level.\(^1\) The discussion of the limit dynamics of growth processes (i.e., the study of the growth dynamics when time approaches infinity) is an essential part of most growth theories, e.g., neoclassical and endogenous growth theories, where the question whether the economy grows in the limit (as in the case of a balanced growth path) or stagnates (as in the case of a steady state) is essential. In this context, the concept of sustainable growth (which refers to sustained growth of per-capita income) arises as a specific variant of the sustainable development definition. In development theory, the question whether a certain development, technology, or institutional level that has been achieved by the help of exogenous forces (e.g., development aid, technical consulting, government, or FDI) can be sustained (when the exogenous forces vanish as in the case of, e.g., liberalization and privatization) is of major interest. These rather theoretical topics are mirrored by popular and applied questions of growth and development theory, e.g., secular stagnation, middle-income trap, and growth slow-down in China, all of which are in some sense questions of sustainability of previous/historical growth episodes/experiences.\(^2\)

In this paper, we propose a system-theoretical model for analyzing the sustainability of growth and development. While many sustainability studies focus on the interactions between the economic system and the ecological system, the development and growth theory implies that the economic system interacts with many other non-economic subsystems in the long run.\(^3\) To capture this fact, we develop a two-system model where the economic system interacts with a non-economic system and the latter encompasses all the non-economic subsystems (e.g., ecological, socio-cultural, political,...) that are relevant for economic dynamics. Although the interactions between the economic and non-economic system are very complex in reality, they can be studied from an abstract conceptual and mathematical perspective, i.e., from a general system-theoretical perspective. By choosing this perspective, we set up a general dynamical model.

---


\(^2\) See, e.g., Wagner (2017a) on the sustainability of China’s development.

\(^3\) See the references listed in Footnote 1 for an overview and Stijepic (2017) for literature references from growth and development theory dealing with the empirical evidence on the interactions between the economic system and the non-economic subsystems.
system describing the dynamics of the economic and non-economic (sub)system(s), their interactions, and the concept of sustainability of economic development. Then, we discuss the major aspects of sustainability in this framework, in particular, drivers of sustainable development and their direct and indirect/cross-system impacts on development indicators, dynamic equilibria in relation to sustainability, cross-system feedbacks, intra-system interactions, critique of non-interdisciplinary studies of sustainability, and design of sustainability policy.

The importance of the study of sustainability in a multi-system framework is obvious. In presence of cross-system interactions, an isolated study of the economic system makes little sense in the context of sustainability, since long-run economic dynamics are not only determined in the economic system but also in the non-economic system. That is, the effects of economic dynamics and policies on the non-economic system and their feedbacks on the economic system must be studied for assessing the (long-run) sustainability of an economic growth strategy/process.

The rest of the paper is organized as follows. In Section 2, we present a model describing the system dynamics and cross-system interactions. Section 3 is devoted to the definition of sustainability in this framework. In Section 4, we discuss the drivers of sustainable development. Section 5 considers briefly the relations between sustainability and dynamic equilibrium in the multi-system framework. In Section 6, we discuss the direct and indirect effects of systems on sustainability. Section 7 focuses on cross-system feedbacks and intra-non-economic system interactions. In Sections 8 and 9, we apply the previous results in a discussion of economic sustainability modeling and sustainability policy design. Concluding remarks are provided in Section 10.

2. A Model of cross-system interactions

The (meta-)model discussed in this section is based on the Stijepic (2017) model.

While there are different mathematical notational conventions, we choose the following notation for reasons of simplicity: small letters (e.g., \( x \)) or small Greek letters (e.g., \( \chi \)) denote scalars; bold small letters (e.g., \( \mathbf{x} \)) denote vectors or vector functions; capital Greek letters (e.g., \( \Phi \)) denote vector functions; capital letters (e.g., \( X \)) denote sets; \( \mathbb{R} \) is the set of real numbers; and a dot indicates a derivative with respect to time (e.g., \( \dot{x} \) is the derivative of \( x \) with respect to time).

A typical economic development model can be described as follows. Let
(a) \( \mathbf{d}(t) = (d_1(t), d_2(t), \ldots, d_\delta(t)) \in D \subseteq \mathbb{R}^\delta \) denote the \( \delta \)-dimensional vector of development indicators (e.g., income per capita, health, education, happiness, \ldots) at time \( t \in [0, \infty) \), where \( D \) is the set of all feasible or meaningful values of the development indicators,

(b) \( \mathbf{e}(t) = (e_1(t), e_2(t), \ldots, e_\varepsilon(t)) \in E \subseteq \mathbb{R}^\varepsilon \) denote the \( \varepsilon \)-dimensional vector of variables describing the state of the economic system at time \( t \in [0, \infty) \), where \( E \) is the set of all feasible or meaningful states of the economic system, and

(c) \( \mathbf{p}(t) = (p_1(t), p_2(t), \ldots, p_\pi(t)) \in P \subseteq \mathbb{R}^\pi \) is a \( \pi \)-dimensional parameter vector at time \( t \in [0, \infty) \), where \( P \) is the parameter space.

In particular, \( \mathbf{e} \) and \( \mathbf{p} \) include all the economic variables and parameters that determine the development indicators \( \mathbf{d} \) as stated by (1).

(1) \( \mathbf{d}(t) = \Phi^d(\mathbf{e}(t), \mathbf{p}(t)) \)

where \( \Phi^d \) is a vector function of the type \( \Phi^d : E \times P \to D \). Note that the vector \( \mathbf{e} \) may also include some variables belonging to the vector \( \mathbf{d} \). In this case, some development indicators (e.g., education indicators) are relevant for determining the dynamics of other development indicators (e.g., per-capita income). For the discussion in our paper, this fact is not of importance, since we do not study causal relations (between the variables), but use the vector \( \mathbf{d} \) to impose some conditions (related to sustainability) on the dynamic system \( \mathbf{e} - \mathbf{n} \).

Without loss of generality, we assume that the dynamics of the economic system \( \mathbf{e} \) are determined by the following differential equation.

(2) \( \dot{\mathbf{e}}(t) = \Gamma^e(\mathbf{e}(t), \mathbf{p}(t)) \)

(3) \( \mathbf{e}(0) = \mathbf{e}_0 \in E \)

where \( \Gamma^e \) is a vector function of the type \( \Gamma^e : E \times P \to \mathbb{R}^\varepsilon \) and \( \mathbf{e}_0 \) is the initial state of the economic system. While standard economic growth models assume that the parameter vector \( \mathbf{p} \) is exogenous, we assume that it is determined in the non-economic system \( \mathbf{n} \), as stated by (4).

(4) \( \mathbf{p}(t) = \Phi^p(\mathbf{n}(t)) \)

where \( \mathbf{n}(t) = (n_1(t), n_2(t), \ldots, n_\eta(t)) \in N \subseteq \mathbb{R}^\eta \) is the \( \eta \)-dimensional vector of variables describing the state of the non-economic system \( \mathbf{n} \) at time \( t \in [0, \infty) \), \( N \) is the set of all feasible or meaningful states of the non-economic system, and \( \Phi^p \) is a vector function of the type \( \Phi^p : N \to P \).

(2) and (4) state that the non-economic system \( \mathbf{n} \) has impacts on the dynamics of the economic system \( \mathbf{e} \) (cf. in Section 1). Moreover, the non-economic system \( \mathbf{n} \) has impacts on the
development indicators $d$ via the parameter vector $p$ (cf. (1) and (4)). This makes sense, since, e.g., the indicators (e.g., air pollution indicators) that indicate the state of the ecological system (which is a non-economic subsystem) can have a direct effect on development indicators (e.g., health indicators).

As we will see in Section 4, it can be useful to reformulate (2) by using (4) as follows:

$$(2') \quad \dot{e}(t) = \Gamma^e(e(t), \Phi^p(n(t))) =: \Phi^e(e(t), n(t))$$

where $\Phi^e$ is a vector function of the type $\Phi^e: E \times N \to R^\varepsilon$.

We assume now that the economic system $e$ has impacts on the dynamics of the non-economic system $n$, which are determined by the differential equation system (5)-(6).

$$\begin{align*}
(5) & \quad \dot{n}(t) = \Phi^p(e(t), n(t)) \\
(6) & \quad n(0) = n_0 \in N
\end{align*}$$

where $\Phi^p$ is a vector function of the type $\Phi^p: E \times N \to R^\eta$ and $n_0$ is the initial state of the non-economic system $n$. Overall, we can see that the model of this section features all the cross-system interactions discussed in Section 1. Moreover, the dynamics of the economic and non-economic system are determined by the autonomous differential equation system (2’) and (5). Thus, we implicitly assume that all the non-autonomous components of the economic system (i.e., the non-constant terms that are not determined in the economic system $e$) are determined in the non-economic system $n$ and vice versa (cf. Stijepic, 2017). This reflects a rather comprehensive definition of the non-economic system.

In general, we can distinguish between two types of cross-system interactions: (a) inter-system interactions, i.e., the interactions between the economic system $e$ and the non-economic system $n$, and (b) intra-system interactions, i.e., the interactions within the economic system $e$ and within the non-economic system $n$. While the model (1)-(6) defines inter-system interactions, we turn, now, to intra-system interactions, where we can distinguish between intra-economic system interactions and intra-non-economic system interactions. Since intra-economic system interactions are not relevant for the discussion here (they are discussed in economic growth and development models), we focus on the intra-non-economic system interactions. For defining these interactions, we partition the non-economic system $n$ into subsystems, as stated by (7)-(8).

$$\begin{align*}
(7) & \quad n(t) \equiv (n_1(t), n_2(t), \ldots, n_\mu(t)) \\
(8) & \quad \forall i \in \{1, 2, \ldots, \mu\} \quad n_i(t) \in N_i \subseteq R^{\eta_i} \\
(9) & \quad \eta_1 + \eta_2 + \ldots + \eta_\mu = \eta
\end{align*}$$
where \( \mathbf{n}_i(t) \) is the \( \eta_i \)-dimensional vector of variables describing the state of the non-economic subsystem \( i \) at time \( t \in [0, \infty) \) and \( N_i \) is the set of all feasible or meaningful states of the non-economic subsystem \( i \). For example, we can set \( \mu = 3 \) and name the three non-economic subsystems as follows: ecological subsystem \( \mathbf{n}_1 \), socio-cultural system \( \mathbf{n}_2 \), and political system \( \mathbf{n}_3 \) (cf. Habermas, 1973, and Wagner, 2014, 2017b).

By using (7)-(9), we can rewrite (5) as follows:

\[
(10) \quad \forall i \in \{1, 2, \ldots \mu\} \quad \dot{\mathbf{n}}_i(t) = \Phi_n^i(e(t), \mathbf{n}(t))
\]

where \( \Phi_n^i \) is a vector function of the type \( \Phi_n^i : E \times N \to R^{\eta_i} \). (10) states that there are intra-non-economic system interactions since the dynamics of each of the non-economic subsystems \( \mathbf{n}_i \) are not only dependent on the state of the respective subsystem \( \mathbf{n}_i \) and the state of the economic system \( \mathbf{e} \) but also on all the other non-economic subsystems \( \mathbf{n}_j, j \neq i \) (cf. (7) and (10)).

As we will see, the intra-system interactions can be neglected for the greatest part of the sustainability discussion in the multi-system framework. In particular, all the major definitions and concepts can be formulated without referring to intra-system interactions. However, intra-system interactions can add interesting aspects to the discussion when determining the thresholds of non-sustainability (cf. Section 7.2).

Now, we turn to the definition of sustainability in the context of the model (1)-(10).

### 3 Definition of sustainability

We distinguish between two versions of sustainability depending on whether sustainability refers to growth rates or levels (cf. Section 1). In the context of the model presented in Section 2, sustainability referring to growth rates (over the period \( T \subseteq [0, \infty) \)) can be defined as follows:

\[
(11a) \quad \forall t \in T \quad \frac{\dot{d}(t)}{d(t)} > \gamma^* \equiv (\gamma_1^*, \gamma_2^*, \ldots, \gamma_\delta^*) \in R^\delta
\]

where \( \gamma^* \) reflects some subjective notion of sustainable growth/development of the development indicators. In particular, the sustainability definition (11a) states that the development indicator \( d_i \) must grow at a minimum growth rate \( \gamma_i^* \) over the period \( T \). The period \( T \) represents (among others) the future and the planning horizon. The strictest definition of sustainability (in our model) refers to \( T = [0, \infty) \).

The alternative definition of sustainability (referring to levels) can be formulated as follows:

\[
(11b) \quad \forall t \in T \quad d(t) > d^* \equiv (d_1^*, d_2^*, \ldots, d_\delta^*) \in D
\]
where $d^*$ are the subjective minimum targets regarding the development indicators. The sustainability definition (11b) implies that the policy makers or the society regards the development as sustainable if a certain minimum development level ($d^*$) is achieved and sustained in future or over the planning period $T$.

Without loss of generality, we will refer to the following special cases of the definitions (11):

(12a) $\lim_{t \to \infty} \frac{\dot{d}(t)}{d(t)} > 0$

(12b) $\lim_{t \to \infty} d(t) > d^*$

That is, we state that the development is sustainable in the sense of dynamic sustainability if (12a) is satisfied. Alternatively, we state that the development is sustainable in the sense of level sustainability if (12b) is satisfied.

It makes sense, to rely on (12a) instead on (11a), since

(a) in general, sustainability means that the development indicators grow at positive rates in the long run (i.e., $\gamma^* > 0$);\(^4\)

(b) the statements of the dynamical systems theory (which we apply in our paper) are relatively general; i.e., in most cases, it is not necessary to distinguish between concrete numerical values of (the entries of) $\gamma^*$ when applying dynamical systems theory; rather, qualitative statements and, in particular, the question whether $\gamma^* = 0$ or $\gamma^* \neq 0$ are of importance;

(c) (12a) is one of the strictest definitions of sustainability, since it requires that the sustainability criterion is satisfied (even) in the limit;

(d) in general, a development path is not regarded as unsustainable if the sustainability criterion is temporarily not satisfied; rather, sustainability is a very long-run concept; in growth theory, the very long run is represented by limit dynamics (in most cases); and

(e) in Sections 4 and 6, the replacement of (11a) by (12a) allows us to simplify the mathematical formulations significantly, while a generalization of these formulations such that they apply to (11a) is straight forward.

The arguments (c) and (d) also support the replacement of (11b) by (12b).

4. Drivers of sustainable development

The sustainability definition (12) refers to the limit (dynamics) of the model (1)-(10). In particular, the sustainability criterion (12a) requires that the development indicators $d$ grow in

\(^4\)Yet, there are some exceptions from this rule, as implied by the example discussed in Section 7.2.
the limit. In the model (1)-(10), dynamics are generated only by the two differential equations (2') and (5). Without these equations the model would be static, i.e., (2') and (5) are the only growth drivers in our model. In particular, the autonomous differential equation system (2')/(5) determines the dynamics of \( e \) and \( n \); all the other model variables and, in particular, \( d \) are determined by the dynamics of \( e \) and \( n \). Autonomous differential equations can generate limit-dynamics (as in the case of a balanced growth path) or not (as in the case of a steady state). Thus, a central requirement for sustained development according to (12a) is that the differential equation system (2')/(5) does not generate a steady state (for the given initial conditions range).

Since (a) our paper deals with cross-system interactions, (b) (2') describes the dynamics of the economic system \( e \), and (c) (5) describes the dynamics of the non-economic system \( n \), it makes sense to analyze (2') and (5) separately. Consider only (2') and assume that \( n \) is given, i.e., \( \forall t \ n(t) = n^o \). We say that given \( n^o \), there is autonomous (limit) development of the economic system \( e \) if \( \lim_{t \to \infty} \dot{e}(t) = \lim_{t \to \infty} \Phi^e(e(t), n^o) \neq 0 \) (cf. (2')). If \( \forall n \in N \subseteq N \lim_{t \to \infty} \dot{e}(t) = \lim_{t \to \infty} \Phi^e(e(t), n) \neq 0 \), we say that there is autonomous (limit) development of the economic system \( e \) on the set \( N \).

Analogously, consider only (5) and assume that \( e \) is given. We say that there is autonomous (limit) development of the non-economic system \( n \) on the set \( E \subseteq E \), if \( \forall e \in E \lim_{t \to \infty} \dot{n}(t) = \Phi^n(e, n(t)) \neq 0 \).

If (12a), \( \lim_{t \to \infty} \dot{e}(t) \neq 0 \), and \( \lim_{t \to \infty} \dot{n}(t) = 0 \) are true, then we say that sustainable development (cf. (12a)) is driven by autonomous economic system development. If (12a), \( \lim_{t \to \infty} \dot{e}(t) = 0 \), and \( \lim_{t \to \infty} \dot{n}(t) \neq 0 \) are true, we say that sustainable development (cf. (12a)) is driven by autonomous non-economic system development. Assume, now, that \( \lim_{t \to \infty} \dot{e}(t) \neq 0 \) and \( \lim_{t \to \infty} \dot{n}(t) \neq 0 \). Then, we distinguish between the following cases (which refer to the differential equation system (2')/(5)), where \( d_i(e(t)) := \sum_{j=1}^\delta \dot{e}_j(t) \partial d_i(t)/\partial e_j(t) \) and \( d_i(n(t)) := \sum_{j=1}^\delta \dot{n}_j(t) \partial d_i(t)/\partial n_j(t) \).

(a) We say that sustained development (cf. (12a)) is driven by both, the economic and the non-economic system, if \( \forall i \in \{1, 2, \ldots, \delta\} \lim_{t \to \infty} \dot{d}_i(e(t)) > 0 \land \lim_{t \to \infty} \dot{d}_i(n(t)) > 0 \).

(b) We say that sustained development (cf. (12a)) is partly driven by the economic system \( e \) and partly driven by the non-economic system \( n \) if: (12a) is true, \( \exists i \in \{1, 2, \ldots, \delta\} \lim_{t \to \infty} \dot{d}_i(e(t)) < 0 \), and \( \exists j \in \{1, 2, \ldots, \delta\} \lim_{t \to \infty} \dot{d}_j(n(t)) < 0 \).

(c) We say that sustained development (cf. (12a)) is driven by the economic system \( e \) but moderated by the non-economic system \( n \) if: (12a) is satisfied, \( \forall i \in \{1, 2, \ldots, \delta\} \lim_{t \to \infty} \dot{d}_i(e(t)) > 0 \), and \( \exists j \in \{1, 2, \ldots, \delta\} \lim_{t \to \infty} \dot{d}_j(n(t)) < 0 \).
We say that sustained development (cf. (12a)) is driven by the non-economic system \( n \) but moderated by the economic system \( e \) if: (12a) is satisfied, \( \forall i \in \{1, 2, \ldots, \delta \} \lim_{t \to \infty} \dot{d}_i(t) > 0, \) and \( \exists j \in \{1, 2, \ldots, \delta \} \lim_{t \to \infty} \dot{d}_j(t) < 0. \)

In case (a), both, the dynamics of the economic system \( e \) and the dynamics of the non-economic system \( n \), have positive impacts on all the development indicators \( d_i \). In case (b), there is at least one development indicator \( d_i \) on which economic dynamics have a positive impact and non-economic dynamics have a negative impact, where the positive impact is stronger than the negative impact such that \( d_i \) grows; analogously, there is at least one development indicator \( d_j \) on which non-economic dynamics have a positive impact and economic dynamics have a negative impact, where the positive impact is stronger than the negative impact such that \( d_j \) grows. In case (c), economic dynamics have positive impacts on all the development indicators \( d_i \) and there is at least one development indicator \( d_j \) on which non-economic dynamics have a negative impact which is, however, weaker than the corresponding positive impact of economic dynamics. In case (d), non-economic dynamics have positive impacts on all the development indicators \( d_i \) and there is at least one development indicator \( d_j \) on which economic dynamics have a negative impact which is, however, weaker than the corresponding positive impact of the non-economic system.

While this discussion focuses on the dynamic definition of sustainability (12a), similar statements can be formulated for the level definition of sustainability (12b). Thus, we omit a detailed discussion of (12b) and summarize the results of Section 4 as follows.

**Result 1.** Sustainable development (cf. (12)) can be driven by autonomous development of the economic system \( e \), by autonomous development of the non-economic system \( n \), or (partly) by both. Moreover, both, the economic system \( e \) and the non-economic system \( n \), may moderate sustainable development.

As we will see in Section 5, the different cases listed in Result 1 correspond to different types of dynamic equilibria.
5 Dynamic equilibria and sustainability

If we define a dynamic equilibrium as a dynamic state (e.g., steady state, balanced growth path, limit cycle, attractor, …) to which a dynamic system converges in the limit (i.e., for \( t \to \infty \)), we can focus our discussion of sustainability on the dynamic equilibrium of the system (1)-(10). The system (1)-(10) can be characterized by (a) a general dynamic equilibrium, i.e., a dynamic state in which both systems (\( e \) and \( n \)) satisfy some dynamic conditions (e.g., both systems are in a steady state), or (b) a partial dynamic equilibrium (cf. Stijepic, 2017), i.e., a dynamic state in which only one of the systems (\( e \) or \( n \)) satisfies some dynamic conditions (e.g., only \( e \) is in a steady state), while the other system may exhibit any sort of dynamic behavior. This classification allows us to formulate the following result.

**Result 2.** Sustainable development requires a general dynamic equilibrium imposing conditions on the dynamics of \( e \) and \( n \) that are consistent with (12).

Result 2 states that sustainability cannot be discussed only within the economic system \( e \), but requires a general multi-system approach including economic and non-economic systems. The proof of Result 2 is quite simple. In Section 3, we have defined sustainable development such that (12) is satisfied. (12) imposes conditions on the dynamics of \( d \). According to (1) and (4), \( d \) is determined by \( e \) and (via \( p \)) by \( n \). Thus, if we seek to satisfy (12), both, the dynamics of \( e \) and the dynamics of \( n \), must satisfy some conditions, i.e., a general dynamic equilibrium is required for satisfying (12). In particular, if we impose only some conditions on the dynamics \( e \) (\( n \)) for satisfying (12) (i.e., if we rely on a partial dynamic equilibrium of \( e \) (\( n \))), then we cannot ensure that the dynamics of \( n \) (\( e \)) do not lead to a violation of (12).

A further interesting aspect of our sustainability discussion related the dynamic equilibrium is that the cases discussed in Result 1 correspond to different concepts of dynamic equilibrium. If sustainability is driven by autonomous economic system development, the limit dynamics of the \( n-e \) system can be described by a steady state of the non-economic system (\( \lim_{t \to \infty} \dot{n}(t) = 0 \)) and, e.g., balanced or exploding growth of the economic system (\( \lim_{t \to \infty} \dot{e}(t) \neq 0 \)). Analogously, if sustainability is driven by autonomous non-economic system development, the limit dynamics of the \( n-e \) system can be described by a steady state of the economic system (\( \lim_{t \to \infty} \dot{e}(t) = 0 \)) and, e.g., balanced or exploding growth of the non-economic system (\( \lim_{t \to \infty} \dot{n}(t) \neq 0 \)).
sustainability is driven by both, economic and non-economic system dynamics, then the limit dynamics of the \textit{n-e} system can be described by, e.g., a balanced growth path of the economic and non-economic system or by exploding growth of both systems ($\lim_{t \to \infty} \dot{e}(t) \neq 0$ and $\lim_{t \to \infty} \dot{n}(t) \neq 0$). If the \textit{n-e} system converges to a steady state ($\lim_{t \to \infty} \dot{e}(t) = \lim_{t \to \infty} \dot{n}(t) = 0$), then the development is not sustainable in the sense of the dynamic sustainability definition (12a) but can be sustainable in the sense of the level sustainability definition (12b).

6 Direct and indirect effects of the systems on sustainability

While the previous sections deal with the direct effects of the systems on sustainability, there are also indirect effects of system dynamics on sustainability due to cross-system interactions.

The non-economic system \textit{n} has a \textit{direct effect} on the development indicators \textit{d} and, thus, on sustainability (cf. (12)). That is, it is not necessary that the economic system \textit{e} transmits the effects of the non-economic system \textit{n} on the development indicators \textit{d} (cf. (1) and (4)). For example, climatic conditions can have a direct effect on the development indicator ‘health’. If there exists a direct effect of the non-economic system \textit{n} on sustainability, then $\exists i \in \{1, 2, \ldots, \delta\}$ $\lim_{t \to \infty} \dot{d}_i(n)(t) \neq 0$ (cf. Section 4).

Moreover, the non-economic system \textit{n} has an impact on the parameters \textit{p} of the economic system \textit{e} (cf. (2) and (4)), which has impacts on the development indicators \textit{d} (cf. (1)) and, thus, on sustainability (cf. (12)). This is a rather \textit{indirect effect} of \textit{n} on \textit{d} and, thus, on sustainability, i.e., the effect of the non-economic system \textit{n} is transmitted via the economic system \textit{e}. Thus, this indirect effect may also be regarded as a \textit{cross-system effect}. For example, disturbances in the political system (indicated by a change in \textit{n}_i) can increase the riskiness of the returns on FDI (indicated by an increase in \textit{p}_j) and, thus, reduce the FDI-growth rate (indicated by a decrease in \textit{e}_k) and, thus, the per-capita income growth rate (indicated by a decrease in \textit{d}_l). This may endanger sustainability (of per-capita income growth). If we define $\dot{d}_{n\to e}(t) := \sum_{f=1}^\varepsilon \partial d_i(t)/\partial e_j(t) \sum_{k=1}^\eta \partial e_j(t)/\partial n_k(t) \dot{n}_k(t)$, then the existence of indirect (cross-system) effects of the non-economic system \textit{n} on the development indicators \textit{d} means that $\exists i \in \{1, 2, \ldots, \delta\}$ $\lim_{t \to \infty} \dot{d}_{n\to e}(t) \neq 0$.

Analogous statements can be made regarding the impact of the economic system \textit{e} on sustainability. According to (1), the economic system \textit{e} can have a \textit{direct effect} on the development indicators \textit{d} and, thus, on sustainability (cf. (12)). In this case, $\exists i \in \{1, 2, \ldots, \delta\}$
\( \lim_{t \to \infty} \dot{d}_e(t) \neq 0 \) (cf. Section 4). Moreover, the economic system \( e \) can have an impact on the dynamics of the non-economic system \( n \) (cf. (5)) and, thus, on the dynamics of the parameters \( p \) (cf. (4)) and, thus, on the development indicators \( d \) (cf. (1)) and, thus, on sustainability (cf. (12)), i.e., the indirect/cross-system effect of the economic system \( e \) on the development indicators \( d \) is transmitted via the non-economic system \( n \). In this case, \( \exists i \in \{1, 2, \ldots, \delta\} \lim_{t \to \infty} \dot{d}_{e \to n}(t) \neq 0 \), where \( \dot{d}_{e \to n}(t) := \sum_{j=1}^{n} \frac{\partial d_i(t)}{\partial n_j(t)} \sum_{k=1}^{e} \frac{\partial n_j(t)}{\partial e_k(t)} \dot{e}_k(t) \).

These facts imply that, even if economic development (indicated by the dynamics of \( e \)) is such that it does not directly affect the development indicators \( d \), it may, nevertheless, contribute to sustainability (cf. (12)) if it has a positive effect on the non-economic system \( n \). For example, an economic policy (e.g., environmental protection policy or income redistribution) that restructures the economic system \( e \) without increasing the per-capita income may increase the development indicators (e.g., health indicators) if it has a positive effect on the non-economic system (e.g., on the ecological system).

Overall, economic and non-economic development can contribute to sustainability via direct and indirect/cross-system channels, as summarized by Results 3 and 4.

**Result 3.** The development of the economic system \( e \) can have

(a) a direct impact on sustainability of development (cf. (12a)) by directly affecting the development indicators \( d \) via (1) such that \( \exists i \in \{1, 2, \ldots, \delta\} \lim_{t \to \infty} d_{e}(t) \neq 0 \); or

(b) an indirect (or cross-system) impact on sustainability of development (cf. (12a)) by affecting the dynamics of the non-economic system \( n \) via (5) and, thus, the dynamics of the parameters \( p \) (cf. (4)) and the development indicators \( d \) (cf. (1)) such that \( \exists i \in \{1, 2, \ldots, \delta\} \lim_{t \to \infty} \dot{d}_{n \to e}(t) \neq 0 \).

**Result 4.** The development of the non-economic system \( n \) can have

(a) a direct impact on sustainability of development (cf. (12a)) by directly affecting the parameters \( p \) via (4) and, thus, the dynamics of the development indicators \( d \) via (1) such that \( \exists i \in \{1, 2, \ldots, \delta\} \lim_{t \to \infty} d_{n}(t) \neq 0 \); or

(b) an indirect (or cross-system) impact on sustainability of development (cf. (12a)) by affecting the dynamics of the economic system \( e \) via (2) and (4) and, thus, the dynamics of the development indicators \( d \) (cf. (1)) such that \( \exists i \in \{1, 2, \ldots, \delta\} \lim_{t \to \infty} \dot{d}_{n \to e}(t) \neq 0 \).
Now, we focus on two further aspects of cross-system interactions, namely, cross-system feedbacks and intra-system interactions.

7 Feedbacks and intra-non-economic system interactions

7.1 Feedbacks

While Section 6 considers the transmission of the impacts of one system via another system (indirect/cross-system impacts), it does not consider cross-system feedbacks. In particular, if the economic system \( e \) has (indirect/cross-system) impacts on the development indicators \( d \) by affecting the non-economic system \( n \) (cf. Result 3b), the changes in \( n \) (i.e., the transmission) may have a feedback effect on the economic system \( e \) (cf. (2')) and, thus, on \( d \) (cf. (1)). This implies that the indirect effect of \( e \) on \( d \) via \( n \) (cf. Result 3b) may be moderated or amplified by the cross-system feedbacks of \( n \) on \( e \) and, thus, on \( d \). Moreover, this feedback may cause further feedbacks (i.e., feedbacks of \( e \) on \( n \)) and so forth.

Analogous statements can be formulated in the case that the non-economic system \( n \) has (indirect/cross-system) impacts on the development indicators \( d \) by affecting the economic system \( e \) (cf. Result 4b). This transmission may cause feedbacks from \( e \) to \( n \), which may moderate or amplify the original effect (i.e., Result 4b). These feedbacks may cause further feedbacks and so forth.

This discussion supports the view that sustainability analysis requires the study of the general dynamic equilibrium in the multi-system framework, since the overall impact of the feedbacks can be calculated only in this type of analysis.

7.2 Intra-non-economic system interactions

All the results of the previous sections apply equally to intra-non-economic system interactions (cf. (7)-(10)). Thus, from the system-theoretical point of view, we need not discussing these results in the context of intra-non-economic system interactions.\(^5\) However, the discussion of the intra-non-economic system interactions in the context of contemporary economic growth and development theory seems very interesting. There are economic growth and development models

---

\(^5\) See Wagner (2017b, pp. 33-37) for a discussion of intra-non-economic system interactions and their effects on economic dynamics in the context of sustainability, where the non-economic system is partitioned into three subsystems: ecological, socio-cultural, and political (cf. Habermas, 1973, and Wagner, 2014).
and studies that incorporate one or another non-economic subsystem. For example, some growth models include resource depletion and environmental pollution (i.e., interactions between the economic system and the ecological subsystem), while others include the interactions between the economic system and the socio-cultural subsystem (by, e.g., studying the impact of economic development on emancipation) or the interactions between the economic system and the political system (by analyzing, e.g., the impact of the political system on the time-preference rate, which is a parameter of most economic growth models).

While such models are highly interesting and most important contributions to development and growth theory, they cannot be regarded as the multi-system models discussed in our paper, since they study the interactions between the economic system \( e \) and a specific non-economic subsystem \( n_i \) (cf. (7) and (8)), while we study the interactions between the economic system \( e \) and the (whole) non-economic system \( n \), where the latter contains all the (relevant) non-economic subsystems (cf. (7)). This distinction is essential in the study of sustainability. If we study the interactions between the economic system \( e \) and a specific non-economic subsystem \( n_i \) and elaborate a development path/strategy that ensures that the development is sustainable within this framework, then this path/policy is not necessarily sustainable according to (12) because of intra-non-economic system interactions (and their interactions with the economic system). In particular, the dynamics of the economic system \( e \) and the dynamics of the considered non-economic subsystem \( n_i \) may have indirect effects on the development indicators \( d \) via other subsystems \( (n_j, j \neq i) \); cf. Results 3 and 4) and feedbacks with the other subsystems \( (n_j, j \neq i) \); cf. Section 7.1) such that the estimated effects of the elaborated path/strategy on the development indicators \( d \) are biased if the indirect effects and feedbacks are not considered. In other words, studies of the interactions between the economics system \( e \) and a specific non-economic subsystem \( n_i \) can only be regarded as studies of partial dynamic equilibria, while the analysis of sustainable development requires the study of the general dynamic equilibrium imposing conditions on \( n \) (i.e., on all subsystems \( n_i \)).

For example, assume that by studying the interactions between the economic system and the socio-cultural subsystem, a policy maker comes to the conclusion that a development program neglecting the socio-cultural development is sustainable because the negative effects of weak development of the socio-cultural system are outweighed by rapid growth in income such that

---

6 See Stijepic (2017) for a brief overview.
overall development can be regarded as sustainable. That is, when considering only the interactions between the economic and socio-cultural system, the elaborated policy program seems to ensure sustained development. However, if we, additionally, take the political system into account, we may come to the conclusion that the elaborated policy program endangers sustainability. In particular, the neglect of socio-cultural development may lead to establishment of backward or radical ideologies, which sooner or later may lead to a change in the political system (e.g., due to some sort of unrest or revolution) that is neither supportive of socio-cultural development nor economic development (due to wars, suppression, or international isolation).

8 Biases of the sustainability predictions of economic models
Economic models can be described by the equation system (1)-(4), where it is assumed that the parameter vector \( p \) (which is determined in the non-economic system \( n \)) is exogenous. Thus, economic models (neglecting the interdisciplinary aspects of sustainability) neglect several aspects discussed in our paper.

First, they neglect the direct effect of the non-economic system on sustainability (cf. Result 4a). In particular, an economic model neglecting these effects may either overestimate or underestimate the economic efforts that are necessary to achieve sustainable development. The efforts are underestimated if the non-economic system dynamics have a negative effect on sustainability of development (cf. Result 4a). For example, autonomous ecological system development (cf. Section 3), e.g., the climate changes that are not caused by human action, may have negative impacts on sustainability. Economic models that neglect this effect may underestimate the resources that are necessary to ensure welfare growth in the long run. On the other hand, positive autonomous change in the socio-cultural system (e.g., autonomous improvement in emancipation) may increase the welfare of the economy directly (via Result 4a). Thus, economic models that do not consider this change may overestimate the resource demands that are necessary to ensure increasing welfare in future (i.e., sustainability).

Second, economic models neglect the indirect effects of economic dynamics on the development indicators \( d \) via the non-economic system \( n \) (cf. Result 3b). These indirect effects may moderate or amplify the effect of economic system dynamics on the development indicators \( d \). For example, on the one hand, economic development (e.g., industrialization) may have a positive effect on the socio-cultural system (e.g., by supporting emancipation), which in turn may have a
positive effect on the development indicators.\textsuperscript{7} On the other hand, industrialization may have a negative impact on the ecological system and, thus, on the development indicators (e.g., health indicators).

Third, \textit{autonomous non-economic system development} can have impacts on the parameters $\mathbf{p}$ of the economic model (cf. (4)) and, thus, bias the economic model predictions regarding the resources that are necessary to ensure sustainability (cf. Result 4b) if the economic model does not incorporate these parameter changes. This problem is particularly relevant in the case of (quasi-)homeostasis of the non-economic system $\mathbf{n}$ such that (a) pure empirical information on the past dynamics of the parameters $\mathbf{p}$ reflecting the non-economic system dynamics is not reliable and (b) interdisciplinary research is necessary to exclude the possibility of (quasi-)homeostasis (cf. Stijepic, 2017). For example, if the ecological system develops autonomously (as in the case of autonomous climate change) parameters of the economic model may change (e.g., the productivity of the agricultural production may be affected by climate change). The models that do not consider such changes may overestimate or underestimate the resources that are necessary to ensure sustained development (e.g., increased input in agricultural production may be necessary due to higher risk of crop failure).

Fourth, even in absence of autonomous non-economic system development, \textit{cross-system feedbacks} (discussed in Section 7.1) may exist. Economic models that assume that the parameters $\mathbf{p}$ are exogenous neglect such feedbacks. As discussed in Section 7.1, cross-system feedbacks may amplify or moderate the impact of the economic system $\mathbf{e}$ on the development indicators $\mathbf{d}$. Thus, economic models neglecting such feedbacks generate, in general, biased predictions of sustainability.

Fifth, because of \textit{intra-non-economic system interactions}, even the models that incorporate a specific non-economic subsystem into analysis, may lead to wrong predictions of sustainability or of the programs that are needed to ensure sustainability, as discussed in Section 7.2.

\textbf{9 Special aspects of economic sustainability policy in the multi-system framework}

While Sections 4, 6, and 7 deal with different aspects of cross-system interactions, we focus now on the implications of these interactions for policy effectivity and design.

\textsuperscript{7} See Stijepic (2017) for a model of this impact channel.
Economic models deal with the question of what happens with the economic system $e$ if some parameters $p_i$ (that are controllable by the policy makers) are varied (exogenously). In contrast, in our model, these policy-parameter variations are determined endogenously. Nevertheless, we can assume for a moment that the policy parameters are exogenous and discuss their effects on the dynamics of the economic system $e$, the non-economic system $n$, the development indicators $d$, and sustainability (12). Under these assumptions, we can derive the following implications of the previous discussion for policy design.

**Implication 1.** Policy programs seeking to ensure sustainable development cannot only rely on economic model predictions regarding the effects of potential policy measures, as discussed in Section 8.

**Implication 2.** It is questionable whether there is autonomous development of the economic system (cf. Section 3). Thus, the development of the non-economic (sub)systems may be important and should not be neglected, since it may drive the development of the economic system and sustainability (cf. Result 4b).

**Implication 3.** Even policies that do not generate significant economic growth (e.g., environmental protection and resources redistribution) may have strong effects on sustainability (cf. (12)) due to the direct effects of non-economic system dynamics on the development indicators (cf. Result 4a).

**Implication 4.** Even the consideration of the direct and indirect effects (cf. Results 4a and 3b) of a policy program on/via the major non-economic subsystems (e.g., via the ecological, socio-cultural, and political subsystem) is not sufficient for ensuring the effectiveness of the program with respect to sustainability of development. The interactions between the non-economic subsystems must be analyzed as well, since they can reverse the impacts of the direct and indirect effects (cf. Section 7.2).

**Implication 5.** The policy decision and the sustainability/enforcement of a policy program are endogenous, since the political system is endogenous, i.e., the policy system depends on all the other non-economic subsystems and on the economic system. Thus, a policy program must be designed such that it ensures the enforcement of the program over the planning period ($T$), i.e., the potential feedbacks from the other (sub)system(s) on the political system that could endanger the enforceability of the program (over the planning period $T$) must be considered.
10 Concluding remarks
The starting point of our analysis is the relatively general two-system model proposed in Section 2 and the discussion of different concepts of sustainability in Section 3, where we have decided to focus on a limit-definition of sustainability (requiring that development indicators grow in the limit). In Sections 4, 6, and 7, we have elaborated on the different types of cross-system interactions existent in our model and their impacts on sustainability (direct effects, indirect effects, cross-system feedbacks, and intra-non-economic system interactions). Sections 8 and 9 were devoted to the application of these results in the discussion of economic modeling and policy design, where we (a) have shown that standard growth and development models neglect important aspects of sustainability by neglecting cross-system interactions and (b) elaborated several implications of our results for the design of sustainability policy.

While our paper is devoted to the conceptual aspects of sustainability in multi-system frameworks (and the development of a general framework for the study of this topic), each of its sections and results offers much potential for further mathematical treatment of the topic. In particular, further research could take two major directions: a system-theoretical one, where it could be tried to specify the basis model of Section 2 further such that the theorems of mathematical dynamical systems analysis become applicable, and an applied one, where concrete economic growth and development models could be generalized to multi-system models such that the aspects discussed in our paper can be analyzed in the context of such models. As demonstrated in our paper, most of the system-theoretical aspects of sustainability of economic development and growth can be studied in a two-system framework, where the economic system and the non-economic system interact. Thus, we have only briefly addressed the interactions between the non-economic subsystems (e.g., ecological, socio-cultural, and political subsystem). Nevertheless, the identification and (system-theoretical) classification of these interactions seems an interesting topic for further research.

11 References

8 See Stijepic (2017) for an example based on the AK growth model.


International Monetary Fund (2017). World Economic Outlook October 2017 - Seeking Sustainable Growth Short-Term Recovery, Long-Term Challenges.


Link: http://onlinelibrary.wiley.com/doi/10.1002/(SICI)1099-1719(199603)4:1%3C1::AID-SD24%3E3.0.CO;2-N/abstract

Link: https://doi.org/10.1007/s10584-007-9267-y

Link: http://www.sciencedirect.com/science/article/pii/S1470160X08000678

Link: http://dx.doi.org/10.2139/ssrn.2946768