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Unemployment, Income Growth and Social Security†

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Abstract

Considering the sustainability of social security in an aging society with fewer children, income growth and population growth are important factors. With a decrease in income growth or population growth, social security transfers such as pension benefits cannot be provided. The intergenerational social security benefit is being reassessed in some OECD countries. In Japan, social security benefits for younger people are small because of an aging society.

This paper presents description of an unemployment model with a minimum wage and social security benefits and presents examination of how unemployment benefits for the younger people affect income growth, fertility, and welfare. The results described herein demonstrate that unemployment benefits raise the capital stock and income level per capita. Therefore, this benefit should be provided to maintain the tax revenue for social security. Moreover, this benefit can increase social welfare.

Keywords: Minimum wage, Social security, Unemployment

JEL Classifications: E24, H55, J64

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1. Introduction

The average unemployment rate in Japan is 2.8% at 2017. In OECD countries, the unemployment rate is decreasing. The factors causing unemployment are demand deficiency and frictional unemployment. Furthermore, because of resistance to downward wage pressure from labor unions, price mechanisms do not function to achieve full employment. This creates conditions of structural unemployment.

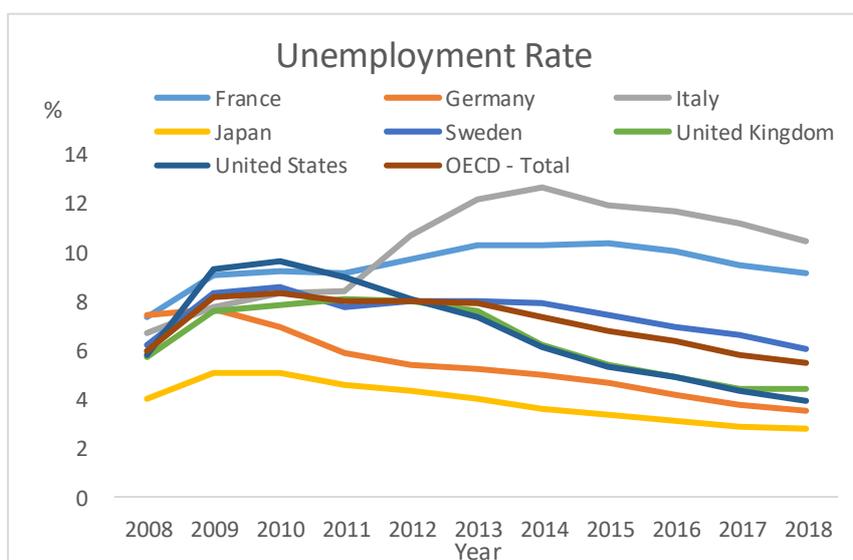


Fig. 1: Unemployment rate in OECD countries (data: OECD Statistics).

Unemployment presents important difficulties in an aging society with fewer children. The government cannot collect sufficient tax revenues to maintain social security programs if many younger people remain unemployed. Therefore, it is important to examine how unemployment occurs and how a government should address the difficulties posed by unemployment.

Many reports of the related literature describe studies of unemployment. Acemoglu (1999), Acemoglu and Shimer (1999), Burdett, Lagos and Wright (2003) and Sinko (2007), Lehmann, Parmentier and van Der Linden (2011), Gomis-Porqueras, Julien and Wang (2013) establish search models and examine how the unemployment rate is determined.

Fanti and Gori (2007, 2011) and Lee and Saez (2012) use minimum wage models to examine how unemployment is determined. In addition, these analyses derive the

optimality of the minimum wage and the respective effects on fertility and income growth. Fanti and Gori (2007) shows that the subsidy for child care increases the unemployment level in the endogenous fertility model. Fanti and Gori (2011) derives that the minimum wage with unemployment benefit raises the income growth even if the unemployment occurs. Tamai (2009) sets the political economy model to examine the income inequality, income growth and unemployment.

Corneo and Marquardt (2000), Imoto (2003), Bräuninger (2005), and Aronsson, Sjögren and Dalin (2009) consider labor union models or wage bargaining models and elucidate how unemployment occurs.

Ono (2007, 2010), after considering a model with unemployment benefit and pension benefit, examines how these policies affect income growth and the unemployment rate. Wang (2015) assesses endogenous fertility and unemployment and how they are affected by a social security system. Reportedly, for any given minimum wage, a pension might improve fertility and decrease unemployment.

We expand the model reported by Wang (2015) and construct a two-term overlapping generation model with a given minimum wage. We analyze a model that includes existing unemployment by the minimum wage and social security. The model includes the assumption of unemployment benefits and a pension for an assessment plan. Furthermore, we analyze how the unemployment benefit as redistribution policy for the young generation affects growth, the birthrate, and welfare. As a main conclusion, the existence of unemployment benefits brings an increase in capital stock and income per capita. In addition to this result, our manuscript can show that the unemployment benefit can improve the birthrate and welfare.

The organization of the remainder of this paper is the following. The next section constructs a perfectly competitive market model with an overlapping-generations model of a la Diamond (1965). Section 3 shows the equilibrium. Section 4 examines the effect of the minimum wage on the unemployment government divide income tax to education and public pension with an exogenous income tax. Section 4 illustrates the dynamics of our model. The final section concludes the paper.

2. The Model

There exist three types of agents: households, firms, and the government.

2.1 Firms

We assume the production function of

$$Y_{i,t} = K_{i,t}^\theta (A_{i,t} L_{i,t})^{1-\theta}, 0 < \theta < 1, i = 1, \dots, I_F \quad (1)$$

In that equation, $K_{i,t}$ and $L_{i,t}$ respectively denote the capital stock and labor input of the firm i in t period. $A_{i,t}$ represents the productivity or knowledge of production of firm i . $Y_{i,t}$ denotes the output of firm i . There exist the number of I_F firms. It is assumed as $A_{i,t} \equiv K_t/N_t$. This assumption is the same as that presented by Romer (1990), Fanti and Gori (2011), Grossman and Yanagawa (1993), and others.

We assume unemployment in this model economy. The labor input is given as

$$L_t = (1 - u_t)N_t, \quad (2)$$

where N_t and u_t respectively denote the population size of younger people in t period and the unemployment rate (or the length of the unemployment). In this model economy, the government sets the minimum wage system as Wang (2015). If $w_{c,t}$ is given by the wage rate of the competitive market without the minimum wage system that is, $u_t = 0$, then the minimum wage $w_{m,t}$ is given as

$$w_{m,t} = \mu w_{c,t}, 1 < \mu, \quad (3)$$

where μ denotes the level of minimum wage. Then, if we assume homogeneous firms, the wage rate $w_{m,t}$ and the interest rate are given as

$$w_{m,t} = (1 - \theta)k_t(1 - u_t)^{-\theta}, \quad (4)$$

$$\delta + r_t = \theta(1 - u_t)^{1-\theta}, 0 < \delta < 1. \quad (5)$$

We notify the aggregate production function $Y_t = K_t^\theta (A_t L_t)^{1-\theta}$, $K_{i,t} = K_{j,t} = K_t$, $L_{i,t} = L_{j,t} = L_t$ and $A_t = K_t/N_t$. Here, δ denotes the capital stock depreciation rate.

Then, with the model of the minimum wage, the employment rate $1 - u_t$ is negatively correlated with the level of μ .

$$1 - u_t = \mu^{\frac{-1}{\theta}}. \quad (6)$$

Considering (5) and (6), the interest rate is given by the fixed value as

$$r_t = \bar{r} = \theta \mu^{\frac{\theta-1}{\theta}} - \delta. \quad (7)$$

We assume $\theta \mu^{\frac{\theta-1}{\theta}} > \delta$ to be a positive value.

2.2. Household

Based on Wang (2015), we set the household behavior. The individuals of households exist in two periods: young and old period and obtain the utility from consumption and the number of children. In the young period, the younger people work or confront unemployment and obtain the wage or unemployment benefit. In the old period, the older people obtain the capital income and pension to consume goods and the services. Then, the utility function v_t is assumed as

$$v_t = \alpha \log c_{y,t} + \beta \log c_{o,t+1} + \gamma \log n_t, \quad (8)$$

where $c_{y,t}$ and $c_{o,t+1}$ respectively denote consumption in the young and old periods. The number of the children given by n_t . α, β and γ denote the parameters in $[0, 1]$. We assume $\alpha + \beta + \gamma = 1$.

The budget constraints of young and old periods are given respectively by the following equation.

$$c_{y,t} + \varepsilon n_t + s_t = I_t \equiv (1 - \tau)(1 - u_t)w_{m,t} + b_t u_t, \quad (9)$$

$$c_{o,t+1} = (1 + r_{t+1})s_t + \lambda_{t+1}. \quad (10)$$

Therein, b_t and λ_{t+1} respectively denote the unemployment benefit and pension benefit. These benefits are financed by income taxation, the tax rate of which is τ . Here, s_t denotes saving; ε is the child-care cost ($0 < \varepsilon$).

The households optimally allocate funds to maximize the utility function (8) subject to the budget constraint (9) and (10) as

$$s_t = \frac{\beta(1 + r_{t+1})I_t - (\alpha + \gamma)\lambda_{t+1}}{(1 + r_{t+1})}, \quad (11)$$

$$n_t = \frac{\gamma[(1 + r_{t+1})I_t + \lambda_{t+1}]}{\varepsilon(1 + r_{t+1})}, \quad (12)$$

$$c_{y,t} = \frac{\alpha}{\gamma} \varepsilon n_t. \quad (13)$$

2.3. Government

The government collects the income tax revenue to provide the unemployment benefit

and pension. As reported by Wang (2015), although the two types of benefits exist, the only mode of financing is income taxation. The government budget constraint is given as

$$\tau(1 - u_t)w_{m,t}N_t = b_t u_t N_t + \lambda_t N_{t-1}. \quad (14)$$

The parts of $\tau(1 - u_t)w_{m,t}N_t$, $b_t u_t N_t$ and $\lambda_t N_{t-1}$ respectively denote the tax revenue, the unemployment benefit, and the pension benefit. Based on a study by Fanti and Gori (2011), unemployment benefit b_t is assumed as

$$b_t = \varphi w_{c,t}, 0 < \varphi < 1. \quad (15)$$

Different from Wang (2015), this model setting assumes an exogenous income tax rate to provide social security benefits of some types. Omori (2009) sets the exogenous income tax rate to provide public education investment and a pension benefit.

3. Equilibrium

The dynamics is given by the capital stock accumulation. Considering $N_{t+1} = n_t N_t$, the dynamics of capital per capita is given as

$$k_{t+1} = \frac{s_t}{n_t}. \quad (16)$$

Considering (3) and (14), the pension benefit is

$$\lambda_t = \left[\tau \mu^{\frac{-1}{\theta}} - \frac{\varphi}{\mu} \left(1 - \mu^{\frac{-1}{\theta}} \right) \right] w_{m,t} n_{t-1}. \quad (17)$$

The following condition is necessary to obtain a positive value of pension benefit.

$$\mu^{\frac{-1}{\theta}} > \frac{\varphi}{\mu} \left(1 - \mu^{\frac{-1}{\theta}} \right). \quad (18)$$

This condition shows that the tax revenue per capita is larger than the unemployment benefit per capita. This condition is usual. Then, the dynamics of capital stock per capita is

$$k_{t+1} = k^* = \frac{\beta \varepsilon}{\gamma(1 + Z)}. \quad (19)$$

In that equation,

$$Z \equiv \left[\tau \mu^{\frac{\theta-1}{\theta}} - \varphi \left(1 - \mu^{\frac{-1}{\theta}} \right) \right] \frac{1 - \theta}{1 + \bar{r}}. \quad (20)$$

With (18), we obtain $Z > 0$.

4. Effects of unemployment

This section presents an examination of how the unemployment benefit affects the capital stock per capita and others. Considering (19) and $\frac{dZ}{d\varphi} < 0$, we obtain the following proposition.

Proposition 1

Unemployment benefits raise the capital stock per capita and the output per capita in the long run.

Unemployment benefits reduce the pension benefit because of the constant income tax rate. A decrease in pension benefits promotes capital accumulation. Then, the capital stock per capita rises. Considering (5), the unemployment benefit does not change the employment rate. Therefore, we obtain Prop 1. The fertility in t period is derived as

$$n_t = \frac{\overbrace{\gamma \left[(1-\tau)\mu^{\frac{\theta-1}{\theta}} + \varphi \left(1 - \mu^{\frac{-1}{\theta}} \right) \right] (1-\theta)k_t}^{\text{income in young period}}}{\varepsilon - \underbrace{\gamma Z k_{t+1}}_{\text{pension benefit}}} \quad (21)$$

$$= n(\text{Income in young period}, \text{pension benefit}).$$

The fertility in the period increases with income in the young period (wage income + unemployment benefit) and pension benefit. The unemployment benefit raises the income in the young period and reduces the pension benefit. Given $k_t > 0$, the effect of unemployment benefit on the fertility is derived as

$$\frac{dn_t}{d\varphi} = \frac{\gamma(1-\theta) \left(1 - \mu^{\frac{-1}{\theta}} \right) k_t}{\underbrace{\varepsilon [1 + (1-\beta)Z]^2}_{>0}} G. \quad (22)$$

where

$$G \equiv \left[(1+Z) - \frac{H}{1+\bar{r}} \right] [1 + (1-\beta)Z] + (1-\beta)(1+Z) \frac{H}{1+\bar{r}}$$

$$H \equiv \left[(1 - \tau)\mu^{\frac{\theta-1}{\theta}} + \varphi \left(1 - \mu^{\frac{-1}{\theta}} \right) \right] (1 - \theta) > 0,$$

Because of (18), we obtain $1 + Z > \frac{H}{1+\bar{r}}$. Furthermore, we obtain $G > 0$ and $\frac{dn_t}{d\varphi} > 0$.

Considering (19) and (20), the fertility in the long run is given as

$$n^* = \frac{\beta \left[(1 - \tau)\mu^{\frac{\theta-1}{\theta}} + \varphi \left(1 - \mu^{\frac{-1}{\theta}} \right) \right] (1 - \theta)}{1 + (1 - \beta)Z}. \quad (23)$$

The numerator of (23) increases with the unemployment benefit. The denominator of (23) decreases with the unemployment benefit because of $\frac{dZ}{d\varphi} < 0$. Therefore, n^* increases with the unemployment benefit. The dynamics reaches the steady state in $t + 1$ period if we consider t period as the initial period. Then, the following proposition can be established.

Proposition 2

The unemployment benefit raises fertility in both the short and long run.

The unemployment benefit reduces the pension benefit in the short and long run. However, the decrease in the pension benefit is smaller than an increase in the income in the young period. Therefore, fertility increases.

As the final study, we derive the effect of unemployment benefit on social welfare. We derived the social welfare at the steady state as

$$v^* = \alpha \log c_y^* + \beta \log c_o^* + \gamma \log n^*, \quad (24)$$

where c_y^* and c_o^* are the consumption in younger period and older period at the steady state, respectively.

From Prop. 2 and (13), we obtain $\frac{dn^*}{d\varphi} > 0$ and $\frac{dc_y^*}{d\varphi} > 0$. Moreover, the consumption in older period $c_{o,t+1}$ increases the unemployment benefit in the short and long run. However, the consumption in the young period at the initial t period decreases with the unemployment benefit.¹ For that reason, the following proposition can be established.

¹ See Appendix for a detailed proof.

Proposition 3

The unemployment benefit raises social welfare at the steady state.

Consumption in the young period and fertility at the steady state increase with the unemployment benefit. Because of the unemployment benefit, the pension benefit decreases and private saving rises. The latter effect is dominant. Therefore, consumption in the old period rises.²

5. Conclusions

Our paper sets a two-period overlapping generations model and examines the effect of social security benefits. The results obtained from our study are shown as follows. The unemployment benefit increases the capital per capita and output per capita. The fertility for any period rises. The welfare at the steady state can be pulled up by the unemployment benefit.

Wang (2015) mainly examines the effect of pension benefit as the redistribution policy of older people. However, our manuscript presents examination of the effect of unemployment benefit as the redistribution policy for younger people. In addition, our manuscript derives the effect on welfare, which was not examined by Wang (2015).

Our manuscript sets labor income taxation for social security. This setting is consistent with those of some OECD countries including Japan, which sets the pay-as-you-go pension. Not to say, we can consider the other types of taxation such as capital income taxation and others.

The result of our manuscript shows that the unemployment benefit does not affect the unemployment rate because of the model setting. As Fanti and Gori (2011) have reported, if a Cobb–Douglas production function considered, then the unemployment rate depends only on the level of the minimum wage.

² Prop 3 considers only the case of the steady state. In the Appendix, we examine the social welfare function in both the steady state and a transitional path.

Appendix

Appendix 1

This appendix presents examination of how the effect of the unemployment benefit affects consumption in the old period. The budget constraint of the household gives the following consumption in the old period:

$$c_{o,t+1} = (1 + r_{t+1})s_t + \lambda_{t+1} = gn_t k_{t+1} = gs_t. \quad (25)$$

In that equation,

$$g \equiv \left\{ (1 + \bar{r}) + \left[\tau \mu^{\frac{\theta-1}{\theta}} - \varphi \left(1 - \mu^{\frac{-1}{\theta}} \right) \right] (1 - \theta) \right\} = (1 + \bar{r})(1 + Z). \quad (26)$$

We obtain $\frac{dc_{ot+1}}{d\varphi} = \left(k_{t+1} \frac{dg}{d\varphi} + g \frac{dk_{t+1}}{d\varphi} \right) n_t + g k_{t+1} \frac{dn_t}{d\varphi}$ because of (25). Then, we obtain

$$\left(k_{t+1} \frac{dg}{d\varphi} + g \frac{dk_{t+1}}{d\varphi} \right) = (1 - \theta) \left(1 - \mu^{\frac{-1}{\theta}} \right) k_{t+1} \left[\frac{g}{(1 + \bar{r})(1 + Z)} - 1 \right].$$

Because of (26), we obtain $g = (1 + \bar{r})(1 + Z)$ and $\left(k_{t+1} \frac{dg}{d\varphi} + g \frac{dk_{t+1}}{d\varphi} \right) = 0$. Finally, $\frac{dc_{ot+1}}{d\varphi} = g k_{t+1} \frac{dn_t}{d\varphi}$ is obtainable. Because of $k_{t+1} = k^* > 0$ and $dn_t/d\varphi > 0$ from Prop. 2. Then, we obtain $\frac{dc_{ot+1}}{d\varphi} > 0$ in the short run and $\frac{dc_o^*}{d\varphi} > 0$ in the long run.

The consumption of older people in the initial t period is $c_{o,t} = gn_{t-1}k_t$, which depends on k_t, n_{t-1} and not φ . Then, $\frac{dc_{ot}}{d\varphi} = \frac{dg}{d\varphi} n_{t-1} k_t < 0$ is obtainable. This result illustrates that the unemployment benefit reduces both the pension benefit and the consumption of older people in t period.

(Q.E.D.)

Appendix 2

This Appendix derives the social welfare function and examines the effect of unemployment benefit. Based on van Groezen, Leers and Meijdam (2003), we define the social welfare function that the initial period is given by t period.

$$\begin{aligned} V_t &\equiv \sum_{j=t}^{\infty} \rho^{j-t} \{ \alpha \log c_{y,j-1} + \beta \log c_{o,j} + \gamma \log n_{j-1} \} \\ &= \frac{\rho}{1 - \rho} \log D + F(n_{t-1}, k_t). \end{aligned}$$

where

$$D \equiv (1 + Z) \frac{\frac{(1-\rho)(\rho+\beta)}{\rho} \left[(1-\tau)\mu^{\frac{\theta-1}{\theta}} + \varphi \left(1 - \mu^{\frac{-1}{\theta}} \right) \right] (1-\theta)}{1 + (1-\beta)Z}$$

$$F(n_{t-1}, k_t) \equiv \log n_{t-1} + (\rho + \beta) \log k_t + \frac{\rho}{1-\rho} \log(1 + \bar{r}) + \frac{\alpha}{1-\rho} \log \alpha$$

$$+ \frac{\alpha + \rho\beta}{1-\rho} \log \varepsilon + \frac{\rho(1-\rho-\beta) - \alpha}{1-\rho} \log \gamma + \frac{\rho(\rho+\beta)}{1-\rho} \log \beta.$$

$$Z \equiv \left[\tau\mu^{\frac{\theta-1}{\theta}} - \varphi \left(1 - \mu^{\frac{-1}{\theta}} \right) \right] \frac{1-\theta}{1+\bar{r}}.$$

ρ denotes the discount factor in $(0, 1)$. n_{t-1} and k_t is given at t period. Because $F(n_{t-1}, k_t)$ is independent of φ , then, with $\frac{dD}{d\varphi} > 0$, we obtain $\frac{dV_t}{d\varphi} > 0$. $\frac{(1-\rho)(\rho+\beta)}{\rho} < \Delta$ is the sufficient condition to hold $\frac{dV_t}{d\varphi} > 0$. Δ is given as

$$\Delta \equiv (1 + Z) \left\{ \frac{1-\beta}{1 + (1-\beta)Z} + \frac{1 + \bar{r}}{\left[(1-\tau)\mu^{\frac{\theta-1}{\theta}} + \varphi \left(1 - \mu^{\frac{-1}{\theta}} \right) \right] (1-\theta)} \right\}.$$

The term of $\frac{(1-\rho)(\rho+\beta)}{\rho}$ shows the effect that the unemployment benefit decreases the consumption of older people and worsens social welfare in the initial t period. However, the term Δ shows the effect by which the unemployment benefit improves the household welfare of individuals born at t or in a later period. Therefore, if the latter effect is dominant, the social welfare rises.

Our paper ignores population size N_t in deriving this result. However, we can derive population size N_t to hold this result. In such a case, the population size increases. The social welfare can be raised because the unemployment benefit raises fertility in the short and long run.

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