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Hattori, Keisuke and Zennyo, Yusuke

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Keisuke Hattori[†1] and Yusuke Zennyo[‡2]

[1]Faculty of Economics, Osaka University of Economics
[2]Graduate School of Business Administration, Kobe University

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Abstract

This paper considers the optimal pricing and diffusion of a durable good that exhibits positive network externalities, when consumers are heterogeneous with respect to their expectations about future network sizes. We consider the existence of naive consumers, as well as of sophisticated consumers who have fulfilled expectations about future network sizes. At the time of purchase, naive consumers presume that the current network size will continue over future periods. We find that the firm charges the sequential-diffusion pricing that makes sophisticated consumers function as early adopters, unless consumers quickly become bored with using the goods and/or unless the firm heavily discounts its future profits. In addition, we show that naive consumers may enjoy a greater surplus than do sophisticated consumers, implying that the firm benefits when more consumers are sophisticated. We also compare the profitability of three possible pricing strategies with different commitment powers: fixed, responsive, and pre-announced pricing.

Keywords: *durable good, network externalities, diffusion process, consumer naivete*

JEL Classification: D21, D42, L12, L14

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†E-mail: hattori@osaka-ue.ac.jp

‡E-mail: xyzennyo@b.kobe-u.ac.jp

1 Introduction

Currently, there is a growing trend for various products to be connected to the Internet, which is known as *Internet of Things (IoT)*, and the connection of these products enables consumers to benefit greatly from the other consumers who join the same network (so-called *network externalities*). A typical example is a home video game system. In the past, people enjoyed playing such games with family or friends at home and, therefore, they were not concerned with the number of other users playing the game. Recently, however, most games have been connected to the Internet, which allows players to enjoy such games with a huge number of other players all over the world.[1] These technological advancements have expanded consumers' *effective network size*, which is the size of network range where consumers actually care about the other consumers. Thus, it is becoming more important to correctly predict how many users are (or will be) playing the same game when making a purchasing decision. However, this is not an easy task. In particular, there might be considerable heterogeneity among consumers with respect to their predictive accuracies. *How will such heterogeneity affect consumer behavior, firm strategies, and the diffusion process of goods?* The main purpose of this paper is to show the optimal pricing and the diffusion process of durable goods that exhibit network externalities, when consumers are heterogeneous with respect to their predictions about future network size.

We develop a simple infinite-period model in which a monopolistic firm produces a durable good that exhibits network externalities. Consumers obtain utilities from the good for infinite periods with discounting. At the same time, they become bored with the good from the continuous use of it, that is, the good is assumed to be imperfectly durable and to depreciate. When consumers decide whether to purchase the good, they must take into account

[1] For example, in 2016, *League of Legends*, a multiplayer online video game developed by Riot Games, had over 100 million monthly active players worldwide, which outnumber the population of Germany. The League of Legends 2017 World Championship attracted approximately 60 million unique viewers. See also the article in the *Economist*: `https://www.economist.com/news/business/21726751-competitive-gaming-advances-next-level-seeking-more-sponsorships-and-fans\-computer-game`

not only the current network size but also the future expansion of the network size. Katz and Shapiro (1985), the seminal work on oligopoly pricing for goods with network externalities, assume fulfilled expectations of consumers to capture this issue in a static one-period model, in which the actual network size is indeed equal to the network size that consumers expected. In reality, however, not all consumers form rational expectations. For example, some consumers may believe that the current network size will continue as a constant over the future, whereas others, who possess greater information, can forecast the future network size perfectly. To consider such heterogeneity among consumers with respect to their beliefs about future network sizes, our model consists of two types of consumers: *sophisticated* and *naive* consumers. Sophisticated consumers have perfect foresight and, thus, can correctly anticipate how many consumers will be participating in the network in future periods. Their expectations will be fulfilled, as assumed in Katz and Shapiro (1985). On the other hand, naive consumers believe that the current network size will continue as a constant in later periods and make their purchase decisions based on such naive expectations. Note that, in this paper, consumers are homogeneous except with regard to their expectations on future network sizes. Therefore, we can show how the heterogeneity of such expectations affects the optimal pricing of a monopolistic provider and the diffusion of the product.

Within the above framework of the model, this paper first considers the simple case of *fixed pricing*, in which the monopoly firm offers its price at the initial period and commits to it being fixed. We show that, unless the consumers become easily bored with the goods and/or the firm heavily discounts its future profits, the firm embraces a *sequential-diffusion* strategy that induces sophisticated consumers to purchase at the initial period and naive consumers to follow them and purchase at the next period. Otherwise, it is optimal for the firm to adopt a *simultaneous-diffusion* strategy, under which all consumers purchase at the initial period. With the simultaneous-diffusion strategy, the firm can obtain all profits at the initial period without them being discounted. However, because the naive consumers believe, at the time of their purchase, that the current market size, which is zero, will continue over the future periods, their willingness to pay for the good would be low. Therefore, a large price discount is required to convince them to purchase along with the sophisticated consumers at the initial period. On the other hand, with the sequential-diffusion strategy, the firm can charge a higher price whereas the profit from the naive consumers will be obtained at the second period. If the higher price is charged, naive consumers do not purchase the good at the initial period. However, the sophisticated consumers rationally expect that if all the sophisticated consumers purchase at the initial period, they will be able to profit from the larger network size hereafter because the naive consumers will join the network at the next period. In sum, the advantages of the sequential-diffusion price would outweigh its disadvantages, unless the consumers become easily bored with using the goods and/or the firm significantly discounts its future profits. This result implies that whether the monopoly firm can exploit the sophisticated consumers as early adopters crucially depends on the durability of the product, the discount rates of the consumers and the firm, and the heterogeneity among consumers with respect to their expectations on the future network path.

In addition, we show an interesting result that highlights *the advantages of naïveté*: in equilibrium, sophisticated consumers never enjoy a greater surplus than do naive consumers. When the simultaneous-diffusion strategy prevails in equilibrium, both sophisticated and naive consumers derive the same surplus. However, when the sequential-diffusion strategy prevails in equilibrium, the surplus of sophisticated consumers is strictly less than that of naive consumers. With the sequential-diffusion price, sophisticated consumers can anticipate that naive consumers never join the network unless all sophisticated consumers do so. Thus, sophisticated consumers have no other way than participating in the network first to lead the subsequent entries of the naive consumers. As a result, at the initial period, sophisticated consumers have to endure the small network externalities among themselves and they become bored with the good to some extent before the network externalities are maximized at the next period. Moreover, of special interest is when consumers become easily bored. In this case, the equilibrium surplus of the sophisticated consumers converges to zero. In other words, the firm can extract surplus from them perfectly.

From the viewpoint of the firm, the above discussion indicates that, with the sequential-diffusion strategy, the firm can extract more surplus from sophisticated consumers. We demonstrate that, as the ratio of sophisticated consumers increases, the firm charges a higher price and, thus, gains greater profits. That is, the firm benefits from more consumers being sophisticated.

We extend the model in a way that the firm chooses different prices for every period to confirm the robustness of our result that the sequential-diffusion strategy is optimal for the monopolistic firm, unless consumers and/or the firm heavily discount their future payoffs. We consider two cases: (i) *responsive pricing*, where the firm adjusts the price at every period, and (ii) *pre-announced pricing*, where the firm commits to the future price path at the initial period. In both cases, we derive a qualitatively similar result regarding the optimality of sequential-diffusion pricing. Furthermore, we confirm that Coase's conjecture holds: committing to the future price path can raise the profits of the firm.

Finally, given that the sequential-diffusion strategy is optimal for the firm, the following question may arise: *What is the best commitment strategy?* The answer is simple. Committing to the future price path (i.e., pre-announced pricing) is naturally the first-best strategy. However, it would be difficult to make such a full commitment in reality. Therefore, next question concerns the second-best commitment strategy: *Which is more profitable for the firm, committing to a fixed price or not?* We show that the answer depends on the rate at which consumers become bored with the good and on the discount factor for consumers. If the firm does not commit to a fixed price, sophisticated consumers have a greater incentive to postpone their purchase to the next period because doing so enables them to win a price discount and enjoy the full network from their first period of use. Therefore, by committing to a fixed price, the firm can effectively extract a larger surplus from sophisticated consumers. However, this does not apply when consumers heavily discount their future payoffs and/or when the effective network size is very small. In these cases, it would be profitable not to commit to a fixed price.

1.1 Related literature

The existence of strategic forward-looking consumers is empirically reported (e.g., Hendel and Nevo, 2013; Li et al., 2014).[2] Hendel and Nevo (2013) estimate the fraction of forward-looking consumers, who can stockpile the goods. Li et al. (2014) estimate that 5.2% to 19.2% of the population is strategic, using data from the air-travel industry. Although the structural model that these papers utilize does not incorporate network externalities, they provide empirical evidence that both strategic and non-strategic consumers exist in the market. The strategic relationship between firms and strategic consumers has been studied using theoretical frameworks of management science (e.g., Besanko and Winston, 1990; Besbes and Lobel, 2015; Cachon and Feldman, 2015; Cachon and Swinney, 2009; Jerath et al., 2010; Liu and Van Ryzin, 2008; Mersereau and Zhang, 2012).[3] Regarding the effect of strategic consumers on firms' profits, mixed results are obtained: Su (2007) and Cho et al. (2009) find that there is a positive effect, whereas Anderson and Wilson (2003), Aviv and Pazgal (2008), and Levin et al. (2009) reveal negative aspects. However, these papers do not examine network externalities.

Ajorlou et al. (2016) studies the model of optimal dynamic pricing with *social network*, where information about goods will be spread through *word-of-mouth* communications among consumers. Papanastasiou and Savva (2017) show that the presence of *social learning* may improve the expected profit of firm. In these studies, even though there do not exist network externalities, consumers form their beliefs about the product's quality from word-of-mouth communications or reviews by those who have already purchase. Therefore, the presence of social network or social learning incentivizes the firm to increase the number of early

[2]Hendel and Nevo (2006) and Nair (2007) assume that all consumers are forward-looking. In contrast, Li et al. (2014) do not assume the existence of strategic consumers a priori.

[3]The revenue losses from ignoring strategic consumers are estimated to vary from 20% in Aviv and Pazgal (2008) to 60% in Besanko and Winston (1990).

adopters, which operates similarly to network externalities of our model.

The present paper is also related to the vast economic literature on network externalities, pioneered by Katz and Shapiro (1985), who studied a static model in which consumers have rational expectations about the network size. In their rational expectations equilibrium, the actual network size does indeed equal the network size that all consumers expected. Because all consumers are assumed to have identical expectations regarding the network size and identical valuations of the good, the model can be reduced to a one-period static model. Some studies that incorporate heterogeneity among consumers with respect to their valuations for the good show that the price of durable goods with network externalities increases over time, contrary to Coase (1972), that is, *introductory* or *penetration pricing* prevails in equilibrium[4] (e.g., Bensaid and Lesne, 1996; Cabral et al., 1999; Cabral, 2011; Fudenberg and Tirole, 2000).[5] These papers assume that all consumers base their purchase decisions on rational expectations.

On the other hand, there are some studies that incorporate consumers who lack the ability to form forward-looking expectations, such that the utility of every consumer depends on the number of users at the moment of purchase, and is not affected by the future network size. (e.g., Arthur, 1989; Chen et al., 2009; Doganoglu, 2003; Makhdoumi et al., 2017; Mitchell and Skrzypacz, 2006).[6] However, there has been little investigation of the dynamic pricing when consumers are heterogeneous with respect to their expectations about the future network size of a durable good. Radner et al. (2014) assume the existence of *bounded rational* consumers who do not pay attention to the monopolist's price announcement, that is, their decisions remain unchanged from the previous period. They show that an optimal

[4]Katz and Shapiro (1985, 1986) and Farrell and Saloner (1985) show that firms offer a low introductory price in equilibrium. In these papers, however, the low price set at the initial period might be caused by competition among firms that struggle to establish installed base customers in advance of their rival firms.

[5]Fudenberg and Tirole (2000) assume that consumers purchase the good only when they are young, to avoid discussing Coasian dynamics. Bensaid and Lesne (1996) allow consumers to choose the timing of purchase in a discrete time model.

[6]Arthur (1989) considers an adoption process for technology, where agents sequentially come into the market and choose which technology to adopt. However, optimal pricing is not investigated, that is, it is assumed that the technologies are not sponsored or strategically manipulated by any profit-maximizing firm.

pricing plan monotonically decreases (increases) to the steady-state price level if the initial subscription rate is above (below) the subscription rate at the steady state. Miao (2010) considers two types of consumers: foresighted and myopic consumers. Foresighted consumers maximize the discounted sum of their utilities, whereas myopic consumers choose to optimize period-by-period. The market they have in mind is for a system good, consisting of two complementary goods (e.g., a printer and cartridges). Consumers are assumed to live for three periods. At the initial period, the consumers choose which printer to purchase. At the remaining two periods, they will consume cartridges only. Unlike the myopic consumers of Miao (2010), we consider *naive* consumers according to Cabral (2011), who defines the naive consumer as 'one who assumes that network size will remain at its current level, i.e. a consumer who fails to "solve" the model and correctly predict the evolution of network size.'

In sum, to our knowledge, no paper provides theoretical predictions on optimal pricing and the diffusion process for durable goods with network externalities when there are naive consumers as well as forward-looking rational consumers.

The rest of this paper proceeds as follows. Section 2 describes the model. Section 3 analyzes the optimal pricing and the diffusion process when the monopoly firm commits to a fixed price. We show that the firm charges the sequential-diffusion price at which sophisticated consumers purchase the good before naive consumers, unless both the firm and the consumers discount their future payoffs too heavily. Section 4 considers responsive and pre-announced pricing strategies. Section 5 considers the first- and second-best commitment strategies and examines the condition under which committing to a fixed price is more profitable for the firm than responsive pricing. We find that whether committing to a fixed price is profitable depends on the parameter values. Section 6 provides a discussion on the implications for management. Section 7 concludes the paper. Proofs are deferred to the Appendix.

2 Model

We consider an infinite-period model in which a monopolistic firm sells a durable good that exhibits positive network externalities. Consumers obtain utilities for infinite periods with discounting. The utility of consumers increases with the number of other consumers using the good. The firm sets the price in the initial period, $t = 1$, and commits to it for the subsequent periods to maximize the present discounted value. We will relax this assumption in Section 4 such that the firm can set different prices in each period. We denote the discount factor for the firm by $\delta_F \in [0, 1)$. We assume a constant marginal cost of producing the good, which is normalized to zero.

Each consumer derives a stand-alone benefit from using the good for each period, which can be normalized to zero without loss of generality. This paper focuses on the utility from network externalities, which is denoted by $v(n_t)$ ($v' > 0$ and $v(0) = 0$), where n_t is the non-negative integer representing the actual number of consumers who join the network at period t. At the beginning of each period, consumers can observe the prior period's network size (or the total number of sales) and then choose whether to purchase the good. At the time of purchasing, each consumer forms an expectation about the continuation path of future network sizes. Katz and Shapiro (1985) assume that consumers' expectations are fulfilled, such that the actual network size does indeed equal the network size that consumers expected ex ante. In reality, however, not all consumers form expectations that will be fulfilled. Some consumers might believe erroneously that the network will remain at its current size during future periods, whereas others correctly anticipate future increases in network sizes.

To investigate the diffusion of durable goods in more realistic situations, this paper considers two types of consumers: *sophisticated* consumers and *naive* consumers. We use n_S and n_N to denote the number of sophisticated and naive consumers, respectively ($n = n_S + n_N$). The two types of consumers are identical except for their expectations. Sophisticated consumers have perfect foresight and, thus, can correctly anticipate how many consumers will be participating in the network for subsequent periods. That is, they can rationally forecast the number of consumers at period t to be n_t for all $t \in T = \{1, 2, \cdots\}$. Let $u_S^e(t)$ and $u_S(t)$, respectively, be the expected and actual present discounted sum of utilities of the sophisticated consumers, who purchase (and start using) the good at (from) period t. In our model, the expected and the actual present discounted values of the sophisticated consumers are equal (i.e., $u_S^e(t) = u_S(t)$). Then, we have:

$$u_S(t) = u_S^e(t) = \sum_{k=t}^{\infty} \left[\{\delta_C(1-\beta)\}^{k-t} \cdot v(n_k) \right],$$

where $\delta_C \in [0, 1)$ is the discount factor for consumers and $\beta \in [0, 1)$ represents the depreciation rate of the durable good, as in Bond and Samuelson (1984) and Suslow (1986).[7] The depreciation rate β can also be interpreted as the rate at which consumers *become bored* with the good. For example, in the video game industry, players become bored with a game after continuous usage, even if the game maintains a huge base of players.

In contrast, naive consumers do not (and cannot) anticipate the network sizes at any future period and simply presume that the current network size will continue.[8] Therefore, the naive consumers decide whether to purchase the good based on the current network size that they perceive at the time of purchase. Let $u_N^e(t)$ and $u_N(t)$, respectively, be the expected and actual present discounted sum of utilities of the naive consumers, who purchase (and start using) the good at (from) period t. These are given as follows.

$$\begin{aligned} u_N(t) &= \sum_{k=t}^{\infty} \left[\{\delta_C(1-\beta)\}^{k-t} \cdot v(n_k) \right] \\ u_N^e(t) &= \sum_{k=t}^{\infty} \left[\{\delta_C(1-\beta)\}^{k-t} \cdot v(n_{t-1}+1) \right] \end{aligned}$$

At the beginning of period t, each naive consumer recognizes the current network size to be n_{t-1} and anticipates that it will be $n_{t-1} + 1$ for the subsequent periods if he/she purchases at that period. Because all naive consumers make the same purchase decision (i.e., $n_k \geq$

[7] Considering δ_C and β separately is important when analyzing the consumers' decisions on whether to postpone their purchases, as analyzed in Section 4.1.

[8] A naive consumer is defined as 'one who assumes that network size will remain at its current level, i.e. a consumer who fails to "solve" the model and correctly predict the evolution of network size.' (Cabral, 2011: p.95)

$n_{t-1}+1$ for all $k \geq t$), it always holds that $u_N^e(t) \leq u_N(t)$, which means that they could eventually obtain a higher value than their willingness to pay they expected ex ante. Let by $V_i(t) = u_i(t) - p$ denote the indirect utility function and let $V_i^e(t) = u_i^e(t) - p$ denote the expected indirect utility function for $i = \{S, N\}$, where p is the price of the durable good set by the firm. Henceforth, for simplicity, we define the consumers' combined discount factor as $\lambda_C \equiv \delta_C(1-\beta) \in [0,1)$.

3 Monopolist commits to a fixed price

This section derives the optimal *fixed pricing* for the durable good with network externalities in equilibrium. In our model, the firm's optimal pricing strategy is one of the following two strategies. The first option is a *simultaneous-diffusion strategy*, in which the firm sets a price such that both sophisticated and naive consumers will simultaneously purchase the good at period 1. The second option is a *sequential-diffusion strategy*, in which the firm offers a price such that sophisticated consumers purchase at period 1 and act as early adopters, whereas naive consumers purchase the good at period 2, following the lead of the sophisticated consumers.

The reason why other strategies are not optimal is as follows. First, there will never be a situation of diffusion in which naive consumers purchase the goods before the sophisticated consumers do. Suppose that the firm offers a price such that naive consumers are willing to purchase. Under that price, sophisticated consumers would form a higher present discounted value than would naive consumers because they can anticipate that all naive consumers will purchase the good. Thus, sophisticated consumers will purchase the good first and they never purchase the good after naive consumers. Next, there will never be a situation of diffusion in which only the sophisticated consumers purchase and no naive consumer purchases. Suppose that there exists a price such that only sophisticated consumers would purchase. Note that this price needs to satisfy $p \leq \sum_{k=t}^{\infty} \left[\lambda_C^{k-t} \cdot v(n_S)\right]$. At the next period, naive consumers will observe that all sophisticated consumers have purchased the good. Here, the expected present discounted value of naive consumers is computed as $\sum_{k=t}^{\infty}\left[\lambda_C^{k-t}\cdot v(n_S+1)\right]$, which is strictly greater than the price offered by the firm. Therefore, the naive consumers will follow the sophisticated consumers after a delay of one period.

We will derive the optimal price among all simultaneous-diffusion strategies in Section 3.1 and among all sequential-diffusion strategies in Section 3.2. Section 3.3 addresses the optimal strategy for the firm by comparing the profits derived in both strategies.

3.1 Simultaneous-diffusion strategy

To diffuse the good to both sophisticated and naive consumers at the same time, the price needs to be less than or equal to the expected present discounted sum of utilities of all consumers. That is, letting p^{sim} be the simultaneous-diffusion price, the price should satisfy the following condition.

$$\begin{aligned} p^{sim} &\le \min\left\{u_S^e(t=1|p^{sim}), u_N^e(t=1|p^{sim})\right\} \\ &= \min\left\{\sum_{t=1}^{\infty}\lambda_C^{t-1}v(n), \sum_{t=1}^{\infty}\lambda_C^{t-1}v(1)\right\} = \min\left\{\frac{v(n)}{1-\lambda_C}, \frac{v(1)}{1-\lambda_C}\right\} = \frac{v(1)}{1-\lambda_C} \end{aligned}$$

The price is set at the expected present discounted sum of utilities of the naive consumers, that is, $p^{sim} = v(1)/(1-\lambda_C)$. Contrary to the expectations of the naive consumers, all consumers actually join the network at period 1. Therefore, the ex-post or actual surplus of consumers is computed as follows.

$$V_i^{sim} = \sum_{t=1}^{\infty}\left[\lambda_C^{t-1}v(n)\right] - p^{sim} = \frac{v(n)-v(1)}{1-\lambda_C} \qquad (i = S, N)$$

The profit of the firm can be derived as $nv(1)/(1-\lambda_C)$. The following proposition summarizes the results.

Proposition 1. *The optimal simultaneous-diffusion price such that both sophisticated and naive consumers purchase at period 1 is as follows:*

$$p^{sim} = \frac{v(1)}{1-\lambda_C}$$

The corresponding profit and consumer surplus, respectively, are as follows:

$$\pi^{sim} = \frac{nv(1)}{1-\lambda_C}, \quad V_S^{sim} = V_N^{sim} = \frac{v(n)-v(1)}{1-\lambda_C}$$

Proposition 1 implies that the firm sets the price at the expected present discounted sum of utilities of the naive consumers in order to diffuse the good to all consumers at period 1. It would be necessary to aggressively discount the price to meet the preferences of the naive consumers.

In addition, we have the following corollary regarding consumer surplus.

Corollary 1. *Both sophisticated and naive consumers derive the same surplus under the simultaneous-diffusion pricing.*

3.2 Sequential-diffusion strategy

Here, we consider the optimal sequential-diffusion prices such that sophisticated consumers purchase the good before naive consumers do.[9] Let p^{seq} be the sequential-diffusion price, which should satisfy the following two conditions.

$$u_N^e(t=1|p^{seq}) < p^{seq} \leq u_S^e(t=1|p^{seq}) \tag{1}$$

$$p^{seq} \leq u_N^e(t=2|p^{seq}) \tag{2}$$

Inequality (1) implies that only sophisticated consumers purchase the good at period 1. Inequality (2) guarantees that naive consumers will purchase at period 2 after observing that all sophisticated consumers purchased at period 1. Given that p^{seq} satisfies both inequalities, $u_S^e(t=1|p^{seq})$, $u_N^e(t=1|p^{seq})$, and $u_N^e(t=2|p^{seq})$ can be computed as follows.

$$u_S^e(t=1|p^{seq}) \quad = \quad v(n_S) + \sum_{t=2}^{\infty}\left[\lambda_C^{t-1}v(n)\right] = v(n_S) + \frac{\lambda_C v(n)}{1-\lambda_C}$$

[9] Note that even if naive consumers purchase later than the sophisticated consumers do, their purchase cannot be delayed by more than one period. That is, naive consumers purchase the good at just the next period following the purchase by the sophisticated consumers. In our model, naive consumers are assumed to make their purchase decisions based on the network size, formed at the beginning of the period. Suppose that sophisticated consumers purchase the good at period 1. If no naive consumers purchased at period 2, it might also be unprofitable for them to purchase at period 3 and later periods.

$$
\begin{aligned}
u_N^e(t=1|p^{seq}) &= \sum_{t=1}^{\infty}\left[\lambda_C^{t-1}v(1)\right] = \frac{v(1)}{1-\lambda_C} \\
u_N^e(t=2|p^{seq}) &= \sum_{t=2}^{\infty}\left[\lambda_C^{t-2}v(n_S+1)\right] = \frac{v(n_S+1)}{1-\lambda_C}
\end{aligned}
$$

The firm offers the highest price that satisfies inequalities (1) and (2), which is equivalent to $\min\{u_S^e(t=1|p^{seq}), u_N^e(t=2|p^{seq})\}$. We can state the following proposition regarding the optimal prices among the sequential-diffusion prices.

Proposition 2. *The optimal sequential-diffusion prices such that sophisticated consumers purchase before naive consumers do is given by:*

$$
p^{seq} = \begin{cases} v(n_S) + \dfrac{\lambda_C v(n)}{1-\lambda_C} & \text{if} \quad 0 \le \lambda_C \le \bar{\lambda}_C, \\ \dfrac{v(n_S+1)}{1-\lambda_C} & \text{if} \quad \bar{\lambda}_C \le \lambda_C < 1, \end{cases}
$$
$$
\textit{where} \quad \bar{\lambda}_C = \frac{v(n_S+1)-v(n_S)}{v(n)-v(n_S)}.
$$

The corresponding profit and consumer surplus, respectively, are as follows.

$$
\begin{aligned}
\pi^{seq} &= p^{seq}\cdot n_S + \delta_F \cdot p^{seq}\cdot n_N \\
V_S^{seq} &= \begin{cases} 0 & \text{if} \quad 0 \le \lambda_C \le \bar{\lambda}_C \\ \dfrac{\lambda_C\{v(n)-v(n_S)\} - \{v(n_S+1)-v(n_S)\}}{1-\lambda_C} & \text{if} \quad \bar{\lambda}_C \le \lambda_C < 1 \end{cases} \\
V_N^{seq} &= \begin{cases} v(n)-v(n_S) & \text{if} \quad 0 \le \lambda_C \le \bar{\lambda}_C \\ \dfrac{v(n)-v(n_S+1)}{1-\lambda_C} & \text{if} \quad \bar{\lambda}_C \le \lambda_C < 1 \end{cases}
\end{aligned}
$$

Note that, under this pricing strategy, there is no incentive for any sophisticated consumer to delay purchase to the next period. The price is set at a level at which naive consumers purchase the good at period 2 only if they observe that all sophisticated consumers have already purchased. If a sophisticated consumer deviated from buying the good at period 1, no naive consumers would purchase at period 2. Therefore, the deviation never enables each sophisticated consumer to enjoy the largest network size from the start of his/her use of the good, and never improves his/her payoffs.

Proposition 2 implies that the optimal price depends on the combined discount factor for consumers λ_C. When consumers do not heavily discount and/or do not quickly become bored of using the good (i.e., $\lambda_C > \bar{\lambda}_C$), the firm offers a price equal to the willingness to pay of naive consumers at period 2, given that sophisticated consumers have already purchased, that is, $p^{seq} = u_N^e(t = 2|p^{seq})$. Note that the willingness to pay of each naive consumer is formed based on the belief that his/her purchase will increase the network size from n_S to $n_S + 1$ because the naive consumers cannot anticipate that other naive consumers also join the network at the same time. In contrast to their forecast, all naive consumers do join the network eventually. Thus, although the price is set at the willingness to pay of the naive consumers, they can eventually derive a positive surplus. In sum, both sophisticated and naive consumers enjoy a positive surplus.

Of special interest is the case in which consumers heavily discount and/or quickly become bored (i.e., $\lambda_C < \bar{\lambda}_C$). In this case, as $u_S^e(t = 1|p^{seq}) < u_N^e(t = 2|p^{seq})$, the firm offers a price equal to the willingness to pay of the sophisticated consumers at period 1, that is, $p^{seq} = u_S^e(t = 1|p^{seq})$. Because their willingness to pay is based on their perfect forecast of the future network sizes, unlike naive consumers, the surplus of the sophisticated consumers becomes zero. On the other hand, naive consumers purchase the good after observing that all sophisticated consumers have purchased at period 1, which implies that a sufficient size of network has been established. Therefore, only naive consumers can gain a positive surplus. Note that the parameter range narrows as the effective network size increases, that is, $\bar{\lambda}_C$ goes to 0 as n increases. As n gets larger, at period 2, there is a larger difference between the network size that the naive consumers expect, n_S+1, and the actually formed network size, n. In other words, naive consumers significantly underestimate the discounted sum of utilities from the network, that is, $u_N^e(t = 2|p^{seq})$ becomes smaller compared with $u_S^e(t = 1|p^{seq})$. Therefore, when the effective network size is large, $u_S^e(t = 1|p^{seq}) < u_N^e(t = 2|p^{seq})$ tends not to be satisfied, which leads to a small value of $\bar{\lambda}_C$.

We have the following corollary regarding the consumer surplus.

Corollary 2. *Naive consumers derive a higher surplus than do sophisticated consumers with the sequential-diffusion price. That is, it always holds that* $V_S^{seq} < V_N^{seq}$.

The intuition behind the advantage of naïveté in Corollary 2 is that sophisticated consumers have to endure the disadvantages of the small network at period 1, whereas naive consumers can enjoy benefits of the full network from the beginning of their use of the product.

3.3 Optimal diffusion strategy

Here, using the results derived in Sections 3.1 and 3.2, we address the optimal pricing for a monopolistic firm selling a durable good that exhibits network externalities. First, comparing the simultaneous- and sequential-diffusion prices yields the following lemma.

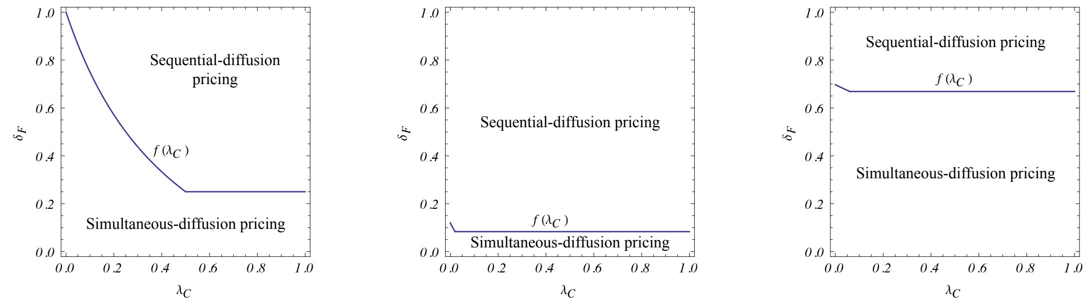
Lemma 1. *The sequential-diffusion price is always higher than the simultaneous-diffusion price. That is,* $p^{sim} < p^{seq}$ *holds.*

Lemma 1 confirms the predictable result that the firm can charge a higher price to diffuse the good sequentially. Next, comparing the profits with simultaneous- and sequential-diffusion prices, we have the following proposition regarding the optimal pricing of the firm.

Proposition 3. *It is optimal for the monopolistic firm to charge the sequential-diffusion price* p^{seq} *if and only if the discount factor for the firm is large enough to satisfy* $\delta_F > \max\{f(\lambda_C)\ ,\ 0\}$*, where*

$$f(\lambda_C) = \begin{cases} \dfrac{nv(1) - n_S\{\lambda_C v(n) + (1-\lambda_C)v(n_S)\}}{n_N\{\lambda_C v(n) + (1-\lambda_C)v(n_S)\}} & \text{if} \quad 0 \le \lambda_C \le \bar{\lambda}_C, \\ \dfrac{nv(1) - n_S v(n_S+1)}{n_N v(n_S+1)} & \text{if} \quad \bar{\lambda}_C \le \lambda_C < 1. \end{cases}$$

Otherwise, it is optimal to charge the simultaneous-diffusion price p^{sim}.



(i) $n_S = 1$; $n_N = 2$; $v(n_t) = n_t$.

(ii) $n_S = 5$; $n_N = 50$; $v(n_t) = n_t$.

(iii) $n_S = 5$; $n_N = 50$; $v(n_t) = n_t^{0.2}$.

Figure 1: Simultaneous- vs. sequential-diffusion pricing

Note that $f(\lambda_C)$ is decreasing for $\lambda_C \in [0, \bar{\lambda}_C]$ and is constant for $\lambda_C \in [\bar{\lambda}_C, 1]$. Figure 1 shows the partition of the equilibrium price by the firm using numerical analysis.

All three panels of the figure show that the firm offers the sequential-diffusion price when δ_F is larger than the critical value $f(\lambda_C)$, indicating that the sequential-diffusion strategy is more profitable for the firm when the firm's discount factor is sufficiently large. It seems to be a natural result. With the simultaneous-diffusion strategy, all profits can be obtained without being discounted, but a large price discount is required to make naive consumers purchase along with sophisticated consumers, as shown in Lemma 1. In contrast, with the sequential-diffusion strategy, the profit from naive consumers will be obtained at period 2 with being discounted. However, as shown in Lemma 1, the firm can charge a higher price because of the leadership of sophisticated consumers. In addition, the left and center panels of the figure illustrate the case of $v(n_t) = n_t$, indicating that the sequential-diffusion pricing would be optimal for the firm if the effective network size is sufficiently large. The left panel shows the case where the effective network size is small, indicating that the simultaneous-diffusion strategy is more likely to be beneficial if the combined discount factor of the consumer is small. Finally, the right panel illustrates the case where $v(n_t) = n_t^{0.2}$, implying weak network externalities. A comparison of the center and right panels of the figure indicates that the stronger is the network effect of the durables, the greater is the advantage of the sequential-diffusion strategy for the firm.

Furthermore, with Corollaries 1 and 2, we obtain the following proposition with respect to the surpluses of both consumer types.

Proposition 4. *Sophisticated consumers never enjoy a greater surplus than do naive consumers.*

Under the simultaneous-diffusion strategy, both types of consumers derive the same surplus because they purchase the good at the same price and at the same time. More interestingly, under the sequential-diffusion strategy, the surplus of sophisticated consumers is less than that of naive consumers. With the sequential-diffusion price, naive consumers never join the network before sophisticated consumers (the early-adopters) do so. Because sophisticated consumers expect such behavior of the naive consumers, they have no option but to participate in the network first. As a result, sophisticated consumers have to endure the disadvantages of a small network at period 1. However, when the naive consumers purchase at the next period, they can immediately enjoy the advantages of the largest network size. Therefore, naive consumers obtain agreater surplus than do sophisticated consumers.

Let us reconsider the above discussion from the viewpoint of the firm. Under the simultaneous-diffusion strategy, the firm extracts the same surplus from every consumer. On the other hand, under the sequential-diffusion strategy, the firm can extract more surplus from the sophisticated consumers by charging a higher price to all consumers. In summary, we can state the following proposition regarding the preceding discussion.

Proposition 5. *Assume that the total number of consumers n is constant. Let α be the ratio of sophisticated consumers, that is, $n_S = \alpha n$ and $n_N = (1-\alpha)n$. Then, the simultaneous-diffusion price p^{sim} does not depend on α, whereas an increase in α raises the sequential-diffusion price p^{seq} and the profit of the firm π^{seq}.*

Proposition 5 provides the comparative statics of the firm's profit with respect to the ratio between the two types of consumers. It shows that the simultaneous-diffusion price does not depend on the ratio between the sophisticated and naive consumers. However, under the sequential-diffusion strategy, as the ratio of sophisticated consumers increases, the firm charges them the higher price and then gains a greater profit. That is, the firm benefits from more consumers being sophisticated.

4 Responsive and pre-announced pricing strategies

The purpose of this section is to confirm the robustness of Proposition 3, that the sequential-diffusion strategy prevails in equilibrium unless the firm heavily discounts its future profits, even if the firm can set different prices at each period. Section 4.1 considers the case of *responsive pricing*, in which the firm can dynamically adjust the price at every period. We also consider the case of *pre-announced pricing* in Section 4.2, in which the firm can commit to the future price path that it will charge. After confirming the robustness of our main result in both cases, in Section 4.3, we confirm that Coase's prediction, that committing to a future price path improves the firm's profit, holds in our model.

4.1 Responsive pricing

Let p_t be the price of the good at period t. As in Section 3, we focus on the simultaneous- and sequential-diffusion strategies. Note that the ability of the firm to change its prices across periods (or equivalently, the lack of a price commitment) does not affect the analysis of the simultaneous-diffusion strategy, under which all consumers purchase the good at period 1. Therefore, we analyze the sequential-diffusion strategy when the firm can adjust its price at every period and then find the condition under which such a sequential-diffusion strategy is optimal for the firm.

Proposition 6. *If the consumers' discount factor and the depreciation rate are small enough to satisfy:*

$$\delta_C \beta < \frac{v(n_S) - v(1)}{v(n) - v(n_S)}, \tag{3}$$

then there exists the price pair $(\hat{p}_1^{seq}, \hat{p}_2^{seq})$ *such that sophisticated consumers purchase at* *period 1 and naive consumers purchase at period 2, where* $\hat{p}_1^{seq}$ *and* $\hat{p}_2^{seq}$ *are given by*

$$\hat{p}_1^{seq} = \frac{v(n_S) - \delta_C \beta \{v(n) - v(n_S)\}}{1-\lambda_C}, \quad \hat{p}_2^{seq} = \frac{v(n_S+1)}{1-\lambda_C}.$$

The corresponding profit and consumer surplus, respectively, are as follows.

$$\hat{\pi}^{seq} = \hat{p}_1^{seq} n_S + \delta_F \cdot \hat{p}_2^{seq} n_N, \quad \hat{V}_S^{seq} = \delta_C \cdot \frac{v(n) - v(n_S)}{1-\lambda_C}, \quad \hat{V}_N^{seq} = \frac{v(n) - v(n_S+1)}{1-\lambda_C}.$$

Equation (3) represents the condition under which the sequential-diffusion strategy prevails in equilibrium. Compared with the fixed-pricing case, in this case, sophisticated consumers have a greater strategic incentive to postpone their purchases in order to match the timing of purchase with naive followers because doing so could reduce the price offered at period 2. Details are provided in the Appendix, but let us explain the intuition briefly as follows. Proposition 6 shows that $\hat{p}_1^{seq}$ is decreasing in β, that is, the period 1 price decreases as consumers become easily bored. When consumers become easily bored with a product, sophisticated consumers have a greater incentive to start using the good when the network is at its largest possible size. In other words, each sophisticated consumer would be likely to delay purchase to the next period in which all the naive consumers will join the network. As a result of these deviation incentives, the firm cannot charge the higher price at period 1. On the other hand, if the firm offers too low a price, naive consumers will also purchase the good at period 1. Therefore, under responsive pricing, the firm cannot perform the sequential-diffusion strategy when consumers become easily bored with the product.

Next, we derive the condition under which the sequential-diffusion strategy is more profitable for the firm when it can adjust its price at every period, as summarized in the following proposition.

Proposition 7. *Suppose that the monopolistic firm can adjust its price at every period. It is optimal for the firm to charge the sequential-diffusion prices* $(\hat{p}_1^{seq}, \hat{p}_2^{seq})$ *if and only if the discount factor for the firm is large enough to satisfy* $\delta_F > \max\left\{\hat{f}(\delta_C, \beta)\ ,\ 0\right\}$*, where*

$$\hat{f}(\delta_C, \beta) \equiv \frac{nv(1) - n_S\left\{v(n_S) - \delta_C \beta\left(v(n) - v(n_S)\right)\right\}}{n_N \cdot v(n_S+1)}.$$

Proposition 7 implies that it is more profitable for the firm to charge the sequential-diffusion prices unless the discount factor for the firm is too small. Therefore, the result of Proposition 3 is qualitatively unchanged even if the firm can adjust its price at every period.

Proposition 7 also implies that $\partial \hat{f}(\delta_C, \beta)/\partial \delta_C = \partial \hat{f}(\delta_C, \beta)/\partial \beta > 0$. With responsive pricing, the sequential-diffusion pricing is more likely to be beneficial to the firm when consumers discount the future payoff more (i.e., when δ_C gets smaller) and/or when consumers become bored less easily (i.e., when β gets smaller). In contrast, with fixed pricing, we have $\partial f(\lambda_C)/\partial \delta_C \leq 0$ and $\partial f(\lambda_C)/\partial \beta \geq 0$. That is, the change in δ_C has the opposite impact on the thresholds under the fixed and responsive pricing strategies. An increase in δ_C directly increases the present discount value and the willingness to pay of all consumers. Under fixed pricing, this direct effect enables the firm to charge the higher sequential-diffusion price because the firm commits to the fixed price such that no sophisticated consumers benefit by delaying their purchase. However, if the firm cannot make such a binding commitment but adopts responsive pricing, the increase in δ_C also affects has the behavior of sophisticated consumers. As δ_C becomes larger, the sophisticated consumers have a greater incentive to delay their purchases to the next period, which would lead the firm to offer a larger price discount. As a result, under responsive pricing, the second indirect effect dominates the first direct effect. That is why an increase in δ_C makes the sequential-diffusion strategy less profitable for the firm.

4.2 Pre-announced pricing

Here, we consider the case in which the firm can perfectly commit to a future price path. For the same reason as in Section 4.1, we first analyze the sequential-diffusion strategy and then find the condition under which the sequential-diffusion strategy is optimal for the firm.

Proposition 8. *When the monopolist can commit to a future price path, the optimal price pair* $(\tilde{p}_1^{seq}, \tilde{p}_2^{seq})$*, such that sophisticated consumers purchase at period 1 and naive consumers* *purchase at period 2, is given by:*

$$\tilde{p}_1^{seq} = v(n_S) + \frac{\lambda_C v(n)}{1-\lambda_C}, \quad \tilde{p}_2^{seq} = \frac{v(n_S+1)}{1-\lambda_C},$$

The corresponding profit and consumer surplus, respectively, are as follows.

$$\tilde{\pi}^{seq} = \tilde{p}^{seq} n_S + \delta_F \cdot \tilde{p}^{seq} n_N, \quad \tilde{V}_S^{seq} = 0, \quad \tilde{V}_N^{seq} = \frac{v(n) - v(n_S+1)}{1-\lambda_C}$$

The firm sets the period 1 price at the level of the sophisticated consumers' willingness to pay. Similarly, the period 2 price is set at the naive consumers' willingness to pay. Because sophisticated consumers form fulfilled expectations, their surplus will be fully extracted by the firm. In contrast, because naive consumers underestimate the network value of the good at the time of purchase, they can eventually gain a positive surplus.

Next, we derive the condition under which the sequential-diffusion strategy is more profitable for the firm when it can commit to a future price path, as summarized in the following proposition.

Proposition 9. *Suppose that the monopolistic firm can commit to a future price path. It is optimal for the firm to charge the sequential-diffusion prices* $(\tilde{p}_1^{seq}, \tilde{p}_2^{seq})$ *if and only if the discount factor for the firm is large enough to satisfy* $\delta_F > \max\left\{\tilde{f}(\lambda_C)\ ,\ 0\right\}$*, where:*

$$\tilde{f}(\lambda_C) \equiv \frac{nv(1) - n_S\left\{(1-\lambda_C)v(n_S) + \lambda_C v(n)\right\}}{n_N \cdot v(n_S+1)}.$$

Proposition 9 confirms the robustness of Proposition 3, that is, the sequential-diffusion strategy is optimal for the firm unless the firm greatly discounts its future profits.

4.3 Profitability of pre-announcement

Here, we check Coase's prediction that committing to a future price path can improve the profit of the monopolist. To this end, comparing $\hat{\pi}^{seq}$ and $\tilde{\pi}^{seq}$ yields the following proposition.

Proposition 10. *Committing to the future price path improves the profit of the monopolistic firm.*

It can be easily shown that $\hat{\pi}^{seq} < \tilde{\pi}^{seq}$, which implies that Coase's prediction holds in our model with naive consumers. From Propositions 6 and 8, we can see $\hat{p}_1^{seq} < \tilde{p}_1^{seq}$ and $\hat{p}_2^{seq} = \tilde{p}_2^{seq}$. When the firm cannot commit to its future prices, it must discount the period 1 price to prevent the sophisticated consumers from postponing their purchases to the next period. However, when the firm precommits to the period 2 price, the sophisticated consumers cannot obtain any price discount by postponing their purchase. As a result, the firm benefits from committing to the future price path because it can charge the higher price at period 1.

5 First- and Second-Best Commitment Strategies

So far, we have analyzed three cases: (i) fixed pricing, (ii) responsive pricing, and (iii) pre-announced pricing. This section demonstrates the first- and second-best strategies for the firm. To this end, we confine our attention to the parameter range in which the firm adopts the sequential-diffusion strategy, which is equivalent to $\delta_F \in [\max\{f(\lambda_C), \hat{f}(\delta_C, \beta), \tilde{f}(\lambda_C), 0\}, 1)$, because it is shown in all cases that the sequential-diffusion strategy is optimal for the firm unless the firm too heavily discounts its future profits.

First, it can easily be seen that pre-announced pricing is the optimal strategy for the firm of the three strategies. Proposition 10 shows that pre-announced pricing is more profitable than responsive pricing. It is also preferable to fixed fixed pricing, as indicated by strategy space of the fixed pricing being a subset of the strategy space of the pre-announced pricing. Therefore, the pre-announced pricing is the first-best strategy if it is possible for the firm to implement it.

However, in reality, it is difficult to the firm to credibly commit to complete path of future prices at the initial period, but committing to a fixed price may be more implementable. Thus, a comparison of fixed and responsive pricing indicates the second-best strategy regarding the firm's commitment. We have the following proposition regarding the order of the sequential-diffusion prices under both cases.

Proposition 11. *The sequential-diffusion prices under fixed pricing (i.e., p^{seq}) and responsive pricing (i.e., $\hat{p}_1^{seq}$ and $\hat{p}_2^{seq}$), respectively, are ordered as follows:*

$$\begin{cases} \hat{p}_1^{seq} < p^{seq} < \hat{p}_2^{seq} & \text{if} \quad 0 \leq \lambda_C < \bar{\lambda}_C \\ \hat{p}_1^{seq} < \hat{p}_2^{seq} = p^{seq} & \text{if} \quad \bar{\lambda}_C \leq \lambda_C < 1. \end{cases}$$

Proposition 11 implies that $\hat{p}_1^{seq}$ is always lower than $\hat{p}_2^{seq}$. In other words, when the firm can adjust the price at every period, the *introductory* or *penetration* pricing prevails in equilibrium, as shown in Bensaid and Lesne (1996) and Cabral et al. (1999).

When λ_C is large enough to satisfy $\lambda_C \geq \bar{\lambda}_C$, the fixed sequential-diffusion price p^{seq} is higher than $\hat{p}_1^{seq}$ and is equal to $\hat{p}_2^{seq}$. Therefore, in this case, it is obviously more profitable to commit to the fixed price. In contrast, when λ_C is smaller than $\bar{\lambda}_C$, p^{seq} is higher than p_1^{seq} and lower than p_2^{seq}, which increases the possibility that the firm will benefit by employing responsive pricing. We find the condition under which responsive pricing is more beneficial than fixed pricing.

Proposition 12. *Suppose that the discount factor for the firm is large enough to satisfy* $\max\{f(\lambda_C), \hat{f}(\delta_C, \beta), 0\} \leq \delta_F < 1$. *If*

$$\bar{\lambda}_C - \lambda_C > \frac{\delta_C}{\delta_F} \cdot \frac{n_S}{n_N}, \tag{4}$$

then it is unprofitable for the firm to commit to the fixed price at period 1. Otherwise, the firm can benefit from committing to the fixed price.

Proposition 12 shows three results: (i) if $\lambda \geq \bar{\lambda}_C$, then the condition (4) is not satisfied, that is, fixed pricing is always more profitable than responsive pricing; (ii) when $\lambda < \bar{\lambda}_C$, as δ_C/δ_F and/or n_S/n_N become smaller, responsive pricing is more likely to be profitable than fixed pricing; and (iii) when the effective network size n is small, $\bar{\lambda}_C$ becomes large, which increases the likelihood that responsive pricing is more likely to be profitable than fixed pricing.

The intuition behind the results is as follows. Result (i) is obvious from Proposition 11. With regard to Result (ii), under sequential-diffusion pricing, some price discounts

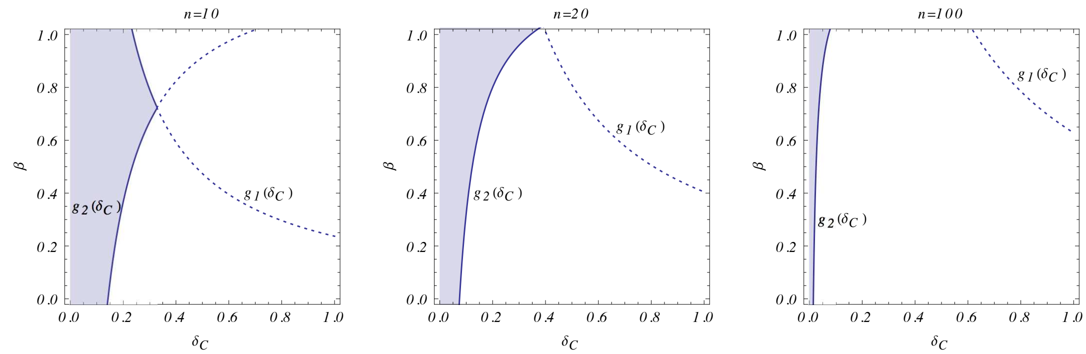


Figure 2: Fixed vs. responsive pricing strategies ($\alpha = 0.2, \delta_F = 0.9$).

are required to prevent sophisticated consumers from delaying their purchases to the next period. Under fixed pricing, these price discounts will be applied not only to sophisticated consumers but also to naive consumers. In contrast, under responsive pricing, the firm can stop discounting for naive consumers by adjusting its price at period 2. If δ_C/δ_F becomes smaller, then larger price discounts are required, which increases the advantage of responsive pricing. If n_S/n_N becomes smaller, then, under fixed pricing, the price discounts would apply to naive consumers, which also increases the advantages of responsive pricing. Finally, regarding Result (iii), $\bar{\lambda}_C$ will take a large value when the effective network size is small, as discussed in Section 3.2. As shown in Proposition 2, when $\bar{\lambda}_C$ is large, the willingness to pay of the sophisticated consumers is lower than that of the naive consumers. As a result, under fixed pricing, the sequential-diffusion price tends to be set at the level of the sophisticated consumers' willingness to pay, and is also applied to naive consumers, despite their higher willingness to pay. In contrast, under responsive pricing, the firm will be able to adjust its price at period 2 to avoid discounting the price offered to naive consumers. Thus, when the effective network effect is small, responsive pricing is more likely to be profitable than fixed pricing.

Finally, we perform numerical analyses to illustrate the above results of Proposition 12 and to discuss how the result would be affected by the effective network size, n. We consider a sufficiently large value of $\delta_F = 0.9$ for the sequential-diffusion strategy to be optimal. In addition, we let $v(n_t) = \sqrt{n_t}$ and $\alpha = 0.2$, that is, $n_S = 0.2n$ and $n_N = 0.8n$. Note that varying α does not qualitatively change our numerical results. In each panel of Figure 2, we define $g_1(\delta_C)$ and $g_2(\delta_C)$ as follows:

$$g_1(\delta_C) \equiv \frac{v(n_S) - v(1)}{v(n) - v(n_S)} \cdot \frac{1}{\delta_C}, \quad g_2(\delta_C) \equiv 1 - \left(\frac{\bar{\lambda}_C}{\delta_C} - \frac{n_S}{\delta_F n_N} \right),$$

where the former comes from condition (3) in Proposition 6 and the latter from condition (4) in Proposition 12. In the region of $\beta < g_1(\delta_C)$, sophisticated consumers purchase at period 1 and naive consumers purchase at period 2. In the region of $\beta > g_2(\delta_C)$, the firm can obtain higher profits under responsive pricing than under fixed pricing. The shaded area of each graph in the figure represents the parameter region where responsive pricing is more profitable for the firm, which shrinks gradually as n increases. In other words, when the effective network size is relatively small, it would be profitable for the firm to make no commitments to future prices.

6 Managerial implications

The central message from our study is that the sequential-diffusion strategy is profitable for the monopolistic firm selling durable goods that exhibit network externalities, unless the firm heavily discounts its future profits. Furthermore, sophisticated consumers, because of their sophistication, get lower surplus than do naive consumers. In other words, the existence of sophisticated consumers enables firm to charge a higher price. The main driver of these results is the existence of naive consumers. They have a lower willingness to pay than do the sophisticated consumers because they always underestimate the future network sizes. Even if the price was set higher than the naive consumers' willingness to pay, sophisticated consumers recognize that simultaneous purchase by all consumers would enable all consumers to gain a positive surplus. However, they also know that, unfortunately, the naive consumers never purchase before the network size has actually expanded. Therefore, sophisticated consumers have no option but to participate in the network first, before its size expands, to entice the naive consumers to subsequently enter into the network. In other words, sophisticated consumers have to take the leadership role to indicate the true value of the network to the naive consumers. This is the reason why the existence of the naive consumers sustains the firm's sequential-diffusion strategy in equilibrium. Moreover, sophisticated consumers have to endure the disadvantages of the small network size before the naive consumers join the network, whereas naive consumers can enjoy the largest network size immediately, as soon as they purchase the good. For this reason, the sophisticated consumers never gain a greater surplus than that of the naive consumers.

An important lesson from our model for firm selling durable network goods is that information about the current network size must be disclosed by the firm at the beginning of every period. If the firm does not construct such an information disclosure system, naive consumers never realize that the sophisticated consumers have purchased the good. Then, this dampens the incentives of the sophisticated consumers to join the network first and induce the subsequent entries of the naive consumers. That is, the firm has to make information about the current number of users and/or the sales volume public.

In addition, it is important not only to disclose information about the current network size, but also to provide an environment that assists more consumers to predict the future network size correctly. Proposition 5 shows that the firm can benefit from an increase in the ratio of sophisticated consumers. Therefore, the firm should supply information to consumers to decrease the proportion of naive consumers who cannot predict the purchase decisions of others. For example, the existence of a word-of-mouth viral site, where consumers can communicate with each other about the good, may help naive consumers to determine other consumers' purchasing intentions.

Moreover, in Proposition 12 and the corresponding numerical analysis, we show that, as the effective network size becomes larger, the advantage of fixed pricing (committing to a fixed price) against responsive pricing (pricing without commitment) becomes greater. Recent movements to establish an IoT might make consumers more sensitive to others' purchase decisions and enhance the managerial significance of committing to a fixed price. Often, in reality, it may be difficult to promise to hold prices constant for long periods. A pre-order system or subscription sales is a good potential strategy for doing so. Using our model, we explain how to use pre-order sales to commit to a fixed price. At the beginning of period 1, the firm announces that it will receive pre-orders at the price p^{seq} and that the deadline for pre-orders is the end of period 2. Sophisticated consumers may pre order at period 1. At the beginning of period 2, the firm should announce the number of pre orders it has received. After observing the announcement, the naive consumers follow the sophisticated consumers to pre order the good. This strategy not only enables the firm to credibly commit to maintaining its price at p^{seq} during the pre order period, but also informs sophisticated consumers of such a commitment. As a result, the firm can sell the good to all consumers at p^{seq}.

7 Conclusion

This paper has investigated the optimal pricing and diffusion of durable goods that exhibit positive network externalities. A key departure in our paper from many existing studies is to consider the existence of naive consumers as well as sophisticated (rational) consumers. In our model, in contrast to sophisticated consumers, naive consumers do not anticipate the network sizes at any future period, simply believing that the current network size will be maintained for future periods. This heterogeneity among consumers with respect to their beliefs about future network sizes significantly alters the optimal pricing strategy and the diffusion of durable goods.

We conclude by noting some limitations of our model and discussing potential avenues for future research. This paper assumes that consumers live for infinite periods, that is, they do not exit from the network that they join. This assumption means naive consumers underestimate the network value of the good compared with sophisticated consumers. In a model where consumers can exit from the network, naive consumers who cannot predict the exit of other consumers would overestimate the network value of the good. In this case, the results obtained in this paper might change. That is a potentially fruitful direction for future research, but it requires a more complex model, which is beyond the scope of this paper.

Appendix

Proof of Proposition 2

The comparison of $u^e_S(t=1|p^{seq})$ and $u^e_N(t=2|p^{seq})$ yields the following condition.

$$\begin{aligned} u^e_S(t=1|p^{seq}) > u^e_N(t=2|p^{seq}) \quad &\Longleftrightarrow \quad v(n_S) + \frac{\lambda_C v(n)}{1-\lambda_C} > \frac{v(n_S+1)}{1-\lambda_C} \\ &\Longleftrightarrow \quad \lambda_C > \frac{v(n_S+1) - v(n_S)}{v(n) - v(n_S)} \equiv \bar{\lambda}_C \end{aligned}$$

Thus, if $\lambda_C > \bar{\lambda}_C$, the price is set at $p^{seq} = u^e_N(t=2|p^{seq})$. Otherwise, the price is set at $u^e_S(t=1|p^{seq})$. Using this, we can obtain the corresponding profit, as shown in Proposition 2. When λ_C is larger than $\bar{\lambda}_C$, the surpluses of the sophisticated and naive consumers, respectively, can be computed as follows.

$$\begin{aligned} V^{seq}_S &= u_S(t=1|p^{seq}) - u^e_N(t=2|p^{seq}) \\ &= \left[v(n_S) + \frac{\lambda_C v(n)}{1-\lambda_C}\right] - \frac{v(n_S+1)}{1-\lambda_C} = \frac{\lambda_C\{v(n) - v(n_S)\} - \{v(n_S+1) - v(n_S)\}}{1-\lambda_C} \\ V^{seq}_N &= u_N(t=2|p^{seq}) - u^e_N(t=2|p^{seq}) = \frac{v(n)}{1-\lambda_C} - \frac{v(n_S+1)}{1-\lambda_C} = \frac{v(n) - v(n_S+1)}{1-\lambda_C} \end{aligned}$$

Similarly, the consumer surplus can be computed when λ_C is smaller than $\bar{\lambda}_C$, as follows.

$$\begin{aligned} V^{seq}_S &= u_S(t=1|p^{seq}) - u^e_S(t=1|p^{seq}) = 0 \\ V^{seq}_N &= u_N(t=2|p^{seq}) - u^e_S(t=1|p^{seq}) = \frac{v(n)}{1-\lambda_C} - \left[v(n_S) + \frac{\lambda_C v(n)}{1-\lambda_C}\right] = v(n) - v(n_S) \end{aligned}$$

$\square$

Proof of Corollary 2

First, when $\lambda_C \leq \bar{\lambda}_C$, $V^{seq}_S = 0$ and $V^{seq}_N = v(n) - v(n_S) > 0$. Thus, it holds that $V^{seq}_S < V^{seq}_N$. Next, when $\lambda_C \geq \bar{\lambda}_C$, we can show $V^{seq}_S < V^{seq}_N$ as follows.

$$V^{seq}_N - V^{seq}_S = \frac{v(n) - v(n_S+1)}{1-\lambda_C} - \frac{\lambda_C\{v(n) - v(n_S)\} - \{v(n_S+1) - v(n_S)\}}{1-\lambda_C}$$

$$= \quad v(n) - v(n_S) > 0$$

□

Proof of Lemma 1

For $\lambda_C \in [\bar{\lambda}_C, 1)$, it is easily seen that $p^{sim} < p^{seq}$ as follows.

$$p^{seq} - p^{sim} = \frac{v(n_S+1)}{1-\lambda_C} - \frac{v(1)}{1-\lambda_C} > 0$$

We can also show $p^{sim} < p^{seq}$ for $\lambda_C \in [0, \bar{\lambda}_C]$ as follows.

$$\begin{aligned} p^{seq} - p^{sim} &= v(n_S) + \frac{\lambda_C v(n)}{1-\lambda_C} - \frac{v(1)}{1-\lambda_C} = \frac{(1-\lambda_C)v(n_S) + \lambda_C v(n) - v(1)}{1-\lambda_C} \\ &> \frac{v(n_S) - v(1)}{1-\lambda_C} > 0 \end{aligned}$$

□

Proof of Proposition 3

Two possible cases should be considered.

(i) $0 \leq \lambda_C \leq \bar{\lambda}_C$

In this case, it holds that $p^{seq} = u_S^e(t=1|p^{seq})$. Comparing the profits under simultaneous- and sequential-diffusion prices, we have the following condition for $\pi^{seq} > \pi^{sim}$.

$$\begin{aligned} \pi^{seq} - \pi^{sim} &= (n_S + \delta_F n_N)\left[v(n_S) + \frac{\lambda_C v(n)}{1-\lambda_C}\right] - n \cdot \frac{v(1)}{1-\lambda_C} \\ &= \frac{(n_S + \delta_F n_N)\{(1-\lambda_C)v(n_S) + \lambda_C v(n)\} - nv(1)}{1-\lambda_C} \\ &> 0 \\ &\iff \delta_F > \frac{nv(1) - n_S\{\lambda_C v(n) + (1-\lambda_C)v(n_S)\}}{n_N\{\lambda_C v(n) + (1-\lambda_C)v(n_S)\}} \equiv f_1(\lambda_C) \\ \text{where} \quad f_1'(\lambda_C) &= -\frac{nv(1)\{v(n) - v(n_S)\}}{n_N\{\lambda_C v(n) + (1-\lambda_C)v(n_S)\}^2} < 0 \end{aligned}$$

(ii) $\bar{\lambda}_C \leq \lambda_C < 1$

In this case, it holds that $p^{seq} = u_N^e(t=2|p^{seq})$. Comparing the profits under simultaneous- and sequential-diffusion prices, we have the following condition for $\pi^{seq} > \pi^{sim}$.

$$\begin{aligned}
\pi^{seq} - \pi^{sim} &= (n_S + \delta_F n_N)\frac{v(n_S+1)}{1-\lambda_C} - n \cdot \frac{v(1)}{1-\lambda_C} \\
&= \frac{n_S v(n_S+1) - nv(1) + \delta_F \cdot n_N v(n_S+1)}{1-\lambda_C} \\
&> 0 \\
&\iff \delta_F > \frac{nv(1) - n_S v(n_S+1)}{n_N v(n_S+1)} \equiv f_2(\lambda_C)
\end{aligned}$$

□

Proof of Proposition 5

Fix the total number of consumers at n. Let $n_S = \alpha n$ and $n_N = (1-\alpha)n$. First, it is obvious that $p^{sim} = v(1)/(1-\lambda_C)$ is independent of α. Next, p^{seq} can be rewritten as

$$p^{seq} = \begin{cases} v(\alpha n) + \dfrac{\lambda_C v(n)}{1-\lambda_C} & \text{if } \ 0 \le \lambda_C \le \bar{\lambda}_C, \\ \dfrac{v(\alpha n+1)}{1-\lambda_C} & \text{if } \ \bar{\lambda}_C \le \lambda_C < 1, \end{cases}$$

which is increasing in α. Finally, we prove that π^{seq} is also increasing in α as follows.

$$\begin{aligned}
\pi^{seq} &= p^{seq} \cdot \alpha n + \delta_F \cdot p^{seq} \cdot (1-\alpha)n = np^{seq}\left\{\alpha + \delta_F(1-\alpha)\right\} \\
\frac{\partial \pi^{seq}}{\partial \alpha} &= n\frac{\partial p^{seq}}{\partial \alpha}\left\{\alpha + \delta_F(1-\alpha)\right\} + np^{seq}(1-\delta_F) > 0
\end{aligned}$$

□

Proof of Proposition 6

We find the optimal price such that all sophisticated consumers purchase at period 1 and all naive consumers purchase at period 2. Then, we will confirm that sophisticated consumers have no incentive to postpone their purchases to the next period. That is, there is no equilibrium in which only a portion of the sophisticated consumers purchase in period 1.

In order for sequential diffusion to occur in equilibrium, the period 1 price must be high enough to satisfy $\hat{p}_1^{seq} > v(1)/(1-\lambda_C)$ because naive consumers also purchase at period 1 if it holds that $\hat{p}_1^{seq} \le v(1)/(1-\lambda_C)$.

First, we consider the period 2 price, $\hat{p}_2^{seq}$, given that all sophisticated consumers purchase at period 1. At the beginning of period 2, all naive consumers observe that the current network size is n_S. In addition, each of them anticipates that if he/she joins the network, the network size will become n_S+1, which will be continued over future periods. Therefore, the firm sets the period 2 price at $\hat{p}_2^{seq} = v(n_S+1)/(1-\lambda_C)$ to make all naive consumers purchase.

Next, let us consider the decision regarding the period 1 price, $\hat{p}_1^{seq}$. The indirect utility of sophisticated consumers at period 1 is given by:

$$V_S = v(n_S) + \frac{\lambda_C v(n)}{1-\lambda_C} - \hat{p}_1^{seq}.$$

Note that $\hat{p}_1^{seq}$ should be set so that each sophisticated consumer has no inventive to postpone purchase to the next period. Consider a situation in which one sophisticated consumer deviates from the equilibrium path. In the situation, at period 2, the naive consumers consider that the network size will become n_S if he/she purchases the good. Therefore, the period 2 price will be set at $p_2' = v(n_S)/(1-\lambda_C)$. Because all remaining consumers actually join the network, the sophisticated consumer who deviated will gain the following surplus:

$$V_S' = \frac{v(n)-v(n_S)}{1-\lambda_C}$$

The condition under which all sophisticated consumers have no inventive to deviate from purchasing at period 1 is that $V_S \geq \delta_C V_S'$ holds. Note that V_S' is discounted by δ_C, not by $\lambda_C = \delta_C(1-\beta)$. We find the condition for $\hat{p}_1^{seq}$ as follows.

$$\begin{aligned} V_S \geq \delta_C V_S' \quad &\iff \quad v(n_S) + \frac{\lambda_C v(n)}{1-\lambda_C} - \hat{p}_1^{seq} \geq \frac{v(n)-v(n_S)}{1-\lambda_C} \times \delta_C \\ &\iff \quad \hat{p}_1^{seq} \leq \frac{(1+\delta_C-\lambda_C)v(n_S) - (\delta_C-\lambda_C)v(n)}{1-\lambda_C} = \frac{v(n_S) - \delta_C\beta\{v(n)-v(n_S)\}}{1-\lambda_C} \end{aligned}$$

Thus, the firm charges the following period 1 price.

$$\hat{p}_1^{seq} = \max\left\{\frac{v(n_S) - \delta_C\beta\{v(n)-v(n_S)\}}{1-\lambda_C}, 0\right\}$$

As described above, this price has to satisfy $p_1^{seq} > v(1)/(1-\lambda_C)$. The condition that the parameters should satisfy is as follows.

$$\frac{v(n_S) - \delta_C\beta\{v(n) - v(n_S)\}}{1-\lambda_C} > \frac{v(1)}{1-\lambda_C} \iff \delta_C\beta < \frac{v(n_S) - v(1)}{v(n) - v(n_S)}$$

Using $\hat{p}_1^{seq}$ and $\hat{p}_2^{seq}$, we obtain the corresponding profit of the firm and the consumer surplus, as shown in Proposition 6.

Finally, we prove that there is no equilibrium in which only a portion of sophisticated consumers purchase at period 1 and the remaining sophisticated consumers do so at period 2. Let n_S^t be the number of sophisticated consumers who purchase at period t. Given that n_S^1 sophisticated consumers purchase at period 1, each naive consumer at period 2 expects that the network size will become $n_S^1 + 1$ if he/she purchases the good. That is, the price of period 2 will be set at $p_2 = v(n_S^1 + 1)/(1-\lambda_C) < p_2^{seq}$. Note that this price is lower than the equilibrium price. Eventually, all naive consumers join the network and derive the following surplus.

$$V_S^2 = V_N = \frac{v(n) - v(n_S^1 + 1)}{1-\lambda_C}$$

In this case, sophisticated consumers who purchase at period 1 obtain the following surplus.

$$V_S^1 = v(n_S^1) + \frac{\lambda_C v(n)}{1-\lambda_C} - p_1$$

Consider a deviation such that one sophisticated consumer postpones purchasing to period 2. Then, each naive consumer at period 2 expects that the network size will become n_S^1 if he/she purchases the good. That is, the price of period 2 will be set at $p_2 = v(n_S^1)/(1-\lambda_C) < p_2^{seq}$. Eventually, all naive consumers join the network and derive the following surplus.

$$V_S' = \frac{v(n) - v(n_S^1)}{1-\lambda_C}$$

There is no incentive for such a deviation if and only if:

$$V_S^1 \geq V_S'$$

$$\iff \quad v(n_S^1) + \frac{\lambda_C v(n)}{1-\lambda_C} - p_1^{seq} \geq \frac{v(n) - v(n_S^1)}{1-\lambda_C} \times \delta_C$$
$$\iff \quad p_1 \leq \frac{(1+\delta_C-\lambda_C)v(n_S^1) - (\delta_C-\lambda_C)v(n)}{1-\lambda_C} < \frac{(1+\delta_C-\lambda_C)v(n_S) - (\delta_C-\lambda_C)v(n)}{1-\lambda_C} = p_1^{seq}.$$

That is, to ensure that such a deviation is unprofitable, the period 1 price should be set at a lower price than the equilibrium price (i.e., $p_1 < p_1^{seq}$). However, setting such a low price never improves the profit of the firm. Therefore, there is no equilibrium such that only a portion of the sophisticated consumers purchase at period 1 and the remaining do so at period 2. □

Proof of Proposition 7

Comparing the profits under simultaneous- and sequential-diffusion prices, we have the following condition for $\hat{\pi}^{seq} > \pi^{sim}$.

$$\hat{\pi}^{seq} > \pi^{sim} \quad \iff \quad n_S \frac{(1+\delta_C-\lambda_C)v(n_S) - (\delta_C-\lambda_C)v(n)}{1-\lambda_C} + \delta_F \cdot n_N \frac{v(n_S+1)}{1-\lambda_C} > n\frac{v(1)}{1-\lambda_C}$$
$$\iff \quad \delta_F > \frac{nv(1) - n_S\left\{(1+\delta_C-\lambda_C)v(n_S) - (\delta_C-\lambda_C)v(n)\right\}}{n_N \cdot v(n_S+1)} \equiv \hat{f}(\delta_C, \beta)$$

□

Proof of Proposition 8

This proof is similar to that for Proposition 6. First, we consider the price of period 2, $\tilde{p}_2^{seq}$, given that all sophisticated consumers purchase at period 1. At the beginning of period 2, all naive consumers observe that the current network size is n_S. In addition, each of them anticipates that if he/she joins the network, the network size becomes $n_S + 1$, which will continue over future periods. Therefore, the firm sets the price of period 2 at $\tilde{p}_2^{seq} = v(n_S+1)/(1-\lambda_C)$ to induce all naive consumers to purchase, which is equal to $\hat{p}_2^{seq}$.

Next, let us consider the decision on the period 1 price, $\tilde{p}_1^{seq}$. When the firm commits to setting the period 2 price at $\tilde{p}_2^{seq}$, sophisticated consumers cannot win any price discount if they postpone their purchase to the next period. Therefore, even if the price of period 1 is set at sophisticated consumers' willingness to pay, they will purchase the good at period 1, which implies that the firm set the price of period 1 at $\tilde{p}_1^{seq} = u_S(t=1|p^{seq}) = v(n_S) + \lambda_C v(n)/(1-\lambda_C)$.

Using $\tilde{p}_1^{seq}$ and $\tilde{p}_2^{seq}$, we obtain the corresponding profit of the firm and consumer surplus as shown in Proposition 8. $\square$

Proof of Proposition 9

Comparing the profits under simultaneous- and sequential-diffusion prices, we have the following condition for $\tilde{\pi}^{seq} > \pi^{sim}$.

$$\begin{aligned}
\tilde{\pi}^{seq} > \pi^{sim} \quad &\Longleftrightarrow \quad n_S \left\{ v(n_S) + \frac{\lambda_C v(n)}{1-\lambda_C} \right\} + \delta_F \cdot n_N \frac{v(n_S+1)}{1-\lambda_C} > n \frac{v(1)}{1-\lambda_C} \\
&\Longleftrightarrow \quad \delta_F > \frac{nv(1) - n_S \left\{ (1-\lambda_C)v(n_S) + \lambda_C v(n) \right\}}{n_N \cdot v(n_S+1)} \equiv \tilde{f}(\lambda_C)
\end{aligned}$$

$\square$

Proof of Proposition 11

We consider the following two cases.

(i) $0 \leq \lambda_C < \bar{\lambda}_C$

In this case, from the definition of $\bar{\lambda}_C$, $p^{seq} < p_2^{seq}$ holds as follows.

$$\lambda_C < \bar{\lambda}_C \quad \Longleftrightarrow \quad v(n_S) + \frac{\lambda_C v(n)}{1-\lambda_C} < \frac{v(n_S+1)}{1-\lambda_C} \quad \Longleftrightarrow \quad p^{seq} < p_2^{seq}$$

Next, we prove that $p_1^{seq} < p^{seq}$, as follows.

$$\begin{aligned}
p^{seq} - p_1^{seq} \quad &= \quad v(n_S) + \frac{\lambda_C v(n)}{1-\lambda_C} - \frac{(1+\delta_C-\lambda_C)v(n_S) - (\delta_C-\lambda_C)v(n)}{1-\lambda_C} \\
&= \quad \frac{(1-\lambda_C)v(n_S) + \lambda_C v(n) - (1+\delta_C-\lambda_C)v(n_S) + (\delta_C-\lambda_C)v(n)}{1-\lambda_C} \\
&= \quad \frac{\delta_C\{v(n) - v(n_S)\}}{1-\lambda_C} > 0
\end{aligned}$$

That is, $p_1^{seq} < p^{seq} < p_2^{seq}$ holds for $\lambda_C \in [0, \bar{\lambda}_C)$.

(ii) $\bar{\lambda}_C \leq \lambda_C < 1$

First, it can be confirmed that $p_2^{seq} = p^{seq} = v(n_S+1)/(1-\lambda_C)$ holds. Next, we prove that $p_1^{seq} < p_2^{seq}$, as follows.

$$p_2^{seq} - p_1^{seq} \quad = \quad \frac{v(n_S+1)}{1-\lambda_C} - \frac{(1+\delta_C-\lambda_C)v(n_S) - (\delta_C-\lambda_C)v(n)}{1-\lambda_C}$$

$$= \quad \frac{v(n_S+1) - v(n_S) + \delta_C \beta \{v(n) - v(n_S)\}}{1-\lambda_C} > 0$$

That is, $p_1^{seq} < p_2^{seq} = p^{seq}$ holds for $\lambda_C \in [\bar{\lambda}_C, 1)$.

$\square$

Proof of Proposition 12

Suppose that the discount factor for the firm δ_F is sufficiently large. We consider the following two cases.

(i) $0 \leq \lambda_C < \bar{\lambda}_C$

In this case, it holds that $\beta > 1 - \bar{\lambda}_C/\delta_C$.

$$\begin{aligned}
\pi^{seq} < \hat{\pi}^{seq} \quad \Longleftrightarrow \quad & (n_S + \delta_F n_N)\left[v(n_S) + \frac{\lambda_C v(n)}{1-\lambda_C}\right] \\
& < n_S \frac{(1+\delta_C - \lambda_C)v(n_S) - (\delta_C - \lambda_C)v(n)}{1-\lambda_C} + \delta_F \cdot n_N \frac{v(n_S+1)}{1-\lambda_C} \\
\Longleftrightarrow \quad & \frac{\delta_C}{\delta_F} \cdot \frac{n_S}{n_N} < \frac{v(n_S+1) - v(n_S)}{v(n) - v(n_S)} - \lambda_C \\
\Longleftrightarrow \quad & \frac{\delta_C}{\delta_F} \cdot \frac{n_S}{n_N} < \bar{\lambda}_C - \lambda_C
\end{aligned}$$

(ii) $\bar{\lambda}_C \leq \lambda_C < 1$

In this case, it holds that $p_1^{seq} < p_2^{seq} = p^{seq}$ as shown in Proposition 11. Therefore, $\pi^{seq} > \hat{\pi}^{seq}$ always holds.

In sum, we obtain the condition for $\pi^{seq} < \hat{\pi}^{seq}$ as follows.

$$\frac{\delta_C}{\delta_F} \cdot \frac{n_S}{n_N} < \bar{\lambda}_C - \lambda_C$$

$\square$

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