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Cognitive stress and learning Economic Order Quantity (EOQ) inventory management: An experimental investigation

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Abstract

We use laboratory experiments to evaluate the effects of cognitive stress on inventory management decisions in a finite horizon Economic Order Quantity (EOQ) model. We manipulate two sources of cognitive stress. First, we vary participants' ability to order inventory from any decision period to only when inventory is depleted. This reduces cognitive stress by restricting the policy choice set. Second we vary participants' participation in a competing pin memorization. This increases cognitive load. Participants complete a sequence of five "annual" inventory management tasks, with monthly ordering decisions. Both sources of cognitive stress negatively impact earnings, with the bulk of these impacts occurring in the first year. Participants' choices in all treatments exhibit trends to near optimal policy adoption. But only in the most favorable treatment do the majority of choices reach the optimal policy. We estimate the learning dynamics of monthly order decisions using a Markov switching model. Estimates suggest increased cognitive load reduces the probability of switching to more profitable policies, and that more complex policy choice sets leads to a greater policy lock-in. Our results suggests that inexperienced individuals will perform more poorly when called upon to make inventory management situations in cognitively stressfully environments, and that the benefits of providing support and task simplicity is greatest when the task is first assigned.

Keywords: *Economic Order Quantity, Cognitive load, Choice set complexity, Learning*

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1 Introduction

Best inventory management practices call for the solution of dynamic optimization problems. This requires inventory managers to parse complex sets of alternative solutions and to use their short-term memory to hold and process information about the past, present, and future values of key variables. Current workplace trends impose increasing demands upon these managers' cognitive resources (Ruderman et al., 2017). Some examples of these trends are increasing complexity of supply chains (Bode and Wagner, 2015), increasing rates and scale of natural disasters and global social upheavals. We assess how increasing cognitive stress through the complexity of the inventory policy choice set and the competition for inventory managers' cognitive resources impacts their decision-making quality.

An extensive literature shows that, even under the best of circumstances, individuals systematically make suboptimal inventory management decisions. Decision-making biases and strategic considerations are often key factors diminishing individual performances in these tasks. When managing the inventory of a perishable good with uncertain demand, i.e. the newsvendor problem, decision makers neither follow the optimal risk neutral or averse policies consistently in experimental studies.¹ When there is a multi-level supply chain for a non-perishable good and certain demand, participants generate large bullwhip effects in beer game experiments. Researcher have shown key factors driving the excessive inventory levels and variance include strategic uncertainty regarding other decision makers (Croson et al., 2014), limited level two thinking (Narayanan and Moritz, 2015) and failure to fully take account of the future deliveries of past orders. In the setting of a durable good with uncertain demand optimal inventory management follows the (S, s) policy. Recent experimental studies by (Magnani et al., 2016; Khaw et al., 2017) demonstrate that individuals take time to find the optimal policy, their policy adaptations are idiosyncratic and often participants abandon the optimal policy once found. Despite all being important inventory management environments, none are ideal to begin an evaluation of how cognitive stress diminishes decision-making quality. The reason being decision-makers' performances are already suboptimal in their respective baseline experimental conditions.

A more suitable inventory management environment should have two properties: the optimal policy is invariant to a decision maker's individual preferences and the majority of decision makers can find the optimal policy under baseline conditions. The finite horizon deterministic economic order quantity environment (EOQ) potentially possesses these properties. Despite being one of the most commonly used models in operations management, behavioral studies have mostly overlooked it. We choose the parameters of

¹See Katok et al. (2011) for an introduction and partial survey of this literature.

our environment such that the optimal inventory policy of the finite horizon matches that of the infinite horizon; when inventory is depleted, the manager orders an optimal quantity that is the multiple of the monthly demand for the good (Schwarz, 1972). We refer to this multiple as an EOQ cycle length. This EOQ environment has several favourable features for our research question: inexperienced participants have a relatively good chance of finding the optimal policy; the solution is invariant to a decision maker's risk attitude; and, it is an individual decision problem absent of strategic considerations.

The EOQ solution in our environment is dynamic, as the manager doesn't make the same decision at each point in time. This gives us an opportunity to observe pure learning behavior in a dynamic programming problem. Our baseline environment provides the most favourable circumstance for inexperienced participants to learn and follow the optimal EOQ policy. The key elements of this baseline case is that we forbid participants from ordering when there is a positive level of inventory - we call this our "EOQ" treatment - and that there is no other tasks competing for the participants' short term memory resources - we call this our "Low" treatment. The majority of participants exposed to this EOQ-Low base level of cognitive stress optimally solve this problem after several "repetitions" of the finite horizon.

From the academic perspective, examining the EOQ environment is a first step in a longer research agenda. However our results still provide managerial insights for a set of practical problems. Individuals without inventory management experience are often called upon to perform such duties; we call these individuals "accidental" inventory managers. Effective inventory management of relief supplies is a key driver of successful response efforts in the aftermath of natural disasters. Two challenges commonly arising are the associated increases in cognitive stress levels and the enlistment of accidental inventory managers. In the ensuing rescue efforts to the 2008 Sichuan earthquake, inexperienced volunteers and government were deputized into inventory managerial roles. During the rescue, excess supplies were delivered to the devastated area due to mismanagement. This led to warehouse overflows and subsequent safety hazards. Similarly, in the wake of Hurricane Katrina in 2005 FEMA and state workers testified that they found their supply chains unable deliver the requested levels of goods, and in response they would order twice as much as needed. In the article "Hurricane Katrina showed importance of logistics" in *Supply & Demand Chain Executive* (2005) it was reported that Wal-Mart, the world's largest multinational retail corporation outperformed the inexperienced inventory managers from FEMA and Red Cross.

Around our EOQ-Low baseline we implement a 2×2 experimental design with two factors that exogenously impose cognitive stress. The first factor we investigate is the complexity of the inventory policies choice set a participant chooses from. In contrast to our EOQ treatment, participants in our "Unrestricted" treatment are allowed to place orders

each month regardless of the current inventory level. A growing and recent literature in economics, e.g. Caplin et al. (2011); Masatlioglu et al. (2012); Abeler and Jäger (2015); Lleras et al. (2017), examines and measures how individual choices are increasingly suboptimal as their choice sets increase in complexity. In our experimental design the policy choice set of our Unrestricted treatment corresponds to the typical case of an unsupported inventory manager while the simpler choice set of the EOQ treatment corresponds to active management intervention. This allows our experiment to provide evidence on the value of this practice.

The second factor we investigate is the presence of a concurrent task that competes for the inventory manager's cognitive resources. This concurrent task is the memorization of a PIN code at the beginning of each inventory year, and successful recall at the end of the year earns a monetary reward. We call this our "High" treatment. The PIN task was first introduced by Miller (1956), and has been successively used in economics and psychology to exogenously shock cognitive load. Some recent examples of its application are in food choice (Shiv and Fedorikhin, 1999), generosity (Roch et al., 2000) and intertemporal choice (Hinson et al., 2003). To the best of our knowledge, we are the first to use this technique in behavioral operations management. Correspondingly this allows our experiment to evaluate the impact of asking inexperienced inventory managers to multi-task.

Our results show that experimental participants earn less when there is a competing task or when the policy choice set is not restricted. We observe there is a trend that participants learned to adopt near optimal EOQ policies in general. The restriction of managers to only place orders when inventories are exhausted and the alleviation of the competing task improved the chance for inexperienced decision makers to reach the optimal inventory policy. It should be noted that these performance differences and suboptimal choices largely occur in the first three iterations of our environment. The inexperienced participants learn to better solve the dynamic optimization problem, we attempt to characterize this learning.

We formulate the learning process as a decision tree in which participants, mainly those in the Unrestictied treatment, learn to avoid choices leading to stock outs and other choices leading to carrying excess inventories. We find that iterations of the task quickly diminish the probability of making such choices and, surprisingly, imposing high cognitive loads doesn't affect these probabilities. Once participants follow the branch to take EOQ policy consistent actions we model the number of monthly demand orders requested, the EOQ cycle length, using a Markov switching model (Shachat and Zhang, 2017) that is particularly well suited for choice sequences made with low levels of rationality. Our estimates of the model suggests that under high cognitive load participants are less likely to choose payoff increasing EOQ cycle lengths. The estimates also suggest that with the

more complicated policy choice sets of the Unrestricted treatment participants are more reluctant to make large changes in EOQ cycle length leading to greater policy lock-in.

Our study is one of the first to experimentally examine a stationary limited horizon EOQ model. But there are two previous studies which examine other EOQ environments. The EOQ is one of the three environments [Stangl and Thonemann \(2017\)](#) consider in their behavioral study of inventory decision-making under two common alternative frames of performance measurement: inventory turnover and the number of days of inventory held. The former leads managers to over-value inventory reductions relative to the latter. [Chen and Wu \(2017\)](#) examine learning in an infinite EOQ environment in which there is varying inventory ordering and holding costs. The experiment consists of fifty rounds of such inventory decisions. For the first fifteen rounds operational costs were constant, and they varied during the last thirty-five rounds. Their result shows that learning occurs over rounds, and participants learn much faster about the optimal choice under stable environment than under changing environment. Suboptimal decisions tend not to be repeated with deterministic feedbacks. It is important to note that their participants' choice sets are even more restricted than those of our EOQ treatment. Participants are required to choose from an EOQ restricted choice set whose elements are the number of weeks, their periodicity of demand, of inventory ordered each time inventory is depleted. Thus, their policy choice set consists only of EOQ policies with fixed EOQ cycle lengths. The feedback [Chen and Wu \(2017\)](#) provide participants is the average operational costs generated per week by their EOQ cycle length choice, and participants' reward metrics are the sum of their average weekly performances. While we provide a monthly reported feedback on each decision made, participants experience and collect rewards on a month-to-month basis, which will vary from months when inventory is ordered to those when it is not.

2 Experiment

2.1 Inventory decision task

In the core decision-making part of our experiment, participants complete a series of six discrete dynamic inventory management tasks. We refer to each tasks as a year, indexed zero to five, and each year consists of twelve months, indexed by t . We use the following context to describe these tasks to a participant.

The participant manages the enterprise ‘S-store’ which sells coffee makers at a price of P7 per unit with a constant demand rate (D) of 10 units per month. S-store sells a new model of coffee makers every year. Coffee maker orders are placed prior to the start of a month, an integer amount denoted q_t , and arrive without lag. Hence are included in the

calculation of a month's opening inventory. The participant chooses the quantity of each monthly order.

Monthly orders and demand determine the changing inventory levels. Let I_t denote the closing inventory for month t . The initial inventory of coffee makers prior to month one is zero, so the first month's opening inventory is the amount of the first month's coffee maker order, i.e. $I_0 + q_1 = q_1$. In general, the opening inventory of coffee makers in month t is $I_{t-1} + q_t$. This inventory is drawn down by the monthly sales, the lesser of the monthly order flow of 10 or the opening inventory (i.e. a stock out.) This results in the closing inventory of $I_t = I_{t-1} + q_t - \min\{10, I_{t-1} + q_t\}$. When the model life cycle concludes at the end of month 12, any remaining inventory is disposed at no cost nor generates but also generates no revenue. Further, we limit a participant's monthly order by its annual demand, i.e., $q_t \in \{0, 1, 2, \dots, 120\}$.

A participant's compensation, excluding a fixed show-up fee, is proportional to S-store's profits, which are expressed - as are all further monetary quantities - in experiment currency units (denoted P). Each coffee maker sells at a price of P7. So revenue in month t is $7 \cdot \min\{10, I_{t-1} + q_t\}$. S-store's cost has two component's: a fixed ordering cost, S , of P45 whenever she places a strictly positive order; and a variable monthly inventory holding cost. The monthly inventory holding costs is calculated by multiplying the average inventory of coffee makers held in t , specifically $\frac{(I_{t-1} + q_t + I_t)}{2}$, and the monthly holding cost, h , of P1 per unit. The monthly profit of S-store is the difference between the revenue and costs, and is calculated

$$\pi_t(q_t, I_{t-1}) = \begin{cases} 7 \cdot 10 - S \cdot \mathbb{1}_{q_t > 0} - \frac{I_{t-1} + q_t + I_t}{2} \cdot 1 & \text{if } I_{t-1} + q_t \geq 10 \\ 7 \cdot (I_{t-1} + q_t) - S \cdot \mathbb{1}_{q_t > 0} - \frac{I_{t-1} + q_t}{2} \cdot 1 & \text{if } I_{t-1} + q_t < 10 \end{cases}$$

where, $\mathbb{1}$ is the indicator function.

A participant i 's inventory policy for year a is the sequence of the twelve monthly quantity orders, $Q_{i,a} = (q_{i,1}, q_{i,2}, \dots, q_{i,12})$. For a given inventory policy S-store's annual profits are,

$$\Pi_{i,a}(Q_{i,a}) = \sum_{t=1}^{12} \pi_t.$$

In the supply chain literature, the set of EOQ policies is the subset of inventory policies which only place a quantity order once inventory reaches zero with no stock outs allowed. In our dynamic decision making environment, stock outs can occur if a non-optimal policy was chosen previously. Correspondingly we adjust the definition of an EOQ policy to classify choices at these points off the optimal path.

Definition 1. *An EOQ action is a temporal inventory management decision satisfying*

the following conditions:

- (1). A participant only orders when the closing inventory of the previous period is less than 10 units, i.e., $q_t > 0$ when $I_{t-1} < 10$;
- (2). A participant doesn't order when the closing inventory of the previous period is more than 10 units, i.e., $q_t = 0$ when $I_{t-1} \geq 10$;
- (3). Participant's order guarantees no stock outs in t , i.e., $I_{t-1} + q_t \geq 10$.

Definition 2. An EOQ policy is a inventory management policy that consists only of EOQ actions.

The original EOQ model solution is derived assuming an infinite demand horizon, in which the average cost minimizing EOQ policy is to order the following quantity whenever the closing inventory of the previous period is zero,

$$q^* = \sqrt{\frac{2DS}{12h}}. \quad (1)$$

If our context then the cost minimizing policy would be to order 30 coffee makers, an EOQ cycle length of three months, whenever closing inventory of the previous period is zero. This would also be the profit maximizing policy as average revenue is constant, up to the monthly demand capacity, and greater than the minimum average cost. In our finite horizon setting the optimal policy does not change. But if an inventory manager deviates from this policy early in the year the optimal course can involve alternative EOQ actions later in the year.

Schwarz (1972) characterizes the optimal EOQ policies for the finite horizon of T months. First, we note the result that average total cost minimizing policy is to order according to Equation 1 if T is an integer multiple of the $\frac{q^*}{D}$. As simply following the EOQ policy of ordering 10 units each period is profitable in our environment, profit maximization will call for satisfying the full annual demand. The EOQ policy of always taking the EOQ action of 30 when inventory is depleted maximizes profit in addition to minimizing average cost.

As individuals can and do fail to act sub-optimally we now consider alternative, i.e. shorter in this case, decision horizons. Let $C(T)$ be total incremental cost over the finite time interval T . We restrict our attention to policies which only place orders when inventory is zero. An EOQ cycle length is the interval of months between such orders, denoted by s_k , which is the interval between the $(k - 1)th$ and the kth order. Let $C(s_k)$ be the total incremental cost for an EOQ cycle, and n be the number of orders over T . We can formulate the problem as

$$\min C(T) = \sum_{k=1}^n C(s_k) \quad s.t. \quad \sum_{k=1}^n s_k = T$$

where

$$C(s_k) = S + hDt^2/2$$

From the quadratic formulation, it is clear that in the optimal solution all of the s_k are of the same length. An EOQ constant inventory policy, denoted \bar{Q}^{s_k} , is one with a constant cycle length.

Let $C^n(T)$ be the total incremental cost for the interval T given n orders,

$$C^n(T) = nS + hDT^2/2n.$$

Minimising $C^n(T)$ gives

$$n^* = \sqrt{\frac{hDT^2}{2S}}.$$

Notice for the first year in our task, i.e. $T = 12$, this yields the same solution as the infinite horizon formulation, $n^* = 4$ and $t^* = 3$. Further investigations on situations when the horizon T is sufficiently small reveals that The optimal number of orders, n^* , is the smallest integer satisfying $n(n+1) \geq \frac{hDT^2}{2S}$.

With the parameter values in our task, the following table gives an overview of the optimal solutions for different values of T :

Table 1: Optimal solutions for different T in our task

Month	T	$\frac{hDT^2}{2S}$	$n^*(n^* + 1)$	The optimal order number (n^*)	The optimal cycle length (s_k^*) sequence	EOQ
12	1	0.111	2	1	{1}	
11	2	0.444	2	1	{2}	
10	3	1	2	1	{3}	
9	4	1.778	2	1	{4}	
8	5	2.778	6	2	{3, 2}	
7	6	4	6	2	{3, 3}	
6	7	5.444	6	2	{3, 4}	
5	8	7.111	12	3	{3, 3, 2}	
4	9	9	12	3	{3, 3, 3}	
3	10	11.111	12	3	{3, 3, 4}	
2	11	13.444	20	4	{3, 3, 3, 2}	
1	12	16	20	4	{3, 3, 3, 3}	

With our finite horizon of one year, the following set of constant EOQ cycles $s_k = \{1, 2, 3, 4, 6, 12\}$ and the corresponding constant EOQ policies are of particular interest.

Table 2 shows for these EOQ constant policies the corresponding annual profits, the number of orders placed annually and the percentage of maximum potential annual profits, i.e. efficiency. Notice that EOQ constant 2 and 4 both generate over 93% of the potential annual profits. Given the minimal loss incurred by adopting these policies we define an alternative decision quality benchmark. When a participant chooses $s_k = \{2, 4\}$ we call this “near optimal” performance.

Table 2: Alternative EOQ constant strategies which do not generate stock-outs or positive closing inventories in month 12 and their respective performance properties.

\bar{Q}^{s_k}	Orders per year	Profit per EOQ cycle	Annual profit	Efficiency
12	1	75	75	15.63%
6	2	195	390	81.25%
4	3	155	465	96.88%
3	4	120	480	100.00%
2	6	75	450	93.75%
1	12	20	240	50.00%

2.2 Experimental design

Our experimental design has two treatment variables, each of which has two categories. This generates a 2×2 factorial experimental design. We adopt a between subject design, a participant only experiences one of the four possible treatment cells.

The first treatment variable is the feasible set of inventory policies a participant can follow. The first category is called “Unrestricted” where a participant can choose any quantity they wish each month as long as the quantity does not exceed 120. The second category is called “EOQ”, where participants are restricted to ordering only once the inventory level is zero. We expect that the larger set of alternatives in the unrestricted category presents participants with a more difficult learning task.

The second treatment variable is the level of exogenous cognitive load burden we induce by introducing a competing task. In the “Low” cognitive load category participants complete the inventory tasks without distractions. In the “High” cognitive load we introduce an incentivized PIN task that is completed along side the inventory management task and requires the utilization of short term memory. At the start of each year a participant is given 15 seconds to memorise a random 6-digit PIN. The PIN is case sensitive, consisting of numbers, upper and lower case letters. After the completion of the year, a participant is prompted to enter the PIN. Entering the correct PIN unlocks an extra reward of P300.

A participant only has one attempt at the PIN task. If a participant actively tries to complete the PIN task successfully we expect the diminished access to short term memory to reduce decision making quality and the speed of any learning.

Table 3 summarizes our experimental design and provides summary statistics on the demographics of the participants. We designate treatment cells by the word pairs $x-y$, where x is feasible set of policies category and y is category of the cognitive load.

Table 3: Summary of the demographic information of participants for each treatment

Treatment cell	Participants	Average age	Male	Postgrad	STEM subjects ¹	Average math level ²
EOQ-Low	39	25	23%	47%	34%	3.26
EOQ-High	36	28	50%	56%	28%	3.53
Unrestricted-Low	41	25	34%	49%	37%	3.68
Unrestricted-High	41	25	37%	44%	56%	3.20

¹ STEM subjects include Engineering & Technology, Life Sciences & Medicine and Natural Sciences. Non-STEM subjects include Arts & Humanities and Social Sciences & Management.

² Math Level was self-assessed, and was categorised into 6 levels. 1 = “Below GCSE”, 2 = “GCSE”, 3 = “A-Levels”, 4 = “Undergraduate”, 5 = “Postgraduate”, 6 = “Above Postgraduate”.

2.3 Experimental procedures

Seven sessions were conducted at Newcastle University Business School experimental economics laboratory during May and July 2017. 162 participants² were recruited via random selection for invitation from a participant pool database of the Behavioural Economics Northeast Cluster. All participants were students from Newcastle University except for three who were from Northumbria University.

Each session lasted no more than sixty minutes, with strict procedures to limit the access to any aides that would provide assistance in calculations or remembering PIN codes. Participants were signed in individually and instructed to leave their personal belongings, including any writing instruments, in the reception area before being escorted to a computer desk placed in a privacy carrel. Each participant was then provided with a

² We excluded five participants from our data analysis and the participant counts given in **Table 3**. One participant, in the EOQ-Low treatment, always submitted the random slider starting position when inventory reached zero. Two other participants, in the EOQ-High treatment, grossly took advantage of the limited liability rule. The final two excluded participants attended the last session and demonstrated behaviour that they had been briefed about the content of the experiment; they clicked through the instructions without reading them and subsequently provided the solution \bar{Q}^3 for all years - even though this was not optimal for the practice year.

pen and two copies of an informed consent document, which they read and signed if they wished to continue their participation. The pen and signed forms were then collected by a monitor. After which participants were sternly informed that no electronic devices - such as mobile phones, calculator, smart watches, etc. - could be used until their session was completed. They were further instructed that the rest of the experimental tasks were fully computerized and they would complete the rest of the experiment only using their mouse. Prior to participants entering the laboratory, all computer keyboards were concealed under a thick opaque cover. This was done to diminish any access to mnemonic devices for remembering PIN codes. These measures were taken in all sessions to provide control between High and Low cognitive load treatments.

The experiment itself was conducted using a self-contained program developed in oTree ([Chen et al., 2016](#)). Access was restricted to other programs on the computer. The sum of these measures eliminated many of the tools participants commonly used to perform mathematical calculations. This dismal work environment was applied to all four treatment cells.

Once instructed to start by the monitor, participants read through the instructions³ at their own pace. After reading the instructions, participants were asked to complete seven multiple choice questions designed to ensure that they understand the calculation of costs and profits. Participants who provided more than two incorrect answers had to review the mistaken questions with one of the experimenters before proceeding to the decision tasks.

Participants then participated in the six year decision task sequence, followed by a short post-experiment survey which collected demographic information. Year 0 was a practice round which used an alternative set of cost parameters⁴ from those of Years 1 through 5, and the performance in this task did not affect a participant's total earnings. The purpose of the practice year was to help familiarize the participants with the task and the decision screen. Orders were entered by moving a slider whose value range was zero to one hundred and twenty. The initial point of the slider was random each month, and in the case of an EOQ treatment with a positive starting inventory it was greyed out. The decision screen included a table providing the entire history of a participant's monthly ordering choices, as well as opening inventory, units sold, closing inventory, sales revenue, ordering costs, holding costs and profits.⁵ For participants who experienced the High cognitive load treatment, we provided an opportunity to practice the PIN task in the practice Year.

Participants then completed the Years 1 through 5 decision tasks. Participants were paid

³In the first Appendix, we provide a complete set of instructions.

⁴ In the practice year the order costs were P45 and the holding costs were P0.5.

⁵ We provide screen captures of these interfaces in the Appendices.

for their accumulated earnings from these decision tasks, at the conversion rate of P300 = £1, as well as a £5 show-up fee. There was limited liability; to ensure the motivation to make profits would not be affected by a large negative earnings made in a particular year, any negative profits made in a year will be treated as 0 earnings.⁶ The average earnings were £13.37 per participant, including the participation fee.

One last important aspect of the experiment was the fixed length of time a participant had to complete the inventory management task for a year. We required that a participant spend exactly four minutes completing each task in Years 1 through 5. This was designed to prevent participants from racing through the monthly decisions in order to reduce the cognitive cost of remembering their PIN. If a participant completed their twelve monthly decisions early they could not advance to the next period (or enter the PIN) until the four minutes expired. If they failed to complete the twelve tasks before the time expired, the computer program executed the remaining months sales with the existing inventory stock.

3 Empirical evaluation of treatment effects

We evaluate the treatment effects of restricted inventory policy choice sets and increased cognitive load by considering their impacts upon participant's earnings in the inventory management tasks, the propensity to choose optimal inventory policies, and then the efficacy of the PIN task and whether performance in that task is correlated with inventory performance.

3.1 Hypotheses

Our motivation of treatment variables leads to several natural hypotheses. Increases in cognitive load reduces short term memory capacity and lead to diminished performance in both the EOQ and Unrestricted policy choice sets, giving the following two hypotheses:

Hypothesis 1. *Average annual earnings are greater in the EOQ-Low treatment than the EOQ-High treatment, as well as in the Unrestricted-Low treatment versus Unrestricted-High treatment.*

As suggested by the number of alternative EOQ constant policies which generate near optimal performance levels, we suggest the following hypothesis may be less likely to confirm:

Hypothesis 2. *The percentage of participants who adopt optimal (near-optimal) inventories is greater in the EOQ-Low treatment than the EOQ-High treatment, as well as in the Unrestricted-Low treatment versus Unrestricted-High treatment.*

⁶ This limited liability only affected the earnings of five participants in five different years.

The set of inventory policies in the unrestricted is much larger than and only adds suboptimal alternatives to the EOQ restricted set of policy choices. The reducing the focalness of EOQ strategies and greatly complicating participants' choice sets in the Unrestricted treatments leads to our next set of hypotheses:

Hypothesis 3. *Average annual earnings are greater in the EOQ-High treatment than the Unrestricted-High treatment, as well as in the EOQ-Low treatment versus Unrestricted-Low treatment.*

Hypothesis 4. *The percentage of participants who adopt optimal (near-optimal) inventories is greater in the EOQ-High treatment than the Unrestricted-High treatment, as well as in the EOQ-Low treatment versus Unrestricted-Low treatment.*

3.2 Annual inventory profits

We test the differences in average annual profit for different treatment groups using two-sided t -tests and non-parametric Wilcoxon rank-sum tests. We report the results of these hypotheses tests in [Table 4](#). The first two rows indicate that both giving participants unrestricted policy choices and shocking their cognitive load each negatively impact average annual profits both statistically and economically. More complicated policy choices cause more profit loss than High cognitive load.

When we examine the effect of exogenously increasing a participant's cognitive load conditional on the policy choice set we find mixed support for Hypothesis 1. There is a statistically significant reduction in average in earnings in the EOQ treatment, but not in the Unrestricted treatment. We do find stronger evidence in support of Hypothesis 3, as we find limiting participant's choices to EOQ restricted policies does lead to statistically greater average earnings in both Low and High cognitive load settings.

A disaggregated view of the average annual profits permit insights into learning over time and how our treatments impact it. [Figure 1](#) presents these time trends for each of the four treatments. There are several prominent features of this figure which provide refined insights into our hypotheses results on the average profit levels. First, performance gains are mostly achieved in Years 1 through 3. Second, average earnings are around 90% of the possible earnings in the last two years; except for the Unrestricted-High treatment which are around 5-10% lower. Third, High cognitive load and Unrestricted policy choice sets both cause the greatest negative performance impact in Year 1.

Table 4: Average annual profits by treatment and hypotheses tests for differences in average annual earnings

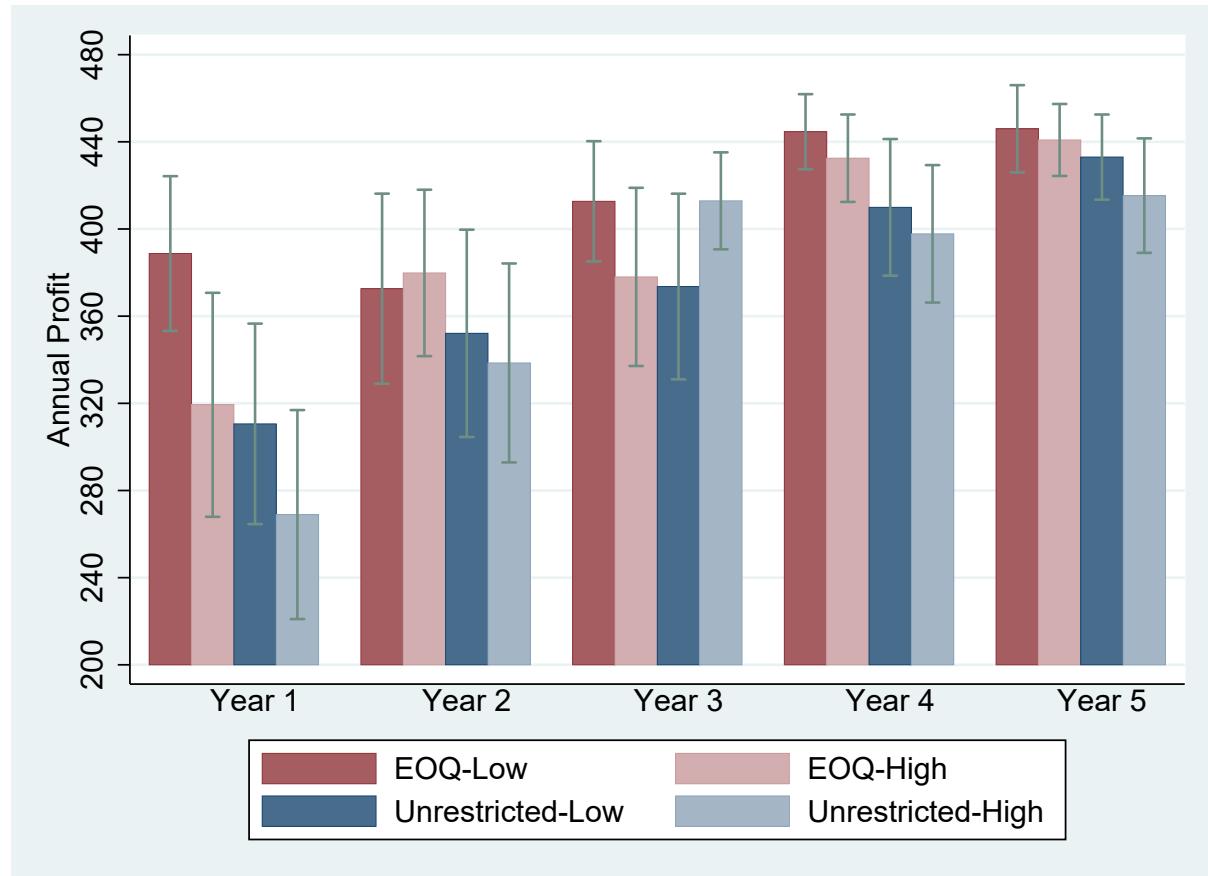
Panel A: Annual profits by treatment

	EOQ-Low	EOQ-High	Unrestricted-Low	Unrestricted-High
Average	412.94	390.10	375.85	366.70
Stand. Dev.	97.35	113.78	129.38	126.87

Panel B: Hypotheses tests for differences in average annual profits (*p*-values reported)

Treatment Comparison	Difference	Profit loss (%)	Two-sided <i>t</i> -tests	Wilcoxon rank-sum
EOQ vs Unrestricted	30.71	7.64%	0.000	0.001
Low vs High	16.29	4.14%	0.055	0.003
EOQ-Low vs EOQ-High	22.84	5.53%	0.038	0.012
Unrestricted-Low vs Unrestricted-High	9.15	2.43%	0.470	0.124
EOQ-Low vs Unrestricted-Low	37.09	8.98%	0.001	0.012
EOQ-High vs Unrestricted-High	23.40	6.00%	0.059	0.052

Figure 1: Annual Profits over individual Years and by treatment: Averages and 95% confidence intervals



We quantify and assess these remarks by conducting a series of dummy variable linear regressions using robust standard errors. We report these results in [Table 5](#). In model (1), we simply regress annual profit on a constant and dummy variables for Years 1 through 4, rendering Year 5 the base level. In model (2) we introduce dummy variables for the Unrestricted and High treatment categories. In this case the constant reflects the average profit level for Year 5 in the EOQ-Low treatment; and the Year 1 through 4 dummy variable coefficients reflect the average annual profits across participants in the EOQ-Low treatment. In the model (3), we add interaction dummy variables for the Unrestricted and High treatment categories to examine if their joint imposition leads to super- or sub-additive impact on annual profit.

Our treatment effects for Unrestricted and High are largely generated by their Year 1 impacts as seen by their individually significant coefficients in models (2) and (3). We conduct a Chow, F -tests, for which the null is model (1) versus the alternative of model (2), i.e. the joint differences of the two treatments are significant. The resulting F -stat is 3.09, the degrees of freedom are (10, 770), and has a p -value of 0.001. We conduct a second F -tests to compare the veracity of model (3) versus model (2). The resulting F -stat in this case is 1.14, the degrees of freedom are (5, 765), and has a p -value of 0.336. Our analyses of annual profits leads us to our first set of results.

Result 1. *Reducing the participants' policy choice sets to EOQ restricted ones leads to higher profits. However, these gains predominantly occur in Year 1 - when the participants face the inventory decision problem for the first time.*

Result 2. *Exogenously increasing participants' cognitive load leads to lower profits. However, these losses predominantly occur in Year 1 - when the participants face the inventory decision problem for the first time.*

Result 3. *There is no super- or sub-additive effect of simultaneously exposing participants to the Unrestricted and High treatment categories.*

3.3 Inventory management policy choices

We turn our analysis towards the inventory policy choices of participants. For each participant we evaluate each of the annual inventory policies, $Q_{i,a}$, for whether it is optimal, \bar{Q}^3 , or if its near-optimal, and EOQ constant strategy of either \bar{Q}^2 or \bar{Q}^4 . [Figure 2](#) depicts the evolution across years of the percentages of participants following optimal and near-optimal policies in each treatment. Inspection of this figure reveals our next set of results.

Result 4. *There is a trend in all treatments for increasing use of optimal and near-optimal policies from Year 1 to Year 4.*

Result 5. *High cognitive loads leads to lower percentage use of these policies for both EOQ and Unrestricted in all five Years.*

Table 5: Dummy variable regressions for annual profit. ($n=785$)

Dummy Variable	(1) Annual Profit	(2) Annual Profit	(3) Annual Profit
Year 1	-112.27*** (12.80)	-67.35*** (18.85)	-57.22*** (20.12)
Unrestricted·Year 1		-45.45* (24.64)	-65.22** (31.90)
High·Year 1		-43.20* (24.86)	-64.31* (33.32)
Unrestricted·High·Year 1			40.40 (49.55)
Year 2	-73.39*** (12.12)	-71.29*** (20.57)	-73.36*** (23.70)
Unrestricted·Year 2		-11.56 (23.99)	-7.53 (34.78)
High·Year 2		8.04 (24.15)	12.35 (31.31)
Unrestricted·High·Year 2			-8.24 (48.05)
Year 3	-38.81*** (10.00)	-54.99*** (15.69)	-33.27** (16.84)
Unrestricted·Year 3		16.24 (20.08)	-26.15 (28.67)
High·Year 3		15.70 (20.12)	-29.56 (27.46)
Unrestricted·High·Year 3			86.59** (39.79)
Year 4	-12.85 (8.47)	-4.49 (12.33)	-1.33 (13.05)
Unrestricted·Year 4		-15.59 (16.38)	-21.75 (22.47)
High·Year 4		-0.45 (16.69)	-7.03 (18.27)
Unrestricted·High·Year 4			12.59 (32.88)
Unrestricted		-19.12* (10.29)	-12.96 (13.84)
High		-11.70 (10.42)	-5.13 (12.80)
Unrestricted·High			-12.58 (20.65)
Constant	433.40*** (5.27)	449.13*** (8.68)	445.97*** (9.89)
R^2	0.12	0.16	0.16
F-statistic	26.02***	10.34***	7.82***

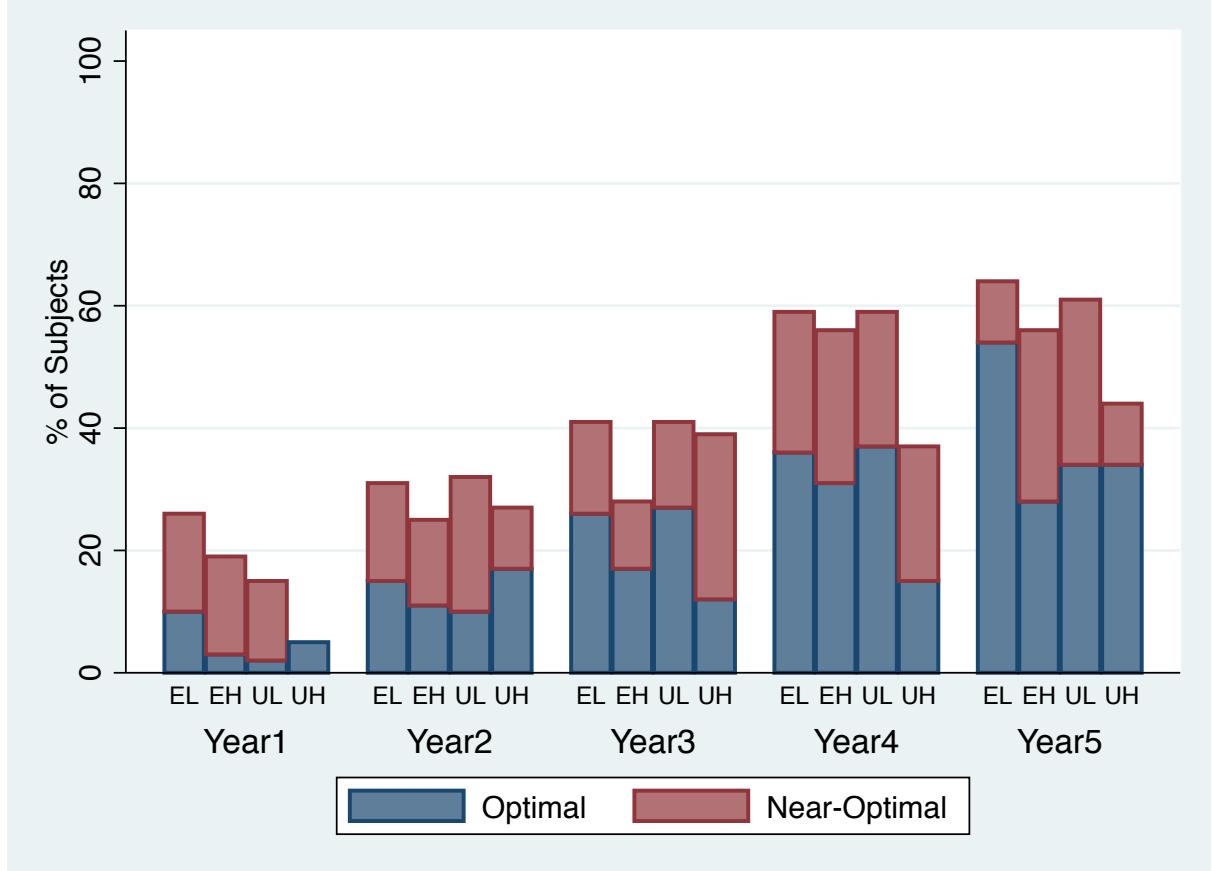
Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

3.4 Efficacy of the PIN reward procedure

Next we evaluate the efficacy of procedure for exogenously increasing the cognitive load. Our experimental design faces a challenging balancing act. If the PIN reward procedure is

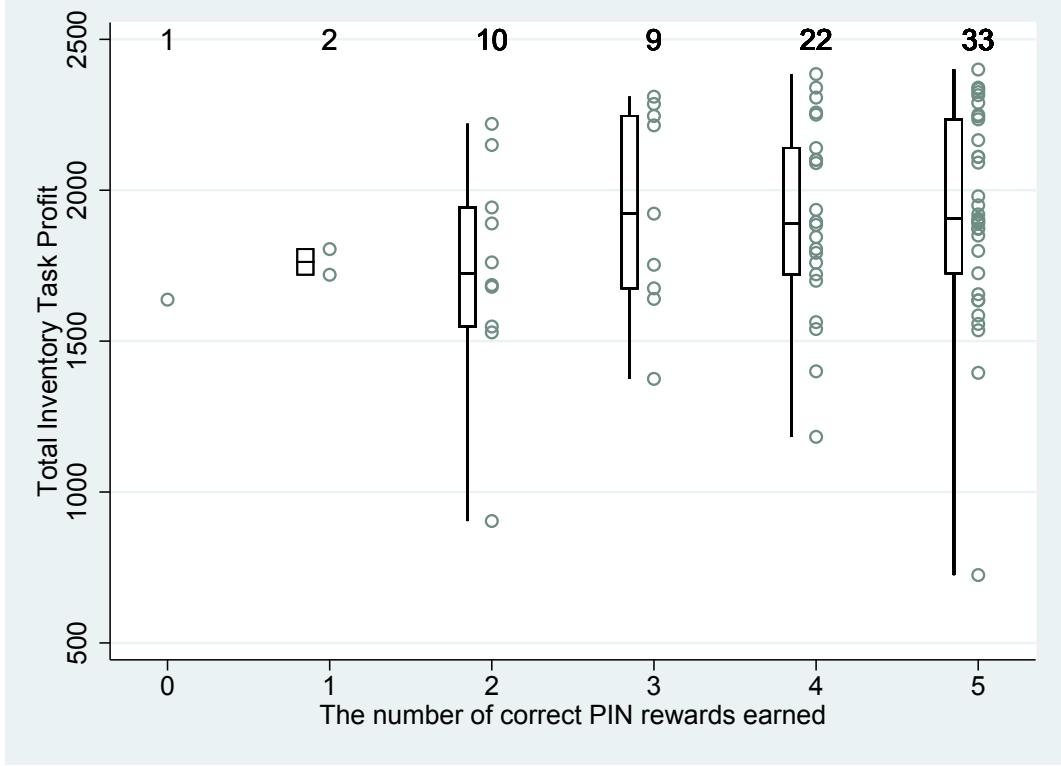
Figure 2: Stacked graph of the percentage of participants following optimal and near-optimal EOQ constant strategies: by Year and treatment



too simple participants will always collect the reward utilizing minimal short run memory resources, and if it is too difficult they could either decide to forgo the mental costs of trying to commit the PIN to short term memory or forgo effort in the Inventory management tasks. A second concern is that raw intelligence is an omitted variable in our analysis which would manifest itself in a strong positive correlation between a participant's performances in the PIN reward and the Inventory management task.

We provide visual evidence that our design successfully addresses this balancing act in [Figure 3](#). First, we observe that only three out of the seventy-seven participants earned one or less PIN rewards; and at the same time thirty-three out of seventy-seven collected all five pin rewards. Second, there doesn't appear to be a clustering of poor Inventory management performers, below the *ad hoc* threshold of ₦1500, on high or low numbers of earned PIN rewards. Third, there is little evident differences in the conditional means of total profits - suggesting the PIN and inventory management tasks performance are independent.

Figure 3: Participants’ total inventory management task profits conditional on the number of PIN rewards earned and the corresponding whisker plots for the 50, 75, and 95% quantiles. The numbers across the top are the counts of participants who earned the corresponding number of PIN rewards.



We quantify the evidence of the independence of PIN and Inventory management task performance by statistically measuring their correlation and testing its statistical significance. [Table 6](#) reports these correlations and the p -values of the hypotheses tests that the correlation is zero. The left portion of the table addresses the correlation between the success of a PIN reward task and the corresponding annual inventory profit. The evidence is mixed. We don’t find correlations significantly different from zero in four out of five years, but do find a highly significant positive correlation when we pool all of the years. This analysis suggests potential positive correlation between a correct PIN tasks and individual reward; however this analysis does not allow for differences in participants’ performances for the PIN task. To address this concern we evaluate the correlations between the total number of PIN rewards earned by a participant j and both j ’s annual profits and her total Inventory tasks profit. We report these correlations in the right side of [Table 6](#). In this analysis we find evidence in favor of no correlation. None of these correlations is significant.

Table 6: Spearman correlations between PIN reward earned in Year a by participant j and j 's corresponding Inventory task profit; Spearman and Pearson Rank correlations between a participant j 's total number of earned PIN rewards and their Inventory task profits

	PIN reward eared in Year a		Number of PIN reward earned	
	Spearman Rank Corr.		Pearson Corr.	Spearman Rank Corr.
	Year 1	0.08 (0.512)	0.13 (0.248)	0.11 (0.324)
Annual Profit	Year 2	0.12 (0.315)	0.08 (0.507)	0.11 (0.338)
	Year 3	0.21 (0.072)	0.10 (0.391)	0.04 (0.750)
	Year 4	0.09 (0.459)	0.15 (0.182)	0.04 (0.725)
	Year 5	0.14 (0.226)	0.10 (0.379)	0.14 (0.218)
	All Years	0.18 (0.001)	N/A N/A	N/A N/A
Total Profit		N/A N/A	0.179 (0.120)	0.182 (0.114)

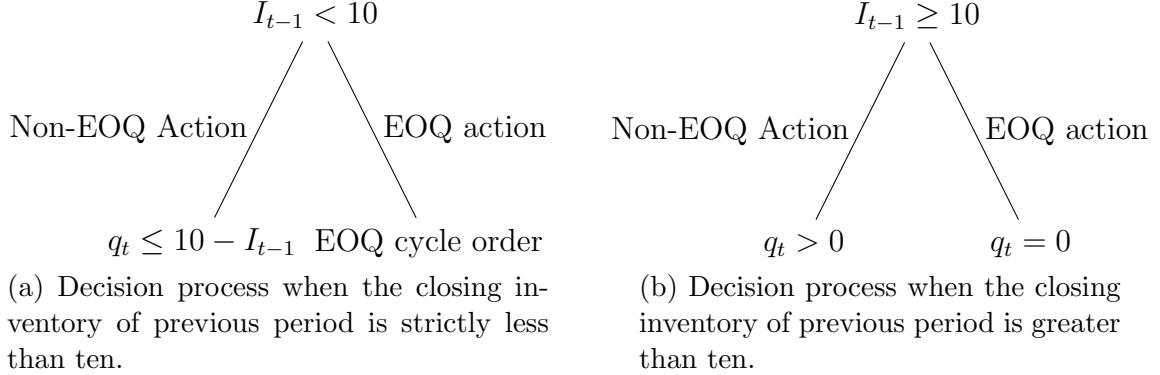
1. The p -values of the respective tests are reported in the parenthesis.
2. We don't report the correlations for Total Profit in column three because the calculation will include multiple repetitions of a participant's total inventory profit.
3. We don't report the correlations for all Years in columns four and five because the calculation will include multiple repetitions of a participant's total number of PIN rewards.

4 Learning Dynamics

In our final analysis we present and estimate a Markovian learning model for participants' monthly order choices. Avoiding stock out - thus not foregoing potential profit - and only ordering when sales have exhausted inventory - thus avoiding excess holding costs - are two key logical motivations for choosing EOQ consistent actions. We formulate a learning process for monthly choices as a decision tree where the first branch is avoiding one of these two pitfalls, and the second branch is the Markov process by which one chooses an EOQ cycle when inventory reaches zero. [Figure 4](#) depicts this process.

We will formulate the probabilities of choosing Non-EOQ actions as simple Logit functions of time, habit formation and whether it is a High cognitive load treatment. As the experimental design prunes Branch 1 for the EOQ treatment for the most part, of key interest here is whether High cognitive load leads to larger probabilities of Non-EOQ actions. Then when an individual chooses an order once inventory reaches zero, we use a low rationality Markov model to specify how participants switch from one EOQ cycle

Figure 4: The branching decision process. First, there is a choice of proceeding to Branch 1 and taking Non-EOQ action or Branch 2 and taking an EOQ action. This formulation depends upon whether the closing inventory of previous period is greater or less than 10.



length to another. In this model we examine the probability of switching to an at least as profitable EOQ action and the viscosity to making large changes to EOQ cycle length.

4.1 Branch Decision 1

To investigate the factors that influence the probability of participants deviating from an EOQ action in any one of the sixty decision rounds with financial incentives we first define an indicator function for

$$NonEOQ_{i,r} = \begin{cases} 1 & \text{if } q_{i,r} \text{ is not an EOQ action in decision round } r, \text{ and} \\ 0 & \text{otherwise.} \end{cases}$$

where $r \in \{1, 2, \dots, 60\}$.

We estimate sets of Logit regressions on the probability a participant chooses a NonEOQ action for two cases; one when the previous month's closing inventory is strictly less than ten and one when it is at least ten. In both cases we consider the following specification

$$Pr(NonEOQ_{i,r} = 1) = F(\beta_0 + \beta_1 Year_r + \beta_2 Month_r + \beta_3 High + \beta_4 NonEOQACC_{i,r-1}).$$

Here F is the logistic cumulative distribution function and $NonEOQACC_{i,r-1}$ is the total number of rounds participant i has deviated from EOQ up through round $r - 1$ - this is intended to capture any habit formation. Note this is a running count of a participant's NonEOQ actions in either state.

The Logit regression results are presented in [Table 7](#): Panel A for the case $I_{t-1} < 10$ and Panel B for the case $I_{t-1} \geq 10$. For the prior case, deviations from an EOQ action occur due to the possibilities of stock outs. While our design was motivated to only allow participants in the Unrestricted to make such NonEOQ actions, it may also happen in

the EOQ treatment when a participant orders less than 10 when the closing inventory of previous period is 0. There are only 40 such observations out of 4500, but we do include these in the Panel A results.⁷ For the latter case - the closing inventory of previous period is at least ten - the only possible deviation from an EOQ action is to order a strictly positive amount, which is not allowed in the EOQ treatment group. For such state only observations from the Unrestricted treatment groups are included.

Table 7: Logit regression on the probability of deviating from an EOQ action

Panel A: $I_{t-1} < 10$				Panel B: $I_{t-1} \geq 10$		
$nonEOQ_{i,r}$	(1)	(2)	(3)	(4)	(5)	(6)
$Year_r$	-0.498*** (0.126)	-0.500*** (0.128)	-0.690*** (0.160)	-0.308*** (0.096)	-0.309*** (0.097)	-0.608*** (0.134)
$Month_r$	0.194*** (0.047)	0.194*** (0.047)	0.153*** (0.054)	-0.115*** (0.027)	-0.114*** (0.027)	-0.141*** (0.029)
High		0.255 (0.425)	0.216 (0.311)		-0.404 (0.376)	-0.193 (0.296)
$NonEOQACC_{i,r-1}$			0.288*** (0.039)			0.307*** (0.040)
Constant	-3.379*** (0.583)	-3.507*** (0.630)	-3.175*** (0.690)	-1.754*** (0.338)	-1.573*** (0.397)	-1.311*** (0.430)
N	3032	3032	2875	3286	3286	3286
χ^2	34.10***	36.06***	97.89***	22.66***	22.90***	75.76***
$Pr(NonEOQ_{i,r}) = 1$	0.034	0.034	0.037	0.035	0.035	0.030

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

First, note the large negative values of the estimated coefficients pushing the argument of the logistic CDF to its far left tail. Thus all estimated probabilities of NonEOQ actions are small as indicated by the last row of the table which reports the estimated probability of a NonEOQ action at the average level of the factors. Second, two significant factors, both statistically and economically, are the number of years and the accumulation of experience of choosing NonEOQ actions. The large estimated coefficient indicates there is significant learning to choose EOQ actions across the five years. The positive estimated value of the coefficient of $NonEOQACC_{i,r-1}$ captures the individual differences in the epiphany of the EOQ logic. The estimated coefficients for Months are statistically significant, but have low magnitude in moving probabilities meaningfully are of opposite signs in two cases. This suggests that stockouts are more likely later a year while ordering when there is excess inventory is less likely later in a year. Surprisingly there is no significant effect of having a high cognitive load on taking NonEOQ actions. Thus the performance differences must come from the types of EOQ actions one takes under high cognitive load.

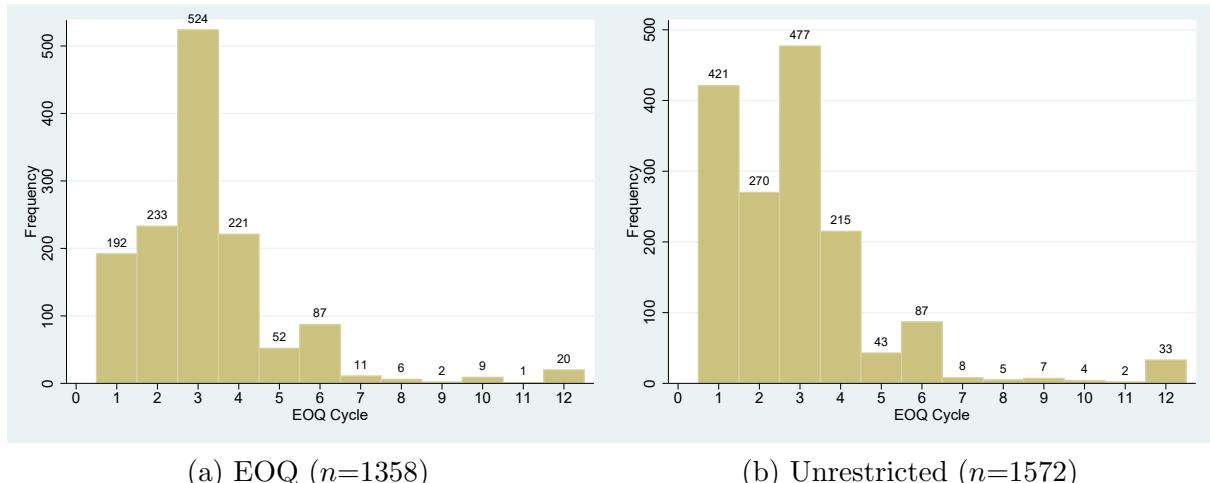
⁷Also there is another possible way to deviate from an EOQ action in the EOQ treatment. Participants may have positive closing inventory of previous period that is less than 10 but are not allowed to place order (138 out 4500 observations). We exclude these observations as they are not by choice.

Overall we interpret this evidence that providing the more complicated choice set does lead to some NonEOQ actions, but these choices diminish with experience.

4.2 Branch Decision 2: A Markov model of EOQ cycle choice

Once an EOQ action is taken, the second branches in [Figure 4](#), we consider how the participant chooses an EOQ cycle length. First, we make a slight modification to our definition of an EOQ cycle to handle situations in the Unrestricted treatment when the previous month's closing inventory is strictly positive but strictly less than ten. Let $\tilde{s}_{i,k}$ denotes the largest integer less than or equal to $\frac{I_{t-1}+q_t}{10}$. To see how this change of definition works consider the following simple example. If a participant has a closing inventory of 2 units from previous period and orders 8 units, then $\tilde{s}_{i,k} = 1$. [Figure 5](#) shows histograms of EOQ cycles choices using this new definition in both Unrestricted and EOQ treatments. This figure illustrates that we see more of the typically optimal EOQ cycles of length three in the EOQ treatment, and more extreme EOQ cycles of lengths one and twelve in the Unrestricted treatment. Using the information of [Figure 5](#) we move forward considering the set of possible EOQ cycle length $\tilde{s}_{i,k} \in \{1, 2, 3, 4, 5, 6, 12\}$.⁸

[Figure 5](#): EOQ cycle choice histograms for EOQ and Unrestricted treatments



Proceeding to the dynamics of a participant's sequence of EOQ cycle choices, we compare the relative ranking of alternative EOQ cycles by their monthly average profit conditional upon month. We denote this monthly average profit as $\bar{\pi}_t(\tilde{s}_{i,k})$. Notice that the pay off function depends upon t and will penalize relatively long EOQ cycles that generate excess inventory at the year's end. We report the values of $\bar{\pi}_t(\tilde{s}_{i,k})$ in [Table 8](#).

⁸Due to the low number of observations we round down EOQ cycles of $\tilde{s}_{i,k} = \{7, 8, 9, 10, 11\}$ to $\tilde{s}_{i,k} = 6$. Also, note that we are including $\tilde{s}_{i,k} = 5$ as an EOQ choice cycle given the high frequency it is chosen despite it not corresponding to a EOQ constant policy.

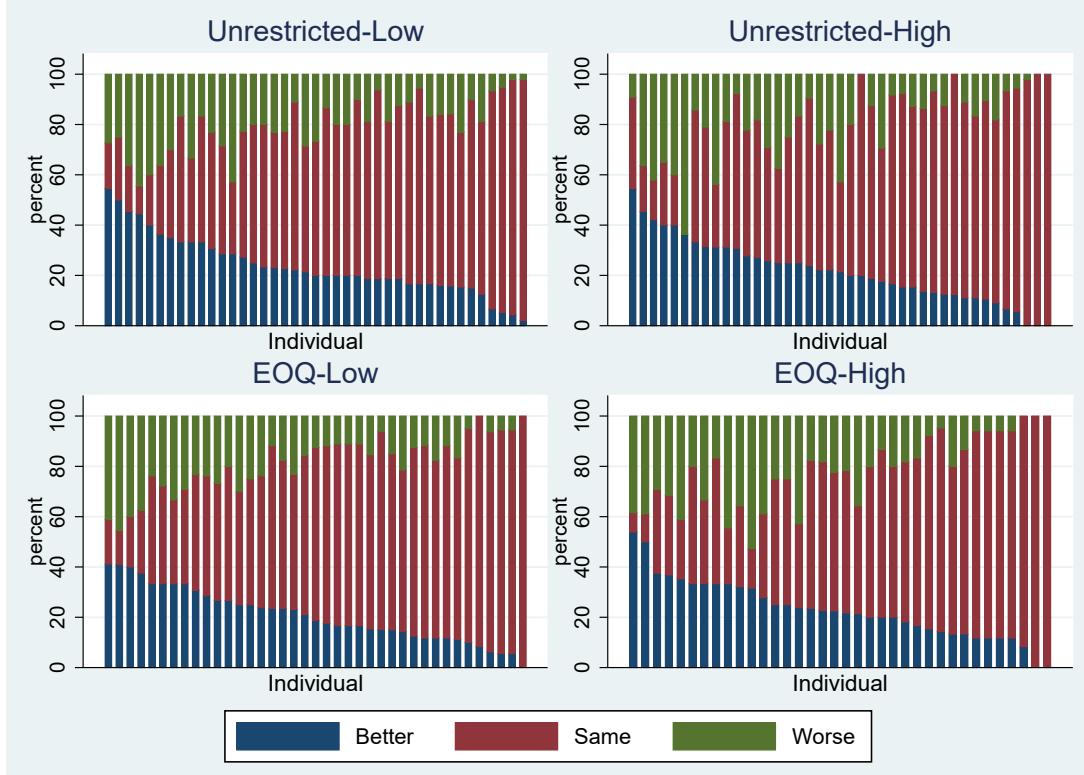
Table 8: Average monthly profit for alternative EOQ cycle choice given the current month

s	1	2	3	4	5	6	12 ¹
Month 1-7	20	37.5	40	38.75	36	32.5	-
Month 8	20	37.5	40	38.75	36	26	-
Month 9	20	37.5	40	38.75	28.75	18.75	-
Month 10	20	37.5	40	30	20	10	-
Month 11	20	37.5	27.5	17.5	7.5	-2.5	-
Month 12	20	10	0	-10	-20	-30	-

¹ $\tilde{s}_{i,k} = 12$ always offers the lowest average monthly payoff

We use this measure to evaluate whether a participant's EOQ cycle choice generates a higher monthly average profit than their previous EOQ cycle choice. For each individual we consider the proportions of transitions to higher, the same, and lower pforit cycles. We plot these proportions by treatment cell in [Figure 6](#) and sort individuals by the proportion of 'better' transitions. This figure illustrates that participants exhibit rather limited individual rationality as their frequency of transitioning to a more profitable EOQ cycle tend to only slightly exceed that of switching to a less profitable cycle. Further there is a large amount of EOQ cycle choice repetition.

Figure 6: Proportions of Better, Same and Worse EOQ cycle transitions - ranked by Better



For a situation in which individuals similarly do not find the subset of higher ranked alternatives salient and there is an ordinal property - but not always monotonic in reward - to the set of alternatives, [Shachat and Zhang \(2017\)](#) introduced a Markov model of limited rationality to describe learning. We adapt that model for our setting. EOQ cycle transitions probabilities are governed by a two-stage process. In the first stage, probability is allocated between two subsets of possible EOQ cycles: NW , the subset of EOQ cycles no worse than $\tilde{s}_{i,k-1}$, and NB , the subset of EOQ cycles no better than $\tilde{s}_{i,k-1}$.⁹ Specifically,

$$NW_t(\tilde{s}_{i,k-1}) = \{j \in \{1, 2, 3, 4, 5, 6, 12\} | \bar{\pi}_t(j) \geq \bar{\pi}_t(\tilde{s}_{i,k-1})\}$$

$$NB_t(\tilde{s}_{i,k-1}) = \{j \in \{1, 2, 3, 4, 5, 6, 12\} | \bar{\pi}_t(j) \leq \bar{\pi}_t(\tilde{s}_{i,k-1})\}$$

NW and NB may not be mutually exclusive; they will share the previous choice of an EOQ cycle when there are sufficient months remaining in the year. We assume that an α measure of probability is allocated to the NW set and a $1 - \alpha$ measure of probability is assigned to the NB set.

In the second stage, probability measure is allocated amongst the elements within each of these subsets. Such allocation is allowed to reflect participants possibly favouring the cycle having a smaller difference in length with the previous cycle. Specially, probability is allocated according to the number of steps between an element and the previous cycle length. The step count between EOQ cycle length j and j' is defined as,

$$\theta(j, j') = |j - j'| + 1.$$

A special case of $j = 12$ is treated as 2 steps from $j' = 6$.

We use the following weighting function to determine an EOQ cycle's assigned share of probability measure,

$$w(j|\tilde{s}_{i,k-1}, Z, \lambda) = \frac{\theta(j, \tilde{s}_{i,k-1})^\lambda}{\sum_{j' \in Z} \theta(j', \tilde{s}_{i,k-1})^\lambda}, \forall j \in Z$$

in which Z is either the NW or NB subset. In the proportional assignment, $\lambda \leq 0$ measures the strength of the bias for small changes within the subset Z . A decrease in λ corresponds to a growing bias. We calculate the transition probability for each EOQ

⁹ These subsets change depending on which month the choice occurs due to finite horizon. For instance, $\tilde{s}_{i,k} = 3$ would be in NW subset of $\tilde{s}_{i,k-1} = 1$ in month 10, but will change to be in NB subset in month 12. A detailed listing on NW and NB subsets for different month can be found in Appendix C.

cycle by adding up the probability measures it is allocated from the NW and NB subsets,

$$\begin{aligned} Pr(\tilde{s}_{i,k} = j | \tilde{s}_{i,k-1}) &= \alpha \times \mathbb{1}_{(j \in NW_t(\tilde{s}_{i,k-1}))} \times w(j | \tilde{s}_{i,k-1}, NW_t(\tilde{s}_{i,k-1}), \lambda) \\ &\quad + (1 - \alpha) \times \mathbb{1}_{(j \in NB_t(\tilde{s}_{i,k-1}))} \times w(j | \tilde{s}_{i,k-1}, NB_t(\tilde{s}_{i,k-1}), \lambda) \end{aligned}$$

For example, if $\tilde{s}_{i,3} = 1$ and $\tilde{s}_{i,4} = 3$, the transition probability is $\alpha \frac{3^\lambda}{\sum_{j=1}^6 j^\lambda}$, while if $\tilde{s}_{i,11} = 1$ and $\tilde{s}_{i,12} = 3$, the transition probability is $(1 - \alpha) \frac{3^\lambda}{\sum_{j=1}^7 j^\lambda}$.

We estimate the two parameters of the Markov choice model for each treatment cell by maximum likelihood estimation and present them in [Table 9](#). In all treatments, the magnitude of approximately 70% of α indicates that participants are more likely to move into their current NW set. However, the ability to order in any month and introducing cognitive load reduce the probability of switching to more profitable actions. The estimate of λ is larger in magnitude for the Unrestricted treatments, indicating a larger bias for small changes within the sets. The ability to order in any month leads to a greater degree of action lock-in. However, the differences of the estimates of the parameters are not statistically significant when we estimate these coefficients jointly and test for differences using likelihood ratio tests - p -values are approximately 0.15 in each case.

[Table 9](#): Parameter estimates for the Markov EOQ cycle choice model, standard errors in parentheses

Parameter	EOQ-Low	EOQ-High	Unrestricted-Low	Unrestricted-High
α	0.760 (0.031)	0.712 (0.032)	0.708 (0.041)	0.676 (0.040)
λ	-0.709 (0.178)	-0.782 (0.137)	-1.104 (0.217)	-1.320 (0.209)

Overall we find the Unrestricted treatment leads to a small percentage of Non-EOQ actions, generating performance diminishing outcomes of excess inventories and stockouts. However, we find the likelihood of these events diminish over time and is surprisingly unaffected by high cognitive loads. The more complex choice sets of the Unrestricted treatment also leads to more inertia in EOQ cycle length choices inducing choice lock-in. This is a likely cause of participants choosing near rather than absolute optimal policies in the last two years. This is a similar phenomenon found in [Caplin et al. \(2011\)](#); as they increase choice set complexity participants tend to switch within a smaller range of values.¹⁰ The effect of the High cognitive load is for participants to exhibit a lower level of rationality once they choose EOQ actions; their probability of choosing EOQ cycles

¹⁰See their Figure 4: Average Value by Selection.

that generate at least the same level of average monthly profit is lower than for those participants who do not have the competing PIN memorization task.

5 Conclusion

We present an experimental investigation to assess the effect of cognitive stress on inventory management decisions in an EOQ model. We exogenously impose cognitive stress from two sources: increased complexity of the inventory policy choice set and increased cognitive load from a PIN task that competes for the participants' short term memory resources. Both sources of cognitive stress negatively impact participants' performance. However, these negative impacts occur predominantly when participants first face the inventory decision problem. While average performance is not statistically different, we note that only in the EOQ-Low treatment cell do we observe the majority of participants eventually learn to use the optimal EOQ policy. We model and then estimate participants learning of monthly action choices using a Markovian learning framework. We find that the availability of the more complicated choice set causes some deviations from EOQ actions, but such deviations diminish with experience. Further, the ability to order in any month leads to a greater degree of EOQ cycle length choice lock-in. Increased cognitive load reduces the probability of switching to more profitable actions.

The EOQ is a prevalent tool of inventory managers in the field. Our results provide managerial insights, particularly in the case of accidental or inexperienced inventory managers. It is clear that asking such individuals to simultaneously complete other tasks impedes their learning of effective inventory management. Further, there is value in restricting the manager's possible actions to those consistent with EOQ policies. Absent this intervention, there is a greater chance of locking in suboptimal EOQ cycles. Of course, in the long run we observe near identical performance with enough experience. But one should proceed with caution thinking that good management will arise eventually with experience; our environment is constant and certain. [Chen and Wu \(2017\)](#) demonstrated that changing ordering and holding costs will slow the learning process.

We believe this is a successful first step in evaluating and developing interventions to minimize the impact of cognitive stress on inventory management performance. Our *ex ante* expectation was that cognitive load would have the more severe impact that would manifest itself in more varied directions that choice set complexity. However, it does appear that presentation of policies has the more complicated, and hence providing more scope for intervention design, impact on the decision-making process. Some natural next steps are to explore how the choice set complexity and corresponding framing impact decision making in the other previously raised inventory management paradigms such as the newsvendor problem, (S, s) inventory management, and multi-tiered supply chains.

References

- Abeler, J. and Jäger, S. 2015. Complex tax incentives. *American Economic Journal: Economic Policy*, 7, 1–28.
- Bode, C. and Wagner, S. M. 2015. Structural drivers of upstream supply chain complexity and the frequency of supply chain disruptions. *Journal of Operations Management*, 36, 215–228.
- Caplin, A., Dean, M., and Martin, D. 2011. Search and satisficing. *American Economic Review*, 101, 2899–2922.
- Chen, D. L., Schonger, M., and Wickens, C. 2016. otree - an open-source platform for laboratory, online, and field experiments. *Journal of Behavioral and Experimental Finance*, 9, 88–97.
- Chen, K.-Y. and Wu, D. 2017. Learning under the deterministic economic order quantity problem. *University of Texas - Arlington*.
- Croson, R., Donohue, K., Katok, E., and Sterman, J. 2014. Order stability in supply chains: Coordination risk and the role of coordination stock. *Production and Operations Management*, 23, 176–196.
- Hinson, J. M., Jameson, T. L., and Whitney, P. 2003. Impulsive decision making and working memory. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 29, p. 298.
- Katok, E. et al. 2011. Using laboratory experiments to build better operations management models. *Foundations and Trends in Technology, Information and Operations Management*, 5, 1–86.
- Khaw, M. W., Stevens, L., and Woodford, M. 2017. Discrete adjustment to a changing environment: Experimental evidence. *Journal of Monetary Economics*, 91, 88–103.
- Lleras, J. S., Masatlioglu, Y., Nakajima, D., and Ozbay, E. Y. 2017. When more is less: Limited consideration. *Journal of Economic Theory*, 170, 70–85.
- Magnani, J., Gorry, A., and Oprea, R. 2016. Time and state dependence in an (S, s) decision experiment. *American Economic Journal: Macroeconomics*, 8, 285–310.
- Masatlioglu, Y., Nakajima, D., and Ozbay, E. Y. 2012. Revealed attention. *American Economic Review*, 102, 2183–2205.
- Miller, G. A. 1956. The magical number seven, plus or minus two: Some limits on our capacity for processing information. *Psychological Review*, 63, p. 81.
- Narayanan, A. and Moritz, B. B. 2015. Decision making and cognition in multi-echelon supply chains: An experimental study. *Production and Operations Management*, 24, 1216–1234.
- Roch, S. G., Lane, J. A., Samuelson, C. D., Allison, S. T., and Dent, J. L. 2000. Cognitive load and the equality heuristic: A two-stage model of resource overconsumption in small groups. *Organizational Behavior and Human Decision Processes*, 83, 185–212.

- Ruderman, M. N., Clerkin, C., and Deal, J. J. 2017. The long-hours culture. In C. L. Cooper and M. P. Leiter eds. *The Routledge Companion to Wellbeing at Work*. Taylor & Francis, Chap. 15, 207–221.
- Schwarz, L. B. 1972. Economic order quantities for products with finite demand horizons. *AIIE Transactions*, 4, 234–237.
- Shachat, J. and Zhang, Z. 2017. The hayek hypothesis and longrun competitive equilibrium: An experimental investigation. *Economic Journal*, 127, 199–228.
- Shiv, B. and Fedorikhin, A. 1999. Heart and mind in conflict: The interplay of affect and cognition in consumer decision making. *Journal of Consumer Research*, 26, 278–292.
- Stangl, T. and Thonemann, U. W. 2017. Equivalent inventory metrics: A behavioral perspective. *Manufacturing & Service Operations Management*, 19, 472–488.
- Supply & Demand Chain Executive. 2005. Hurricane Katrina showed importance of logistics. Retrievable from <https://www.sdcexec.com/sourcing-procurement/news/10356958/hurricane-katrina-showed-importance-of-logistics-supply-chain-management>.

A Experiment Instructions and Interface

Instructions for different treatments are presented as the texts/sentences in italics and square brackets below.

A.1 Instruction Page

Welcome

Welcome to today's experiment. Please read the following instructions carefully as they are directly relevant to how much money you will earn today. Please do not communicate with other people during the experiment. Please note that **you are not permitted to use pen and paper or a mobile phone**. Please kindly switch your mobile phone off or put it on silent mode. Students causing a disturbance will be asked to leave the room. You will enter all of your decisions in todays experiment using only the computer mouse. Please do not attempt to use the keyboard or remove the keyboard cover. The information displayed on your computer monitor is private and specific to you. All monetary amounts in todays experiment are expressed as experimental currency units (ECU). The conversion rate for ECU and GBP is 300 ECU = £1 cash payment. Your payment will be rounded up to the nearest ten pence.

If you have any questions at any point during today's session, please raise your hand and one of the monitors will come to help.

Task

In todays experiment, you will be making **inventory management decisions** for an enterprise called S-Store. S-Store sells coffee makers. You will perform this role for a sequence of 6 years. Every month you will decide how many coffee makers to order from the coffee maker supplier. Your earnings in this experiment will be proportional to the total profitability of S-Store. S-store will sell a new coffee maker model every year. Thus in the first month of a year your inventory always starts from zero. Further, any coffee makers remaining in inventory at the end of month 12 will be disposed of. To summarise, you will be making 12 monthly decisions for a year, and you will do this for 6 years in total.

You will have up to **4 minutes** to complete your task for each year. Year 1 is a practice round, and you will have up to 7 minutes to complete the task for this year. You should use this as an opportunity to familiarize yourself with the software and decision tasks. If you dont finish within the time allowed, the computer will automatically execute the remaining month(s) sales with the existing inventory. You will not be able to add inventory. A 'wait page' displays automatically if you spend less than the allowed time in a year. You will only be able to proceed to the next year when the remaining time runs out.

Before the decision making portion of the experiment begins, there will be a **Quiz** consisting of 7 simple questions to check your understanding of the task. Please answer the questions carefully. If you missed 3 or more questions, you would be asked review the correct answers before you can proceed to the task.

[The following italic texts are additional for treatments with High Cognitive Loads]

PIN

In addition to the task, you will be given a 7-digit PIN at the beginning of each year. The PIN is case sensitive, and consisting of numbers, uppercase and lowercase letters. You will have 15 seconds to remember the PIN. This is your KEY to unlock an account which contains an extra reward of 300 ECU. You can open the account at the end of each year by correctly entering the PIN. You will only have one attempt to correctly enter the pin to claim this extra reward.

Payment

Year 0 is a practice round, and you will receive no earnings from your decisions in this year. For **Years 1 through 5**, your earnings will accumulate across years. At the end of the experiment you will be paid £5 show-up fee and your accumulated earnings, converted to Pounds. Note, negative profit may occur if poor coffee maker ordering decisions are made. To ensure that no one will leave the experiment with a payment less than £5, a negative total profit made in Year 1 to Year 5 will be treated as 0 earnings.

A.2 Background Information

[The following Background Information section shows up on every decision page.]

Your Role:

S-Store is open 360 days per year. You are the inventory manager for S-Store. In your role, you will control S-Stores inventory level which determines the stores total profits.

We now explain how S-Stores, and correspondingly you, earns profit. While we are explaining how the calculations are made, during the decision tasks the computer will carry out these calculations and report the results to you.

S-Store sells coffee makers at a price of **7 ECU** per unit. S-Store can sell up to **10 coffee makers per month**. A coffee maker can only be sold if there is a unit held in inventory. If you hold 10 or more units in inventory at the start of the month, S-Store will sell 10 coffee makers that month. However, if there are less than 10 units held in inventory at the start of the month then S-Store will only sell that amount. For example, if there are 2 units held in inventory at the beginning of a month then S-Store only sells 2 units that month. *[(For EOQ treatment only) You can only place an order when the current months opening inventory is 0. For example, if the current months opening inventory is 3 units, you cannot place an order this month, S-Store only sells 3 units this month.]* S-Stores sales revenue for a month is calculated as follows:

$$\text{Sales revenue} = 7 \text{ ECU} * \text{Number of units sold.}$$

Your job is to manage the stores inventory levels by each month choosing an inventory order. Prior to the start of each month you can order coffee makers from the supplier to add to the inventory. Your inventory management determines the S-Stores total costs. S-Store pays two types of costs. One is the **ordering cost**. Every time you order a positive amount you have to pay an order cost. This ordering cost is **45 ECU**, and does not depend upon the size of the order. If you order zero coffee makers then you do not pay the 45 ECU ordering cost. Holding coffee makers in inventory is costly so S-Store pays a monthly **inventory holding cost**. S-Store pays monthly inventory holding cost

is based on the average number of coffee makers held in inventory multiplied by the per unit monthly inventory holding cost of **1 ECU**. This is calculated as follows:

$$\text{Inventory holding costs} = 1 \text{ ECU} * (\text{Opening inventory} + \text{Order Quantity} + \text{Closing inventory})/2.$$

Calculation of S-Stores profits

$$\text{Profits} = \text{Sales revenue} - \text{Ordering costs} - \text{Inventory holding costs}$$

Your monthly earnings are equal to S-Stores monthly profits.

Examples:

1. Alices closing inventory of last month is 20 units, she placed an order of 0 units in this month.

The demand for each month is 10 units.

She made sales of 10 units.

Her closing inventory of this month is $20 - 10 = 10$ units.

Her profit in this month is equal to: $7 * 10 - 0 - 1 * (20 + 0 + 10)/2 = 55$.

2. Alices closing inventory of last month is 4 units, she placed an order of 5 units in this month.

The demand for each month is 10 units.

She only made sales of 9 units. Her closing inventory of this month is 0 units. Her profit in this month is equal to: $7 * 9 - 45 - 1 * (4 + 5 + 0)/2 = 13.5$.

A.3 Multiple Choice Questions prior to Decision Task

There are a couple of questions for you before the task, please use the information:

The demand for each month is 10 units.

Price of each coffee maker is 7.

Ordering cost is 45 per order.

Monthly inventory holding cost is 1 per unit.

Question 1 of 7

If the inventory level was 5 and you ordered 0 units. How many units will you SELL this month?

- A 0
- B 5
- C 10
- D 15

Question 2 of 7

If the inventory level was 0 and you ordered 15 units. How many units will you SELL this month?

- A 0
- B 5
- C 10
- D 15

Question 3 of 7

If you made sales of 10 units. What will be your SALES REVENUE this month?

- A 0
- B 10
- C 25
- D 70

Question 4 of 7

If you ordered 0 units. What will be your ORDERING COST this month?

- A 0
- B 1
- C 45
- D 70

Question 5 of 7

If you ordered 1 unit. What will be your ORDERING COST this month?

- A 0
- B 1
- C 45
- D 70

Question 6 of 7

If the inventory level was 0 and you ordered 10 units. You made sales of 10 units. What will be your HOLDING COST this month?

- A 0
- B 1
- C 5
- D 10

Question 7 of 7

If your sales revenue is 70. Your ordering cost is 0 and your holding cost is 10. What will be your PROFIT this month?

- A 15
- B 25
- C 60
- D 70

Figure 7 shows the result page of the multiple choice questions when participants had given more than 2 incorrect answers. Under such circumstances, they had to raise their hands to go through incorrectly answered questions with the experimenter in order to obtain a passcode to proceed to the decision tasks.

A.4 Decision Tasks

Prior to each year's decision tasks, a mini-instruction page appears. *Figure 8* is an example with PIN task. For treatments with high cognitive loads, the pin page follows (*Figure 9*).

Figure 7: Result Page of the Multiple Choice Questions

Results

Question	Your answer	Correct answer	Answered correctly?
If the inventory level was 5 and you ordered 0 units. How many units will you SELL this month?	B. 5	B. 5	True
If the inventory level was 0 and you ordered 15 units. How many units will you SELL this month?	D. 15	C. 10	False
If you made sales of 10 units. What will be your SALES REVENUE this month?	B. 10	D. 70	False
If you ordered 0 units. What will be your ORDERING COST this month?	D. 70	A. 0	False
If you ordered 1 unit. What will be your ORDERING COST this month?	B. 1	C. 45	False
If the inventory level was 0 and you ordered 10 units. You made sales of 10 units. What will be your HOLDING COST this month?	C. 5	C. 5	True
If your sales revenue is 70. Your ordering cost is 0 and your holding cost is 10. What will be your PROFIT this month?	D. 70	C. 60	False

Explanation of answers:

1. There are 5 units in total held in inventory this month then S-Store sells 5 units that month.
2. There are 15 units in total held in inventory this month, the demand is 10 units, then S-Store sells 10 units that month.
3. S-Store's sales revenue for a month is calculated as follows: 7 ECU * Number of units sold = $7*10 = 70$
4. You ordered 0 coffee makers then you do not pay the ordering cost.
5. The ordering cost is 45 ECU, and does not depend upon the size of the order.
6. S-Store's holding cost for a month is calculated as follows: 1 ECU inventory holding cost per unit * (Opening inventory + Order Quantity + Closing inventory) / 2 = $1 * (0+10+0)/2 = 5$
7. S-Store's profit for a month is calculated as follows: Sales revenue – Ordering costs – Holding costs = $70 - 0 - 10 = 60$

You answered 2 out of 7 questions correctly.

*Caution! You have missed a large number of questions. This suggests that you may struggle in this task. We suggest you raise your hand so that you can review the correct answers with the monitor.

Please ask the monitor for the passcode, when you are confident about the questions, please enter your passcode and click 'Next' to continue.

Please enter your PASSCODE:

[Next](#)

An example of the ordering decision page is shown in Figure 10. Participants move the horizontal bar to enter their decision of order quantity for each month. Order quantities, costs, and profits of previous months are also displayed on the page. If participants completed the year's decision task within 4 minutes, they had to wait until the end of 4 minutes.

They were then prompted to enter the PIN (Figure 11), followed by the end of the year result page (Figure 12).

Figure 8: Year 4 Instruction

Year 4 Instructions

- On the next page, you will be given a 6-digit PIN. This is your KEY to unlock an account at the end of the year, to claim an extra reward of 300 ECU. You will have 15 seconds to remember the PIN. After the PIN Page, you will be making monthly orders for S-Store from the supplier for this year.
- To help you with understanding the task, at the beginning of the Order Page, you can find the basic formulas we introduced to you in the instructions.
- Next, you will be given information regarding the current month to remind you of the key information you will need.
- There will be a Monthly Record Table displayed on the screen to calculate the Sales revenue, Costs, and Profits for you. The table headings will be look like the following, and the content generates as you proceed to the next months:

Monthly Record

Month	Opening Inventory (units)	Order Q (units)	Sale (units)	Closing Inventory (units)	Sales Revenue	Ordering Costs	Inventory Holding Costs	Profits
-------	---------------------------	-----------------	--------------	---------------------------	---------------	----------------	-------------------------	---------

- Also, there will be an Annual Profit Table displayed on the screen to record your profits made in each year from Year 2 to Year 6. The table headings will be look like the following, and the content generates as you proceed to the next months:

Annual Profit Table

Year 2	Year 3	Year 4	Year 5	Year 6	Total Profit	Total Earnings(€)
--------	--------	--------	--------	--------	--------------	-------------------

Click "Next" to proceed to the next page. You will have up to 4 minutes to complete your task for the year.

[Next](#)

Figure 9: PIN Page prior to Ordering Page

Year4 PIN - Reward

Time left to complete this page: 0:04

Please remember the 6-digit PIN displayed on your screen. This is your KEY to unlock the account with an extra reward of 300. You can open the account at the end of the year by correctly entering the PIN.

7 Q 4 K B t

Figure 10: Ordering Page

You are making inventory orders for Year 4

Time left to complete this year: 2 minutes 58 seconds

Basic Formula:

$$\text{Profits} = \text{Sales revenue} - \text{Ordering costs} - \text{Inventory holding costs}$$

$$\text{Sales revenue} = 7 \text{ ECU} * \text{Number of units sold}$$

$$\text{Ordering costs} = 0 \text{ or } 45$$

$$\text{Inventory holding costs} = 1 \text{ ECU inventory holding cost per unit} * (\text{Opening inventory} + \text{Order Quantity} + \text{Closing inventory}) / 2$$

Period Information:

- This is Month 7 of the 12 months in Year 4.
- The demand for each month is 10 units.
- Price of each coffee maker is 7.
- Ordering cost is 45 per order.
- Monthly inventory holding cost is 1 per unit.

Monthly Record

Month	Opening Inventory (units)	Order Q (units)	Sale (units)	Closing Inventory (units)	Sales Revenue	Ordering Costs	Inventory Holding Costs	Profits
1	0	41	10	31	70.00	-45.00	-36.00	-11.00
2	31	0	10	21	70.00	0.00	-26.00	44.00
3	21	0	10	11	70.00	0.00	-16.00	54.00
4	11	0	10	1	70.00	0.00	-6.00	64.00
5	1	74	10	65	70.00	-45.00	-70.00	-45.00
6	65	0	10	55	70.00	0.00	-60.00	10.00

Your Opening Inventory of this month is 65 units.

How many units (coffee makers) would you like to order for this month?

100

(Please move the slider to select the number of coffee makers you would like to order from the supplier this month. The slider starts from a random point every month. Choose 0 if you do not wish to make an order for this month.)

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Annual Profit Table

Year 2	Year 3	Year 4	Year 5	Year 6	Total Profit	Total Earnings(€)
-134.00	254.00				254.00	0.85

Figure 11: Enter the PIN Page

Year4 PIN - Reward

Please enter the combination of your KEY to open the reward of 300.

PIN1 :

C ▼

PIN2 :

6 ▼

PIN3 :

m ▼

PIN4 :

----- ▼

PIN5 :

----- ▼

PIN6 :

----- ▼

[Next](#)

Figure 12: End of the Year Result Page

Year 4 Result

You guess C6mGEB, the PIN was 7Q4kbt. PIN wrong. You won 0.

Month	Opening Inventory (units)	Order Q (units)	Sale (units)	Closing Inventory (units)	Sales Revenue	Ordering Costs	Inventory Holding Costs	Profits
1	0	41	10	31	70.00	-45.00	-36.00	-11.00
2	31	0	10	21	70.00	0.00	-26.00	44.00
3	21	0	10	11	70.00	0.00	-16.00	54.00
4	11	0	10	1	70.00	0.00	-6.00	64.00
5	1	74	10	65	70.00	-45.00	-70.00	-45.00
6	65	0	10	55	70.00	0.00	-60.00	10.00
7	55	0	10	45	70.00	0.00	-50.00	20.00
8	45	0	10	35	70.00	0.00	-40.00	30.00
9	35	0	10	25	70.00	0.00	-30.00	40.00
10	25	0	10	15	70.00	0.00	-20.00	50.00
11	15	0	10	5	70.00	0.00	-10.00	60.00
12	5	0	5	0	35.00	0.00	-2.50	32.50
PIN WRONG							+ 0	
Total:							348.50	

Annual Profit Table

Year 2	Year 3	Year 4	Year 5	Year 6	Total Profit	Total Earnings(€)
-134.00	254.00	348.50			602.50	2.01

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B Post-Experimental Survey, Demographics and Summary Statistics of Participants

Participants were asked to fill a simple questionnaire at the end of the experiment for us to collect some demographic information.

Figure 13: Post-Experimental Survey

Questionnaire

Please answer the following questions.

1. What is your age?

..... ▾

2. What is your gender?

Male

Female

3. What is your country of citizenship?

United Kingdom ▾

4. Please indicate your current level of education :

Undergraduate

Postgraduate

5. Please select your subject area :

..... ▾

6. How would you describe your mathematical skill level?

..... ▾

7. On a scale of 1-5, how strongly were you motivated by the PIN and the bonus? (1 - I only cared about the PIN; 3 - I cared about the PIN and the inventory decision task equally; 5 - I cared about the inventory decision task only and disregarded the PIN):

..... ▾

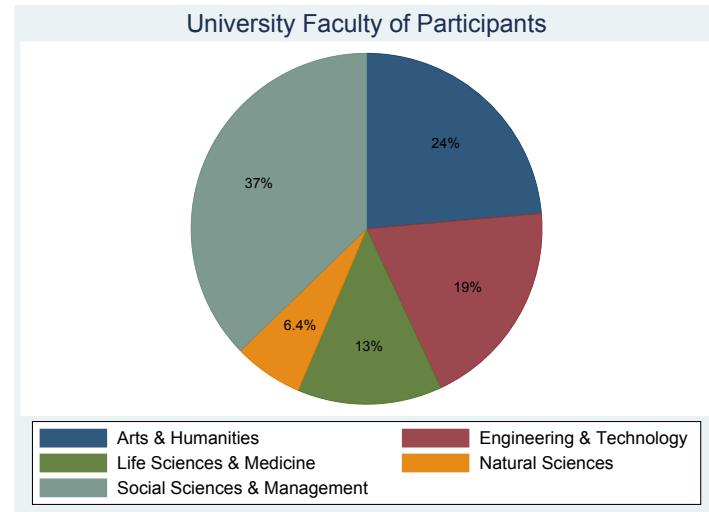
[Next](#)

The following are some summary statistics of the participants.

Table 10: Demographics in Participants

Age (mean)	25.6
Gender (% female)	65%
Education (%Undergraduate)	51%

Figure 14: Distribution of University Programs Participants study



One can observe that 37% of the participants are from Social Science & Management, among which they may have training in operations management or have been exposed to the EOQ model before.

Figure 15: Mathematics levels of participants - self reported

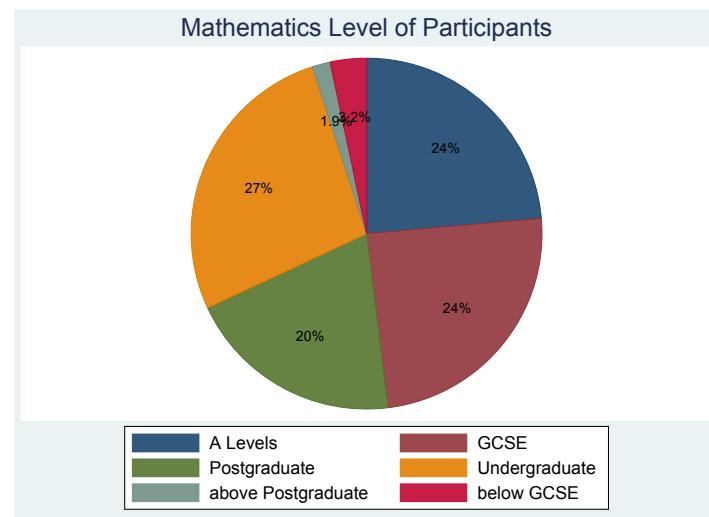


Table 11: Regression on PIN and demographic information

	(1)	(2)
	Annual Profit	Annual Profit
Year 1	-129.08*** (19.30)	-130.78*** (18.84)
Year 2	-71.18*** (16.84)	-70.65*** (16.90)
Year 3	-31.00** (13.76)	-30.90** (13.23)
Year 4	-13.95 (12.28)	-13.74 (11.89)
Unrestricted	-22.32** (11.22)	-33.26*** (12.56)
High·Correct PIN	27.26* (15.96)	19.09 (16.13)
Age		-2.42*** (0.78)
Male		16.14 (11.88)
Postgrad		-7.44 (12.40)
STEM		8.66 (13.73)
Math level		-5.98 (5.20)
Constant	417.18*** (15.61)	505.75*** (33.00)
N	385	385
R ²	0.18	0.22

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

C Possible NW NB sets by month

Table 12: The No worse than and No better than sets for each EOQ cycle by month

	$\tilde{s}_{i,k-1} = 1$	$NW = \{1, 2, 3, 4, 5, 6\}$	$NB = \{1, 12\}$
	$\tilde{s}_{i,k-1} = 2$	$NW = \{2, 3, 4\}$	$NB = \{1, 2, 5, 6, 12\}$
	$\tilde{s}_{i,k-1} = 3$	$NW = \{3\}$	$NB = \{1, 2, 3, 4, 5, 6, 12\}$
Months 2-8	$\tilde{s}_{i,k-1} = 4$	$NW = \{3, 4\}$	$NB = \{1, 2, 4, 5, 6, 12\}$
	$\tilde{s}_{i,k-1} = 5$	$NW = \{2, 3, 4, 5\}$	$NB = \{1, 5, 6, 12\}$
	$\tilde{s}_{i,k-1} = 6$	$NW = \{2, 3, 4, 5, 6\}$	$NB = \{1, 6, 12\}$
	$\tilde{s}_{i,k-1} = 12$	$NW = \{1, 2, 3, 4, 5, 6, 12\}$	$NB = \{12\}$
Month 9	$\tilde{s}_{i,k-1} = 1$	$NW = \{1, 2, 3, 4, 5\}$	$NB = \{1, 6, 12\}$
	$\tilde{s}_{i,k-1} = 2$	$NW = \{2, 3, 4\}$	$NB = \{1, 2, 5, 6, 12\}$
	$\tilde{s}_{i,k-1} = 3$	$NW = \{3\}$	$NB = \{1, 2, 3, 4, 5, 6, 12\}$
	$\tilde{s}_{i,k-1} = 4$	$NW = \{3, 4\}$	$NB = \{1, 2, 4, 5, 6, 12\}$
	$\tilde{s}_{i,k-1} = 5$	$NW = \{2, 3, 4, 5\}$	$NB = \{1, 5, 6, 12\}$
	$\tilde{s}_{i,k-1} = 6$	$NW = \{1, 2, 3, 4, 5, 6\}$	$NB = \{6, 12\}$
Month 10	$\tilde{s}_{i,k-1} = 12$	$NW = \{1, 2, 3, 4, 5, 6, 12\}$	$NB = \{12\}$
Month 11	$\tilde{s}_{i,k-1} = 1$	$NW = \{1, 2, 3, 4, 5\}$	$NB = \{1, 5, 6, 12\}$
	$\tilde{s}_{i,k-1} = 2$	$NW = \{2, 3\}$	$NB = \{1, 2, 4, 5, 6, 12\}$
	$\tilde{s}_{i,k-1} = 3$	$NW = \{3\}$	$NB = \{1, 2, 3, 4, 5, 6, 12\}$
	$\tilde{s}_{i,k-1} = 4$	$NW = \{2, 3, 4\}$	$NB = \{1, 4, 5, 6, 12\}$
	$\tilde{s}_{i,k-1} = 5$	$NW = \{1, 2, 3, 4, 5\}$	$NB = \{1, 5, 6, 12\}$
	$\tilde{s}_{i,k-1} = 6$	$NW = \{1, 2, 3, 4, 5, 6\}$	$NB = \{6, 12\}$
Month 12	$\tilde{s}_{i,k-1} = 12$	$NW = \{1, 2, 3, 4, 5, 6, 12\}$	$NB = \{12\}$