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Abstract

In this article, I incorporate the anchoring-and-adjustment heuristic into the Black-Scholes option pricing framework, and show that this is equivalent to replacing the risk-free rate with a higher interest rate. I show that the price from such a behavioralized version of the Black-Scholes model generally lies within the no-arbitrage bounds when there are transaction costs. The behavioralized version explains several phenomena (implied volatility skew, countercyclical skew, skew steepening at shorter maturities, inferior zero-beta straddle return, and superior covered-call returns) which are anomalies in the traditional Black-Scholes framework. Six testable predictions of the behavioralized model are also put forward.

JEL Classification: G13, G12

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Behavioralizing the Black-Scholes Model

Black-Scholes model (Black and Scholes 1973) is one of the most well-known models in finance. Anchoring-and-Adjustment (Tversky and Kahneman 1974) is one of the most robust decision-making heuristics. In this article, I behavioralize the Black-Scholes model by incorporating anchoring-and adjustment into the model. I show that the price generated by the behavioralized version of the Black-Scholes model always lies within the transaction-cost induced bounds derived in Constantinides and Perrakis (2002), and is generally within Leland (1985) bounds. Hence, anchoring bias does not create arbitrage opportunities in the presence of transaction costs. The behavioralized version generates the implied volatility skew, which is countercyclical and steepens at shorter maturities. It also explains superior covered-call return (Whaley 2002), and inferior zero-beta straddle return (Coval and Shumway 2001), which are anomalies in the Black-Scholes/CAPM framework. Testable predictions of the behavioralized model are also put forward.

The original Black-Scholes article (Black and Scholes 1973) presents an alternate derivation that relies on CAPM. Here, I use the same route to introduce the anchoring-and-adjustment heuristic. The starting point is the observation that a call option magnifies the corresponding gains and losses in the underlying security. Hence, call option beta is a scaled-up version of the beta of the underlying security. This fact makes the underlying security beta, a natural starting point for call option beta. This starting point needs to be appropriately scaled-up. However, relying on a starting point and attempting to make appropriate adjustments exposes investors to anchoring bias, which is the robust finding that such adjustments typically do not go far enough (see Tversky and Kahneman (1974) for early exposition, and Furnham and Boo (2011) for a survey of a large literature on anchoring).

Anchoring-and-adjustment is a heuristic that we rely upon frequently (Epley and Gilovich 2001). In fact, it may be considered an optimal response of a Bayesian decision-maker with finite computational resources (Lieder, Griffiths, and Goodman 2013). What is the orbital-period of Mars? When did George Washington become the first president of USA? What is the freezing point of Vodka? When asked these questions, people typically reasons as follows: Mars is farther from the Sun than Earth is, and Earth's orbital period is 356 days, so Mars probably takes longer. So respondents use 356 days as a starting point and add to it. USA became a country in 1776. The first president could only be elected after that, so respondents start from 1776 and add to it. Vodka is still liquid when water freezes, so respondents start from 0 Celsius and subtract from it. In all cases, the adjustments do not go far enough with the final answers remaining too close to the starting points (Epley and Gilovich 2006, 2001).

Index beta (such as S&P 500 index) is usually taken as 1 by investors. It follows that a call option on an index must have a beta greater than 1. So, 1 is a clear starting point, which needs to be scaled-up. As adjustments to starting points do not go far enough (anchoring-bias), it follows that, starting from 1, insufficient scaling-up is applied. In other words, a call option beta is typically underestimated. This in turn implies that the magnitude of the corresponding put option beta (usually negative) is overestimated (put option beta follows deductively from corresponding call and underlying beta). Hence, anchoring-bias makes both types of options more expensive. Underestimation of call option beta is apparent in *stock replacement with call option* strategy, which is quite popular among market professionals.[1] In this strategy, stocks are replaced with call options to take advantage of embedded leverage; however, the resulting increase in portfolio beta is not properly appreciated.

This article is organized as follows: Section 2 derives the behavioralized versions of Black-Scholes formulas applicable to European call and put options. Section 3 shows that prices generated by the behavioralized versions generally lie within no-arbitrage bounds in the presence of transaction costs. Section 4 shows that the behavioralized version generates the implied volatility skew similar to what is observed with index options. Section 5 shows that superior covered-call returns, and inferior zero-beta straddle returns, are consistent with the behavioralized model. Section 6 derives six testable predictions of the behavioralized model. Section 7 shows how to behavioralize other option pricing models. Section 8 concludes.

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[1] http://www.minyanville.com/mvpremium/2013/11/29/swapping-stock-for-options/
http://www.etf.com/sections/features-and-news/nations-1?nopaging=1
http://www.optionsuniversity.com/blog/stock-replacement-options-mastery-series-lesson-24/

2. Anchoring in the Black-Scholes Framework

To derive the behavioralized PDE, I assume the existence of three instruments:

1) A riskless bond that evolves as $dB = rBdt$ where r is the risk-free rate.
2) An underlying security which follows the Ito process: $dS = \mu Sdt + \sigma SdW$.
3) A call option written on the underlying security which, by Ito's Lemma, follows the following process:

$$dC = \left(\frac{\partial C}{\partial t} + \mu S\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 S}{\partial S^2}\right)dt + \left(\sigma S\frac{\partial C}{\partial S}\right)dW \tag{2.1}$$

Time subscripts on $C, S,$ and B are suppressed for notational convenience. It follows that $\beta_C = \frac{\partial C}{\partial S}\frac{S}{C}\beta_S$ where β_C and β_S are call option and the underlying security beta respectively (Black and Scholes 1973).

By applying CAPM, we can write, for a small time increment dt:

$$E\left[\frac{dS}{S}\right] = rdt + \beta_S[\bar{r}_M - r]dt \tag{2.2}$$

$$E\left[\frac{dC}{C}\right] = rdt + \beta_C[\bar{r}_M - r]dt \tag{2.3}$$

where $\bar{r}_M$ is the expected return on the aggregate market portfolio.

As call beta is a scaled-up version of the underlying security beta, it must be true that for some $\bar{A}_t$: $\beta_C = (1 + \bar{A}_t)\beta_S$. Substituting this in (2.3), realizing $E\left[\frac{dS}{S}\right] = \mu dt$, and then substituting for $\beta_S[\bar{r}_M - r]$ from (2.2) leads to:

$$E\left[\frac{dC}{C}\right] = rdt + (1 + \bar{A}_t)\{\mu dt - rdt\} \tag{2.4}$$

The correct scaling-up factor is $\bar{A}_t = \frac{\partial C}{\partial S}\frac{S}{C} - 1$. With this substitution, (2.4) becomes:

$$E\left[\frac{dC}{C}\right] = rdt + \frac{\partial C}{\partial S}\frac{S}{C}\{\mu - r\}dt \tag{2.5}$$

From (2.1), $E[dC] = \left(\frac{\partial C}{\partial t} + \mu S\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 S}{\partial S^2}\right)dt$. Substituting this in (2.5) leads to:

$$\frac{\partial C}{\partial t} + rS\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 S}{\partial S^2} = rC \tag{2.6}$$

(2.6) is the original Black-Scholes PDE as derived in Black and Scholes (1973).

Next, I introduce anchoring-bias into the picture. With anchoring-bias, using the underlying beta as a starting point, insufficient scaling-up is applied to estimate call option beta. That is, $A_t = m\bar{A}_t$ where $0 < m < 1$. So, (2.4) becomes:

$$E\left[\frac{dC}{C}\right] = rdt + (1 + m\bar{A}_t)\{\mu dt - rdt\}$$

$$=> E\left[\frac{dC}{C}\right] = rdt + (1 + \bar{A}_t)\{\mu dt - rdt\} - (1 - m)\bar{A}_t\{\mu dt - rdt\}$$

$$=> E\left[\frac{dC}{C}\right] = rdt + \left[\frac{\partial C}{\partial S}\frac{S}{C}\right]\{\mu dt - rdt\} - (1 - m)\left[\frac{\partial C}{\partial S}\frac{S}{C} - 1\right]\{\mu dt - rdt\}$$

$$=> E\left[\frac{dC}{C}\right] = [r + (1 - m)(\mu - r)]dt + \left[\frac{\partial C}{\partial S}\frac{S}{C}\right][\mu - \{r + (1 - m)(\mu - r)\}]dt$$

$$=> E\left[\frac{dC}{C}\right] = r^* dt + \frac{\partial C}{\partial S}\frac{S}{C}\{\mu - r^*\}dt \tag{2.7}$$

(2.7) is identical to (2.5) with r replaced with $r^* = r + (1 - m)\delta$ where $\delta = \mu - r$ is the risk-premium on the underlying security. Note that with correct adjustment, that is, in the absence of anchoring-bias ($m = 1$), $r^* = r$. The effect of introducing anchoring-bias is equivalent to replacing the risk-free rate, r, with a higher interest rate, $r^* = r + (1 - m)\delta$.

From (2.1), $E[dC] = \left(\frac{\partial C}{\partial t} + \mu S \frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 S}{\partial S^2}\right) dt$. Substituting this in (2.7) leads the following behavioralized version of the Black-Scholes PDE:

$$\frac{\partial C}{\partial t} + r^* S \frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 S}{\partial S^2} = r^* C \qquad (2.8)$$

where $r^* = r + (1-m)\delta$. With correct adjustment, that is, with $m = 1$, the behavioralized version converges to the original Black-Scholes PDE.

The behavioralized PDE can be solved in the same way as the original Black-Scholes PDE. Proposition 1 presents the solution.

Proposition 1: *The behavioralized Black-Scholes formula for the price of a European call option with strike K is given by:*

$$\boldsymbol{C = SN(d_1^*) - Ke^{-\{r+(1-m)\delta\}(T-t)}N(d_2^*)}$$

where $\boldsymbol{d_1^* = \frac{ln\left(\frac{S}{K}\right)+\left\{r+(1-m)\delta+\frac{\sigma^2}{2}\right\}(T-t)}{\sigma\sqrt{t-t}}}$, $\boldsymbol{d_2^* = \frac{ln\left(\frac{S}{K}\right)+\left\{r+(1-m)\delta-\frac{\sigma^2}{2}\right\}(T-t)}{\sigma\sqrt{t-t}}}$*, and* $\boldsymbol{0 \le m \le 1}$

Proof:

By solving (2.8) in the same way as the original Black-Scholes PDE is solved.■

Corollary 1: *The behavioralized Black-Scholes formula for the price of a European put option with strike K is given by:*

$$\boldsymbol{P = Ke^{-r(T-t)}\left\{1 - e^{-\delta(1-m)(T-t)}N(d_2^*)\right\} - S\{1 - N(d_1^*)\}}$$

Proof:

There are two equivalent ways.

First method: $\beta_P = -\beta_S \left\{\frac{S}{P} - \frac{C}{P}(1 + m\bar{A})\right\}$. Use this expression to calculate the expected put return from CAPM. Then, use Ito's Lemma to substitute out $E[dP]$ in CAPM. Solve the resulting PDE.

Second Method: Use put-call parity.■

Corollary 2 *Anchoring-bias makes both types of options more expensive than the corresponding Black-Scholes benchmark.*

Proof:

Follows from direct comparison.■

3. Behavioralized Model and No-Arbitrage Bounds

Constantinides and Perrakis (2002) derive option pricing bounds in the presence of proportional transaction costs. They show that their bounds are generally tighter than Leland (1985) bounds. Here, I show that the price generated by the behavioralized model always lies within the bounds derived in Constantinides and Perrakis (2002). Note that as anchoring-bias makes options more expensive than the Black-Scholes benchmark, we only need to consider the upper bound (as lower bound lies below the Black-Scholes price). If the proportional transaction cost is $k > 0$, then the

Constantinides and Perrakis (2002) upper bound in the Black-Scholes context is given by:

$$\bar{C} = \frac{(1+k)}{(1-k)}\left\{SN\left(d_1^{\mu}\right) - Ke^{-\mu(T-t)}N\left(d_2^{\mu}\right)\right\} \tag{3.1}$$

where $d_1^{\mu} = \frac{\boldsymbol{ln\left(\frac{S}{K}\right)+\left\{\mu+\frac{\sigma^2}{2}\right\}(T-t)}}{\boldsymbol{\sigma\sqrt{t-t}}}$, $d_2^{\mu} = \frac{\boldsymbol{ln\left(\frac{S}{K}\right)+\left\{\mu-\frac{\sigma^2}{2}\right\}(T-t)}}{\boldsymbol{\sigma\sqrt{t-t}}}$, and $\mu = r + \delta$

In other words, call upper bound is the price at which the expected return from a call option is equal to the expected return from the underlying security net of round trip transaction costs (see Proposition 1 in Constantinides and Perrakis (2002)).

It is easy to see that:

$SN(d_1^*) - Ke^{-\{r+(1-m)\delta\}(T-t)}N(d_2^*) < SN\left(d_1^{\mu}\right) - Ke^{-\mu(T-t)}N\left(d_2^{\mu}\right)$ for $0 < m \leq 1$

And

$SN(d_1^*) - Ke^{-\{r+(1-m)\delta\}(T-t)}N(d_2^*) = SN\left(d_1^{\mu}\right) - Ke^{-\mu(T-t)}N\left(d_2^{\mu}\right)$ for $m = 0$

It follows:

$SN(d_1^*) - Ke^{-\{r+(1-m)\delta\}(T-t)}N(d_2^*) \leq SN\left(d_1^{\mu}\right) - Ke^{-\mu(T-t)}N\left(d_2^{\mu}\right)$ for $0 \leq m \leq 1$

And,

$SN(d_1^*) - Ke^{-\{r+(1-m)\delta\}(T-t)}N(d_2^*) < \frac{1+k}{1-k}\left\{SN\left(d_1^{\mu}\right) - Ke^{-\mu(T-t)}N\left(d_2^{\mu}\right)\right\}$ for $0 \leq m \leq 1$

Hence, the behavioralized model price is always less than the Constantinides and Perrakis (2002) upper bound. As Constantinides and Perrakis (2002) upper bound is generally less than the corresponding Leland (1985) bound, it also follows that the

Table 1

Behavioralized model price vs Constantinides & Perrakis (2002) and Leland (1985) bounds

Strike-to-price ratio	**0.95**	**1.0**	**1.05**
Black-Scholes price	6.07	2.99	1.19
Behavioralized Black-Scholes (price range)	(6.07-6.79)	(2.99-3.50)	(1.19-1.48)
Constantinides and Perrakis price	6.93	3.57	1.50
Leland price with trading interval 1/250 years	7.69	4.90	2.91
Leland price with trading interval 1/52 years	6.88	3.98	2.05

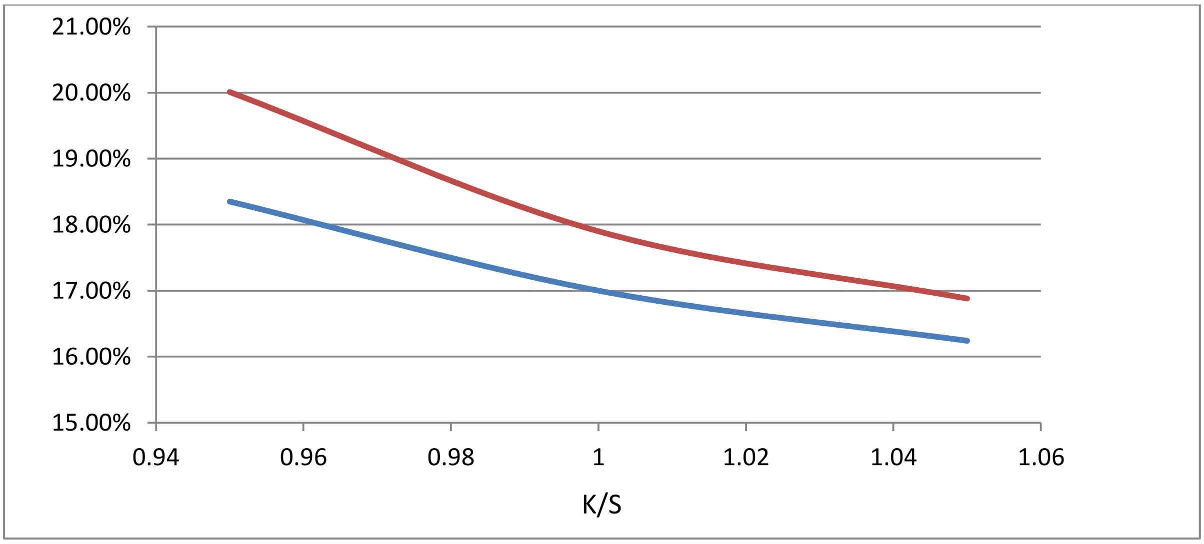
behavioralized price is generally smaller than the Leland (1985) upper bound as well. Note that for trading interval, Δt, the Leland (1985) upper bound is obtained by making the following substitution in the Black-Scholes formula: $\sigma^2 \rightarrow \sigma^2 + \sqrt{\frac{8}{\pi}}\sigma\frac{k}{\sqrt{\Delta t}}$

As an illustrative example, table 1 shows Black-Scholes price, price range from the behavioralized model ($\forall m\ 0 \leq m \leq 1$), Constantinides and Perrakis (2002) upper bound, as well as Leland (1985) upper bounds with daily and weekly trading intervals. The parameter values are: $S = 100, T - t = 0.25\ year, r = 0, \mu = 0.04,\ \sigma = 0.15, and\ k = 0.01$. Three different levels of money-ness are considered: 0.95, 1.0, and 1.05. As can be seen, the behavioralized model price range is less than the Constantinides and Perrakis (2002) upper bound as well as Leland (1985) bound throughout. The fact that the behavioralized price is always below the Constantinides and Perrakis (2002) upper bound implies that anchoring, as modelled here, can be considered a mechanism that further tightens the option pricing bounds.

4. Implied Volatility Skew in the Behavioralized Model

If actual prices are generated in accordance with the behavioralized model, and the Black-Scholes model is used to back-out implied volatility, then a skew is observed. Continuing with the previous example with $S = 100, T - t = 0.25\ year, r = 0, \delta = 0.04,\ \sigma = 0.15,$ and $m = 0.75$, figure 1 plots the skew (blue curve). The skew is also plotted at a higher value of $\delta = 0.06$ (red curve). It is easy to verify that the skew steepens as the risk-premium on the underlying security increases (as δ goes up). The behavioralized model predicts that the skew should steepen during recessions

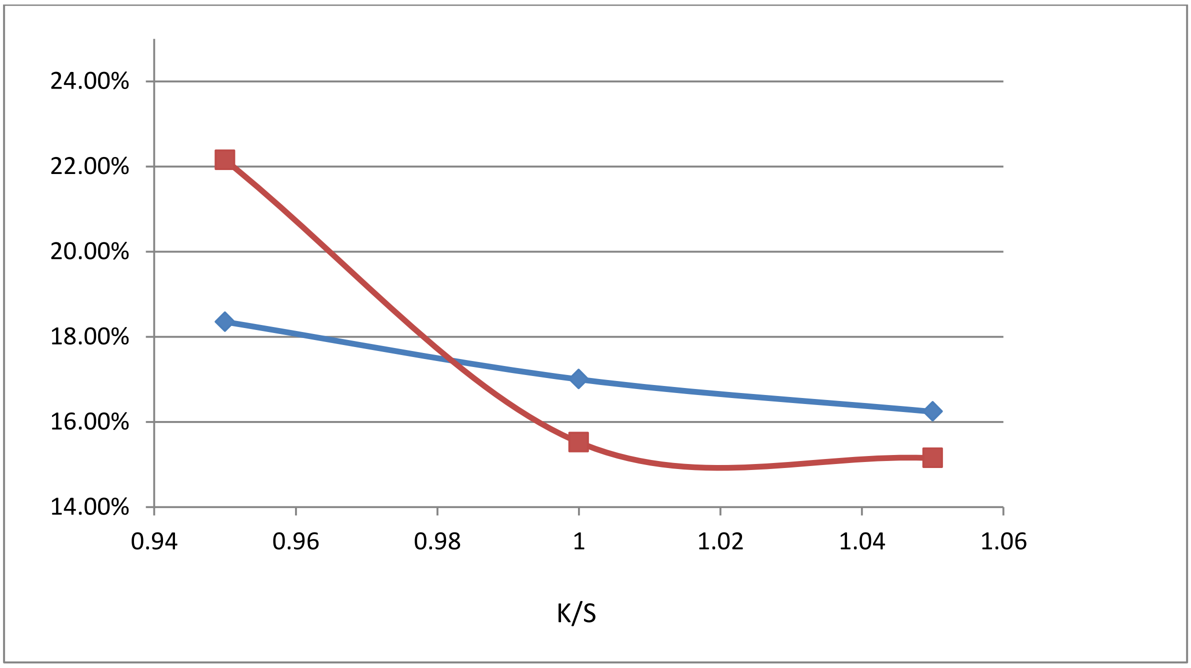
Implied Volatility Skew Steepens with Risk-Premium



Red curve corresponds to $\delta = 0.06$. Blue curve corresponds to $\delta = 0.04$. Other parameters are: $S = 100, T - t = 0.25\ year, r = 0,\ \sigma = 0.15,$ and $m = 0.75$

Figure 1

Implied Volatility Skew Steepens at Shorter Maturity



Red curve corresponds to $T - t = 1\ week$. Blue curve corresponds to $T - t = 3\ months$. Other parameters are: $S = 100, \delta = 0.04, r = 0,\ \sigma = 0.15$, and $m = 0.75$

Figure 2

as δ is higher during recessions. This is consistent with empirical findings (Rosenberg and Engle 2002). Another prediction of the behavioralized model is steepening of the skew at shorter maturities. This is also consistent with empirical evidence (Derman and Miller 2016). Figure 2 plots the skew at maturities of 3 months, and 1 week. The steepening at shorter maturity is seen.

5. Zero-Beta Straddle and Covered-Call Returns

Coval and Shumway (2001) find that zero-beta straddles earn negative returns in sharp contrast with the prediction of the Black-Scholes/CAPM framework that they should earn the risk-free rate. Anchoring-bias makes both call and put options more expensive (corollary 2) compared to the Black-Scholes values. This lowers the return from zero-beta straddle to below the risk-free rate. Another way of seeing the same thing is as follows: If anchoring-bias is present and we set-up a zero-beta straddle with the assumption that there is no anchoring-bias, then the weight on the call option in our portfolio would be lower than what it should be (the weight on the put option would be higher than what it should be) to achieve the risk-free rate. Consequently, the return would be lower than the risk-free rate. Zero-beta straddle is set-up as follows:

$$\theta\beta_C + (1-\theta)\beta_P = 0$$

where $\beta_C = \beta_S(1+\bar{A})$ and $\beta_P = -\beta_S\left\{\frac{S}{P} - \frac{C}{P}(1+\bar{A})\right\}$ for $\bar{A} > 0$.

It follows that the weight on the call option to achieve the risk-free rate is:

$$\theta = \frac{\frac{S}{P} - \frac{C}{P}(1+\bar{A})}{(1+\bar{A}) + \left\{\frac{S}{P} - \frac{C}{P}(1+\bar{A})\right\}}$$

If underlying security beta is insufficiently scaled-up to estimate call option beta, then the weight on the call option to achieve the risk-free would be:

$$\theta_m = \frac{\frac{S}{P} - \frac{C}{P}(1 + m\bar{A})}{(1 + m\bar{A}) + \left\{\frac{S}{P} - \frac{C}{P}(1 + m\bar{A})\right\}} \; where \; 0 \leq m < 1$$

$$=> \theta_m = \frac{\frac{S}{P} - \frac{C}{P}(1 + \bar{A}) + \frac{C}{P}(1 - m)\bar{A}}{(1 + \bar{A}) + \left\{\frac{S}{P} - \frac{C}{P}(1 + \bar{A})\right\} - (1 - m)\bar{A} + \frac{C}{P}(1 - m)\bar{A}} > \theta$$

It follows that, if there is anchoring-bias and we ignore it while setting up a zero-beta straddle, then the weight on the call option is too low and the weight on the put option is too high resulting in portfolio return being less than the risk-free rate. Hence, the inferior historical performance of zero-beta straddles is consistent with anchoring-bias. This suggests that there is a simpler explanation for inferior performance of zero-beta straddles than assuming the existence of additional risk factors. Nevertheless, empirical work must carefully consider the possibility that the inferior performance may be due to the anchoring-bias.

Whaley (2002) documents superior returns from covered-call writing when compared with the Black-Scholes/CAPM benchmark. Again, this is exactly what one expects from the behavioralized model. As a call option is overpriced when compared with the Black-Scholes benchmark, the initial value of covered-call writing portfolio is smaller, which increases returns.

6. Behavioralized Model: Six Testable Predictions

Anchoring-bias implies that investors insufficiently scale-up underlying security beta to estimate call option beta. If anchoring-bias is present and we ignore it while deleveraging a call option, then we would over deleverage. Hence, the resulting deleveraged portfolio would have a beta smaller than the intended target beta of the underlying security. Similarly, we would under-deleverage a put option creating a portfolio with a beta greater than that of the underlying security. Hence, option deleveraging exercises provide a fertile testing ground for the behavioralized model.

The expected return from a call option over dt under Black-Scholes model is:

$$\frac{1}{dt}E\left[\frac{dC}{C}\right] = r + \frac{\partial C}{\partial S}\frac{S}{C}(\mu - r)$$

Deleveraging means combining a call option with the risk-free asset so that the portfolio beta is equal to the beta of the underlying security. This is achieved by creating a portfolio with the weight of $\frac{1}{\frac{\partial C S}{\partial S C}}$ on a call option and the weight of $1 - \frac{1}{\frac{\partial C S}{\partial S C}}$ on the risk-free asset. So, under the Black-Scholes model:

$$\frac{r + \frac{\partial C}{\partial S}\frac{S}{C}(\mu - r)}{\frac{\partial C}{\partial S}\frac{S}{C}} + \left(1 - \frac{1}{\frac{\partial C}{\partial S}\frac{S}{C}}\right)r = \mu \qquad (6.1)$$

So, under the Black-Scholes model, the expected return from a deleveraged call option is equal to the expected return from the underlying security.

Under the behavioralized Black-Scholes model, the expected return from a call option is:

$$\frac{1}{dt}E\left[\frac{dC}{C}\right] = r^* + \frac{\partial C}{\partial S}\frac{S}{C}(\mu - r^*)\ where\ r^* = r + (1-m)\delta$$

So, the expected return from a deleveraged call option is:

$$\frac{r^* + \frac{\partial C}{\partial S}\frac{S}{C}(\mu - r^*)}{\frac{\partial C}{\partial S}\frac{S}{C}} + \left(1 - \frac{1}{\frac{\partial C}{\partial S}\frac{S}{C}}\right)r = \mu - (1-m)\delta\left\{1 - \frac{1}{\frac{\partial C}{\partial S}\frac{S}{C}}\right\} \qquad (6.2)$$

Hence, the following two predictions directly follow from the behavioralized model regarding call options:

1) Deleveraged call option return must be lower the return on the underlying security.
2) Deleveraged call option return must fall as the ratio of strike-to-spot increases. This is because $\frac{\partial C}{\partial S}\frac{S}{C}$ rises as strike-to-spot increases, which increases $1 - \frac{1}{\frac{\partial C S}{\partial S C}}$

The expected return from a put option under the Black-Scholes model is:

$$\frac{1}{dt}E\left[\frac{dP}{P}\right] = r - \left[\frac{S}{P} - \frac{C}{P}(1+\bar{A})\right](\mu - r)\ where\ \bar{A} = \frac{\partial C}{\partial S}\frac{S}{C} - 1$$

Deleveraging a put option requires combining a writing position in the put option with a long position in the risk-free asset with the weight of $\frac{1}{\frac{S}{P} - \frac{C}{P}(1+\bar{A})}$ on the put writing position and the weight of $1 + \frac{1}{\frac{S}{P} - \frac{C}{P}(1+\bar{A})}$ on the risk-free asset.

Hence, under the Black-Scholes model, a deleveraged put should yield:

$$\frac{r-\left[\frac{S}{P}-\frac{C}{P}(1+\bar{A})\right](\mu-r)}{-\left[\frac{S}{P}-\frac{C}{P}(1+\bar{A})\right]}+\left(1+\frac{1}{\frac{S}{P}-\frac{C}{P}(1+\bar{A})}\right)r=\mu \tag{6.3}$$

Just like a deleveraged call option, a deleveraged put option should also yield a return equal to the underlying security return if the Black-Scholes model is correct.

Under the behavioralized Black-Scholes model:

$$\frac{1}{dt}E\left[\frac{dP}{P}\right]=r-\left[\frac{S}{P}-\frac{C}{P}(1+\bar{A})\right](\mu-r)-\frac{C}{P}(1-m)\bar{A}\delta$$

So, deleveraged put option return is:

$$\frac{r-\left[\frac{S}{P}-\frac{C}{P}(1+\bar{A})\right](\mu-r)-\frac{C}{P}(1-m)\bar{A}\delta}{-\left[\frac{S}{P}-\frac{C}{P}(1+\bar{A})\right]}+\left(1+\frac{1}{\frac{S}{P}-\frac{C}{P}(1+\bar{A})}\right)r$$

$$=\mu+(1-m)\delta\frac{C\left(\frac{\partial C}{\partial S}\frac{S}{C}-1\right)}{S\left(1-\frac{\partial C}{\partial S}\right)} \tag{6.4}$$

The following two testable predictions follow:

1) Deleveraged put option return must be larger than the return on the underlying security.
2) Deleveraged put option return must fall as the ratio of strike-to-spot increases.

Apart from the two testable predictions pertaining to the deleveraged call option and the two predictions pertaining to the deleveraged put option, it is possible to derive further predictions by subtracting deleveraged call return from deleveraged put return. That is, eq. (6.4) minus eq. (6.2) results in:

$$Deleveraged\ Put - Deleveraged\ Call = (1-m)\delta \frac{\left(\frac{\partial C}{\partial S}\frac{S}{C} - 1\right)}{\left(1 - \frac{\partial C}{\partial S}\right)\left(\frac{\partial C}{\partial S}\frac{S}{C}\right)} \tag{6.5}$$

It follows that, according to the behavioralized model:

1) The difference between corresponding deleveraged put and deleveraged call returns must fall as the ratio of strike-to-spot increases.
2) The difference between corresponding deleveraged put and deleveraged call returns must rise as time-to-expiry nears.

In this section, six testable predictions of the behavioralized Black-Scholes model are derived. Careful empirical testing of these predictions is the subject of future research. Intriguingly, Constantinides , Jackwerth and Savov (2013) present empirical results consistent with the first four predictions derived in this section.

7. Behavioralizing Other Option Pricing Models

The approach used in behavioralizing the Black-Scholes model can be easily generalized to behavioralize other option pricing models such as the ones developed in Heston (1993) and Bates (1996). The only change is that instead of scaling-up the beta of underlying security, one scales-up the risk of underlying security to estimate the risk of a call option. Anchoring-bias then implies that call option risk is underestimated. It is straightforward to see that the call option formulas pertaining to Heston (1993) and Bates (1996) models would change in only one way: replacing the risk-free rate, r, with a higher interest rate, r^*, which is equal to $r + (1 - m)\delta$.

8. Conclusions

In this article, anchoring-and-adjustment heuristic is incorporated into the Black-Scholes model and behavioralized versions of call and put option pricing formulas are put forward. It is shown that the behavioralized price generally lies within no-arbitrage bounds with proportional transaction costs. The behavioralized model explains several implied volatility and option return puzzles. Six testable predictions of the behavioralized model are also derived. The technique shown here can be used to behavioralize any option pricing model such as stochastic volatility model of Heston (1993) and stochastic volatility with jumps model of Bates (1996).

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