A Note on Stable Cartels

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Abstract

In noncooperative cartel formation games, it is usually assumed that cartel members will maximize their joint payoffs. Through an example, this note shows that this assumption is problematic, because it imposes some unnecessary restrictions on cartel members' actions.

Keywords: cartel formation, stable cartel, self-enforcing agreement
JEL codes: D79, H41, C79

1 Introduction

The noncooperative coalition (cartel) formation model has been widely applied in many economic situations, such as collusion in oligopolistic markets (d'Aspremont et al., 1983; Diamantoudi, 2005), R&D joint ventures (Katz, 1986), customs unions (Yi, 1996), international environmental agreements (Carraro and Siniscalco, 1993; Barrett, 1994), and sharing of natural resources (Miller and Nkuiya, 2016). In a typical application of this model, cooperation among players may potentially create a surplus. However, the existence of externalities and lack of binding agreements may cause free rider problem, which can hinder cooperation and lead to inefficient outcomes.

One possible method to overcome this problem is to form a cartel that regulates its members' actions. Those players that voluntarily choose to be a member will form the cartel and sign a self-enforcing agreement. When payoffs are transferable, a commonly used assumption about the agreement is that all cartel members should coordinate their actions, so as to maximize their joint payoffs. We call it the MJP assumption, and call the agreement derived by it the MJP agreement. This assumption is intuitive, since otherwise the cartel members are likely to renegotiate among themselves to replace the agreement by the MJP agreement so that they could all get larger payoffs.

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Nevertheless, one may still wonder whether the MJP assumption is indeed reasonable. For example, why the MJP assumption requires that members of all possible cartels, rather than only the stable ones, to maximize their joint payoffs? After all, only stable cartels matter, since non-stable ones will not be formed. Therefore, it seems that the MJP assumption is making too much restrictions on players' actions than necessary.

To justify the MJP assumption, we should show that the MJP agreement can be found to be optimal or is an equilibrium outcome when the agreement is endogenously determined. There are many studies in the coalition formation literature that discuss the endogenous determination of coalition agreement. In cooperative game setting, Zhou (1994), Okada (1996), and Ray and Vohra (1999) highlight that the formation of a coalition and the allocation of payoffs in this coalition should be determined simultaneously and endogenously.[1] However, to the best of my knowledge, few study in the noncooperative coalition formation literature has shown explicit evidences that support or falsify the MJP assumption.

In this note, we present a simple open membership cartel formation model[2] where the MJP agreement will lead to a stable cartel in which all members are willing to adopt a different agreement. This example shows that the MJP assumption is indeed problematic.

2 An example

Suppose that there is a public good, which may be produced by a set $N = \{1, 2, \ldots, 5\}$ of homogeneous players. Let x_i denote player i's output. Player i's payoff is[3]

$$u_i = \sum_{k \in N} x_k - \frac{1}{300}\Big(\sum_{k \in N} x_k\Big)^2 - \frac{1}{2}x_i^2, \tag{1}$$

which depends on the total output of the good $\sum_{k \in N} x_k$ and i's individual cost $\frac{1}{2}x_i^2$.

The social welfare is the sum of all players' payoffs $\sum_{k \in N} u_k$, which is maximized when $x_i^* = 4.23$ for all $i \in N$. However, each player's Nash equilibrium output is $x_i^0 = 0.97 < x_i^*$. This commonly known social dilemma of insufficient provision of public good is caused by the free rider problem.

To overcome this problem, we can form a cartel so as to coordinate its members' actions. Consider a two-stage cartel formation game. In stage one,

[1] See Bloch (2003) for a review.

[2] Open membership means no player can be prevented from becoming a coalition member.

[3] This payoff function is a special case of that used in Barrett (1994) and many others.

all players simultaneously decide whether or not to join the cartel. Those choosing to join become cartel members. In stage two, all members coordinate actions to maximize their joint payoffs (MJP assumption), while simultaneously non-members choose their own actions.

This game can be solved by backward induction. Suppose that the cartel formed in stage one is M, with cardinality $|M| = m$. In stage two, each non-member $j \notin M$ chooses output x_j to maximize u_j, while each member $i \in M$ chooses x_i to maximize $\sum_{k \in M} u_k$. In equilibrium, these lead to

$$x_j = \frac{150}{155 - m + m^2}, \quad j \notin M \tag{2}$$

$$x_i = \frac{150m}{155 - m + m^2}, \quad i \in M \tag{3}$$

if $|M| = m$.

We apply the stability concept introduced by d'Aspremont et al. (1983) to predict which cartel will form in stage one. When there are $|M| = m$ members, let $u^C(m)$ and $u^I(m)$ denote the payoff of a member and a non-member, respectively. A cartel $M \notin \{\emptyset, N\}$ is said to be stable if $u^I(m) > u^C(m+1)$, and $u^C(m) \geq u^I(m-1)$. Further, N is stable if $u^C(n) \geq u^I(n-1)$, while $\emptyset$ is stable if $u^I(0) > u^C(1)$. A stable cartel is one in which no player has an incentive to unilaterally deviate from his or her participation decision.

From (1) (2) and (3), the condition for M to be stable is $m = 2$. Therefore, the payoff of a cartel member is $u^C(2) = 4.71$; the payoff of a non-member is $u^I(2) = 6.08$; and the social welfare is $2u^C(2) + 3u^I(2) = 27.67$.

In this example, the MJP assumption requires each member $i \in M$ to follow a specific agreement in stage two — to produce $x_i = \frac{150m}{155-m+m^2}$ if $|M| = m$. But is this a reasonable assumption? Let us examine this cartel formation game with the following agreement: each member $i \in M$ should produce $x_i = 0.5m$ if $|M| = m$. As a reaction to this alternative agreement, the output of each non-member $j \notin M$ is $x_j = \frac{300-m^2}{160-2m}$ if $|M| = m$. Thus, it is easy to verify that the condition for M to be stable is $m = 5$, the payoff of each member is $u^C(5) = 8.85$, and the social welfare is $5u^C(5) = 44.27$.

This outcome shows that everyone (including members, non-members, and the social planner who cares about social welfare) will agree to change the cartel agreement from $x_i = \frac{150m}{155-m+m^2}$ to $x_i = 0.5m$. The new agreement is better than the MJP agreement, irrespective of the criterion used to evaluate it. Intuitively, this is because the new agreement provides a smaller incentive for players to free ride on other players' effort than the MJP agreement

does[4]. As a result, more players choose to join the cartel and a more efficient outcome is realized.

3 Discussion

We have shown that the MJP assumption is problematic despite seeming quite intuitive and being widely applied. At least under some situations, everyone has an incentive to replace the agreement based on the MJP assumption by one that provides less free-riding incentive. The problem with the MJP assumption is that it imposes some unnecessary restrictions on members' actions. We should only care about players' payoffs in a stable cartel, rather than the payoffs in all possible cartels.

Since agreements are not binding, some readers may wonder whether the non-MJP agreement $x_i = 0.5m$ is renegotiation-proof against the MJP agreement $x_i = \frac{150m}{155-m+m^2}$. That is, once the cartel is formed and all players receive their payoffs under $x_i = 0.5m$, will the members have incentives to switch this agreement to the MJP agreement? In fact, none of the members will choose to do so, since otherwise their payoffs will either decrease from 8.85 to 4.71 (as a member), or decrease from 8.85 to 6.08 (as a non-member).

A question is whether the new agreement ($x_i = 0.5m$) is "optimal". The point is that an explicit criterion is needed to establish whether or not an agreement is "optimal". For future studies, a lesson we can learn from this note is that a cartel agreement should be endogenously determined, rather than exogenously given. Under noncooperative coalition formation setting, some studies (Carraro et al., 2009; Köke and Lange, 2017; Mao, 2017) have already discussed endogenous agreements of cartel formation models in some specific applications, but more work is needed in more general situations.

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[4]Let $\Delta(m) = u^I(m-1) - u^C(m)$ characterizes the degree of a cartel member's free-riding incentive when $|M| = m$. The value of $\Delta(m)$ under agreement $x_i = \frac{150m}{155-m+m^2}$ is larger than that under agreement $x_i = 0.5m$ for all $m \geq 1$.

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