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A Mixed Integer Linear Programming Model to Regulate the Electricity Sector

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Abstract

This paper introduces the concept of market design and make the distinction between the three different levels of market design such as industry structure, wholesale and marketplace design. We present a mixed-integer linear programming (MILP) model for the optimal long-term electricity planning of the Greek wholesale generation system. In order to capture more accurately the technical characteristics of the problem, we have divided the Greek territory into a number of individual interacted networks (geographical zones). In the next stage we solve the system of equations and provide simulation results for the daily/hourly energy prices based on the different scenarios adopted. The empirical findings reveal an inverted-M shaped curve for electricity demand in Greece, while the SMP curve is also non-linear. Lastly, given the simulations results, we provide the necessary policy implications for government officials, regulators and the rest of the marketers.

JEL codes: C60; Q40; L94;

Keywords: Electricity market; Linear programming; Constraints; Day-ahead scheduling; Mathematical programming.

1. Introduction

During the last years, there is a process in the European Union (EU) towards the integration of electricity markets, through market coupling and the establishment of a common Target Model. This is done mainly through the introduction of wholesale electricity markets (exchange type) and the unbundling of the traditional vertically integrated monopolies. The pioneer in the electricity sector reform was Chile, commencing its efforts in 1982. Since then, many EU countries (i.e. Germany, France, United Kingdom, Belgium, etc) deregulated their electricity markets, following different paths. The differences in the pace and extent of market reforms are mainly related to the starting point of each reform and the problems associated with the internal environment of the market. This is more evident in Europe, where although a goal for a single market has been set back in 1996 (Directive 96/92/EC), different levels of unbundling and introduction of competition have been implemented across the member states (Fiorio and Florio, 2013).

There is a substantial body of literature estimating the optimal planning of the wholesale electricity market. One strand of literature tries to investigate the price responsiveness of electricity consumers, based on non-dynamic electricity prices neglecting demand response to real-time market prices (Wolak, 2011; Genc, 2016; Clastres and Khalfallah, 2015). The other strand of literature identifies that demand response resources may have noticeable impact on the electricity markets' operation (Magnago et. al., 2015; Jiang et. al., 2014; Philpott et. al., 2000; Downward et. al., 2016; Dagoumas and Polemis, 2017).

The aim of this paper is to build a MILP model that will be used mainly in order to simulate the daily/hourly energy prices in the Greek electricity industry. For this reason, it attempts to quantify all the parameters of an optimization problem, integrating a unit commitment model, which is applied in case of the Greek power system. The Unit Commitment (UC) problem identifies the units that will operate in the day-ahead electricity market based on an optimization approach that considers their variable costs, their bidding strategy, the ancillary services and other technical criteria required by the Transmission System Operator (TSO). In order to capture more accurately the technical characteristics of the problem, we have divided the Greek territory into a number of individual interacted networks (geographical zones).optimization problem the computation of daily/hourly energy prices.

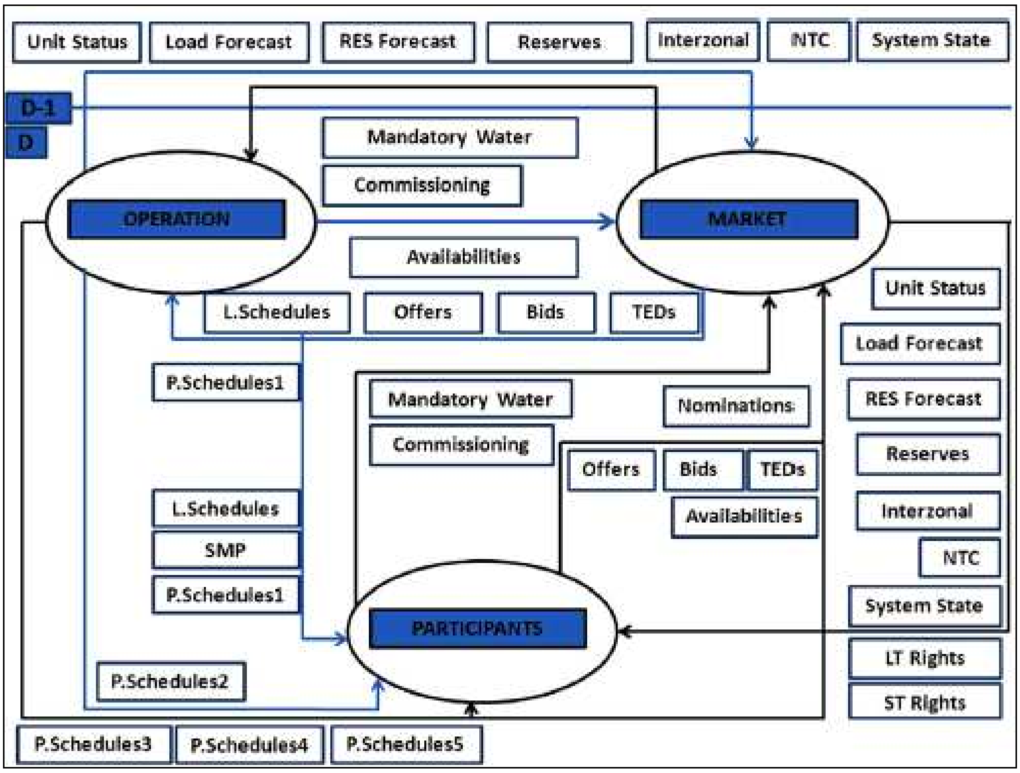
This paper contributes to the relevant literature in several ways. First, our model, tries to identify the main determinants of the optimal planning of the wholesale electricity market. For this reason, we measure the impact of certain main parameters such as the selection of the power generation technologies, the type of fuels used in the electricity generation procedure and lastly the power plant locations. Second, it provides a price signal on the profitability of retailers. Third, our model identifies the retailers risk generated from their price responsive customers. Based on the above, it is worth mentioning that the Greek electricity market incorporates a complex mathematical algorithm, considering economic and technical characteristics. The motivation of this paper is to present the formulation of the Day-Ahead Scheduling (DAS) problem in the Greek mandatory wholesale market. We also tried to sketch some of the most important issues that market designers have to deal with in Greece's wholesale electricity market such as the role of imports/exports, hydro plants, renewable energy sources and priced demand.

The rest of the paper is organised as follows. Section 2 describes the functioning of the electricity industry in Greece, while the mathematical formulation of the model is provided in Section 3. In Section 4, we present the simulation results of our energy model using different scenarios. Finally, Section 5 concludes the paper offering some useful policy recommendations.

2. The electricity sector in Greece

The Greek wholesale electricity market has been organised as a pure mandatory pool since its inception in 2005, so as to allow competition to emerge, in a context which, however, had a severe constraint. In particular, the incumbent remained dominant in both the generation and the retail sectors, retaining exclusive access to cheap lignite and hydro resources, while retail prices, despite the gradual removal of cross-subsidies, remained unlinked to wholesale prices. This combination of unfavourable market conditions posed severe obstacles to new entrants in the early years of market liberalisation, resulting in capacity shortage over the subsequent years. The Greek electricity market incorporates a complex mathematical algorithm, considering economic and technical characteristics. Figure 1 provides an overview of the Greek electricity market, showing the linkages between the Day-ahead market and the real-time dispatch schedule. The main responsibility of the Organised Electricity Market (OEM) is the determination of the Day-Ahead electricity price, considering the energy offers and the load declarations of participants as well as the technical characteristics of the system.

Figure 1: Overview of the Greek wholesale market



Source: Polemis and Dagoumas, (2013)

More specifically, the day-ahead procedure (Day Ahead Market Clearing) produces a System Marginal Price (SMP) for each settlement period (one hour) and a 24 hour production schedule for each unit. The solution of the day-ahead procedure will be based on the co-optimization of the energy offers (energy market) and reserve offers (balancing market) in order to satisfy the energy demand and reserve requirements, while the transmission system zonal constraint mechanism will introduce an additional constraint. The liberalization of the electricity market and the incentives given by the Greek state, have led to a change in the fuel mix through the on-going penetration of natural gas and renewables. Moreover, the operation of the electricity market has led to re-adjustments of the electricity tariffs, as the suppliers were in position to compete with the tariffs of the PPC and have taken an important share of the market. This was highly influenced by the level of demand. In a neoclassical market, which "*obeys*" the laws of demand and supply as the Greek electricity industry is operating, if the demand is decreased, the SMP either remains stable or is decreased. The decrease can be significant due to the significant difference in the variable cost (and consequently in the energy offers) between lignite and natural gas units. The usage of the interconnection capacity is also playing important role in the determination of the SMP. Therefore the price is highly dependent on the economic offers of the participants and on the level of the electricity demand. On the other hand, the electricity demand is highly influenced by the electricity prices.

3. Model

In this section, we present the Day Ahead Scheduling (DAS) problem under a MILP framework in order to determine the strategic (e.g., construction of new plants, capacity expansion) and operational (e.g., flows of electricity and energy resources) decisions[1]. In order to preserve space and to enhance the readability of the model, the description of the variables (nomenclature) is included in the Appendix A (see Tables A1 and A2).

[1] The size of the problem is rather large since it contains 50 dispatchable units (31 thermal and 19 hydro). The variables are calculated up to 31.000 (7.000 of them are integer) and the number of constraints up to 38.000. For 60 dispatchable units (20% increase) the size of the problem becomes 36.000 x 46.000 (16% x 20%), while for 75 dispatchable units (50% increase) the size is 42.500 x 58.000 (41% x 49%).

This study constitutes an integrated approach which combines a unit commitment model (MILP) well-grounded at an hourly level with the three distinct aspects of market design (i.e industry structure, wholesale and marketplace design). This approach is based on similar studies in the literature (Koltsakis and Georgiadis, 2015, Koltsakis et al, 2014; Koltsakis, et, al, 2015; Dagoumas and Polemis, 2017; Lu et al, 2018), which presented a market-based medium-term power systems planning model. However, this work is further extended to capture some of the most important issues that regulators and system operators have to deal with in Greece's wholesale electricity market such as the role of imports/exports, hydro plants, renewable energy sources and priced demand.

3.1 Objective function

In the DAS problem we face a discrete type of auction where offers and bids refer to quantities of energy (blocks). Thus, the objective function has to describe the maximization of the difference between the value of all energy blocks which demand side would pay and the amount of money that supply side would be paid in order to generate these energy blocks. Actually, it is the maximization of the difference between the demand and supply revenue streams. The maximization though is not a simple difference since in the market participate many players and each of them submits bids or offers for many energy blocks. Additionally, though all the offer and bid quantities refer to one-hour time period, the optimization is conducted for a wider time frame which is typically 24 hours. A simplified illustration of the objective function of the DAS problem is given by:

$$\max \quad \sum_{t}^{24}\left(\sum_{D=1}^{D}\sum_{j=1}^{j} Q_{D_j}^{t} P_{D_j}^{t} - \sum_{S=1}^{S}\sum_{j=1}^{j} Q_{S_j}^{t} P_{S_j}^{t}\right) \qquad (1)$$

where energy blocks are denoted with *j*, demand side players with *D* and supply side players with *S*. Actually, the objective function is the difference between the demand and the supply *P*x*Q* products where *P* is the price and *Q* the quantity of each energy block within the 24- hours time frame.[2]

3.2 Model constraints

Model constraints concern the power system as a whole and refer to issues such as load satisfaction, power flow congestion, exchanges with other power systems through interconnections and system requirements for ancillary services. These are described in the next sub sections.

3.2.1 Load Constraints

These constraints are formulated for each specific zone that has been defined by the corresponding study done by OEM. The power system is divided in geographical zones in such a way that reflects possible appearance of congestion in power flow. It is assumed that there are two geographical regions that define the zones: northern and southern Greece. Load constraints are set for both zones and their purpose is to express and at the same time to assure the balance between load and generation. These load constraints, in their general form, are expressed by the following simplified relation:

(Zonal Demand – Zonal Generation + Zonal Exports – Zonal Imports) = 0 (2)

The structure of the analytical form of these constraints is similar to the structure of the objective function with some differences: loss factors are applied to all load and generation quantities and reserve quantities for ancillary services are not included in the load constraints. The last two variables expressing zonal imports and exports are mutually excluded (i.e. if one of them takes a positive value the other is set equal to zero).

[2] For an analytical presentation of the objective function see Appendix B.

For z =1 (North) and for everyDispatch Period t

$$\sum_{p=1}^{p}\frac{1}{a_p^t}x_p^{t,1} + \sum_{pp=1}^{pp}\frac{1}{a_{pp}^t}\sum_{s=1}^{s}x_{pp_s}^{t,1} + \sum_{k=1}^{k}\frac{1}{a_k^t}\sum_{s=1}^{s}x_{k_s}^{t,1} + \sum_{pm=1}^{pm}\frac{1}{a_{pm}^t}\sum_{s=1}^{s}x_{pm_s}^{t,1} - \sum_{u=1}^{u}a_u^t\left(y_u^{t,1}Q_{\min_u} + \sum_{s=1}^{s}x_{u_s}^{t,1}\right) - \sum_{j=1}^{j}a_j^t\sum_{s=1}^{s}x_{j_s}^{t,1} - \sum_{con=1}^{con}a_{con}^t x_{con}^{t,1} - \sum_{r=1}^{r}a_r^t x_r^{t,1} + x_{z1\to z2}^t - x_{z2\to z1}^t = 0$$

(3)

Non-Priced Priced Exports Pumped Storage Dispatchable Imports Contracted

non-Priced Zonal Zonal

Load Load Units Units Units

Generation Exports Imports

Demand ***Supply***

For z =2 (South) and and for everyDispatch Period t

$$\sum_{p=1}^{p}\frac{1}{a_p^t}x_p^{t,2} + \sum_{pp=1}^{pp}\frac{1}{a_{pp}^t}\sum_{s=1}^{s}x_{pp_s}^{t,2} + \sum_{k=1}^{k}\frac{1}{a_k^t}\sum_{s=1}^{s}x_{k_s}^{t,2} + \sum_{pm=1}^{pm}\frac{1}{a_{pm}^t}\sum_{s=1}^{s}x_{pm_s}^{t,2} - \sum_{u=1}^{u}a_u^t\left(y_u^{t,2}Q_{\min_u} + \sum_{s=1}^{s}x_{u_s}^{t,2}\right) - \sum_{j=1}^{j}a_j^t\sum_{s=1}^{s}x_{j_s}^{t,2} - \sum_{con=1}^{con}a_{con}^t x_{con}^{t,2} - \sum_{r=1}^{r}a_r^t x_r^{t,2} + x_{z2\to z1}^t - x_{z1\to z2}^t = 0 \quad (4)$$

3.3 Transmission Constraints

Interzonal power flow constraints refer to the capability of the transmission lines that connect the zones mentioned above:

$$\forall t \qquad x^{t}_{z1\rightarrow z2} \leq FL_{z1\rightarrow z2} \tag{5}$$

$$\forall t \qquad x^{t}_{z2\rightarrow z1} \leq FL_{z2\rightarrow z1} \tag{6}$$

Equation (5) denotes that the power flow from zone 1 (northern Greece) to zone 2 (southern Greece) cannot exceed a certain amount ($FL_{z1\rightarrow z2}$). Equation (6) denotes the same for the flow from zone 2 to zone 1. If the transmission constraints are activated during the Day-Ahead Scheduling procedure then the differential variable generation cost of each unit is used instead of the offered prices declared in the Injection Offers. If still the transmission constraints are activated then the problem is solved with these constraints on and different marginal prices are calculated for different system regions.

3.3.1 System Interconnections Capacity Constraints

These constraints are set in order to control the import and export flow regarding to the capacity of the interconnection transmission lines. The first constraint (Eq. 7) is set at node level. More specifically, it denotes that for each Dispatch Period *t*, the sum of all quantities to be exported from node *m*, from all exporters *k,* is less than or equal to the exporting capacity of the specific node $IntC_{exp}{}^{t}{}_{m}$. Constraint (Eq. 8) expresses the same, but for a set (m*) of interconnection nodes (e.g. all the north interconnections).

$$\forall m,t \qquad \sum_{k^{\mathrm{m}}=1}^{k^{\mathrm{m}}} \sum_{s=1}^{s} x^{t,z}_{k^{\mathrm{m}}{}_{s}} \leq IntC_{Exp_{m}}{}^{t} \tag{7}$$

$\forall m^*, t \qquad \sum_{k^{\mathrm{m}}=1}^{k^{\mathrm{m}}} \sum_{s=1}^{s} x_{k^{\mathrm{m}}{}_s}^{t,z} \leq IntC_{Exp_{m^*}}^{\;\;t}$ (8)

where $m \in m^*$

Constraint (9) denotes that for each Dispatch Period *t*, the sum of all quantities to be imported from node *m*, from all importers *j,* is inferior to the importing capacity of the specific node $IntC_{imp}{}^{t}{}_{m}$. Constraint (10) expresses the same, but for a set (m*) of interconnection nodes (e.g. all the north interconnections).

$\forall m, t \qquad \sum_{j^{\mathrm{m}}=1}^{j^{\mathrm{m}}} \sum_{s=1}^{s} x_{j^{\mathrm{m}}{}_s}^{t,z} \leq IntC_{imp_m}^{\;\;t}$ (9)

$\forall m^*, t \qquad \sum_{j^{\mathrm{m}}=1}^{j^{\mathrm{m}}} \sum_{s=1}^{s} x_{j^{\mathrm{m}}{}_s}^{t,z} \leq IntC_{imp_{m^*}}^{\;\;t}$ (10)

where $m \in m^*$

Additionally, two more constraints, (11) and (12), are set for the total exporting and importing capacity of the system. Constraint (11) implies that for each Dispatch Period *t*, the sum of all quantities to be exported from all nodes, from all exporters *j,* is less than or equal to the exporting capacity of the system $exp^{t}{}_{sys}$. Respectively, constraint (12) implies that that for each Dispatch Period *t*, the sum of all quantities to be imported from all nodes, from all exporters *j,* is less than or equal to to the importing capacity of the system $imp^{t}{}_{sys}$.

$\forall t \qquad \sum_{k^{m}=1}^{k^{m}} \sum_{s=1}^{s} x_{k^{\mathrm{m}}{}_s}^{t,z} \leq exp_{sys}^{t}$ (11)

$\forall t \qquad \sum_{j^{\mathrm{m}}=1}^{j^{\mathrm{m}}} \sum_{s=1}^{s} x_{j^{\mathrm{m}}{}_s}^{t,z} \leq imp_{sys}^{t}$ (12)

3.3.1 System Requirements for Ancillary Services Constraints

The following set of constraints refers to system's requirements for ancillary services (primary reserve, secondary range reserve and tertiary reserve – spinning and non spinning).

3.4 Primary Reserve Requirements Constraints

Constraint (13) denotes that for a specific Dispatch Period *t*, the sum of all reserve quantities for primary reserve from all units *u* must be equal to or greater than the system total requirement for primary reserve $Q_{PR}{}^{t}$.

$$\forall t \qquad \sum^{u} x_{PR_u}{}^{t,z} \geq Q_{PR}{}^{t} \tag{13}$$

3.4.1 Secondary Reserve Requirements Constraints

Constraint (15) refers to the upward reserve range for secondary control and denotes that for each Dispatch Period *t*, the sum of all reserve quantities for upward secondary reserve from all units *u* must be equal to or greater than system's required generation increase for secondary control $Q^{up}{}_{SEC}{}^{t}$.

$$\forall t \qquad \sum^{u} x^{up}_{SEC_u}{}^{t,z} \geq Q^{up}_{SEC}{}^{t} \tag{14}$$

Similarly, for the downward reserve range for secondary control constraint (15) denotes that for each Dispatch Period *t*, the sum of all reserve quantities for downward secondary reserve from all units *u*, must be equal to or greater than system's required generation decrease for secondary control $Q^{dw}{}_{SEC}{}^{t}$.

$$\forall t \qquad \sum^{u} x^{dw}_{SEC_u}{}^{t,z} \geq Q^{dw}_{SEC}{}^{t} \tag{15}$$

In that case the total generation output variations, within a Dispatch Period t, must respect system ramp-up and ramp-down capability. Constraint (16) implies that, for each Dispatch Period t, the maximum expected increase of total generation for secondary control, calculated as the sum of the upward secondary reserve of all generation units, must not exceed system's ramp-up rate R^{up}_{sys}, expressed in MW/h per 60 min.

$$\forall t \qquad \sum^{u} x^{up}_{SEC_u}{}^{t,z} \leq R^{up}_{sys} \tag{16}$$

Respectively, constraint (17) denotes that, for each Dispatch Period t, the maximum expected decrease of total generation for secondary control, calculated as the sum of the downward secondary reserve of all generation units, must not exceed system's ramp-down rate R^{dw}_{sys}, expressed in MW per 60 min.

$$\forall t \qquad \sum^{u} x^{dw}_{SEC_u}{}^{t,z} \leq R^{dw}_{sys} \tag{17}$$

3.4.2 Tertiary Reserve Requirements Constraints

Constraint (18) implies that for each Dispatch Period t, the spinning (x_{ST}) and non spinning (x_{NST}) reserves of all units, for tertiary control must be equal to or greater than system's total requirements for tertiary reserve $Q_{TER}{}^{t}$.

$$\forall t, reg \qquad \sum^{u^{reg}} x_{ST}{}^{t,z}_{u^{reg}} + \sum^{u^{reg}} x_{NST}{}^{t,z}_{u^{reg}} \geq Q^{reg\ t}_{TER} \tag{18}$$

Index *reg* here denotes the constraint may be implemented once for the whole system or more than once for different sub regions of the system, for operational reasons. These sub regions are not identical with the zones, mentioned above which are related to transmission flow restrictions.

4. Assumptions and simulation results

This section provides assumptions and simulation results of the various scenarios under consideration.[3] Specifically, the scenarios examined in this study concern the cases where the retailers' customers are price sensitive or not. Similar to Koltsaklis and Georgiadis (2015), Koltsakis et al, (2016) and Dagoumas and Polemis, (2017) the problem to be addressed is concerned with the hourly energy balance of a specific power system including the optimal dispatch of power generating units (UCP). Therefore the problem under consideration is formally defined under the following assumptions:

a) The scheduling period includes hourly time steps $t \in T$, where the electricity market operator determines the optimal scheduling plan for the 24 hours of the next day (day-ahead market).

b) The power system under consideration is split into a number of subsystems $s \in S$. These subsystems are further divided into a certain number of zones $z \in Z$ to better represent the system's regional/spatial characteristics.

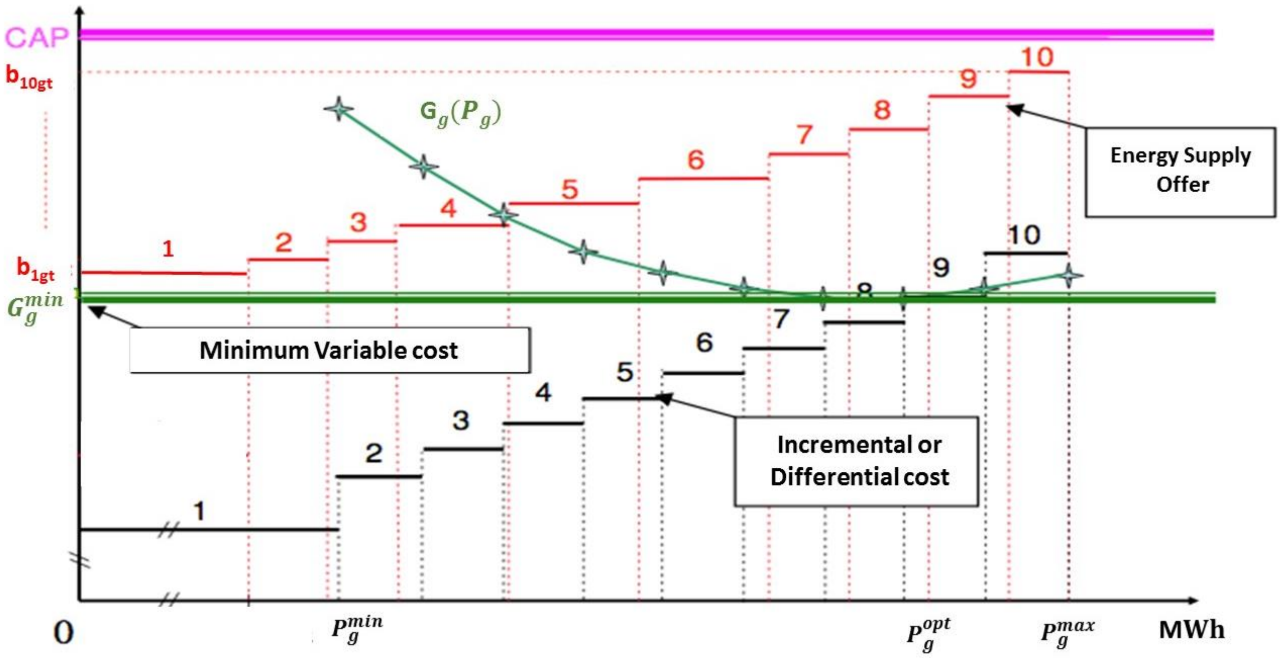
c) A set of power generating units $g \in G$ is installed in each subsystem $g \in G^s$ (or zone $g \in G^z$). This set includes thermal units $g \in G^{th}$, hydroelectric units $g \in G^h$, (both referred to as hydrothermal ones $g \in G^{hth}$) and renewable units $g \in G^{res}$. Each renewable unit $g \in G^{res}$ is characterized by a specific availability factor in each zone and time period, $AF_{g,z,m,t}$. Each unit $g \in G$ is characterized by a specific available power capacity $PC_{g,m,t}$.

d) The available power capacity of each hydrothermal unit $g \in G^{hth}$ is divided into a number of blocks $b \in B$, $PCB_{g,b,m,t}$ to fully represent the operational characteristics of each unit and the real operation of power markets. In each time period and for each power capacity block, each hydrothermal power generating unit provides a specific amount of energy (to be determined by the optimization process) at a specific price (marginal cost), $CB_{g,b,m,t}$ (incorporating variable operating

[3] This mathematical problem has been solved to global optimality making use of the ILOG CPLEX 24.7.2.

and maintenance cost, fuel cost, and CO_2 emissions cost) in order for the power demand in each subsystem and time period, $D_{s,m,t}$, to be covered. Figure 2 presents the energy supply offer for a thermal unit u, compared to its incremental cost and its minimum variable cost, for different power outputs, among unit's technical minimum P_u^{min} and technical maximum P_u^{max}.

Figure 2: Energy supply offer for a thermal unit (Euro/MWh)



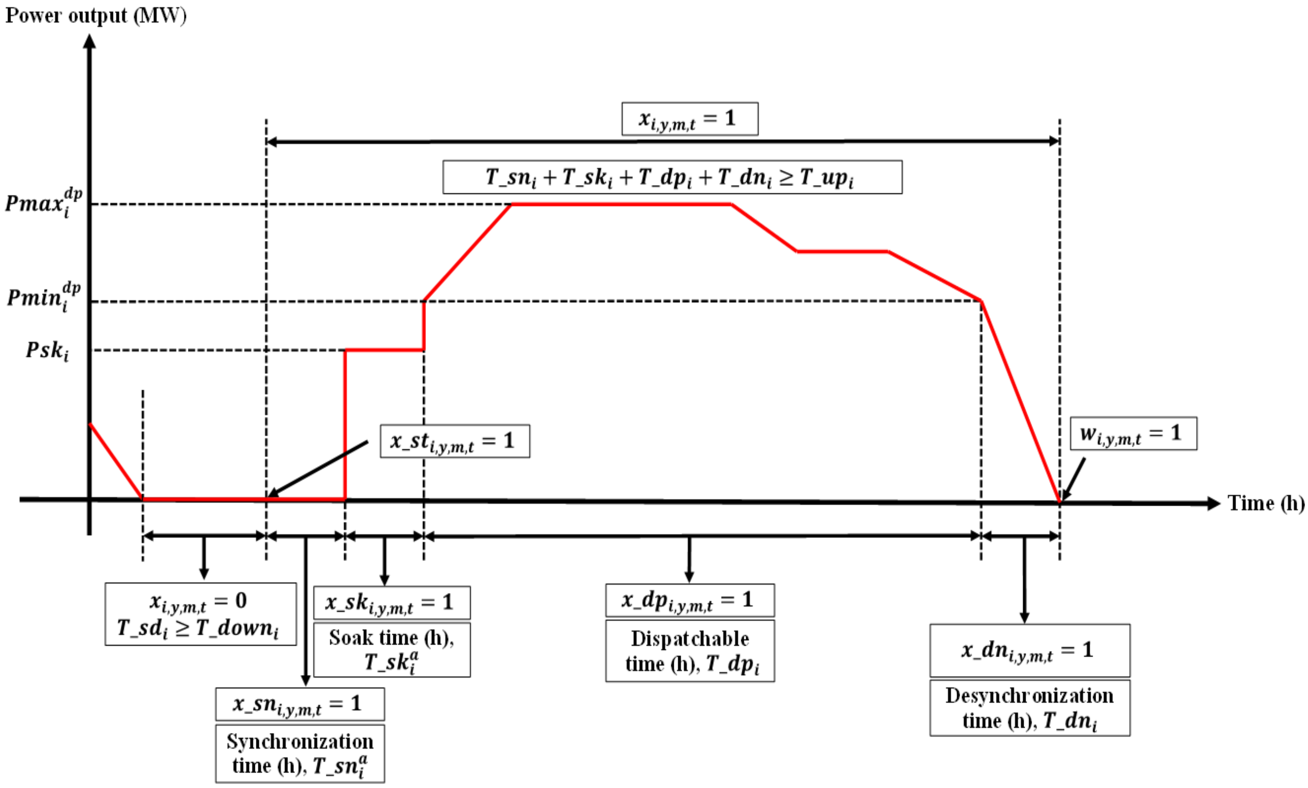
e) The same rule applies to both electricity imports from each interconnected system, $n \in N^s$ (or zone $n \in N^z$), and load representatives such as power exports to other interconnected systems $n \in N^s$ (or zone $n \in N^z$), and pumping load $e \in E$. More specifically, the power capacities of each interconnection $n \in S^s$ and pumping load $e \in E^s$, is divided into certain blocks ($IP_{n,b,m,t}$ for imports, $EP_{n,b,m,t}$ for exports, and $PMB_{e,b,m,t}$ for pumping load), having a certain marginal cost $CIP_{n,b,m,t}$ for imports, and a given bid $CEP_{n,b,m,t}$ for exports and $CPM_{e,b,m,t}$ for pumped storage units.

f) Apart from the priced component of each unit's energy offer function, there can be a non-priced one (zero marginal cost), $NP_{g,m,t}$, including mandatory hydroelectric injection, power injection from renewable units, and power contribution from commissioning units.

g) With reference to the operational cycle of each hydrothermal unit $g \in G^{hth}$: after a shut-down decision has been taken for each unit, it has to remain off (non-operational) for at least T_g^{down} hours, i.e., it is associated with specific minimum down time. A certain cost is associated with the shut-down decision of each unit $g \in G^{hth}$, SDC_g.

h) According to the real non-operational time of each unit $g \in G^{hth}$, T_g^{rdn}, there are three available start-up types $w \in W$ $\{W: hot, warm, cold\}$ when a start-up decision is determined by the model. There are specific time limits after which each unit $g \in G^{hth}$ changes stand-by condition, including time before going from hot to warm (T_g^{htw}) and warm to cold stand-by condition (T_g^{wtc}) respectively. After the determination of the appropriate start-up type decision, each unit enters the synchronization phase followed by the soak phase, which have a duration of $T_g^{sync,w}$ and $T_g^{soak,w}$ hours respectively, during which phases unit's power output is zero and P_g^{soak} respectively. The duration of both phases is dependent on the selected start-up type decision $w \in W$. After the completion of the soak phase, each unit $g \in G^{hth}$ enters the dispatchable phase, wherein its power output range from its technical minimum, P_g^{min}, to its technical maximum, P_g^{max}, or from $P_g^{min,sc}$ to $P_g^{max,sc}$, if that unit is selected for providing secondary reserve. During that phase, each unit is characterized by specific up, R_g^{up}, and down, R_g^{down}, ramp rates, or, R_g^{sc}, when providing secondary reserve (up and down). The last operational stage of each unit $g \in G^{hth}$ is that of desynchronization with a duration of T_g^{desyn} hours. A unit $g \in G^{hth}$ is considered operational when it operates in each one of the aforementioned phases, i.e., synchronization, soak, dispatch and desynchronization. The total operational time of each unit must be greater than or equal to its minimum up time, T_g^{up} in order to be allowed to shut-down. The different phases of a thermal unit are represented in Figure 3.

Figure 3: Different phases of the operation of a thermal unit (MW)



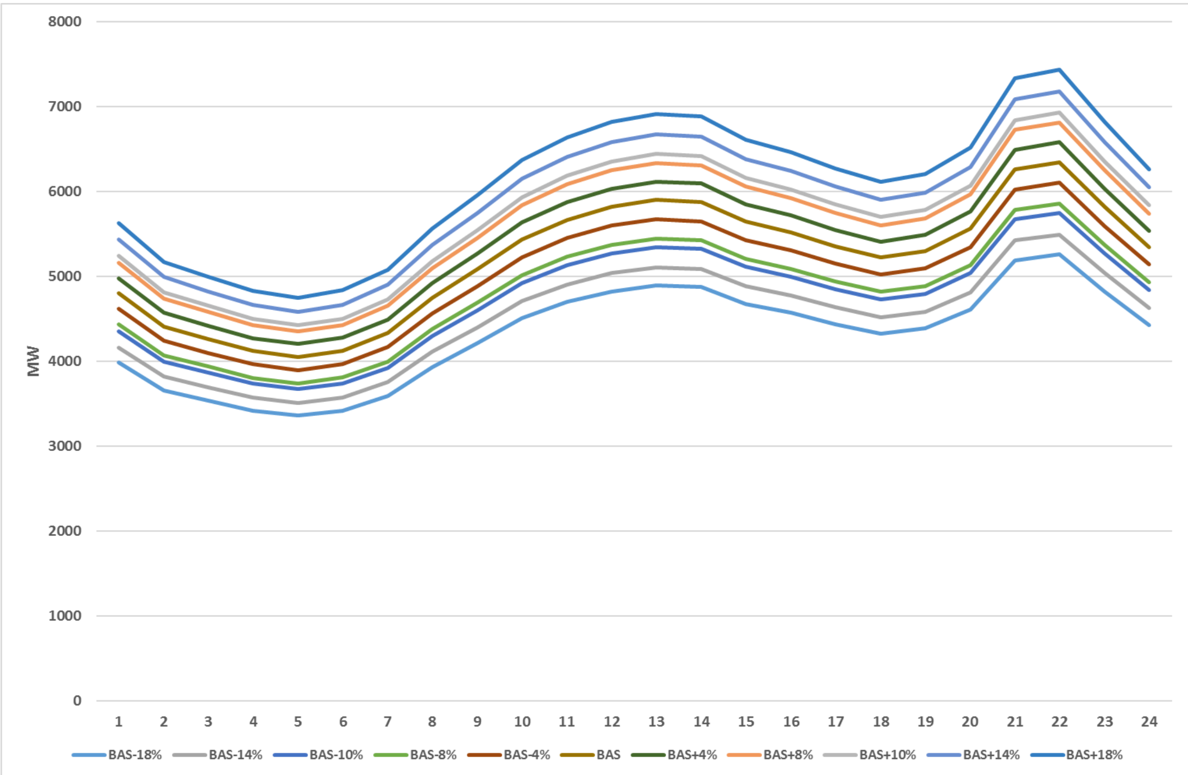
i) The power system's requirements include: (i) electricity demand requirements in each subsystem and time period, $D_{s,t}$, (ii) primary-up reserve requirements in each time period, $R1_t^{up}$, (iii) secondary-up, $R2_t^{up}$, and secondary-down, $R2_t^{down}$, reserve requirements in each time period, (iv) fast secondary-up, $FR2_t^{up}$, and fast secondary-down, $FR2_t^{down}$, reserve requirements in each time period, and (v) tertiary reserve requirements in each time period, $R3_t$.

j) When it comes to power reserve provision capabilities, each unit $g \in G^{hth}$ is identified based on: (i) upper bound on the provision of primary reserve, $R1_g$, (ii) upper bound on the provision of secondary reserve, $R2_g$, (iii) upper bound on the provision of tertiary spinning, $R3_g^{sp}$, and non-spinning reserve, $R3_g^{nsp}$. Each unit's energy reserve offer has a certain price, i.e., $RC1_{g,t}$ for the primary energy reserve, and $RC2_{g,t}$ for the secondary range energy offer, while tertiary energy offer is non-priced.

k) The electricity demand is considered to be responsive to price signals. The final consumers respond to fluctuations of the $SMP_{s,t}$, when a tolerance level $TOL_{s,c,t}$ is activated for a customer type $c \in C$. This tolerance concerns the percentage of change between the $TARIFF_{s,r,c,t}$ and the $SMP_{s,t}$. Practically, when final consumers find a price spike, positive or negative, where they respond by decreasing or increasing respectively their consumption.

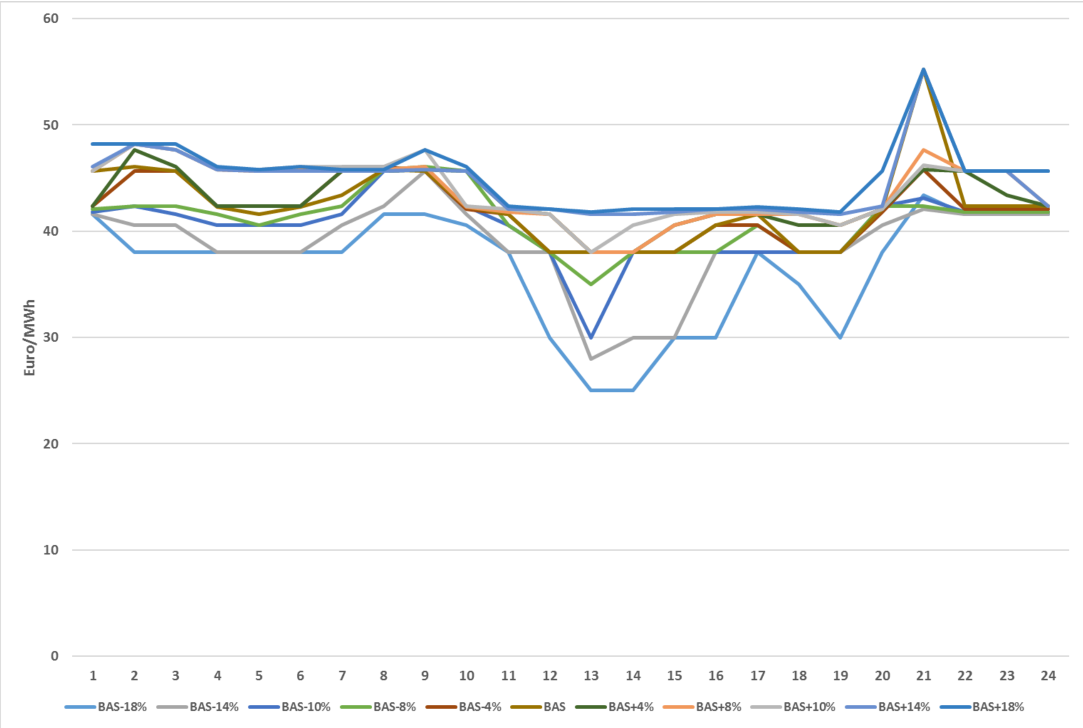
Based on the above considerations, we proceed with the simulations results. For the purpose of our study we implement a Monte Carlo analysis, assuming a ± 20% deviation over its reference prices (Dagoumas and Polemis, 2017). In the following figures the simulation results (compared to the baseline scenario) of the total electricity demand and the SMP evolution over a 24hour period are portrayed. From the inspection of Figure 4, it is obvious that the day-ahead electricity market is characterized from non-linearity in the effect of demand response.

Figure 4: Simulated demand curve evolution over a 24-hour period (Euro/MWh)



This finding which has also been found to other studies (see Dagoumas and Polemis, 2017), stipulates that electricity demand in Greece follows an inverted-M shape. In other words, the simulated pattern implies that total electricity demand is characterized by strong cyclicality effects. Based on the existence of such effects, a decrease in electricity consumption is evident late at nights or early in the evening. On the contrary, electricity demand spikes are existent during the day or late in the evening hours. This result raises important policy implications in terms of market regulation toward a more effective electricity management in Greece. Specifically, the existence of a cyclical pattern in the electricity demand is important primarily to the Transmission System Operator (TSO) which must match electricity supply to demand in real time. Changes in electricity demand levels are generally predictable and have daily, weekly, and seasonal patterns. In this case, electricity demand levels rise throughout the day and tend to be highest during a block of hours ("on-peak") which usually occurs between 7:00 a.m. and 10:00 p.m. on weekdays and lower during the “off-peak” hours (between 22:00 a.m. and 5:00 p.m. and during the weekends). Moreover, a stable predictable pattern of electricity demand is also useful to the regulator which may lower any discrepancies in the transmission system (i.e brown outs) in order to achieve one of its primary goals namely the energy security supply. Lastly, the existence of an inverted-M shape curve in the electricity consumption, may also affect the electricity supply side since the stakeholders of a power plant (i.e investors, stockholders, etc) may address the demand fluctuations in a more efficient way. Similarly, the SMP follows a non-monotonic pattern during the 24hour period (see Figure 5). However, in this case, cyclicality effects are absent. This raises important implications. Firstly, similarly to other studies (see for instance Lu et, al, 2018; Dagoumas and Polemis, 2017; Koltsakis et, al, 2016) a linear fluctuation of total electricity demand, due to demand response, leads to non-linear evolution the SMP. Secondly, the non-linear evolution of SMP is strongly linked to a number of factors such as the marginal cost of the power plants, the bidding strategies of the market players during the Day Ahead Market and finally the technical characteristics of the power generators.

Figure 5: Simulated SMP variation over a 24-hour period (Euro/MWh)



Based on the simulation results of the MILP some important policy implications emerge. First, the non-monotonic relationship between aggregate demand (appeared in an inverted-M shape) and the evolution of intra-day SMP (expressed in a non-cyclical pattern), stimulates risk for the incumbent firms in the retail segment of the industry (i.e retailers, suppliers, importers and exporters of electricity). This outcome might lead even to short-term losses for some short-term periods, affecting strongly the variability of the undertakings. Moreover, this may negatively affect the decision of the private firms to enter the Greek electricity industry by incurring high market entry and investment costs. Second, the MILP model provides the necessary price signals on the profitability of retailers, in their effort to formulate the necessary tariff rates. In this case, the proposed model may act as a pivotal study in order to uncover possible distortions and flaws of the Day Ahead Market. Third, the model is also useful for policy makers, government officials and regulatory bodies (i.e. transmission and distribution system operators), considering that it identifies the effect of demand responsiveness to the fluctuations of wholesale prices (SMP). Moreover, it shapes the electricity demand pattern in Greece and the formulation of the SMP during the daily hour fluctuations giving important level of information to the market participants (incumbent, independent power plants, retailers, etc) and the possible entrants.

5. Conclusions

In this paper, we present a MILP model for the optimal long-term electricity planning of the Greek wholesale generation system. In order to capture more accurately the technical characteristics of the problem, we have divided the Greek territory into a number of individual interacted networks (zones). The proposed model determines the optimal planning of the wholesale electricity market, the selection of the power generation technologies, the type of fuels and the plant locations so as to meet the expected electricity demand and possible environmental concerns. Despite the fact that the formulations of the model components are not introduced for the first time in the literature, their combination form a model with many significant parameters and restrictions suitable for policy modelling. For this reason, we assure that the model was implemented and thoroughly tested on a real data set from a recently liberalized electricity market.

Based on the above analysis, we argue that the Greek wholesale electricity market is a day-ahead mandatory pool scheme that provides a day ahead firm price based upon the supply/demand balance that ensures efficient short term dispatch taking in to account generation unit constraints, reserve requirements and a simplified transmission system zonal constraint mechanism. The day-ahead procedure (Day Ahead Market Clearing) produces a SMP for each settlement period (one hour) and a 24 hour production schedule for each unit. The solution of the day-ahead procedure is based on the co-optimization of the energy offers (energy market) and reserve offers (balancing market) in order to satisfy the energy demand and reserve requirements, while the transmission system zonal constraint mechanism will introduce an additional constraint. A regulated SMP price cap will be determined in order to prevent excessive price spikes in the event that insufficient capacity is declared available to meet the demand. Offers will be firm at the day-ahead market. Generators must maintain their availability as declared and be able to generate at the level set according to their day-ahead schedule. Changes in availability will result in an exposure to the imbalance price. Likewise, levels of demand declared by suppliers will also be firm and deviations will be liable for settlement at the imbalance price. Therefore, during the Day Ahead Settlement, generators are paid at the day-ahead SMP for their scheduled generation while there is no remuneration for the scheduled reserves. On the other hand, suppliers pay the day ahead SMP for their declared load.

Lastly, the proposed model provides useful insights into the risk of retailers and therefore acts as a pivotal study to policy makers and practitioners (i.e. regulators, TSO, DSO) active in the Greek electricity market.

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Appendix A

Nomenclature

1. General Symbols and Indexes

x: real variables

y, dy: integer variables

t: Dispatch Period (t = 1,2, .., 24)

s: priced energy blocks of generation offers or Load Declarations (s = 1,2, …, 10)

u: dispatchable generation units

r: refers to non-priced generation (e.g. RES, CHP, must-run hydro etc.)

u(hydro): refers to hydroelectric generation units (subset of **u**)

pm: refers to pumped storage units

i: refers to the prohibited (due to oscillations) generation level zones

pp: refers to Load Representatives that submit priced Load Declarations

p: refers to Load Representatives that submit non-priced Load Declarations

j: refers to importers

k: refers to exporters

con: refers to contracted units

m: interconnection nodes

z: refers to system's zones

$\boldsymbol{\alpha}$: refers to loss factor

PR: refers to primary reserve

SEC: refers to secondary reserve

ST, NST: refers to spinning and non spinning tertiary reserve

TER: refers to system tertiary reserve requirements

Table A1: Variable description

VARIABLE	DESCRIPTION	CODE SYMBOL
Real Variables		
$x_{pp_s}^{t,z}$	Priced load of block ***s***, in the Load Declaration of Load Representative pp, to be satisfied in zone ***z*** during the Dispatch Period ***t***.	$\sum^{s} x_{pp_s}^{t,z}$ = DASQpt
$x_{p}^{t,z}$	Non Priced load to be satisfied in zone ***z*** during the Dispatch Period ***t***, corresponding to the Load Declaration of Load Representative ***p***.	DASQpt (1 grade only)
$x_{k_s}^{t,z}$	Dispatched quantity of the offer block ***s*** of the exporter ***k*** to be exported from an interconnection node of zone ***z***, during the Dispatch Period ***t***.	$\sum^{s} x_{k_s}^{t,z}$ = DASQkmt
$x_{pm_s}^{t,z}$	Priced load to be satisfied in zone ***z*** during the Dispatch Period ***t***, corresponding to the Load Declaration of the pumped storage unit ***pm***.	-
$x_{j_s}^{t,z}$	Dispatched quantity of the offer block ***s*** of the importer ***j*** to be injected in an interconnection node of zone ***z***, during the Dispatch Period ***t***.	$\sum^{s} x_{j_s}^{t,z}$ =DASQjmt
$x_{r}^{t,z}$	Dispatched non-priced quantity of the generation unit ***r***, located in zone ***z***, during the Dispatch Period ***t***.	$\sum^{s} x_{r}^{t,z}$ =DASQrt
$x_{con}^{t,z}$	Dispatched quantity of the contracted generation unit ***u***, located in zone ***z***, during the Dispatch Period ***t***.	-
$x_{u_s}^{t,z}$	Dispatched quantity of the offer block ***s*** of the generation unit ***u***, located in zone ***z***, during the Dispatch Period ***t***.	$\sum^{s} x_{u_s}^{t,z}$ = DASQut
$x_{PR_u}^{\;t,z}$	Generation quantity reserve corresponding to the Primary Reserve for the generation unit ***u***, located in zone ***z***, during the Dispatch Period ***t***.	-
$x_{SEC_u}^{up\;t,z}$	Generation quantity reserve corresponding to the upward Secondary Reserve range for the generation unit ***u***, located in zone ***z***, during the Dispatch Period ***t***.	-
$x_{SEC_u}^{dw\;t,z}$	Generation decrease corresponding to the downward Secondary Reserve range for the generation unit ***u***, located in zone ***z***, during the Dispatch Period ***t***.	-
$x_{ST_{u^{reg}}}^{\;t,z}$	Generation quantity reserve corresponding to the Tertiary Spinning Reserve for the generation unit ***u***, located in zone ***z***, during the Dispatch Period ***t***.	-
$x_{NST_{u}^{reg}}^{\;t,z}$	Generation quantity reserve corresponding to Tertiary non-Spinning Reserve for the generation unit ***u***, located in zone ***z***, during the Dispatch Period ***t***.	-
Integer Variables		
$y_{u}^{t,z}$	Commitment status of generation unit ***u***, located in zone ***z***, during the Dispatch Period ***t*** (1: online, 0: offline)	-
$y_{AGC_u}^{\;t,z}$	AGC operating mode status for provision of Secondary Reserve) of	-

	generation unit u, located in zone z, during the Dispatch Period t. (1: operation in AGC mode, 0: operation not in AGC mode)	
${y_{DEC}}_u^{t,z}$	Decommissioning status of generation unit ***u***, located in zone ***z***, during the Dispatch Period ***t*** (1: the unit has just been decommitted, 0: all other statuses)	-
${dy_{COM_u}}^{t,z}$	Auxiliary integer variable denoting change in the operating status of generation unit ***u***, located in zone ***z***, during the Dispatch Period ***t*** (2: from offline to online, 1: no change, 0: from online to offline).	-
${dy_{DEC_u}}^{t,z}$	Auxiliary integer variable denoting change in the operating status of generation unit ***u***, located in zone ***z***, during the Dispatch Period ***t*** (2: from online to offline, 1: no change, 0: from offline to online).	-
${dz_{u_u}}^{i}$	Auxiliary integer variable to formulate the either-or constraints for the prohibited zones of a hydro electric unit.	-
Dependent Variables		
SMP^t	System's Marginal Price for the Dispatch Period ***t***.	DASMPt
LMP_1^t	Locational Marginal Price in zone 1, for the Dispatch Period ***t***.	-
LMP_2^t	Locational Marginal Price in zone 2, for the Dispatch Period ***t***.	-
$x_{z1\rightarrow z2}^t$	Transmission flow from zone 1 to zone 2 during the Dispatch Period ***t***.	-
$x_{z2\rightarrow z1}^t$	Transmission flow from zone 2 to zone 1 during the Dispatch Period ***t***.	-

Table A2: Input variables

VARIABLE	DESCRIPTION	CODE SYMBOL
P_{cap}	Price Cap	-
$P^{t,z}_{pp_s}$	Bid price of the load block ***s*** in the Load Declaration of Load Representative ***pp*** in zone ***z***, during the Dispatch Period ***t***.	-
$P^{t,z}_{k_s}$	Bid Price of the generation block **s** in the Load Declaration of the exporter ***k*** to be exported from an interconnection node in zone ***z***, during the Dispatch Period ***t***.	-
$P^{t,z}_{pm_s}$	Bid price of the load block ***s*** in the Load Declaration of pumped storage unit ***pm***, located in zone ***z***, during the Dispatch Period ***t***.	-
$P^{t,z}_{j_s}$	Price of the generation block **s** in the Injection Offer of importer ***j*** to be imported from an interconnection node in zone ***z***, during the Dispatch Period ***t***.	-
P_{syn_u}	Start up cost of unit ***u***	-
$P^{t,z}_{u_s}$	Offer Price for the generation block **s** in the Injection Offer of generation unit ***u***, located in zone ***z***, during the Dispatch Period ***t***.	STEPPsut
$P_{PR_u}{}^{t,z}$	Offer Price for the generation block corresponding to the Primary Reserve of the generation unit ***u***, located in zone ***z***, during the Dispatch Period ***t***.	-
$P_{SEC_u}{}^{t,z}$	Offer Price for the block corresponding to the Secondary Reserve range of the generation unit ***u***, located in zone ***z***, during the Dispatch Period ***t***.	-
$P_{u_{NST}}^{t,z}$	Offer Price for the generation corresponding to non-spinning Tertiary Reserve of the generation unit ***u***, located in zone ***z***, during the Dispatch Period ***t***.	-
a^t_p	Loss factor applied to the non-priced load, located in zone ***z*** and declared by the Load representative ***p***, during the Dispatch Period ***t***.	-
a^t_{pp}	Loss factor applied to the priced load, located in zone ***z*** and declared by the Load representative ***pp***, during the Dispatch Period ***t***.	-
a^t_k	Loss factor applied to the quantity to be exported by the exporter ***k*** from an interconnection node in zone ***z***, during the Dispatch Period ***t***.	-
a^t_{pm}	Loss factor applied to the load declared by the pumped storage unit ***pm***, located in zone ***z***, during the Dispatch Period ***t***.	-
a^t_u	Loss factor applied to the quantity generated by the generation unit ***u***, located in zone ***z***, during the Dispatch Period ***t***.	-
a^t_j	Loss factor applied to the quantity to be imported by the importer ***j*** from an interconnection node in zone ***z***, during the	-

	Dispatch Period ***t***.	
a_{con}^{t}	Loss factor applied to the quantity generated by the contracted unit ***con***, located in zone *z*, during the Dispatch Period ***t***.	-
a_{r}^{t}	Loss factor applied to the non-priced quantity generated by the unit ***r***, located in zone *z*, during the Dispatch Period ***t***.	-
$FL_{z1\rightarrow z2}$	Limit of transmission flow from zone 1 to zone 2.	-
$FL_{z2\rightarrow z1}$	Limit of transmission flow from zone 2 to zone 1.	-
$IntC_{Exp_m}{}^{t}$	Interconnection transfer capability for exports at node ***m***, during the Dispatch Period ***t***.	-
$IntC_{Exp_{m^*}}{}^{t}$	Interconnection transfer capability for exports for the set of nodes ***m****, during the Dispatch Period ***t***.	-
$IntC_{imp_m}{}^{t}$	Interconnection transfer capability for imports at node ***m***, during the Dispatch Period ***t***.	-
$IntC_{imp_{m^*}}{}^{t}$	Interconnection transfer capability for imports for the set of nodes ***m****, during the Dispatch Period ***t***.	-
exp_{sys}^{t}	Total system export capability, during the Dispatch Period ***t***.	-
imp_{sys}^{t}	Total system import capability, during the Dispatch Period ***t***.	-
$Q_{PR}{}^{t}$	Total system requirements for Primary Reserve, during the Dispatch Period ***t***.	-
$Q_{SEC}^{up}{}^{t}$	Total system requirements for upward Secondary Reserve, during the Dispatch Period ***t***.	-
$Q_{SEC}^{dw}{}^{t}$	Total system requirements for downward Secondary Reserve, during the Dispatch Period ***t***.	-
R_{sys}^{up}	System's overall ramp-up rate capability	-
R_{sys}^{dw}	System's overall ramp-down rate capability	-
$Q_{TER}^{reg}{}^{t}$	System requirements for Tertiary Reserve in geographical region ***reg***, during the Dispatch Period ***t***.	-
$Q_{\min_u}$	Technical minimum of unit ***u***.	-
$Q_{\max_u}$	Technical maximum output capability of unit ***u***.	-
R_{u}^{up}	Rump-up rate of unit ***u***	-
R_{u}^{dw}	Rump-down rate of unit ***u***	-
$Q_{\min_u}^{AGC}$	Technical minimum of unit ***u***, in AGC mode.	-
$Q_{\max_u}^{AGC}$	Technical maximum output capability of unit ***u***, in AGC mode.	-

RR_{AGC_u}	Rump rate of unit ***u***, in AGC mode.	-
E_u	Total daily production capability of the hydroelectric unit ***u***.	-
$FZ_u^{i,lw}$	Lower limit of the prohibited continuous operation zone ***i*** (due to oscillations), of the hydroelectric unit ***u***.	-
$FZ_u^{i,up}$	Upper limit of the prohibited continuous operation zone ***i*** (due to oscillations), of the hydroelectric unit ***u***.	-
$Q_{u_s}^{t,z}$	Generation quantity of block **s** in the Injection Offer of generation unit ***u***, located in zone *z*, located in zone *z*, during the Dispatch Period ***t***.	$\sum^{s} Q_{u_s}^{t,z}$ =STEPQsut
$Q_{PR_u}{}^{t,z}$	Generation quantity corresponding to the Offer for Primary Reserve of generation unit ***u***, located in zone *z*, during the Dispatch Period ***t***.	-
$Q_{SEC_u}{}^{t,z}$	Quantity corresponding to the Offer for Secondary Reserve range of generation unit ***u***, located in zone ***z***, during the Dispatch Period ***t***.	-
$R_{AGC_u}^{t}$	Ramp Rate of unit ***u***, in AGC mode, during the Dispatch Period ***t***.	-
$Q_{NST u}{}^{t,z}$	Generation quantity corresponding to non-spinning Tertiary Reserve capability of generation unit ***u***, located in zone ***z***, during the Dispatch Period ***t***.	-
$t_{u\ \min}^{\ dw}$	Minimum down time of generation unit ***u***	-
$t_{u\ \min}^{\ up}$	Minimum up time of generation unit ***u***	-

Appendix B

Two variables with no cost assignment are introduced at the end of the objective function corresponding to the total energy transferred from one zone to the other. In fact these variables are dependent each other: when one of them is greater than zero the other is set equal to zero. The objective function is of the following form:

$$\max \sum_{t}^{24}\Bigg\{\sum_{z}^{2}\Bigg[\sum_{pp=1}^{pp}\sum_{s=1}^{s} x_{pp_s}^{t,z} P_{pp_s}^{t,z} + \sum_{p=1}^{p} x_{p}^{t,z} P_{cap} + \sum_{k=1}^{k}\sum_{s=1}^{s} x_{k_s}^{t,z} P_{k_s}^{t,z} + \sum_{pm=1}^{pm}\sum_{s=1}^{s} x_{pm_s}^{t,z} P_{pm_s}^{t,z} - \sum_{j=1}^{j}\sum_{s=1}^{s} x_{j_s}^{t,z} P_{j_s}^{t,z} - \sum_{r=1}^{r} x_r^{t,z}\cdot 0 - \sum_{con=1}^{con} x_{con}^{t,z} P_{cap} -$$

Priced Load non-Priced Exports Pumped Storage Imports non-Priced Contracted Load

Units Generation Units

Demand ***Supply from non Dispatchable Units***

$$-\sum_{u=1}^{u}\Bigg(y_{DEC_u}^{t,z}\cdot P_{syn_u} + y_u^{t,z} Q_{\min_u} P_{u_1}^{t,z} + \sum_{s=1}^{s} x_{u_s}^{t,z} P_{u_s}^{t,z} + x_{PR_u}^{t,z} P_{PR_u}^{t,z} + x_{SEC_u}^{up\,t,z} P_{SEC_u}^{t,z} + x_{SEC_u}^{dw\,t,z} P_{SEC_u}^{t,z} + y_{AGC_u}^{t,z}\cdot 0 + x_{ST_u}^{t,z}\cdot 0 + x_{NST_u}^{t,z} P_{u_{NST}}^{t,z}\Bigg)\Bigg] + x_{z1\to z2}^{t}\cdot 0 + x_{z2\to z1}^{t}\cdot 0\Bigg\}$$

Synchronization Technical Blocks Primary Secondary Reserve Spinning Non-

Spinning ***Interzonal***

Minimum Reserve Tertiary Tertiary

Power Flow

Supply from Dispatchable Units

Appendix C

Unit Constraints

Unit constraints are the set of constraints that concern each unit that participates in the Day-Ahead Scheduling and refer to minimum and maximum generation output capability, ramp-up and ramp-down capability, reserves for ancillary services, commitment and decommitment statuses and some special restrictions for hydroelectric units.

Synchronization Status Constraints

Constraint (C1) refers to the online/offline status of each unit, defining whether for the certain Dispatch Period *t,* the unit provides energy and reserve for ancillary services. More specifically, for each Dispatch Period *t*, the binary variable $y^{t,z}_{u}$ denotes if the unit is synchronized or not. If this variable is set equal to zero (unit not synchronized) then all the other variables of the constraint, which correspond to the energy blocks and the reserve quantities for ancillary services, are also set equal to zero since the coefficient *M* of the variable is a sufficiently big number (e.g. one thousand times the largest value of the technological parameters and the right hind side of the mathematical problem). Its value is in purpose set, so that all the variables take zero values each time the binary variable is equal to zero. In case that the binary variable is equal to 1 (unit synchronized) then it is obvious that all the left-hand side of the constraint will be negative due to the very big value of the coefficient *M*. When the binary variable is equal to 1, at least the technical minimum of the unit is dispatched.

$$\forall u,t \qquad \sum_{s=1}^{s} x_{u_s}^{t,z} + x_{PR_u}^{t,z} + x_{SEC_u}^{dw\ t,z} + x_{SEC_u}^{up\ t,z} + x_{ST_u}^{t,z} - y_u^{t,z} \cdot M \le 0 \tag{C1}$$

Technical Minimum Constraints with/without Automatic Generation Control (AGC)

Constraint (C2), which is related with the technical minimum of the unit and the Automatic Generation Control mode operation for secondary control provision, has a twofold scope. First, it does not allow the binary variable $y^{t,z}_{AGCu}$ which indicates if the unit operates in AGC mode ($y^{t,z}_{AGCu} = 1$), to take value equal to 1, when the synchronization status variable ($y^{t,z}_{u}$) is set to offline (equal to zero).

$$\forall u,t \qquad y_u^{t,z} Q_{\min_u} + \sum_{s=1}^{s} x_{u_s}^{t,z} - y_{AGCu}^{t,z} \cdot Q_{\min_u}^{AGC} \ge 0 \tag{C2}$$

Second, when the unit operates in AGC mode ($y^{t,z}_{AGCu} = 1$) the technical minimum of the unit has a different value ($Q_{min}{}^{AGC}{}_{u}$); in that case, constraint (21) artificially increases the technical minimum by setting the difference of the two minimums as a quantity that will be included in the variables that correspond to the energy blocks.

Maximum Capacity Constraints with/without Automatic Generation Control (AGC)

This constraint restrains the sum of all the variables that represent generation to exceed the technical maximum of the unit (Q_{maxu}), for each Dispatch Period t. This is also restrained in the case where the unit operates in AGC mode and the technical maximum has a different value ($Q_{max}{}^{AGC}{}_{u}$).

$$\forall u,t \qquad y_u^{t,z} Q_{\min_u} + \sum_{s=1}^{s} x_{u_s}^{t,z} + x_{PR_u}{}^{t,z} + x_{SEC_u}^{up}{}^{t,z} + x_{ST_u}{}^{t,z} + y_{AGC_u}^{t,z} \cdot \left(Q_{\max_u} - Q_{\max_u}^{AGC}\right) \le Q_{\max_u} \qquad \text{(C3)}$$

Ramp-Up and Ramp-Down Capability Constraints

Constraint (C4) refers to the ramp-up capability rate of a generation unit and restrains the unit from increasing its generation output more than its technical capability within a Dispatch Period. It is valid when the unit operates in both normal and AGC mode, where the ramp-up rate capability has a different value.

$$\forall u,t \qquad \sum_{s=1}^{s} x_{u_s}^{t,z} - \sum_{s=1}^{s} x_{u_s}^{t-1,z} \le R_u^{up} + y_{AGC_u}^{t,z}\left(RR_{AGC_u} - R_u^{up}\right) \qquad \text{(C4)}$$

Respectively, constraint (C5) refers to the ramp-down capability rate of a generation unit and restrains the unit from decreasing its generation output more than its technical capability within a Dispatch Period. It is also valid when the unit operates in normal and AGC mode.

$$\forall u,t \qquad \sum_{s=1}^{s} x_{u_s}^{t-1,z} - \sum_{s=1}^{s} x_{u_s}^{t,z} \le R_u^{dw} + y_{AGC_u}^{t,z}\left(RR_{AGC_u} - R_u^{dw}\right) \qquad \text{(C5)}$$

Special Constraints concerning the Hydroelectric Units

Constraint (C6) refers to the total generation capability of a hydroelectric unit within a Dispatch Day and restrains the unit to be dispatched for a quantity than it cannot generate that Dispatch Day. The constraint takes into account both the quantities included in the Injection Offer of the unit and the non-priced (must-run hydro) quantities that have to be generated by the same unit.

$$\forall u_{hydro} \qquad \sum_{t=1}^{24}\left(\sum_{s=1}^{s} x_{u_{(hydro)_s}}^{t,z} + x_{r_{(hydro)}}^{t,z}\right) \le E_u \qquad \text{(C6)}$$

The second constraint (C7) refers to the prohibited (due to oscillations) generation level zones i for the hydroelectric units and restrains the total generation output of the unit, for each Dispatch Period t, from being within these zones (where M is a sufficiently big number).

$$\forall u_{hydro}, t, i \quad \sum_{s=1}^{S} x_{u_s}^{t,z} \le FZ_u^{i,lw} - \left(1 - dz_u^i\right) \times M \tag{C7}$$

$$\forall u_{hydro}, t, i \quad \sum_{s=1}^{S} x_{u_s}^{t,z} \ge FZ_u^{i,up} - dz_u^i \times M \tag{C8}$$

Ancillary Services Reserve Constraints

The following constraint (C9), for each Dispatch Period *t*, does not allow assigning to the unit primary reserve more than the quantity that represents unit's capability to provide primary reserve (according its Techno-economic Declaration) and is included in its Reserve Offer ($Q_{PRI}{}^{t,z}{}_u$). If the unit is not dispatched, the primary reserve quantity is automatically set equal to zero.

$$\forall u, t \qquad x_{PR_u}^{\ t,z} - y_u^{t,z} \cdot Q_{PR_u}^{\ \ t,z} \le 0 \tag{C9}$$

Similarly, for each Dispatch Period *t*, constraint (C10) does not allow assigning to the unit secondary reserve more than the range that represents unit's capability to provide secondary reserve (according its Techno-economic Declaration) and is included in its Reserve Offer ($Q_{SEC}{}^{t,z}{}_u$). If the unit is not dispatched or the unit does not operate in AGC mode (the binary variable $y_{AGC}{}^{t,z}{}_u = 0$), the secondary reserve range is automatically set equal to zero.

$$\forall u, t \qquad (x_{SEC\ u}^{up\ \ t,z} + x_{SEC\ u}^{dw\ \ t,z}) - y_{AGC_u}^{\ \ t,z} \cdot Q_{SEC_u}^{\ \ t,z} \le 0 \tag{C10}$$

Constraints (C11) and (C12) assure that for each Dispatch Period *t* the downward or upward variation in the generation output for the provision of secondary control will not exceed the ramp rate (expressed in MW per hour) of the unit whet it operates in AGC mode (RR_{AGCu}).

$$\forall u, t \qquad x_{SEC\ u}^{up\ \ t,z} - y_{AGC_u}^{\ \ t,z} \cdot RR_{AGC_u} \le 0 \tag{C11}$$

$$\forall u, t \qquad x_{SEC\ u}^{dw\ \ t,z} - y_{AGC_u}^{\ \ t,z} \cdot RR_{AGC_u} \le 0 \tag{C12}$$

The next constraint (C13), set for each Dispatch Period *t*, does not allow any decrease in unit's generation output for the provision of secondary reserve that would led the generation level below the technical minimum of the unit.

$$\forall u, t \qquad \sum_{s=1}^{S} x_{u_s}^{t,z} - y_{AGC_u}^{\ \ t,z} \cdot (x_{SEC\ u}^{dw\ \ t,z} - Q_{\min\ u}^{AGC}) \ge 0 \tag{C13}$$

Constraint (C14) assures that the non-spinning reserve that a unit may provide within the Dispatch Period *t* does not exceed its corresponding capability. Also, it excludes the unit from the provision of non spinning tertiary reserve if the unit is dispatched.

$$\forall u,t \qquad x_{NST_u}^{t,z} + y_u^{t,z} \cdot Q_{NST_u}^{t,z} \le Q_{NST_u}^{t,z} \tag{C14}$$

Constraint (C15) does not allow the quantity of the non spinning reserve provided by a unit during the Dispatch Period *t*, to be less than the technical minimum of the unit.

$$\forall u,t \qquad x_{NST_u}^{t,z} - Q_{\min_u} \ge 0 \tag{C15}$$

Unit Commitment Constraints

For every unit and Dispatch Period *t,* the commissioning (dy^t_{COM}) and decommissioning (dy^t_{DEC}) indicator dependent (integer) auxiliary variables are calculated (C16, C17). The possible values for the first one (dy^t_{COM}) is either 0 (decommissioning), 1 (the unit remains online or offline) or 2 (commissioning). Respectively for the second variable (dy^t_{DEC}) the possible values are either 0 (commissioning), 1 (the unit remains online or offline) or 2 (decommissioning).

$$\forall u,t \qquad dy_{COM_u}^{t} - (y_u^t - y_u^{t-1} + 1) = 0 \tag{C16}$$

$$\forall u,t \qquad dy_{DEC_u}^{t} - (y_u^{t-1} - y_u^t + 1) = 0 \tag{C17}$$

For every Dispatch Period *t*, if a unit is synchronized the constraint (C18) does not allow to the unit to desynchronize before the minimum up time is elapsed.

$$\forall u,t \qquad y_u^t + \ldots + y_u^{t+(t_{u_{\min}}^{up}-1)} - (dy_{COM_u}^{t} - 1) \cdot t_{u_{\min}}^{up} \ge 0 \tag{C18}$$

Respectively constraint (C19), if a unit is desynchronized the constraint (C20) does not allow to the unit to synchronize before the minimum down time is elapsed.

$$\forall u,t \qquad y_u^t + \ldots + y_u^{t+(t_{u_{\min}}^{dw}-1)} + (dy_{DEC_u}^{t} - 1) \cdot t_{u_{\min}}^{dw} \le t_{u_{\min}}^{dw} \tag{C19}$$

In the objective function these two auxiliary variables are not used but another dependent binary variable ($y_{DEC}{}^{t}{}_{u}$) is used instead. Constraint (38) actually links each dy_{DEC} variable with the corresponding y_{DEC} variable and at the same time keeps the binary character of y_{DEC}.

$$\forall u,t \qquad y_{DEC_u}^{\ t} - (dy_{DEC_u}^{\ t} - 1) \geq 0 \tag{C20}$$

Other Constraints

The following constraints set for every unit the first dispatched quantity variable (x_1) less than or equal to the difference between the first block in the Injection Offer and the technical minimum of the unit.

$$\forall t,u \qquad x_{u_1}^{t,z} \leq Q_{u_1}^{t,z} - Q_{\min_u} \tag{C21}$$

For the rest variables ($x_u{}^{t,z}{}_s$) that correspond to the dispatched quantities of every offered block they can not exceed the maximum quantity offered per block ($Q_u{}^{t,z}{}_s$).

$$\forall t,u,s \qquad x_{u_s}^{t,z} \leq Q_{u_s}^{t,z} \qquad \text{s=2,…, 10} \tag{C22}$$

For the contracted units the dispatched quantities must be less than or equal to the contracted ones.

$$\forall t,con \qquad x_{con}^{t,z} \leq Q_{con}^{t,z} \tag{C23}$$

All the variables must be greater than or equal to zero and the integer binary variables equal to or less than:

$$\forall t,u,s$$

$$x_{u_s}^{t,z}, x_{PR_u}^{\ t,z}, x^{dw}_{SEC_u}{}^{t,z}, x^{up}_{SEC_u}{}^{t,z}, x_{ST_u}^{\ t,z}, x_{NST_u}^{\ t,z}, x_{z1\to z2}^{t}, x_{z2\to z1}^{t}, y_u^{t,z}, y_{AGC_u}^{\ t}, y_{DEC_u}^{\ t}, dy_{DEC_u}^{\ t}, dy_{COM_u}^{\ t} \geq 0 \tag{C24}$$

$$\forall t,u \qquad \text{dz}, y_u^{t,z}, y_{AGC_u}^{\ t}, y_{DEC_u}^{\ t} \leq 1 \tag{C25}$$

Finally: $\qquad y_u^{t,z}, y_{AGC_u}^{\ t}, y_{DEC_u}^{\ t}, dy_{DEC_u}^{\ t}, dy_{COM_u}^{\ t} \in Z$