Decomposition of Value-Added in Gross Exports:Unresolved Issues and Possible Solutions

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Abstract: To better understand trade in the context of global value chains, it is important to have a full and explicit decomposition of value-added in gross exports. While the decomposition proposed by Koopman, Wang and Wei (2014) is a first step in this direction, there are still three outstanding issues that need to be further addressed: (1) the nature of double counting in gross exports; (2) the calculation of the foreign value-added net of any double counting; and (3) the decomposition of gross exports at the industry level (the industry where exports take place). In this paper, we propose a new accounting framework that addresses these different issues and clarifies the definition of exports in inter-country input-output (ICIO) tables. It contributes to the literature: (i) by refining the definition of double-counted value-added in gross exports; (ii) by providing new expressions for the foreign value-added and double-counted terms; and (iii) by indicating how the new framework can be used to decompose exports at the industry level.

Keywords: Trade accounting, input-output table, Value-added decomposition, Global value chains

JEL Codes: E01, E16, F14, F23, L14

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The recent availability of inter-country input-output (ICIO) tables has created new opportunities for analyzing the intricate flows of value-added that are embedded in international trade. A first approach consists in following the Leontief model and looking at the origin of value-added in the final demand of countries (Johnson and Noguera, 2012). The resulting decomposition identifies as 'exports of value-added' the value-added contributed by a given country and industry to final demand abroad. Such decomposition does not depart from the foundations of input-output analysis as it multiplies the Leontief inverse by a vector of final demand. It can provide results at the country level (exports of value-added to the world), bilaterally (exports of value-added to a given partner) and by industry (but based on the industry of origin of value-added in the exporting economy).

A second approach, proposed by Koopman, Wang and Wei (KWW, 2014), aims at decomposing gross exports, which is the basic aggregate used in trade economics and reported by countries in their national accounts and balance of payments. This approach has to deal with the fact that gross exports are made both of final products and intermediate goods and services. The latter also end up in final products at the end of the production process. It explains why the decomposition cannot simply be the multiplication of the Leontief inverse by a vector of gross exports and why there is some "double counting" as some of the intermediate goods and services exported can also be part of the value of exports of final products in the case of vertical specialization trade.

However, it is also possible to use the Leontief model and input-output relationships to derive mathematical expressions for the value-added embodied in gross exports, as it is done by KWW. In a comment, Los, Timmer and de Vries (LTV, 2016) provide an alternative decomposition based on 'hypothetical extraction' where the domestic value-added in exports is expressed in a way fully consistent with the Leontief model.

But despite the sound theoretical support provided to the concept of domestic value-added in exports, the comment by LTV has left unanswered the question of the calculation of the foreign value-added in exports.[3] And beyond the domestic and foreign value-added consistent with value-added measured in GDP, gross exports are also made of value-added that has already been accounted for before in the domestic and foreign value-added and therefore corresponds to some double counting.

The KWW framework introduces ‘pure double counted terms’ (corresponding to term 6 and term 9 in their decomposition). These terms multiply by a coefficient the gross exports of the exporting economy (domestic term) and the exports of partner countries (foreign term). They are indicated as not being part of the GDP of any country (KWW, p. 469) and related to “two-way intermediate trade from all bilateral routes” (KWW, p. 481).

There is no consensus at this stage on how to calculate the domestic and foreign double counting, leaving also unanswered the question of the foreign value-added net of any double counting. Three recent papers in particular question the KWW result. Nagengast and Stehrer (2016) argue that there is some arbitrariness in the decomposition of intermediate and final gross exports in KWW and that they do not correctly identify multiple border crossings. Nagengast and Stehrer propose an alternative decomposition for the domestic value-added in exports (terms 1, 2 and 3 of KWW) but do not explore further the implications for double counting and the foreign value-added, as the focus of their paper is on bilateral gross exports and trade balances. However, they introduce the distinction between the ‘source-based’ and ‘sink-based’ approaches that lead to a different double-counting in bilateral gross exports. Borin and Mancini (2017) also look at the decomposition of bilateral gross exports and are more explicit about how a definition of double-counting as any VA that crosses the same (domestic) border more than once affects the calculation of the foreign value-added. They propose a decomposition where the foreign value-added at the aggregate level (summing across partners) is the same in the source-based and sink-based approach. Their decomposition points to a different foreign double counted

[3] The authors indicate that it is left for future research and requires a complete decomposition of world GDP.

term as compared to KWW. Lastly, Johnson (2017) also notes that KWW and LTV have not fully solved the question of the domestic and foreign content of exports and offers additional insights on the foreign value-added in a framework similar to Los, Timmer and de Vries (2016). The paper only includes a two-country decomposition of aggregate exports but with results departing from KWW for the foreign value-added (and foreign double counting).

In this paper, we are also interested in providing a decomposition of value-added in a country's gross exports, leaving aside the bilateral decomposition. As emphasized by LTV, we also believe that such decomposition should be consistent with the foundations of input-output analysis. Moreover, from our point of view, the decomposition of the foreign value-added terms should be symmetric with the domestic ones, since the foreign value-added in the exports of a given country is domestic value-added in the exports of another. For instance, in the decomposition framework, there should be terms to identify the foreign value-added that returns to the exporting country, similar to the terms indicating the domestic value-added that returns home.

In addition to this discussion on the measurement of the foreign value-added in aggregate exports, neither the KWW framework nor the hypothetical extraction method can be easily extended to decompose the value-added in gross exports at the industry level. Here, it is important to specify the industry from the point of which value-added is measured. There are (at least) 3 industry dimensions in the gross exports decomposition: the source industry (i.e. the industry of origin of primary inputs used to generate the value-added in exports), the gross exports industry (i.e. the industry that has produced the gross exports which are decomposed into different value-added terms) and the final demand industry (i.e. the last industry using the value-added identified in exports before final consumption).[4]

[4] More industries can be involved when the intermediate inputs exported are further processed in different industries across countries before being incorporated into a final product. The incorporation in the final product can take place either in the last exporting economy or in the importing country

A decomposition of gross exports at the industry level means that the starting point of the decomposition is the value of gross exports for a specific industry (and country), i.e. the exports industry. In an extension of KWW to the industry level, Wang, Wei and Zhu (WWZ, 2013) point out that there is an additional layer of complexity when decomposing industry-level gross exports. Instead of 9 terms, their decomposition has to rely on 16 terms to cover all the complex inter-industry interactions across countries in the ICIO. For the hypothetical extraction method as well, while it is possible to calculate an hypothetical GDP where only the exports of a single industry are removed, the different terms of the LTV framework are also not easily obtained at the industry level. Therefore, there is also a need to better explain how the results of the trade in value-added literature can be derived for specific industries.

In this paper, we explore some solutions to the issues mentioned above. We first clarify the relationship between gross exports and final demand in the inter-country input-output framework and how we can express the domestic and foreign value-added in exports in some new input-output framework focusing on gross exports rather than gross output. Then, we use the Ghosh insight to provide a more straightforward decomposition of gross exports that gives the initial domestic value-added, first round foreign value-added and later rounds double counted value-added in a consistent input-output framework. This decomposition is fully consistent with the one that is derived from the Leontief model. It provides a domestic value-added in exports equal to KWW and LTV but new foreign value-added terms which are different from KWW. Finally, we show how this framework can accommodate analysis at the industry level.

The paper is organized as follows. In section I, we introduce an alternative mathematical framework to clarify the relationship between gross exports and final demand in the ICIO model and explain how it can be used to express the domestic and foreign value-added in exports (consistent with GDP and net of any double counting). In section II, we use the Ghosh insight to define value-added trade flows and

(‘transiting’ through different domestic industries).

decompose gross exports into domestic value-added, foreign value-added and double counted terms. In section III, we explain how our decomposition differs from KWW and how it can be extended to provide terms similar to their framework that distinguishes intermediates from final products, as well as the country of absorption of value-added. Section IV deals with the extension of the framework to the industry level. Section V concludes.

I. Clarifying the relationship between gross exports and final demand in inter-country input-output tables

The input-output model comes from the work of Leontief (1936) who demonstrated that the amount and type of intermediate inputs needed in the production of one unit of output can be estimated based on the input-output (IO) structure across industries. The model allows tracing gross output in all stages of production needed to produce one unit of final goods (or services[5]). When the gross output flows associated with a particular level of final demand are known, the value-added generated and 'traded' can simply be derived by multiplying these flows with the value added to gross output ratio in each industry.

In the IO table, all gross output must be used either as an intermediate or a final good,

$$X = AX + Y \qquad (1)$$

where, X is the $N \times 1$ gross output vector, Y is the $N \times 1$ final demand vector, and A is the $N \times N$ IO coefficients matrix.

A. The input-output framework for exports

If we split the output in the ICIO table into exports (E) and domestic sales (H), the following accounting equations can be obtained: $E = A^F(E+H)+Y^F$ and $H = A^D(E+H)+Y^D$, where A^D is a matrix of the domestic coefficients in the global

[5] We use the expression 'goods' in a generic way. Input-output tables cover all types of products or industries, i.e. goods and services.

ICIO table (i.e. the block diagonal of the A matrix) and A^F is the export matrix (the elements of the A matrix off the block diagonal) including the IO coefficients for the use of intermediate inputs from one country into another country, so that we have $A = A^D + A^F$. Y^D is the domestic final demand and Y^F is the foreign final demand, so that $Y = Y^D + Y^F$.

After re-arrangement (see Appendix A), the accounting relationship between gross exports and the final demand in destination countries in the ICIO model can be expressed as:

$$E = \tilde{A}E + \tilde{Y} \tag{2}$$

with $\tilde{Y} = Y^F + \tilde{A}Y^D$ and $\tilde{A} = A^F(I - A^D)^{-1}$.

Equation (2) is to gross exports what equation (1) is to gross output. It suggests a different type of input-output table where gross exports have replaced gross output.[6] Conceptually, we have a new type of Leontief matrix $\tilde{A}$ and a new final demand $\tilde{Y}$ with interpretations similar to the original *A* and *F* but in the context of gross exports. The elements of the $\tilde{A}$ matrix describe the units of intermediate goods produced and exported that are used in the production of one unit of exports in the destination country. For example, the element $\tilde{A}_{ij}$ means that in order to produce one unit of exports in country *j*, country *i* needs to produce $\tilde{A}_{ij}$ units of intermediate goods that are shipped to *j* and embodied in exports of *j*. $\tilde{A}_{ij}E_j$ indicates country's *i* intermediate inputs used in country *j*'s exports. Therefore, we can call $\tilde{A}$ the 'direct inputs requirement matrix' for exports. The term $\tilde{A}_{ij}E_j$ also corresponds to the vertical specialization (VS) exports as defined in Hummels, Ishii and Yi (2001).

[6] Another way of introducing equation (2) is to think about the elements extracted from gross output in the hypothetical extraction method proposed by LTV. As such, the two frameworks are consistent and they provide the same results as illustrated in Appendix B.

Re-arranging equation (2), we can also obtain equation $E=\tilde{B}\tilde{Y}$, and $\tilde{B}=(I-\tilde{A})^{-1}$, similar to $B=(I-A)^{-1}$ in the traditional IO model. Matrix $\tilde{B}$ is the 'total inputs requirement matrix' for exports.

$\tilde{Y}$ is the vector of final demand for exports. For country *i*, the element $\tilde{Y}_i$ in the vector $\tilde{Y}$ is simply other countries' final demand for exports of *i*. But since the perspective is the destination country (i.e. the final demand in the partner country), $\tilde{Y}_i$ includes both intermediate goods and final goods produced in country *i*. It combines the demand for final goods Y_i^F manufactured in *i* (and exported as final goods) and the demand for intermediate goods $\tilde{A}Y_i^D$ that are used to produce final goods in the destination country that are consumed domestically. In this case, trade in intermediate goods takes place between country *i* and country *j* but in order to produce final goods in *j*.

Therefore, $\tilde{A}E$ is the intermediate demand for gross exports that covers all trade in intermediate inputs that are further embodied in exports, while $\tilde{Y}$ is a final demand for gross exports combining all trade flows in final goods but also trade in intermediates that are directly used to produce final goods in the partner country. Intermediate and final are defined from the point of view of the partner country in exports.

If we extend the expression $E=\tilde{B}\tilde{Y}$ to the G countries and N industries case, exports of country *i* can be decomposed as follows:

$$E_i=\sum_{t}^{G}\sum_{j\neq t,i}^{G}\tilde{B}_{it}Y_{tj}+\sum_{j\neq i}^{G}\tilde{B}_{ij}Y_{jj}+\sum_{t\neq i}^{G}\tilde{B}_{it}Y_{ti}+(\tilde{B}_{ii}-I)Y_{ii} \tag{3}$$

In this equation, each term clarifies what is the destination of country *i's* exports and whether exports are for intermediate or final use. Subscript *j* indicates in this case country *i*'s ultimate export destination. Term 1 and term 2 correspond to country *i*'s exports of goods to country *j* that are finally absorbed by country *j*. Term 1 describes the goods exported by i (intermediate or final) and absorbed by j as final goods. These goods can be first exported as intermediates to a third-country before coming as final goods in j. Term 2 indicates the intermediate goods from country i that are exported and processed in country j into final goods before being absorbed by country j. Again, they can transit through different countries to be further processed before reaching j, which is the ultimate destination. But they reach j as intermediate goods.

The next two terms are about exports of inputs that come back to country i (after transiting through one or several other countries). Term 3 indicates the exports from country i that finally return back to country i as final goods (and directly absorbed by country i) while term 4 describes the exports from country i that come back to country i as intermediate goods and are processed in country i into final goods before being absorbed.

B. How to measure the domestic value-added in exports

In addition to the 'direct inputs requirement matrix for exports' and 'total inputs requirement matrix for exports', we can also derive a concept similar to the value-added ratio in our IO framework for exports. We call it $\tilde{V}$, the exports value-added multiplier.

Theorem 1: For country i's exports, the domestic value-added multiplier coefficient is $\tilde{V}_i = u(I - \sum_{j \neq i}^{G} \tilde{A}_{ji}) = V_i(I - A_{ii})^{-1}$.

Here, we define $V_i = u[I - A_{ii} - \sum_{j \neq i}^{G} A_{ji}]$ as a 1×N direct value-added coefficient vector in the IO table and u is a 1×N unity vector. Each element of V_i gives the share of direct domestic value-added in total output.

Accordingly, when working with the new matrix $\tilde{A}$, we can see that in country i's exports E_i, all of intermediate inputs are $\sum_{j \neq i}^{G} \tilde{A}_{ji} E_i$. Therefore, country i's value-added in exports is $u \times VaE(i) = u(E_i - \sum_{j \neq i}^{G} \tilde{A}_{ji} E_i)$. The domestic value-added multiplier coefficient is $\tilde{V}_i = u(I - \sum_{j \neq i}^{G} \tilde{A}_{ji}) = V_i (I - A_{ii})^{-1}$. This is equal to one minus the intermediate input share from all countries (including domestically produced intermediates). The domestic value-added in country i can be expressed as: $uVaE(i) = \tilde{V}_i E_i = V_i (I - A_{ii})^{-1} E_i$. This expression is consistent with KWW and LTV (more details after and in Appendix A).

To better understand the domestic value-added multiplier, we can deduce the consistent expression for the domestic value-added (or GDP) from the initial ICIO model. In the ICIO model, country i's gross output can be written as:

$$X_i = A_{ii} X_i + Y_{ii} + \sum_{j \neq i}^{G} A_{ij} X_j + \sum_{j \neq i}^{G} Y_{ij} = A_{ii} X_i + Y_{ii} + E_i \quad (4)$$

Rearranging equation (4), we get:

$$X_i = (I - A_{ii})^{-1} Y_{ii} + (I - A_{ii})^{-1} E_i \quad (5)$$

Matrix $(I - A_{ii})^{-1}$ is sometimes called the 'local' Leontief inverse in the ICIO model. From there, country i's value-added (or GDP) can be calculated as follows:

$$VA_i (GDP_i) = V_i X_i = V_i (I - A_{ii})^{-1} Y_{ii} + V_i (I - A_{ii})^{-1} E_i \quad (6)$$

According to equation (6), country i's value-added (or GDP) is divided into two parts: one part is the value-added in country's i final demand and the other part $V_i (I - A_{ii})^{-1} E_i$ is the value-added in exports of country i. From there, we can also get the expression of the domestic value-added in exports which is consistent with the discussion before, and regard $V_i (I - A_{ii})^{-1}$ as the value-added multiplier coefficient for a country's exports.

C. How to measure the foreign value-added in exports

The next issue is how to measure the foreign value-added in exports. From the above analysis, we already know that $\tilde{A}_{ji} E_i$ are the intermediate inputs exported from country j to country i and used in country i's exports. Therefore, if we want to measure country j's value-added in country i's exports, we can just multiply this expression by the value-added multiplier coefficient: $V_j(I-A_{jj})^{-1}\tilde{A}_{ji}E_i$. The same expression can also be derived from the initial ICIO model.

Similarly, we can express country j's value-added (GDP) as: $VA_j(GDP_j)=V_jX_j=V_j(I-A_{jj})^{-1}Y_{jj}+V_j(I-A_{jj})^{-1}E_j$. Meanwhile, we have country j's exports equal to: $E_j=E_{ji}+\sum_{s\neq i,j}^{G}E_{js}$. Therefore, country j's value-added (or GDP) exported into country i is $V_j(I-A_{jj})^{-1}E_{ji}$. We can then expand the bilateral exports expression from j to i as follows:

$$\begin{aligned} E_{ji} &= A_{ji}X_i+Y_{ji}=A_{ji}(I+(I-A_{ii})^{-1}A_{ii})E_i+A_{ji}(I-A_{ii})^{-1}Y_{ii}+Y_{ji} \\ &= \tilde{A}_{ji}E_i+\tilde{A}_{ji}Y_{ii}+Y_{ji} \end{aligned} \tag{7}$$

In this expression, country j's value-added exported to country i, $V_j(I-A_{jj})^{-1}E_{ji}$, can be divided into 3 parts: $V_j(I-A_{jj})^{-1}\tilde{A}_{ji}E_i$, $V_j(I-A_{jj})^{-1}\tilde{A}_{ji}Y_{ii}$ and $V_j(I-A_{jj})^{-1}Y_{ji}$.[7] And these parts can be described as: country j's value-added (or GDP) in country i's exports ($V_j(I-A_{jj})^{-1}\tilde{A}_{ji}E_i$), country j's value-added entered into country i as part of an intermediate good, processed and absorbed by country i ($V_j(I-A_{jj})^{-1}\tilde{A}_{ji}Y_{ii}$), and country j's value-added entered into country i as part of a final good and then absorbed by country i directly ($V_j(I-A_{jj})^{-1}Y_{ji}$). If we sum up the value-added from all countries, except country i, in country i's exports, we obtain the foreign

[7] This decomposition is similar to what Johnson (2017) develops for two countries in the supplemental appendix of his paper. These terms link value-added in exports to an overall decomposition of GDP along the lines suggested by LTV but left for future research. This decomposition is also what distinguishes our results from other papers that unlike Johnson (2017) follow the original KWW framework where the starting point is exports rather than GDP and where value-added in intermediate or final exports is defined from the point of view of the exporting economy rather than the destination country.

value-added in country i's exports, expressed as $\sum_{j \neq i}^{G} V_j (I - A_{jj})^{-1} \tilde{A}_{ji} E_i$.

II. Tracing value-added and double counting in gross exports: the Ghosh insight

The previous section has already provided an expression for the domestic and foreign value-added in gross exports. Now we need to give a full decomposition of gross exports and deal with the issue of the double counting.

Because intermediate inputs can 'travel' several times across countries before being incorporated into final products and come back to their source country before being exported again, the sum of the domestic and foreign value-added as defined above is different from gross exports. Gross exports include some 'double counting' in the sense that the same value-added (already defined as domestic or foreign) is counted twice or more.

As a first approach, the double counting is the difference between gross exports and the domestic and foreign value-added consistent with the GDP of countries (where primary factors of production cannot contribute two times to the same value). KWW refer to some 'pure double counting' because any foreign value-added is in a way already double counted in gross exports statistics. The foreign value-added of one country in the exports of another is also domestic value-added in the exports of this country. Also, the domestic value-added that returns home (but without being incorporated in exports again) is part of the double counting in trade statistics. But any concept of 'double counting' is relative to the aggregate to which it is applied. Therefore, when working with the gross exports of a specific country, it seems reasonable to identify a domestic and foreign value-added consistent with GDP (both in the domestic economy and in foreign countries) and a residual called 'double counting' which is split into a domestic and foreign part. But still we need some explanation and economic interpretation for this residual and why we regard it as double counting.

The objective in this section is to provide explicit expressions for the domestic, foreign and double counted value-added terms in gross exports, but also an interpretation based on the Ghosh insight. Ghosh (1958) has introduced what is known as the 'supply–driven' input-output model, where value-added is the exogenously specified driving force in the framework. As the Ghosh model describes the generation of value-added in successive rounds, it seems more appropriate to trace flows of value-added in exports. There are some debates in the input-output literature on the interpretation and 'plausibility' of the Ghosh model (Oosterhaven, 1988; Dietzenbacher, 1997). However, the way we use it in this section does not depend on these debates, as we are discussing an accounting framework for the decomposition of gross exports and not an economic model where we have to identify exogenous and endogenous variables.

In the Ghosh framework, output coefficients are defined as $l_{ij} = x_{ij} / x_i$. An output coefficient gives the percentage of output of industry *i* that is sold to industry *j*. The accounting equation can be rewritten as:

$$X^T = VA^T + X^T L = VA \cdot G \tag{8}$$

where $G = (I - L)^{-1}$ is the Ghosh inverse; meanwhile, in $G = \hat{X}^{-1} B \hat{X}$, $\hat{X}$ is a $N \times N$ diagonal matrix with output on the diagonal.

Transposing the model to the 'export ICIO table' described in Section II, exports can be written as $E^T = VaE^T + E^T \tilde{L} = VaE^T \cdot \tilde{G}$. Here $\tilde{G} = \hat{E}^{-1} \tilde{B} \hat{E}$, $\tilde{L} = \hat{E}^{-1} \tilde{A} \hat{E}$ and $\tilde{L}_{ij} = \hat{E}_i^{-1} \tilde{A}_{ij} \hat{E}_j$. $\tilde{L}_{ij}$ measures the share of country *i*'s output in country *j*'s exports.

To illustrate the relationship between exports and value-added, we can refer to the Taylor expansion:

$$E^T = VaE^T (I + \tilde{L} + \tilde{L}^2 + \tilde{L}^3 + \cdots) \tag{9}$$

As before, we use the traditional concepts of input-output analysis linking output and value-added, transposed to the relationship between gross exports and value-added. The export value E^T can be decomposed into different rounds where value is added.

In particular, we can distinguish three value-added inputs: an initial input VaE^T, a direct input $VaE^T \cdot \tilde{L}$ in the first round and indirect inputs in subsequent rounds amounting to $VaE^T(\tilde{L}^2 + \tilde{L}^3 + \cdots)$. We can give the full expression for the specific exports of country *i* as follows:

$$\begin{aligned} E_i^T &= VaE(i)^T + VaE(i)^T \tilde{L}_{ii} + \sum_{j \neq i}^{G} VaE(j)^T \tilde{L}_{ji} \\ &+ VaE(i)^T [\tilde{L}]^2{}_{ii} + VaE(i)^T [\tilde{L}]^3{}_{ii} + \cdots \\ &+ \sum_{j \neq i}^{G} VaE(j)^T [\tilde{L}]^2{}_{ji} + \sum_{j \neq i}^{G} VaE(j)^T [\tilde{L}]^3{}_{ji} + \cdots \end{aligned} \quad (10)$$

The above expression provides an explicit interpretation of the decomposition of gross exports (including the 'double counting') in an input-output context, following the Ghosh insight.

The initial effect is country *i*'s value-added in exports, which is equal to $VaE(i)^T \times u^T = V_i (I - A_{ii})^{-1} E_i$. This term is the domestic value-added in exports (consistent with GDP) and we call it 'initial domestic value-added' as a reference to the Ghosh framework but also to make it clear that it is the first time this value is generated and that subsequently it can be double counted because it comes back in later rounds in the production process. For simplicity (and to follow KWW and LTV), we will just call it domestic value-added in the rest of the paper.[8]

In the first round, the direct effect can be divided into two parts, the effect from the domestic country *i* and from the foreign country *j*. Because $\tilde{L}_{ii}$ is equal to **0**, we have $VaE(i)^T \tilde{L}_{ii} = 0$. We are left only with the effect from country *j*. Since the foreign value-added is in the intermediate goods imported from country *j*, this term is equal to:

[8] While we are not dealing with the decomposition of bilateral exports in this paper, it should be noted that a bilateral domestic value-added can be calculated by simply replacing E_i by bilateral exports E_{ij}. It would be what Nagengast and Stehrer (2016) describe as a source-based approach. All the subsequent terms described in this section can be derived at the bilateral level the same way as they all include E_i.

$$\sum_{j\neq i}^{G} VaE(j)^T \tilde{L}_{ji} u^T = \sum_{j\neq i}^{G} V_j (I - A_{jj})^{-1} \hat{E}_j \times \hat{E}_j^{-1} \tilde{A}_{ji} E_i = \sum_{j\neq i}^{G} V_j (I - A_{jj})^{-1} \tilde{A}_{ji} E_i \qquad (11)$$

which is the foreign value-added in exports. We can therefore call it 'first round' foreign value-added (and will refer to it simply as foreign value-added in exports).

It should be noted that the initial and first rounds already provide the domestic and foreign value-added in exports, consistent with GDP and net of any double-counting. From equation (10), we have derived the same equations as in Section I. They are not dependent on the Ghosh framework since they were previously derived from the Leontief model.

But the Ghosh insight offers an interpretation for the 'residual' or why we have further value-added in gross exports and why we can reasonably call it 'double counting'. Since the initial and first rounds have already exhausted the domestic and foreign value-added in country *i*'s exports, what we measure as domestic value-added and foreign value-added in the later rounds of equation (10), when continuing the Taylor expansion, is something that was already measured in the initial and first rounds and is coming back.

In the second round, the additional value-added can also be divided into a domestic part and a foreign part. It includes the value-added passed over from country *i*'s exports to foreign countries which has returned back home before being exported again. In this domestic part, country *i*'s value-added is $VaE(i)^T \sum_{k}^{G} \tilde{L}_{ik} \tilde{L}_{ki} u^T$ and reflects country *i*'s value-added $VaE(i)^T \tilde{L}_{ik} u^T$ that has propagated to country *k* before coming back home. This value-added has already been measured in the initial round, so it is part of the domestic double counting. We have

$$\begin{aligned} &VaE(i)^T [\tilde{L}]^2_{ii} u^T = VaE(i)^T \sum_{k}^{G} \tilde{L}_{ik} \tilde{L}_{ki} u^T \\ &= V_i (I - A_{ii})^{-1} \hat{E}_i \sum_{k}^{G} \hat{E}_i^{-1} \tilde{A}_{ik} \hat{E}_k \times \hat{E}_k^{-1} \tilde{A}_{ki} E_i = V_i (I - A_{ii})^{-1} \sum_{k}^{G} \tilde{A}_{ik} \tilde{A}_{ki} E_i \end{aligned} \qquad (12)$$

For the foreign part of the second round, country *j*'s value-added is

$VaE(j)^T \sum_{k}^{G} \tilde{L}_{jk} \tilde{L}_{ki} u^T$, corresponding to country *j*'s value-added $VaE(j)^T \tilde{L}_{jk} u^T$ that has propagated to country *k* before coming back to country *i*. This value-added has also already been counted in the first round, so it is part of the foreign double counted term. We have:

$$\begin{aligned} &\sum_{j\neq i}^{G} VaE(j)^T [\tilde{L}]^2{}_{ji} u^T = VaE(j)^T \sum_{k}^{G} \tilde{L}_{jk} \tilde{L}_{ki} u^T \\ &= V_j (I - A_{jj})^{-1} \hat{E}_j \sum_{k}^{G} \hat{E}_j^{-1} \tilde{A}_{jk} \hat{E}_k \cdot \hat{E}_k^{-1} \tilde{A}_{ki} E_i = V_j (I - A_{jj})^{-1} \sum_{k}^{G} \tilde{A}_{jk} \tilde{A}_{ki} E_i \end{aligned} \tag{13}$$

Therefore, in the second round, the foreign double counted value-added is: $\sum_{j\neq i}^{G} V_j (I - A_{jj})^{-1} \sum_{k}^{G} \tilde{A}_{jk} \tilde{A}_{ki} E_i$.

Summing up all the domestic double counted value-added (from the second and later rounds), we can obtain an expression for the full domestic double counting in gross exports:

$$\begin{aligned} &VaE(i)^T [\tilde{L}]^2{}_{ii} u^T + VaE(i)^T [\tilde{L}]^3{}_{ii} u^T + \cdots = \\ &V_i (I - A_{ii})^{-1} (\sum_{j}^{G} \tilde{A}_{ij} \tilde{A}_{ji} + \sum_{k}^{G} \sum_{j}^{G} \tilde{A}_{ij} \tilde{A}_{jk} \tilde{A}_{ki} + \cdots) E_i = V_i (I - A_{ii})^{-1} (\tilde{B}_{ii} - I) E_i \end{aligned} \tag{14}$$

Theorem 2: The domestic double counted value-added in this framework is equal to the 'pure domestic double counted term' in KWW (see proof in Appendix A).

$$V_i (I - A_{ii})^{-1} (\tilde{B}_{ii} - I) E_i = V_i \sum_{j\neq i}^{G} B_{ij} A_{ji} (I - A_{ii})^{-1} E_i$$

The derivation we propose confirms the KWW result for the domestic double counting (the 'double counted intermediate exports produced at home' part of the 'pure double counted terms'). However, the Ghosh insight explains how this double counting is built through successive rounds of value-added inputs.

Similarly, the foreign double counted value-added in gross exports is (summing the second and later rounds):

$$\sum_{j\neq i}^{G} VaE(j)^T[\tilde{L}]^2{}_{ji}u^T + \sum_{j\neq i}^{G} VaE(j)^T[\tilde{L}]^3{}_{ji}u^T + \cdots =$$

$$\sum_{j\neq i}^{G} V_j(I-A_{jj})^{-1}(\sum_{k}^{G}\tilde{A}_{jk}\tilde{A}_{ki} + \sum_{t}^{G}\sum_{k}^{G}\tilde{A}_{jk}\tilde{A}_{kt}\tilde{A}_{ti} + \cdots)E_i = \sum_{j\neq i}^{G} V_j(I-A_{jj})^{-1}(\tilde{B}_{ji} - \tilde{A}_{ji})E_i \quad (15)$$

We can also show that in this decomposition of gross exports, the sum of the initial domestic value added and later rounds double counted domestic value-added is equal to the domestic content of exports:

$$V_i(I-A_{ii})^{-1}E_i + V_i(I-A_{ii})^{-1}(\tilde{B}_{ii} - I)E_i = V_iB_{ii}E_i \quad (16)$$

Also, the sum of the first round foreign value-added and later rounds double counted foreign value added in gross exports is equal to the foreign content of exports:

$$\sum_{j\neq i}^{G} V_j(I-A_{jj})^{-1}\tilde{A}_{ji}E_i + \sum_{j\neq i}^{G} V_j(I-A_{jj})^{-1}(\tilde{B}_{ji} - \tilde{A}_{ji})E_i = \sum_{j\neq i}^{G} V_jB_{ji}E_i \quad (17)$$

III. The value-added decomposition of gross exports: additional terms and comparison with KWW

In the KWW decomposition of gross exports, the domestic value-added and foreign value-added are decomposed into further terms (a total of 9). Our decomposition can also provide similar terms if one is interested in distinguishing the domestic and foreign value-added imported via intermediate or final goods, or the value-added that returns home. Merging equations (3), (16) and (17), we can obtain the terms detailed in the table below:

Table 1. A 10-term decomposition of gross exports

	Terms
Domestic value-added absorbed by foreign countries in final imports (T1)	$V_i(I-A_{ii})^{-1}\sum_{t}^{G}\sum_{j\neq t,i}^{G}\tilde{B}_{it}Y_{tj}$
Domestic value-added absorbed by foreign countries in intermediate imports (T2)	$V_i(I-A_{ii})^{-1}\sum_{j\neq i}^{G}\tilde{B}_{ij}Y_{jj}$
Domestic value-added that returns home via final imports (T3)	$V_i(I-A_{ii})^{-1}\sum_{j\neq i}^{G}\tilde{B}_{ij}Y_{ji}$

Domestic value-added that returns home via intermediate imports (T4)	$V_i(I-A_{ii})^{-1}(\tilde{B}_{ii}-I)Y_{ii}$
Domestic double counted value-added (T5)	$V_i(I-A_{ii})^{-1}(\tilde{B}_{ii}-I)E_i$
Foreign value-added absorbed by foreign countries in final imports (T6)	$\sum_{j\neq i}^{G} V_j(I-A_{jj})^{-1}\tilde{A}_{ji}\sum_{t}^{G}\sum_{k\neq t,i}^{G}\tilde{B}_{it}Y_{tk}$
Foreign value-added absorbed by foreign countries in intermediate imports (T7)	$\sum_{j\neq i}^{G} V_j(I-A_{jj})^{-1}\tilde{A}_{ji}\sum_{k\neq i}^{G}\tilde{B}_{ik}Y_{kk}$
Foreign value-added that returns via final imports (T8)	$\sum_{j\neq i}^{G} V_j(I-A_{jj})^{-1}\tilde{A}_{ji}\sum_{t\neq i}^{G}\tilde{B}_{it}Y_{ti}$
Foreign value-added that returns via intermediate imports (T9)	$\sum_{j\neq i}^{G} V_j(I-A_{jj})^{-1}\tilde{A}_{ji}(\tilde{B}_{ii}-I)Y_{ii}$
Foreign double counted value-added (T10)	$\sum_{j\neq i}^{G} V_j(I-A_{jj})^{-1}(\tilde{B}_{ji}-\tilde{A}_{ji})E_i$

As compared to the KWW decomposition, there are two differences in the above table. First, the domestic terms are defined slightly differently because our perspective is not the same when identifying intermediate and final trade flows. The KWW decomposition is motivated by how often value-added crosses international borders. More specifically, $V_i\sum_{j\neq i}^{G} B_{ii}Y_{ij}$ is the value-added in country *i*'s final exports; $V_i\sum_{j\neq i}^{G} B_{ij}Y_{jj}$ is the value-added in country *i*'s intermediate exports used by the direct importer to produce final goods consumed by the direct importer; and $V_i\sum_{j\neq i}^{G}\sum_{s\neq i,j}^{G} B_{ij}Y_{js}$ is the value-added in country *i*'s intermediate exports used by the direct importer to produce final goods for third countries. In contrast, the decomposition in our framework is based on the destination country. Final or intermediate flows are defined relative to the importing economy. The two approaches remain nonetheless consistent on the domestic side.[9] We can show below that the formulas are the same if we consider the domestic value-added absorbed by other countries, the domestic value-added that returns home and the domestic double counted value-added (additional proof in Appendix A).

1) Domestic value-added absorbed by other countries:

$$V_i(I-A_{ii})^{-1}\sum_{t}^{G}\sum_{j\neq t,i}^{G}\tilde{B}_{it}Y_{tj}=V_i\sum_{j\neq i}^{G}B_{ii}Y_{ij}+V_i\sum_{j\neq i}^{G}\sum_{k\neq i,j}^{G}B_{ij}Y_{jk}$$

When t=i, we have $V_i(I-A_{ii})^{-1}\sum_{j\neq i}^{G}\tilde{B}_{ii}Y_{ij}=V_i\sum_{j\neq i}^{G}B_{ii}Y_{ij}$;

$$V_i(I-A_{ii})^{-1}\sum_{j\neq i}^{G}\tilde{B}_{ij}Y_{jj}=V_i\sum_{j\neq i}^{G}B_{ij}Y_{jj}$$

2) Domestic value-added that returns home:

$$V_i(I-A_{ii})^{-1}\sum_{j\neq i}^{G}\tilde{B}_{ij}Y_{ji}=V_i\sum_{j\neq i}^{G}B_{ij}Y_{ji}$$

$$V_i(I-A_{ii})^{-1}(\tilde{B}_{ii}-I)Y_{ii}=V_i\sum_{j\neq i}^{G}B_{ij}A_{ji}(I-A_{ii})^{-1}Y_{ii}$$

3) Domestic double counted value-added:

$$V_i(I-A_{ii})^{-1}(\tilde{B}_{ii}-I)E_i=V_i\sum_{j\neq i}^{G}B_{ij}A_{ji}(I-A_{ii})^{-1}E_i$$

When it comes to the foreign value-added in exports, two new terms emerge in our decomposition related to the foreign value-added that returns back to the exporting country I (where it is absorbed). These terms provide a full symmetry between the analysis of the domestic value-added and foreign value-added in our gross exports decomposition. In the KWW framework, we can assume that these terms are part of the 'foreign value added in final goods exports' and the 'foreign value added in

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[9] Referring to Figure 1 in KWW, T1 in Table 1 is equal to (1) 'DV in direct final goods exports' and (3) 'DV in intermediates re-exported to third countries' in KWW, while T2 is equal to (2) 'DV in intermediates absorbed by direct exporters'. In our destination country framework, the third term of KWW corresponds to value added entering the last country as a final product and is therefore similar to the first term. But we have the same sum for the three first terms describing the value-added absorbed by other countries (see Appendix B for an empirical illustration).

intermediate gross exports' since unlike what they do for the domestic value-added, the authors do not specifically identify the foreign value added that returns to the exporting economy.

Beyond differences in the definition of the foreign value added terms, our framework does not provide the same foreign double counting (and therefore not the same foreign value added net of any double counting). It is a more fundamental difference and not related to the Ghosh insight and our 10-term decomposition. Already in Section I, we have defined the domestic value-added in exports consistent with GDP and the foreign value-added in exports consistent with GDP. The difference between these two terms and gross exports is by definition the double counting. Summing the domestic and foreign double counted terms in KWW does not provide this double counting as defined in Section I. And since we have exactly the same domestic double counting, the foreign double counting is the reason why it is not the case. An illustration of these differences can be found in Appendix B where the gross exports of 6 countries in 2014 are decomposed according to the different methodologies reviewed.

IV. From country-level to industry-level analysis: the source, gross export and final demand industry dimension

In order to extend the gross exports decomposition to the industry level, we need first to clarify what are the source industry, gross exports industry and final demand industry in the input-output framework and its gross exports version. The source and gross exports industries are similar to the concepts of forward linkages and backward linkages introduced by Wang, Wei and Zhu (2013) in the paper that transposes to the industry level the KWW method. The source industry decomposition is about measuring the value-added originating in a specific sector while the gross exports industry decomposition aims at measuring the value-added (domestic or foreign) in a specific exporting industry. The exporting industry relies on value-added from all other (source) industries in the domestic economy and foreign countries supplying inputs. As for the final demand industry decomposition, the objective is to measure the value-added absorbed by a specific sector (i.e. the industry of the final product which is imported or manufactured with imported inputs). This later approach is not commonly used in the literature but could also be interesting from an analytical point of view to analyze value-added trade flows related to specific final products. The source industry approach is the one followed by Johnson and Noguera (2012) in the calculation of the sectoral VAX ratio [10], while the gross exports industry decomposition is the purpose of the WWZ paper. In the gross exports industry decomposition, all terms sum to the sectoral exports of a specific country.

In this section, we first show how we can decompose gross exports by industry in a similar way to the approach we have suggested at the country level in Section I. Then, we illustrate how the same can be done for possibly all terms presented in Table 1. The process is more tedious but there is no particular difficulty once one has clearly identified the industry dimension (source, exports or final demand) in the equations.

From Section I, we know that the (initial) domestic value-added in gross exports can be expressed as $V_i(I - A_{ii})^{-1}E_i$. For the convenience of writing, we denote the local Leontief inverse matrix $(I - A_{ii})^{-1}$ as L_i. The subscript *i* means country *i*. To better explain the value-added generation at the industry level, we introduce a sectoral superscript.

At the industry level, country *i*'s value added in exports can be expressed with the local Leontief inverse as follows:

[10] VAX is defined by Johnson and Noguera as the ratio of value-added to gross exports.

$$\hat{V}_i L_i \hat{E}_i = \begin{bmatrix} v_i^1 & 0 & \cdots & 0 \\ 0 & v_i^2 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & v_i^n \end{bmatrix} \begin{bmatrix} l_i^{11} & l_i^{12} & \cdots l_i^{1n} \\ l_i^{21} & l_i^{22} & \cdots l_i^{2n} \\ \vdots & \vdots & \ddots \vdots \\ l_i^{n1} & l_i^{n2} & \cdots l_i^{nn} \end{bmatrix} \begin{bmatrix} e_i^1 & 0 & \cdots & 0 \\ 0 & e_i^2 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e_i^n \end{bmatrix}$$

$$= \begin{bmatrix} v_i^1 l_i^{11} e_i^1 & v_i^1 l_i^{12} e_i^2 & \cdots v_i^1 l_i^{1n} e_i^n \\ v_i^2 l_i^{21} e_i^1 & v_i^2 l_i^{22} e_i^2 & \cdots v_i^2 l_i^{2n} e_i^n \\ \vdots & \vdots & \ddots \quad \vdots \\ v_i^n l_i^{n1} e_i^1 & v_i^n l_i^{n2} e_i^2 & \cdots v_i^n l_i^{nn} e_i^n \end{bmatrix} \quad (18)$$

The matrix in equation (18) provides estimates of domestic value-added in exports by industry. Each element in the matrix accounts for the value-added from a source industry directly or indirectly embodied in the exports of a specific industry. In this matrix, the values along the rows indicate the distribution of value-added originating from a specific industry across all sectors. Therefore, summing up the *s*th row of the matrix, we can have total value-added originating from country *i*'s *s*th sector in country *i*'s exports. In other words, we have the source industry value-added decomposition which can be expressed mathematically as $v_i^s(l_i^{s1}e_i^1 + l_i^{s2}e_i^2 + \cdots\cdots + l_i^{sn}e_i^n)$. In the same matrix but along the columns, we have the distribution of value-added from all industries to the exports of a specific industry. Summing up all the elements in the *h*th column, $(v_i^1 l_i^{1h} + v_i^2 l_i^{2h} + \cdots\cdots + v_i^n l_i^{nh})e_i^h$, provides the total domestic value-added in the gross exports industry.

To put it in a nutshell, the sum of the $\hat{V}_i L_i \hat{E}_i$ matrix across columns along a row traces the forward linkages across all downstream sectors from a supply-side perspective and provides the source industry decomposition. And the sum of the $\hat{V}_i L_i \hat{E}_i$ matrix across rows along a column traces backward linkages across upstream sectors from a users' perspective and provides the gross exports industry decomposition. If we apply similar matrix arrangements into the other terms in equations (16) and (17), we can obtain an industry-level decomposition of gross exports similar to the one described at the country level.

When considering the destination of exports, the industry-level extension is more tedious but straightforward. We can illustrate this with term 1 and term 6 in Table 1, as an example. Assuming that domestic value-added from country *i* is going to country *t* before being finally absorbed by country *j*, we can expand the elements in the expression $V_i(I-A_{ii})^{-1}\tilde{B}_{it}Y_{tj}$ as $\hat{V}_i L_i \tilde{B}_{it}\hat{Y}_{tj}$. For the elements in the matrix above, we have the universal expression $v_i^s l_i^{sh} \tilde{b}_{it}^{hf} y_{tj}^f$ where superscripts *s*, *h* and *f* identify respectively the source, gross exports and final demand industries. Therefore, if we extend the decomposition term in the source industry dimension (country *i*'s *s*th industry), the other two dimensions have to be summed up. The equation becomes $\sum_h \sum_f v_i^s l_i^{sh} \tilde{b}_{it}^{hf} y_{tj}^f$. In contrast, the extension to the gross exports decomposition (country *i*'s *h*th industry) is $\sum_s \sum_f v_i^s l_i^{sh} \tilde{b}_{it}^{hf} y_{tj}^f$ and the extension to the final demand decomposition $\sum_s \sum_h v_i^s l_i^{sh} \tilde{b}_{it}^{hf} y_{tj}^f$ (country *j*'s *f*th industry).

Similarly, we can also decompose country *i*'s first round foreign value-added by industry. We introduce superscript *m* for the industry in country *i* that imports from country *j*. The expression $v_j^s l_j^{sm} \tilde{a}_{ji}^{mh} \tilde{b}_{it}^{hf} y_{tk}^f$ is the value-added flow from country *j* to country *i* that goes through country *t* before being finally absorbed by country *k*. Country *i*'s foreign value-added (from country *j*) in exports is $\sum_m \sum_h \sum_f v_j^s l_j^{sm} \tilde{a}_{ji}^{mh} \tilde{b}_{it}^{hf} y_{tk}^f$ in the source industry decomposition (the value-added from country *j*'s *s*th industry). It becomes $\sum_s \sum_m \sum_f v_j^s l_j^{sm} \tilde{a}_{ji}^{mh} \tilde{b}_{it}^{hf} y_{tk}^f$ in the gross exports industry dimension (country *i*'s *h*th industry) and $\sum_s \sum_m \sum_h v_j^s l_j^{sm} \tilde{a}_{ji}^{mh} \tilde{b}_{it}^{hf} y_{tk}^f$ in the final demand industry decomposition (country *k*'s *f*th industry).

For the later rounds double counted terms, the industry expansion is a bit different. In Section II, we have derived these terms from the Ghosh insight. If we write

$V_i(I-A_{ii})^{-1}(\tilde{B}_{ii}-I)E_i$ as $\hat{V}_i L_i(\tilde{B}_{ii}-I)\hat{E}_i$, the elements in the matrix can be expressed as: $v_i^s l_i^{sm}(\tilde{b}_{ii}^{mh}-\delta)e_i^h$, Here, δ is equal to 1 when $m=h$, and 0 otherwise. In this industry level expression, the element $\tilde{b}_{ii}^{mh}-\delta$ indicates how value-added has returned home (i.e. been re-imported) and been re-exported again. Superscript *m* also defines the import sector of the returned domestic value-added. Therefore, for country *i*'s domestic later rounds double counted value added, the formula in the source industry (country *i*'s *s*th industry) decomposition is $\sum_m \sum_h v_i^s l_i^{sm}(\tilde{b}_{ii}^{mh}-\delta)e_i^h$; and the formula in the gross exports industry (country *i*'s *h*th industry) decomposition is $\sum_s \sum_m v_i^s l_i^{sm}(\tilde{b}_{ii}^{mh}-\delta)e_i^h$. Also, we can obtain similar industry-level expressions for the foreign later rounds double counted value added as $\sum_{j\neq i} \sum_m \sum_h v_j^s l_j^{sm}(\tilde{b}_{ji}^{mh}-\tilde{a}_{ji}^{mh})e_i^h$ (source industry) or $\sum_{j\neq i} \sum_s \sum_m v_j^s l_j^{sm}(\tilde{b}_{ji}^{mh}-\tilde{a}_{ji}^{mh})e_i^h$ (gross exports industry).

The KWW framework can also provide a source industry decomposition and a final demand industry decomposition in a consistent way by following the same logic (the gross exports industry decomposition being explained in WWZ). As soon as the source, gross exports and final demand industries are clearly identified, it is straightforward to derive industry-level formulas.

But the more sophisticated and detailed the gross exports decomposition is, the more complicated it becomes to track the different industry dimensions. As an illustration, we provide below the full expansion of our 10-term decomposition in Table 1 at the gross exports industry level. Country *i*'s gross exports in industry *h* can be decomposed as:

$$
\begin{aligned}
e_i^h &= \sum_t \sum_{j\neq t,i} \sum_s \sum_f v_i^s l_i^{sh} \tilde{b}_{it}^{hf} y_{tj}^f + \sum_{j\neq i} \sum_s \sum_f v_i^s l_i^{sh} \tilde{b}_{ij}^{hf} y_{jj}^f \\
&+ \sum_{j\neq i} \sum_s \sum_f v_i^s l_i^{sh} \tilde{b}_{ij}^{hf} y_{ji}^f + \sum_s \sum_f v_i^s l_i^{sh} (\tilde{b}_{ii}^{hf} - \delta) y_{ii}^f \\
&+ \sum_s \sum_m v_i^s l_i^{sm} (\tilde{b}_{ii}^{mh} - \delta) e_i^h \\
&+ \sum_{j\neq i} \sum_t \sum_{k\neq t,i} \sum_s \sum_m \sum_f v_j^s l_j^{sm} \tilde{a}_{ji}^{mh} \tilde{b}_{it}^{hf} y_{tk}^f + \sum_{j\neq i} \sum_{k\neq i} \sum_s \sum_m \sum_f v_j^s l_j^{sm} \tilde{a}_{ji}^{mh} \tilde{b}_{ik}^{hf} y_{kk}^f \\
&+ \sum_{j\neq i} \sum_{t\neq i} \sum_s \sum_m \sum_f v_j^s l_j^{sm} \tilde{a}_{ji}^{mh} \tilde{b}_{it}^{hf} y_{ti}^f + \sum_{j\neq i} \sum_s \sum_m \sum_f v_j^s l_j^{sm} \tilde{a}_{ji}^{mh} (\tilde{b}_{ii}^{hf} - \delta) y_{ii}^f \\
&+ \sum_{j\neq i} \sum_s \sum_m v_j^s l_j^{sm} (\tilde{b}_{ji}^{mh} - \tilde{a}_{ji}^{mh}) e_i^h
\end{aligned}
\tag{19}
$$

Here, δ is equal to 1 when $h = f$ and 0 otherwise. For sub-term $\sum_s \sum_m v_i^s l_i^{sm} (\tilde{b}_{ii}^{mh} - \delta) e_i^h$, δ is equal to 1 when $m = h$ and 0 otherwise.

V. Concluding remarks

This paper has introduced a new framework for the decomposition of value-added in gross exports that has a firm foundation in input-output analysis and provides terms with a clear economic interpretation, including for the double counted elements. It confirms the results of earlier literature for the decomposition of the domestic value-added in exports but brings new results for the foreign value-added and the foreign double counting.

The starting point is a reinterpretation of the input-output model in terms of a relationship between gross exports and intermediate and final demand for exports in the destination country. Using the Ghosh insight, the framework allows to fully decompose gross exports into an initial domestic value-added consistent with GDP, a first round foreign value-added also consistent with GDP and later rounds domestic and foreign double counted terms that account for some value-added coming back to the exporting economy and entering again into exports. The generation of this multiple counting in successive rounds of value addition is explicit in the Ghosh framework but the initial domestic value-added and first round foreign value-added do not depend on the Ghosh insight.

The domestic and foreign value-added can be further decomposed to distinguish, for example, the value-added that returns home (before being absorbed in the domestic economy) or whether value-added is entering the destination country via a final or intermediate product. Such distinctions, as introduced by KWW, can be useful for trade economics or policymaking. But we believe it is important to have some symmetry in the domestic and foreign terms. For example, the foreign value-added that returns to the country where it was first embodied in exports is interesting to identify some 'circular' trade.

Also, it seems more practical to use a destination country perspective in the gross exports decomposition to avoid some overlap in the terms. When the global Leontief inverse is introduced in a term, value-added can cross borders several times before being absorbed abroad or returning back, transiting through different countries and leading to ambiguous interpretations with respect to flows of final or intermediate goods.

Finally, also having in mind the popularity of trade in value-added indicators among economists and policymakers, it seems important to provide industry-level formulas for the decomposition of gross exports. It requires a careful analysis of the industry dimension in input-output relationships and in particular to clearly distinguish the source industry, the gross exports industry and the final demand industry. We show that our framework can be extended to decompose the value-added in gross exports of a specific industry but also to track the value-added originating in a specific industry or ending up in the final products of a specific industry. But it is not a feature specific to this framework and can be done for other decompositions of gross exports proposed in the literature.

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Appendix A

Proposition 1: The accounting relationship between gross exports E and final demand in destination in an Inter-Country Input-Output (ICIO) model can be expressed as:

$$E = \tilde{A}E + \tilde{Y}$$

Here, $\tilde{A} = A^F(I - A^D)^{-1}$, A^D is the matrix of domestic coefficients in the global ICIO table (i.e. the block diagonal matrix of the A matrix). A^F is the matrix of export coefficients (i.e. the elements of the A matrix off the block diagonal that indicate the use of intermediate inputs from one country into another country). In addition, $\tilde{Y} = Y^F + \tilde{A}Y^D$, with Y^D the domestic final demand and Y^F the final demand in foreign countries.

Proof: According to the description of the matrixes above, we can obtain the following accounting equalities:

$$E = A^F(E + H) + Y^F$$
$$H = A^D(E + H) + Y^D$$

with H the vector of gross domestic shipments (and E the vector of exports). Solving for H, we obtain:

$$H = (I - A^D)^{-1}A^D E + (I - A^D)^{-1}Y^D$$

Merging the expression for H and the expression for E, we have:

$$\begin{aligned}
E &= A^F(E + H) + Y^F \\
&= A^F[E + (I - A^D)^{-1}A^D E + (I - A^D)^{-1}Y^D] + Y^F \\
&= A^F[I + (I - A^D)^{-1}A^D]E + A^F(I - A^D)^{-1}Y^D + Y^F \\
&= A^F(I - A^D)^{-1}E + A^F(I - A^D)^{-1}Y^D + Y^F \\
&= \tilde{A}E + \tilde{Y}
\end{aligned}$$

here, we define $\tilde{A} = A^F(I - A^D)^{-1}$, for the elements in the matrix $\tilde{A}$,

$$\tilde{A}_{ij} = \begin{cases} \mathbf{0} & i = j \\ A_{ij}(I - A_{jj})^{-1} & i \neq j \end{cases} \quad \text{and} \quad \tilde{Y} = \tilde{A}Y^D + Y^F .$$

Proposition 2：The 'total inputs requirement matrix for exports' $\tilde{B} = (I - \tilde{A})^{-1}$, for the elements in matrix $\tilde{B}$, $\tilde{B}_{ij} = (I - A_{ii})B_{ij}$.

Proof: We can express $\tilde{B}$ as

$$\begin{aligned}
\tilde{B} &= (I - \tilde{A})^{-1} = [I - A^F(I - A^D)^{-1}]^{-1} = [(I - A^D)(I - A^D)^{-1} - A^F(I - A^D)^{-1}]^{-1} \\
&= [(I - A^D - A^F)(I - A^D)^{-1}]^{-1} \\
&= (I - A^D)B
\end{aligned}$$

So for the elements in the matrix, we have $\tilde{B}_{ij} = (I - A_{ii})B_{ij}$.

Theorem 1: For country *i*'s exports, the domestic value-added multiplier coefficient is

$$u(I - \sum_{j \neq i}^{G} \tilde{A}_{ji}) = V_i(I - A_{ii})^{-1}$$

Proof: Based on the definition of $\tilde{A}$, we already know that for country *i*'s exports E_i, all of intermediate inputs are $\sum_{j \neq i}^{G} \tilde{A}_{ji}E_i$, so country *i*'s value-added in exports is $uVaE(i) = u(E_i - \sum_{j \neq i}^{G} \tilde{A}_{ji}E_i) = u(I - \sum_{j \neq i}^{G} \tilde{A}_{ji})E_i$.

Expanding the equation $u(I - \sum_{j \neq i}^{G} \tilde{A}_{ji})$, we have:

$$\begin{aligned}
u(I - \sum_{j \neq i}^{G} \tilde{A}_{ji}) &= u[I - \sum_{j \neq i}^{G} A_{ji}(I - A_{ii})^{-1}] \\
&= u[(I - A_{ii})(I - A_{ii})^{-1} - \sum_{j \neq i}^{G} A_{ji}(I - A_{ii})^{-1}] \\
&= u(I - A_{ii} - \sum_{j \neq i}^{G} A_{ji})(I - A_{ii})^{-1} = u(I - \sum_{j}^{G} A_{ji})(I - A_{ii})^{-1} \\
&= V_i(I - A_{ii})^{-1}
\end{aligned}$$

Here, if we want to extend the value-added multiplier coefficient at the industry level, we can just transform the value-added coefficient vector V_i into a diagonal matrix $\hat{V}_i$.

Theorem 2: The later rounds domestic double-counting value-added term in our framework is equal to the domestic 'pure double counting' term in the KWW framework:

$$V_i(I-A_{ii})^{-1}(\tilde{B}_{ii}-I)E_i = V_i\sum_{j\neq i}^{G} B_{ij}A_{ji}(I-A_{ii})^{-1}E_i$$

Proof: Based on the definition of the Leontief inverse matrix in the ICIO model, we have:

$$\begin{bmatrix} I-A_{11} & -A_{12} & \cdots & -A_{1G} \\ -A_{21} & I-A_{22} & \cdots & -A_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ -A_{G1} & -A_{G2} & \cdots & I-A_{GG} \end{bmatrix}\begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1G} \\ B_{21} & B_{22} & \cdots & B_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ B_{G1} & B_{G2} & \cdots & B_{GG} \end{bmatrix} = \begin{bmatrix} I & 0 & \cdots & 0 \\ 0 & I & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & I \end{bmatrix}$$

$$= \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1G} \\ B_{21} & B_{22} & \cdots & B_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ B_{G1} & B_{G2} & \cdots & B_{GG} \end{bmatrix}\begin{bmatrix} I-A_{11} & -A_{12} & \cdots & -A_{1G} \\ -A_{21} & I-A_{22} & \cdots & -A_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ -A_{G1} & -A_{G2} & \cdots & I-A_{GG} \end{bmatrix}$$

Then, we can obtain the following two equations:

$$B_{ii}-\sum_{k}^{G} A_{ik}B_{ki} = B_{ii}-\sum_{k}^{G} B_{ik}A_{ki} = I$$

$$B_{ij}-\sum_{k}^{G} A_{ik}B_{kj} = 0,\ j\neq i$$

Therefore, we already have the equation $B_{ii}-\sum_{k}^{G} B_{ik}A_{ki} = I$. Re-writing this equation, we can obtain:

$$B_{ii}-\sum_{j}^{G} B_{ij}A_{ji} = B_{ii}-B_{ii}A_{ii}-\sum_{j\neq i}^{G} B_{ij}A_{ji} = B_{ii}(I-A_{ii})-\sum_{j\neq i}^{G} B_{ij}A_{ji} = I$$

Re-arranging the equation above, we have:

$$\sum_{j\neq i}^{G} B_{ij}A_{ji}(I-A_{ii})^{-1} = B_{ii}-(I-A_{ii})^{-1} = (I-A_{ii})^{-1}[(I-A_{ii})B_{ii}-I] = (I-A_{ii})^{-1}(\tilde{B}_{ii}-I).$$

Proposition 3.1 The sum of the initial domestic value-added and the later rounds domestic double counted value-added are equal to the domestic content in exports.

$$V_i(I-A_{ii})^{-1}E_i + V_i(I-A_{ii})^{-1}(\tilde{B}_{ii}-I)E_i = V_iB_{ii}E_i$$

Proof: Because $V_i(I-A_{ii})^{-1}E_i + V_i(I-A_{ii})^{-1}(\tilde{B}_{ii}-I)E_i = V_i(I-A_{ii})^{-1}\tilde{B}_{ii}E_i$. Then according to the Proposition 2, we have $\tilde{B}_{ii} = (I-A_{ii})B_{ii}$.

Therefore, $V_i(I-A_{ii})^{-1}E_i + V_i(I-A_{ii})^{-1}(\tilde{B}_{ii}-I)E_i = V_i(I-A_{ii})^{-1}\tilde{B}_{ii}E_i = V_iB_{ii}E_i$.

Proposition proved.

Proposition 3.2 The sum of the first round foreign value-added and the later rounds foreign double counted value-added are equal to the foreign content in export.

$$\sum_{j\neq i}^{G} V_j(I-A_{jj})^{-1}\tilde{A}_{ji}E_i + \sum_{j\neq i}^{G} V_j(I-A_{jj})^{-1}(\tilde{B}_{ji}-\tilde{A}_{ji})E_i = \sum_{j\neq i}^{G} V_jB_{ji}E_i$$

Proof: Similar with Proposition 3.1.

Proposition 4.1 In the decomposition framework of this paper, for the domestic value-added absorbed by other countries, we have

$$V_i(I-A_{ii})^{-1}\sum_{t}^{G}\sum_{j\neq t,i}^{G}\tilde{B}_{it}Y_{tj} = V_i\sum_{j\neq i}^{G}B_{ii}Y_{ij} + V_i\sum_{j\neq i}^{G}\sum_{k\neq i,j}^{G}B_{ij}Y_{jk}$$

When t=i, we have $V_i(I-A_{ii})^{-1}\sum_{j\neq i}^{G}\tilde{B}_{ii}Y_{ij} = V_i\sum_{j\neq i}^{G}B_{ii}Y_{ij}$; and

$$V_i(I-A_{ii})^{-1}\sum_{j\neq i}^{G}\tilde{B}_{ij}Y_{jj} = V_i\sum_{j\neq i}^{G}B_{ij}Y_{jj}$$

Proof: According to Proposition 2, we have $\tilde{B}_{it} = (I-A_{ii})B_{it}$. Therefore,

$$V_i(I-A_{ii})^{-1}\sum_{t}^{G}\sum_{j\neq t,i}^{G}\tilde{B}_{it}Y_{tj} = V_i\sum_{t}^{G}\sum_{j\neq t,i}^{G}B_{it}Y_{tj}$$

Re-writing the subscript, $V_i\sum_{t}^{G}\sum_{j\neq t,i}^{G}B_{it}Y_{tj}=V_i\sum_{j}^{G}\sum_{k\neq j,i}^{G}B_{ij}Y_{jk}=V_i\sum_{j\neq i}^{G}B_{ii}Y_{ij}+V_i\sum_{j\neq i}^{G}\sum_{k\neq i,j}^{G}B_{ij}Y_{jk}$.

Obviously, when t=i, $V_i(I-A_{ii})^{-1}\sum_{j\neq i}^{G}\tilde{B}_{ii}Y_{ij}=V_i\sum_{j\neq i}^{G}B_{ii}Y_{ij}$;

For the equation $V_i(I-A_{ii})^{-1}\sum_{j\neq i}^{G}\tilde{B}_{ij}Y_{jj}=V_i\sum_{j\neq i}^{G}B_{ij}Y_{jj}$, the proof is similar.

Proposition 4.2 In the decomposition framework of this paper, for the domestic value-added that returns home, we have:

$$V_i(I-A_{ii})^{-1}\sum_{j\neq i}^{G}\tilde{B}_{ij}Y_{ji}=V_i\sum_{j\neq i}^{G}B_{ij}Y_{ji}$$

$$V_i(I-A_{ii})^{-1}(\tilde{B}_{ii}-I)Y_{ii}=V_i\sum_{j\neq i}^{G}B_{ij}A_{ji}(I-A_{ii})^{-1}Y_{ii}$$

Proof: For equation $V_i(I-A_{ii})^{-1}\sum_{j\neq i}^{G}\tilde{B}_{ij}Y_{ji}=V_i\sum_{j\neq i}^{G}B_{ij}Y_{ji}$, the proof is similar to Proposition 4.1.

For equation $V_i(I-A_{ii})^{-1}(\tilde{B}_{ii}-I)Y_{ii}=V_i\sum_{j\neq i}^{G}B_{ij}A_{ji}(I-A_{ii})^{-1}Y_{ii}$, the proof is similar to Theorem 2.

Appendix B

This appendix compares the decomposition of gross exports according to the KWW methodology, LTV methodology and the methodology we propose in this paper. We use the publicly available data from the 2016 release of the World Input-Output Database (Timmer et al., 2015). We decompose gross exports in 2014 (the latest year available in the dataset) for 6 exporting economies: China, France, Germany, Mexico, Japan and the United States. We pick these countries because they are major exporters but also illustrate different cases in terms of the prevalence of double counting, thus helping to understand how the different methodologies point to different results.

Table B.1 first provides a comparison for the domestic and foreign value-added, including the double counting terms. DVA is the domestic value-added without double counting, DVAD is the double counted domestic value-added, FVA is the foreign value-added without double counting and FVAD the double counted foreign value-added. In the case of the LTV decomposition, the 3 last terms are not distinguished. The authors only provide DVA and the rest is a residual (RES).

From Table B.1 it is clear that there is a consensus on the share of the domestic value-added in exports consistent with GDP with no double counting (DVA). Moreover, our methodology provides the same share as KWW for the domestic double counted VA (DVAD), which is consistent with the proof provided in Appendix A. But the two methodologies offer different results for the foreign value-added net of double counting (FVA) and the double counted foreign value-added (FVAD).

One can see in particular that our FVA is not systematically higher or lower as compared to KWW. In the case of China, Germany, France and Mexico, our FVA is lower and the KWW methodology underestimates the double counting. But it is higher (and there is a lower double counting) in the case of Japan and the United States.

To further compare our methodology with KWW, we show in Table B.2 the results of the full decomposition as described in Table 1 of the main text. The decomposition has 9 terms in the case of KWW and 10 terms in our case as we have symmetry between the domestic and foreign VA terms. To facilitate the comparison and account for the difference in the origin and destination approach in terms of trade in intermediate and final products, we split our first term (T1) to match the KWW framework so that T1.1, T2 and T1.2 in our framework are equivalent to T1, T2 and T3 in KWW. As proved in Appendix A, our decomposition yields exactly the same results for all domestic terms (T1 to T6 in KWW, T1 to T5 in our framework).

But when moving to the foreign value-added decomposition, our approach points to different results. Even if we split T6 into T6.1 (the foreign VA absorbed by foreign countries in final imports and exported as final) and T6.2 (the foreign VA absorbed by foreign countries in final imports and exported as intermediate), we cannot really match the KWW terms, in particular because T8 and T9 in our framework (the foreign VA that returns to the exporting country) have no equivalent in KWW. But the calculations confirm that the foreign double counting terms (T9 in KWW and T10 in our framework) are different independently of how we can re-arrange the foreign value-added terms.

Lastly, in Table B.3, we provide the full decomposition of the domestic value-added by LTV (domestic VA in final exports, domestic VA in intermediate exports, domestic VA reflected back to the home country and residual) and compare with our framework. The two methodologies provide exactly the same percentages in the decomposition. The only difference is that T1 in our framework captures the value-added which is entering the destination country in a final product and not exported as a final product. As done before, we have to split T1 into T1.1 (the VA absorbed by foreign countries in final imports and exported in a final product) and T1.2 (the VA absorbed by foreign countries in final imports and exported in an intermediate product) to match the categories of LTV (domestic VA in final exports and domestic VA in intermediate exports). Otherwise, the results are the same.

Table B.1 Basic decomposition: Domestic and foreign value-added (selected countries, 2014)

Country	Gross exports (million USD)	Koopman, Wang and Wei (percent)				Los, Timmer and de Vries (percent)		Our framework (percent)			
		DVA	DVAD	FVA	FVAD	DVA	RES	DVA	DVAD	FVA	FVAD
China	2,425,464	83.15	0.94	12.69	3.22	83.15	16.85	83.15	0.94	11.68	4.23
Germany	1,682,253	71.85	1.39	19.22	7.53	71.85	28.15	71.85	1.39	18.77	7.98
France	759,654	72.28	0.46	19.96	7.30	72.28	27.72	72.28	0.46	19.44	7.82
Japan	817,514	76.41	0.32	17.19	6.09	76.41	23.59	76.41	0.32	17.89	5.38
Mexico	368,185	66.44	0.26	29.70	3.59	66.44	33.56	66.44	0.26	25.43	7.86
United States	1,927,091	87.15	0.70	8.84	3.32	87.15	12.85	87.15	0.70	9.45	2.71

Source: Authors' calculations based on WIOD. DVA = Domestic value-added; DVAD = Double counted domestic value-added; FVA = foreign value-added; FVAD = Double counted foreign value added; RES = Residual in the case of the Los, Timmer and de Vries decomposition, i.e. gross exports minus DVA (corresponding to DVAD + FVA + FVAD).

Table B.2 Full decomposition: comparison between KWW and our framework

Panel A. Koopman, Wang and Wei (percent)

Country	Domestic value-added						Foreign value-added		
	T1	T2	T3	T4	T5	T6	T7	T8	T9
China	42.05	32.30	6.36	0.85	1.58	0.94	7.97	4.72	3.22
Germany	31.58	30.33	7.86	1.24	0.84	1.39	11.09	8.14	7.53
France	30.11	32.28	8.66	0.67	0.56	0.46	11.79	8.16	7.30
Japan	32.13	35.28	8.02	0.46	0.52	0.32	7.80	9.39	6.09
Mexico	29.23	32.37	4.32	0.19	0.34	0.26	18.90	10.80	3.59
United States	30.39	42.23	8.17	3.18	3.18	0.70	4.18	4.65	3.32

Panel B. Our framework (percent)

Country	Domestic value-added						Foreign value-added					
	T1.1	T2	T1.2	T3	T4	T5	T6.1	T7	T6.2	T8	T9	T10
China	42.05	32.30	6.36	0.85	1.58	0.94	5.89	4.44	0.96	0.14	0.25	4.23
Germany	31.58	30.33	7.86	1.24	0.84	1.39	7.99	7.94	2.22	0.37	0.27	7.98
France	30.11	32.28	8.66	0.67	0.56	0.46	8.49	8.10	2.49	0.20	0.16	7.82
Japan	32.13	35.28	8.02	0.46	0.52	0.32	5.96	9.55	2.11	0.12	0.16	5.38
Mexico	29.23	32.37	4.32	0.19	0.34	0.26	14.48	9.44	1.37	0.06	0.09	7.86
United States	30.39	42.23	8.17	3.18	3.18	0.70	4.56	4.56	0.80	0.44	0.40	2.71

Source: Authors' calculations based on WIOD. Panel A (KWW): T1 = Domestic VA in direct final goods exports; T2 = Domestic VA in intermediates absorbed by direct exporters; T3 = Domestic VA in intermediates re-exported to third countries; T4 = Domestic VA in intermediates that returns via final imports; T5 = Domestic VA in intermediates that returns via intermediate imports; T6 = Double counted intermediate exports produced at home; T7 = Foreign VA in final goods exports; T8 = Foreign VA in intermediate goods exports; T9 = double counted intermediate exports produced abroad. Panel B (our framework): T1.1 = Domestic VA absorbed by foreign countries in final imports (exported as final); T2 = Domestic VA absorbed by foreign countries in intermediate imports; T1.2 = Domestic VA absorbed by foreign countries in final imports (exported as intermediate, equivalent to T3 in KWW); T3 = Domestic VA that returns home via final imports; T4 = Domestic VA that returns home via intermediate imports; T5 = Domestic double counted VA; T6.1 = Foreign VA absorbed by foreign countries in final imports (exported as final); T7 = Foreign VA absorbed by foreign countries in intermediate imports; T6.2 = Foreign VA absorbed by foreign countries in final imports (exported as intermediate);T8 = Foreign VA that returns via final imports; T9 = Foreign VA that returns via intermediate imports; T10 = Foreign double counted VA.

Table B.3 Full decomposition: comparison between LTV and our framework

Country	*Los, Timmer and de Vries (percent)*				
	DVA(A,Fin)	DVA(A,Int)		DVA(R)	RES
China	42.05	38.66		2.43	16.85
Germany	31.58	38.20		2.08	28.15
France	30.11	40.94		1.23	27.72
Japan	32.13	43.30		0.98	23.59
Mexico	29.23	36.68		0.54	33.56
United States	30.39	50.39		6.36	12.85
Country	*Our framework (percent)*				
	T1.1	T1.2	T2	T3 + T4	T5-T10
China	42.05	6.36	32.30	2.43	16.85
Germany	31.58	7.86	30.33	2.08	28.15
France	30.11	8.66	32.28	1.23	27.72
Japan	32.13	8.02	35.28	0.98	23.59
Mexico	29.23	4.32	32.37	0.54	33.56
United States	30.39	8.17	42.23	6.36	12.85

Source: Authors' calculations based on WIOD. Los, Timmer and de Vries: DVA(A,Fin) = Domestic VA in exports of final goods; DVA(A,Int) = Domestic VA in exports of intermediate goods; DVA(R) = Domestic VA reflected back to the home country; RES = Residual (gross exports minus the other terms). Our framework: T1.1 = Domestic VA absorbed by foreign countries in final imports (exported as final); T1.2 = Domestic VA absorbed by foreign countries in final imports (exported as intermediate and final in a third country); T2 = Domestic VA absorbed by foreign countries in intermediate imports; T3 + T4 = Domestic VA that returns home (via final and intermediate imports); T5-T10 = Residual (all other terms).

Appendix C

Measuring the foreign value-added in gross exports with the hypothetical extraction method

In this Appendix, we provide an alternative method to measure the foreign value-added in gross exports, consistent with the one we have developed but based on a hypothetical extraction. The hypothetical extraction method was first proposed by Timmer et al. (2016) to measure the domestic value-added in exports. However, the authors left for future research the question of the foreign value-added (net of any double counting). Borin and Mancini (2017) also use in their framework some form of extraction in the ***A*** matrix by setting to 0 the coefficients for specific countries. Lastly, Johnson (2017) suggested an extraction method to obtain an expression for the foreign value-added in exports in the two-country case. Our objective in this Appendix is to extend the extraction method to an arbitrary number of countries in the ICIO and to show that the result is consistent with the foreign value-added calculated in our framework.

We can start from equation (1) and re-organise it to separate out the exports of a given country. Here, we take country 1 as an example:

$$\begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_j \\ \vdots \\ X_G \end{bmatrix} = \begin{bmatrix} A_{11} & 0 & \cdots & 0 & \cdots & 0 \\ A_{21} & A_{22} & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ A_{j1} & 0 & \cdots & A_{jj} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ A_{G1} & 0 & \cdots & 0 & \cdots & A_{GG} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_j \\ \vdots \\ X_G \end{bmatrix} + \begin{bmatrix} Y_{11} & 0 & \cdots & 0 & \cdots & 0 \\ Y_{21} & Y_{22} & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ Y_{j1} & 0 & \cdots & Y_{jj} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ Y_{G1} & 0 & \cdots & 0 & \cdots & Y_{GG} \end{bmatrix} + \begin{bmatrix} E_1 \\ E_2^- \\ \vdots \\ E_j^- \\ \vdots \\ E_G^- \end{bmatrix}$$

In this extraction matrix A^*, we keep the diagonal block matrix and the column corresponding to inputs exported to country 1 (a difference with the extraction matrix used by other authors). The notation E_j^- refers to country *j*'s exports to all countries except country 1. In this equation, the extraction matrix fully accounts for the propagation of output (including for domestic use) and of exports in the ICIO.

The extraction matrix described by Timmer et al. (2016) and Johnson (2017) just removes intermediate inputs from country 1 in the production of country 2, since they assume only two countries. It is expressed as $\begin{pmatrix} A_{11} & \mathbf{0} \\ A_{21} & A_{22} \end{pmatrix}$. Borin and Mancini (2017) extend the extraction matrix to an arbitrary number of countries by setting to zero the coefficients that identify the requirement of inputs imported from country *s* within the input matrix. Their extraction matrix can be expressed as $A^{g} = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1s} & \cdots & A_{1G} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & A_{ss} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ A_{G1} & A_{G2} & \cdots & A_{Gs} & \cdots & A_{GG} \end{bmatrix}$. This expression should also correspond to the many-country case in an extension of Timmer et al. (2016). However, we believe that there are two issues with this type of extraction. First, because of the intermediate inputs blocks from one country to another (e.g. from country *j* to country *k*), it is not consistent with the Ghosh insight decomposition. Second, it is not consistent with the measurement of global GDP in the context of exports ICIO tables as explained in Section 2 of our paper. This is why we start from a different extraction matrix, A^*.

We then re-arrange the equation to calculate the gross output required to produce country 1's exports. We pre-multiply by the value-added ratios and obtain the value-added embodied in country 1's exports:

$$\begin{bmatrix} VaE_1 \\ VaE_{21} \\ \vdots \\ VaE_{j1} \\ \vdots \\ VaE_{G1} \end{bmatrix} = \hat{V}^{-1}\left[I - A^*\right]^{-1} \begin{bmatrix} E_1 \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} V_1(I - A_{11})E_1 \\ V_2(I - A_{22})^{-1} A_{21}(I - A_{11})^{-1} E_1 \\ \vdots \\ V_j(I - A_{jj})^{-1} A_{j1}(I - A_{11})^{-1} E_1 \\ \vdots \\ V_G(I - A_{GG})^{-1} A_{G1}(I - A_{11})^{-1} E_1 \end{bmatrix}$$

where VaE_{j1} is the value-added from country *j* embodied in exports of country 1. In the two-country case, we have $E_2^- = 0$ and our extraction method is fully equal to the framework of Johnson (2017). This result is not only consistent with the expression introduced in our paper but also an extension to many countries of the foreign value-added expression proposed by Johnson (2017).