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Strict Duality and Overlapping Productivity Distributions between Formal and Informal Firms *

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Abstract

This paper develops a multi-industry general equilibrium model where entrepreneurs within each industry can decide to operate formally or informally. The model generates a rich set of predictions including productivity cut-offs for formal and informal firms to operate within different industries. In doing so, it matches empirical research that finds an overlap in the aggregate productivity distributions of formal and informal firms, while being consistent with theoretical predictions of strict duality within industries. Our explanation for this outcome is that it is natural result of fixed costs varying across industries. We offer evidence that the overlap between formal and informal firms in the aggregate is larger than the overlaps within industries for the case of Indian manufacturing establishments. Our model is also consistent with other features of the data in that it can explain high levels of competition between formal and informal firms that decrease with formal firm size.

Keywords: Informality; Competition; Dual View; Productivity

JEL Classification Numbers: O17, E26

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1 Introduction

Informal firms are a ubiquitous feature in developing countries and are an important step in the process of development. However, despite their prominence within developing countries, their relationship to and interactions with the formal sector remain an area of active inquiry. A concrete understanding of how the formal and informal sectors interact is of first-order importance to policy makers and informs questions ranging from optimal taxation to the effects of trade liberalization.

Advances in the surveys of informal firms have led to a wealth of informative micro-level data. This data has helped bring some clarity to the nature of informality, but it has also led to new questions related to the interactions between formal and informal firms. Prominent among these questions was the seeming contradiction between the theoretical literature and empirical facts. The canonical theoretical model developed by Rauch (1991) explains which entrepreneurs decide to operate informal firms. A central result of the model is a strict size and productivity dualism. The smallest (least productive) formal firm is still larger (more productive) than the largest (most productive) informal firm. However, empirical research shows that there is a clear overlap between formal and informal firms in productivity and/or firm size distributions.[1]

The principal contribution of this paper is to model an alternative explanation for the overlapping productivity distributions at the aggregate level. We show that this overlap is the natural result of the relative entry costs of different industries. Importantly, our formulation also allows for a version of strict duality *within* industry, consistent with theoretical predictions such as Rauch (1991). Our explanation is

[1] Examples include Taymaz (2009), Hsieh and Klenow (2010), Nataraj (2011), Busso et al. (2012) Meghir et al. (2015), and Ulyssea (2017).

broadly consistent with two salient features of the aggregate data: (1) on average lower productivity firms tend to be informal and (2) high productivity informal firms in some industries will be more productive than low productivity formal firms in others. One central prediction of the model is that there should be greater overlap between formal and informal productivity distributions in the aggregate than within industries. We test this prediction using Indian manufacturing data and show that there is indeed greater overlap in the aggregate distributions than in all but a handful of very small industries.

Our work builds on two recent studies that make significant contributions in explaining the overlap in productivity distributions. Meghir et al. (2015) addresses this empirical fact that we observe both formal and informal firms with the same productivity through formal and informal labor markets. In equilibrium firms may be equally profitable being formal or informal by tailoring their wage offers to different institutional frameworks. At certain productivity levels, firms may be indifferent between hiring employees from formal or informal labor markets; thus we observe both formal and informal firms with the same productivity. Alternatively, Ulyssea (2017) shows that even within industries, there is a productivity overlap generated by the entry of firms who do not know their *ex ante* productivity levels. Moreover, his model draws policy conclusions by modeling both the status of the firm (formal vs. informal) and the amount of labor hired from informal labor markets.

Our model builds on these works along several margins. First, we embrace a multi-industry framework in order to evaluate how entry costs into different industries and sectors (i.e. formal vs. informal) influence productivity distributions and competition. Second, our model more closely follows the framework of Melitz (2003), where firms make the typical decisions about entry, production, and pricing, but also must choose whether to operate formally or informally. We abstract away from the dual margins in Ulyssea (2017), instead creating strict productivity cut-offs for each sector within each industry. These two elements generate across industry overlaps while predicting strict duality within sectors. Third, we pay particular attention to the within-industry productivity distributions and show that there is a smaller degree of overlap within industries than in the aggregate productivity distributions for formal and informal firms.

The intuition of the model is straightforward. Suppose there are two industries H and L that have the same additional cost of being formal (i.e. the fee that must be paid to register with the government), but industry H has a slightly higher fixed cost of production. In this example, it would be less expensive for firms to enter both the informal and formal sectors in industry L (relative to the formal and informal sectors in industry H), and therefore the cut-off productivities for each sector in industry L would be lower as well. However, because the fixed costs of production do not differ greatly, the ordering of the cut-offs would most likely be: Informal entry into L < Informal entry into H < Formal entry into L < Formal entry into H. These cut-offs create a range of productivities where firms would decide to be formal in the low-cost industry, but would not be productive enough to operate formally in the high-cost industry. Therefore firms in this range of productivities between the last two cut-offs will be formal in industry L but informal in industry H leading to an observed overlap in the aggregate productivity distributions for formal and informal firms.

This research also contributes by looking at within-industry levels of competition. We motivate our model using two stylized facts that highlight key features of informal firms. First, we document that over half of the formal firms in our data report competing against informal firms. This fact suggests that formal and informal firms often inhabit the same economic space rather than producing in and serving distinct markets. This high level of competition is relatively consistent across different measurements, geographic regions, and time. Second, the level of competition is lower for larger, more productive firms. Together, these facts suggest a nuanced view of across sector competition that we explore in our model.

The broader literature on informality provides a helpful context for this work. It generally falls into three categories. On one end of the spectrum, De Soto (1989, 2000) argues that the informal sector exists due to restrictive institutional constraints. In his view, institutional reforms would unleash the creativity and entrepreneurial spirit of the informal sector. Other authors argue that informality is simply a profit-driven decision, and informal entrepreneurs operate informally to gain a competitive advantage by avoiding costly regulations and taxation. (Farrell, 2004; Levy, 2008)

Tracing back to Lewis (1954), the dual view suggests that informal firms exist in a separate economic space, distinct from formal firms. In his verbiage, there exists a "subsistence" sector (informal sector) and a more productive "capitalist" sector (formal sector). Firms in the capitalist sector do not truly compete with subsistence firms due to their superior productivity. His work suggests that informality exists as a stage in the process of development, and, importantly, dissipates as countries develop. This branch of the literature includes seminal theoretical contributions such as Harris and Todaro (1970) and Rauch (1991). These theoretical models have been largely substantiated by empirical investigations done by La Porta and Shleifer (2008, 2014). For instance, they find that 91% of formal, or registered, firms began that way (although *entrepreneurs* could have started firms in different sectors). There is also evidence that suggests that informal firms may produce different goods than formal firms. La Porta and Shleifer (2008) find considerable differences in value added between the formal and informal sector, suggesting that formal firms produce higher value goods than informal firms.

This research is not intended to be a test between these viewpoints. Important work by Ulyssea (2017) shows that there are firms that fit each of these "opposing" viewpoints. Instead, we see our work as providing additional understanding about how firms compete within industries, and how we should understand firm productivity across sectors. In summary, this paper reaches several important conclusions that help frame how we should view informality. First, within a given industry, the largest, most productive firms will be formal. Second, across industries, there will be some informal firms that are more productive than formal firms. Third, informal and formal firms compete, with the degree of competition decreasing as a function of formal firms' productivity.

This paper is divided into six sections. Section 2 presents our empirical motivation. Section 3 develops our model of the macroeconomy and Section 4 presents our main theoretical results. Section 5 examines a central prediction of the model using data on Indian manufacturing establishments. Finally, Section 6 concludes.

2 Empirical Motivation

Our empirical motivation is divided into three sections. The first section documents and describes our data sources. The second illustrates the high level of reported competition between formal and informal firms and explores the degree to which this finding is robust to different measures, survey years, and regions. The final section investigates the relationship between competition, firm size (a proxy for productivity), and development.

2.1 The Data

We use survey data from the World Bank's Enterprise Surveys to motivate our theoretical model.[2] These surveys are a stratified random sampling of manufacturing and retail firms with five or more employees in the formal sector. We supplement this survey data with macroeconomic indicators from the World Bank and the United Nations Development Programme.

Our vintage of the Standardized Data contains countries surveyed between 2006 and mid-2016.[3] In total, the data spans 140 different countries and over 124,000 firms. We use four main survey questions to capture competition and proxy for firm productivity. The first measure of competition is a binary indicator for whether a formal firm competes with informal firms. A second related measure asks formal firms the degree to which informal competition is an obstacle to their operations. We refer to these measures as our binary and categorical measures of competition, respectively. Finally, we use the reported number of *permanent* full-time employees and the number of full-time employees (permanent plus temporary) to condition our findings on firm size and as a proxy for firm productivity.[4] When appropriate, we utilize the Enterprise Survey's probability weights so that individual firms can be properly weighted to reconstruct an approximation of the universe of manufacturing and retail firms in a given country.[5]

[2] This is one of the same data sources used by La Porta and Shleifer (2014).

[3] Our data is the Standardized Data set released August 1st, 2016.

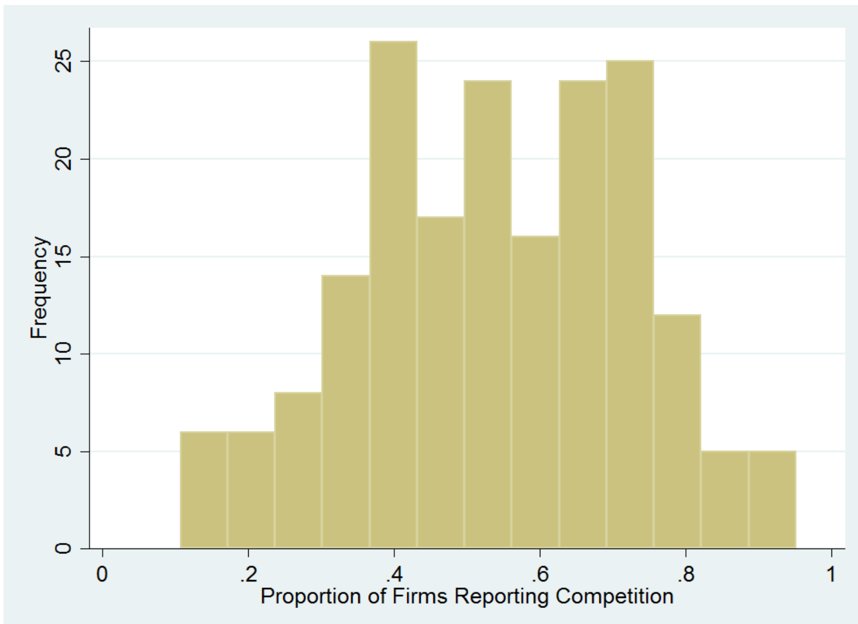
[4] Summary statistics for these variables can be found in Appendix A.

[5] A more in-depth discussion of the appropriate use of these weights can be found in the implementation notes for each country-year survey. These notes can be found online at http://www.enterprisesurveys.org.

2.2 Competition

Stylized Fact 1. *A significant proportion of formal sector firms compete with informal firms in developing countries.*

Figure 1: Country-Level Means



The unit of observation is a country-year. All country means utilize the probability weights provided by the World Bank Enterprise Survey. We drop all firms that did not answer either "yes" or "no" to the survey question.

Nearly 51% of all frims in the data report that they compete with informal firms.[6] Figure 1 shows the proportion of firms that compete with informal firms for each of the countries surveyed. While there is a broad range of competition, all surveyed countries exhibit competition between formal and informal firms. We perform a simple cross-country regression exercise to investigate important drivers of

[6] Even if *all* non-responses and typos are included as facing no competition from informal firms, about 43% of all firms say they compete.

informality. It indicates that that competition is remarkably stable across regions and time. Appendix B contains a complete regression table documenting the statistically insignificant role of time and most region dummies. On average, countries in the Americas tend to have slightly higher levels of competition, while countries in Asia tend to have slightly lower levels of competition.

The degree of competition is not sensitive to the measure of competition. An analysis of the categorical measure of competition yields similar conclusions. Across countries and time, the average ranking is 1.33, meaning that the average formal sector firm surveyed sees informal competition as being a minor to moderate obstacle. Roughly 22% of surveyed firms reported that informal firms were a “very severe” or “major” obstacle to their operations.[7]

Between these two measures of competition, we prefer the binary measure. First, the categorical measure of competition is highly subjective. “Minor” may mean something very different for different firms. Second, the binary nature of the first measure allows for an easy interpretation of the country-level mean: the percentage of firms that compete with informal firms. This result is of importance because of the link it provides to productivity. Subject to the necessary caveats with respect to survey data and measuring competition from only a formal perspective, we find the evidence quite compelling that formal and informal firms tend to compete in a meaningful way.[8] In the broader debate about informality, our data has clear parallels in Maloney (2004) which suggests there is a larger degree of integration between the formal and informal sectors than is suggested by a strict interpretation of the dual view.

[7] This statistic excludes non-responses. It is likely an underestimate of informal competition since small firms are underrepresented and are more likely to cite informal firms as significant obstacles.

[8] Making inferences about informal firms based on survey data collected from formal sector firms is not uncommon. See Dabla-Norris et al. (2008) for a typical example.

2.3 Firm Size and Competition

Stylized Fact 2. *There is an inverse relationship between firm size and whether a formal firm competes with informal firms.*

The Enterprise Survey data also offers evidence to suggest that smaller formal firms are more likely to compete with informal firms. This result's importance is magnified by the link between firm size and a firm's productivity. Many theoretical papers assume a direct link between firm size and productivity. This is the case for instance in Melitz (2003). Empirical work has also tended to support this conclusion.[9] Throughout the remainder of the paper, we take the positive relationship between firm size and productivity as given and often use them interchangeably.

Pooling the data across countries and survey years and regressing our binary competition measure on firm size shows that firm size is negatively related to whether a formal firm reports competing with informal firms. Table 1 reports the weighted regression results across several different specifications. The linear probability model in column (3) suggests that increasing firm size by 1% would decrease the probability that a firm competes with informal firms by .41%. We prefer this specification due to its straightforward interpretation. The negative relationship is seen in our other less involved specifications (columns (1) and (2)) which allow for a larger sample. Columns (4) and (5) estimate our preferred specification using logit and probit regressions, respectively, to account for the well known deficiencies of hypothesis testing with the linear probability model. Finally, these results are also robust to our more expansive definition of labor: permanent and temporary full-time workers. Those results are documented in Table 4 in Appendix C.

In both of our regression exercises, the cross-country regressions and the pooled

[9] For an example pertaining to the developing world see Söderbom and Teal (2004).

Table 1: Competition and Firm Size: Pooled Regressions

Variable	(1) OLS	(2) OLS	(3) OLS	(4) Logit	(5) Probit
Log firm size	-0.0298***	-0.0263***	-0.0386***	-0.1637***	-0.1010***
	(0.0052)	(0.0056)	(0.0059)	(0.0254)	(0.0156)
Log GDP/capita		-0.0396***	-0.0516***	-0.2183***	-0.1352***
		(0.0056)	(0.0064)	(0.0277)	(0.0170)
Industry fixed effects	no	no	yes	yes	yes
Region fixed effects	no	no	yes	yes	yes
Time fixed effects	no	no	yes	yes	yes
N	103,747	88,783	82,587	82,587	82,587
Conditional Means					
Small (< 20)	55.6%	56.2%	55.5%		
Medium (20-99)	49.2%	49.7%	49.4%		
Large (100+)	42.4%	42.9%	42.7%		

The dependent variable is a binary measure of whether a firm competes with informal firms. Firm size is measured as the number of permanent full-time employees. All specifications utilize probability weighting. Regional dummies are Africa, Americas, Asia, Europe, or Oceania. Sector fixed effects are based on two-digit ISIC codes. We drop industries that cannot be estimated with a logit/probit to maintain the same sample across specifications (3), (4), and (5). Standard errors are provided in parenthesis. Asterisks denote significance at the 1% (***), 5% (**) and 10% (*) levels.

firm-level regressions, show a negative relationship between income per capita and informality. We are wary of any causal interpretation of this result as there are a host of potential channels and/or mitigating factors that we cannot account for with our data. The observed relationship is important because it illustrates that the data we use to derive our stylized facts is consistent with the data used by others to substantiate the dual view of informality, particularly La Porta and Shleifer (2008, 2014).

Our review of the data leaves us with two motivating facts that must be reflected in our theoretical model. First, the data is clear that at least some formal sector firms face competition from informal firms. In truth, a large percentage of formal sector firms likely face this type of competition. Second, the likelihood that a firm faces informal competition decreases with its size (productivity).

3 Model

Our theoretical model builds on the framework developed by Melitz (2003), with two significant differences. First, we add an industry layer to the economy. Second, we embed the choice for firms to be informal or formal. We refer to this decision as a firm's sectoral choice. These two additions generalize the work of Melitz (2003) and build the necessary framework to examine whether and how formal and informal firms compete.

3.1 Model Set-Up

3.1.1 Households

Suppose there is a representative household that is endowed with income, Y. The household seeks to maximize its consumption, c, which is a Cobb-Douglas composite of goods produced in the $\mathscr{I}$ industries of the economy. Therefore, the household's problem can be written as:

$$\begin{aligned} \underset{C_i}{\text{maximize}} \quad & c = \sum_{i=1}^{\mathscr{I}} \eta_i log(C_i) \\ \text{subject to} \quad & \sum_{i=1}^{\mathscr{I}} P_i C_i = Y, \end{aligned}$$

where C_i, P_i, and η_i represent aggregate consumption, the price level, and preferences for goods in industry i, respectively. We assume that industry preferences (η_i) sum to one across all industries. The standard solution to the household's consumption problem is

$$P_i C_i = \eta_i Y = R_i, \; i \in \{1, \mathscr{I}\}, \tag{1}$$

where R_i is the total revenue of firms in industry i. Within each industry there is a mass of firms that produce distinct varieties, $\omega \in \Omega_i$. Industry production of the consumption good, C_i, and prices, P_i, are CES aggregates of the varieties produced in industry i:

$$C_i = \left(\int_{\omega \in \Omega_i} \xi_i(\omega) C_i(\omega)^{\epsilon} d\omega \right)^{\frac{1}{\epsilon}} \tag{2}$$

$$P_i = \left(\int_{\omega \in \Omega_i} p_i(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}}, \tag{3}$$

where $p_i(\omega)$ is the price of variety ω and $\xi_i(\omega)$ is a preference parameter for formal and informal goods. We assume that individual's have lower preference for informal goods, such that:

$$\xi_i = \begin{bmatrix} 1 & \text{if formal} \\ \xi_i \leq 1 & \text{if informal} \end{bmatrix}.$$

Varieties are substitutes for each other such that $0 < \epsilon < 1$ and the elasticity of substitution between varieties is $\sigma = \frac{1}{1-\epsilon} > 1$.

Using equations (1) and (2) it can be shown that the demand for each variety is given by

$$C_i(\omega) = C_i \left(\frac{P_i \xi_i(\omega)}{p_i(\omega)} \right)^{\sigma}. \tag{4}$$

3.1.2 Government

Government in the model plays two roles. First, it collects tax revenue from firms in the formal sector. Each formal sector firm is taxed τ percent of their profits. Second, the government locates and fines informal firms at a rate μ. Informal firms that are caught forfeit the entirety of their profits. The government's total revenue, T, is described by

$$T = \sum_{i=1}^{\mathscr{I}} \left[\tau \int_{\omega \in F_i} \pi(\omega) d\omega + \mu \int_{\omega \in I_i} \pi(\omega) d\omega \right], \tag{5}$$

where $\pi(\omega)$ is the profitability of the firm producing good ω and F_i and I_i refer to set of firms operating in the formal and informal sectors in industry i, respectively.[10] All revenue collected by the government is transferred back to the household as a lump-sum payment.

[10] We assume that all firms are either entirely formal or entirely informal. Possible extensions to the model may allow formal firms to hide part of their operation or allow informal firms to avoid detection by paying bribes. We abstract away from this complications to prioritize tractability of the model.

3.1.3 Aggregate Market Clearing

After accounting for taxes and fines, firms' net profits are transferred back to the household as a dividend:

$$D = \sum_{i=1}^{\mathscr{I}} (1-\tau) \int_{\omega \in F_i} \pi(\omega) d\omega + (1-\mu) \int_{\omega \in I_i} \pi(\omega) d\omega. \tag{6}$$

Finally, market clearing implies that:

$$Y = w^I L^I + w^F L^F + T + D = \sum_{i=1}^{\mathscr{I}} R_i. \tag{7}$$

Total wage labor, $w^I L^I + w^F L^F$, is determined by a nominal wage rate w^s that differs between formal and informal firms ($w^I < w^F$), and the labor supply $L = L^I + L^F$.[11] Total income Y, in the economy is equivalent to total firm revenue. Note that Equations (1) and (7) imply that the total revenue of each industry is fixed and is determined entirely by household income and preferences.

3.2 Firm Structure

3.2.1 Industry Entry

Each firm decides whether to enter industry i and produce its own variety ω. Entry into an industry requires the firm to pay a fixed cost f^e.[12] After a firm enters an industry and pays the associated fixed cost, our model parallels Melitz (2003). All firms receive a productivity draw ϕ from a common distribution $g(\phi)$. This distribution is accompanied by the usual assumptions, mainly that $g(\phi)$ has support

[11] Below we discuss the impact of formal and informal firms paying different wage rates.
[12] As in Melitz (2003) all fixed costs are written in terms of units of labor.

on the interval $(0, \infty)$, and we represent the continuous cumulative distribution for firm productivity as $G(\phi)$. Upon realizing its productivity, a firm may either decide to produce or immediately exit. In an effort to conserve notation, the industry indicator i will be used only when deemed necessary.

3.2.2 Sectoral Entry

Firms may decide to operate either formally or informally. Throughout the paper, firms operating informally are denoted with an I and firms operating formally with an F. Superscripts are used to denote variables that are unique to the formal or informal sector. Both types of firms *can choose* to operate in each industry i and enjoy market power by producing a unique variety, ω. Within an industry, firms compete via monopolistic competition.

It is worth emphasizing how our assumptions relate to our research question. Of central interest is how and whether formal and informal firms compete. Assuming that all firms are monopolistically competitive does not *ex ante* assume the answer to this question. In fact, our model is general enough to support a variety of views of the informal economy. In particular, it could underscore the view that informal firms operate in their own economic space. In this way, informal firms would produce goods in the first n industries such that $i \in \{1, n\}$ and formal firms produce goods in the remaining industries $i \in \{n+1, \mathscr{I}\}$. In equilibrium both types of firms would exist; however, formal and informal firms would not compete within industries.

3.2.3 Production

Producing firms have access to an increasing returns production function that is different across industries and depends on whether a firm is formal or informal. As a result, the firm's labor demand function is:

$$l_i^s = f_i^s + \frac{C_i(\omega)}{\phi} \text{ for } s \in \{I, F\}, \tag{8}$$

where s denotes the firm's sectoral choice (informal or formal). Additionally, we assume the following inequalities hold:

$$f_i^I \leq f_i^F, f_{i-1}^s \leq f_i^s, \frac{f_i^F}{f_i^I} \leq \frac{f_{i-1}^F}{f_{i-1}^I}.$$

The first inequality says that formal firms face higher costs than informal firms regardless of industry. This assumption is rather intuitive as formal firms face higher costs due to registration fees and other government regulation. The second inequality orders the industries in terms of increasing levels of cost, this is done without loss of generality in order to compare outcomes across industries. Finally, the third inequality says that the ratio of formal and informal costs are falling as overall firm costs increase. This follows from the fact that administrative costs for formal firms become an ever decreasing share of production costs as industries become more complex. For instance, the higher costs associated with operating in the manufacturing industry would mean that administration costs represent a smaller share of total costs when compared to the retail industry. Additionally, we assume that the substitutability between formal and informal firms is falling in industry complexity ($\xi_{i-1} \geq \xi_i$). The thought behind this is that as fixed costs increase, so does the complexity of the goods that those firms produce. Therefore, consumers are more discriminating over how their goods are made. For example, a person may care very little whether a pineapple is from a grocery store or a fruit vendor, but they probably do care if their laptop was made by Apple or by someone in their basement. Because all firms face the same residual demand curve, they choose a price equal to a constant mark-up over marginal cost:

$$p(\phi) = \frac{w^s}{\epsilon\phi}. \tag{9}$$

We proceed by assuming that there is a minimum wage (normalized to 1) that all formal firms must pay, while informal firms can pay a wage $w^I < 1$ (the informal wage is shared across industries due to the free movement of labor).

The labor demand function in Equation (8) along with the pricing rule in Equation (9), imply that the firm's profits and revenues depend both upon their productivity as well as their sector s. The expression for profits is given by

$$\pi_i^s(\phi) = \frac{R_i}{\sigma} w^s (P_i \epsilon\phi)^{\sigma-1} \left(\frac{\xi_i^s}{w^s}\right)^{\sigma} - w^s f_i^s \text{ for } s \in \{I, F\}. \tag{10}$$

$$r_i(\phi) = R_i w^s (P_i \epsilon\phi)^{\sigma-1} \left(\frac{\xi_i^s}{w^s}\right)^{\sigma} \tag{11}$$

3.2.4 Cut-offs

There is a large pool of possible entrants into each industry. Firms pay a fixed cost in order to receive a productivity draw from the cumulative distribution $G(\phi)$.[13] Once the firm receives their draw, they immediately decide whether or not to exit. Firms will exit if their profits are negative and because firm profits are increasing in the firm's productivity, there exists a productivity (ϕ^*) below which all firms exit. This implies that the distribution of productivities among operating firms is:

$$\gamma_i(\phi) = \left[\begin{array}{cc} \frac{g(\phi)}{1-G(\phi_i^*)} & \text{if } \phi \geq \phi_i^* \\ 0 & \text{otherwise} \end{array} \right. . \tag{12}$$

[13] The distribution is the same across sectors.

The firm's decision to enter and produce depends upon the profitability of production and the expected value of entry. As shown in Equation 10, a firm's profitability depends upon productivity as well as sector. This implies that the zero profit condition depends upon the structure of the industry.[14] In contrast, because paying the fixed cost of entry takes place prior to the realization of productivity and thus sectoral choice, there is a single free entry condition (FE), regardless of the industry structure.

It should be noted that the sectoral choice by the firm means that the relationship between average profits/revenues and productivity depends upon the distribution, $G(\cdot)$. It will prove to be mathematically convenient to express the zero profit condition (henceforth ZPC) in terms of average revenue, $\bar{r}$, rather than average profits. Additionally, because the ZPC is written down in terms of average revenue, the free entry condition must be as well.

Lemma 1. *The zero profit and free entry conditions can be written in terms of average revenue as a function of the minimum productivity level, ϕ^*. For an industry i where formal and informal firms coexist (a mixed industry is denoted $\mathcal{I}$) these conditions are:*

$$\bar{r} = \left(\frac{\tilde{\phi}(\phi^*)}{\phi^*}\right)^{\sigma-1} \sigma f^I \left(\frac{w^I}{\xi}\right)^{\sigma} \qquad \textit{(ZPC)} \tag{13}$$

where $\tilde{\phi}(\phi^*)$ is defined as:

$$\tilde{\phi}(\phi_i^*) = \left[\frac{1}{1-G(\phi_i^*)}\left(\int_{\bar{\phi}}^{\infty} \phi^{\sigma-1} g(\phi) d\phi + w^I \left(\frac{\xi}{w^I}\right)^{\sigma} \int_{\phi^*}^{\bar{\phi}} \phi^{\sigma-1} g(\phi) d\phi\right)\right]^{\frac{1}{\sigma-1}}. \tag{14}$$

[14] In an industry with both formal and informal firms, firms require lower revenue to be willing to produce as they have the option of paying the lower fixed costs of operation associated with informality.

$$\bar{r} = \frac{\sigma(f^e + (G(\bar{\phi}) - G(\phi^*))(1-\mu)f^I + (1-G(\bar{\phi}))(1-\tau)f^F)}{(G(\bar{\phi}) - G(\phi^*))w^I \left(\frac{\xi}{w^I}\right)^{\sigma}(1-\mu)\left[\frac{\tilde{\phi}^I}{\tilde{\phi}}\right]^{\sigma-1} + (1-G(\bar{\phi}))(1-\tau)\left[\frac{\tilde{\phi}^F}{\tilde{\phi}}\right]^{\sigma-1}} \qquad \textit{(FE)} \tag{15}$$

where $\bar{\phi}$ is the formality cut-off productivity (defined in Lemma 2), and $\tilde{\phi}^F$ and $\tilde{\phi}^I$ refer to the average productivity of formal and informal firms, respectively.

Proof. See appendix A. □

In an industry with only formal firms ($\mathcal{F}$) the ZPC instead depends on the fixed cost of operation for formal firms, f^F and $\bar{\phi} = \phi^*$. The minimum productivity cut-off, ϕ^*, is determined by the intersection of the free entry and zero profit conditions. However, without additional assumptions regarding the distribution of productivities it is not possible to determine where the intersection occurs, if it occurs at all. The next section will place additional structure on $G(\cdot)$ and derive the main results of the paper.

The second cut-off of interest is the formality cut-off which we designate $\bar{\phi}$. In particular, the formality cut-off within an industry occurs where $E\pi^I(\bar{\phi}) = \pi^F(\bar{\phi})$.

Lemma 2. *Suppose that within industry i there are both informal and formal firms. There exists a productivity level, $\bar{\phi}$, such that firms who draw a productivity level ϕ greater than $\bar{\phi}$ enter the formal sector. Moreover, this formality cut-off, $\bar{\phi}$, can be explicitly written as:*

$$\bar{\phi} = \hat{F}\phi^* \tag{16}$$

where

$$\hat{F} = \left[\frac{\left(\frac{\xi}{w^I}\right)^{\sigma}((1-\tau)f^F - (1-\mu)w^I f^I)}{((1-\tau) - (1-\mu)w^I\left(\frac{\xi}{w^I}\right)^{\sigma})f^I}\right]^{\frac{1}{\sigma-1}}.$$

In order for both informal and formal firms to exist in the same industry ($\hat{F} > 1$), *we require:*

$$w^I\left(\frac{f^I}{f^F}\right)^{1/\sigma} < \xi < w^I\left(\frac{1-\tau}{w^I(1-\mu)}\right)^{1/\sigma}$$

Before moving on it is should be noted that one additional constraint must be imposed in order for there to be both formal and informal firms in the economy:

$$\xi < (w^I)^{\frac{\sigma-1}{\sigma}}$$

which ensures that $r^F(\phi) > r^I(\phi) \ \ \forall\phi$.

3.2.5 Aggregation

Firm level prices and productivities can be aggregated in the same manner as Melitz (2003). The industry price given in Equation (3) can be written as:

$$P_i = \left[\frac{1}{1-G(\phi_i^*)}\int_{\phi_i^*}^{\infty} p(\phi)^{1-\sigma} M_i g(\phi) d\phi\right]^{\frac{1}{1-\sigma}}, \tag{17}$$

where M_i is the mass of firms operating in industry i. Using Equations (14) and (17) the industry specific aggregate price level, quantity produced, revenue, and profits can be written as:

$$P_i = M_i^{\frac{1}{1-\sigma}} p(\tilde{\phi}_i) \tag{18}$$

$$C_i = M_i^{\frac{1}{\epsilon}} C_i(\tilde{\phi}_i) \tag{19}$$

$$R_i = P_i C_i = M_i r_i(\tilde{\phi}_i) = M_i \bar{r}_i. \tag{20}$$

4 Results

The results in this section can be divided into two categories. The first pertain to results *within* industries. For clarity, we continue to omit the industry subscript and specify whether an industry is mixed or strictly formal. In order to clarify notation and avoid piece-wise definitions, equations default to a mixed industry ($\mathcal{I}$). We address any differences with a strictly formal industry ($\mathcal{F}$) in the text or footnotes. The second set of results pertains to comparisons *across* industries. Accordingly, we reintroduce the industry subscript, i, for those results.

4.1 Equilibrium

This section analytically derives the main results of the model using the Pareto distribution with minimum value k and shape parameter α. Appendix E includes numerical results that show that results from this section hold for a wide range of well-behaved distributions (log-normal, weibull, exponential). The first step in solving the model is to find the average productivity for the entire industry, as well as for informal and formal firms.

Lemma 3. *The average productivities of a mixed industry ($\tilde{\phi}$), and for formal ($\tilde{\phi}^F$) and informal ($\tilde{\phi}^I$) firms within that industry, are directly proportional to the cut-off value, ϕ^*.*[15] *They are given by*

- $\tilde{\phi} = \left(\frac{\alpha}{1+\alpha-\sigma}\right)^{\frac{1}{\sigma-1}} \left[\hat{F}_{\mathcal{I}}^{-1-\alpha+\sigma} + w^I \left(\frac{\xi}{w^I}\right)^{\sigma} \left(1 - \hat{F}_{\mathcal{I}}^{-1-\alpha+\sigma}\right)\right] \phi^*,$
- $\tilde{\phi}^I = \left[\left(1 - \hat{F}_{\mathcal{I}}^{\alpha}\right)^{-1} \left(1 - \hat{F}_{\mathcal{I}}^{-1-\alpha+\sigma}\right) \left(\frac{\alpha}{1+\alpha-\sigma}\right)\right]^{\frac{1}{\sigma-1}} \phi^*,$
- $\tilde{\phi}^F = \left(\frac{\alpha}{1+\alpha-\sigma}\right)^{\frac{1}{\sigma-1}} \hat{F}_{\mathcal{I}}\phi^*.$

[15] In order to ensure that the cut-offs are positive we assume that $\sigma < 1 + \alpha$

Proof. All proofs can be found in Appendix D. □

In an industry where all firms are formal, Lemma 3 collapses to its first point. From Lemma 3 we can determine how the zero profit condition and the free entry condition depend on the cut-off value ϕ^*.

Proposition 1. *Suppose that a given industry is mixed in that it has both formal and informal firms. If the underlying distribution of firm productivities is Pareto with minimum value k and shape parameter α, then the zero profit condition is horizontal with respect to ϕ^*, and it is given by:*

$$\bar{r} = \left(\frac{\alpha}{1+\alpha-\sigma}\right)\left[\hat{F_{\mathcal{I}}}^{-1-\alpha+\sigma} + w^I\left(\frac{\xi}{w^I}\right)^{\sigma}(1-\hat{F}_{\mathcal{I}}^{-1-\alpha+\sigma})\right]\sigma f^I\left(\frac{\xi}{w^I}\right)^{-\sigma}. \qquad \textit{(ZPC)} \tag{21}$$

The free entry condition is upward sloping with respect to ϕ^* and is given by:

$$\bar{r} = \left[\hat{F_{\mathcal{I}}}^{-1-\alpha+\sigma} + w^I\left(\frac{\xi}{w^I}\right)^{\sigma}(1-\hat{F}_{\mathcal{I}}^{-1-\alpha+\sigma})\right]\sigma\left[\left(\frac{\phi^*}{k}\right)^{\alpha}\left(\frac{f^e}{\bar{F}_{\mathcal{I}}}\right) + f^I\left(\frac{\xi}{w^I}\right)^{-\sigma}\right], \qquad \textit{(FE)} \tag{22}$$

where

$$\bar{F}_{\mathcal{I}} = \hat{F}_{\mathcal{I}}^{-1-\alpha+\sigma}(1-\tau) + w^I\left(\frac{\xi}{w^I}\right)^{\sigma}\left(1-\hat{F}_{\mathcal{I}}^{-1-\alpha+\sigma}\right)(1-\mu).$$

Proof. The proof is a direct application of Lemmas 1 and 3. □

Proposition 1 can be readily applied to industries that are strictly formal by setting all $f^I = f^F$ and letting $\bar{F}_{\mathcal{F}} = (1-\tau)$. We use Proposition 1 to calculate the cut-off value ϕ^* for a mixed industry:[16]

$$\phi^* = k \left[\frac{f^I \bar{F}_{\mathcal{I}}(\sigma - 1)}{f^e(1 + \alpha - \sigma)} \left(\frac{\xi}{w^I} \right)^{-\sigma} \right]^{\frac{1}{\alpha}}. \tag{23}$$

Note that in order to ensure that $\phi^* > k$, we assume that the fixed cost of entry is sufficiently low that

$$f^e < \frac{f^I \bar{F}_{\mathcal{I}}(\sigma - 1)}{1 + \alpha - \sigma} \left(\frac{\xi}{w^I} \right)^{-\sigma}.$$

Not surprisingly, the cut-off value is increasing in the fixed cost of production, as greater fixed costs require more productive firms to cover them.

There are two key questions that can be answered within the framework above. First, how does the equilibrium with both informal and formal firms within an industry differ from one with only formal firms? If formal firms are thought to be beneficial for the process of development, then it is important to understand what effect informality has on an industry. The second question deals with the empirical results found in the literature. Can this model demonstrate the size and productivity dualism that we would expect, while still showing the productivity overlap between formal and informal firms seen in the data? Starting with the first question, we proceed with Proposition 2.

Proposition 2. *Suppose that we have two industries that share the same fixed costs, revenues, and tax rates, but they differ in their probability of closing and fining informal firms. In the first industry $\mu = 1$ such that there are no informal firms ($\mathcal{F}$). In the second industry $\mu < 1$ such that there are both formal and informal firms*

[16] Other papers have shown that there is a overlap in productivities within an industry (see Ulyssea, 2017 for a recent example). Our model is consistent theoretical duality of Rauch (1991), however it would be possible to generate an overlap in productivities within an industry by adding some noise to the productivity draw. This would cause some firms at the margin to opt into the "wrong" sector.

within the industry ($\mathcal{I}$). Under such conditions, each of the following is true:

1. $\phi^*_{\mathcal{I}} < \phi^*_{\mathcal{F}} < \bar{\phi}_{\mathcal{I}}$
2. $\bar{r}_{\mathcal{I}} < \bar{r}_{\mathcal{F}}$
3. $M_{\mathcal{I}} > M_{\mathcal{F}}$
4. $M^F_{\mathcal{I}} < M^F_{\mathcal{F}}$

Proof. See Appendix D.[17] □

These results describe the main effects of informality. First, Propostion 2.1 shows that informality allows lower productivity firms to enter into the market. This is a result of the lower fixed costs associated with being informal. Productivity draws that previously could not afford to enter the market, now can. However, the effect of having lower productivity firms enter the market is that the cut-off to become formal increases. Firms need a higher productivity draw to enter into the formal sector. Proposition 2.2 reflects that with lower average productivity and greater competition, average revenue falls due to informality. Proposition 2.3 ties the greater range of entering firms and lower average productivity to a larger mass of firms in the mixed industry. Finally Proposition 2.4, is driven by the decrease in average revenue. Lower average revenue makes it more difficult to cover the fixed costs of the formal sector, which in turn shrinks the formal sector relative to the industry with no informal competition. It is important to note that the results found in Proposition 2 are consistent with Rauch (1991) as there is a distinct cut-off between the formal and informal sectors within an industry.

Another empirical result from the survey data is that smaller formal firms indicate that they face more competition from informal firms. We can measure informal

[17] Note: $\phi^*_x, \bar{\phi}_x, \bar{r}_x, M_x, and\ M^F_x$ stand for the productivity cut-off for informal firms, productivity cut-off for formal firms, average revenue, total mass of firms, and the mass of firms that are formal, respectively. x indicates whether the economy is formal only ($\mathcal{F}$) or mixed ($\mathcal{I}$).

competition indirectly by comparing the profits that formal firms make when they face informal competition to those that they make when there is only formal competition. This measure gives us an idea of how much profit firms "lose" as a result of informal competition. Explicitly, for a given productivity ϕ, the percentage of profit lost (PPL) due to informal competition can be calculated as:

$$PPL(\phi) = \frac{\pi_{\mathcal{F}}^{F}(\phi) - \pi_{\mathcal{I}}^{F}(\phi)}{\pi_{\mathcal{F}}^{F}(\phi)} \tag{24}$$

which brings us to our next proposition.

Proposition 3. *The percentage of profit lost due to informality is declining in firm productivity.*

Proof. See Appendix D. □

Proposition 3 states that more productive (larger) firms lose a smaller share of their profits as a result of informal competition. This is consistent with the data in that larger (more productive) firms are less likely to compete against informal firms. The reason for this result is that larger more productive firms charge lower prices than their competition. When smaller, less productive informal firms enter the market they charge high prices posing little competitive threat to their more productive formal counterparts. However, those firms on the margin between formal and informal face greater competition because their prices are more similar.

Several studies have documented an overlap in the size and productivity distributions for formal and informal firms. Based on the model, it is clear that *within* industry there will be no overlap between formal and informal firms. This result echos the seminal theoretical contribution of Rauch (1991). However, it is highly likely that there will be overlapping productivity distributions *across* industries, which brings us to our final proposition.

Proposition 4. *Assume that industries are ordered from lowest fixed costs of operation to highest, such that $f^s_{i-1} < f^s_i, \forall i$ and $s \in \{I, F\}$. Under such conditions, the following are true:*

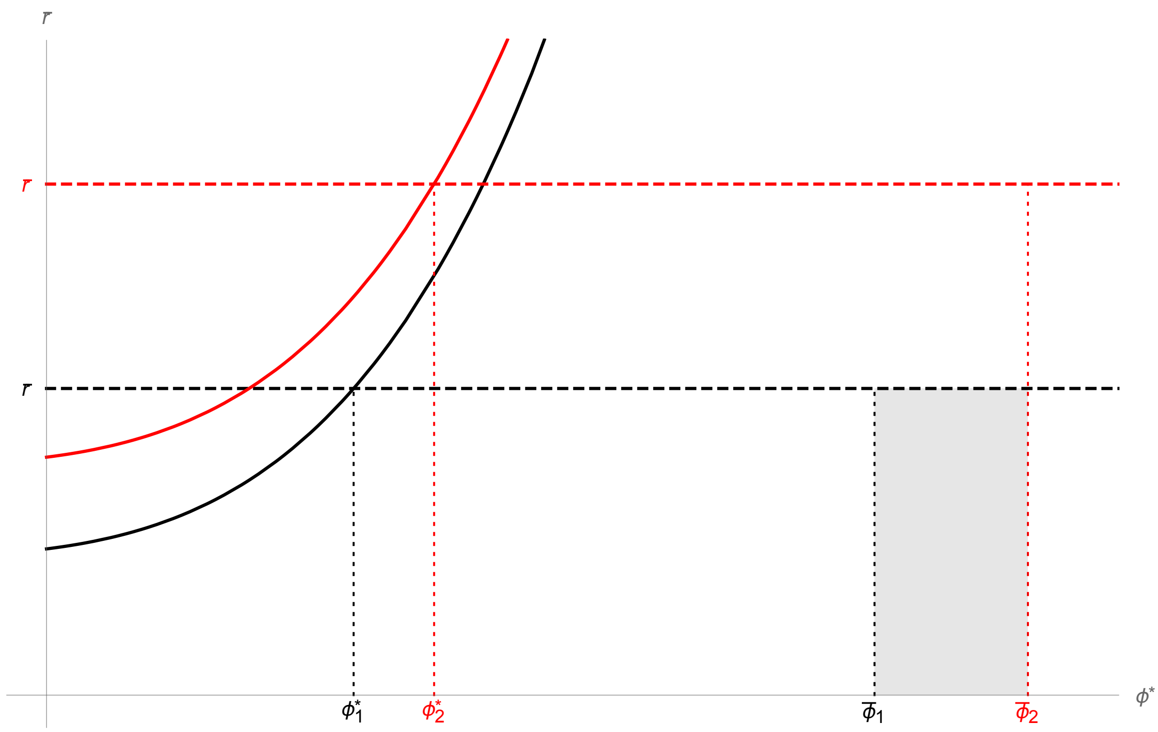
1. $\phi^*_{i-1} < \phi^*_i$
2. $\bar{r}_{i-1} < \bar{r}_i$
3. $\frac{M^F_{i-1}}{M_{i-1}} < \frac{M^F_i}{M_i}$
4. *If* $\left(\frac{f^I_i}{f^I_j}\frac{\bar{F}_i}{\bar{F}_j}\right)^{\frac{1}{\alpha}} \neq \frac{\hat{F}_j}{\hat{F}_i}$ *for some* $\{i, j\}$*, then there is an overlap between the informal and formal sectors across industries.*

Proof. See Appendix D. □

These results describe the effects of different fixed costs across industries. First, Proposition 4.1 shows that higher cost industries have higher productivity cut-off values. This is quite intuitive as higher costs necessitate higher productivities in order to be profitable. The higher cut-off values result in higher average productivities (see Equation 14), increasing the average revenue of firms (Proposition 4.2). Additionally, as industry costs increase they cause the ratio of formal to informal costs to fall, and the percentage of firms that are formal increases (Proposition 4.3). This result is driven by the ratio f^F/f^I. As it falls, the economic benefits of being informal diminish, resulting in a greater percentage of firms opting to be formal.

The final result in Proposition 4 deals with the overlap in firm productivities across industries. From above, we know that the formal cut-off, $\bar{\phi}$, for an industry is a function of the cut-off value, $\bar{\phi} = \hat{F}\phi^*$. As long as $\bar{\phi}_i \neq \bar{\phi}_j$, we will observe an overlap in productivity distributions across industries. In this case, the overlap would occur across the productivity range $\{\bar{\phi}_i, \bar{\phi}_j\}$.[18] Intuitively, this follows from the different costs of entry. Some industries will have lower cut-offs for formality than others, ultimately resulting in lower productivities for formal firms in those industries relative to informal firms in others.

Figure 2: Overlap of Informal and Formal Firms across Industries.



The figure shows the ZPC (horizontal line) and FE (upward sloping line) for two industries. The vertical axis is average industry revenue, and the horizontal axis measures productivity. The intersection of the two lines in each industry determine the informality productivity cut-off (ϕ_i^*). The formality cut-off is determined using ϕ_i^* and $\hat{F}_i$ for each industry. The range between the formality cut-offs in each industry is the overlap region (indicated by the grey box).

Figure 2 shows the overlap (grey shaded area) region that is described in Propo-

[18] For instance, assume that $\bar{\phi}_i < \bar{\phi}_j$. Industry i the firms with productivities in the range $\{\bar{\phi}_i, \bar{\phi}_j\}$ would be formal but they would be informal in industry j.

sition 4 for industries 1 and 2. It should be noted that Figure 2 is only one of three possible outcomes for the overlap region. In the figure, industry 2 not only has the higher entry cut-off, but also a higher formality cut-off. For firms with productivity in the range $\{\bar{\phi}_1, \bar{\phi}_2\}$, they will be capable of operating formally in industry 1, as they exceed the formality cut-off. At the same time, their productivity is too low to operate in industry 2, such that they will have to operate informally in industry 2. We find this to be the most intuitive outcome as high fixed cost industries would likely have higher fixed costs for entry in the formal sector. However, it is possible that for a given set of fixed costs that the formality cut-off in industry 2 can be less than the the cut-off in industry 1, this would still result in an overlap across the two industries.[19]

This model ultimately predicts that there is strict productivity and size duality within an industry, but an overlap across industries. It should be noted that while the strict duality is a strong result, it comes about because firms perfectly observe their productivity. If they observed their productivity with noise, firms around the formality cut-off would misallocate themselves creating an overlap within industries as well. Overall, even in a model with noise, we would expect that the aggregate overlap across industries would be greater than the overlap between informal and formal firms within an industry.

[19] It should be noted that if the equation in result 4 of Proposition 4 holds with equality, there will be no overlap and the formality cut-off will be the same across industries. This is a knife edge condition. In order for this model not to be consistent with the empirically observed overlap the equation: $\left(\frac{f_i^I}{f_j^I}\frac{\bar{F}_i}{\bar{F}_j}\right)^{\frac{1}{\alpha}} = \frac{\hat{F}_j}{\hat{F}_i}$ would have to hold for all $\{i, j\}$.

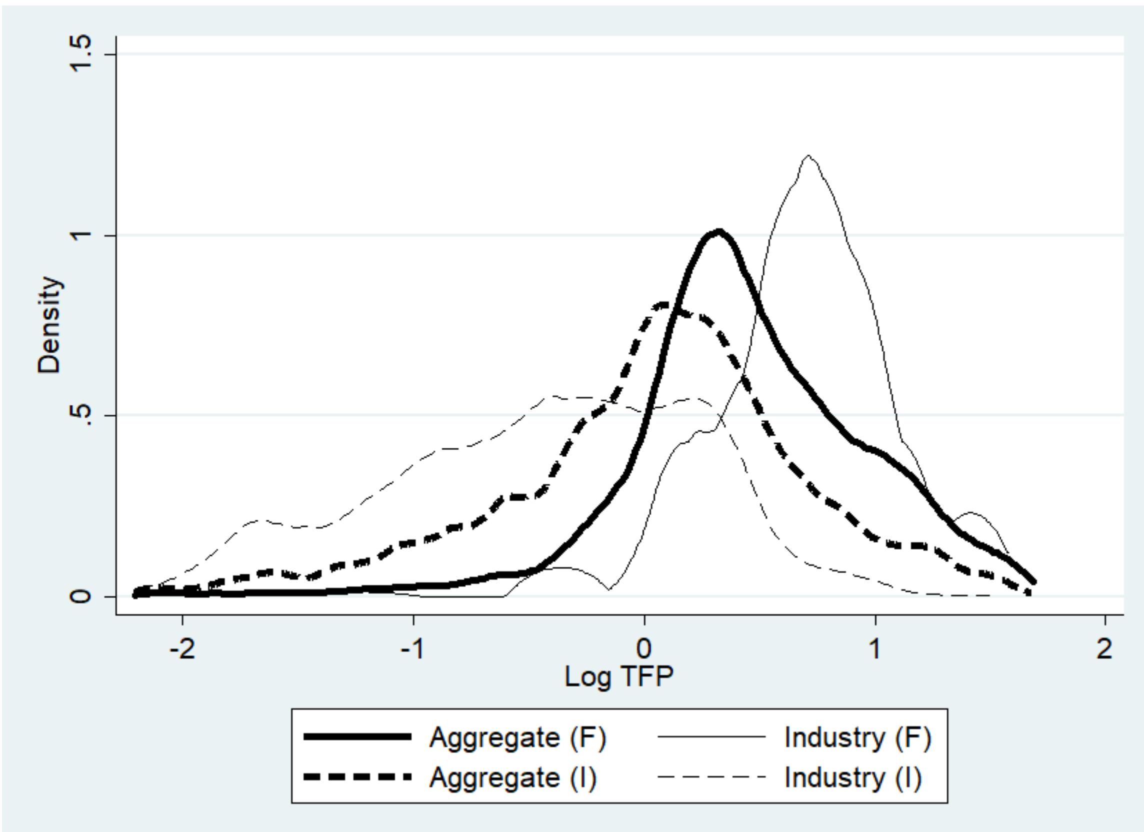
5 A Case Study of Indian Manufacturing Establishments

In this section, we test a central conclusion of our model: the degree of overlap between the aggregate distributions of productivity for informal and formal firms should be larger than the overlap within industries. While our model predicts that the latter should not overlap at all, i.e. strict duality, this result stems from the lack of uncertainty in the model. In reality, firms do not perfectly know their productivity. This naturally leads to overlaps even when theory predicts strict cut-offs. For instance, work that has looked at the productivity cut-offs for exporters based on Melitz (2003) have identify "fuzzy" cutoffs (Delgado et al. (2002), Cassiman et al. (2010), Girma et al. (2005)). Ulyssea (2017) documents that, even after controlling for variation in productivity across industries using a set of industry dummies, there is overlap in the residual productivity distributions of formal and informal firms. We take that analysis one step further and look at whether the *degree* of overlap within individual industries exceeds the overlap in the aggregate. Such a finding would be consistent with our theoretical model given uncertainty in the real world.

Figure 3 provides an example of our problem. It shows the aggregate productivity distributions for formal and informal firms (bold) relative to the formal and informal productivity distributions for the largest individual industry in our data (manufacture of boxes and crates) for the years 1999-2001. Visually, it appears that the industry-level overlap is substantially smaller than for the aggregate distributions. We test whether this pattern holds across industries in our data.

To capture the degree of overlap between the productivity distributions of the formal and informal sectors, we calculate two-sample Kolmogorov-Smirnov test statistics (henceforth K-S). This statistic captures the extent to which two distributions differ from each other. A larger K-S statistic implies a greater difference between

Figure 3: Aggregate vs. Industry-Level Overlaps



Log TFP is as calculated in Nataraj (2011). Both the aggregate and industry distributions are limited to 1999-2001 data. The plotted industry is the largest industry in our sample for 1999-2001 (manufacture of wooden boxes and crates).

the two distributions and a smaller degree of overlap. Let $F(\phi)$ and $I(\phi)$ represent the cumulative distribution functions (CDFs) of productivity for formal and informal firms respectively, either within an industry or in the aggregate. The K-S test statistic then is

$$D = max(D_{upper}, D_{lower}), \tag{25}$$

where

$$D_{upper} = |max_{\phi}\{F(\phi) - I(\phi)\}| \tag{26}$$

$$D_{lower} = |min_{\phi}\{F(\phi) - I(\phi)\}|. \tag{27}$$

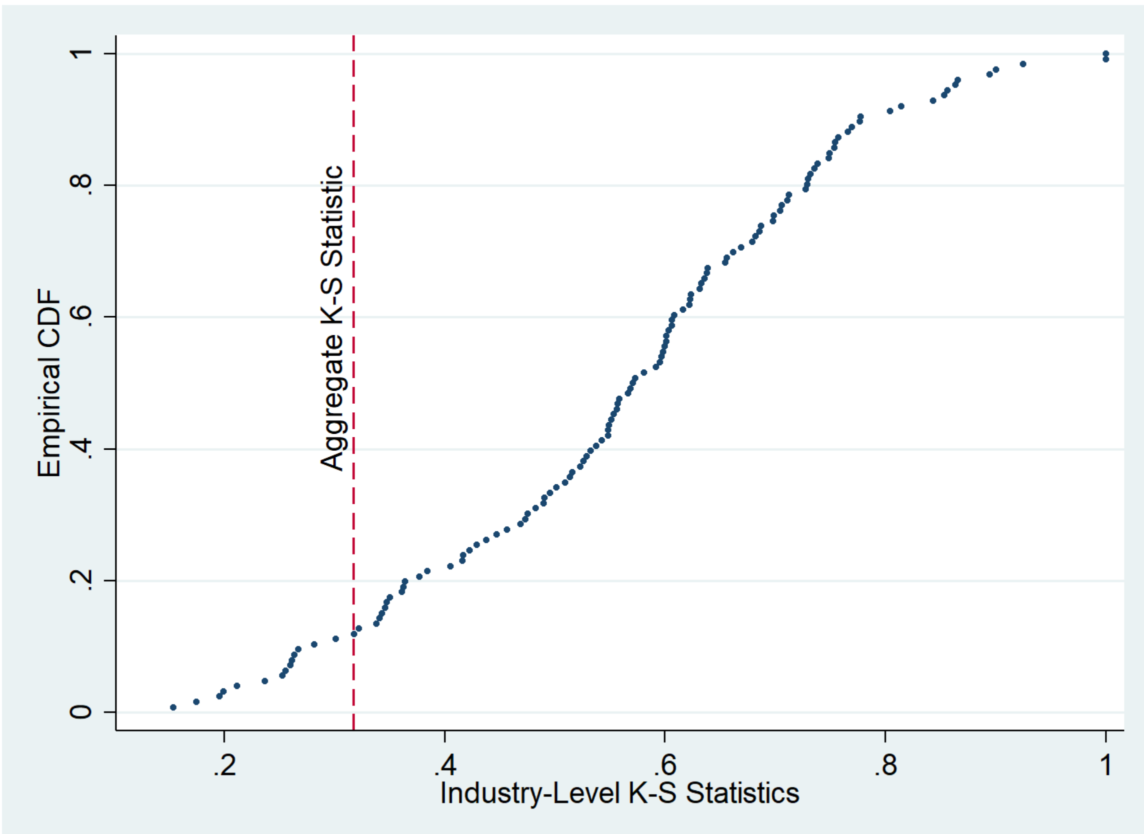
Intuitively, the K-S statistic captures the largest difference between the two CDFs. Since the informal sector tends to have a large number of relatively low productivity firms, the informal CDF is stochastically dominated by the formal CDF at low productivity levels, making D_{lower} the relevant metric in almost all industries.

To generate the empirical cumulative distribution functions, $F(\phi)$ and $I(\phi)$, we use productivity estimates for Indian manufacturing establishments from Nataraj (2011). Formal sector data come from the Survey of Industries (ASI) and informal sector data come from the National Sample Survey Organisation (NSS). Our sample contains over 282,000 establishments over three time periods: 1989-1990, 1994-1995, and 1999-2001, and covers firms in 139 different industries (by India's National Industrial Classification (NIC-87)).[20] It also contains sampling weights allowing us to scale our sample to reflect the productivity distributions for all manufacturing establishments in India.[21]

[20] For the final time period, formal firms are survey from 1999-2000 and informal firms are surveyed from 2000-2001.

[21] Firms with more than one establishment in a given state and industry may provide a combined return, but few do so.

Figure 4: Distribution of Industry-Level K-S Statistics



The horizontal axis plots the K-S statistic, D, for each industry in 1999-2001. The figure contains data on 126 manufacturing industries. The aggregate K-S statistic is 0.317.

We calculate the K-S statistic for each industry that includes both formal and informal establishments for every time period in our sample. Figure 4 shows the distribution of the industry-level K-S statistics relative to the aggregate K-S statistics.[22] Recall that the K-S statistic is a measure of the difference between two distributions. Hence, a smaller K-S statistic indicates a greater degree of overlap between the formal and informal productivity distributions. The figure clearly shows that there is a much smaller difference between the formal and informal productivity distributions in the aggregate (i.e. a larger overlap) as the aggregate K-S statistic is in the bottom tail of the distribution of K-S statistics for industries. The pattern presented in Figure 4 is robust to survey periods. Roughly 11% of industries have a smaller K-S statistic than the aggregate. However, the industries that tend to be below the aggregate are much smaller and are therefore estimated subject to greater error in estimating their productivities.[23] This evidence provides an empirical validation of our theoretical prediction that there should be greater overlap in the aggregate productivity distributions that within individual industries. It is consistent with strict duality within industries that is subject to firms determining their productivity through a noisy process.

6 Conclusion

The principle contribution of this paper is to offer an alterantive explanation for the seeming contradiction in the literature on informality. Even as the standard theoretical model of Rauch (1991) suggests strict size and productivity dualism, empirical work has clearly shown overlapping firm size and productivity distributions. Using

[22] This exercise is similar to Chetty et al. (2009) which plots placebo estimates relative to a reported treatment effect.

[23] Appendix F contains a similar plot where K-S statistics are scaled by industry size.

a multi-industry general equilibrium model that nests a sectoral choice decision, we show that this observation is a natural result of fixed costs that vary across industries. Within industries, formal sector firms will be larger and more productive than their informal counterparts. Across industries though, there will be informal firms that are more productive than some formal firms, reflecting the productivity overlap documented in the empirical literature. We test this central prediction of our model using data on Indian manufacturing establishments. We find convincing evidence that the overlap in aggregate productivity distributions between formal and informal firms is larger than the overlaps within most industries. While the overlaps still exists, they are consistent with the theory of strict duality subject to a noisy process through which firms determine their productivity.

By construction, our model reflects several key features from the World Bank Enterprise Surveys. The data indicate that at least half of formal sector firms report competing with informal firms and informal competition is even greater for smaller firms. Under very general parameters, industries in our model have both formal and informal firms; however, the existence of these informal firms does not necessarily imply that they "compete." We are able to show competition along two margins. First, we show that larger more productive formal firms lose fewer profits to informal firms. Undoubtedly, this reflects the data in that these firms are less likely to report competing with informal firms. Second, we show that industries with higher fixed costs will contain more productive firms and a smaller percentage of informal firms, again reflecting the negative relationship between firm size and competition.

We view this paper as an important contribution to the understanding of informality and the dual view. The tractability of our model, as a natural extension of Melitz (2003), lends itself to future investigation of other important dimensions of informality, in particular questions of welfare and impacts of trade policy. We have left these as avenues for future research as they likely require a dynamic version of our model in order to capture costs association with transitions between steady states. At the same time, a dynamic version of the model may obfuscate important facts about how formal and informal firms interact. Our model and stylized facts emphasize that formal and informal firms often compete and operate in the same markets. By better understanding how firms compete, we can better investigate the effect of policy on existing firms and the decision of new entrepreneurs to enter the formal or informal sectors.

Bibliography

Busso, M., Fazio, M. V., and Algazi, S. L. (2012). (In)formal and (un)productive: The productivity costs of excessive informality in Mexico. *IDB Working Paper Series*, IDB-WP-341.

Cassiman, B., Golovko, E., and Martínez-Ros, E. (2010). Innovation, exports, and productivity. *International Journal of Industrial Organization*, 28:372–476.

Chetty, R., Looney, A., and Kroft, K. (2009). Salience and taxation: Theory and evidence. *American Economic Review*, 99(4):1145–1177.

Dabla-Norris, E., Gradstein, M., and Inchauste, G. (2008). What causes firms to hide output? The determinants of informality. *Journal of Development Economics*, 85:1–27.

De Soto, H. (1989). *The Other Path: The Invisible Revolution in the Third World.* Harper and Row, New York.

De Soto, H. (2000). *The Mystery of Capital: Why Capitalism Triumphs in the West and Fails Everywhere Else.* Basic Books, New York.

Delgado, M. A., Fariñas, J. C., and Ruano, S. (2002). Firm productivity and export markets: A non-parametric apprach. *Journal of International Economics*, 57:397–422.

Farrell, D. (2004). The hidden dangers of the informal economy. *McKinsey Quarterly*, 3:27–37.

Girma, S., Kneller, R., and Pisu, M. (2005). Exports versus FDI: An empirical test. *Review of World Economics*, 141(2):193–218.

Harris, J. R. and Todaro, M. P. (1970). Migration, unemployment and development: A two-sector analysis. *American Economic Review*, 60(1):126–142.

Hsieh, C. and Klenow, P. J. (2010). Development accounting. *American Economic Journal: Macroeconomics*, 2(1):207–223.

La Porta, R. and Shleifer, A. (2008). The unofficial economy and economic development. *Brookings Papers on Economic Activity*, pages 275–352.

La Porta, R. and Shleifer, A. (2014). Informality and development. *Journal of Economic Perspectives*, 28(3):109–126.

Levy, S. (2008). *Good Intentionas, Bad Outcomes: Social Policy, Informality, and Economic Growth in Mexico*. Brookings Institution Press, Washington, D.C.

Lewis, A. W. (1954). Economic development with unlimited supply of labor. *Manchester School of Economic and Social Studies*, 22(2):139–191.

Maloney, W. F. (2004). Informality revisited. *World Development*, 32(7):1159–1178.

Meghir, C., Narita, R., and Robin, J.-M. (2015). Wages and informality in developing countries. *American Economic Review*, 105(4):1509–1546.

Melitz, M. J. (2003). The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica*, 71(6):1695–1725.

Nataraj, S. (2011). The impact of trade liberalization on productivity: Evidence from India's formal and informal manufacturing sectors. *Journal of International Economics*, 85(2):292–301.

Rauch, J. E. (1991). Modeling the informal sector formally. *Journal of Development Economics*, 35(1):33–47.

Söderbom, M. and Teal, F. (2004). Size and efficiency in African manufacturing firms: Evidence from firm-level panel data. *Journal of Development Economics*, 73(1):369—-394.

Taymaz, E. (2009). Informality and productivity: Productivity differentials between formal and informal firms in Turkey. ERC Working Papers in Economics 0901, Economic Research Center.

Ulyssea, G. (2017). Firms, informality and development: Theory and evidence from Brazil. *American Economic Review, Forthcoming.*

Appendix A Summary Statistics

Table 2 provides pooled summary statistics for our variables of interest.

Table 2: Summary Statistics

Variable	Observations	Weighted
Binary Competition	104,560	50.9% (mean)
Categorical Competition	118,677	1 (median)
Firm Size (perm. full-time)	124,034	65.59 (mean)
Firm Size (all full-time)	120,085	123.73 (mean)

Observations exclude probable typos and non-responses. This standard is used throughout the paper unless otherwise noted. Measures of central tendency are calculated with the sampling weights. A value of one for our categorical competition measure implies that the median firm sees informal firms as a minor obstacle. We drop several observations for temporary labor that are extreme outliers (10 standard deviations from the mean).

Appendix B Competition: Cross-Country Regressions

Table 3: Competition: Cross-Country Regressions

Variable	(1)	(2)	(3)	(4)
Log GDP per capita	-0.051***	-0.054***	-0.043***	-0.041***
	(0.011)	(0.011)	(0.013)	(0.013)
Time fixed effects				
2007		-0.078		0.063
2008		-0.161**		0.097
2009		-0.082*		0.086*
2010		-0.066		0.016
2011		-0.113		0.034
2012		-0.106		0.168
2013		-0.149***		0.042
2014		-0.083		0.058
2015		-0.206***		0.024
Region fixed effects				
Africa			0.012	0.040
Americas			0.099	0.159*
Asia			-0.182***	-0.171**
Europe			-0.106	-0.098
Constant	0.937***	1.039***	0.923***	0.831***
	(0.089)	(0.090)	(0.119)	(0.133)
N	185	185	183	183

The dependent variable is the probability-weighted level of competition for each country surveyed. All regressions are estimated using OLS. Each country is put into one of five regions: Africa, Americas, Asia, Europe, or Oceania. Standard errors are provided in parenthesis. All coefficients are relative to a country from Oceania in 2006. Asterisks denote significance at the 1% (***), 5% (**) and 10% (*) levels for a standard two-tailed t-test.

Appendix C Additional Firm Size and Competition Results

Table 4: Competition and Firm Size: Pooled Regressions

Variable	(1)	(2)	(3)	(4)	(5)
	OLS	OLS	OLS	Logit	Probit
Log firm size	-0.0275***	-0.0249***	-0.0377***	-0.1593***	-0.0113***
	(0.0054)	(0.0056)	(0.0061)	(0.0264)	(0.0041)
Log GDP/capita		-0.0371***	-0.0442***	-0.1877***	-0.1132***
		(0.0057)	(0.0061)	(0.0266)	(0.0163)
Time fixed effects					
2007			-0.0829	-0.3577	0.063
2008			0.0015	-0.0018	0.097
2009			0.0032	0.0029	0.086*
2010			0.1085**	0.4610*	0.016
2011			-0.0469	-0.2023	0.034
2012			0.1254*	0.5164*	0.168
2013			-0.0870	-0.3736	0.042
2014			-0.0532	-0.2330	0.058
2015			0.0125	0.0374	0.024
Region fixed effects					
Africa			0.0040	0.0214	0.040
Americas			0.0669	0.2900	0.159*
Asia			-0.0223	-0.0886	-0.171**
Europe			-0.0596*	-0.2421*	-0.098
Sector fixed effects	no	no	yes	yes	yes
Constant	0.5996***	0.9009***	1.0201***	2.2060***	1.1546***
	(0.0181)	(0.0509)	(0.0820)	(0.3599)	(0.2164)
N	99,349	84,665	78,621	78,621	78,621

The dependent variable is the binary measure of whether a firm competes with informal firms. Firm size is measured as the number of permanent full-time employees. All regressions utilize probability weighting. Regional dummies are Africa, Americas, Asia, Europe, or Oceania. Sector fixed effects are based on two-digit ISIC codes. We drop industries that cannot be estimated with a logit/probit to maintain the same sample across specifications (3), (4), and (5). The country of Malaysia is also dropped due to implausible levels of temporary workers (15 firms have greater than 100,000 temporary workers) Standard errors are provided in parenthesis. Asterisks denote significance at the 1% (***), 5% (**) and 10% (*) levels.

Appendix D Proofs

D.1 Proof of Lemma 1

D.1.1 Zero Profit Condition

The zero profit condition is given by:

$$\pi^s(\phi^*) = 0 \Rightarrow r(\phi^*) = \sigma w^s f^s \text{ for } s \in \{I, F\}$$

and depends on the state of the industry. From here it is straightforward to show that the average revenue in the sector is a function of the cut-off value:

$$\bar{r} = r(\tilde{\phi}) = \left[\frac{\tilde{\phi}}{\phi^*}\right]^{\sigma-1} r(\phi^*) = \left[\frac{\tilde{\phi}(\phi^*)}{\phi^*}\right]^{\sigma-1} \frac{\sigma f^I}{w^I}\left(\frac{\xi}{w^I}\right)^{-\sigma} \tag{28}$$

D.1.2 Free Entry

The free entry condition is given by:

$$E\pi = f^e, \tag{29}$$

where the expected profit is equal to:

$$E\pi = p^I(1-\mu)\bar{\pi}^I + p^F(1-\tau)\bar{\pi}^F, \text{ for } j \in \{I, F\}.$$

p^j and $\bar{\pi}^j$ are the probability of entry and the average profit for sector j, respectively. The probability of a firm being informal or formal is simply the probability of drawing a productivity within the range $[\phi^*, \bar{\phi})$ for informal and the range $[\bar{\phi}, \infty)$ for formal. Explicitly, these probabilities are given by:

$$\begin{aligned} p^I &= G(\bar{\phi}) - G(\phi^*) \\ p^F &= 1 - G(\bar{\phi}). \end{aligned} \tag{30}$$

The free entry condition will also be written in terms of average revenue, rather than average profit. To do this, first define the average productivity of informal ($\tilde{\phi}^I$) and formal ($\tilde{\phi}^F$) firms as:

$$\tilde{\phi}^I = \left[\frac{1}{G(\bar{\phi}) - G(\phi^*)}\int_{\phi^*}^{\bar{\phi}} \phi^{\sigma-1} g(\phi)d\phi\right]^{\frac{1}{\sigma-1}}, \quad \tilde{\phi}^F = \left[\frac{1}{1-G(\bar{\phi})}\int_{\bar{\phi}}^{\infty} \phi^{\sigma-1} g(\phi)d\phi\right]^{\frac{1}{\sigma-1}} \tag{31}$$

From here, the average revenue of informal and formal firms can be written as:

$$r(\tilde{\phi}^I) = \left[\frac{\tilde{\phi}^I}{\tilde{\phi}}\right]^{\sigma-1} w^I \left(\frac{\xi}{w^I}\right)^{\sigma} r(\tilde{\phi}), \quad r(\tilde{\phi}^F) = \left[\frac{\tilde{\phi}^F}{\tilde{\phi}}\right]^{\sigma-1} r(\tilde{\phi}) \tag{32}$$

This ultimately means that the average profits of informal and formal firms can be written as functions of average revenue:

$$\bar{\pi}^I = \left[\frac{\tilde{\phi}^I}{\tilde{\phi}}\right]^{\sigma-1} \frac{w^I \left(\frac{\xi}{w^I}\right)^{\sigma} \bar{r}}{\sigma} - f^I, \quad \bar{\pi}^F = \left[\frac{\tilde{\phi}^F}{\tilde{\phi}}\right]^{\sigma-1} \frac{\bar{r}}{\sigma} - f^F \tag{33}$$

Combining Equations (29), (30), and (33), the free entry condition becomes:

$$(G(\bar{\phi})-G(\phi^*))(1-\mu)\left[\left[\frac{\tilde{\phi}^I}{\tilde{\phi}}\right]^{\sigma-1} \frac{w^I \left(\frac{\xi}{w^I}\right)^{\sigma} \bar{r}}{\sigma} - f^I\right] + (1-G(\bar{\phi}))(1-\tau)\left[\left[\frac{\tilde{\phi}^F}{\tilde{\phi}}\right]^{\sigma-1} \frac{\bar{r}}{\sigma} - f^F\right] = f^e \tag{34}$$

Finally, Equation (34) can be solved for $\bar{r}$ in order to determine the relationship between average profits and the cut-off ϕ^*.

$$\bar{r} = \frac{\sigma(f^e + (G(\bar{\phi}) - G(\phi^*))(1-\mu)f^I + (1-G(\bar{\phi}))(1-\tau)f^F}{(G(\bar{\phi}) - G(\phi^*))(1-\mu)w^I \left(\frac{\xi}{w^I}\right)^{\sigma} \left[\frac{\tilde{\phi}^I}{\tilde{\phi}}\right]^{\sigma-1} + (1-G(\bar{\phi}))(1-\tau)\left[\frac{\tilde{\phi}^F}{\tilde{\phi}}\right]^{\sigma-1}} \tag{35}$$

D.2 Proof of Lemma 2

Formality means that the firm will face a higher fixed cost of production and a certain tax rate, however they avoid the possibility of losing the entirety of their profit due to fines in the informal sector. Therefore the expected profit for a given productivity for each type of firm is:

$$\begin{aligned} E\pi^I(\phi) &= (1-\mu)\left[\left(\frac{\phi}{\phi^*}\right)^{\sigma-1} w^I f^I - w^I f^I\right] \\ \pi^F(\phi) &= (1-\tau)\left[\left(\frac{\phi}{\phi^*}\right)^{\sigma-1} f^I \left(\frac{\xi}{w^I}\right)^{-\sigma} - f^F\right] \end{aligned} \tag{36}$$

Based on the assumption above, clearly $E\pi^I(0) > \pi^F(0)$ and $\frac{\partial E\pi^I(\phi)}{\partial\phi} < \frac{\partial \pi^F(\phi)}{\partial\phi}$, which means that there is a single crossing point below which firms will become informal and above which firms become formal. Define $\bar{\phi}$ such that: $E\pi^I(\bar{\phi}) = \pi^F(\bar{\phi})$, then the cut-off is explicitly given by:

$$\bar{\phi} = \left[\frac{\left(\frac{\xi}{w^I}\right)^\sigma \left(f^F(1-\tau) - f^I(1-\mu)w^I\right)}{f^I\left((1-\tau) - (1-\mu)w^I\left(\frac{\xi}{w^I}\right)^\sigma\right)} \right]^{\frac{1}{\sigma-1}} \phi^* \tag{37}$$

D.3 Proof of Lemma 3

Proof. First note that:

$$1 - G(\phi) = \left(\frac{k}{\phi}\right)^\alpha$$

$$G(\bar{\phi}) - G(\phi^*) = \left(\frac{k}{\phi^*}\right)^\alpha - \left(\frac{k}{\bar{\bar{\phi}}}\right)^\alpha = \left(\frac{k}{\phi^*}\right)^\alpha - \left(\frac{k}{\hat{F}_s\phi^*}\right)^\alpha = \left(\frac{k}{\phi^*}\right)^\alpha \left(1 - \hat{F}_s^{-\alpha}\right)$$

This implies that:

$$\begin{aligned}
\tilde{\phi} &= \left[\left(\frac{\phi^*}{k}\right)^\alpha \left(\int_{\bar{\phi}}^\infty \phi^{\sigma-1} g(\phi) d\phi + w^I \left(\frac{\xi}{w^I}\right)^\sigma \int_{\phi^*}^{\bar{\phi}} \phi^{\sigma-1} g(\phi) d\phi \right) \right]^{\frac{1}{\sigma-1}} \\
&= \left[\left(\frac{\phi^*}{k}\right)^\alpha \left(\frac{\alpha k^\alpha}{1+\alpha-\sigma} (\hat{F}\phi^*)^{-1-\alpha+\sigma} + \frac{\alpha k^\alpha}{1+\alpha-\sigma} w^I \left(\frac{\xi}{w^I}\right)^\sigma (\phi^*)^{-1-\alpha+\sigma} (1 - \hat{F}^{-1-\alpha+\sigma}) \right) \right]^{\frac{1}{\sigma-1}} \\
&= \left(\frac{\alpha}{1+\alpha-\sigma}\right)^{\frac{1}{\sigma-1}} \left[\hat{F}^{-1-\alpha+\sigma} + w^I \left(\frac{\xi}{w^I}\right)^\sigma (1 - \hat{F}^{-1-\alpha+\sigma}) \right] \phi^*
\end{aligned}$$

$$\begin{aligned}
\tilde{\phi}^I &= \left[\left(\frac{k}{\phi^*}\right)^{-\alpha} \left(1 - \hat{F}_s^{-\alpha}\right)^{-1} \int_{\phi^*}^{\bar{\phi}} \phi^{\sigma-1} g(\phi) d\phi \right]^{\frac{1}{\sigma-1}} \\
&= \left[\left(\frac{k}{\phi^*}\right)^{-\alpha} \left(1 - \hat{F}_s^{-\alpha}\right)^{-1} \left(\frac{\alpha k^\alpha (\phi^*)^{-1-\alpha+\sigma}}{1+\alpha-\phi} \right) \left(1 - \hat{F}_s^{-1-\alpha+\sigma}\right) \right]^{\frac{1}{\sigma-1}} \\
&= \left[\left(1 - \hat{F}_s^{-\alpha}\right)^{-1} \left(1 - \hat{F}_s^{-1-\alpha+\sigma}\right) \left(\frac{\alpha}{1+\alpha-\sigma}\right) \right]^{\frac{1}{\sigma-1}} \phi^*
\end{aligned}$$

$$\tilde{\phi}^F = \left[\left(\frac{k}{\bar{\bar{\phi}}}\right)^{-\alpha} \int_{\bar{\phi}}^\infty \phi^{\sigma-1} g(\phi) d\phi \right]^{\frac{1}{\sigma-1}} = \left[\left(\frac{k}{\bar{\bar{\phi}}}\right)^{-\alpha} \left(\frac{\alpha k^\alpha \bar{\phi}^{-1-\alpha+\sigma}}{1+\alpha-\sigma} \right) \right]^{\frac{1}{\sigma-1}} = \left(\frac{\alpha}{1+\alpha-\sigma}\right)^{\frac{1}{\sigma-1}} \hat{F}_s \phi^*$$

$\square$

D.4 Proof of Proposition 2

Proof. 1. The proof for the first inequality follows from the ratio of $\phi^*_{\mathcal{F}}/\phi^*_{\mathcal{I}}$:

$$\frac{\phi^*_{\mathcal{F}}}{\phi^*_{\mathcal{I}}} = \left(\left(\frac{f^I w^I(1-\mu)}{f^F(1-\tau)}\right) + \hat{F}_I^{-\alpha}\left(1 - \frac{f^I w^I(1-\mu)}{f^F(1-\tau)}\right)\right)^{-\frac{1}{\alpha}} > 1$$

where the inequality follows from the fact that $\left(\frac{f^I w^I(1-\mu)}{f^F(1-\tau)}\right) < 1$ and $\hat{F}_I^{-\alpha} < 1$.

The second inequality comes from the ratio of $\bar{\phi}_I/\phi^*_F$:

$$\frac{\bar{\phi}_{\mathcal{I}}}{\phi^*_{\mathcal{F}}} = \hat{F}_I\left(\left(\frac{f^I w^I(1-\mu)}{f^F(1-\tau)}\right) + \hat{F}_I^{-\alpha}\left(1 - \frac{f^I w^I(1-\mu)}{f^F(1-\tau)}\right)\right)^{\frac{1}{\alpha}} > 1$$

To see where the inequality comes from, let $x = \left(\frac{f^I w^I(1-\mu)}{f^F(1-\tau)}\right)$, then we need:

$$\hat{F}_I((x + \hat{F}_I^{-\alpha}(1-x))^{1/\alpha} > 1$$

which can be rearranged to be:

$$x + \hat{F}_I^{-\alpha}(1-x) > \hat{F}_I^{-\alpha}$$

rearranging one more time, yields:

$$\hat{F}_I^{-\alpha} < 1$$

which is unambiguously true.

2. Let $\rho = (\xi/w^I)$. This result comes about by taking the ratio of the revenues in each economy:

$$\frac{\bar{r}_{\mathcal{F}}}{\bar{r}_{\mathcal{I}}} = \frac{f^F}{f^I}\frac{\rho^\sigma}{\hat{F}^{-1-\alpha+\sigma} + w^I\rho^\sigma(1-\hat{F}^{-1-\alpha+\sigma})} \geq 1$$

The inequality can be seen by setting $\rho = (f^I/f^F)$ (its minimum value), which results in:

$$\frac{\bar{r}_{\mathcal{F}}}{\bar{r}_{\mathcal{I}}} = 1$$

from there it is straightforward to show that:

$$\frac{\partial \bar{r}_{\mathcal{F}}/\bar{r}_{\mathcal{I}}}{\partial \rho} > 0$$

3. We know that the mass of firms equals $M = R/\bar{r}$, then this result is simply an application of the second result.

4. First, we know that total revenue is equal to average revenue times the mass of firms:

$$M\bar{r} = R$$

This can also be written as the sum of the total revenue in each sector:

$$M^I \bar{r}^I + M^F \bar{r}^F = R$$

Note that $M = M^I + M^F$, which means the above equation can be written as:

$$(M - M^F)\bar{r}^I + M^F \bar{r}^F = R$$

Solving for M^F yields:

$$M^F = \frac{R\left(1 - \frac{\bar{r}^I}{\bar{r}}\right)}{\bar{r}^F - \bar{r}^I}$$

where the substitution $M = R/\bar{r}$ is made. Using the definitions of $\bar{r}, \bar{r}^I$, and $\bar{r}^F$ above, we can write the mass of firms in the formal sector as:

$$M^F = \hat{F}^{-\alpha} \frac{R}{\bar{r}}$$

To get the result above, take the ratio of M^F in each economy:

$$\frac{M^F_{\mathcal{I}}}{M^F_{\mathcal{F}}} = \frac{\frac{R}{\bar{r}_{\mathcal{I}}} \hat{F}_{\mathcal{I}}^{-\alpha}}{\frac{R}{\bar{r}_{\mathcal{F}}}} = \frac{f^F}{f^I} \frac{\rho^\sigma}{\hat{F}_{\mathcal{I}}^{-1-\alpha+\sigma} + w^I \rho^\sigma (1 - \hat{F}_{\mathcal{I}}^{-1-\alpha+\sigma})} \hat{F}_{\mathcal{I}}^{-\alpha} \leq 1$$

It should be noted that when $\rho = (f^I/f^F)^{1/\sigma}$, that $M^F_{\mathcal{I}}/M^F_{\mathcal{F}} = 1$. It can also be shown that:

$$\lim_{\rho \to \frac{1-\tau}{w^I(1-\mu)}} \frac{M^F_{\mathcal{I}}}{M^F_{\mathcal{F}}} = 0$$

from there it is easy to show that

$$\frac{\partial M^F_{\mathcal{I}}/M^F_{\mathcal{F}}}{\partial \rho} < 0$$

□

D.5 Proof of Proposition 3

Proof. First note that a firm's profit can be written as:

$$\pi^s(\phi) = \frac{r^s(\phi)}{\sigma} - f^s$$

We know that a firm's revenue can be expressed as a function of the cut-off value ϕ_s^*:

$$r(\phi) = \left(\frac{\phi}{\phi_s^*}\right)^{\sigma-1} r(\phi_s^*) = \left(\frac{\phi}{\phi_s^*}\right)^{\sigma-1} \sigma w^s f^s$$

Therefore the ratio of revenue for a given ϕ across economies is:

$$\frac{r_F(\phi)}{r_I(\phi)} = \frac{\left(\frac{\phi}{\phi_F^*}\right)^{\sigma-1} \sigma f^F}{\left(\frac{\phi}{\phi_I^*}\right)^{\sigma-1} \sigma f^I} = \left(\frac{\phi_I^*}{\phi_F^*}\right)^{\sigma-1} \frac{f^F}{w^I f^I} \equiv \chi > 1$$

Note that the revenue in an economy with only formal firms is proportional to the revenue in an economy with informal and formal firms. To see how we get the inequality, substitute for ϕ_s^* and rearrange to get:

$$\chi = \left(\frac{\xi}{w^I}\right)^{\frac{-\sigma(\sigma-1)}{\alpha}} \left[w^I \left(\frac{1-\mu}{1-\tau}\right)\left(\frac{\xi}{w^I}\right)^{\sigma} + \left(\left(\frac{\xi}{w^I}\right)^{\sigma}\left(\frac{f^F}{f^I} - w^I\left(\frac{1-\mu}{1-\tau}\right)\right)\left(1 - w^I\left(\frac{1-\mu}{1-\tau}\right)\left(\frac{\xi}{w^I}\right)^{\sigma}\right)\right.\right.$$

First, let $f^F = f^I$, then $\chi > 1$, if:

$$\left[\frac{1 - w^I\left(\frac{1-\mu}{1-\tau}\right)\left(\frac{\xi}{w^I}\right)^{\sigma}}{\left(\frac{\xi}{w^I}\right)^{\sigma}\left(1 - w^I\left(\frac{1-\mu}{1-\tau}\right)\right)}\right]^{\frac{\alpha}{\sigma-1}} > \frac{(w^I)^{\frac{\alpha}{\sigma-1}} - w^I\left(\frac{1-\mu}{1-\tau}\right)}{\left(1 - w^I\left(\frac{1-\mu}{1-\tau}\right)\right)}$$

This is clearly true if $\xi < w^I$. If $\xi > w^I$, then $\chi > 1$ if:

$$\xi < w^I \left[w^I\left(\frac{1-\mu}{1-\tau}\right) + \left(\frac{(w^I)^{\frac{\alpha}{\sigma-1}} - w^I\left(\frac{1-\mu}{1-\tau}\right)}{1 - w^I\left(\frac{1-\mu}{1-\tau}\right)}\right)^{\frac{\sigma-1}{\alpha}}\left(1 - w^I\left(\frac{1-\mu}{1-\tau}\right)\right)\right]^{-1/\sigma}$$

We know that $\xi < (w^I)^{\sigma-1}\sigma$, the above inequality holds if:

$$\left[\frac{(w^I)^{\frac{\alpha}{\sigma-1}} - w^I\left(\frac{1-\mu}{1-\tau}\right)}{\left(1 - w^I\left(\frac{1-\mu}{1-\tau}\right)\right)}\right]^{\frac{\sigma-1}{\alpha}} \leq 1$$

which is clearly true. It is rather straightforward to show that:

$$\frac{\partial \chi}{\partial f^F} > 0$$

Therefore $\chi > 1$.

Going back to the ratio of the two revenues, it implies that the percentage of profit loss is equal to:

$$PPL(\phi) = \frac{r_I(\phi)(\chi - 1)}{\chi r_I(\phi) - f^s}$$

Taking the derivative w.r.t. ϕ yields:

$$\frac{\partial PPL(\phi)}{\partial \phi} = -\frac{r_I^{'}(\phi)(\chi - 1)f^s}{(\chi r_I(\phi) - f^s)^2} < 0$$

where $r_I^{'}(\cdot) < 0$.

□

D.6 Proof of Proposition 4

Proof. 1. Result 1

Using the definition of ϕ^* above, the ratio of ϕ^* between two industries can be written as:

$$\frac{\phi^*_{i-1}}{\phi^*_i} = \left(\frac{\rho_i^\sigma f^I_{i-1}\bar{F}_{i-1}}{\rho_{i-1}^\sigma f^I_i \bar{F}_i}\right)^{\frac{1}{\alpha}}$$

By definition:

$$\frac{f^I_{i-1}}{f^I_i} < 1$$

To show that

$$\left(\frac{\rho_i}{\rho_{i-1}}\right)^\sigma \frac{\bar{F}_{i-1}}{\bar{F}_i} < 1$$

We need to note that because $\xi_{i-1} > \xi_i$ $\exists$ $0 < x < 1$, such that $\xi_{i-1} = x\xi_i$. Also because $\frac{f^F_{i-1}}{f^I_{i-1}} > \frac{f^F_i}{f^I_i}$ $\exists 0 < y < 1$, such that $\frac{f^F_{i-1}}{f^I_{i-1}} = y\frac{f^F_i}{f^I_i}$. Using these definitions, the inequality above can be written as:

$$\left[\frac{1-w^I\left(\frac{1-\mu}{1-\tau}\right)\left(\frac{\xi}{w^I}\right)^\sigma}{1-w^I\left(\frac{1-\mu}{1-\tau}\right)\left(\frac{x\xi}{w^I}\right)^\sigma}\right]^{\frac{\alpha}{\sigma-1}}(x^\sigma)^{\frac{\alpha}{\sigma-1}} < \left[\frac{y\frac{f^F}{f^I}-w^I\left(\frac{1-\mu}{1-\tau}\right)}{\frac{f^F}{f^I}-w^I\left(\frac{1-\mu}{1-\tau}\right)}\right]^{\frac{-1-\alpha+\sigma}{\sigma-1}}$$

It is clear that $LHS < 1 < RHS$ because $-1-\alpha+\sigma < 1$.

Therefore

$$\frac{\phi^*_{i-1}}{\phi^*_i} = \left(\frac{\rho_i^\sigma f^I_{i-1}\bar{F}_{i-1}}{\rho^\sigma_{i-1} f^I_i \bar{F}_i}\right)^{\frac{1}{\alpha}} < 1$$

2. Result 2

   We know that:

$$\frac{\bar{r}_i}{\bar{r}_{i-1}} = \frac{\left(\hat{F}_i^{-1-\alpha+\sigma} + w^I\rho^\sigma(1-\hat{F}_i^{-1-\alpha+\sigma}\right)}{\left(\hat{F}_{i-1}^{-1-\alpha+\sigma} + w^I\rho^\sigma(1-\hat{F}_{i-1}^{-1-\alpha+\sigma}\right)}\left(\frac{\rho_{i-1}}{\rho_i}\right)^\sigma \frac{f^I_i}{f^I_{i-1}}$$

   We know that $f^I_{i-1} < f^I_i$, we need to check if:

$$\frac{\left(\hat{F}_i^{-1-\alpha+\sigma} + w^I\rho^\sigma(1-\hat{F}_i^{-1-\alpha+\sigma}\right)}{\left(\hat{F}_{i-1}^{-1-\alpha+\sigma} + w^I\rho^\sigma(1-\hat{F}_{i-1}^{-1-\alpha+\sigma}\right)}\left(\frac{\rho_{i-1}}{\rho_i}\right)^\sigma > 1$$

   Simplifying yields:

$$\left[\frac{1-w^I\left(\frac{x\xi}{w^I}\right)^\sigma}{1-w^I\left(\frac{\xi}{w^I}\right)^\sigma}\right] > x^\sigma\left[\frac{1-w^I\left(\frac{1-\mu}{1-\tau}\right)\left(\frac{x\xi}{w^I}\right)^\sigma}{1-w^I\left(\frac{1-\mu}{1-\tau}\right)\left(\frac{\xi}{w^I}\right)^\sigma}\right]^{\frac{-1-\alpha+\sigma}{\sigma-1}}\left[\frac{\frac{f^F}{f^I}-w^I\left(\frac{1-\mu}{1-\tau}\right)}{y\frac{f^F}{f^I}-w^I\left(\frac{1-\mu}{1-\tau}\right)}\right]^{\frac{-1-\alpha+\sigma}{\sigma-1}}$$

   where x and y are as defined above. It is clear that $LHS > 1 > RHS$, because $-1-\alpha+\sigma < 0$. Therefore $\bar{r}_{i-1} < \bar{r}_i$.

3. Result 3

   The percentage of firms that are formal is:

$$\frac{M^F}{M} = \hat{F}^{-\alpha}$$

   Now we check if

$$\frac{M^F_i}{M^F_{i-1}}\frac{M_{i-1}}{M_i} > 1$$

$$\frac{M_i^F}{M_{i-1}^F}\frac{M_{i-1}}{M_i} = \left[x^{\frac{\sigma}{\sigma-1}} \left(\frac{1-\tau-(1-\mu)w^I\left(\frac{\xi}{w^I}\right)^\sigma}{1-\tau-(1-\mu)w^I\left(\frac{\xi x}{w^I}\right)^\sigma} \right)^{\frac{1}{\sigma-1}} \left(\frac{y\frac{f^F}{f^I}(1-\tau)-(1-\mu)w^I}{\frac{f^F}{f^I}(1-\tau)-(1-\mu)w^I} \right)^{\frac{1}{\sigma-1}} \right]^{-1/\alpha} > 1$$

Based on the definitions of x and y above, this inequality is clearly true.

4. Result 4
   As long as $\bar{\phi}_i \neq \bar{\phi}_j$, then there is a range $\{\bar{\phi}_j, \bar{\phi}_i\}$ such that productivities in that range are formal in one industry and informal in the other. Making the appropriate substitutions for $\bar{\phi}$ and ϕ^*, yields:

$$\left(\frac{\rho_j f_i^I \bar{F}_j}{\rho_i f_j^I \bar{F}_i} \right)^{\frac{1}{\alpha}} \neq \frac{\hat{F}_j}{\hat{F}_i}$$

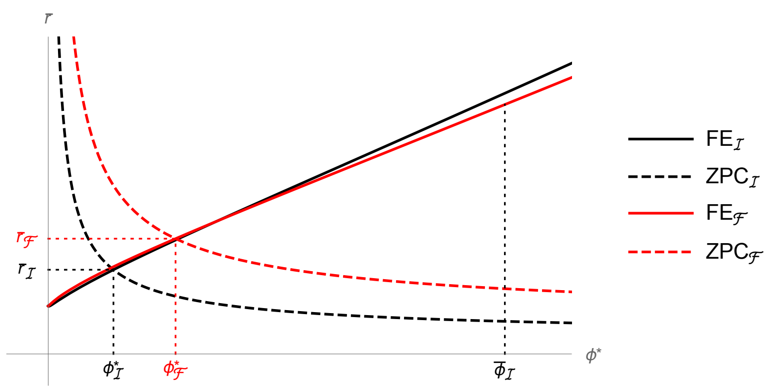
□

Appendix E General Distribution Results

In this appendix we generalize the results found in the main section of paper to other distributions using numerical methods. The figures below are created using a log-normal distribution but the results are robust to a wide-range of "well-behaved" distributions (exponential, gamma, Weibull) and parameter choices.[24]

Starting with Proposition 1, Figure 6 below graphs the free entry and zero profit cut-off lines for two economies: one with both informal and formal ($\mathcal{I}$) firms and the other with only formal firms ($\mathcal{F}$).

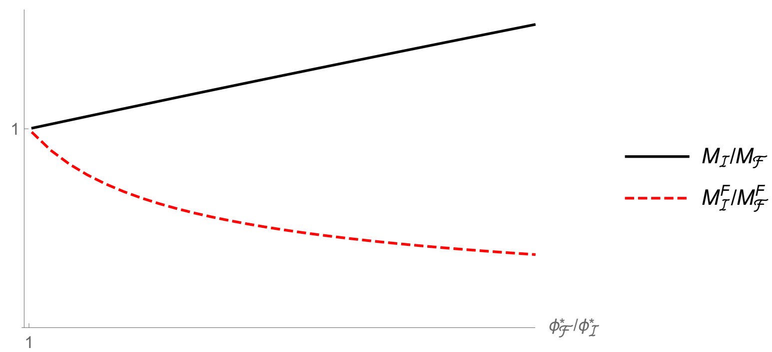
Figure 5: Equilibrium cut-off (ϕ^*) and average revenue ($\bar{r}$)



The first two results of Proposition 2 are readily apparent. The $\mathcal{I}$ economy has a lower cut-off and average revenue than the economy without informal firms. In addition, based on average revenue it is clear that the mass of firms is smaller in the economy with only formal firms. These results together confirm the underlying intuition of the model, informality allows those firms who otherwise be priced on the the market to enter. The entry of additional firms creates increased competition, resulting in lower revenues on average. However, this figure cannot speak to is the relative size of the formal sector in each economy. This result is given in the figure below. It should be noted that the figure below is generated by varying the fixed cost associated with being formal within the range $\{f^I, 1.5 * f^I\}$. The ratios of the mass of firms ($M_{\mathcal{I}}/M_{\mathcal{F}}$) and the mass of formal firms ($M^F_{\mathcal{I}}/M^F_{\mathcal{F}}$) are plotted against the ratio of the cut-off values ($\phi^*_{\mathcal{F}}/\phi^*_{\mathcal{I}}$). It is expected that the ratio of the cut-offs would rise as the fixed cost of formality rises relative to the fixed cost of being informal.

[24] We did not come across a set of parameters that qualitatively changed the results presented in this section, despite our honest attempts to do so.

Figure 6: The ratios of the mass of firms ($M_{\mathcal{I}}/M_{\mathcal{F}}$) and the mass of formal firms ($M^F_{\mathcal{I}}/M^F_{\mathcal{F}}$) is plotted against the ratio of the cut-off values ($\phi^*_{\mathcal{F}}/\phi^*_{\mathcal{I}}$) for an economy with both formal and informal firms ($\mathcal{I}$) and with only formal firms ($\mathcal{F}$).



Not surprisingly, the mass of firms in the mixed economy rises relative to the economy with only formal firms as f^F increases. This is because as the fixed cost of formality increases it lowers the number of formal firms. In the formal only economy, firms that do not become formal exit (lowering the overall number of firms), while in the $\mathcal{I}$ economy these firms simply become informal. The numerical results confirm that the $\mathcal{I}$ economy will see very little change in the mass of firms while the formal only economy will see a significant decline. What is particularly interesting in this figure is that despite the fact that the mass of firms in the formal only economy is falling relative to the mixed economy, the mass of formal firms is relatively increasing. This means that as the fixed costs of formality increase it causes more firms to abandon formality in the $\mathcal{I}$ economy than those who exit in the formal only economy.

Proposition 3 showed that the firms that were most affected by informal competition were the ones with relatively low productivity. In Figure 7 below, we consider this result more generally. The metric of increased competition continues to be the percentage of profit lost due to informal firms. Consistent with what was shown under the Pareto distribution, the percentage of profit loss is falling in firm productivity. The most significant losses occur for those firms who would be near the cut-off in a formal only economy. Those least productive formal firms in the all formal economy would be most affected by informality. It should be noted that these firms would no longer be formal in a mixed economy.

The final proposition in the previous section showed that between industries there is a TFP overlap between formal and informal firms. Figure 8 generalizes this result for other distributions. To see this suppose there are two industries with different costs of entry (WLOG assume that $f^s_1 < f^s_2$). In Figure 8, the productivity range

Figure 7: Percentage of profit loss due to informal competition

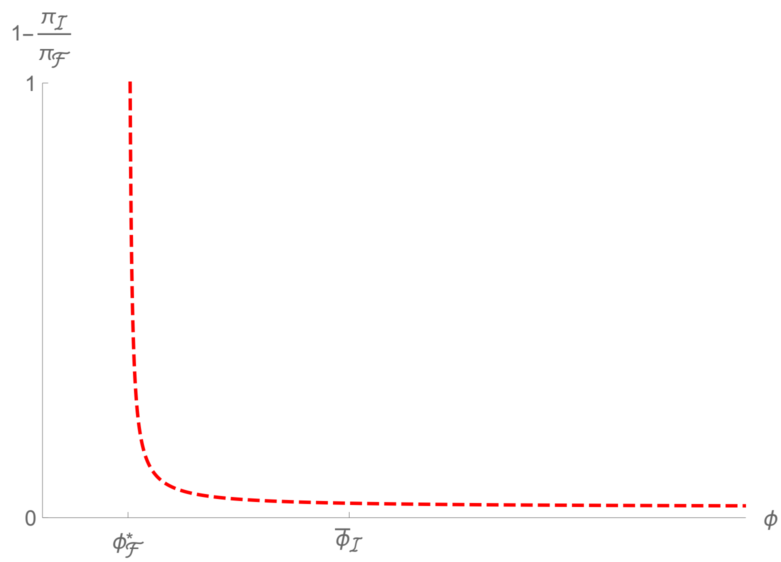
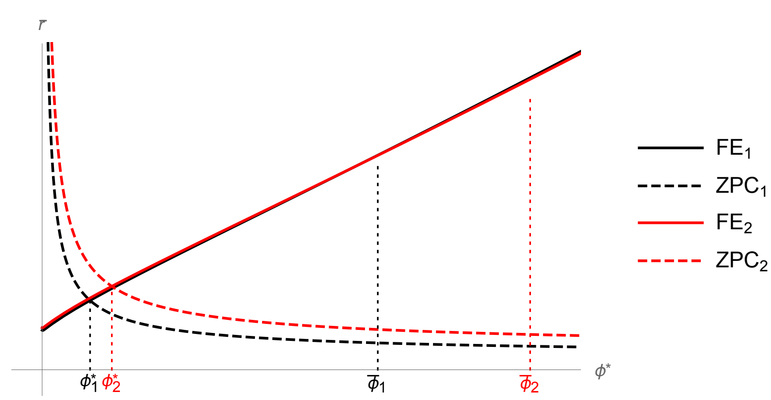


Figure 8: TFP overlap across industries

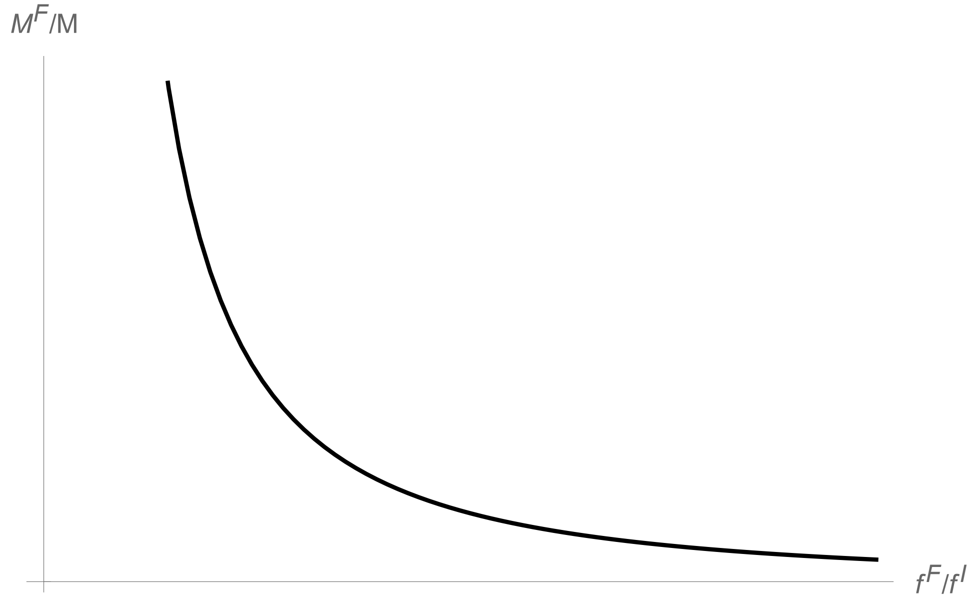


$\{\bar{\phi}_2, \bar{\phi}_1\}$ is comprised of formal firms in industry 1 and informal firms in industry 2. The reason for this overlap is that the higher cost of entry shrinks the mass of firms, thus lowering competition and raising the overall price level. The higher price level means that less productive firms can cover the costs of formality. One of the questions that arises from Figure 8 is how the percentage of firms that are formal changes with the fixed costs. Figure 8 shows that increasing the fixed costs of production results in a higher cut-off, but it is unclear how this changes the distribution of formal and informal firms. In order to see how the distribution changes, we solve for the percentage of firms that are formal:

$$\frac{M^F}{M} = \frac{\tilde{\phi}^{\sigma-1} - (\tilde{\phi}^I)^{\sigma-1}}{(\tilde{\phi}^F)^{\sigma-1} - (\tilde{\phi}^I)^{\sigma-1}}.$$

Because $\tilde{\phi}^F > \tilde{\phi}$ the expression above is clearly between zero and one. Figure 9 below shows how the percentage of formal firms changes as the fixed cost of formal production changes relative to informal production.

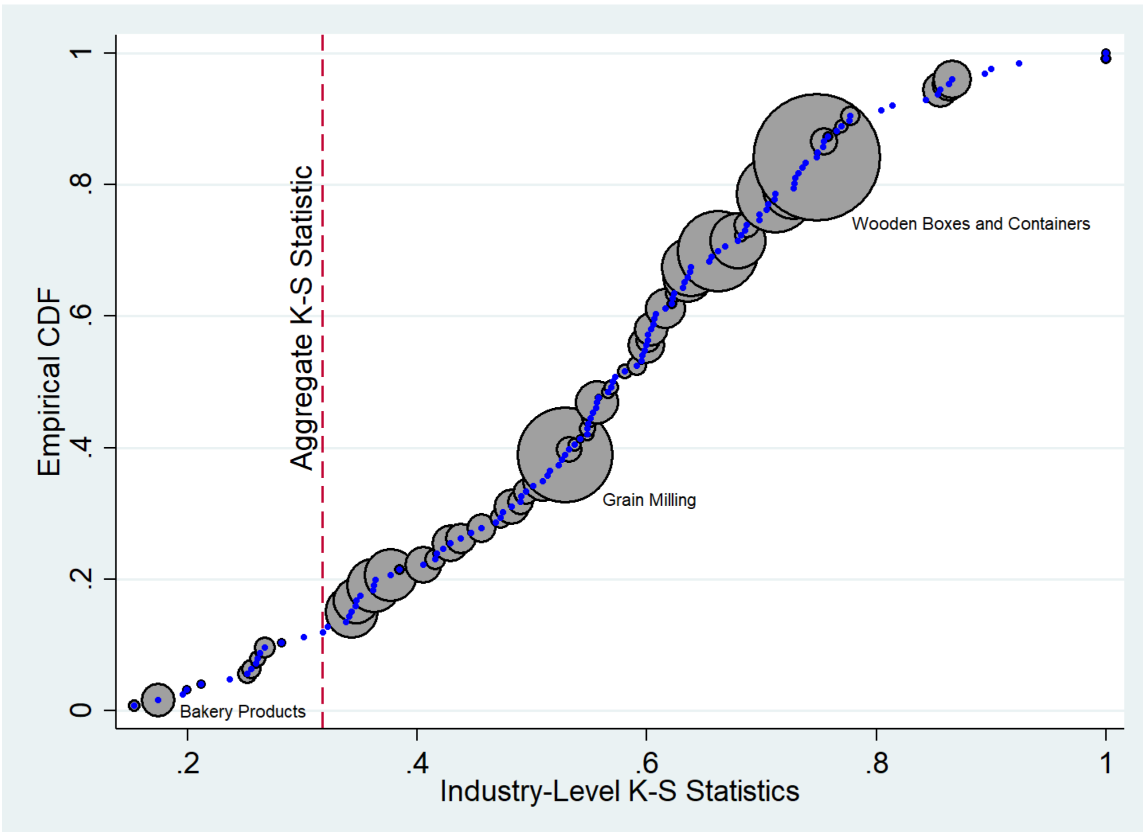
Figure 9: Percent of firms that are formal plotted against quality of institutions.



Clearly, the percentage of firms that are formal is decreasing as the fixed costs increase.

Appendix F K-S Statistics with Industry Size

Figure 10: Distribution of Industry K-S Statistics by Industry Size



The horizontal axis plots the K-S statistic, D, for each industry in 1999. The figure contains data on 126 manufacturing industries. The aggregate K-S statistic is 0.317. Small points represent non-scaled industries, while "bubbles" represent industries that are scaled by the number of firms in each industry. Note that the size aspect of some industries is covered by larger "nearby" industries.