Superstars in Team Sports: An Economic Model

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Abstract

In professional sports, superstars can earn extremely high incomes compared with those of other players. The existence of superstars in team sports is interesting because, unlike in individual sports, it is the teams that compete, not the individual players. This paper examines the mechanism of how an individual player can be a superstar even in the context of team sports. The key to the mechanism is that the probability of winning differs tremendously depending on whether or not a team has top-ranked players (i.e., those with relatively high abilities). This factor, combined with the effect of ranking preference, means that the salaries of players increase exponentially from the bottom- to the top-ranked player. As a result, a top-ranked player can be a superstar even in team sports.

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1 INTRODUCTION

In professional sports, superstars can earn extremely high incomes as compared with those of other players. Several models have been presented to explain why a superstar can be generated. An early and important work was that of Rosen (1981), which was followed by those of Adler (1985, 2006), Frank and Cook (1995), and Borghans and Groot (1998). Rosen (1981) attributed the extremely high incomes of superstars to differences in talent (quality) and to a special market structure (i.e., non-rivalry). Adler (1985) discussed "consumption capital," and Borghans and Groot (1998) discussed the topic of "endogenous property rights" and monopoly power. Harashima (2016, 2017) presented an alternative model based on the concepts of ranking value and preference. People have a ranking preference because ranking is an important element in people's lives and economic activities. Ranking preference is deeply rooted in the process of evolution of human beings, who have dominance hierarchies (cf., Landau, 1951; Bayly et al., 2006). An important point about ranking is that ranking value and preference provide monopoly powers and profits to the producers of products that have high ranking values.

The question arises, however, that even though superstars in individual sports can be explained by the above models, can superstars in team sports also be explained? In team sports, teams compete and championships are won by the team, not any individual player. Of course, individual achievements are possible in team sports (e.g., most valuable player), but they are not the main objective. Individual accolades are merely by-products of the team sport, but there are clearly superstars in team sports.

Why do individual players become superstars and earn disproportionately large salaries in team sports? In a team sport, player salaries may have to be relatively more evenly distributed because championships are won by collaborative works of players. Unless a team intentionally distributes salaries extraordinarily unevenly to its players, a superstar will not be generated. In this sense, the idea of a superstar in team sports is puzzling. In this paper, I examine the generation mechanism of superstars in team sports on the basis of the concepts of ranking value and preference presented by Harashima (2016, 2017).

In Harashima's model of ranking value and preference, there are two kinds of value: practical value and ranking value. Practical value is the value that people feel when consuming a good or service for practical purposes, and ranking value is the value that people feel from the rank of a good or service in a set of similar types of goods or services that people use, possess, or observe. An important point is that ranking value and preference provide monopoly powers to the producers of high-rank goods and services. Hence, if households' ranking preference is strong enough, the highest-ranked producer can be a “superstar.”

In this paper, I show that the probability of a win for a team differs largely depending on whether or not a team has high-ability (i.e., high-ranked) players. Because a win or loss is determined by relative differences in the performances of teams, players that are even a little better can play an important role in a team win. Because of this nature of the probability of winning, as well as the effect of ranking preference, salaries of players increase exponentially from the bottom- to the top-ranked player, and superstars can be generated in a team sport.

2 RANKING VALUE AND PREFERENCE

The model is based on the concepts of ranking value and preference that were first presented by Harashima (2016, 2017) and are briefly explained below.

2.1 Ranking value

Value is regarded as reflecting something useful. People feel, obtain, or consume value when using, enjoying, or consuming goods and services. Values derived from practical use have usually been considered in economics, but people also consume or feel value derived from ranking. For example, if a curio is evaluated to be the best among a set of similar types of curios, its price will be much higher than that of the others, even if the object is not practically useful. Its price is high only because of its top rank. People therefore obtain utility not only from practical uses but also from a sense of ranking.

Therefore, there are two kinds of value: practical value and ranking value. Practical value is the value that people feel when consuming a good or service for practical purposes. Ranking value is the value that people feel from the rank of a good or service in a set of similar types of goods or services that people use, possess, or observe. In other words, ranking value is the value people place on goods or services on the basis of their ranks (e.g., the ranking of a book on a best-seller list or that of a baseball team in a professional league).

2.2 Ranking preference

Goods and services have three properties: quantity, quality, and ranking. Quality is related to practical value, ranking is related to ranking value, and quantity is related to both. Suppose that the quality and ranking of each good or service are given exogenously and fixed. Here, for simplicity, I assume that there is only one type of good or service in the economy (these goods or services are hereafter called “goods”), and that all goods belong to this type and are substitutable for each other for households’ practical uses. Although the goods are substitutable from the point of view of practical uses, they are differentiated from the point of view of ranking.

Let R (= 1, 2, 3, …) be the rank of goods. The good with $R = 1$ is most preferred by households. $R = 2$ indicates the next most preferred, and so on. For simplicity, no tied ranks are assumed. A household's utility derived from consuming the good with rank R is

$$u(q_{n,R}, q_{l,R}, R)$$

where $q_{n,R}$ and $q_{l,R}$ are the quantity and quality of the good with rank R, respectively. For simplicity, the utility of the household is modified to

$$u(\tilde{q}_R, R)$$

where $\tilde{q}_R$ is the "quality-adjusted quantity" of the good with rank R, and $\tilde{q}_R = q_{n,R}\, q_{l,R}$.

The utility function has the following conventional characteristics:

$$\frac{\partial u(\tilde{q}_R, R)}{\partial \tilde{q}_R} > 0$$

and

$$\frac{\partial^2 u(\tilde{q}_R, R)}{\partial \tilde{q}_R^2} < 0 \ .$$

In addition, for any $r \in R$,

$$u(\tilde{q}_r, r+1) < u(\tilde{q}_r, r)$$

and

$$u(\tilde{q}_r, r+2) - u(\tilde{q}_r, r+1) > u(\tilde{q}_r, r+1) - u(\tilde{q}_r, r) \ .$$

2.3 Monopoly power

Ranking value and preference provide monopoly powers to the producers of high-ranked goods and services because selling ranking value to consumers requires no additional cost; that is, the marginal cost of producing a ranking value is zero, and thereby such producers can set prices above the marginal costs. If households' ranking preference is strong enough, the producer of the highest-ranked good can be a superstar.

3 MECHANISM OF GENERATION OF SUPERSTARS IN TEAM SPORTS

3.1 The environment

Suppose that there is a professional sports league that consists of M teams and P players. Each team is assigned a "rank" of 1, 2, 3, etc., where the "rank 1 team" indicates the best team (the champion team), the "rank 2 team" indicates the next-best team, and so on. "Rank" is also assigned to players. It is assumed for simplicity that the roles of players on teams are identical. A "rank 1 player" indicates the best player, "rank 2 player" indicates the second best, and so on. Let a_r be the ability of a player with rank r ($r \in P$). It is assumed that a_r is measurable and additive, and that all players have different abilities. Each team equally consists of n players, so $P = nM$. The "ability of a team" is the sum of the abilities of the n players who belong to the team. If the ability of a team is higher than that of an opposing team, the probability that that team will win the championships is higher than that of the opposing team.

It is assumed that each team's revenue (ticket sales, the sale of broadcasting rights, license fees, etc.) depends entirely on its rank. Teams with a higher rank can enjoy stronger monopoly powers and obtain more revenue. Hence, the revenue (y) of a team is a function of the team's rank m ($\in M$), such that

$$y = f(m) \ . \tag{1}$$

By the nature of ranking value and preference,

$$\frac{dy}{dm} < 0$$

and

$$\frac{d^2y}{dm^2} > 0 \ .$$

As the ranking preference of people increases, y is larger for smaller m (i.e., for higher-ranking teams). Therefore, equation (1) reflects the strength of people's ranking preference.

Because the revenue of a team is determined solely by its rank, teams' revenues can be very different even if their abilities are almost the same. It is assumed for simplicity that there is no capital or other cost related to owning and operating a sports team except for player salaries. Hence, through competition among teams, all of the revenue of a team will be used for player salaries, and team revenue will eventually be equal to the sum of the salaries of the players on the team.

The abilities of people approximately follow a normal distribution, and top athletes correspond to the tail of the distribution of ability with regard to sports. Therefore, in a professional league, the abilities of players will increase approximately exponentially from the lowest-ability player to the highest, even though the differences will be small because there is an upper limit on human ability. Hence, it is assumed for simplicity that the ability of the players increases exponentially from the rank *P* player to the rank 1 player, even though the absolute differences may be small.

3.2 Player salary examples

3.2.1 Example 1

Suppose that there are only two teams and four players (player *A*, *B*, *C*, and *D*) in a league (i.e., $M = 2$, $P = 4$, and $n = 2$). The abilities of the four players are

$$a_A = e^{0.15} = 1.1618,$$
$$a_B = e^{0.1} = 1.1052,$$
$$a_C = e^{0.05} = 1.0512, \text{ and}$$
$$a_D = 1 \quad ,$$

where a_A, a_B, a_C, and a_D are the abilities of players *A*, *B*, *C*, and *D*, respectively. Abilities increase exponentially from players *D* to *A*, but the differences among their abilities are not large; in this case, the ability of *A* is 1.16 times that of *D*.

As assumed in Section 3.1, the probability of winning the championship is determined by the relative difference between the abilities of teams (i.e., between the sums of the two players' abilities on the two teams). The probability of a win with each combination of players in the two teams is assumed as follows.

- The probability that the team with players *A* and *B* ($a_A + a_B = 2.267$) wins over the team with players *C* and *D* ($a_C + a_D = 2.051$) is 1. Thereby, the probability that the team with players *C* and *D* wins over the team with players *A* and *B* is 0.
- The probability that the team with players *A* and *C* ($a_A + a_C = 2.213$) wins over the team with players *B* and *D* ($a_B + a_D = 2.105$) is 0.8. Thereby, the probability

that the team with players *B* and *D* wins over the team with players *A* and *C* is 0.2.

- The probability that the team with players *A* and *D* ($a_A + a_D = 2.162$) wins over the team with players *C* and *D* ($a_B + a_C = 2.156$) is 0.7. Thereby, the probability that the team with players *C* and *D* wins over the team with players *A* and *D* is 0.3.

These probabilities mean that even a small relative difference between the two teams' abilities generates a large difference in the probabilities of winning.

The team that wins the championship has a revenue of 2, and the losing team has a revenue of 1. Hence, the total revenue of the two teams is 2 + 1 = 3. In addition, for any player, the probability of belonging to either of the two teams is 50%; that is, the players are indifferent between the two teams. Therefore, if a team hires player *A*, the expected revenue of the team (Ω_A) is calculated by the average of revenues weighted by the probabilities of winning in the cases that player *A* is teamed with players *B*, *C*, and *D*. Hence, by using the above probabilities of winning the championship and the given revenue conditions,

$$\Omega_A = \frac{(2\times 1+0)+(2\times 0.8+0.2)+(2\times 0.7+0.3)}{3} = 1.83 \ . \tag{2}$$

Similarly, the expected revenues of the team if it hires the other players are

$$\Omega_B = 1.5 \ , \tag{3}$$
$$\Omega_C = 1.36 \ , \text{ and} \tag{4}$$
$$\Omega_D = 1.3 \tag{5}$$

where Ω_B , Ω_C , and Ω_D are the expected revenues of the teams that hire players *B*, *C*, and *D*, respectively.

Because both teams use all of their revenue for player salaries (as assumed in Section 3.1) and the probability of belonging to a team is identical for any team and player, the expected salary of a player is the average of the remaining three players' salaries. Hence,

$$\Omega_A = z_A + \frac{z_B + z_C + z_D}{3} \ , \tag{6}$$

$$\Omega_B = z_B + \frac{z_A + z_C + z_D}{3} \ , \tag{7}$$

$$\Omega_C = z_C + \frac{z_A + z_B + z_D}{3} \quad \text{, and} \tag{8}$$

$$\Omega_D = z_D + \frac{z_A + z_B + z_C}{3} \quad . \tag{9}$$

where z_A, z_B, z_C, and z_D are the salaries of players *A*, *B*, *C*, and *D*, respectively. By equations (2) to (9),

$$z_A - z_B = \frac{3}{2}(\Omega_A - \Omega_B) = 0.5 \quad , \tag{10}$$

$$z_B - z_C = \frac{3}{2}(\Omega_B - \Omega_C) = 0.2 \quad \text{, and} \tag{11}$$

$$z_C - z_D = \frac{3}{2}(\Omega_C - \Omega_D) = 0.1 \quad . \tag{12}$$

By equations (10), (11), and (12),

$$z_A = 0.5 + 0.2 + 0.1 + z_D \ , \tag{13}$$

$$z_B = 0.2 + 0.1 + z_D \ \text{, and} \tag{14}$$

$$z_C = 0.1 + z_D \ . \tag{15}$$

Because both teams use all of their revenue for player salaries, the total salary of all players is equal to the total revenue of the two teams (in this case, 3). Therefore, by equations (13), (14), and (15),

$$z_A + z_B + z_C + z_D = 1.2 + 4\, z_D = 3 \ . \tag{16}$$

By equation (16), $z_D = 0.45$, and thereby, by equations (13), (14), (15), and (16),

$z_A = 1.25$,
$z_B = 0.75$,
$z_C = 0.55$, and
$z_D = 0.45$.

The calculated salaries indicate that (1) the differences in salaries among players are far larger than those in their abilities (i.e., $\frac{z_A}{z_D} = 2.78$ while $\frac{a_A}{a_D} = 1.16$), and (2) the salary increases approximately exponentially as the rank of players rises from *D* to *A*.

3.2.2 Example 2

In this example, the probabilities used in Example 1 are modified such that the probability of winning the championship is generalized as follows:

- The probability that the team with players *A* and *B* wins over the team with *C* and *D* is *u*. Thereby, the probability that the team with players *C* and *D* wins over the team with *A* and *B* is $1 - u$.
- The probability that the team with players *A* and *C* wins over the team with *B* and *D* is *v*. Thereby, the probability that the team with players *B* and *D* wins over the team with *A* and *C* is $1 - v$.
- The probability that the team with players *A* and *D* wins over the team with *C* and *D* is *w*. Thereby, the probability that the team with players *C* and *D* wins over the team with *A* and *D* is $1 - w$.

Here, $0.5 < u \leq 1$, $0.5 < v < 1$, $0.5 < w < 1$, and $w < v < u$ because of the differences in abilities of players. The other conditions are the same as in Example 1, and by the same procedure used in Example 1,

$$z_A = \frac{u+v+z}{2} \quad ,$$

$$z_B = \frac{2+u-v-w}{2} \quad ,$$

$$z_C = \frac{2-u+v-w}{2} \quad , \text{and}$$

$$z_D = \frac{2-u-v+w}{2} \quad .$$

Hence,

$$z_A - z_B = v + w - 1, \tag{17}$$

$$z_B - z_C = u - v \quad , \text{and} \tag{18}$$

$$z_C - z_D = v - w\,. \tag{19}$$

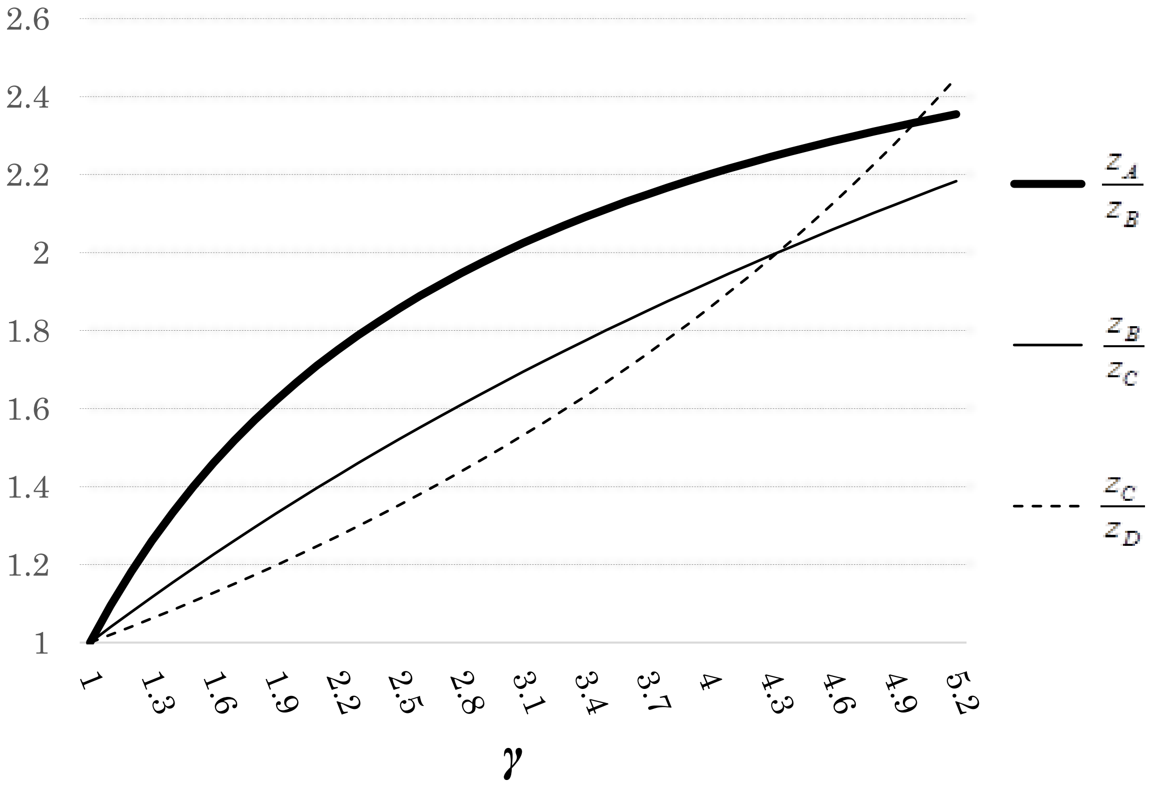
Because $w < v < u$, $z_B - z_C > z_C - z_D$. Thereby, by equations (18) and (19), the sequence of z_D, z_C, and z_B can be approximated by an exponential increase from z_D to z_B. In addition, equations (17) and (18) indicate that, if the values of *u*, *v*, and *w* are sufficiently close (i.e., if they decrease gradually from *u* to *w*), then z_A is far larger than z_B. The gradual decrease from u to w means that the probability of a win for the team to which player A belongs will not sharply decrease, even when the next highest-ranking teammate (B) is replaced with the lowest ranking player (D); that is, player A plays the pivotal role regardless of the teammate. If this condition is satisfied, the sequence of z_D, z_C, z_B, and z_A can be approximated by an exponential increase from z_D to z_A.

3.2.3 Example 3

In this example, the revenue used in Example 1 is generalized. A team's revenue when the team wins the championship is changed from 2 to γ (> 1), whereas the revenue when it loses remains 1. The value of γ varies depending on equation (1); that is, γ reflects the strength of the ranking preference of people. The other conditions are the same as in Example 1. By the same procedure as used in Example 1,

$$z_A = 0.75(\gamma - 1) - 0.5 = 0.75\gamma - 0.25,$$
$$z_B = 0.25(\gamma - 1) + 0.5 = 0.25\gamma + 0.25,$$
$$z_C = 0.05(\gamma - 1) + 0.5 = 0.05\gamma + 0.45, \text{ and}$$
$$z_D = -0.05(\gamma - 1) + 0.5 = -0.05\gamma + 0.55.$$

Figure 1: Values of the ratios $\frac{z_A}{z_B}$, $\frac{z_B}{z_C}$, and $\frac{z_C}{z_D}$



As γ increases, z_A increases the most, followed by z_B and z_C; z_D decreases. Figure 1 shows how the ratios $\frac{z_A}{z_B}$, $\frac{z_B}{z_C}$, and $\frac{z_C}{z_D}$ change as γ increases. All three ratios increase as γ increases, and if γ is less than about 4, then

$$\frac{z_A}{z_B} > \frac{z_B}{z_C} > \frac{z_C}{z_D} \quad .$$

Therefore, if γ is less than about 4, the sequence of z_D, z_C, z_B, and z_A can be approximated by an exponential increase from z_D to z_A. The case where γ is greater than about 4 is discussed in Section 3.3.3.

3.2.4 Example 4

Example 4 combines the generalizations given in Examples 2 and 3. Variables γ, u, v, and w are the same as those used in Examples 2 and 3, and the other conditions are same as in Example 1. By the same procedures used in Examples 1, 2, and 3,

$$z_A = \frac{(\gamma-1)(u+v+w-1)+1}{2} \quad ,$$

$$z_B = \frac{(\gamma-1)(1+u-v-w)+1}{2} \quad ,$$

$$z_C = \frac{(\gamma-1)(1-u+v-w)+1}{2} \quad \text{, and}$$

$$z_D = \frac{(\gamma-1)(1-u-v+w)+1}{2} \quad .$$

The basic features of Examples 2 and 3 are preserved in this more generalized example.

3.3 Probability of winning and player salary

The examples discussed in Section 3.2 show that player salaries will generally increase exponentially from the bottom- to the top-ranked player. In this section, I explore whether these results are to be expected, or whether they are an exceptional outcome or stem from unnatural parameter values.

3.3.1 Probability of winning

As was the case in Section 3.1, suppose that there are M teams and P players and each team consists equally of n players. The probability that a player of rank r belongs to a team of rank m is identical for any m and r. The ability of a team is the sum of the abilities of all players who belong to the team, and the team with the higher total ability has a higher probability of winning the championship than its opponent. Although each team consists of n players, there are many possible combinations of n players on a team. Suppose that the number of possible combinations in which a player with rank r is included as one of the n players on a team is Λ. A natural number is assigned to each of the Λ possible combinations in order from 1 to Λ. Note that the number of possible combinations is commonly Λ for any r.

Let $\tilde{a}_r$ be the expected ability of a team to which rank r player belongs, and let $a_{r,\lambda}$ be the ability of team for a combination λ ($\in \Lambda$). Because the probability that a player with rank r belongs to a team with rank m is identical for any m and r, all of the combinations have an equal probability of being realized. Hence, $\tilde{a}_r$ is calculated by using the simple average of $a_{r,\lambda}$ such that

$$\tilde{a}_r = \Lambda^{-1} \sum_{\lambda=1}^{\Lambda} a_{r,\lambda} \ .$$

$\tilde{a}_r$ can be divided into two parts. One part is attributed to the combinations in which the player with rank r + 1 belongs to the team, and the other is attributed to the combinations in which the rank r + 1 player does not. Let $\tilde{a}_{r,r+1}$ be the former and $\tilde{a}_{r,r}$ be the latter. Thereby,

$$\tilde{a}_r = \tilde{a}_{r,r+1} + \tilde{a}_{r,r} \ . \tag{20}$$

$\tilde{a}_{r+1}$ can similarly be divided into two parts: one attributed to the combinations in which the rank r player belongs to the team and the other in which the rank r player does not. Let $\tilde{a}_{r+1,r}$ be the former and $\tilde{a}_{r+1,r+1}$ be the latter. Thereby,

$$\tilde{a}_{r+1} = \tilde{a}_{r+1,r} + \tilde{a}_{r+1,r+1} \ . \tag{21}$$

Because the combinations in which both the rank r and r + 1 players belong to the team is common in $\tilde{a}_r$ and $\tilde{a}_{r+1}$, then

$$\tilde{a}_{r,r+1} = \tilde{a}_{r+1,r} \ . \tag{22}$$

By equations (20), (21), and (22),

$$\tilde{a}_r - \tilde{a}_{r+1} = \tilde{a}_{r,r} - \tilde{a}_{r+1,r+1} \tag{23}$$

for any r. Here, because the abilities of players increase exponentially from the bottom to the top player,

$$\tilde{a}_{r,r} - \tilde{a}_{r+1,r+1} > \tilde{a}_{r+1,r+1} - \tilde{a}_{r+2,r+2} \quad . \tag{24}$$

for any r. Therefore, by equation (23) and inequality (24),

$$\tilde{a}_r - \tilde{a}_{r+1} > \tilde{a}_{r+1} - \tilde{a}_{r+2} \tag{25}$$

for any r. Inequality (25) indicates that $\tilde{a}_r$ can be approximated by an exponentially increasing function of $P - r$; that is, $\tilde{a}_r$ increases exponentially as the rank of the player increases.

Because a win or loss is determined by the relative differences in teams' abilities, as shown in Section 3.1, by inequality (25) the probability of a win for a team to which a player with rank r belongs can also be approximated by an exponentially increasing function of $P - r$. In addition, because a team's revenue (Ω_r) is determined by the probability of winning the championship as shown in the previous examples, Ω_r can also be approximated by an exponentially increasing function of $P - r$.

3.3.2 Amplified probabilities of a win

3.3.2.1 Amplification

Because only slight differences in teams' abilities are decisive for winning and losing, even a small relative difference between the teams' abilities will result in a large difference in the probabilities of winning and losing. This means that the ability differences are amplified in the differences in teams' probabilities of winning the championship, as shown in Example 1.

Let Ψ (> 0) be the difference in the abilities of a team and an opposing team, and let $p(\Psi)$ be the probability that the team with greater ability wins. Considering the amplification effect, $p(\Psi)$ can be modeled as

$$p(\Psi) = \rho\frac{1}{2} + (1-\rho)\frac{\exp(\tau\Psi)}{2} \quad \text{if } \frac{\rho}{2} + (1-\rho)\frac{\exp(\tau\Psi)}{2} \leq 1$$

$$p(\Psi) = 1 \quad \text{if } \frac{\rho}{2} + (1-\rho)\frac{\exp(\tau\Psi)}{2} > 1 \tag{26}$$

where ρ $(0 < \rho < 1)$ is the probability that a win or loss is determined by chance (in which case the probability of a win is $\frac{1}{2}$), and τ (> 0) is a parameter and represents the degree of amplification. If τ is sufficiently large, $p(\Psi)$ is around unity even if the difference in abilities (Ψ) is very small. This means that relative superiority is the key for a win, even when the absolute difference in abilities is small. It seems natural that, in general, equation (26) holds and τ is sufficiently large. That is, the difference in abilities is greatly amplified in the difference in the probability of winning the championship. As a result, revenues (Ω_r) will increase more sharply than players' abilities (a_r) do as r decreases from P to 1.

3.3.2.2 Moderation

However, even if τ is sufficiently large, it will not be too large; that is, cases in which Ψ is such that $p(\Psi)$ is nearly equal to 1 will not often be observed in most professional sport leagues. Professional teams are generally also competing as profit-seeking firms and, as a result, the abilities of teams will nearly converge to a similar level. More importantly, even if the abilities of teams diverge greatly for some reason, this situation will be corrected by the governing authority of the league, because overly large differences in teams' abilities will greatly reduce the total ranking value people feel from league games.

People may lose interest in a professional league if one particular team always wins, because they may judge that teams from different categories are competing in the games. If the result is evident before the games are played, people may not feel and extract enough ranking value from watching the games, so that the league may not be able to function successfully as a mechanism that provides ranking value to consumers. Therefore, overly large differences in abilities are problematic for sports leagues. Hence, if the abilities of teams become too divergent, the league authority may take some of the following measures:

1) Introduce a draft for rookies.
2) Introduce a cap on a team's total player salaries.
3) Pool some parts of teams' revenues (e.g., broadcasting rights) that are later distributed to teams so as to equalize the abilities of teams.
4) Change the game rules to increase ρ (i.e., a win or loss is more largely determined by chance).

The above corrections will continue to be strengthened until the total ranking value people feel and extract from league games is maximized.

3.3.3 Strength of ranking preference

Examples 3 and 4 indicate that ability differences are amplified in salary differences through the effect of ranking preference (i.e., γ) because $\gamma > 1$. As the ranking preference of people becomes stronger (i.e., γ is larger), the salary differences grow larger, even if the players' abilities are unchanged. Ω_r and z_r also increase more sharply than players' abilities (a_r) do as r decreases from P to 1. The strength of ranking preference is therefore a key factor that generates superstars in team sports.

Note, however, that Example 3 indicates that if γ (> 1) is very large (e.g., if $\gamma > 5$ in Example 3), the salary of a player cannot be approximated by an exponentially increasing function of $P - r$, because z_D decreases to zero and eventually becomes negative as γ increases. If a team offers negative salaries to players with lower ranks (i.e., $z_D < 0$), those players will not join the league and teams will not be able to employ a sufficient number of players to sustain the league. Therefore, a league can be sustained only when the ranking value of a win relative to that of a loss (i.e., γ in Example 3) is not above some threshold value.

Normally, however, in a professional league the ranking value of a win for the top-ranked team will most likely not be tremendously greater than that of the bottom-ranked team, because the players are all top athletes. There are usually many minor and amateur leagues for most sports, and the teams in a top-level professional sport league are all among the few top teams in the sport as a whole (i.e., including those in minor and amateur leagues). Hence, people generally will regard even the bottom-ranked team in a professional league as ranking sufficiently highly compared with all other teams. Therefore, the ranking value of a rank 1 team should generally not be much greater than that of the bottom team to exceed the sustainable threshold value.

3.4 Mechanism of generation of superstars in team sports

On the basis of the examples presented and examined in Sections 3.2 and 3.3, the salary of a player in a team sport can be modeled as follows. The ability of a rank r player is

$$a_r = \exp[\alpha(P - r)] \tag{27}$$

where α (> 0) is a parameter and $a_P = 1$; that is, the ability of the rank P (lowest-ranked) player is normalized to be unity. As shown in Section 3.3.1, $\tilde{a}_r$ and Ω_r increase exponentially as r decreases (i.e., as rank rises), and as shown in Sections 3.3.2 and 3.3.3, any increase in a_r is amplified in increases in Ω_r by the effect of γ and $p(\Psi)$.

Ω_r consists of the salary of the rank r player (z_r) and the salaries of the other $n - 1$ teammates. If P and n are sufficiently large, the salaries of the $n - 1$ teammates are approximately equal to the average salary of all players ($\bar{z}$) times $n - 1$. Therefore, the salary of the rank r player is approximately

$$z_r = \Omega_r - (n-1)\bar{z} \quad .$$

Hence,

$$z_r - z_{r+1} = \Omega_r - \Omega_{r+1} \quad ,$$

and by iterations,

$$z_r - z_P = \Omega_r - \Omega_P \quad . \tag{28}$$

By equation (28), the salary of the rank r player (z_r) is

$$z_r = \Omega_r + (z_P - \Omega_P) \quad . \tag{29}$$

Equation (29) indicates that z_r is positively proportionate to Ω_r. Because Ω_r increases exponentially as r decreases from P to 1, as shown in Section 3.3.1, z_r also increases exponentially. In addition, Ω_r increases more sharply than does a_r, as shown in the examples and discussed in Sections 3.3.2 and 3.3.3. As a result, the salary of the rank r player can be modeled as

$$z_r = (z_R - \delta) + \delta \exp[\alpha\beta(P - r)] \tag{30}$$

where β (> 1) and δ (> 0) are parameters.

Compare equation (27), which describes a player's ability, with equation (30), which describes a player's salary. Both ability (a_r) and salary (z_r) increase exponentially from the bottom to the top player, but an increase in a player's ability is amplified greatly in the corresponding increase in the player's salary by β through the effect of γ and $p(\Psi)$. Therefore, even if α (> 0) is very small, $\alpha\beta$ can be very large (i.e., even if the differences in players' abilities are small, differences in their salaries can be very large). A few top players can thereby obtain extremely large salaries as compared with those of many other players and can become superstars in team sports.

4 CONCLUDING REMARKS

In this paper, the mechanism of generation of superstars in team sports is examined on the basis of the model of ranking value and preference presented by Harashima (2016, 2017). In this model, there are two kinds of value: practical value and ranking value. Ranking value and preference provide monopoly powers to the producers of highly ranked goods and services. If households' ranking preferences are strong enough, the highest-ranked producer can be a "superstar."

In team sports, unlike in individual sports, the monopoly producer is the team, not the individual player. Even so, I showed here that an individual player can be a superstar and earn a disproportionately large salary than other players in a team sport. An essential point in this mechanism is that the probability of a team winning differs depending on whether or not the team has high-ability players. This characteristic, combined with the effect of ranking preference, means that player salaries can increase exponentially from the bottom- to top-ranked players. As a result, top-ranked players can be superstars even in team sports.

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