Time-inconsistent environmental policies with a consumer-friendly firm: tradable permits versus emission tax

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Abstract
This study considers the timing of environmental policies with a consumer-friendly firm having abatement technology, and compares two market-based regulations: tradable permits and emission tax regulations. When the government can credibly commit its policy, we show that the equilibrium outcomes under both policies are equivalent in terms of permits price and tax rate. Under the non-committed policy, however, the equivalence breaks down because firms have different incentives to induce time-consistent policy to be adjusted ex post. In particular, compared to pre-committed government, firms abate less emission to induce higher emission quotas under the permits policy while a consumer-friendly firm abates more emissions to reduce tax rate under the tax policy. Finally, we show that tax policy can induce higher welfare and lower environmental damage when the concern on consumer surplus is moderate.

Keywords: abatement technology; consumer-friendly firm; environmental policy; tradable permits; emission tax

JEL classification: L13; L31; Q5

1. Introduction

During the last generation, the waves of market-based environmental protection have been salient features of economic policies in polluting industries around the world. In light of the increasing importance of such environmental policy, the government has continuously conducted various environmental regulation by using emission standards, quotas, subsidies, taxes and tradable permits. In particular, the widespread acceptance of permits trading program generates an interesting debate among researchers on the efficiency of environmental and climate change policy.\(^1\) Many economists have shown that

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\(^1\)Kato (2006) introduced some useful real-world discussions on the tradable emission permits as climate change policy instrument in the United Nations. Since the United States implemented permits trading system experimentally in 1980s, many countries including the European Union and China introduced this program gradually during the last decade. Nowadays, it becomes a successful international experiment for controlling a large amount of greenhouse gases in the
governments can promote social welfare by implementing market allocation of tradable permits or, equivalently, emission taxes since they can minimize abatement costs when they differ between firms.\textsuperscript{2}

Recent works on tradable permits have been conducted in a different situation of mixed market where the objectives among firms differ. For example, Kato (2006, 2011) examined a standard mixed market where a welfare-maximizing public firm competes with private firms whose objectives are to maximize profits with homogeneous products in the same market.\textsuperscript{3} He showed that this asymmetry in their objectives can work for improving social welfare even if the abatement technologies are the same because tradable permits can induce the equalization of the marginal costs among firms.

The present paper extends the analysis into the other context of mixed market which are characterized by the co-existence of for-profit firms and not-for-profit firms. In particular, we consider a consumer-friendly firm which competes with a for-profit firm in a mixed duopoly market under the same abatement technologies. We define the objective of the consumer-friendly firm as a combination of consumer surplus and its profits. Thus, the firm puts a higher weight on output in order to commit to a higher output than rival firm. This type of formulation can be viewed as one way of adopting corporate social responsibility (CSR) initiatives, in which it utilizes consumer surplus as a proxy of CSR concern.\textsuperscript{4} Owing to the current expansion of CSR, the heterogeneity of objectives among the firms in this mixed market configuration is an essential part of our analysis.

This study examines tradable permits policy and compares its efficiency with that of emission tax policy in the presence of a consumer-friendly firm. Further, we allow the possible ability of a government to commit credibly to an environmental policy when a pollution-reducing abatement activity is present. When the government determines its policy ex-ante, i.e., before the firms make their abatement decisions, it can be credible only when the regulator possesses a commitment mechanism. Thus, if the government can commit the level of permits or tax, both policies are fully equivalent under perfect competition. However, once abatement has been chosen by the firms, the optimal policy determined ex-ante is not ex-post optimal. This is because firms’ abatement costs are already sunk and thus the

\textsuperscript{2}For example, Borenstein (1988), Malueg (1990) and Sartzetakis (1997) showed that the tradable permits can increase social welfare in the competitive production and emission markets when there are differences with respect to the abatement technologies among regulated firms. However, Requate (1993), Sartzetakis (2004) and Lee and Park (2005) demonstrated that if firms differ in both production and abatement technologies, the tradable permits cannot always assure efficiency.

\textsuperscript{3}On the other hand, recent works on mixed markets with environmental tax policies can be found in Ohori (2006), Wang and Wang (2009), Pal and Saha (2015), Xu and Lee (2015, 2018) and Xu et al. (2016).

\textsuperscript{4}Numerous theoretical studies have recently analyzed this type of formulation for analyzing the CSR activities in different competition models. For example, see Goering (2012, 2014), Kopel and Brand (2012), Chang et al. (2014), Matsumura and Ogawa (2014), Brand and Grothe (2015) and Lambertini and Tampieri (2015) among others.
ex post government objective function differs from the ex-ante. Thus, such policies which require firms to invest may be subject to a time-consistency problem. It resembles a hold-up problem, caused by the strategic behavior of firms. Therefore, if the government cannot react to the firm’s abatement activities, the equivalence of both policies might break down. This suggests that the ability of credible commitment to an environmental policy in the policy-making process has significant implications to support the equivalence between the two policies associated with a committed policy.

In the previous literature of environmental economics, Denicolò (1999), Gersbach and Glazer (1999), Requate and Unold (2003) investigated the commitment problem when the regulator is not able to commit credibly to permits policy and shed light on the ex ante and ex post welfare effects of strategic behaviors of the regulated firms. Petrakis and Xepapadeas (1999, 2001) and Poyago-Theotoky and Teerasuwanajak (2002) pointed out the time-inconsistency problem under emission tax policy and showed that if the regulator cannot commit credibly to the stringency of the tax, firms have strategic incentives because the regulator has an ex-post possibility to ratchet up regulation. D’Amato and Dijkstra (2015) also examined environmental technology adoption with/without long-term commitment in which the government ex-ante commits a tax rate before the innovation takes place or adjusts an ex-post tax rate after the firms invest.\(^5\) Regarding the strategic choices in a dynamic setting with comparison between taxes (price regulation) and permits (quantity regulation), Montero (2011), Wirl (2014) and Moner-Colonques and Rubio (2015) examined firm’s innovation incentives when the government has time-consistent temptations to revise its policy design after innovation and showed that the performance of two policy instruments differ.

The present paper adopts the recent analysis of the comparison between the two market-based environmental policies, tradable permits and emission tax policies, in a mixed duopoly with a consumer-friendly firm in order to examine the efficiency between committed ex-ante policy and non-committed ex-post policy. As a closely related work, Moner-Colonques and Rubio (2015) examined the timing of the environmental policy in a private duopoly model where both firms have the same profit functions, and showed that the welfare results between emission standard and emission tax depend on the efficiency of abatement technology. However, our analysis considers a mixed duopoly model where both firms have asymmetric payoff functions, and examines trading emission permits policy rather than non-tradable emission permits policy. Hence, these two differences lead to different policy implications.

Under the committed policy, we show that the equilibrium outcomes under both policies are equivalent in terms of permit price and tax rate. We also show that both permits price and tax increase as

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\(^5\)Recent analysis of the timing of ex-post tax policies in mixed oligopolies can be found in Xu et al. (2017), Hsu et al. (2017), Lian et al. (2018), Leal et al. (2018) and Lee et al. (2018).
the concern on consumer surplus rises because higher productions from the larger concern on consumer surplus cause higher total emissions. Further, we show that both firms can earn higher profits with permits rather than a tax, but the profit of a consumer-friendly firm is always larger than rival’s profit irrespective of policy instruments. However, both welfare and environmental damage are simultaneously decreasing or increasing depending on the degree of consumer-friendliness. Thus, higher asymmetry between the firms might result in the welfare loss, which can outweigh the gains obtained from the environmental regulation. This result represents a typical trade-off between welfare and environmental damage under the committed policy regime.

We also compare permits and tax policies under the non-committed policy and show that some results under the committed policy still hold. However, we also find several different results in which the equivalence breaks down. First, both market price of permits and tax rate increase as the concern on consumer surplus rises but permits price is always higher than the tax rate. This is because firms have different incentives to induce ex-post time-consistent policy under the non-committed policy. In particular, compared to the outcomes under the committed policy, both firms reduce abatement levels under the permits policy to increase their emission quotas unless the concern is too large while a consumer-friendly firm increases its abatement levels under the tax policy to reduce tax rate unless the concern is too small. Thus, the significance of strategic incentives of the firms affect the equivalence properties of both policies.

Second, the abatement activities and emissions of a consumer-friendly firm are always larger than those of a for-profit firm, but its relative amounts depend on the concern on consumer surplus and policy instruments. In particular, both firms’ abatement activities under permits are always lower than tax, and thus, total emissions under permits might be more significant than those under tax when the consumer-friendliness is high.

Finally, under both policies, it is possible that welfare increases but environmental damage decreases when the consumer-friendliness is not too large. This result sharply contrasts to the result under the committed policy. Furthermore, under the non-committed policy, tax policy induces higher welfare and lower environmental damage than permits policy when the consumer-friendliness is moderate. Therefore, due to the larger time-consistent distortion of abatement activities with permits policy, tax can be a better policy than permits in the presence of a consumer-friendly firm.

The remainder of this paper is organized as follows. In section 2, we formulate a Cournot duopoly model with a consumer-friendly firm having abatement technology. We analyze tradable permits and tax, respectively, in section 3 and 4. Finally, section 5 compares the results and provide main findings. Section 6 concludes the paper.
2. Model

We consider a quantity-setting Cournot duopoly model. One of the firms is a consumer-friendly (CF) firm (hereafter referred to as firm 0) that cares for not only its profits but consumers surplus. The other is a for-profit (FP) firm (hereafter referred to as firm 1) that maximizes its profits only. Firms sell homogeneous output, $q_0 > 0$ and $q_1 > 0$, respectively, at the market clearing price $p(Q) = 1 - Q$ where $Q = q_0 + q_1$. We assume that both firms have identical technologies and the production cost function takes a quadratic form, $c(q_i) = q_i^2$, $i \in \{0, 1\}$.

Production leads to pollution, $e_i > 0$, but each firm can reduce pollution by undertaking abatement activities. Suppose that firm $i$ chooses pollution abatement level $z_i > 0$. Then, the emission level can be reduced to $e_i = q_i - z_i$ by investing an amount of $\frac{z_i^2}{2}$ in abatement.

The government has a responsibility to regulate emissions. We consider two policy options that the regulator may use to protect the environment. The first policy is a tradable permits regulation: The government decides the total emission levels to maximize the social welfare and then assigns emission quotas (permits) $E_i$ to each firm. At the same time, the government allows the firms to trade emission permits at the emission trading market. We assumed that the emission trading market is competitive and thus the emission trading occurs by the market clearing price. Thus, if we define the net demand of firm $i$ as $D_i = e_i - E_i$, total net demand of emission permits is zero at the market equilibrium $D_0 + D_1 = 0$.

The profit of firm $i$ is given by:

$$\pi_i = p \cdot q_i - q_i^2 - \frac{1}{2} z_i^2 - \lambda \cdot (e_i - E_i), \quad i = 0, 1$$

(1)

The second policy consists in an emission tax regulation: The government imposes a tax on the emission level, for which the tax rate is $t$. The resulting total tax revenue collected by the government is $T = t \sum_i e_i$. The profit of firm $i$ is given by:

$$\pi_i = p \cdot q_i - q_i^2 - \frac{1}{2} z_i^2 - t \cdot e_i, \quad i = 0, 1$$

(2)

We assume that the FP firm seeks only for profit maximization. However, the CF firm maximizes profits plus a fraction of consumer surplus, $CS = \frac{Q^2}{2}$. Thus, the payoff that CF firm maximizes is as

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6Our model could be extended to the oligopoly model without further insights gained.
7The specific function guarantees the interior solutions in the equilibrium for $\theta \in [0, 1]$.
8This is the only difference with the command-and-control regulation of assigning emission quotas. We compared non-tradable emission permits and tradable emission permits policies in Appendix B.
9It implies that both firms do not have market powers in the emission market, but product market is in a duopolistic competition. Regarding the interaction of a competitive market for emission permits with an imperfectly competitive product market, see Borenstein (1988), Sartzetakis (1997, 2004), Lee and Park (2005) and Kato (2006).
follows:

\[ V_0 = \pi_0 + \theta CS \] (3)

The parameter \( \theta \in [0, 1] \) measures the degree of concern on consumer surplus that the CF firm has, as a CSR-initiative, which is exogenously given.

The extent of environmental damage due to pollution by the industry is given by \( ED = \frac{(\sum_i e_i)^2}{2} \). Note that total environmental damage under permits policy with an emission quota \( E_i \) to each firm becomes \( ED = \frac{(\sum_i E_i)^2}{2} \). Then, the social welfare is the sum of consumer surplus (CS), the profits of both firms \((\pi_0 + \pi_1)\) and the total taxes collected by the government \((T)\) minus environmental damage \((ED)\):

\[ W = CS + \pi_0 + \pi_1 + T - ED \] (4)

where \( T = 0 \) if the government implements a tradable permits policy.\(^{10}\)

We shall consider two alternative policy regimes, each featuring a three-stage game between a welfare maximizing regulator and two firms with different objectives. Both policies are analyzed in the context of committed or non-committed policy. In the **committed policy**, the regulator sets the environmental instrument value such as emission quotas or tax, depending on policy implemented. Taking the instrument value as given, the firms choose investment in abatement effort simultaneously and independently. In the **non-committed policy**, firms first select its abatement level, simultaneously and independently, and then the regulator sets the emission quotas or tax. Finally, the firms select output in the third stage.

\(^{10}\)We assume that the government utilizes a grandfathering approach for initial allocating of costless permits to the firms. It has by far been the dominant allocation approach in practice, both because it can offset the costs of emission reduction as well as for political reasons. See Stavins (1998) and Fowlie (2010). However, if the government implements an auctioning for the initial allocating of permits, \( T \) is a fixed lump-sum amount and thus we can treat it as a constant number. On the other hand, while the emission tax revenue can provide double-dividend effect, which can be used for not only externality but for public finance to eliminate other distorting tax system such as income tax, the emission tax has its own distorting effects on labor supply, which can have the excess burden as a tax on labor income. In the following analysis, we ignore these effects of the public finance.
3. Committed policy

3.1. Tradable emission permits

In the third stage firms 0 and 1 choose their outputs to maximize (3) and (1), respectively. By solving these problems the equilibrium output as a function of the permit price, $\lambda$, is obtained:

$$q_0 = \frac{(3 + \theta)(1 - \lambda)}{3(5 - \theta)}, \quad q_1 = \frac{(3 - \theta)(1 - \lambda)}{3(5 - \theta)}, \quad Q = \frac{2(1 - \lambda)}{5 - \theta} \quad (5)$$

Note that each firm’s output decreases in the permit price. Also if the concern on consumer surplus rises, the CF firm is more aggressive and thus increases its output while the FP firm decreases the output. However, the total outputs increases.

In the second stage, firms choose abatement efforts to maximize their payoffs. Firm 0 chooses $z_0$ that maximizes (3) while firm 1 chooses $z_1$ that maximizes (1). Solving these problems gives the equilibrium abatement level as a function of the permit price:

$$z_i = \lambda, \quad i \in \{0, 1\} \quad (6)$$

that defines a positive relationship between abatement and the permit price. It simply states that tradable emission permits make the firm’s marginal cost in abatement equal to the permit price.

Substituting (5) and (6) into the total net demand of the emission quota, where $D_0 + D_1 = \sum_i ((q_i - z_i) - E_i) = 0$, yields:

$$\lambda = \frac{2 - (5 - \theta) (E_0 + E_1)}{2(6 - \theta)} \quad (7)$$

One can easily check a negative relationship that the market price of permits is higher if the regulator reduces the emission quota of firm $i$, i.e., $\frac{\partial \lambda}{\partial E_i} < 0$.

Substituting (7) into (5) and (6), we obtain outputs and abatement levels as functions of the emission quotas. In this stage the regulator assigns the emission quota to each firm that maximizes social welfare, given by expression (4). This maximization yields the following condition:

$$(1 - Q (E_i, E_j)) \left( \frac{\partial Q}{\partial E_j} - 2 \sum_{i=0}^1 q_i (E_i, E_j) \frac{\partial q_i}{\partial E_j} - \sum_{i=0}^1 z_i (E_i, E_j) \frac{\partial z_i}{\partial E_j} \right) = \sum_{i=0}^1 E_i, \quad j = 0, 1 \quad (8)$$

where the first term on the left-hand side measures the increase in consumer surplus coming from the increase in total market outputs when the regulator raises the emission quotas. The second term stands for the increase in production cost coming from the increase in each firm’s output, and the third term stands for the decrease in abatement cost from the decrease in each firm’s abatement effort, respectively, when the regulator raises the emission quotas. The right-hand side implies the increase in environmental damages coming from increase in emission quotas.
From (8), we obtain the total emission quota

$$\sum_{i=0}^{1} E_i = \frac{2(63 - 18\theta - 2\theta^2)}{H}$$  \hspace{1cm} (9)$$

where $H = 909 - 306\theta + 29\theta^2 > 0$. When the government sets a non-discriminatory equal emission quota, we have:

$$E_{tpc}^0 = E_{tpc}^1 = \frac{63 - 18\theta - 2\theta^2}{H}$$  \hspace{1cm} (10)$$

We employ superscript $tpc$ to denote the equilibrium under the tradable permits policy with the commitment. Then, we have $\frac{\partial E_{tpc}^i}{\partial \theta} > 0$ if $\theta \leq \frac{3}{7}$. It states that the relationship between the regulator’s optimal emission quota and the concern on consumer surplus is non-monotonic. When the concern on consumer surplus is low, the optimal emission quota increases with the concern. However, when the concern on consumer surplus is increased past a certain level, the optimal emission quota begins decreasing with the concern.

From (10) the equilibrium permit price, output, abatement levels and emissions are obtained:

$$\lambda^c = \frac{99 - 9\theta + 2\theta^2}{H}, \quad z_{tpc}^0 = z_{tpc}^1 = \lambda^c$$

$$q_{tpc}^0 = \frac{9(6 - \theta)(3 + \theta)}{H}, \quad q_{tpc}^1 = \frac{9(6 - \theta)(3 - \theta)}{H}$$

$$e_{tpc}^0 = \frac{63 + 36\theta - 11\theta^2}{H}, \quad e_{tpc}^1 = \frac{63 - 72\theta + 7\theta^2}{H}$$  \hspace{1cm} (11)$$

In equilibrium under the committed permits policy, the output of CF firm is larger than that of FP firm, but both firms make the same abatement effort; therefore the CF firm’s emission level is also larger than its rival’s. Note that $\frac{\partial q_{tpc}^i}{\partial \theta} > 0$, $\frac{\partial z_{tpc}^i}{\partial \theta} < 0$ and $\frac{\partial e_{tpc}^i}{\partial \theta} > 0$. Also, we have $\frac{\partial \lambda^c}{\partial \theta} > 0$. Thus, the equilibrium permit price increases as the concern on consumer surplus rises. Furthermore, $D_0 > 0$ which implies that the CF firm buys emission permits.

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\[11\] In Appendix C, we examine the case where the government sets a discriminatory emission quota under the committed policy and show that our analysis remains except the firms’ profits. In reality, however, the information burden is quite demanding for a discriminatory quotas system and also politically it might be very costly to manage the discriminatory regulation especially under the large number of interest groups. Regarding the informational asymmetry between the government and firms, Lee (1996) proposed an optional permits regulation while Lee and Kim (1995) and Lee and Kim (2000) analyzed non-linear emission tax regulations.

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Finally, we have the resulting profits of the firms, environmental damage and social welfare:

\[
\begin{align*}
\pi_{0}^{\text{tpc}} &= 127251 - 76464\theta - 8091\theta^2 + 6732\theta^3 - 652\theta^4 \\
\pi_{1}^{\text{tpc}} &= 127251 - 111456\theta + 38565\theta^2 - 5904\theta^3 + 320\theta^4 \\
ED^{\text{tpc}} &= \frac{2(63 - 18\theta - 2\theta^2)^2}{H^2} \\
W^{\text{tpc}} &= \frac{189 - 54\theta - 6\theta^2}{H}
\end{align*}
\]  

(12)

**Proposition 1.** \(\pi_{1}^{\text{tpc}} < \pi_{0}^{\text{tpc}}\) for any \(0 < \theta \leq 1\);

It states that the profit of CF firm is always larger than that of FP firm in equilibrium under the committed permits policy. This is because the CF firm is more aggressive in production to increase consumer surplus, which induces less production of FP firm but more production in total outputs.\(^{12}\)

**Proposition 2.** \(\frac{\partial ED^{\text{tpc}}}{\partial \theta} > 0\) and \(\frac{\partial W^{\text{tpc}}}{\partial \theta} > 0\) if \(0 \leq \theta < \frac{3}{7}\)

It states that both welfare and environmental damage are increasing in \(\theta\) unless \(\theta\) is so high. This result implies that higher degree of \(\theta\) might deteriorate the welfare because of excessive production. This is because trading of emission permits induces excessive output redistribution from the FP firm to CF firm in controlling emissions. Thus, higher asymmetry between the firms results in the inefficiency, which can outweigh the gains obtained from the environmental regulation.\(^{13}\) It also states that both welfare and environmental damage are simultaneously decreasing or increasing depending on the values of \(\theta\). This result represents a typical trade off between welfare and environmental damage in the literature.

### 3.2. Emission tax

In the third stage, firms choose their outputs to maximize their payoffs in (3) and (2), respectively. Given the emission tax rate, \(t\), the first-order conditions get the following equilibrium output level of each firm and total outputs:

\[
q_0 = \frac{(3 + \theta)(1 - t)}{3(5 - \theta)}, \quad q_1 = \frac{(3 - \theta)(1 - t)}{3(5 - \theta)}, \quad Q = \frac{2(1 - t)}{5 - \theta}
\]

(13)

Note that each firm’s output coincides with that under permits policy in (5) if \(t = \lambda\).

\(^{12}\)For more discussion on this point, see Lambertini and Tampieri (2015) and Leal et al. (2018).

\(^{13}\)It also happens in a symmetric duopoly when both firms have different costs. For more discussion on this point, see Borenstein (1988), Malleg (1990) and Sartzetakis (1997).
In the second stage, firms choose abatement efforts to maximize their payoffs in (3) and (2), respectively. The first-order conditions give the equilibrium abatement levels as a function of the tax:

\[ z_i = t, \quad i \in \{0, 1\} \quad (14) \]

It also defines a positive relationship between abatement and the tax, which also states that emission tax make the firm’s marginal cost in abatement equal to the tax rate.

In the first stage the government sets the emission tax that maximizes social welfare in (4). Solving the first-order condition yields the optimal emission tax, which is given by

\[ t^c = \frac{99 - 9\theta + 2\theta^2}{H} \quad (15) \]

where \( H \) is defined as before. We have \( \frac{\partial t^c}{\partial \theta} > 0 \). Thus, the optimal tax increases as the concern on consumer surplus rises. From (11) and (15), it is easy to check that the equilibrium output, abatement and emission levels, environmental damage and social welfare coincide with those of the committed permit policy. Therefore, under regulatory commitment both policy instruments are equivalent in the sense that they yield the same equilibrium outcomes.\(^{14}\)

The profits of both firms under the committed tax regime are the followings:

\[
\begin{align*}
\pi_{t^c0} & = \frac{114777 - 71766\theta - 82716\theta^2 + 6768\theta^3 - 644\theta^4}{2H^2} \\
\pi_{t^c1} & = \frac{114777 - 106758\theta + 38385\theta^2 - 5868\theta^3 + 328\theta^4}{2H^2}
\end{align*}
\quad (16)
\]

We employ superscript \( t^c \) to denote the equilibrium under the tax policy with the commitment.

**Proposition 3.** \( \pi_{t^c1} < \pi_{t^c0} \) for any \( 0 < \theta \leq 1 \);

It states that in equilibrium under the committed tax, the profit of CF firm is always larger than that of FP firm. This result is the same with that under the committed permit policy.

3.3. Comparing permits and tax

**Proposition 4.** \( \pi_{0t^c} < \pi_{0pc} \) and \( \pi_{1t^c} < \pi_{1pc} \) for any \( \theta \in [0, 1] \)

Under the committed policy both firms can earn higher profits with permits policy. This is because both firms can save tax payments under the same outcomes. It also implies that both firms prefer permits policy to tax policy when they can choose or lobby for the policy instruments.

\(^{14}\)Note that this result does not depend on \( \theta \). Moner-Colonques and Rubio (2015) examined the timing of the environmental policy in a private duopoly model with symmetric payoff functions, and showed that the equivalence between emission standard and emission tax holds irrespective of the efficiency of abatement costs and environmental damage parameter.
4. Non-committed policy

4.1. Tradable emission permits

The last stage is the same as in subsection 3.1. In the second stage, the regulator assigns the welfare maximizing emission quota to each firm taking as given the firms’ abatement levels. Welfare defined in (4) becomes:

\[ W = Q(\lambda, \theta) - \frac{Q^2(\lambda, \theta)}{2} - \sum_{i=0}^{1} q_i^2(\lambda, \theta) - \frac{1}{2} \sum_{i=0}^{1} z_i^2 - \lambda \sum_{i=0}^{1} D_i(E_i; z, q_i(\lambda, \theta)) - \frac{1}{2} \left( \sum_{i=0}^{1} E_i \right)^2 \]  

(17)

The first-order conditions are:

\[-\lambda \frac{\partial (D_0 + D_1)}{\partial E_j} = \sum_{i=0}^{1} E_i, \quad j = 0, 1 \]  

(18)

where \( \frac{\partial (D_0 + D_1)}{\partial E_j} = -1 \). Solving (18) we obtain the total emission quota

\[ \sum_{i=0}^{1} E_i = \lambda \]  

(19)

Contrary to the committed policy, it defines a positive relationship between emission quotas and the permits price, that is, the regulator increases the emission quota in response to an increase in the market clearing permits price.

Substituting (5) and (18) into the total net demand of the emission quota, where \( D_0 + D_1 = \sum_i ((q_i - z_i) - E_i) = 0 \), yields:

\[ \lambda = \frac{2 - (5 - \theta) (z_0 + z_1)}{7 - \theta} \]  

(20)

Contrary to the committed policy, we have a negative relationship that permits price is higher if firm \( i \) reduces its abatement level, i.e., \( \frac{\partial \lambda}{\partial z_i} < 0 \) or if the concern on consumer surplus increases, i.e., \( \frac{\partial \lambda}{\partial \theta} > 0 \).

In the first stage, firms choose their abatement efforts taking into account how the regulator is going to respond. Under the non-discriminatory emission quota,\(^{15} \) where \( \bar{E}_0 = \bar{E}_1 = \lambda \frac{\Omega}{2} \), firm 0 chooses \( z_0 \) that maximizes (3) while firm 1 chooses \( z_1 \) that maximizes (1). Solving these problems yields the following equilibrium abatement levels:

\[ z_{tpn}^0 = \frac{2(9 - 2\theta) \left( 12 + 6\theta - \theta^2 \right)}{\Omega} \]
\[ z_{tpn}^1 = \frac{2(9 - \theta) \left( 12 - 6\theta + \theta^2 \right)}{\Omega} \]  

(21)

\(^{15}\)In Appendix C, we also examine the case where the government sets a discriminatory emission quota under the non-committed policy and show that our analysis remains except the firms’ profits.
where \( \Omega = (6 - \theta)R > R = 432 - 135\theta + 11\theta^2 > 0 \). We also employ superscript \( tpn \) to denote the equilibrium under the tradable permits policy with non-commitment. It is easy to show that \( z_{0}^{tpn} > z_{1}^{tpn} \). Thus, CF firm is more aggressive in investing abatement technology, which induces a larger amount of abatement under the non-committed permits policy. Note that \( \frac{\partial z_{0}^{tpn}}{\partial \theta} > 0 \), \( \frac{\partial z_{1}^{tpn}}{\partial \theta} < 0 \), and \( \frac{\partial (z_{0}^{tpn} + z_{1}^{tpn})}{\partial \theta} > 0 \).

From (21), the equilibrium permit price, outputs and emission quotas are obtained:

\[
\lambda_{n} = \frac{2(36 - 9\theta + \theta^2)}{R}, \quad E_{0}^{tpn} = E_{1}^{tpn} = \frac{\lambda_{n}}{2},
\]

\[
q_{0}^{tpn} = \frac{3(8 - \theta)(3 + \theta)}{R}, \quad q_{1}^{tpn} = \frac{3(8 - \theta)(3 - \theta)}{R},
\]

\[
e_{0}^{tpn} = \frac{216 - 42\theta + 9\theta^2 - \theta^3}{(6 - \theta)(432 - 135\theta + 11\theta^2)}, \quad e_{1}^{tpn} = \frac{216 - 138\theta + 21\theta^2 - \theta^3}{(6 - \theta)(432 - 135\theta + 11\theta^2)}
\]

In equilibrium under the non-committed permits policy, the output of CF firm is larger than that of FP firm. We can also show that \( e_{0}^{tpn} > e_{1}^{tpn} \) in (22). Thus, the emission level of CF firm is larger than that of FP firm. Note that \( \frac{\partial q_{0}^{tpn}}{\partial \theta} > 0 \), \( \frac{\partial q_{1}^{tpn}}{\partial \theta} < 0 \) and \( \frac{\partial Q^{tpn}}{\partial \theta} > 0 \). Also, we have \( \frac{\partial \lambda_{n}}{\partial \theta} > 0 \) and thus \( \frac{\partial E_{i}^{tpn}}{\partial \theta} > 0 \). It states that in equilibrium under the non-committed permits policy, if the concern on consumer surplus rises so does the permit price, and the emission quotas set by the regulator. Furthermore, \( D_{0} > 0 \) which implies that the CF firm buys emission permits.

Finally, we have the resulting profits of the firms, environmental damage and social welfare:

\[
\pi_{0}^{tpn} = \frac{2(268272 - 222912\theta + 25452\theta^2 + 17064\theta^3 - 5625\theta^4 + 633\theta^5 - 256\theta^6)}{\Omega^2}
\]

\[
\pi_{1}^{tpn} = \frac{2(268272 - 305856\theta + 145548\theta^2 - 36396\theta^3 + 5031\theta^4 - 366\theta^5 + 116\theta^6)}{\Omega^2}
\]

\[
ED^{tpn} = \frac{2(36 - 9\theta + \theta^2)^2}{R^2}
\]

\[
W^{tpn} = \frac{2(225504 - 199584\theta + 60024\theta^2 - 6156\theta^3 - 306\theta^4 + 99\theta^5 - 5\theta^6)}{\Omega^2}
\]

**Proposition 5.** \( \pi_{1}^{tpn} < \pi_{0}^{tpn} \) for any \( 0 < \theta \leq 1 \)

It states that in equilibrium under the non-committed permits policy, the profit of CF firm is always larger than that of FP firm. The economic reason is the same with the committed case where the CF firm is more aggressive in production, which induces less production of FP firm.

**Proposition 6.** \( \frac{\partial ED^{tpn}}{\partial \theta} > 0 \) for any \( \theta \in [0, 1] \) but \( \frac{\partial W^{tpn}}{\partial \theta} \leq 0 \) if \( \theta \leq \theta_{W}^{tpn} \approx 0.544 \)

It state that welfare decreases and environmental damage increases as the concern on consumer surplus increases when \( \theta \) is large. Thus, the emergence of a consumer-friendly firm is not always desirable to both the society and environment when its concern on consumer surplus is large under the
non-committed tradable permits policy. This is because higher degree of $\theta$ induces excessive production of outputs, which deteriorates the welfare, and induces large emission quotas by strategic behaviors, which also deteriorates environmental quality. Thus, higher asymmetry between the firms under the non-committed tradable permits policy results in the welfare loss and environmental damage. This is sharply contrast to the results in the committed policy.

4.2. Emission tax

The last stage is the same as in subsection 3.2. In the second stage, the regulator chooses the welfare maximizing emission tax taking as given the firms’ abatement levels. The first order condition of this problem yields:

$$t = \frac{9 + 9\theta + 2\theta^2 - 9(5 - \theta)(z_0 + z_1)}{2(27 + \theta^2)}$$

(24)

This expression defines a negative relationship between firms’ abatement investments and the tax, that is, the regulator decreases the tax rate in response to an increase in the firms’ abatement levels. Thus, firms can strategically use its choice of abatement to influence taxation: by increasing investment in emission-reducing activities, the firms can expect a lower emission tax. Also as the concern on consumer surplus increases, so does the emission tax.

In the first stage, firms choose their abatement efforts taking into account how the regulator is going to respond. Firm 0 chooses $z_0$ that maximizes (3) while firm 1 chooses $z_1$ that maximizes (2). Solving these problems we derive the following optimal abatement efforts:

$$z_{taxn}^{0} = \frac{40095 + 42768\theta - 2592\theta^2 + 1782\theta^3 + 63\theta^4 + 4\theta^6}{D}$$

$$z_{taxn}^{1} = \frac{40095 - 1944\theta + 10530\theta^2 + 387\theta^4 + 4\theta^6}{D}$$

(25)

where $D = (99 - 9\theta + 2\theta^2)S > S = 4779 - 729\theta + 261\theta^2 - 27\theta^3 + 2\theta^4 > 0$. We also employ superscript $taxn$ to denote the equilibrium under the tax policy with non-commitment. Then, we have that $z_{0}^{taxn} > z_{1}^{taxn}$. It states that CF firm is more aggressive in investing abatement technology, which induces a larger amount of total abatement under the tax policy. Unlike the non-committed permits policy, it is noteworthy that $\frac{\partial z_{i}^{taxn}}{\partial \theta} > 0$. Thus, both firms increase more abatement activities under tax policy when $\theta$ increases.
From (25), the optimal emission tax, and the equilibrium output and emission levels are obtained:
\[
  t^n = \frac{243 + 810\theta + 171\theta^2 + 4\theta^4}{2S}, \\
  q_{0}^{tax n} = \frac{9(3 + \theta)(69 - 3\theta + 2\theta^2)}{2S}, \\
  q_{1}^{tax n} = \frac{9(3 - \theta)(69 - 3\theta + 2\theta^2)}{2S}, \\
  \epsilon_{0}^{tax n} = \frac{104247 - 48843\theta + 6723\theta^2 - 945\theta^3 - 234\theta^4 + 36\theta^5 - 8\theta^6}{2D}, \\
  \epsilon_{1}^{tax n} = \frac{104247 - 82377\theta - 2997\theta^2 - 3915\theta^3 - 450\theta^4 - 36\theta^5 - 8\theta^6}{2D}.
\] (26)

In equilibrium under the non-committed tax, the CF firm’s output and abatement levels are larger than those of the FP firm. But, we have \(\epsilon_{0}^{tax n} > \epsilon_{1}^{tax n}\). Thus, the emissions generated by the CF firm are higher than those generated by the FP firm. Note that \(\frac{\partial q_{0}^{tax n}}{\partial \theta} > 0\), \(\frac{\partial q_{1}^{tax n}}{\partial \theta} < 0\), and \(\frac{\partial q_{tax n}}{\partial \theta} > 0\). Also, \(\frac{\partial t^n}{\partial \theta} > 0\). Thus, the tax rate increases as \(\theta\) increases.

Finally, we have the resulting profits of the firms, environmental damage and social welfare:
\[
  \pi_0^{tax n} = \frac{33373963359 + \psi_0}{2D}, \\
  \pi_1^{tax n} = \frac{33373963359 + \psi_1}{2D}, \\
  ED^{tax n} = \frac{(3 - 2\theta)(351 + 45\theta + 12\theta^2 + 2\theta^3)^2}{2S^2}, \\
  W^{tax n} = \frac{3(30801788919 + \psi_2)}{2D^2}. 
\] (27)

**Proposition 7.** \(\pi_1^{tax n} < \pi_0^{tax n}\) if \(0 < \theta < \theta_\pi \approx 0.925\)

It states that in equilibrium under the non-committed tax policy, the profit of CF firm can be larger than that of FP firm except the case when the consumer-friendliness is too high. Thus, similarly to our previous results, higher production by the CF firm leads to higher profits to the CF firm in most cases of \(\theta\). However, if the consumer-friendliness is sufficiently high, higher production induces higher tax rate, which might reduce the profits of the CF firm more than that of the FP firm. This is contrast to our previous results.

**Proposition 8.** \(\frac{\partial ED^{tax n}}{\partial \theta} < 0\), but \(\frac{\partial W^{tax n}}{\partial \theta} \geq 0\) if \(\theta \leq \theta_W \approx 0.43\)

It states that welfare increases but environmental damage decreases as the concern on consumer surplus increases when \(\theta\) is small. Thus, the emergence of a consumer-friendly firm might be desirable.

---

\(^{16}\)For the sake of expositional convenience, we provide \(\psi_j\) (\(j = 0, 1, 2\)) in ’Appendix A’
to both the society and environment when its concern on consumer surplus is small under the non-committed tax policy. This is because as $\theta$ increases, both firms not only increase outputs production but undertake more abatement activities strategically under tax policy. However, as $\theta$ is higher, excessive production of outputs deteriorates the welfare but induces large abatement activities by strategic behaviors, which improves environmental quality. This result sharply contrasts to the results under the non-committed permits policy and committed tax policy.

4.3. Comparing permits and tax

**Proposition 9.** $t^n < t^e = \lambda^e < \lambda^n$ for any $\theta \in [0, 1]$

![Figure 1: Permits price and tax comparison.](image)

Therefore, under the regulator’s inability to commit the equivalence between the two policy instruments breaks down.\(^{17}\) In particular, it states that under the non-committed policy where the time-inconsistency occurs, the optimal tax is smaller than the equilibrium permits price. Figure 1 shows that both permits price and tax rate increase as the concern on consumer surplus rises. It also shows that, compared to the equivalent tax rate or permits price, the tax rate becomes smaller and permits price becomes larger.

This finding also indicates that firms’ strategic incentives depends not only on policy instruments but the degree of consumer-friendliness. This is because firms would expect the regulator to change its

\(^{17}\)Note that this result does not depend on $\theta$. Moner-Colonques and Rubio (2015) also showed that the equivalence between emission standard and emission tax breaks down in a private duopoly model when the government cannot commit the policy.
policy ex post and thus they have different incentives to induce time-inconsistent policy to be adjusted ex post. In particular, compared to pre-committed government, firms abate less emission to induce higher emission quotas under the permits policy while a consumer-friendly firm abates more emissions to reduce tax rate under the tax policy. We will show these findings in the following proposition.

**Proposition 10.**

1. $q^\text{taxn}_0 > q^\text{tpn}_0$, $q^\text{taxn}_1 > q^\text{tpn}_1$ and $Q^\text{taxn} > Q^\text{tpn}$ for any $\theta \in [0, 1]$;

2. $z^\text{taxn}_0 > z^\text{tpn}_0$ and $z^\text{taxn}_1 > z^\text{tpn}_1$ for any $\theta \in [0, 1]$;

3. $e^\text{taxn}_0 < e^\text{tpn}_0$ and $e^\text{taxn}_1 < e^\text{tpn}_1$ if $\theta > \theta^*_e \approx 0.471$.

It states that under the non-committed policy, both firms produce more outputs but more abatement investments with tax policy than permits policy. Thus, both firm’s emission levels become more significant under permits policy when the concern on consumer surplus is relatively high. Fig. 2 shows how both firms choose abatement levels strategically by expecting ex-post policies. In particular, under permits policy, both firms reduce abatement levels unless the concern on consumer surplus is too large, which induces ex-post permits policy with more emission quotas. Under tax policy, a consumer-friendly firm increases its abatement levels to reduce tax rate unless the concern on consumer surplus is too small, which induces a higher tax rate. Thus, these opposite incentives break down the equivalence between permits and tax policies.

![Figure 2: Abatement comparison.](image)

**Proposition 11.** $\pi^\text{taxn}_0 < \pi^\text{tpn}_0$ and $\pi^\text{taxn}_1 < \pi^\text{tpn}_1$ for any $\theta \in [0, 1]$.

It implies that under the non-committed policy both firms can earn higher profits with permits policy due to the strategic effects on ex-post policy. Thus, as like in the committed policy, both firms...
still prefer permits policy to tax policy when they can choose or lobby for the policy. It also implies that irrespective of the timing of environmental policies, tradable permits policy can be an endogenous choice of rent-seeking equilibrium in a political process.\textsuperscript{18}

**Proposition 12.**

1. $ED_{tax} < ED_{tpn}$ for any $\theta_E \approx 0.466577 < \theta \leq 1$ where $\theta_E$ satisfies $ED_{tax} = ED_{tpn}$;

2. $W_{tax} > W_{tpn}$ for any $0 < \theta < \theta_W \approx 0.957481$ where $\theta_W$ satisfies $W_{tax} = W_{tpn}$.

This proposition states that under the non-committed policy, tax policy induces higher welfare and lower environmental damage than permits policy when the consumer-friendliness is moderate, i.e., $ED_{tax} < ED_{tpn}$ and $W_{tax} > W_{tpn}$ if $\theta_E < \theta < \theta_W$. Because there are opposite strategic effects between the two policies if the government can not make credible pre-commitment, we have larger time-consistent distortion of abatement activities with permits policy. Therefore, from the policy perspective of both welfare and environmental quality, tax policy can be a better policy than permits policy when the concern on consumer surplus is moderate. Note that the welfare-superiority in the non-committed tax policy holds when $\theta = 0$ where both firms have the same objectives in a duopoly market.\textsuperscript{19}

5. Concluding remarks

We provided the analysis of different policy timing with respect to the two environmental regulatory measures between tradable permits and emission tax in a mixed duopoly with a consumer-friendly firm. We examined the strategic choices on abatement technologies and showed that the equilibrium outcomes under both policies are equivalent in terms of permits price and tax rate only when the government can credibly commit its policy. Also, we showed that the profit of a consumer-friendly firm is always larger than rival’s profit but both firms can earn higher profits with permits.

Under the non-committed policy, however, the equivalence breaks down because firms have different incentives to induce time-inconsistent policy to be adjusted ex post. In particular, compared to pre-committed government, firms abate less emission to induce higher emission quotas under the permits policy while a consumer-friendly firm abates more emissions to reduce tax rate under the tax policy.

\textsuperscript{18}Regarding rent-seeking behaviors over tradable permits policy, Rode (2014) examined the opportunity cost for a rent-seeking lobby in EU’s CO\textsubscript{2} ETS (Emissions Trading System).

\textsuperscript{19}In a private duopoly model Moner-Colonques and Rubio (2015) examined the time-inconsistent problem in both emission standard and emission tax policy and showed that the welfare-superiority of tax policy over emission standard depends on the relative efficiency of abatement technology.
We can summarize our findings under the non-committed policy. First, both firms’ abatement activities under permits are lower than those under tax, but total emissions under tax are smaller than those under permits when the concern on consumer surplus is high enough. Second, both permits price and tax increase as the concern on consumer surplus rises, but due to the strategic incentive to increase emission quotas under the permits policy, permits price is always higher than the equivalent tax. Finally, tax policy induces more outputs, more abatements and less emissions than permits when the concern on consumer surplus is moderate, and thus tax is a better policy than permits in the presence of a consumer-friendly firm.

Our findings show that not only the regulator’s inability to commit but the firm’s CSR initiatives can play significant roles in the design and implementation of environmental policy and have detrimental effects on social welfare. However, our analysis has a limitation because of the simple structure of our modelling with linear demand and quadratic cost functions. Thus, the importance of CSR should be further examined in more general settings under the alternative market structure with the efficiency parameter of abatement technology, product differentiation, dominant market power in the permits market, different timing of the game and so on. This has to be left for future research.

References


20In Appendix B, we examine non-tradable emission permits regulation and compare the welfare consequences with those under tradable emission permits regulation. Regarding this issue, see Sartzetakis (1997) and Kato (2006, 2011).


Appendix A. The values of $\psi_i$

$$
\psi_0 = -19755079146\theta + 1776371067\theta^2 + 252749403\theta^3 - 547148034\theta^4 + 143738388\theta^5 - 34205499\theta^6 \\
+ 4527819\theta^7 - 486567\theta^8 + 25272\theta^9 + 360\theta^{10} - 144\theta^{11} + 16\theta^{12};
$$

$$
\psi_1 = -28584262773\theta + 13764085704\theta^2 - 3714674175\theta^3 + 1124686620\theta^4 - 1809458199\theta^5 + 36948636\theta^6 \\
- 3873177\theta^7 + 563193\theta^8 - 36936\theta^9 + 4248\theta^{10} - 144\theta^{11} + 16\theta^{12};
$$

$$
\psi_2 = -1241658752\theta + 3686527485\theta^2 - 1334901060\theta^3 + 129079274\theta^4 - 64796436\theta^5 + 10120956\theta^6 \\
- 1744740\theta^7 - 76626\theta^8 - 19440\theta^9 - 3168\theta^{10} - 32\theta^{12}
$$

(A.1)
Appendix B. Non-tradable Emission Permits

We examine non-tradable emission permits (NTEP), in which the government prohibits the firms from trading emission permits. In this case, each firm can discharge emissions as long as it obeys its own emission constraint, i.e., \( e_i = q_i - z_i \leq E \). We derive and compare the equilibrium of committed NTEP policy and that of non-committed NTEP policy, respectively. Recall that firms choose \( q_i \) in the last stage under the effective regulatory constraint that \( q_i = E + z_i \).

Appendix B.1. Committed policy

Under \( \text{ex-ante} \) NTEP, firms choose \( z_i \) after the government chooses \( E \). Maximization problems of CF firm and FP firm are given respectively by

\[
\begin{align*}
\text{Max } & z_0 \pi_0 + \theta CS + \mu_0 (E - q_0 + z_0) \\
\text{Max } & z_1 \pi_1 + \mu_1 (E - q_1 + z_1)
\end{align*}
\]  

(B.1)

Let \( \mu_i \) be the shadow price of the emission constraint of firm \( i \). Solving the maximization problem of each firm with the binding constraint, we derive the following equilibrium abatement levels:

\[
\begin{align*}
z_0^* &= \frac{4 + \theta - 5E(4 - \theta)}{4(6 - \theta)} \\
z_1^* &= \frac{4 - \theta - E(20 - 3\theta)}{4(6 - \theta)}
\end{align*}
\]  

(B.2)

Then, the social welfare under NTEP is

\[
W^* = \frac{112 - 32\theta - 3\theta^2 - E^2 (1616 - 544\theta + 51\theta^2) + E (224 - 64\theta - 6\theta^2)}{16(6 - \theta)^2}
\]  

(B.3)

In this stage the regulator assigns the welfare-maximizing emission quota to each firm. Solving (B.3) yields the optimal emission quota

\[
E^* = \frac{112 - 32\theta - 3\theta^2}{1616 - 544\theta + 51\theta^2}
\]  

(B.4)

Regarding welfare comparisons, we have for any \( \theta \in [0, 1] \)

\[
\begin{align*}
W^*(E^*) - W^{tpc} &= \frac{45(6 - \theta)^2\theta^2}{(909 - 306\theta + 29\theta^2)(1616 - 544\theta + 51\theta^2)} \geq 0 \\
W^*(E_i^{tpc}) - W^{tpc} &= \frac{405(6 - \theta)^2\theta^2}{16 (909 - 306\theta + 29\theta^2)^2} \geq 0 \\
W^*(E^*) - W'(E^*) &= \frac{80(6 - \theta)^2\theta^2}{(1616 - 544\theta + 51\theta^2)^2} \geq 0
\end{align*}
\]  

(B.5)

where \( W'(E^*) \) is obtained inserting (5)-(7) and (B.4) into (4). These results support the findings in Kato (2006, 2011) who showed that the command-and-control regulation might be superior to market-based instruments depending on the firm’s objective function in a mixed market.

22
Appendix B.2. Non-committed policy

Under ex-post NTEP, firms choose $z_i$ before the government chooses $E$. Then, the social welfare is given by

$$W^\star\star = 2(1 - 3E)E - 2z_0^2 + (1 - 4E)z_1 - 2z_1^2 + z_0 (1 - 4E - z_1) \quad (B.6)$$

In the second stage, the regulator assigns the welfare-maximizing emission quota to each firm. Solving (B.6) yields the optimal emission quota

$$E = \frac{1}{6} (1 - 2z_0 - 2z_1) \quad (B.7)$$

Finally, making use of $q_i = E + z_i$ and (B.7) and inserting them into (B.1), firms chooses $z_i$ that solve the maximization problem. This yields the following optimal abatement levels:

$$z_0^\star\star = \frac{3(8 + 5\theta)}{16(18 - \theta)} \quad \text{and} \quad z_1^\star\star = \frac{24 + \theta}{16(18 - \theta)} \quad (B.8)$$

The resulting equilibrium emission quota and welfare are:

$$E^\star\star = \frac{4 - \theta}{2(18 - \theta)}, \quad W^\star\star(E^\star\star) = \frac{3(5696 - 384\theta - 113\theta^2)}{256(18 - \theta)^2} \quad (B.9)$$

Regarding welfare comparison between ex ante NTEP and ex post NTEP, we have:

$$W^\star\star(E^\star\star) - W^\star(E^\star) = -\frac{3(84992 + 32768\theta - 242032\theta^2 - 22432\theta^3 + 4995\theta^4)}{256(18 - \theta)^2 (1616 - 544\theta + 51\theta^2)} > 0 \quad \text{if} \quad \theta > \frac{6}{0.6459}.$$ 

Therefore, the welfare comparison between ex ante NTEP and ex post NTEP depends on the degree of consumer-friendliness.

Appendix C. Tradable emission permits with discriminatory quota

We examine discriminatory emission quota, in which the government allocates emission permits to the firms differentially and then allows firms to trade emission permits. In this case, we assume that $E_0 = \alpha \sum_{i=0}^1 E_i$ and $E_1 = (1 - \alpha) \sum_{i=0}^1 E_i$, where $\alpha$ is exogenously given as $0 \leq \alpha \leq 1$. We derive and compare the equilibrium of committed policy and that of non-committed policy, respectively.

Appendix C.1. Committed policy

Using the total emission quota in (9), we can set $E_0 = \alpha \frac{2(63 - 18\theta - 2\theta^2)}{\theta}$ and $E_1 = (1 - \alpha) \frac{2(63 - 18\theta - 2\theta^2)}{\theta}$. Given that the permit price in (7) depends on the total emission quotas, the equilibrium permit price, output and abatement levels in (11) remain the same. It is also easy to show that $ED^{tpc}$ and $W^{tpc}$ in (12) remain as well.
However, the profits of the firms can be written as follows:

\[
\pi_{t^0_{tpc}} = \frac{114777 - 71766\theta - 8271\theta^2 + 6768\theta^3 - 644\theta^4}{2\sqrt{H}} + \alpha L, \\
\pi_{t^1_{tpc}} = \frac{114777 - 106758\theta + 38385\theta^2 - 5868\theta^3 + 328\theta^4}{2\sqrt{H}} + (1 - \alpha)L. 
\]

(C.1)

where \( L = \frac{2(63 - 18\theta - 2\theta^2)(99 - 9\theta + 2\theta^2)}{H^2} > 0. \)

The difference between the firm’s profits is

\[
\pi_{t^0_{tpc}} - \pi_{t^1_{tpc}} = \frac{2(-6237 + 11097\theta - 11754\theta^2 + 3177\theta^3 - 239\theta^4)}{H^2} + 2\alpha L. 
\]

Let \( \alpha \equiv \frac{6237 - 11097\theta + 11754\theta^2 - 3177\theta^3 + 239\theta^4}{2(99 - 9\theta + 2\theta^2)(63 - 18\theta - 2\theta^2)} > 0. \) Note that \( \alpha \) is a convex function of \( \theta \) where \( \alpha(0) = \frac{1}{2} \) and \( \alpha(1) = \frac{1}{2} \).

**Proposition 13.** \( \pi_{t^0_{tpc}} \geq \pi_{t^1_{tpc}} \) if and only if \( \theta \in [0, 1] \) and \( \alpha \leq \alpha \leq 1. \)

From equation (16) and (C.1), we have \( \pi_{t^i_{tpc}} = \pi_{t^i_{taxc}} + \lambda E_i, i = 0, 1 \) where \( \lambda \) is the equilibrium price permit given in (11). This implies that Proposition 4 is satisfied under the discriminatory emission quota.

Finally, given that the environmental damage, \( ED_{tpc} \), and welfare, \( W_{tpc} \), are the same that in equation (12), we can observe that they don’t depend on the distribution of the emission quotas. Hence, Proposition 2 remains under discriminatory emission quota.

**Appendix C.2. Non-committed policy**

Using the total emission quota in (19), we can set \( E_0 = \alpha\lambda \) and \( E_1 = (1 - \alpha)\lambda \). Then, firms choose their abatement efforts \( z_i \) to maximize their objective functions in (3) and (1), respectively:

\[
z_{t^0_{tpn}}^* = \frac{396 - 21\theta - 28\theta^2 + 3\theta^3 - 2\alpha(5 - \theta)(36 - 9\theta + \theta^2)}{\Omega}, \\
z_{t^1_{tpn}}^* = \frac{(12 - \theta)(3 - \theta)(1 - \theta) + 2\alpha(5 - \theta)(36 - 9\theta + \theta^2)}{\Omega}. 
\]

(C.3)

Since the total abatement of \( z_{t^0_{tpn}}^* + z_{t^1_{tpn}}^* \) computed from (C.3) is the same total abatement of \( z_{t^0_{tpn}} + z_{t^1_{tpn}} \) computed from (21), the abatement values of the discriminatory case yield the same equilibrium permit price, therefore, they yield the same equilibrium outputs as in (22).

The equilibrium emission quotas are however:

\[
E_{t^0_{tpn}} = \alpha\lambda, \quad E_{t^1_{tpn}} = (1 - \alpha)\lambda \n\]

(C.4)
By replacing (C.3) and (C.4) in (1), the equilibrium profits are:

\[
\pi_{0\star} = \frac{931824 - 828360 \theta + 102735 \theta^2 + 62436 \theta^3 - 21010 \theta^4 + 2368 \theta^5 - 93 \theta^6}{20\Omega^2}
\]

\[
\pi_{1\star} = \frac{1149552 - 1297512 \theta + 615087 \theta^2 - 153708 \theta^3 + 21290 \theta^4 - 1556 \theta^5 + 47 \theta^6}{20\Omega^2}
\]

The difference between the profits of the firms under the discriminatory quota are:

\[
\pi_{0\star} - \pi_{1\star} = \frac{2 \left( 36 - 9 \theta + \theta^2 \right) \left( 36 - 9 \theta + \theta^2 \right) \left( 252 - 54 \theta - 3 \theta^2 + \theta^3 \right) \left( 6 - \theta \right) \left( 2 \alpha - A \right)}{\Omega^2}
\]

where \( A \equiv \frac{9072 - 18036 \theta + 18342 \theta^2 - 5049 \theta^3 + 771 \theta^4 - 35 \theta^5}{(36 - 9 \theta + \theta^2)(252 - 54 \theta - 3 \theta^2 + \theta^3)}. \) For any \( 0 \leq \theta \leq 1, \) we have \( 0 < A \leq 1. \)

**Proposition 14.** \( \pi_{0\star} \geq \pi_{1\star} \) if and only if \( \alpha \geq \frac{A}{2}, \) for any \( 0 \leq \theta \leq 1 \) and \( 0 \leq \alpha \leq 1. \)

Regarding profit's comparison between tradable emission permits with discriminatory quota and emission tax, we can draw Figure C.3. It shows that the tradable emission permit yields higher profits to both firms regardless of the degree of consumer-friendliness, if the distribution of the allowed quotas is more equitable. But, if the consumer friendliness is small, for high (small) values of \( \alpha, \) the firm 1 (0) would obtain larger profits with the tax while the rival would obtain larger profits with the permits.

**Proposition 15.** For any \( 0 \leq \theta \leq 1: \)

1. \( \pi_0^{\text{taxn}} < \pi_0^{\text{pn} \star} \) if \( \alpha > \max \{ \alpha_{nc0}(\theta), 0 \}, \) where \( \alpha_{nc0}(\theta) \) is such that \( \pi_0^{\text{taxn}}(\alpha_{nc0}) = \pi_0^{\text{pn} \star}(\alpha_{nc0}). \)

2. \( \pi_1^{\text{taxn}} < \pi_1^{\text{pn} \star} \) if \( \alpha < \min \{ \alpha_{nc1}(\theta), 1 \}, \) where \( \alpha_{nc1}(\theta) \) is such that \( \pi_1^{\text{taxn}}(\alpha_{nc1}) = \pi_1^{\text{pn} \star}(\alpha_{nc1}). \)
Finally, we can see that the environmental damage is the same as in (23) regardless of the value of \( \alpha \) while the welfare depends on \( \alpha \):

\[
W_{tpn}^* = \frac{1320624 - 1202904\theta + 377055\theta^2 - 43908\theta^3 - 508\theta^4 + 466\theta^5 - 25\theta^6}{\Omega^2} + 4\alpha(5 - \theta)(36 - 9\theta + \theta^2)(180 + 15\theta - 22\theta^2 + 2\theta^3) - 4\alpha^2(5 - \theta)^2(36 - 9\theta + \theta^2)^2
\]

(C.7)

**Proposition 16.** \( \frac{\partial E_D^{tpn}}{\partial \theta} > 0 \) for any \( \theta \in [0, 1] \) but \( \frac{\partial W_{tpn}^*}{\partial \theta} \leq 0 \) if \( \theta \leq \theta_{Wd}(\alpha) \).

Note that \( W_{tpn}^* \) is concave on \( \alpha \) and thus it is maximized at \( \alpha_{tpn}^* = \frac{180 + 15\theta - 22\theta^2 + 2\theta^3}{2(180 - 81\theta + 14\theta^2 - \theta^3)} \geq \frac{1}{2} \), which increases in \( \theta \) (\( \alpha = \frac{1}{2} \) if \( \theta = 0 \)). This represents that discriminatory emission quota is efficient in the presence of a consumer-friendly firm. It also implies that the regulator should allow a larger emission quota to the consumer-friendly firm and the quota should be higher for higher values of consumer-friendliness.