Optimal financial contracts with unobservable investments

Tirelli, Mario

Università di Roma Tre

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OPTIMAL FINANCIAL CONTRACTS WITH UNOBSERVABLE INVESTMENTS

MARIO TIRELLI*

Abstract. In this article we propose a security-design problem in which risk neutral entrepreneurs make unobservable investment decisions while employing the investment funds of risk-neutral outside investor/creditor(s). Contracts are restricted to satisfy limited liability and monotonicity of the payment schedule. The model we present extends the classical one proposed by Innes (1990, *Journal of Economic Theory* **52**, 47-67) along three main directions: agents' decisions may be restricted by their initial capital and outside financial opportunities; their investment decisions may also consist in hiding funds in an asset placed outside their firms; initial firms' capital, which identifies entrepreneur types, may only be imperfectly observed by creditors (*i.e.* types are private information). We motivate our interest in this security-design problem referring to the 'opacity' that often characterizes the financial situation and decisions of small firms, a particularly large fraction of the non-financial sector in most developed countries.
(*JEL Classification Numbers*: D820, D860, G110, G320.)
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Contents

*Department of Economics, University of Rome 3. Via Silvio D'Amico 77, 00145 Rome (Italy).
E-mail: `mario.tirelli@uniroma3.it`. Tel no.: +39 06 5733 5735. Fax no.: +39 06 5733 5771.
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1. Introduction

Consider a competitive market or industry in which there is a single production activity whose outcome (gross profit) is uncertain. Each entrepreneur (or firm-owner) can increase the likelihood of higher profit realizations by investing in his firm. Investment raises firm's expected profitability or productivity when, for example, it consists in R&D expenditure and in expenditure on human capital formation or organization. The level of investment, in turn, is affected *a)* by the amount of the firm's initial capital, *b)* by the entrepreneur's access to outside finance and *c)* by the value of an 'outside option', which (for simplicity) in our analysis consists in an anonymous, safe deposit.

In this context, the our main goal is to characterize the efficient menu of financial contracts and the implied optimal financial and real investment decisions, when entrepreneurs (agents) are risk-neutral and the regulator/principal faces the following restrictions:

- contracts must satisfy limited liability (LL), possibly, be subject to a monotonicity condition of the payment schedule with respect to profits (M), and yield no expected losses to creditors (*i.e.* individual lenders participate - IP);
- entrepreneurs take investment decisions after financial contracts are signed, but before states of nature realize; investments, both in the firm and in an outside deposit are unverifiable;
- creditors may be unable to observe firms' initial capital (*i.e.* there may be asymmetric information on agent types).

To characterize optimal contract we consider a principal-agent model that builds on the classical one due to Innes (1990). It follows Innes in the assumption that agents' actions are unobservable and in some of the main restrictions on contracts. Instead, it differs from Innes' in three relevant aspects. First, a real investment decision replaces an 'effort' choice. This implies that, in our context, an agent decision is subject to budget-feasibility, while in Innes' it is not and does only produce a subjective welfare cost. Second, entrepreneurs can divert funds from productive investment and hide them outside their firms. Third, firms' capital (net-worth), identifying entrepreneur types, is private information.

Given the presence of *moral hazard* and, possibly, of *adverse selection*, the space of contracts we consider is multidimensional, consisting not only of a payment schedule, as in Innes', but also of a loan size and of an initial, down-payment. The down-payment can assume different forms configuring, for example, a participation fee (*e.g.* in the spirit of franchising), a security deposit (*e.g.* bank credit credit lines), a collateral requirement (*e.g.* in secured credit card and bank credit lines) and a minimum capital requirement (*e.g.* a minimum capital stock required by corporate law to public companies). Alternative forms of this instrument have various impact on agents' incentives and decisions. A third contractual dimension is the loan size that may, possibly, be used to induce a dose of '*credit rationing.*'[1]

[1]Some path-breaking articles on the role of: *a)* collateral are Stiglitz & Weiss (1981) and Chan & Thakor (1987); *b)* security (or bond) deposit is Lewis & Sappington (2000); *c)* franchise fee is Mathewson & Winter (1984). Stigliz and Weiss cit. is also a classic of credit rationing.

In the context considered, our main results are gathered in a theorem and can be summarized as follows. The menu of optimal contracts have the following characteristics:

1.i) A payment schedule of *standard debt contracts* (SDCs) if monotonicity is imposed;[2] or live-or-die contracts (LDCs), otherwise;

1.ii) A loan size that induces no credit 'rationing' or 'restriction' to the agent;

1.iii) A down-payment in the form of maximal capital participation by the agent, if monotonicity is imposed; or in the form of a full collateralization, otherwise;

1.iv) An interest rate schedule that is monotonically decreasing in the firm's initial capital and in its debt size, up to the point that financial opportunities sustain the first-best (*i.e.* the level of investment that would be optimal in the absence of asymmetric information).

Moreover, the optimal menu of contracts supports entrepreneurial policies with the following characteristics:

2.i) The financial structure of each firm is uniquely determined and is one in which firm's capital (net-worth) is the most preferred source of finance followed by outside debt: if initial capital is sufficient to fund the first-best level of individual investment, the entrepreneur does not demand outside finance; otherwise, he subscribes a financial contract and reduces investments below the first-best (*i.e.* the market incurs in underinvestment);

2.ii) Entrepreneurs neither effectively implement hidden saving nor (if feasible) have incentive to pursue hidden borrowing;

2.iii) Entrepreneurs who operate higher capitalized firms invest more and borrow at a lower implicit interest rate.

The fact that the optimal menu is composed of either SDCs or LDCs, depending on whether or not monotonicity is assumed, confirms Innes (1990) and is in line with most of the theoretical literature.[3] In the presence of moral hazard, these contracts reinforce incentives to invest by redistributing most of the firm's financial cost from 'good' to 'bad' states ('bankruptcy' states).

Optimal contracts entail no credit 'rationing' or 'restriction' (in 1.ii). Moreover, although the resulting investment projects are below the first-best level (in 2.i), entrepreneurs neither find optimal to subscribe a loan of larger size, nor to try to secretly collect savings from

[2]SDCs are defined as in Gale & Helwig (1985); Informally speaking, they are financial contracts whose payment schedule dictates a fixed (non-contingent) payment for high enough, profit realizations and a 'default-payment' (realized profits + collateral) otherwise.

[3]Innes (1990) uses the reduced-form pioneered by Mirrlees (1976) and Holmstrom (1979). More recently, an equivalent characterization has been obtained by Poblete & Spulber (2012) with a state-space approach. Debt contracts are, in general, not optimal if the entrepreneur is risk averse. Though, even in this case an initial debt contract is optimal, according to Matthews (2001), provided that it can be renegotiated after the agent has chosen his level of effort/investment, but before profits are realized. Finally, debt contracts are optimal in models of costly state verification initiated by Townsend (1979), Diamond (1984), and Gale and Hellwig (1985).

other firms (2.ii).[4] Indeed, we show that if an entrepreneur were to be offered a contract that makes such policies budget-feasible, he would find optimal to renegotiate the contract with a feasible one that is characterized by a lower size and a lower implicit interest rate (by 1.iv). An analogous reasoning allows to explain why optimal contracts defeat hidden savings and borrowing in 2.ii).

Optimal contracts prescribe full down-payment (in 1.iii). This can essentially be explained based on two considerations. First, the down-payment, either in the form of collateral or of capital participation, allows to mitigate moral hazard by increasing the power of the incentive scheme. In fact, in order to avoid the payment loss, occurring at low profit realizations, each agent tends to increase real investment; this, in turn, allows creditors to offer a loan contract with a lower interest rate, thereby sustaining incentives. Thus, even if initial capital were common knowledge (essentially, as in Innes, cit.), optimal contracts would prescribe full down-payment. The incentive effects of collateral are consistent with the findings of some contributions in the literature on banking and credit markets (see, for example, Chan and Thakor, 1987), which however –to my knowledge- do not approach the problem as a general one of security design.[5] Second, when initial capital is private information, collateral is an efficient *screening* devise.[6] In absence of any down-payment, entrepreneurs with low capitalized firms would claim to have high capital in order to access to contracts with a lower interest rate and a larger size (by 1.iv).[7]

The uniqueness of the firm financial structure in 2.i), unsurprisingly, contrasts with traditional neoclassical theory of the firm and invalidates Modigliani-Miller theorem. Moreover, the prescribed financial structure is in line with a modern version of the *pecking order theory*: when a firm experiences an imbalance of internal cash-flow and real investment opportunities, it resorts to external finance raising the debt-to-equity ratio. In other words, rather than having in mind a static optimal debt-equity ratio, everything else equal, our entrepreneurs adjust their financial structure to the initial net-worth position and investment opportunities.[8] In this paper debt is preferred to equity (*i.e.* to issue a participation to the firm's profits) as debt produces a more powerful incentive scheme, supporting higher

[4]Unlike in this paper, it is sometimes the case in this literature that credit-rationing is identified with underinvestment (*e.g.* Gale & Hellwig, 1985).

[5]For example, in Chan & Thakor. cit. the mechanism design problem restricts the space of contracts to that of bank loans. Moreover, they assume that agents/borrowers have unlimited collateral that can be used to finance an investment of exogenously fixed amount.

[6]This is so, for example, in Gale & Hellwig (1985), where they consider initial capital as equity and they find optimal to have debt contracts with 'maximum capital participation'.

[7]This view is not consistent with some contributions in the same literature, which highlight the adverse selection effects caused by higher collateral values (see, for example, Stiglitz and Weiss, cit.). However, in Stiglitz & Weiss cit. collateral is added to contracts designed with respect to the interest rate alone; thus, again contracts are not defined as a solution of a general mechanism design problem.

[8]Although the goal of this paper is not matching empirical facts, the implied financial structure is coherent with (at least) part of the empirical evidence; see, for example, Shyam-Sunder & Myers (1999). Yet there is also evidence rejecting the empirical content of the pecking order theory; see, for example, Frank & Goyal (2003) and the literature review therein.

investments and firm's expected profits.[9] Thus, although entrepreneurs would rather finance investments through internal funds, a contract based on a combination of a SDC and a minimum capital requirement can reproduce an efficient equity-based compensation scheme; a scheme that reduces agency costs and boosts firm investments (in 2.iii). The fact that firms whose managers have high equity-based compensation schemes and high cash flow experience higher operating performance has recently been documented by Chen and Chen (2017). Moreover, a consolidated empirical literature documents that companies with sounder, initial wealth conditions tend to have access to better credit opportunities and have higher likelihood of success in business.[10]

To our knowledge, the only paper that has investigated a security-design problem similar to ours, with both moral hazard and asymmetric information on agents' capital, is Lewis and Sappington (2000).[11] Yet, their focus and model differ substantially from ours. First, the problem they analyze is one of optimal delegation in which a principal, the owner of a project, sicks to select one or more agents for its implementation; agents' effort and initial wealth are unobservable to the principal. Our problem, instead, is one in which a regulator (principal) designs financial contracts that maximize the surplus generated by entrepreneurs (agents) investments subject to the participation of a financier (principal) conditional to the specified information asymmetries.[12] This distinction is not purely semantic: the introduction of incentive-compatibility constraints makes the underline optimal design problems non-concave; hence, the solution attained from its primal and dual formulations may not just consist in a different sharing rule of social surplus. Second, the model considered in Lewis and Sappington is binomial; a project either success or fail. This restriction, in our perspective, severely limits the possibility to distinguish between a SDC and more complex forms of non-linear financial agreements. Third, contrary to this paper, their agent-choice model does not account for (hidden) financial decisions; ex-post entering a contract, agents only choose their effort level (investment in our terminology). The option to divert cash flows is valuable under asymmetric information on initial wealth as it allows agents to select contracts with higher loan size, designed for lower capitalized firms, and divert part of the funds from the firm. Finally, the space of contracts we consider encompasses the one in Lewis and Sappington's. We argue later that the type of down-payments they consider (the 'up front bond' payment in their language) is feasible for our designer but not optimal when collateral can instead be required. This is again due to the fact that collateral has the effect of increasing the incentive to invest in the firm (*i.e.* impairing moral hazard), instead

[9]In more traditional models of pecking order theory (*e.g.*, Myers & Majluf, 1984), equity is the most disfavor mean of outside finance because it produces the worst form of adverse selection.

[10]The positive correlation between firms' investment and their internal cash flow has been documented in the strand of empirical studies started by Fazzari, Hubbard, & Petersen (1988), Hoshi, Kashyap, & Sharfstein (1991), Gertler & Gilchrist (1994). See also see Hubbard, 1998 for an up to date review of this literature.

[11]See also Lewis & Sappington (2001) and the references therein.

[12]Apart from Innes (1990), our formulation is common to many other contributions in the literature on optimal security design (see, for example, problem (2) in Gale & Hellwig, 1985). It is mostly natural under perfect competition on financial markets.

of just working as a screening device; something that, again, is in line with the literature on banking and credit markets under asymmetric information (see, for example, Chan and Thakor, 1987).

Another closely related work is De Marzo and Fishman (2006) that analyzes the investment problem of an agent under moral hazard, in a multi-period, dynamic setting. However, their investment model differs substantially from ours in that investment can exclusively affect the scale of a project. Instead, in our model investment modifies the whole distribution of cash flows. As we detail latter, in our setting higher investment level increases the probability of higher profit realizations, changing both the expected return to a project and its risk. Apart from Innes cit., a specific model of firm decision like ours is presented, for example, in Greenwald and Stiglitz (1990) and better captures situations in which investment affects both a firm scale and productivity; this, we think, is much closer, in spirit, to the neoclassical growth literature on R&D and on human capital.

Motivation. Beside a pure theoretical interest, our motives for extending a classical agency model to hidden financial decisions and unobservable financial state/situation is the 'opacity' of small companies, which form a particularly large fraction of the non-financial sector, in most developed countries (see Kushnir et al., 2010). This 'opacity' is largely determined by three factors: a very simplified governance, softer regulatory requirements on accounting and transparency (information reporting), a low capitalization (*e.g.* see the discussion in Beger and Udell, 1998). Indeed, small companies often take the form of a simple partnership or are characterized by an elementary owner-manager structure, which makes harder for outside investors to distinguish the financial situation of a firm from the personal one of its owners. Such problem is then worsen by a consolidated tendency of regulators to reduce the administrative burden for small business. Even recently, a EU Directive (34/2013)[13] has limited the amount of information required to an annual balance sheet and a profit/loss account, even with reference to companies with limited-liability. In addition, depending on their dimension (small or micro firms), the Directive, allows EU Members to introduce further significative simplifications at national level.[14] Similarly, in the US, small companies are not required to release financial information on 10K forms. Finally, small companies (and, more generally, companies which are not publicly traded) are often less capitalized; something that exacerbates agency costs. Clearly, opacity can be reduced by monitoring; yet, especially for private companies (*i.e.* non-quoted on the stock market) monitoring financial positions is costly and sometimes impossible, especially because it is difficult (or impossible) to assess their cash management policies (e.g., see Gale and Hellwig, 1985).

As a partial evidence of the importance of asymmetric information and financial frictions on small firms' policies, it has been documented that investment projects tend to be more

[13]The Directive is part of the Responsible Business package (see European Parliament, Directive 2013/34/EU).

[14]More specifically, EU Member State can require these firms to prepare only abridged balance sheets and profit/loss accounts, with a consistent reduction of financial information.

sensitive to cash-flow and are, only residually, financed with outside debt, mostly in the form of bank credit. Such evidence is sharper for companies at their early stage and intensively active in R&D. Moreover, asymmetric information seems to persist even in economies with thick financial markets, a well developed system of specialized intermediaries and venture capitalists[15]. Finally, a number of studies suggests that firms not in their early stage, who rely on bank credit, tend to establish long lasting relationship with one, or very few counterparts, and to purchase a multiplicity of financial services. These relationships allow intermediaries to acquire 'soft' information on the kind of entrepreneurs they are dealing with, including 'ability' or intrinsic 'productivity', that might not be easily verified (*e.g.*, see Petersen and Rajan, 1994). Yet, for these firms asymmetries on 'hard' information, including certain policies and actions, may be more relevant; something that reinforces the interest for moral hazard.

Organization. The paper unfolds as follows. Section 2 presents the model and some preliminary results. Section 3 defines the optimal mechanism-design problem and states our theorem. The proof of the theorem is then split in two subsections in which, for expositional reasons, we first solve the optimal mechanism problem for the case of pure moral hazard (*i.e.* assuming common knowledge of entrepreneur types). Then, we show that the menu of optimal contracts obtained is incentive-compatible.

2. Basic structure and preliminary results

The model is one with a single consumption good, two time periods and uncertainty over the second period. Agents are risk neutral entrepreneurs. Each entrepreneur is the single-owner of a firm and his type is determined by the firm's first period, endowment of capital (net-worth) a in $A := [\underline{a}, \bar{a}]$. Initial net-worth is the consequence of an unrepresented past and may both reflect the return of the entrepreneur's past production and financial decisions.

In the first period there is uncertainty on the outcome of each firm's production activity, which is represented by the realization of (gross) profits, π in $\Pi := [0, \infty)$, occurring in the second period. Although, profit opportunities Π are equal across firms, their likelihood depends on each entrepreneur's investment choice, $x \geq 0$. We, respectively, denote by $g(\pi|x)$ and $G(\pi|x)$ the (conditional) density and the distribution functions of profits for an entrepreneur who has invested x.

The representation of the state space Π can also be used to identify specific 'innovation states'. We define an *innovation* state π_s, as the threshold-profit such that an increase of x raises the probability of each and every $\pi \geq \pi_s$ and reduces that of all $\pi < \pi_s$. According

[15]See Hall & Lerner (2010) for an up-to-date survey on the empirical evidence, and Borisova & Brown (2013, 2015) for a more recent empirical test on the impact of financing frictions on corporate investments.

to this definition, $1 - G(\pi_s|x)$ measures the probability that an entrepreneur, investing x, successfully innovates.[16]

Throughout the paper the following assumptions will be maintained, unless differently specified.

Assumption 1. *The distributions g and G satisfy the following properties.*

(1) *$g(\pi|x)$ and $G(\pi|x)$ are twice continuously differentiable functions of x.*

(2) *Innovation requires investment, $G(\pi_s|0) = 1$;*

(3) *Entrepreneur's investment increases the likelihood of higher profits; for all x,*

$$\frac{\partial}{\partial \pi}\left(\frac{g_x(\pi|x)}{g(\pi|x)}\right)$$

is positive on Π and strictly positive for almost all (a.a.) $\pi \geq \pi_s$;

Property (3) is a *monotone likelihood ratio condition* (MLR) (Milgrom, 1981) and implies that the class of distributions considered satisfy *first-order stochastic dominance* over firms' outcomes. By (3), investment increase the likelihood of innovation, $G_x(\pi|x) \leq 0$, for all $\pi > 0$, holding with strict inequality at a.a. $\pi \leq \pi_s$.

An additional, technical assumption is the convexity of the distribution function:

Assumption 2. *For all $(x, z) > 0$, $\int_z^\infty g(\pi|x)d\pi = 1 - G(z|x)$ is strictly convex in x.*

This assumption prescribes a form of stochastic diminishing return to scale in investment, $G_{xx}(z|x) \leq 0$ for all $(x, z) > 0$. It is common in the moral hazard literature, as it makes possible to exploit the 'first-order approach' (see Rogerson, 1985). Conditional distributions satisfying the two assumptions and, particularly, MLR and convexity, can be found in the class of bivariate exponential, often used in applications.[17]

Financial contracts. A financial contract l in $\mathcal{L}$ is characterized by a triplet $(B_l, \boldsymbol{P}_l, \alpha_l)$. Namely, an amount of funds (or loan size) $B_l \geq 0$; a down-payment, in the form of a fraction $0 \leq \alpha_l \leq 1$ of the initial capital of the counterpart, which is secured in a safe deposit with return $R > 0$; a payment schedule $\boldsymbol{P}_l$ is a function of the firm's verifiable net-worth, given by the firm's profits π and the down-payment. We further restrict contracts by assuming that $\boldsymbol{P}_l$: *(a)* is piecewise smooth and right differentiable on Π; *(b)* satisfies *limited liability*; namely, for all π in Π, and l in $\mathcal{L}$,

$$\text{(LL)} \qquad 0 \leq \boldsymbol{P}_l \leq \pi + \alpha_l a_l R$$

The upper bound in (LL) is often motivated by legal provisions, or by the ability of the entrepreneurs (especially in small business) to hide funds and other sources of income. Instead, $0 \leq \boldsymbol{P}_l$ establishes that creditors' liability is limited by the loan offered to the firm

[16]The identification of one, or more, of such innovation-states is only for expositional reasons and can be removed without altering any of our results.

[17]For example, $1 - G(z|x) = a(bcx + 1)\exp[-za(bcx + 1)]$, $(a, b, c) > 0$, $z > 0$, known as Arnold & Strauss (1988) model.

(*i.e.* in the worse possible scenario, a creditor looses the amount of the loan). This excludes contracts in which the financier buys equity rights from the entrepreneur, which she may eventually waive, transfer back or loose.[18]

Up-front payments are secured in a bank deposit and, unless some form of confiscation is prescribed by the contract, they remain a firm's property. Confiscation may occur, according to (LL), either unconditionally or conditionally to some future states. In the first case, we have contracts such as those prescribing a participation (or franchise) fee; in the second case we have contracts such as collateralized loans, for which confiscation occurs, for example, in 'bankruptcy states' (states with 'particularly' low profit realizations).

We say that a financial contract l in $\mathcal{L}$ is a *standard debt contract* (SDC), in the sense of Gale & Hellwig (1985), if there exists a state z_l in Π such that, for all $\pi > z_l$, some constant payment is required; while, for all states $\pi \leq z_l$, a 'maximum payment' is required to the firm; this, in our context, correspond to the observable firm's assets, that is, firms' realized profit and the down-payment. Thus, in this respect, when $\pi \leq z_l$ we have the equivalent of what are usually called 'bankruptcy states'. More formally,

$$\text{(SDC)} \qquad \boldsymbol{P}_l = \begin{cases} z_l, & \text{for all } \pi > z_l \\ \pi + \alpha_l a R, & \text{for all } \pi \leq z_l \end{cases}$$

A financial contract l in $\mathcal{L}$ is a *live-or-die* (LDC) in the sense of Innes (1990), if at all $\pi > z_l$, a zero payment is required,

$$\text{(LDC)} \qquad \boldsymbol{P}_l = \begin{cases} 0, & \text{for all } \pi > z_l \\ \pi + \alpha_l a R, & \text{for all } \pi \leq z_l \end{cases}$$

A financial contract l in $\mathcal{L}$ is *monotone* (M) if the payment schedule is non-decreasing. Monotonicity has been explained as a way to prevent situations in which profits are strategically 'manipulated'. For example, an agent who can ex-post access to (hidden) borrowing or saving can fictitiously increase profits so as to escape bankruptcy and the consequent loss of collateral. Similarly, in the absence of (M), a creditor might find convenient to claim lower (or to 'sabotage') profits, so as to confiscate collateral.[19] Perhaps, the most relevant reason to impose monotonicity is that this property is simply observed in most real-world contracts.

Clearly, collateralized SDCs and LDCs, above, are both examples of contracts violating (M) (for SDC, see the dashed line in figure 1). Monotonicity is restored for collateralized SDCs if they prescribe to pay $\boldsymbol{P}_l = z_l + \alpha_l a_l R$, at all states $\pi \geq z_l$; and the new contract has a payment schedule that replicates the typical SDC defined in the literature (SDC + (M) in figure 1). However, imposing monotonicity has the effect to make the down-payment equivalent to a requirement of maximal capital participation by the agent.

The principal finances every contract, contextually, collecting deposits (*i.e.* borrowing) at the market interest rate R. He participates in a contract l financing an investment x if

[18] An equity contract of this type and size κ_l imposes a LL on the side of the financier, $-\kappa_l \leq \min_\pi \boldsymbol{P}_l$.
[19] See, also the discussion in Innes (1990), p.50.

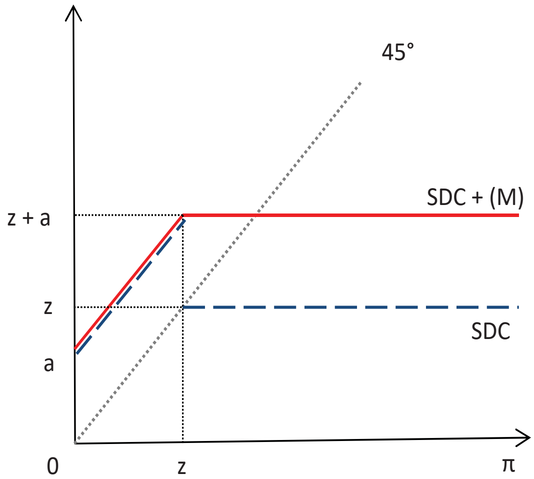

FIGURE 1. Contracts with full down-payment ($\alpha = 1$)

and only if,

$$\text{(IP)} \qquad \int \boldsymbol{P}_l g(\pi|x) d\pi - RB_l \geq 0$$

Later, we shall often refer to the (implicit) *interest rate* of a contract l; by this we simply the average payment, $\int \boldsymbol{P}_l g(\pi|x) d\pi / B_l$.

Entrepreneurs (agents). For every contract l, a *project* of an entrepreneur of type a, is characterized by a plan (a', x) that solves,

$$(\mathcal{F}(l,a)) \qquad \max_{(a',x)\in\mathbb{R}^2_+} \mathbb{E}[V(a', x; \pi, a, l)|x] :=$$

$$= \left\{ \int [\pi - \boldsymbol{P}_l] g(\pi|x) d\pi + \alpha_l a R + R a' : a' + x \leq B_l + (1 - \alpha_l) a \right\}$$

More precisely, we shall denote the project of a associated to contract l,

$$\big(a'(l,a), x(l,a)\big)$$

Implicit in this formulation is that firms cannot monitor other firms and that putting money into a safe deposit is the only feasible financial investment. Also, in line with the finance literature, we interpret the entrepreneur's decision to set $a' = 0$ as one in which he commits to maximal *equity financing* of a risky investment x. Indeed, $a' > 0$ occurs if the entrepreneur uses part of his assets for a financial investment, which, only for simplicity, we assumed to be the safe deposit with return R.

We say that entrepreneur a is willing to participate to a (non-trivial) contract l (*i.e.* with $B_l > 0$) if for some x and a' it yields an higher expected profits than implementing any other budget-feasible project $(\widetilde{a}', \widetilde{x})$ with self-financing. The following assumption ensures that investment decisions x are interior.

Assumption 3. *For every contract l in $\mathcal{L}$ to which entrepreneur a in A would participate,*

$$\frac{\partial}{\partial x}\mathbb{E}[V(a', x; \pi, a, l)|x] > 0$$

at $(a', x) = (B_l + a(1-\alpha_l),\ 0)$.

Remark 2.1 (On SDCs: collateral versus equity)**.** *Observe that, by definition of, the entrepreneur's problem, under a collateralized SDC, expected profits take the form:*

$$\int [\pi - \boldsymbol{P}_l] g(\pi|x) d\pi + \alpha_l a R + R a' = \int_{z_l}^{\infty} [\pi - z_l] g(\pi|x) d\pi + R\left[a(1 - \alpha_l \mathbf{1}_{\pi \le z_l}) + B_l - x\right]$$

This clarifies why we define such contracts 'collateralized' SDC: contracts which impose the payment of realized profits and collateral in 'bankruptcy' states (when $\pi \le z_l$ and the indicator function $\mathbf{1}_{\pi \le z_l}$ equals 1) and a fixed payment z_l, otherwise.

*SDC with capital participation are instead standard debt contracts which require the entrepreneur deposits a fraction α_l of the firm existing assets to the financier; although he fully maintains the control rights of the firm (*i.e. *he remains the only one entitled to receive the firm's net profits). Such contracts emerge when collateralized SDC are required to be monotone and satisfy (M) (see SDC+(M) in figure 1).*

Some preliminary results.

Lemma 1 (Investment)**.** *Assume that G is a continuous distribution. Then, for every entrepreneur of type a in A, contract l in $\mathcal{L}$ and investment x, such that (IP) holds, a solution to $\mathcal{F}(l, a)$ exists. Moreover, under assumption 2, the solution is unique if contracts are either SDCs or LDCs. Moreover, under assumption 3, $x > 0$.*

Proof. The first part is the result of the objective function being continuous and bounded on a compact domain of the choice variables. By (IP) and (LL), $RB_l \le \mathbb{E}[\boldsymbol{P}_l|x] \le \mathbb{E}[\pi|x] < \infty$.[20] Individual budget constraint implies that $0 \le a' \le a + B - x \le a + B < \infty$. Thus,

$$\mathbb{E}[V(a', x; \pi, t, l)|x] \le \mathbb{E}[\pi|x] + Ra' < \infty$$

Hence, by continuity, the objective is bounded from above. It is also bounded from below by the return from inaction, Ra. Therefore, existence of a solution follows from Weierstrass Theorem. Uniqueness holds for both SDCs and LDCs under assumption 2 and LL. ∎

Under assumption 1(1) and 2 we can characterize a project (a', x) in $(a'(l, a), x(l, a))$, as the pair of continuous functions that satisfy the following condition: whenever $a' = B_l + (1-\alpha)a - x > 0$,

$$(x) \qquad \frac{\partial}{\partial x}\mathbb{E}[V(a', x; \pi, a, l)|x)] = 0$$

[20]Hereafter, expectations are all computed with respect to the densities g.

Let $\mathbb{E}\left[V(a', x; \pi, a, R)|x\right]$ denote the expected profits of a typical entrepreneur who can borrow/save at the risk-free rate R. As we argue later, this contract is the optimal one in the absence of information asymmetries, hence it is hereafter addressed as the *first-best* contract. The following lemma defines the first-best level of investment.

Lemma 2 (Investment efficiency)**.** *Under assumption 1(1) and 2, for all a in A, there exists a unique efficient (first-best) investment x^*,*

$$\frac{\partial}{\partial x}\mathbb{E}\left[V(a', x; \pi, a, R)|x\right] = 0$$

Finally, to focus on situations in which outside financing is demanded by a non-negligible portion of entrepreneurs, we assume that a 'large' fraction of them would not be able to implement their projects without borrowing (*i.e.* choosing to subscribe the null contract $l = 0$). This is summarized in the following assumption.

Assumption 4. *There is an open interval A' of A such that, for all a in A', at a solution (a', x) to $\mathcal{F}(0, a)$,*

$$\frac{\partial}{\partial x}\mathbb{E}\left[V(a', x; \pi, a, 0)|x\right] > 0$$

Throughout the rest of the paper we shall redefine the type space A as equal to A'.

3. Optimal financial contracts

We define and characterize optimal contracts and analyze entrepreneurial policy decisions, assuming that: *i)* before contracting, each agent type sends a message concerning his type a and project (a', x); *ii)* once a contract l is signed, each agent chooses the project to implement; *iii)* profit realizations are publicly observed after projects have been finalized.

Contracts and entrepreneurial policies are defined based on a revelation mechanism, which establishes a sharing-rule over the verifiable outcome of projects and firm net-worth.

Definition 1 (Mechanism $\mathcal{M}$)**.** *A* ***mechanism*** *in $\mathcal{M}$ is:*

- ***i)*** *a set of messages $M \subset A \times \mathbb{R}^2_+$, of typical element $m = (a, a', x)$, that each entrepreneur can send to the principal,*
- ***ii)*** *a profile of outcome functions $M \to \mathcal{L} \times \mathbb{R}^2_+$ whose values identify a feasible contract l in $\mathcal{L}$ and an actual implementation of the the project, $(a'(m), x(m))$; where a contract l in $\mathcal{L}$ identifies a triplet $(\boldsymbol{P}_{l(m)}, B_{l(m)}, \alpha_{l(m)})$ defined above.*

$\mathcal{M}$ is a ***direct-mechanism*** *if for each type $\widetilde{a}$ in A, the message sent is truthful, $m \equiv (\widetilde{a}, \widetilde{a}', \widetilde{x})$, $l(m) \equiv \widetilde{a}$ and the announced project is implemented, $(a'(m), x(m)) \equiv (a'(\widetilde{a}), x(\widetilde{a}))$.*

Definition 2 (Optimal mechanism)**.** *An* ***optimal mechanism*** *is a direct-mechanism in $\mathcal{M}$ such that, for every entrepreneur of type a, each value of the outcome functions (l, a', x)*

solves problem $\mathcal{P}$:

$$\max_{(l,(a',x))\in(\mathcal{L},\mathbb{R}^2_+)} \mathbb{E}[V(a',x;\pi,a,l)|x] \quad s.t.$$

$$\text{(CC)} \qquad (l,a',x) \in \arg\max_{\{\bar{l},\bar{a}',\bar{x}\}} \mathbb{E}[V(\bar{a}',\bar{x};\pi,a,\bar{l})|\bar{x}]$$

$$\text{(IP)} \qquad \mathbb{E}[\boldsymbol{P}_l|x] - RB_l \geq 0$$

(CC) is both a *commitment* and an *incentive-compatibility* constraint ensuring that the agent does not deviate with respect to the project (a',x) and spontaneously subscribes the contract designed for his type.

Therefore, we define the *menu of optimal contracts* and *optimal entrepreneurial policies* as the outcomes of our optimal, direct mechanism. Restricting to direct-mechanism is without loss of generality and follows from a standard application of the *Revelation Principle*.

Theorem. *Let assumptions 1 through 4 hold. The menu of optimal contracts in $\mathcal{L}$ and optimal entrepreneurial policies is the outcome of a direct mechanism in $\mathcal{M}$ such that, for all a in A,*

(1) *entrepreneur a subscribes a financial contract l $(\equiv a)$ that i) if monotonicity (M) is assumed, takes the form of a SDC with maximal capital participation of the agent, $\alpha_l a = a$, or ii) if (M) is not assumed, takes the form of a LDC with 'full collateral', $\alpha_l a = a$;*
(2) *contract l brakes even (*i.e. *(IP) holds with equality);*
(3) *entrepreneur a chooses an inefficient investment project, $0 < x(l,a) \leq x^*$, with strict inequality if (CC) binds;*
(4) *entrepreneur a does neither resort to hidden saving nor to hidden borrowing ($a'(l,a) = 0$ is non-binding).*

For expositional reasons, it is useful to, first, deal with the case in which types are common knowledge (*i.e.* the case of 'pure moral hazard') and then to introduce asymmetric information on types. This will be, respectively, done in the next two subsections in which proposition 1 and 2 will be established. The proof of our theorem follows directly from these propositions.

3.1. **Pure moral hazard.** When a type a becomes common knowledge before contracting, the message space M is reduced to entrepreneur a projects and $\mathcal{P}$ is redefined simply by substituting (IC) with,

$$\text{(CC)} \qquad (a',x) \in \arg\max \mathbb{E}[V(a',x;\pi,a,l)|x]$$

which, in our language, is the *commitment-constraint* (CC).

For notational simplicity, in the rest of this section, we shall drop all the indices that are unnecessary, namely the entrepreneur's type a, and the index of contracts, l.

Proposition 1. *The menu of optimal contracts in $\mathcal{L}$ and optimal entrepreneurial policies is such that, for each entrepreneur a, properties (1)-(4) in the theorem hold.*

We denote by $\mathcal{L}^*$ the menu of optimal contracts.

To prove this result, consider the Lagrangian of problem $\mathcal{P}$, in which (M) is dropped, budget-balance is used to eliminate a' and (CC) is substituted with,

$$\text{(CC')} \qquad \frac{\partial}{\partial x}\mathbb{E}[V|x] \equiv \int [\pi - \boldsymbol{P}] g_x(\pi|x) d\pi - R \geq 0$$

Also, restrict to no hidden-borrowing, assume $a' \geq 0$, and denote the candidate multipliers by $(\psi, \gamma, \mu, \eta, \theta, \xi, \zeta)$,

$$\begin{aligned}
\max_{x,P,B,\alpha} \int [\pi - \boldsymbol{P}] g(\pi|x) d\pi + \alpha a R + (1+\psi) R\left[(1-\alpha)a + B - x\right] + \\
+ \gamma \left[\int \boldsymbol{P} g(\pi|x) d\pi - RB \right] + \\
+ \mu \left[\int [\pi - \boldsymbol{P}] g_x(\pi|x) d\pi - R \right] + \\
+ \int \eta(\pi)[\pi + \alpha a R - \boldsymbol{P}] g(\pi|x) d\pi + \int \theta(\pi) \boldsymbol{P} g(\pi|x) d\pi + \\
+ \xi B R + \zeta_0 \alpha a R + \zeta_1 (1-\alpha) a R
\end{aligned}$$

Necessary conditions for optimality are:

$$\text{(f1)} \qquad x: \ \frac{\partial}{\partial x}\mathbb{E}[V|x] + \gamma \int \boldsymbol{P} g_x(\pi|x) d\pi + \mu \frac{\partial^2}{\partial x^2}\mathbb{E}[V|x] - \psi R = 0,$$

$$\psi: \ \psi \geq 0, \ \psi\left[(1-\alpha)a + B - x\right] = 0$$

$$\text{(f2)} \qquad P: \ g(\pi|x)\left\{ -1 + \gamma - \mu \frac{g_x(\pi|x)}{g(\pi|x)} - [\eta(\pi) - \theta(\pi)] \right\} = 0, \quad \textit{for all } \pi$$

$$\text{(f3)} \quad B, \xi: \ 1 + \psi - \gamma + \xi = 0, \ \xi \geq 0, \ \xi B = 0$$

$$\text{(f4)} \quad \alpha, \zeta: \ 1 - (1+\psi) + \int \eta(\pi) g(\pi|x) d\pi = \zeta_1 - \zeta_0, \ \zeta_0, \zeta_1 \geq 0, \ \zeta_0 \alpha = 0, \ \zeta_1(1-\alpha) = 0.$$

Like in Innes (1990), risk-neutrality implies that the problem is linear in P. Let $\varphi(\pi, x, \cdot) := -1 + \gamma - \mu \frac{g_x(\pi|x)}{g(\pi|x)} - [\eta(\pi) - \theta(\pi)]$; then, for any given α, the optimal payment schedule satisfies the following,

$$\begin{aligned}
\varphi(\pi, x, \cdot) > 0 \quad &\textit{only if} \quad P = \pi + \alpha a, \quad \eta(\pi) \geq 0, \ \theta(\pi) = 0 \\
\varphi(\pi, x, \cdot) = 0 \quad &\textit{only if} \quad P \in [0, \pi + \alpha a], \quad \eta(\pi) \geq 0, \ \theta(\pi) \geq 0 \\
\varphi(\pi, x, \cdot) < 0 \quad &\textit{only if} \quad P = 0, \quad \eta(\pi) = 0, \ \theta(\pi) \geq 0
\end{aligned}$$

The proof of proposition 1 follows from four lemmas that are presented next.

Proof of proposition 1.

Lemma 3. *For any non-trivial contract in $\mathcal{L}^*$ with a non-binding commitment constraint (CC'), properties (1)-(3) in the theorem hold, with the investment level equal to x^*. $a' \geq 0$ is non-binding.*

Proof. We aim at showing that, at a solution, $\mu = 0$ implies $\gamma > 0$ (precisely, $\gamma = 1$) and $\psi = 0$. First, since a first-best entails a non-contingent payment R, we claim that, w.l.o.g., $B = x - a > 0$ and $\alpha = 0$. This, by complementary slackness, implies $\xi = 0$. $(f3)$ implies $\gamma \geq 1 + \psi \geq 1$. Next, we show that $\gamma \leq 1$. By contradiction, suppose $\gamma > 1$, then $P = \pi$ binding at all π occurring with positive probability; otherwise, $\eta(\pi) = 0$ would imply $\varphi(\pi, x, \cdot) = -1 + \gamma + \theta(\pi) > 0$, contradicting $(f2)$. However, $P = \pi$ at all π occurring with positive probability violates (CC'). Indeed, if $P = \pi$ at all π, $\frac{\partial}{\partial x}\mathbb{E}[V|x] = -R < 0$. Therefore, we conclude that $\mu = 0$ implies $\gamma = 1$.
By complementary slackness, $\gamma = 1 > 0$ implies that (IP) holds with equality.
Next, $\mu = 0$ and $\gamma = 1$, by $(f3)$, imply that $\psi = 0$ (at $B > 0$).
Finally, $\mu = 0, \gamma = 1$ and $(f1)$ imply that,

$$0 = \frac{\partial}{\partial x}\mathbb{E}[V|x] + \int \boldsymbol{P} g_x(\pi|x)d\pi = \int \pi g_x(\pi|x)d\pi - R$$

Hence, $x = x^*$. (CC) holds by Assumption 3. Clearly, in this case, there is some indeterminacy of the mechanism at the first-best: a mechanism with payment R, size $B = x - a$, $\alpha = 0$ and $a' = 0$ is equivalent to one with -same uncontingent payment- loan size $B' = a' + B + \alpha a$, $0 < \alpha \leq 1$ and $a' \neq 0$. ∎

Lemma 4. *For any non-trivial contract in $\mathcal{L}^*$ with a binding commitment constraint (CC'), properties (1)-(3) in the theorem hold and $a' \geq 0$ is non-binding.*

Proof. We proceed in steps.

Step 1: A solution satisfies (IP) with equality.
$(f3)$ implies $\gamma = 1 + \psi > 0$. By complementary slackness (IP) binds.

Step 2: A solution satisfies (CC') with equality.
From $\mu > 0$ and complementary slackness, $0 = \frac{\partial}{\partial x}\mathbb{E}[V|x]$.

Step 3: A solution is a LD contract.
$\mu > 0$ and MLR imply that $\varphi(\pi, x, \cdot)$ is decreasing in π, strictly decreasing for some $\pi \geq \pi_s$. Moreover $\varphi(\pi, x, \cdot) \geq 0$ at some π occurring with positive probability, otherwise $P = 0$ would be chosen violating (IP). Moreover, we know that, generically, a solution satisfies $\varphi(\pi, x, \cdot) > 0$ at some π. Also, (CC') implies that we cannot have $P = \pi + \alpha a$ at all π and thus, at a solution, it must be that $\varphi(\pi, x, \cdot) \leq 0$ at some π. By continuity and the Intermediate Value Thm., we conclude that there exists a $z \geq \pi_s > 0$ such that $\varphi(\pi, x, \cdot) = 0$. The fact that $z \geq \pi_s$ follows from Step 1, $\gamma \geq 1$. Moreover, z is unique by strict monotonicity of g_x/g on $[\pi_s, \infty)$. The optimal payment schedule is typical of a LDC, in the sense of Innes (1990),

$$\boldsymbol{P} = \begin{cases} 0, & \pi > z \\ \pi + \alpha a R & \textit{otherwise} \end{cases}$$

Step 4: We now show that, because at a solution $\int \boldsymbol{P} g_x(\pi|x)d\pi > 0$, the solution is inefficient, $0 < x < x^*$, and satisfies (CC) with a non-binding constraint $a' \geq 0$.
First, because (CC') binds ($\mu > 0$), $0 = \frac{\partial}{\partial x}\mathbb{E}V[\pi|x]$ is a necessary optimality condition for

the agent at a non-binding constraint $a' \geq 0$; implying $\psi = 0$. Moreover,

$$(o) \qquad \begin{aligned} 0 &= \frac{\partial}{\partial x}\mathbb{E}[V|x] \\ &= \frac{\partial}{\partial x}\left[\int [\pi - \boldsymbol{P}]g(\pi|x)d\pi] + \alpha a R + R(B + a(1-\alpha) - x)\right] \\ &= \frac{\partial}{\partial x}\int [\pi - Rx]g(\pi|x)d\pi - \int \boldsymbol{P} g_x(\pi|x)d\pi \end{aligned}$$

Using, $0 = \frac{\partial}{\partial x}\mathbb{E}[V|x]$, $\gamma = 1, \psi = 0$, $(f1)$ reads,

$$\frac{\partial^2}{\partial x^2}\mathbb{E}[V|x] = -\frac{1}{\mu}\int \boldsymbol{P} g_x(\pi|x)d\pi < 0$$

with the latest inequality following from assumption 2 and $\mu > 0$. Using, $\int \boldsymbol{P} g_x(\pi|x)d\pi > 0$ in (o), one finds,

$$(*) \qquad \frac{\partial}{\partial x}\int [\pi - Rx]g(\pi|x)d\pi > 0$$

As (CC') holds with equality, a solution x satisfies necessary and sufficient conditions for individual optimality (CC), with a non-binding constraint $a' \geq 0$. Finally, by assumption 2, $\int [\pi - R]g(\pi|x)d\pi$ is strictly concave in x; hence, $(*)$ implies $x < x^*$. $0 < x$ follows from assumption 3.

Step 5: Full collateralized LDC contracts.
For any non-trivial contract, we have seen that, $\xi = 0 = \psi$, $\gamma = 1$, irrespectively of $\mu \geq 0$. Hence, (f4) reduces to,

$$(\text{f4'}) \qquad \int \eta(\pi)g(\pi|x)d\pi = \zeta_1 - \zeta_0 \gtreqless 0$$

which, respectively, implies an optimal presciption: $\alpha = 1$ if $lhs > 0$, $\alpha \in [0,1]$ if $lhs = 0$ and $\alpha = 0$ if $lhs < 0$. Next, integrating (f2) on Π,

$$\int [\eta(\pi) - \theta(\pi)]g(\pi|x)d\pi = \gamma - 1 = 0$$

Since, by (f2), $\theta(\pi) \geq 0$ holds strictly at some π, occurring with positive probability, we conclude that a solution of $\mathcal{P}$ yields $\zeta_1 > 0 = \zeta_0$ and, by complementary slackness, it delivers $\alpha = 1$. ■

Lemma 5. *In the context of our theorem, restrict contracts to have a monotone payment schedule (M). Then, any non-trivial solution to $\mathcal{P}$ is a SDC such that properties (1)-(3) stated in the theorem hold.*

Proof. The proof reiterates the ones given for lemma 3 and 4. It is immediate to verify that the only difference is for the latest lemma, when it comes to the definition of the payment schedule $P(\cdot)$, for which (M) dictates, $P = z + aR$, for all $\pi > z$. This pins down the optimal SDC. ■

The following establishes property (4) in the theorem.

Lemma 6. *In the context of our theorem, any optimal financial contract l in $\mathcal{L}^*$ entails no hidden financial decisions.*

The fact that an optimal contract does not produce hidden borrowing follows directly from lemma 3. Thus, we are left to prove that a solution to $\mathcal{P}$ does never entail hidden saving. This proof, we shall give next, has an intuitive argument. Indeed, reasoning by contradiction, suppose that a solution to $\mathcal{P}$ is a contract of size B such that the agent finds feasible and rewarding to save $a' > 0$. There exists an alternative contract with a lower size $\widetilde{B}$ and a lower cost $\widetilde{z}$ such that, if subscribed by the agent, would finance a project characterized by a higher real investment $\widetilde{x}$ and a lower saving $\widetilde{a}'$. Indeed, if the agent, at the optimal contract, were indifferent between investing in the real asset and in the hidden deposit, now a reduction of the interest rate on outside financing (given R) makes profitable for him to increase the first and reduce the second. Thus, a contradiction to optimality is attained.

Proof. By contradiction, suppose that an optimal SDC, (B, z), entails $a' > 0$. Let us marginally reduce B and increase x: $dx = -\epsilon dB > 0$, $\epsilon > 0$ arbitrarily small. This is feasible if $da' = dB - dx = dB(1+\epsilon)$, and if it satisfies (IP). (IP) holds if,

$$\left(\frac{\partial}{\partial z}\mathbb{E}[\boldsymbol{P}|x]\right)\frac{dz}{dB} + \left(\int \boldsymbol{P} g_x(\pi|x)d\pi\right)\frac{dx}{dB} - R = 0$$

where, under (M), $\frac{\partial}{\partial z}\mathbb{E}[\boldsymbol{P}|x] := \int_z^\infty g(\pi|x)d\pi > 0$.[21] Or, equivalently,

$$(\star) \qquad \frac{dz}{dB} = \frac{R + \left(\int \boldsymbol{P} g_x(\pi|x)d\pi\right)\epsilon}{\frac{\partial}{\partial z}\mathbb{E}[\boldsymbol{P}|x]}$$

whose rhs, evaluated at an optimal contract, is positive. Next, totally differentiating the agent's objective,

$$d\mathbb{E}[V|x] = -\left(\frac{\partial}{\partial z}\mathbb{E}[\boldsymbol{P}|x]\right)dz + \left(\frac{\partial}{\partial x}\mathbb{E}[V|x]\right)dx + RdB$$

which, evaluated at the optimal contract (entailing $\frac{\partial}{\partial x}\mathbb{E}[V|x] = 0$), reduces to,

$$\frac{d\mathbb{E}[V|x]}{dB} = -\left(\frac{\partial}{\partial z}\mathbb{E}[\boldsymbol{P}|x]\right)\frac{dz}{dB} + R$$

Using $(\star)$,

$$\begin{aligned}\frac{d\mathbb{E}[V|x]}{dB} &= -\left(\frac{\partial}{\partial z}\mathbb{E}[\boldsymbol{P}|x]\right)\left[\frac{R + \left(\int \boldsymbol{P} g_x(\pi|x)d\pi\right)\epsilon}{\frac{\partial}{\partial z}\mathbb{E}[\boldsymbol{P}|x]}\right] + R \\ &= -\left(\int \boldsymbol{P} g_x(\pi|x)d\pi\right)\epsilon < 0\end{aligned}$$

[21]Notice that $\frac{d}{dz}D_x\mathbb{E}[\boldsymbol{P}|x] := \int_z^\infty g_x d\pi > 0$, because $g_x > 0$ for a.a. $\pi \geq z \geq \pi_s$, by assumption 1.

for any $\epsilon > 0$. It remains to check that (CC') continues to hold, for at least some $\epsilon > 0$. At the initial, optimal contract, (CC') holds if,

$$-\left(\frac{\partial}{\partial z} D_x \mathbb{E}[\boldsymbol{P}|x]\right) dz + \left(\frac{\partial^2}{\partial x^2} E[V|x]\right) dx \geq 0$$

where, under (M) (*i.e.* for collateralized SDCs), $\frac{\partial}{\partial z} D_x \mathbb{E}[\boldsymbol{P}|x] := \int_z^\infty g_x(\pi|x) d\pi$. Using this to determine ϵ:

$$\epsilon \equiv -\frac{dx}{dB} \geq \left(\frac{\frac{\partial}{\partial z} D_x \mathbb{E}[\boldsymbol{P}|x]}{\frac{\partial^2}{\partial x^2} E[V|x]}\right) \frac{dz}{dB}$$

By ($\star$), $dz/dB > 0$ implying that the right hand side of the latter expression is negative, when evaluated at the initial optimal contract. Therefore, (CC') holds for all $\epsilon > 0$. To sum up, a marginal reduction of the loan size, accompanied with a feasible adjustment of z (decreasing) and x (increasing) improves the agent's welfare; which provides the desired contradiction. As the initial contract delivering $a' > 0$, was arbitrary, this concludes our proof. It is straightforward to reiterate the argument for the case of a LDC. ■

Finally, it is important to remark that the above lemmata can be restated and proved for the case in which $\mathcal{L}$ is restricted to admit only contracts that do not allow for down-payments. If we denote this subset of contracts by $\mathcal{L}_0$, it is then straightforward to verify that the following corollary of our theorem holds.

Corollary 1. *The menu of optimal contracts in $\mathcal{L}_0$ and optimal entrepreneurial policies is such that, for each entrepreneur a, properties (1)-(4) in the theorem hold.*

3.2. **Discussion.**

On the firm's financial policy. Which are the specific characteristics of a firm's financial structure under optimal contracts? Our theorem says that an optimal mechanism is a loan contract whose size exactly matches the agent's demand of credit. Hence, we can conclude that if the agent were allowed to secretly borrow at the same contract (or issuing a bond at a fixed market rate R) after the optimal financial contract is signed, he would not do so. Moreover, the fact that the entrepreneur does not use part of the loan to build up (hidden) savings, confirms the idea that the entrepreneurs' demand of outside funds is somewhat residual, equal to that fraction of projects that they cannot cover by internal sources.

Is the optimal financial contract unique? Answering this question is relevant to assess if the financial structure of the firm matters. In standard neoclassical theory of the firm, Modigliani-Miller's theorem holds: regardless financial markets are complete or incomplete, the financial structure of the firm is indeterminate. In our model instead, the optimal financial structure of the firm is unique. Indeed, we have defined an optimal contract as one in which the entrepreneur finds individually optimal to commit to his announced project (a', x). We have also established that, at any optimal contract, the firm's second-order-condition holds with strict inequality (by assumption 2 and lemma 3), hence (a', x) is unique and so is the threshold z. Finally, since at a solution to $\mathcal{P}$, the entrepreneur budget balances, the loan size is also uniquely determined, $B = x$ (recall that $a' = 0$ and $\alpha = 1$).

The relevance of hidden saving. Let us assume that an entrepreneur's saving a' is verifiable (*i.e.* it cannot be hidden). Then, omitting down-payments, financial contracts can be written contingently on,

$$w(a') := \pi + Ra'$$

Consequently, the entrepreneur saving decision becomes indeterminate and can be disregarded. Indeed, it is easy to verify that every optimal contract l in $\mathcal{L}$ takes again the form of a LDC. Precisely, if z is unique, for a type a, with actions (a', x) in $(a'(l,a), x(l,a))$, the payment schedule of l is,

$$\boldsymbol{P}_l(w(a')) = \begin{cases} 0, & w(a') \geq z \\ w(a') & otherwise \end{cases}$$

and $RB_l = \mathbb{E}[P_l(w)|x]$. Therefore, (IP) holds also at $(B_l - a')$ and $\boldsymbol{P}_l$, implying that entrepreneur t is indifferent between $(a', x, B_l, \boldsymbol{P}_l(w))$ and $(0, x, B_l - a', \boldsymbol{P}_l)$.

To summarize, whenever contracts can also be written on saving, saving decisions become redundant/indeterminate and, in absence of private information, the whole analysis falls back into Innes' (1990).

Down-payments improve incentives. Let us go back to Innes's (1990). Even if we eliminate the possibility of hidden savings/borrowing, an optimal mechanism prescribes maximum down-payment. Restrict contracts to have a security deposit that is unconditionally returned to the firm in the next period; or, alternatively, a down-payment that is non-refundable (*e.g.* a non-refundable capital participation, or a franchise fee). In this new setting the size and nature of the down-payment is indeterminate, as the principal can always neutralize them by adjusting the debt size, when necessary.[22] These types of contracts are clearly feasible in our model, as the principal can always implement them by, respectively, choosing $P \leq \pi$ and $P \leq \pi + \alpha a$, at all π. Yet, based on our theorem, we conjecture that (unless the first-best is implementable) contracts with a down-payment are 'preferred' or more efficient, because they strengthen agents' incentives to invest; something that is ultimately due to the fact that down payments allow to relax the limited liability constraint.

To further understand why down-payment can implement more powerful incentive scheme, we next argue that, at an optimal mechanism with entrepreneur a investing x', any alternative contract in $\mathcal{L}$, with either no down payment or a full security deposit,[23] supporting x', carries 'weaker incentives' (*i.e.* induces a to choose $x < x'$). More precisely, suppose that an entrepreneur of type a subscribes a collateralized LDC $(B', z', 1)$ in $\mathcal{L}^*$,

[22] Again, this is trivially so by the uncontingent nature of the payments and hold inasmuch agents do not suffer of credit rationing.

[23] By security deposit we mean that the up-front payment is returned to the firm unconditionally.

supporting a project, x', and $x' = B'$. Consider a LDC with no down payment $(B, z, 0)$ supporting the same project $x' = B' = B$ with (IP) holding with equality.

Claim 1: *Collateralized LDC deliver stronger incentives: $z' < z$.*

The best contract LDC with no down-payment, in the eyes of the entrepreneur, is one that makes (IP) hold with equality. So assume that,

$$\int_0^z \pi g(\pi|x')d\pi = RB'$$

Combined with the (IP) holding with equality at the optimal collateralized contract, yields,

$$\int_0^{z'} \pi g(\pi|x')d\pi + RaG(z'|x') = RB' = \int_0^z \pi g(\pi|x')d\pi$$

The latest is satisfied if and only if $z' < z$.

Claim 2: *Consider the same two contracts above, $(B', z', 1)$, $(B, z, 0)$ with $z' < z$. Although x' is feasible under the contract with no down-payment the entrepreneur of type a prefers $x < x'$.*

By contradiction, suppose that the entrepreneur prefers $x \geq x'$. Then, by individual optimality,

$$\int_z^\infty \pi g_x(\pi|x')d\pi \geq R$$

also by the optimality of x' at the collateralized contract $(B', z', 1)$,

$$\int_{z'}^\infty \pi g_x(\pi|x')d\pi - RaG_x(z'|x') = R$$

Combining the two conditions,

$$\int_z^\infty \pi g_x(\pi|x') - \int_{z'}^\infty \pi g_x(\pi|x')d\pi \geq -RaG_x(z'|x') > 0$$

Implying,

$$-\int_{z'}^z \pi g_x(\pi|x')d\pi > 0$$

The left hand side of the latest is negative for all optimal mechanisms with a binding incentive-compatibility constraint.[24] Therefore, we achieve a contradiction.

Clearly, the two claims above are true also for SDCs.

To summarize, the introduction of collateralized contracts allows to weaken (LL) and to redistribute the loan cost from states with high profits (successful states) to those with low profits (unsuccessful or 'bankruptcy' states). This is illustrated graphically, for SDCs, in figure 2. Therefore, provided that collateral does not ration the amount of funds available

[24]This is not immediate and follows from the optimal definition of the payment schedule and of z' in lemma 4. For all $\pi \geq z' \geq \pi_s$, yields $\varphi(\pi, x', \cdot) \leq 0$; namely, $\mu g_x(\pi|x') \geq \theta(\pi) \geq 0$, when $\mu > 0$; where the inequality for a.a. $\pi \geq z'$.

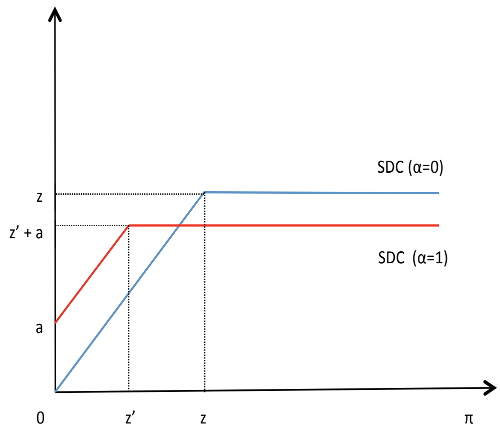

FIGURE 2. Contracts with and without down-payment

to investors, such contracts lower the implicit interest rate, raise the firm's expected profits and boost real investments.

Investment policy predictions over the cross-section of firms. Consider entrepreneurs who subscribe collateralized SDCs in $\mathcal{L}^*$ and, for expositional simplicity, refer each contract optimally designed for entrepreneur a as contract a; the space of optimal contracts is indexed over $A := [\underline{a}, \bar{a}]$. The next result establishes an important property of optimal SDC; namely, contracts incentivate firms with higher initial capital to invest more in the firm. The reason why this occurs is that higher capitalized firms participate in the project with an higher down-payment (or collateral); this, according to our previous discussion (see Claims 1,2) makes feasible for the intermediary to decrease the loan threshold z; which -for fixed R- increases the entrepreneur incentive to invest and x. Before establishing this formally, we present a useful technical result.

Lemma 7. *At any monotone SDC a in* $\mathcal{L}^*$*,*

$$R - \int \boldsymbol{P}_a g_x(\pi|x(a))d\pi > \left[\frac{1 - G(z|x(a))}{\int_z^\infty g_x(z|x(a))d\pi}\right] \frac{\partial^2}{\partial x^2}\mathbb{E}[V|x(a)] \tag{oo}$$

Proof. By contradiction, suppose that, at an optimal contract (oo) fails to hold. We are going to prove that by, proportionally, increasing B and x and lowering z, we can increase the value of the objective without violating (IP) and the commitment constraint (CC'). Indeed, let $dB = dx > 0$ (and $da' = 0$), be an arbitrarily small change. Since (IP) holds with equality at a.a. a in $\mathcal{L}^*$, differentiating and evaluating at the optimal contract, we find,

$$\frac{dz}{dx} = \frac{R - \int \boldsymbol{P} g_x(\pi|x)d\pi}{1 - G(z|x)} \tag{$o*$}$$

which is negative when (oo) fails. We now show that our feasible adjustment dz/dx does not violate incentives. For (CC') to hold,

$$-\left(\frac{\partial}{\partial z}\int \boldsymbol{P} g_x(\pi|x)d\pi\right)dz+\left(\frac{\partial^2}{\partial x^2}E[V|x]\right)dx\geq 0$$

Yielding,

$$\frac{dz}{dx}\leq\frac{\frac{\partial^2}{\partial x^2}E[V|x]}{\int_z^{\infty}g_x(z|x)d\pi}<0$$

This does not contradict ($o*$) iff,

$$\frac{R-\int \boldsymbol{P} g_x(\pi|x)d\pi}{1-G(z|x)}\leq\frac{\frac{\partial^2}{\partial x^2}E[V|x]}{\int_z^{\infty}g_x(z|x)d\pi}$$

that we can rewrite as,

$$R-\int \boldsymbol{P} g_x(\pi|x)d\pi\leq\left[\frac{1-G(z|x)}{\int_z^{\infty}g_x(z|x)d\pi}\right]\frac{\partial^2}{\partial x^2}E[V|x]$$

This holds because (oo) does not. Finally, it is immediate to check that an increase of x, B and a decrease of z raise the value of the objective, delivering the desired contradiction. ■

Lemma 8. *The menu of optimal, monotone SDCs is characterized by an investment schedule $x(a)$ and a loan rate schedule $z(a)$ that are, respectively, monotonically increasing and decreasing for almost all a in $\{A : x(a) < x^*\}$ and constant otherwise.*

Proof. We essentially verify that the Implicit Function Theorem holds and that it delivers the desired monotonicity properties of $(z, x)(a)$. From (CC') holding with equality at almost all (a.a.) a, we find,

$$(+)\qquad \frac{dx}{da}(a)=\left(\frac{\frac{\partial}{\partial z}\int \boldsymbol{P}_a g_x(\pi|x(a))d\pi}{\frac{\partial^2}{\partial x^2}\mathbb{E}[V|x(a)]}\right)\frac{dz}{da}(a)=-\mu(a)\left(\frac{\frac{\partial}{\partial z}\int \boldsymbol{P}_a g_x(\pi|x(a))d\pi}{\int \boldsymbol{P}_a g_x(\pi|x(a))d\pi}\right)\frac{dz}{da}(a)$$

For any feasible change dz/da, (+) measures the slope of the optimal investment schedule on A. This is zero in the case of no-moral hazard (*i.e.* when the commitment constraint is non-binding, $\mu(a) = 0$), corresponding to efficient investment x^*, for all firms, independently of theirs initial capital. The numerator of (+) is, $(\int_z^{\infty} g_x(z|x)d\pi)(dz/da)$, evaluated at (a). This implies that -given the capital requirements- it is individually optimal to increase real investments for entrepreneurs with higher capitalized firms if and only if they face a decreasing loan threshold z. Next, we prove that indeed, at any optimal, monotone SDC a, $dz/da < 0$.

Recall that (IP) holds with equality at a.a. contracts in $\mathcal{L}^*$. Hence, the total differential (with respect to a, z, x, B) must be equal to zero almost everywhere (a.e.). Moreover, since for a.a. a, the individual budget constraint balances at zero saving, $\frac{dB}{da} = \frac{dx}{da}$. Hence, passing to SDCs and using $g := g(\pi|x(a))$, for brevity,

$$\begin{aligned}0 &= \frac{\partial}{\partial a}\left(\int \boldsymbol{P}_a g d\pi\right) da + \frac{\partial}{\partial z}\left(\int \boldsymbol{P}_a g d\pi\right) dz + \frac{\partial}{\partial x}\left(\int \boldsymbol{P}_a g d\pi\right) dx - RdB \\ &= Rda + \frac{\partial}{\partial z}\left(\int_0^z \pi g d\pi + z\int_z^\infty g d\pi\right) dz + \left(\int \boldsymbol{P}_a g_x(\pi|x(a)) d\pi - R\right) dx \\ &= R + [1 - G(z|x(a))]\frac{dz}{da} + \left(\int \boldsymbol{P}_a g_x(\pi|x(a)) d\pi - R\right)\frac{dx}{da}\end{aligned}$$

Next, using (+) into the latest expression and rearranging terms,

$$\frac{dz}{da}(a) = \frac{-R}{1 - G(z|x(a)) + \left(\int \boldsymbol{P}_a g_x(\pi|x(a)) d\pi - R\right)\left(\frac{\int_z^\infty g_x(z|x(a))d\pi}{\frac{\partial^2}{\partial x^2}\mathbb{E}[V|x(a)]}\right)}$$

The right hand side of this expression is negative iff the denominator is strictly positive, which is true at a.a. optimal, monotone SDCs by lemma 7. ■

3.3. **Private information on firms' initial capital.** So far, we have assumed that the regulator/principal does not observe entrepreneurs' types. In this section, we relax this assumption and prove that the optimal mechanism delivers a menu of contracts, in $\mathcal{L}$, that is incentive-compatible, in the sense of (IC): it induces all entrepreneurs to truthfully reveal their types, implement their announced projects and subscribe the contracts which are designed to sustain such projects. For expositional reasons, we assume that financial contracts are exclusive; in the sense that every entrepreneur can subscribe a single financial contract. Later, in remark 3.1, we explain why –at an optimal menu of contracts- entrepreneurs do actually prefer to subscribe a single optimal contract than to shop for a portfolio of different contracts.

Consider contracts that require the entrepreneur who sends a message a to dispose a down-payment of a. Clearly, under full down-payment, an entrepreneur of type $\widetilde{a}$ can only send messages with $a \leq \widetilde{a}$, which rules out the possibility that he can access to (hence deviate to) SDCs designed for higher capitalized firms. Then, the following is true.

Proposition 2. *Under private information on types in A, a.a. contracts l in $\mathcal{L}^*$ are incentive-compatible.*

Proof. For any agent $\widetilde{a}$ in A, we have to establish that he prefers to follow the behavior prescribed by the mechanism $\widetilde{a}$ with an expected payoff $\nu(\widetilde{a},\widetilde{a})$, instead of taking an alternative mechanism $\ell \neq \widetilde{a}$ in $\mathcal{L}^*$ and use it to implement an individually optimal project $(a'(\ell,\widetilde{a}), x(\ell,\widetilde{a}))$.[25] Since for a type $\widetilde{a}$ the only feasible messages are those with $a \leq \widetilde{a}$, consider these deviations.

First, we show that for all $\ell \leq \widetilde{a}$, $x(\ell,\widetilde{a}) = x(\ell)$. Indeed, $x(\ell)$ is feasible for type $\widetilde{a} > \ell$, who can claim to have a firm with capital $a_l < \widetilde{a}$ and implement $x(\ell) = B_\ell < B_\ell + \widetilde{a} - a_\ell$. This implies that $x(\ell,\widetilde{a}) \geq x(\ell)$. Next, because types have the same objectives and type ℓ

[25]Later, we use the notation $(a'(\ell), x(\ell)) := (a'(\ell,\ell), x(\ell,\ell))$.

is not rationed at the optimal contract ℓ, $x(\ell)$ is also individually optimal for $\widetilde{a}$, implying that $a'(\ell, \widetilde{a}) = \widetilde{a} - a_\ell$; namely, $x(\ell, \widetilde{a}) \leq x(\ell)$.

Second, recall that for a.a. ℓ in $\mathcal{L}^*$, $a'(\ell) = 0$ and $x(\ell) = B_\ell$.

We are now going to exploit these two last properties of $\mathcal{L}^*$ to verify that the menu of contracts found in proposition 1 is incentive-compatible. For all $a \leq \widetilde{a}$ and entrepreneur $\widetilde{a}$,

$$
\begin{aligned}
\nu(\widetilde{a}, \widetilde{a}) - \nu(a, \widetilde{a}) &= \int [\pi - \boldsymbol{P}_{\widetilde{a}}] g(\pi|x)(\widetilde{a}) d\pi - \int [\pi - \boldsymbol{P}_a] g(\pi|x)(a, \widetilde{a}) d\pi - Ra - Ra'(a, \widetilde{a}) \\
&= \int [\pi - \boldsymbol{P}_{\widetilde{a}}] g(\pi|x)(\widetilde{a}) d\pi - \int [\pi - \boldsymbol{P}_a] g(\pi|x)(a) d\pi - R[a + a'(a, \widetilde{a})] \\
&= \int \pi [g(\pi|x)(\widetilde{a}) - g(\pi|x)(a)] d\pi - RB_{\widetilde{a}} + RB_a - R\widetilde{a} \\
&= \int \pi [g(\pi|x)(\widetilde{a}) - g(\pi|x)(a)] d\pi - R(B_{\widetilde{a}} + \widetilde{a}) + Rx(a) \\
&= \int \pi [g(\pi|x)(\widetilde{a}) - g(\pi|x)(a)] d\pi - R[x(\widetilde{a}) - x(a)]
\end{aligned}
$$

This, by the Fundamental Theorem of Calculus, for all $a \leq \widetilde{a}$, can be written as,

$$
\int_a^{\widetilde{a}} \left[\left(\int \pi g_x(\pi|x)(\ell) d\pi - R \right) \frac{dx}{da}(\ell) \right] d\ell
$$

For the latest to be positive, it suffices that for all $a \leq \ell \leq \widetilde{a}$, the argument in the square brackets is positive. Indeed, this is true since, first, by (CC'),

$$
\int \pi g_x(\pi|x)(\ell) d\pi - R = \int \boldsymbol{P}_\ell(\pi) g_x(\pi|x)(\ell) d\pi \geq 0
$$

with strict inequality if $x(\ell) < x^*$; second, for all $\ell \leq \bar{a}$, $(dx/da)_\ell \geq 0$ by lemma 8 above, holding with strict inequality for a.a. contracts ℓ for which $x(\ell) = x^*$. Therefore, $\nu(\widetilde{a}, \widetilde{a}) - \nu(a, \widetilde{a}) > 0$ if $x(\ell) < x^*$ and is zero otherwise. ■

It is interesting to remark that the introduction of private information on the initial firms' capital does not alter the original incentive scheme provided by the optimal mechanism in the pure moral hazard case (proposition 1). Thus, contracts retain the original properties and deliver an interest rate schedule, $(\int \boldsymbol{P}_a g(\pi|x) d\pi / B_a)_{a \in \mathcal{L}^*}$, that is decreasing in the initial firms' capital a. This, again, proves that higher capitalized firms have better financial opportunities. The down payment is used here, both as an optimal incentive devise and as an efficient screening devise. The down payment increases incentives to invest, by relaxing the limited liability, and preserves incentive-compatibility by eliminating the possibility that agents lie on their type. In fact, because contracts for higher capitalized firms are cheeper and offer a loan of bigger size, they would certainly attract individuals with lower capitalized firms.

Remark 3.1 (Exclusivity)**.** *In the context of our proposition 2 the optimal menu of contracts is such that no entrepreneur has an incentive to subscribe multiple contracts; that is,* *the restriction that contracts are exclusive is non-binding. In fact, any entrepreneur's demand of credit financed by a portfolio of loans (designed for lower capitalized firms), instead of a single contract, is characterized by a higher interest rate.*

4. Conclusions

In this article we propose a security-design problem of a setting in which risk-neutral entrepreneurs sick to finance an investment project that is unobservable by risk-neutral, outside investor/creditors. A larger investment in the firm increases its productivity by raising the likelihood of higher realizations of future profits. However, entrepreneurs may also find rewarding to divert firm's funds and hide them in an asset. Since investments are unobservable and the exact consistency of firms' net-worth is unverifiable, creditors are exposed to both moral hazard and adverse selection. Contracts can only be written contingently on future profits, which are publicly observable, and on initial down-payments, in the form of a security deposit. We focus on contracts that are written in the best interest of the entrepreneurs and that satisfy some canonical restrictions: limited liability and, possibly, the monotonicity of the payment schedule the firms verifiable net-worth.

In this context, an optimal menu of contracts is formed by standard-debt-contracts and a maximum down-payment. The down-payment takes either the form of collateral or that of a capital requirement, respectively, depending on whether one we assume monotonicity of the payment schedule. Their interest rate schedule is decreasing in the agents' initial capital. Therefore, higher capitalized firms access to better credit conditions. Contracts induce no credit rationing, but typically sustain a level of productive investments that is only second-best efficient. The occurrence of agency costs has some key implications on the entrepreneurs' decisions. First, entrepreneurs choose a financial structure that privileges internal to external funds. In particular, only entrepreneurs who have an insufficient initial capital decide to borrow. These entrepreneurs are exactly those who decide to underinvest in their firms. Second, at the optimal contracts, entrepreneurs prefer to invest in their firms, rather than hide funds. Third, these properties of optimal contracts are robust to the introduction of private information on entrepreneurs initial net-worth. The presence of down-payments grants incentive-compatibility, by preventing agents to lie on their capital and subscribe contracts designed for higher capitalized firms. Finally, our findings hold regardless contracts are assume to be exclusive. The fact that the interest rate schedule is decreasing in the down-payment, represents a disincentive to finance a project by entering multiple contracts.

We have motivated our interest in this security-design problem also referring to the 'opacity' that typically characterizes the financial situation and decisions of small firms, representing a particularly large fraction of the non-financial sector in most developed countries. These firms heavily rely on bank credit and raise outside funds in the form of standard credit lines. The interest rate conditions on credit lines are typically decreasing in the loan size, which is highly positively correlated with firms' dimension (in general, capital). This type of financial structure has often be considered to cause inefficiency and underinvestment. Instead, to some extent, our analysis questions this conclusion. A proper combination of standard debt contracts and collateral or minimum capital requirement may constitute an efficient (in sense of *second-best*) corporate finance. It is so, in the presence of moral hazard, essentially because standard-debt-contracts may provide entrepreneurs with a reward scheme that is equivalent to the best equity-base one, compatible with incentives. It is also so in the presence of adverse selection due to the positive effect of down-payments in separating firms with different levels of initial capital.

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