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Abstract

This study considers the politics of public education and its impacts on economic growth and welfare across generations. Public education is funded by taxing the labor income of the working generation and capital income of the retired. We employ probabilistic voting to demonstrate the politics of taxes and expenditure and show that aging results in a shift of the tax burden from the old to the young and a slowdown of economic growth. We then consider three alternative constraints that limit the choice of taxes and/or expenditure: a minimum level of public education expenditure, an upper limit of the capital income tax rate, and a combination of the two. These constraints all create a trade-off between current and future generations in terms of welfare.

- Keywords: Public education, Economic growth, Capital income tax, Political equilibrium
- JEL Classification: D70, E24, H52.

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1 Introduction

This study considers the politics of public education and its impacts on economic growth and welfare across generations, a topic widely researched since Saint-Paul and Verdier (1993). Given population aging and the increasing share of the elderly in voting in many developed countries (OECD, 2007), public education expenditure in these countries is expected to decrease in the future (see Cattaneo and Wolter, 2009 and the references therein). Gradstein and Kaganovich (2004) and Kunze (2014) examine this prediction in a two-period-lived overlapping-generations model including a conflict of interest between generations over public education. In their analyses, public education expenditure is funded by consumption tax (Gradstein and Kaganovich, 2004) or labor income tax (Kunze, 2014). Capital income tax on interest income, paid by the retired elderly who benefit little from public education, is abstracted from their analyses.[1]

In many OECD countries, income tax accounts for the largest share of tax revenue. Income tax includes all forms of labor and capital income taxes. In a lifecycle model, labor income tax is a burden on the working generation, while capital income tax is a burden on the retired elderly. The model suggests that a conflict of interest between the working generation and retired arises when we consider an option for financing public education expenditure. In addition, taxing capital income discourages saving and investment and thus may negatively affect economic growth (Judd, 1985; Chamley, 1986; Atkeson, Chari, and Kehoe, 1999). The argument thus far suggests that capital income tax should be included in the analysis when we consider the politics of public education funding and its impact on economic growth in the lifecycle framework.

Motivated by the argument above, this study presents a two-period-lived overlapping-generations model with physical and human capital accumulation (Lambrecht, Michel, and Vidal, 2005; Kunze, 2014; Ono and Uchida, 2016). Public education contributes to human capital formation, and it is funded by taxing the labor income of the working generation (i.e., the young) and capital income of the retired generation (i.e., the old). Within this framework, we employ probabilistic voting à la Lindbeck and Weibull (1987). In each period, the government representing the young and old chooses taxes and expenditure to maximize the weighted sum of the utility of the young and old. Based on this voting mechanism, we demonstrate the political determinant of both taxes and expenditure and its impacts on economic growth. In particular, we show that increased political weight on the old, stemming from population aging, results in a shift of the tax burden from the old

[1]Apart from the works mentioned above, a number of studies present an intergenerational conflict over public education expenditure (Gonzalez-Eiras and Niepelt, 2012; Kaganovich and Meier, 2012; Kaganovich and Zilcha, 2012; Naito, 2012; Lancia and Russo, 2016; Ono and Uchida, 2016; Bishnu and Wang, 2017). However, their analyses also lack a discussion on capital income tax.

to the young and a reduction in public education expenditure. The model thus predicts that aging has a negative impact on economic growth via the choice of fiscal policy.

The negative growth effect of population aging suggests that aging developed countries are under pressure to take policy action against lower growth rates. For this policy purpose, the present study considers three alternative constraints that limit the choice of taxes and/or expenditure: a minimum level of public education expenditure, an upper limit of the capital income tax rate, and a combination of the two. We show that the minimum constraint increases public education expenditure, promotes economic growth, and thus benefits future generations. However, it forces the government to raise the capital income tax rate to finance its increased expenditure, and therefore worsens the initial old generation. Thus, the constraint creates a trade-off between current and future generations in terms of welfare.

Such a trade-off still arises when we alternatively assume the upper limit constraint on capital income tax. The initial old are made better off because the constraint reduces their tax burden. However, future generations are made worse off because the constraint reduces tax revenue and thus public education expenditure as an engine of economic growth. In addition, we show that the combination of the two constraints fails to resolve each scenario's shortcoming. Therefore, our analysis indicates that constraints on fiscal policy choice involve a trade-off between current and future generations in terms of welfare.

The presented analysis of policy constraints uses the Pareto criterion. To view the welfare implications from an alternative perspective, we consider a benevolent planner who can commit to all his or her choices at the beginning of a period, subject to the resource constraint. Assuming such a planner, we evaluate the political equilibrium in the absence and presence of policy choice constraints by comparing it with the planner's allocation. Based on a numerical method, we find that the political equilibrium attains a higher (lower) growth rate than the planner's allocation for most discount factors of the planner when the minimum constraint of public education is active (inactive). In addition, in the presence of the minimum constraint, earlier generations are made worse off, while later generations are made better off by the political decision making.

The present study relates to the literature on the politics of capital income taxation in the overlapping-generations framework. Earlier studies assumed no production sector and said nothing about the effect of capital income (i.e., interest rate income) taxation on economic growth (Renström, 1996; Huffman, 1996; Dolmas and Huffman, 1997). In addition, these studies drop the conflict of interest between generations from their analyses by assuming that either the young or the old generation is decisive in voting. Notable exceptions are Soares (2006) and Razin and Sadka (2007), but they cannot fully present the generational conflict because they assume equal tax rates on labor and capital income. Razin, Sadka, and Swagel (2004) and Mateos-Planas (2010) overcome this point by assuming different tax rates on capital and labor income and investigate the effect of population aging on the capital income tax choice. The last four studies, however, focus on the politics of capital income taxation and ignore economic growth and welfare across generations. The present study contributes to the literature by addressing this neglected issue.

The rest of this paper is organized as follows. Section 2 presents the model and characterizes an economic equilibrium. Section 3 characterizes a political equilibrium and investigates the effects of political conflict on economic growth. Section 4 evaluates three alternative policy constraints in terms of growth and welfare. Section 5 compares the political equilibrium with the planner's allocation. Section 6 provides concluding remarks.

2 Model

The discrete time economy starts in period 0 and consists of overlapping generations. Individuals are identical within a generation and live for three periods: youth, middle, and elderly ages. Each middle-aged individual gives birth to $1+n$ children. The middle-aged population for period t is N_t and the population grows at a constant rate of $n(>-1)$: $N_{t+1}=(1+n)N_t$.

2.1 Individuals

Individuals display the following economic behavior over their lifecycles. During youth, they make no economic decisions and receive public education financed by the government. In middle age, individuals work, receive market wages, and make tax payments. They use after-tax income for consumption and savings. Individuals retire in their elderly years and receive and consume returns from savings.

Consider an individual born in period $t-1$. In period t, the individual is middle-aged and endowed with h_t units of human capital. He or she supplies them inelastically in the labor market and obtains labor income $w_t h_t$, where w_t is the wage rate per efficient unit of labor in period t. After paying tax $\tau_t w_t h_t$, where $\tau_t \in (0,1)$ is the period t labor income tax rate, the individual distributes the after-tax income between consumption c_t and savings invested in physical capital s_t. Therefore, the period t budget constraint for the middle age becomes

$$c_t + s_t \leq (1-\tau_t) w_t h_t.$$

The period $t+1$ budget constraint in elderly age is

$$d_{t+1} \leq \left(1-\tau_{t+1}^k\right) R_{t+1} s_t,$$

where d_{t+1} is consumption, τ_{t+1}^k is the period $t+1$ capital income tax rate, $R_{t+1}(>0)$ is the gross return from investment in capital, and $R_{t+1}s_t$ is the return from savings.[2]

Period t middle-aged individuals care about their children's income , $w_{t+1}h_{t+1}$. Children's human capital in period $t+1$, h_{t+1}, is a function of government spending on public education, x_t, and parents' human capital, h_t. In particular, h_{t+1} is formulated by using the following equation:

$$h_{t+1} = D\left(x_t\right)^{\eta}\left(h_t\right)^{1-\eta},$$

where $D(>0)$ is a scale factor and $\eta \in (0,1)$ denotes the elasticity of education technology with respect to education spending.

We note that private investment in education may also contribute to human capital formation. For example, parents' time (Glomm and Ravikumar, 1995, 2001, 2003; Glomm and Kaganovich, 2008) or spending (Glomm, 2004; Lambrecht, Michel, and Vidal, 2005; Kunze, 2014) devoted to education may complement public education. In the present study, we abstract private education from the main analysis to simplify the presentation of the model and focus on the effect of public education on growth and utility.

We assume that parents are altruistic toward their children and concerned about their income in middle age, $w_{t+1}h_{t+1}$. The preferences of an individual born in period $t-1$ are specified by the following expected utility function of the logarithmic form:

$$U_t = \ln c_t + \beta\left\{\ln d_{t+1} + \gamma \ln w_{t+1}h_{t+1}\right\},$$

where $\beta \in (0,1)$ is a discount factor and $\gamma(>0)$ denotes the intergenerational degree of altruism. We substitute the budget constraints and human capital production function into the utility function to form the following unconstrained maximization problem:

$$\max_{\{s_t\}} \ln\left[(1-\tau_t)w_t h_t - s_t\right] + \beta\left\{\ln R_{t+1}s_t + \gamma \ln w_{t+1}D\left(x_t\right)^{\eta}\left(h_t\right)^{1-\eta}\right\}.$$

By solving this problem, we obtain the following savings and consumption functions:

$$s_t = \frac{\beta}{1+\beta}(1-\tau_t)w_t h_t,$$
$$c_t = \frac{1}{1+\beta}(1-\tau_t)w_t h_t,$$
$$d_{t+1} = \frac{\beta\left(1-\tau_{t+1}^k\right)R_{t+1}}{1+\beta}(1-\tau_t)w_t h_t.$$

2.2 Firms

Each period contains a continuum of identical firms that are perfectly competitive profit maximizers. According to Cobb–Douglas technology, they produce a final good Y_t us-

[2]The results are qualitatively unchanged if capital income tax is on the net return from saving rather than the gross return from saving.

ing two inputs: aggregate physical capital K_t and aggregate human capital $H_t \equiv N_t h_t$. Aggregate output is given by

$$Y_t = A (K_t)^{\alpha} (H_t)^{1-\alpha},$$

where $A(> 0)$ is a scale parameter and $\alpha \in (0, 1)$ denotes the capital share.

Let $k_t \equiv K_t/H_t$ denote the ratio of physical to human capital. The first-order conditions for profit maximization with respect to H_t and K_t are

$$w_t = (1-\alpha)A (k_t)^{\alpha}, \text{ and } \rho_t = \alpha A (k_t)^{\alpha-1},$$

where w_t and ρ_t are labor wages and the rental price of capital, respectively. The conditions state that firms hire human and physical capital until the marginal products are equal to the factor prices. We assume the full depreciation of capital.

2.3 Government Budget Constraint

Public education expenditure is financed by taxes on labor income and capital income. The government budget constraint in period t is

$$\tau_t w_t h_t N_t + \tau_t^k R_t s_{t-1} N_{t-1} = N_{t+1} x_t,$$

where $\tau_t w_t h_t N_t$ is aggregate labor income tax revenue, $\tau_t^k R_t s_{t-1} N_{t-1}$ is aggregate capital income tax revenue, and $N_{t+1} x_t$ is aggregate expenditure on public education. By dividing both sides of the above expression by N_t, we obtain a per capita form of the constraint:

$$\tau_t w_t h_t + \frac{\tau_t^k R_t s_{t-1}}{1+n} = (1+n) x_t. \tag{1}$$

2.4 Economic Equilibrium

The market-clearing condition for capital is $K_{t+1} = N_t s_t$, which expresses the equality of total savings by the middle-aged population in period t, $N_t s_t$, to the stock of aggregate capital at the beginning of period $t+1$, K_{t+1}. By using $k_{t+1} \equiv K_{t+1}/H_{t+1}$, $h_{t+1} = H_{t+1}/N_{t+1}$, and the savings function, we can rewrite the condition as

$$(1+n) k_{t+1} h_{t+1} = \frac{\beta}{1+\beta}(1-\tau_t) w_t h_t. \tag{2}$$

The following defines the economic equilibrium in the present model.

Definition 1. Given a sequence of policies, $\left\{\tau_t, \tau_t^k, x_t\right\}_{t=0}^{\infty}$, an *economic equilibrium* is a sequence of allocations $\{c_t, d_t, s_t, k_{t+1}, h_{t+1}\}_{t=0}^{\infty}$ and prices $\{\rho_t, w_t, R_t\}_{t=0}^{\infty}$ with the initial conditions $k_0(> 0)$ and $h_0(> 0)$, such that (i) given $\left(w_t, R_{t+1}, \tau_t, \tau_t^k, x_t\right)$, (c_t, d_{t+1}, s_t)

solves the utility maximization problem; (ii) given (w_t, ρ_t), k_t solves a firm's profit maximization problem; (iii) given (w_t, h_t, k_t), $\left(\tau_t, \tau_t^k, x_t\right)$ satisfies the government budget constraint; (iv) the arbitrage condition holds, $\rho_t = R_t$; and (v) the capital market clears: $(1+n)k_{t+1}h_{t+1} = s_t$.

In the economic equilibrium, the indirect utility of the middle-aged in period t, V_t^M, and that of the old in period t, V_t^o, can be expressed as functions of fiscal policy and physical and human capital as follows:

$$
\begin{aligned}
V_t^M &= V^M\left(A\left(k_t\right)^\alpha, h_t, x_t, \tau_t^k, \tau_{t+1}^k\right) \\
&\equiv \left(1+\alpha\beta\left(1+\gamma\right)\right)\ln\left[(1-\alpha)A\left(k_t\right)^\alpha h_t - (1+n)x_t + \tau_t^k \alpha A\left(k_t\right)^\alpha h_t\right] \\
&\quad + \beta\eta\left(1-\alpha\right)\left(1+\gamma\right)\ln x_t + \beta \ln\left(1-\tau_{t+1}^k\right) + \beta\left(1-\alpha\right)\left(1+\gamma\right)\ln D\left(h_t\right)^{1-\eta} + C, \quad (3)\\
V_t^o &= V^o\left(A\left(k_t\right)^\alpha, h_t, \tau_t^k\right) \\
&\equiv \ln\left(1-\tau_t^k\right) + \ln \alpha A\left(k_t\right)^\alpha h_t(1+n) + \gamma\ln(1-\alpha)A\left(k_t\right)^\alpha h_t, \quad (4)
\end{aligned}
$$

where C includes constant terms and is defined by

$$
C \equiv \beta\left(\alpha - 1 + \gamma\alpha\right)\ln\frac{\beta}{(1+n)(1+\beta)} + \beta\left(\ln \alpha A + \gamma\ln(1-\alpha)A\right) + \left(\ln\frac{1}{1+\beta} + \beta\ln\frac{\beta}{1+\beta}\right).
$$

We use the government budget constraint in (1) to replace τ_t with τ_t^k and x_t. The derivations of (3) and (4) are provided in Appendix B.1.

3 The Politics

In this section, we consider voting on fiscal policy. In particular, we employ probabilistic voting à la Lindbeck and Weibull (1987). In this voting scheme, there is electoral competition between two office-seeking candidates. Each candidate announces a set of fiscal policies subject to the government budget constraint. As demonstrated in Persson and Tabellini (2000), the two candidates' platforms converge in the equilibrium to the same fiscal policy that maximizes the weighted-average utility of voters.

In the present framework, the young, middle-aged, and elderly have an incentive to vote. While the young may benefit from public education expenditure in the future, we assume that they are unable to vote because they are below the voting age. Thus, the political objective is defined as the weighted sum of the utility of the middle-aged and old, given by $\tilde{\Omega}_t \equiv \omega V_t^o + (1+n)(1-\omega)V_t^M$, where $\omega \in [0,1]$ and $1-\omega$ are the political weights placed on the old and middle-aged in period t, respectively. The weight on the middle-aged is adjusted by the gross population growth rate, $(1+n)$, to reflect their share of the population. To gain the intuition, we divide $\tilde{\Omega}_t$ by $(1+n)(1-\omega)$ and redefine the objective function as follows:

$$\Omega_t = \frac{\omega}{(1+n)(1-\omega)} V_t^o + V_t^M,$$

where the coefficient $\omega/(1+n)(1-\omega)$ of V_t^o represents the relative political weight on the old.

We substitute V_t^M in (3) and V_t^o in (4) into Ω_t. By rearranging the terms, we obtain

$$\begin{aligned}\Omega_t \simeq & \underbrace{\frac{\omega}{(1+n)(1-\omega)}}_{(*1)} \ln\left(1-\tau_t^k\right) + \underbrace{(1+\alpha\beta(1+\gamma))}_{(*2)} \ln Z_t \\ & + \underbrace{\beta\eta(1-\alpha)(1+\gamma)}_{(*3)} \ln x_t + \underbrace{\beta}_{(*4)} \ln\left(1-\tau_{t+1}^k\right), \end{aligned} \tag{5}$$

where Z_t, representing the disposable income of the young, is defined as

$$Z_t \equiv \left((1-\alpha) + \alpha\tau_t^k\right) A (k_t)^\alpha h_t - (1+n)x_t. \tag{6}$$

We use the notation $\simeq$ because the irrelevant terms are omitted from the expression of Ω_t. The terms (*1), (*2), (*3), and (*4) in (5) represent the relative political weight on the old, the weight on the young's utility of lifetime consumption, the altruism toward children, and the weight on the marginal cost of capital income taxation in terms of utility, respectively. As demonstrated below, the first three terms play key roles in determining fiscal policy.

The political objective function above suggests that the current policy choice affects the decision on future policy via physical and human capital accumulation. In particular, the period t choice of τ_t^k and x_t affects the formation of physical and human capital in period $t+1$. This in turn influences the decision making on fiscal policy in period $t+1$. To demonstrate such an intertemporal effect, we employ the concept of the Markov-perfect equilibrium under which fiscal policy today depends on the current payoff-relevant state variables. In the present framework, the payoff-relevant state variables are physical capital, k_t, and human capital, h_t. Thus, the expected rate of capital income tax for the next period, τ_{t+1}^k, is given by the function of the next period stock of physical and human capital, $\tau_{t+1}^k = T^k(k_{t+1}, h_{t+1})$. By using recursive notation with z' denoting the next period z, we can define a Markov-perfect political equilibrium as follows:

Definition 2. A *Markov-perfect political equilibrium* is a set of functions, $\langle T, T^k, X\rangle$, where $T : \Re_{++} \times \Re_{++} \to [0,1]$ is a labor income tax rule, $\tau = T(k,h)$, $T^k : \Re_{++} \times \Re_{++} \to [0,1]$ is a capital income tax rule, and $\tau^k = T^k(k,h)$, $X : \Re_{++} \times \Re_{++} \to \Re_{++}$ is a public education expenditure rule, $x = X(k,h)$, such that the following conditions are satisfied:

(i) The capital market clears,

$$(1+n)k'h' = \frac{\beta}{1+\beta}\left(1-T(k,h)\right)(1-\alpha)\, A\,(k)^{\alpha}\, h, \tag{7}$$

(ii) Given k and h, $\langle T(k,h), T^k(k,h), X(k,h) = \arg\max \Omega$ subject to $\tau^{k\prime} = T^k\left(k', h'\right)$, the capital market-clearing condition in (7), the government budget constraint,

$$T(k,h)\,(1-\alpha)\, A\,(k)^{\alpha}\, h + T^k(k,h)\alpha A\,(k)^{\alpha}\, h = (1+n)X(k,h), \tag{8}$$

and the human capital formation function, $h' = D(h)^{1-\eta}\left(X(k,h)\right)^{\eta}$.

3.1 Characterization of the Political Equilibrium

To obtain a set of functions in Definition 2, we conjecture that the capital income tax rate in the next period, $\tau^{k\prime}$, is independent of physical and human capital:

$$\tau^{k\prime} = \bar{\tau}^k,$$

where $\bar{\tau}^k \in (0,1)$ is a constant parameter. Based on this conjecture, the first-order conditions with respect to τ^k and x are

$$\tau^k : (-1)\frac{\frac{\omega}{(1+n)(1-\omega)}}{1-\tau^k} + \frac{\left(1+\alpha\beta\,(1+\gamma)\right)\alpha A\,(k)^{\alpha}\, h}{Z} \leq 0, \tag{9}$$

$$x : (-1)\frac{\left(1+\alpha\beta\,(1+\gamma)\right)(1+n)}{Z} + \frac{\beta\eta(1-\alpha)\,(1+\gamma)}{x} = 0. \tag{10}$$

A strict inequality holds in (9) if $\tau^k = 0$. By using these conditions, we can verify the conjecture and obtain the following result.

Proposition 1. *Suppose that the following conditions hold:*

$$\max\left\{0, \frac{\alpha}{1-\alpha}\left\{1+\beta\,(1+\gamma)\,(\alpha+\eta\,(1-\alpha))\right\} - \beta\eta\,(1+\gamma)\right\}$$
$$\leq \frac{\omega}{(1+n)(1-\omega)} \leq \frac{\alpha}{1-\alpha}\left\{1+\beta\,(1+\gamma)\,(\alpha+\eta\,(1-\alpha))\right\}. \tag{11}$$

There is a Markov-perfect political equilibrium such that the policy functions are given by

$$T^k(k,h) = \tau^k_{un} \equiv 1 - \frac{1}{\alpha\Lambda} \times \frac{\omega}{(1+n)(1-\omega)},$$
$$X(k,h) = \frac{X_{un}}{1+n} A\,(k)^{\alpha}\, h,$$
$$T(k,h) = \tau_{un} \equiv 1 - \frac{1+\alpha\beta\,(1+\gamma)}{(1-\alpha)\Lambda},$$

where

$$\Lambda \equiv \frac{\omega}{(1+n)(1-\omega)} + 1 + \beta(1+\gamma)(\alpha + \eta(1-\alpha)),$$
$$X_{un} \equiv \frac{\beta\eta(1-\alpha)(1+\gamma)}{\Lambda}.$$

Proof. See Appendix A.1.

The subscript "*un*" implies that the choice of fiscal policy is unconstrained. In the next section, we consider several cases of restrictions on the choices and compare them with the unconstrained case. The result in Proposition 1 suggests that the tax rates, τ_{un}^k and τ_{un}, and public education expenditure, X_{un}, are affected by the weights of the political objective function, (*1), (*2), and (*3) in (5). To understand these effects, we reformulate τ_{un}^k, τ_{un}, and $(1+n)x/A(k)^{\alpha} h$ as follows:

$$\tau_{un}^k = 1 - \frac{1}{\alpha}\left[1 + \frac{\overbrace{1+\beta(1+\gamma)(\alpha+\eta(1-\alpha))}^{\text{(*2)+(*3)}}}{\underbrace{\frac{\omega}{(1+n)(1-\omega)}}_{\text{(*1)}}}\right]^{-1},$$

$$\tau_{un} = 1 - \frac{1}{1-\alpha}\left[1 + \frac{\overbrace{\frac{\omega}{(1+n)(1-\omega)}}^{\text{(*1)}} + \overbrace{\beta\eta(1-\alpha)(1+\gamma)}^{\text{(*3)}}}{\underbrace{1+\alpha\beta(1+\gamma)}_{\text{(*2)}}}\right]^{-1},$$

$$X_{un} = \left[1 + \frac{\overbrace{\frac{\omega}{(1+n)(1-\omega)}}^{\text{(*1)}} + \overbrace{1+\alpha\beta(1+\gamma)}^{\text{(*2)}}}{\underbrace{\beta\eta(1-\alpha)(1+\gamma)}_{\text{(*3)}}}\right]^{-1}.$$

These expressions indicate the following properties. First, the term (*1) implies that the greater political power of the old leads to a larger weight of the utility of consumption for the old. This incentivizes the government to shift the tax burden from the old to the young and reduce public education expenditure. Second, the term (*2) implies that an increase in the weight on the young's utility of lifetime income gives the government an incentive to shift the tax burden from the young to the old and reduce the tax burden on the young by cutting public education expenditure. Third, the term (*3) implies that greater altruism toward children provides the government with an incentive to increase public education expenditure by raising capital and labor income tax rates. Finally, as a corollary to the first point, population aging, because of decreased fertility, results in a shift of the tax burden from the old to the young and a reduction in public education expenditure.

3.2 Steady-state Growth

Based on the result in the previous subsection, we derive the steady-state growth rate of the economy and investigate how it is affected by population aging. To present the analysis, consider per capita output, y_t, which is defined by $y_t \equiv Y_t/N_t = A\left(k_t\right)^{\alpha} h_t$. Then, the growth rate of per capita output is

$$\frac{y'}{y} = \frac{A\left(k'\right)^{\alpha} h'}{A\left(k\right)^{\alpha} h},$$

where z' denotes the next period $z(= k, h, y)$. In the steady state with $k' = k$, the growth rate of per capita output, y'/y, is equal to the growth rate of human capital, h'/h. Therefore, in what follows, we focus on the steady-state growth rate of human capital.

To derive the steady-state growth rate of human capital, we recall the human capital formation function, $h' = D(h)^{1-\eta}(x)^{\eta}$. Given the policy function of x presented in Proposition 1, we can reformulate the formation function as

$$\frac{h'}{h} = D\left(\frac{X_{un}}{1+n} A\left(k\right)^{\alpha}\right)^{\eta}. \tag{12}$$

By substituting this into the capital market-clearing condition in (7) and rearranging the terms, we obtain the law of motion of physical capital as

$$k' = \frac{\frac{\beta}{1+\beta}\left(1-\tau_{un}\right)\left(1-\alpha\right)}{(1+n)D\left(\frac{X_{un}}{1+n}\right)^{\eta}}\left(A\left(k\right)^{\alpha}\right)^{1-\eta}.$$

This equation implies that a unique and non-trivial steady state exists and that for any initial condition $k_0 > 0$, the sequence of k stably converges to the unique steady state. By computing the steady-state value of k and substituting it into (12), we can write the law of motion of human capital as

$$\frac{h'}{h} = \left.\frac{h'}{h}\right|_{un} \equiv \left[D\left(\frac{X_{un}}{1+n}\right)^{\eta}\right]^{\frac{1-\alpha}{1-\alpha(1-\eta)}} \left[\frac{\frac{\beta}{1+\beta}\left(1-\tau_{un}\right)\left(1-\alpha\right)}{1+n}\right]^{\frac{\alpha\eta}{1-\alpha(1-\eta)}} (A)^{\frac{\eta}{1-\alpha(1-\eta)}}. \tag{13}$$

This equation suggests that the growth rate is affected by the relative political weight of the old, $\omega/(1+n)(1-\omega)$, through public education expenditure, X_{un}, and the labor income tax rate, τ_{un}. As described above, an increase in the political weight on the old, $\omega/(1+n)(1-\omega)$, results in a shift of the tax burden from the old to the young and a reduction in public education expenditure. In other words, population aging raises the labor income tax rate τ_{un} and lowers the ratio of public education expenditure to GDP, X_{un}. Therefore, aging has a negative impact on the steady-state growth rate via the choice of fiscal policy.

4 Fiscal Policy Constraints

The analysis in the previous section showed that population aging affects fiscal policy formation and that this in turn reduces the steady-state growth rate. This result suggests that given that populations grow older in most developed countries, these countries are under pressure to take action against low growth rates. For this policy purpose, we here consider three alternative policy choice constraints: a minimum level of public education expenditure, an upper limit of the capital income tax rate, and a combination of the two constraints. We compare these three cases of constraints with the unconstrained case presented in the previous section in terms of steady-state growth and welfare across generations.

4.1 Minimum Level of Public Education Expenditure

We first consider the following minimum constraint on public education expenditure, which is introduced as an unchangeable rule of law:

$$\frac{N_{t+1}x_t}{Y_t} \geq X_{xc}(> X_{un}),$$

where $X_{xc} \in (0, 1-\alpha)$ is an exogenously given lower bound of the ratio of public education expenditure to GDP. The minimum constraint, X_{xc}, is bounded above by $1-\alpha$ since $X_{xc} = 1-\alpha$ is feasible as long as the labor income tax rate is 100%, that is, $\tau = 1$.

The problem of the government under the minimum constraint on public education expenditure is to choose a set of fiscal policy to maximize the political objective function in (5) subject to the above constraint. Given the assumption of $X_{xc} > X_{un}$, the constraint is binding at an optimum: $(1+n)x = X_{xc} \cdot A(k)^{\alpha}h$. The associated capital and labor income tax rates are given as follows.

Proposition 2. *Suppose that the ratio of public education expenditure to GDP is constrained by $N_{t+1}x_t/Y_t \geq X_{xc}(> X_{un})$ where $X_{xc} \in (0, 1-\alpha)$. If the following conditions hold,*

$$\left(\frac{1-X_{xc}}{1-\alpha} - 1\right)(1+\alpha\beta(1+\gamma)) \leq \frac{\omega}{(1+n)(1-\omega)} \leq \frac{\alpha(1+\alpha\beta(1+\gamma))}{(1-\alpha)-X_{xc}},$$

then there is a Markov-perfect political equilibrium such that the policy functions are *given by*

$$T^k(k,h) = \tau^k_{xc} \equiv 1 - \frac{1-X_{xc}}{\alpha}\left[1 + \frac{1+\alpha\beta(1+\gamma)}{\frac{\omega}{(1+n)(1-\omega)}}\right]^{-1},$$

$$X(k,h) = \frac{X_{xc}}{1+n} A(k)^{\alpha} h,$$

$$T(k,h) = \tau_{xc} \equiv 1 - \frac{1-X_{xc}}{1-\alpha}\left[1 + \frac{\frac{\omega}{(1+n)(1-\omega)}}{1+\alpha\beta(1+\gamma)}\right]^{-1}.$$

Proof. See Appendix A.2.

The subscript "*xc*" in the expressions of the policy functions in Proposition 2 means that the ratio of public education expenditure to GDP is binding at the minimum level, X_{xc}. Based on the characterization of the political equilibrium in Proposition 2, we compare the cases with and without the minimum constraint on public education expenditure in terms of the capital income tax rate, economic growth, and welfare across generations. Hereafter, the old at the timing of the introduction of a constraint is called the initial old.

Proposition 3. *Consider the political equilibrium in the presence of the minimum constraint on public education expenditure presented in Proposition 2.*

(i) *The growth rate and capital and labor income tax rates are higher in the political equilibrium in the presence of the constraint than in the political equilibrium in the absence of the constraint:* $h'/h|_{xc} > h'/h|_{un}$, $\tau^k_{xc} > \tau^k_{un}$, *and* $\tau_{xc} > \tau_{un}$.

(ii) *The initial old are made worse off, whereas the steady-state generations are made better off by the introduction of the constraint:* $V^o_0|_{xc} < V^o_0|_{un}$ *and* $\lim_{t\to\infty} V^M_t\big|_{xc} > \lim_{t\to\infty} V^M_t\big|_{un}$.

Proof. See Appendix A.3.

The introduction of the minimum constraint forces the government to increase public education expenditure. This action stimulates human capital accumulation and thus works to increase the growth rate. However, the increased expenditure incentivizes the government to raise the capital income tax rate. This lowers the welfare of the initial old because they owe the tax burden but benefit nothing from fiscal policy. In addition, an increased labor income tax rate lowers the disposable income of current and future generations, implying a negative income effect on economic growth. Thus, there are two opposing effects on economic growth, and the result in Proposition 3(i) shows that the former positive effect outweighs the latter negative effect. This fact implies that future generations benefit from increased income and thus are made better off by the introduction of the constraint.

4.2 Upper Limit of the Capital Income Tax Rate

Alternatively, we assume that the following upper limit of the capital income tax rate is introduced as an unchangeable rule of law:

$$\tau_t^k,\ \tau_{t+1}^k \leq \tau_{kc}^k \left(< \tau_{un}^k\right),$$

where $\tau_{kc}^k \in \left[0, \tau_{un}^k\right)$ is an exogenously given upper limit of the capital income tax rate. This constraint enables the government to lower the tax burden of the old, which is expected to improve the welfare of the initial old. The problem of the period t government is choosing a set of fiscal policy, $\left\{\tau_t^k, \tau_t, x_t\right\}$, that maximizes the political objective function in (5) subject to the above upper limit constraint. The constraint is binding at an optimum: $\tau_t^k = \tau_{t+1}^k = \tau_{kc}^k$. The corresponding policy functions of x and τ are summarized in the following proposition.

Proposition 4. *Suppose that the capital income tax rate is constrained by $\tau_t^k = \tau_{t+1}^k \leq \tau_{kc}^k \left(< \tau_{un}^k\right)$. There is a Markov-perfect political equilibrium such that the policy functions are given as follows:*

$$\begin{aligned}
T(k,h) &= \tau_{kc}^k,\\
X(k,h) &= \frac{X_{kc}}{1+n} A (k)^\alpha h,\\
T(k,h) &= \tau_{kc} \equiv 1 - \frac{1}{1-\alpha} \times \frac{(1+\alpha\beta(1+\gamma)) \cdot \left(1-\alpha\left(1-\tau_{kc}^k\right)\right)}{1+\beta(1+\gamma)(\alpha+\eta(1-\alpha))},
\end{aligned}$$

where

$$X_{kc} \equiv \frac{\beta(1+\gamma)\eta(1-\alpha)\left(1-\alpha\left(1-\tau_{kc}^k\right)\right)}{1+\beta(1+\gamma)(\alpha+\eta(1-\alpha))}.$$

Proof. See Appendix A.4.

The subscript "kc" implies that the capital income tax rate is constrained and binding at the upper limit constraint, τ_{kc}^k. We compare the result in Proposition 4 with that in Proposition 1 and obtain the following result.

Proposition 5. *Consider the steady-state political equilibrium in the presence of the upper limit constraint of the capital income tax rate, $\tau_t^k = \tau_{t+1}^k \leq \tau_{kc}^k \left(< \tau_{un}^k\right)$, as in Proposition 4.*

(i) *The ratio of public education expenditure to GDP and the growth rate are lower and the labor income tax rate is higher in the presence of the constraint than in the absence of the constraint: $X_{kc} < X_{un}$, $h'/h|_{kc} < h'/h|_{un}$, and $\tau_{kc} > \tau_{un}$.*

(ii) *The initial old are made better off, whereas the steady-state generations are made worse off by the introduction of the constraint:* $V_0^o|_{kc} > V_0^o|_{un}$ *and* $\lim_{t\to\infty} V^M\big|_{kc} < \lim_{t\to\infty} V^M\big|_{un}$.

Proof. See Appendix A.5.

The government wants to set the capital income tax rate to $\tau^k = \tau_{un}^k$, but it is now constrained by the upper limit, $\tau_{kc}^k (< \tau_{un}^k)$. Because of this constraint, the government is forced to cut public education expenditure and raise the labor income tax rate. This in turn lowers the growth rate of human capital.

Next, consider the welfare of the initial old and steady-state generations. The welfare of the initial old is improved by the introduction of the constraint because their capital income tax burden is reduced. The welfare of the steady-state generations is given by

$$
\begin{aligned}
V_t^M \simeq & \left(1+\alpha\beta\left(1+\gamma\right)\right)\ln\left[\left(1-\alpha\right)-X_j+\alpha\tau_j^k\right]A\left(k_j\right)^\alpha \\
& +\beta\eta\left(1-\alpha\right)\left(1+\gamma\right)\ln\frac{X_j}{1+n}A\left(k_j\right)^\alpha+\beta\ln\left(1-\tau_j^k\right) \\
& +\left\{1+\beta\left(1+\gamma\right)\right\}\ln h_{t,j},
\end{aligned}
$$

where $j = un$ (kc) holds in the absence (presence) of the constraint. The first three terms are constant along the steady-state paths, and the last term grows along the paths. Because the growth rate of human capital is lower in the presence of the constraint than in its absence, the negative effect of the constraint via the fourth term becomes larger over time. Therefore, the steady-state generations are made worse off by the introduction of the constraint.

4.3 Combination of the Two Constraints

The analyses in the previous two subsections show that the introduction of a constraint on either public education expenditure or capital income tax, but not on both, is not Pareto-improving. In particular, the minimum constraint on public education expenditure leads to an increase in the capital income tax rate and thus lowers the welfare of the initial old; the upper limit constraint of the capital income tax rate leads to a decrease in the public education expenditure and thus lowers the welfare of the steady-state generations.

To overcome these shortcomings, we here consider the case where both constraints are introduced. The minimum constraint of public education expenditure prohibits the government from reducing expenditure in response to the introduction of the upper limit constraint of the capital income tax rate. In addition, the upper limit constraint prohibits the government from raising the capital income tax rate in response to the introduction of the minimum constraint of public education expenditure. Therefore, the combination is expected to compensate for each other's limitations.

The problem of the government in period t is choosing a set of fiscal policy to maximize the political objective function in (5) subject to the two constraints. There are at most four possible solutions to the problem: (i) both τ^k and x are non-binding; (ii) only x is binding; (iii) only τ^k is binding; and (iv) both x and τ^k are binding. The first three cases do not appear to exist because they contradict the assumption of the constraints. Therefore, there is a political equilibrium where both constraints are binding, as described in the following proposition.

Proposition 6. *Suppose that public education expenditure and the capital income tax rate are constrained by* $N_{t+1}x_t/Y_t \geq X_{xc}(> X_{un})$ *and* $\tau^k \leq \tau^k_{kc}(< \tau^k_{un})$, *where* $\left(X_{xc}, \tau^k_{kc}\right)$ *satisfies*

$$\frac{1}{\alpha}\left(X_{xc} - (1-\alpha)\right) < \tau^k_{kc} \leq \frac{1}{\alpha} X_{xc}.$$

If the following condition holds:

$$\frac{\omega}{(1+n)(1-\omega)} \leq \frac{\alpha}{1-\alpha}\left\{1+\beta\left(1+\gamma\right)\left(\alpha+\eta\left(1-\alpha\right)\right)\right\},$$

then there is a Markov-perfect political equilibrium such that the policy functions are given as follows:

$$\begin{aligned}
T^k(k,h) &= \tau^k_{kc}, \\
X(k,h) &= \frac{X_{xc}}{1+n} A\left(k\right)^{\alpha} h, \\
T(k,h) &= \tau_{xkc} \equiv \frac{1}{1-\alpha}\left(X_{xc} - \alpha\tau^k_{kc}\right) \in [0,1).
\end{aligned}$$

Proof. See Appendix A.6.

The subscript "xkc" in τ_{xkc} implies that both constraints are binding. To understand the growth and welfare implications of the combination of the two constraints, let us first compare the growth rates in the absence and presence of the constraints. The growth rate in the presence of the two constraints, denoted by $h'/h|_{xkc}$, is given by replacing X_{un} and τ_{un} in (13) with X_{xc} and τ_{xkc}, respectively. By direct calculation, we have

$$\begin{aligned}
\left.\frac{h'}{h}\right|_{un} \gtreqless \left.\frac{h'}{h}\right|_{xkc} &\Leftrightarrow \left(X_{un}\right)^{1-\alpha}\left(1-\tau_{un}\right)^{\alpha} \gtreqless \left(X_{xc}\right)^{1-\alpha}\left(1-\tau_{xkc}\right)^{\alpha} \\
&\Leftrightarrow \underbrace{\left(X_{un}\right)^{1-\alpha}\left(\left(1-X_{un}\right)-\alpha\left(1-\tau^k_{un}\right)\right)^{\alpha}}_{LHS} \gtreqless \underbrace{\left(X_{xc}\right)^{1-\alpha}\left(\left(1-X_{xc}\right)-\alpha\left(1-\tau^k_{kc}\right)\right)^{\alpha}}_{RHS},
\end{aligned} \tag{14}$$

where the second line comes from $\tau_{un} = \left(X_{un} - \alpha\tau^k_{un}\right)/(1-\alpha)$ (Proposition 1) and $\tau_{xkc} = \left(X_{xc} - \alpha\tau^k_{kc}\right)/(1-\alpha)$ (Proposition 6).

The expression in (14) suggests that the growth rate in the presence of the two constraints lowers as the upper limit of τ^k, τ^k_{kc}, declines. Keeping this in mind, we first consider the case of $\tau_{kc}^{k}=\tau_{un}^{k}$ and illustrate the graph of (14), taking X_{xc} on the horizontal axis as in Figure 1. From the figure, we can find a critical level of X_{xc}, denoted by $\tilde{X}_{xc}$, such that $LHS \gtrless RHS \Leftrightarrow X_{xc} \lessgtr \tilde{X}_{xc}$. Thus, $LHS < RHS$ holds in (14) if $X_{xc} < \min\left\{1-\alpha, \tilde{X}_{xc}\right\}$. In other words, the introduction of the two constraints increases the steady-state growth rate if $\tau_{kc}^{k}=\tau_{un}^{k}$ and $X_{xc} \in \left(X_{un}, \min\left\{1-\alpha, \tilde{X}_{xc}\right\}\right)$. However, as illustrated in Figure 1, the introduction of the two constraints is less likely to attain a higher growth rate as the upper limit of the capital income tax rate decreases.

[Figure 1 is here.]

The result established thus far has the following welfare implications. First, the initial old are made better off because the capital income tax burden is lowered by the upper limit constraint. As a consequence of this improvement, the initial young generation is made worse off. Second, future generations in the steady state are made better off (worse off) if the steady-state growth rate increases (decreases) Thus, there still arises an intergenerational trade-off in terms of welfare. The combination of the two constraints does not solve the problem that arises when each constraint is independently engaged.

5 Planner's Allocation

In the previous section, we use the Pareto criterion to evaluate the welfare consequence of alternative constraints. In this section, we take an alternative approach by deriving an optimal allocation that maximizes an infinite discounted sum of generational utilities for an arbitrary social discount factor (e.g., Bishnu, 2013). In particular, consider a benevolent planner who can commit to all his or her choices at the beginning of a period, subject to the resource constraint. Assuming such a planner, we evaluate the political equilibrium by comparing it with the planner's allocation in terms of long-run growth rates, the ratio of public education expenditure to GDP, and welfare across generations.

5.1 Characterization of the Planner's Allocation

The planner is assumed to value the welfare of all generations. In particular, the objective of the planner is to maximize a discounted sum of the lifecycle utility of all current and future generations:

$$SW=\sum_{t=-1}^{\infty}\theta^{t}U_{t},$$

under the resource constraint:

$$N_{t}c_{t}+N_{t-1}d_{t}+K_{t+1}+N_{t+1}x_{t}=A\left(K_{t}\right)^{\alpha}\left(H_{t}\right)^{1-\alpha},$$

or

$$c_t + \frac{1}{1+n} d_t + (1+n) k_{t+1} h_{t+1} + (1+n) x_t = A (k_t)^{\alpha} h_t,$$

where k_0 and h_0 are given. The parameter $\theta \in (0,1)$ is the planner's discount factor.

In the present framework, the state variable h_t does not lie in a compact set because it continues to grow along an optimal path. To reformulate the problem into one in which the state variable lies in a compact set, we undertake the following normalization:

$$\tilde{c}_t \equiv c_t / h_t, \tilde{d}_t \equiv d_t / h_t, \text{ and } \tilde{x}_t \equiv x_t / h_t.$$

Then, the above resource constraint is rewritten as

$$\tilde{c}_t + \frac{1}{1+n} \tilde{d}_t + (1+n) k_{t+1} D (\tilde{x}_t)^{\eta} + (1+n) \tilde{x}_t = A (k_t)^{\alpha}, \tag{15}$$

and the utility function becomes

$$\begin{aligned}
U_{-1} &= \beta \ln \tilde{d}_0 + \beta (1+\gamma) \ln h_0 + \alpha\beta\gamma \ln k_0 + \beta\gamma \ln(1-\alpha)A, \\
U_0 &= \ln \tilde{c}_0 + \beta \ln \tilde{d}_1 + \alpha\beta\gamma \ln k_1 + \eta\beta (1+\gamma) \ln \tilde{x}_0 + (1 + \beta (1+\gamma)) \ln h_0 \\
&\quad + \beta\gamma \ln(1-\alpha)A + \beta (1+\gamma) \ln D, \\
U_t &= \ln \tilde{c}_t + \beta \ln \tilde{d}_{t+1} + \alpha\beta\gamma \ln k_{t+1} + \beta\eta (1+\gamma) \ln \tilde{x}_t + \eta (1 + \beta (1+\gamma)) \sum_{j=0}^{t-1} \ln \tilde{x}_j \\
&\quad + (1 + \beta (1+\gamma)) \ln h_0 + \beta\gamma \ln(1-\alpha)A + \{\beta (1+\gamma) + t (1 + \beta (1+\gamma))\} \ln D, \ t \geq 1.
\end{aligned}$$

The planner's objective function is now given by

$$SW(k_0) \simeq \frac{\alpha\beta\gamma}{\theta} \ln k_0 + \sum_{t=0}^{\infty} \theta^t \left[\ln \tilde{c}_t + \frac{\beta}{\theta} \ln \tilde{d}_t + \alpha\beta\gamma \ln k_{t+1} + \eta \left\{ \beta (1+\gamma) + (1 + \beta (1+\gamma)) \frac{\theta}{1-\theta} \right\} \ln \tilde{x}_t \right] \tag{16}$$

where the constant terms are omitted from the expression. Thus, we can express the Bellman equation for the problem as follows:

$$V(k) = \max_{\{\tilde{c}, \tilde{d}, k', \tilde{x}\}} \left\{ \ln \tilde{c} + \frac{\beta}{\theta} \ln \tilde{d} + \alpha\beta\gamma \ln k' + \eta \left\{ \beta (1+\gamma) + (1 + \beta (1+\gamma)) \frac{\theta}{1-\theta} \right\} \ln \tilde{x} + \theta V(k') \right\}, \tag{17}$$

subject to (15), where k' denotes the next period stock of capital and $V(\cdot)$ is the optimal value function. Solving the problem in (17) leads to the following result.

Proposition 7. *Given k_0 and h_0, the planner's allocation, $\{c_t, d_t, k_{t+1}, x_t\}_{t=0}^{\infty}$, is char-*

acterized by

$$\begin{aligned}
c_t &= \frac{1}{\phi} A\left(k_t\right)^{\alpha} h_t, \\
d_t &= \frac{(1+n)\beta}{\phi\theta} A\left(k_t\right)^{\alpha} h_t, \\
x_t &= \frac{1}{1+n}\left[\phi - \left(1+\frac{\beta}{\theta}\right) - \left(\alpha\beta\gamma + \theta\phi_1\right)\right] \frac{1}{\phi} A\left(k_t\right)^{\alpha} h_t, \\
k_{t+1} &= \frac{\alpha\beta\gamma + \theta\phi_1}{(1+n)D\left[\frac{1}{1+n}\left\{\phi - \left(1+\frac{\beta}{\theta}\right) - \left(\alpha\beta\gamma+\theta\phi_1\right)\right\}\right]^{\eta}} \left(\frac{1}{\phi} A\left(k_t\right)^{\alpha}\right)^{1-\eta},
\end{aligned}$$

where

$$\begin{aligned}
\phi &\equiv \frac{1}{1-\alpha\theta(1-\eta)}\left[\left(1+\frac{\beta}{\theta}\right) + \alpha\beta\gamma(1-\eta) + \eta\left\{\beta(1+\gamma) + (1+\beta(1+\gamma))\frac{\theta}{1-\theta}\right\}\right], \\
\phi_1 &\equiv \alpha\phi.
\end{aligned}$$

Proof. See Appendix A.7.

5.2 Numerical Analysis

Based on the result in Proposition 7, we compute the corresponding steady-state capital, ratio of public education expenditure to GDP, growth rate, and welfare across generations. We then compare these with those in the political equilibrium in the presence and absence of the constraints and evaluate the constraints in terms of growth and welfare. To proceed with the numerical analysis, we calibrate the model in the absence of the constraint in Section 3 to the Japanese economy; note that the result would be qualitatively unchanged when using other countries' data.

We fix the share of capital in production at $\alpha = 1/3$ and assume that each period lasts 30 years; these assumptions are standard in quantitative analyses of the two-period overlapping-generations model (e.g., Gonzalez-Eiras and Niepelt, 2008, 2012; Song, Storesletten, and Zilibotti, 2012; Lancia and Russo, 2016). Our selection of β is 0.98, which is also standard in the literature. Since the agents in the present model plan over generations that span 30 years, we discount the future by $(0.98)^{30}$.

We assume that the gross population growth rate for each period is 1.0232. This figure comes from the average rate in Japan during 1995–2014. For η, the estimate in Card and Krueger (1992) implies an elasticity of school quality of 0.12. In addition, recent simulation studies suggest that η is in the range of $0.1-0.3$ (Cardak, 2004) and $0.05-0.15$ (Glomm and Ravikumar, 1998). Following these studies, we set $\eta = 0.12$. For ω, we set $\omega = 0.4$ to attain interior solutions for τ^k and τ.

To determine γ, we focus on the ratio of public education expenditure to GDP. In particular, the ratio is given by

$$\frac{N_{t+1}x_t}{Y_t} = X_{un} \equiv \frac{\beta\eta(1-\alpha)(1+\gamma)}{\frac{\omega}{(1+n)(1-\omega)} + 1 + \beta(1+\gamma)(\alpha + \eta(1-\alpha))},$$

when we assume no fiscal constraint. Given $\alpha = 1/3$, $\beta = (0.98)^{30}$, $1 + n = 1.0232$, $\eta = 0.12$, and $\omega = 0.4$, we can solve this expression for γ by using the ratio observed in Japan. The average ratio during 1995–2014 is 0.0349. We can determine γ by solving the above expression and obtain $\gamma = 0.611$.

The productivity of human capital, D, is normalized to $D = 1$. For the productivity of final goods, we use the data on the per capita GDP gross growth rate of 1.249 in Japan during 1995–2014. We substitute this figure and the values of α, β, n, η, γ, and D into the equation expressing the per capita growth rate of human capital and solve the expression for A to obtain $A = 58.215$. The source of the data for the gross population growth rate, ratio of public education expenditure to GDP, and per capita growth rate is the World Development Indicators.[3]

Figure 2 illustrates the numerical results for the ratio of public education expenditure to GDP (Panel (a)), steady-state capital (Panel (b)), and steady-state growth rates (Panel (c)). In each panel, we compare the planner's allocation with the political equilibrium in the presence and absence of the constraints by taking the planner's discount factor θ from 0 to 1 on the horizontal axis.

[Figure 2 is here.]

To illustrate the numerical examples of the three cases of constraints in Section 4, we consider the following three scenarios. In the first scenario, the minimum constraint on public education expenditure is set to maximize the steady-state growth rate: $X_{xc} = \arg\max\left(h'/h|_{xc}\right)$. In the second scenario, the upper limit constraint on the capital income tax rate is set to zero: $\tau^k_{kc} = 0$. In the third scenario, the two constraints are both in play, but the upper limit constraint on the capital income tax rate is set to the rate in the absence of the constraints: $X_{xkc} = \arg\max\left(h'/h|_{xkc}\right)$ and $\tau^k_{xkc} = \tau^k_{un}$. In Table 1, the values of the constraints are marked with asterisks and the resulting fiscal policy variables are given with no asterisk.

[Table 1 is here.]

Panel (a) shows that in the planner's allocation, the ratio of public education expenditure to GDP increases as the planner's discount factor θ increases. A higher θ implies that

[3]Source: https://data.worldbank.org/products/wdi (Accessed on August 26, 2017).

the planner attaches a larger weight to future generations, who thus have more incentive to invest in human capital through public education. Because of this incentive, the planner's allocation is more likely to attain a higher ratio of public education expenditure to GDP than in the political equilibrium as his or her discount factor increases. However, when the minimum constraint is introduced into the political equilibrium, the planner's allocation always attains a lower ratio than that in the political equilibrium, regardless of the planner's discount factor. This result is straightforward since the minimum constraint is set sufficiently high to maximize growth.

Panel (b) plots steady-state capital. In the planner's allocation, steady-state capital is hump-shaped, which peaks around $\theta = 0.75$. This fact suggests two opposing effects of θ on capital accumulation: a positive effect produced by the planner's incentive to bequeath more physical capital to future generations and a negative effect caused by the crowding out effect of human capital investment. The corresponding effects also appear in the political equilibrium, but the negative effect is strengthened by the minimum constraint of public education expenditure. This then implies that the political equilibrium attains lower steady-state capital than that in the planner's allocation in the presence of the minimum constraint for most discount factors. However, the negative effect is not as strong in the absence of the minimum constraint. Thus, the political equilibrium in the absence of the minimum constraint attains higher steady-state capital than the planner's allocation for most discount factors. The presence of the minimum constraint is thus crucial to determining the relative size of the steady-state capital stock in the planner's allocation and the political equilibrium.

Panel (c) plots the steady-state growth rate of human capital. In the planner's allocation, the steady-state growth rate increases as the planner's discount factor increases. A higher θ provides an incentive for the planner to invest more in education. In addition, as argued above, a higher θ creates a positive effect on capital accumulation, which works to increase public education expenditure. Because of these two positive effects on education expenditure, the planner's allocation attains a higher growth rate as his or her discount factor increases. When the planner's allocation is compared with the political equilibrium, we find that the political equilibrium attains a higher (lower) growth rate than the planner's allocation for most discount factors when the minimum constraint of public education is active (inactive). This is because the constraint pushes up spending on public education. Thus, the minimum constraint is crucial to determining the performance of economic growth in the political equilibrium relative to the planner's allocation.

Figure 3 plots the evolution of economic growth and distribution of utility across generations when $\theta = 0.5$.[4] In particular, we take the ratio of the growth rate in the

[4]The result is qualitatively unchanged when θ varies from 0.1 to 0.9.

political equilibrium allocation to that in the planner's allocation for each period. We also take the corresponding ratio of utility for each generation. The terms un, xc, kc, xkc, and pl in the figure correspond to the subscripts introduced earlier. The line denoted by j/pl ($j = un, xc, kc, xkc$) implies the ratio of a concerned variable in the political equilibrium to that in the planner's allocation. For example, the line denoted by un/pl in Panel (a) shows the ratio of the growth rate in the political equilibrium in the absence of any constraints to that in the planner's allocation. Each ratio implies that the political equilibrium outweighs the planner's allocation when the ratio is above unity.

[Figure 3 here.]

For the reasons already stated, the political equilibrium attains a higher (lower) growth rate than the planner's allocation in the presence (absence) of the minimum constraint on public education expenditure across periods, as illustrated in Panel (a) of Figure 3. Panel (b) illustrates the distribution of utility across generations. The figure indicates that in the presence of the minimum constraint on public education expenditure, the first eight generations are made worse off by the political decision making because these generations suffer from a high tax burden to pay for public education expenditure. However, the generations from nine onward benefit from past public education expenditure via human capital accumulation. This positive effect outweighs the negative tax burden effect, and thus the political equilibrium outweighs the planner's allocation in terms of utility from generation nine onward. Such a positive effect does not arise in the absence of the minimum constraint; generations from period three onward are made worse off by the political decision making.

6 Conclusion

The present study develops a two-period-lived overlapping-generations model with physical and human capital accumulation. Public education contributes to human capital formation and it is funded by taxing the labor income of the working generation and capital income of the retired generation. Within this framework, we employ probabilistic voting to demonstrate the political determinant of both taxes and expenditure and investigate its impacts on economic growth. The model predicts that aging has a negative impact on economic growth via the choice of fiscal policy.

To resolve the negative growth effect, we propose three alternative constraints that limit the choice of taxes and/or expenditure: a minimum level of public education expenditure, an upper limit of the capital income tax rate, and a combination of the two. We show that the minimum constraint benefits future generations at the expense of the current old. The upper limit constraint benefits the current old at the expense of future generations. In addition, a combination of the two cannot solve the trade-off inherent to each constraint. Our analysis therefore indicates that the constraints on the fiscal policy choice involve a trade-off between current and future generations in terms of welfare.

We also consider a benevolent planner's allocation to view the welfare implications of the political equilibrium from an alternative perspective. We find that the political equilibrium attains a higher (lower) growth rate than the planner's allocation for most discount factors of the planner when the minimum constraint of public education is active (inactive). In addition, in the presence of the minimum constraint, earlier generations are made worse off, while later generations are made better off by the political decision making. Hence, political decision making on taxes and expenditure inevitably involve an intergenerational trade-off in terms of welfare.

A Proofs and Supplementary Explanations

A.1 Proof of Proposition 1

Assume first that the first-order condition with respect to τ^k holds with an equality. We substitute (10) into (9). By rearranging the terms, we obtain

$$(1+n)x = \beta\eta(1-\alpha)(1+\gamma)\left(\frac{\omega}{(1+n)(1-\omega)}\right)^{-1}\alpha A(k)^{\alpha} h\left(1-\tau^k\right). \tag{18}$$

We substitute this into (9) and solve for τ^k to obtain

$$\tau^k = \tau^k_{un} \equiv 1 - \frac{1}{\alpha\Lambda}\cdot\frac{\omega}{(1+n)(1-\omega)},$$

where Λ is defined in Proposition 1. Thus, the conjecture is verified as long as $\tau^{k\prime} = \bar{\tau}^k = \tau^k_{un}$.

The corresponding public education expenditure becomes

$$(1+n)x = \frac{X_{un}}{1+n}A(k)^{\alpha} h,$$

where X_{un} is defined as follows:

$$X_{un} \equiv \frac{\beta\eta(1-\alpha)(1+\gamma)}{\Lambda}.$$

With the government budget constraint in (8), we can compute the labor income tax rate as

$$\tau = \tau_{un} \equiv 1 - \frac{1+\alpha\beta(1+\gamma)}{(1-\alpha)\Lambda}.$$

We immediately find that the tax rates τ^k_{un} and τ_{un} are below one. They are greater than or equal to zero if the following conditions hold:

$$\tau^k_{un} \geq 0 \Leftrightarrow \frac{\omega}{(1+n)(1-\omega)} \leq \frac{\alpha}{1-\alpha}\left\{1+\beta(1+\gamma)(\alpha+\eta(1-\alpha))\right\},$$
$$\tau_{un} \geq 0 \Leftrightarrow \frac{\omega}{(1+n)(1-\omega)} \geq \frac{\alpha}{1-\alpha}\left\{1+\beta(1+\gamma)(\alpha+\eta(1-\alpha))\right\} - \beta\eta(1+\gamma).$$

Therefore, $\tau^k_{un} \in [0,1)$ and $\tau_{un} \in [0,1)$ hold if the assumption in (11) holds.

■

A.2 Proof of Proposition 2

Conjecture that the constraint is binding: $(1+n)x = X_{xc}\cdot A(k)^{\alpha}h$. The first-order condition with respect to τ^k in (9) holds with an equality since the choice of τ^k is unconstrained.

We assume an interior solution of τ^k and substitute the conjecture $(1+n)x = X_{xc} \cdot A(k)^\alpha h$ into (9) to obtain

$$(-1)\frac{\frac{\omega}{(1+n)(1-\omega)}}{1-\tau^k} + \frac{(1+\alpha\beta(1+\gamma))\,\alpha A(k)^\alpha h}{((1-\alpha)+\alpha\tau^k)\,A(k)^\alpha h - X_{xc}\cdot A(k)^\alpha h} = 0.$$

By rearranging the terms, we obtain $\tau^k = \tau^k_{xc}$.

We substitute $(1+n)x = X_{xc} \cdot A(k)^\alpha h$ and $\tau^k = \tau^k_{xc}$ into the first-order condition with respect to x in (10) and rearrange the terms. Then, we obtain $X_{un} \leq X_{xc}$. This condition holds with a strict inequality by assumption. Thus, the conjecture is verified.

To derive the labor income tax rate, we substitute $(1+n)x = X_{xc} \cdot A(k)^\alpha h$ and $\tau^k = \tau^k_{xc}$ into the government budget constraint in (8) and then obtain $\tau = \tau_{xc}$. These tax rates imply that

$$\tau^k_{xc} \geq 0 \Leftrightarrow \frac{\omega}{(1+n)(1-\omega)} \leq \frac{\alpha(1+\alpha\beta(1+\gamma))}{(1-\alpha)-X_{xc}},$$

$$\tau_{xc} \geq 0 \Leftrightarrow \left(\frac{1-X_{xc}}{1-\alpha} - 1\right)(1+\alpha\beta(1+\gamma)) \leq \frac{\omega}{(1+n)(1-\omega)}.$$

■

A.3 Proof of Proposition 3

We first compare the growth rates. The growth rate in the absence of the constraint is given by (13). The growth rate in the presence of the constraint, denoted by $h'/h|_{xc}$, is given by replacing X_{un} and τ_{un} with X_{xc} and τ_{xc}, respectively. By direct comparison, we have

$$\begin{aligned}
\left.\frac{h'}{h}\right|_{un} \gtreqless \left.\frac{h'}{h}\right|_{xc} &\Leftrightarrow (X_{un})^{1-\alpha}(1-\tau_{un})^\alpha \gtreqless (X_{xc})^{1-\alpha}(1-\tau_{xc})^\alpha \\
&\Leftrightarrow \left[\frac{\frac{1+\alpha\beta(1+\gamma)}{(1-\alpha)\Lambda}}{\frac{1-X_{xc}}{1-\alpha}\cdot\left[1+\frac{\frac{\omega}{(1+n)(1-\omega)}}{1+\alpha\beta(1+\gamma)}\right]^{-1}}\right]^\alpha \gtreqless \left(\frac{X_{xc}}{X_{un}}\right)^{1-\alpha} \\
&\Leftrightarrow \left[\frac{1+\alpha\beta(1+\gamma)+\frac{\omega}{(1+n)(1-\omega)}}{\Lambda}\right]^\alpha (X_{un})^{1-\alpha} \gtreqless (1-X_{xc})^\alpha (X_{xc})^{1-\alpha} \\
&\Leftrightarrow (1-X_{un})^\alpha (X_{un})^{1-\alpha} \gtreqless (1-X_{xc})^\alpha (X_{xc})^{1-\alpha}.
\end{aligned}$$

The right-hand side of the last expression, denoted by RHS, has the following properties:

$$\frac{\partial RHS}{\partial X_{xc}} = RHS \cdot \frac{(1-\alpha)-X_{xc}}{(1-X_{xc})X_{xc}}, \left.\frac{\partial RHS}{\partial X_{xc}}\right|_{X_{xc}=X_{un}} > 0, \left.\frac{\partial RHS}{\partial X_{xc}}\right|_{X_{xc}=1-\alpha} = 0.$$

These properties imply that

$$\left.\frac{h'}{h}\right|_{un} < \left.\frac{h'}{h}\right|_{xc} \forall X_{xc} \in (X_{un}, 1-\alpha).$$

We next compare the capital income tax rates. Direct comparison leads to the following result:

$$\begin{aligned}
\tau_{un}^{k} \gtreqless \tau_{xc}^{k} &\Leftrightarrow 1 - \frac{1}{\alpha\Lambda} \cdot \frac{\omega}{(1+n)(1-\omega)} \gtreqless 1 - \frac{1-X_{xc}}{\alpha} \cdot \left[1 + \frac{1+\alpha\beta(1+\gamma)}{\frac{\omega}{(1+n)(1-\omega)}}\right]^{-1} \\
&\Leftrightarrow \frac{1-X_{xc}}{\frac{\omega}{(1+n)(1-\omega)} + 1 + \alpha\beta(1+\gamma)} \gtreqless \frac{1}{\Lambda} \\
&\Leftrightarrow \frac{\beta(1+\gamma)\eta(1-\alpha)}{\Lambda} \gtreqless X_{xc} \\
&\Leftrightarrow X_{un} \gtreqless X_{xc}.
\end{aligned}$$

Given the assumption of $X_{un} < X_{xc}$, we obtain $\tau_{un}^{k} < \tau_{xc}^{k}$.

The labor income tax rates are compared as follows:

$$\begin{aligned}
\tau_{un} \lesseqgtr \tau_{xc} &\Leftrightarrow 1 - \frac{1+\alpha\beta(1+\gamma)}{(1-\alpha)\Lambda} \lesseqgtr 1 - \frac{1-X_{xc}}{1-\alpha} \cdot \frac{1+\alpha\beta(1+\gamma)}{\Lambda - \beta(1+\gamma)\eta(1-\alpha)} \\
&\Leftrightarrow (1-X_{xc})\Lambda \lesseqgtr \Lambda - \beta(1+\gamma)\eta(1-\alpha) \\
&\Leftrightarrow \frac{\beta(1+\gamma)\eta(1-\alpha)}{\Lambda} \lesseqgtr X_{xc} \\
&\Leftrightarrow X_{un} \lesseqgtr X_{xc},
\end{aligned}$$

where the last line comes from the definition of X_{un}. Given the assumption of $X_{un} < X_{xc}$, we obtain $\tau_{un} < \tau_{xc}$.

Finally, we compare the two cases in terms of the welfare of the initial old and steady-state generations. The initial old are made worse off by the introduction of the constraint because the capital income tax rate increases. The welfare of the middle-aged in some generation t is, from (3),

$$\begin{aligned}
V_t^M &= (1+\alpha\beta(1+\gamma)) \ln\left[(1-\alpha)A(k_t)^{\alpha} h_t - (1+n)x_t + \tau_t^k \alpha A(k_t)^{\alpha} h_t\right] \\
&\quad + \beta\eta(1-\alpha)(1+\gamma)\ln x_t + \beta\ln\left(1-\tau^k\right) + \beta(1-\alpha)(1+\gamma)\ln D(h_t)^{1-\eta} + C.
\end{aligned}$$

Given that $x_t = X_j A(k_j)^{\alpha} h_t$, $j = un, xc$, in the steady state, the above expression is reformulated as

$$\begin{aligned}
V_{t,j}^M &\simeq (1+\alpha\beta(1+\gamma)) \ln\left[(1-\alpha) - X_j + \tau_j^k \alpha\right] A(k_j)^{\alpha} \\
&\quad + \beta\eta(1-\alpha)(1+\gamma)\ln \frac{X_j}{1+n} A(k_j)^{\alpha} + \beta\ln\left(1-\tau_j^k\right) + \{1+\beta(1+\gamma)\}\ln h_{tj}.
\end{aligned}$$

Recall that k_j and τ_j^k are constant stationary along the steady-state path. However, human capital h_{tj} grows along the steady-state path and the difference between $h_{t,un}$ and $h_{t,xc}$ rises over time. Therefore, $V_{t,un}^M < V_{t,xc}^M$ holds in the steady state.

■

A.4 Proof of Proposition 4

Conjecture that the capital income tax rate is binding at $\tau_t^k = \tau_{t+1}^k = \tau_{kc}^k$. The first-order conditions with respect to τ^k and x in (9) and (10) are rewritten, respectively, as

$$\tau^k : (-1)\frac{\frac{\omega}{(1+n)(1-\omega)}}{1-\tau_{kc}^k} + \frac{(1+\alpha\beta(1+\gamma))\,\alpha A(k)^\alpha h}{\left((1-\alpha)+\alpha\tau_{kc}^k\right)A(k)^\alpha h-(1+n)x} \geq 0, \tag{19}$$

$$x : (-1)\frac{(1+\alpha\beta(1+\gamma))(1+n)}{\left((1-\alpha)+\alpha\tau_{kc}^k\right)A(k)^\alpha h-(1+n)x} + \frac{\beta\eta(1-\alpha)(1+\gamma)}{x} = 0. \tag{20}$$

The condition in (20) is reformulated as

$$(1+n)x = \frac{\beta\eta(1-\alpha)(1+\gamma)\left((1-\alpha)+\alpha\tau_{kc}^k\right)}{1+\beta(1+\gamma)(\alpha+\eta(1-\alpha))}A(k)^\alpha h. \tag{21}$$

We substitute (21) into (19) and rearrange the terms to obtain

$$\tau_{un}^k \equiv 1-\frac{1}{\alpha\Lambda}\cdot\frac{\omega}{(1+n)(1-\omega)} \geq \tau_{kc}^k.$$

This condition holds by assumption. Thus, the conjecture is verified.

We next derive the labor income tax rate. The substitution of $\tau^k = \tau_{kc}^k$ and x in (21) into the government budget constraint in (1) leads to

$$\tau = \tau_{kc} \equiv 1-\frac{1}{1-\alpha}\cdot\frac{\{1+\beta(1+\gamma)\alpha\}\left((1-\alpha)+\alpha\tau_{kc}^k\right)}{1+\beta(1+\gamma)(\alpha+\eta(1-\alpha))}.$$

This expression implies that $\tau_{kc} \geq 0$ if

$$\tau_{kc}^k \leq \frac{1-\alpha}{\alpha}\cdot\frac{\beta\eta(1-\alpha)(1+\gamma)}{1+\alpha\beta(1+\gamma)}. \tag{22}$$

With $\tau_{kc}^k \leq \tau_{un}^k$, τ_{kc}^k must satisfy

$$\tau_{kc}^k \leq \min\left\{\tau_{un}^k, \frac{1-\alpha}{\alpha}\cdot\frac{\beta\eta(1-\alpha)(1+\gamma)}{1+\alpha\beta(1+\gamma)}\right\}.$$

Direct comparison leads to

$$\begin{aligned}\tau_{un}^k &\gtrless \frac{1-\alpha}{\alpha}\cdot\frac{\beta\eta(1-\alpha)(1+\gamma)}{1+\alpha\beta(1+\gamma)}\\ \Leftrightarrow \frac{\omega}{(1+n)(1-\omega)} &\lessgtr \frac{\alpha}{1-\alpha}\cdot\{1+\beta(1+\gamma)(\alpha+\eta(1-\alpha))\}-\beta(1+\gamma)\eta.\end{aligned}$$

Under the assumption of (11), the last expression holds with an inequality, ">". Therefore, we have

$$\tau_{kc}^k \leq \min\left\{\tau_{un}^k, \frac{1-\alpha}{\alpha}\cdot\frac{\beta\eta(1-\alpha)(1+\gamma)}{1+\alpha\beta(1+\gamma)}\right\} = \tau_{un}^k,$$

implying that $\tau_{kc} \geq 0$ holds under the assumption of $\tau_{kc}^k < \tau_{un}^k$.

■

A.5 Proof of Proposition 5

We first compare the growth rates. The growth rate in the absence of the upper limit constraint of the capital income tax rate is given by (13). The growth rate in the presence of the constraint, denoted by $h'/h|_{kc}$, is given by replacing X_{un} and τ_{un} in (13) by X_{kc} and τ_{kc}, respectively. By direct comparison, we have

$$\left.\frac{h'}{h}\right|_{un} \gtrless \left.\frac{h'}{h}\right|_{kc} \Leftrightarrow \left(X_{un}\right)^{1-\alpha}\left(1-\tau_{un}\right)^{\alpha} \gtrless \left(X_{kc}\right)^{1-\alpha}\left(1-\tau_{kc}\right)^{\alpha}.$$

By comparing X_{un} and X_{kc}, we have

$$\begin{aligned}
X_{un} > X_{kc} &\Leftrightarrow \frac{\beta\eta\left(1-\alpha\right)\left(1+\gamma\right)}{\frac{\omega}{(1+n)(1-\omega)}+1+\beta\left(1+\gamma\right)\left(\alpha+\eta\left(1-\alpha\right)\right)} \\
&> \frac{\beta\eta\left(1-\alpha\right)\left(1+\gamma\right)\left\{\left(1-\alpha\right)+\alpha\tau_{kc}^{k}\right\}}{1+\beta\left(1+\gamma\right)\left(\alpha+\eta\left(1-\alpha\right)\right)} \\
&\Leftrightarrow 1+\beta\left(1+\gamma\right)\left(\alpha+\eta\left(1-\alpha\right)\right) \\
&> \left\{\left(1-\alpha\right)+\alpha\tau_{kc}^{k}\right\}\underbrace{\left\{\frac{\omega}{(1+n)(1-\omega)}+1+\beta\left(1+\gamma\right)\left(\alpha+\eta\left(1-\alpha\right)\right)\right\}}_{=\Lambda} \\
&\Leftrightarrow \alpha\left(1-\tau_{kc}^{k}\right)\Lambda > 0,
\end{aligned}$$

which holds for any $\tau_{kc}^{k} \in \left[0, \tau_{un}^{k}\right)$. We also compare τ_{un} with τ_{kc} and obtain

$$\begin{aligned}
1-\tau_{un} &> 1-\tau_{kc} \\
&\Leftrightarrow \frac{1}{1-\alpha}\cdot\frac{1+\alpha\beta\left(1+\gamma\right)}{\Lambda} > \frac{1}{1-\alpha}\cdot\frac{\left\{1+\alpha\beta\left(1+\gamma\right)\right\}\left\{\left(1-\alpha\right)+\alpha\tau_{kc}^{k}\right\}}{1+\beta\left(1+\gamma\right)\left(\alpha+\eta\left(1-\alpha\right)\right)} \\
&\Leftrightarrow 1+\beta\left(1+\gamma\right)\left(\alpha+\eta\left(1-\alpha\right)\right) \\
&> \left\{\left(1-\alpha\right)+\alpha\tau_{kc}^{k}\right\}\left\{\frac{\omega}{(1+n)(1-\omega)}+1+\beta\left(1+\gamma\right)\left(\alpha+\eta\left(1-\alpha\right)\right)\right\} \\
&\Leftrightarrow 0 > \frac{\omega}{(1+n)(1-\omega)}-\alpha\left(1-\tau_{kc}^{k}\right)\Lambda \\
&\Leftrightarrow \tau_{un}^{k} \equiv 1-\frac{1}{\alpha}\cdot\frac{\frac{\omega}{(1+n)(1-\omega)}}{\Lambda} > \tau_{kc}^{k}.
\end{aligned}$$

The inequality on the last line holds by assumption. Therefore, $\left(X_{un}\right)^{1-\alpha}\left(1-\tau_{un}\right)^{\alpha} > \left(X_{kc}\right)^{1-\alpha}\left(1-\tau_{kc}\right)^{\alpha}$ holds, and thus we obtain $h'/h|_{un} > h'/h|_{kc}$.

The labor income tax rates τ_{un} and τ_{kc} are compared as follows:

$$\begin{aligned}
\tau_{un} \lesseqgtr \tau_{kc} &\Leftrightarrow 1-\frac{1+\alpha\beta\left(1+\gamma\right)}{\left(1-\alpha\right)\Lambda} \lesseqgtr 1-\frac{1}{1-\alpha}\cdot\frac{\left(1+\alpha\beta\left(1+\gamma\right)\right)\left(1-\alpha\left(1-\tau_{kc}^{k}\right)\right)}{1+\beta\left(1+\gamma\right)\left(\alpha+\eta\left(1-\alpha\right)\right)} \\
&\Leftrightarrow \left(1-\alpha\left(1-\tau_{kc}^{k}\right)\right)\Lambda \lesseqgtr 1+\beta\left(1+\gamma\right)\left(\alpha+\eta\left(1-\alpha\right)\right) \\
&\Leftrightarrow \frac{\omega}{(1+n)(1-\omega)}-\alpha\left(1-\tau_{kc}^{k}\right)\Lambda \lesseqgtr 0 \\
&\Leftrightarrow \tau_{kc}^{k} \lesseqgtr 1-\frac{\omega}{(1+n)(1-\omega)}\cdot\frac{1}{\alpha\Lambda} = \tau_{un}^{k}.
\end{aligned}$$

Given the assumption of $\tau^k_{kc} < \tau^k_{un}$, we obtain $\tau_{un} < \tau_{kc}$.

Next, consider the welfare of the initial old. Eq. (4) indicates that given k and h, the welfare of the initial old is decreasing in τ^k. Given that $\tau^k_{kc} < \tau^k_{un}$, their welfare is improved by the introduction of the constraint.

Finally, consider some generation t in the steady state. From Eq. (3), its lifetime welfare is

$$\begin{aligned} V^M_{t,j} &\simeq (1+\alpha\beta(1+\gamma)) \ln\left[(1-\alpha) - X_j + \alpha\tau^k_j\right] A(k_j)^\alpha \\ &\quad + \beta\eta(1-\alpha)(1+\gamma)\ln \frac{X_j}{1+n} A(k_j)^\alpha + \beta \ln\left(1-\tau^k_j\right) \\ &\quad + \{1+\beta(1+\gamma)\} \ln h_{t,j}, \end{aligned}$$

where $j(= un, kc)$ denotes the status of the constraint, k_j is the steady-state level of capital, and $h_{t,j}$ is the period t human capital level along the steady-state path. The first three terms on the right-hand side are constant along the steady-state path, whereas the last term grows over time. In the steady state, $h_{t,un} > h_{t,kc}$ holds, and the difference between them becomes larger over time. Therefore, $V^M_{t,un} > V^M_{t,kc}$ holds in the steady state.

■

A.6 Proof of Proposition 6

When x is binding at the minimum constraint X_{xc}, the government wants to set the capital income tax rate above τ^k_{un} (Proposition 3). However, this choice does not satisfy the upper limit constraint on τ^k, $\tau^k \leq \tau^k_{kc} (< \tau^k_{un})$. Alternatively, when τ^k is binding at the upper limit constraint, $\tau^k \leq \tau^k_{kc}$, the government wants to choose $(1+n)x/y$ below X_{un} (Proposition 5). Such a choice is inconsistent with the minimum requirement on X, $X_{kc} (> X_{un})$. Thus, x is binding at the minimum constraint X_{xc} and τ^k is binding at the upper limit constraint, $\tau^k \leq \tau^k_{kc}$, in the presence of the two constraints.

Recall the first-order condition with respect to τ^k and x in (9) and (10). These are rearranged as follows:

$$\tau^k : (-1) \frac{\frac{\omega}{(1+n)(1-\omega)}}{1-\tau^k} + \frac{(1+\alpha\beta(1+\gamma))\alpha A(k)^\alpha h}{Z} \geq 0, \tag{23}$$

$$x : (-1) \frac{(1+\alpha\beta(1+\gamma))(1+n)}{Z} + \frac{\beta\eta(1-\alpha)(1+\gamma)}{x} \leq 0, \tag{24}$$

where $Z \equiv \left((1-\alpha)+\alpha\tau^k\right) A(k)^\alpha h - (1+n)x$. The strict inequalities hold in (23) and (24) since the two constraints are binding at $\tau^k = \tau^k_{kc}$ and $(1+n)x = X_{xc} A(k)^\alpha h$.

We substitute $\tau^k = \tau^k_{kc}$ and $(1+n)x = X_{xc} A(k)^\alpha h$ into (23) and obtain

$$\frac{(1+\alpha\beta(1+\gamma))\alpha}{(1-\alpha)+\alpha\tau^k_{kc} - X_{xc}} \geq \frac{\frac{\omega}{(1+n)(1-\omega)}}{1-\tau^k_{kc}}. \tag{25}$$

Given $\tau_{kc}^k < \tau_{un}^k$, (25) holds with a strict inequality if

$$\frac{(1+\alpha\beta(1+\gamma))\alpha}{(1-\alpha)+\alpha\tau_{un}^k - X_{xc}} \geq \frac{\frac{\omega}{(1+n)(1-\omega)}}{1-\tau_{un}^k}.$$

This is reformulated as $X_{xc} \geq X_{un}$, which holds by assumption.

Next, we substitute $\tau^k = \tau_{kc}^k$ and $(1+n)x = X_{xc}A(k)^\alpha h$ into (24) and obtain

$$\frac{\beta\eta(1-\alpha)(1+\gamma)}{X_{xc}} \leq \frac{(1+\alpha\beta(1+\gamma))}{(1-\alpha)+\alpha\tau_{kc}^k - X_{xc}}. \tag{26}$$

Given $\tau_{kc}^k < \tau_{un}^k$, (26) holds with a strict inequality if

$$\frac{\beta\eta(1-\alpha)(1+\gamma)}{X_{xc}} \leq \frac{(1+\alpha\beta(1+\gamma))}{(1-\alpha)+\alpha\tau_{un}^k - X_{xc}}.$$

This is reformulated as $X_{un} \leq X_{xc}$, which holds by assumption. The argument thus far suggests that both constraints are binding. By substituting $\tau^k = \tau_{kc}^k$ and $(1+n)x = X_{xc}A(k)^\alpha h$ into the government budget constraint, we obtain

$$\tau = \tau_{xkc} \equiv \frac{1}{1-\alpha}\left(X_{xc} - \alpha\tau_{kc}^k\right).$$

Our final task is to determine the conditions for which $\tau_{kc}^k \in [0,1)$ and $\tau_{xkc} \in [0,1)$ hold. Recall that the capital income tax rate is assumed to satisfy $\tau_{kc}^k < \tau_{un}^k$. Thus, $\tau_{kc}^k \in [0,1)$ holds if $\tau_{un}^k \in [0,1)$, that is, if

$$\frac{\omega}{(1+n)(1-\omega)} \leq \frac{\alpha}{1-\alpha}\cdot\{1+\beta(1+\gamma)(\alpha+\eta(1-\alpha))\}.$$

The labor income tax rate, τ_{xkc}, satisfies $\tau_{xkc} \in [0,1)$ if the following conditions hold:

$$\tau_{xkc} < 1 \Leftrightarrow \frac{1}{\alpha}\cdot(X_{xc}-(1-\alpha)) < \tau_{kc}^k,$$
$$\tau_{xkc} \geq 0 \Leftrightarrow \tau_{kc}^k \leq \frac{1}{\alpha}\cdot X_{xc}.$$

Thus, we have $\tau_{xkc} \in [0,1)$ if

$$\frac{1}{\alpha}\cdot(X_{xc}-(1-\alpha)) < \tau_{kc}^k \leq \frac{1}{\alpha}\cdot X_{xc}.$$

■

A.7 Proof of Proposition 7

We substitute (15) into (17) to reformulate the problem as

$$V(k) = \max_{\{\tilde{d},k',\tilde{x}\}}\left\{\ln\left[A(k)^\alpha - \frac{1}{1+n}\tilde{d} - (1+n)k'D(\tilde{x})^\eta - (1+n)\tilde{x}\right]\right.$$
$$\left. + \frac{\beta}{\theta}\ln\tilde{d} + \alpha\beta\gamma\ln k' + \eta\left\{\beta(1+\gamma) + \frac{\theta}{1-\theta}(1+\beta(1+\gamma))\right\}\ln\tilde{x} + \theta\cdot V(k').\right. \tag{27}$$

The first-order conditions with respect to $\tilde{d}, k'$, and $\tilde{x}$ are

$$\tilde{d}: \frac{1/(1+n)}{\tilde{c}} = \frac{\beta/\theta}{\tilde{d}}, \tag{28}$$

$$k': \frac{(1+n)D(\tilde{x})^{\eta}}{\tilde{c}} = \frac{\alpha\beta\gamma}{k'} + \theta \cdot V'(k'), \tag{29}$$

$$\tilde{x}: \frac{\eta(1+n)k'D(\tilde{x})^{\eta-1} + (1+n)}{\tilde{c}} = \frac{\eta\left\{\beta(1+\gamma) + \frac{\theta}{1-\theta}(1+\beta(1+\gamma))\right\}}{\tilde{x}}. \tag{30}$$

We make the guess $V(k') = \phi_0 + \phi_1 \ln k'$, where ϕ_0 and ϕ_1 are undetermined coefficients. For this guess, (29) becomes

$$(1+n) \cdot D(\tilde{x})^{\eta} \cdot k' = (\alpha\beta\gamma + \theta\phi_1) \cdot \tilde{c}. \tag{31}$$

From (30) and (31), we obtain

$$(1+n)\tilde{x} = \eta \cdot \left[\beta(1+\gamma) + \frac{\theta}{1-\theta}(1+\beta(1+\gamma)) - (\alpha\beta\gamma + \theta\phi_1)\right] \cdot \tilde{c}. \tag{32}$$

The substitution of (28), (31), and (32) into the resource constraint in (15) leads to

$$\tilde{c} = \frac{1}{\phi} A(k)^{\alpha},$$

where

$$\phi \equiv \left(1 + \frac{\beta}{\theta}\right) + (\alpha\beta\gamma + \theta\phi_1)(1-\eta) + \eta\left\{\beta(1+\gamma) + \frac{\theta}{1-\theta}(1+\beta(1+\gamma))\right\}.$$

The corresponding functions of $\tilde{d}, \tilde{x}$, and k' become

$$\tilde{d} = (1+n) \cdot \frac{\beta}{\theta} \cdot \frac{1}{\phi} A(k)^{\alpha},$$

$$\tilde{x} = \frac{1}{1+n} \cdot \left[\phi - \left\{\left(1 + \frac{\beta}{\theta}\right) + (\alpha\beta\gamma + \theta\phi_1)\right\}\right] \cdot \frac{1}{\phi} A(k)^{\alpha}, \tag{33}$$

$$k' = \frac{\alpha\beta\gamma + \theta\phi_1}{(1+n)D\left[\frac{1}{1+n} \cdot \left\{\phi - \left(\left(1+\frac{\beta}{\theta}\right) + (\alpha\beta\gamma + \theta\phi_1)\right)\right\}\right]^{\eta}} \cdot \left(\frac{1}{\phi} A(k)^{\alpha}\right)^{1-\eta}. \tag{34}$$

Substituting these policy functions into the Bellman equation gives

$$V(k) = Cons(\phi_0, \phi_1) + \alpha\phi \ln k,$$

where $Cons(\phi_0, \phi_1)$ includes constant terms. The guess is verified if $\phi_0 = Cons(\phi_0, \phi_1)$ and $\alpha\phi = \phi_1$. Therefore, ϕ_1 and ϕ_0 are given by

$$\phi_1 = \frac{\alpha}{1-\alpha\theta(1-\eta)} \cdot \left[\left(1+\frac{\beta}{\theta}\right) + \alpha\beta\gamma(1-\eta) + \eta\left\{\beta(1+\gamma) + \frac{\theta}{1-\theta}(1+\beta(1+\gamma))\right\}\right],$$

$$\phi_0 = \frac{1}{1-\alpha\theta(1-\eta)} \cdot \left[\left(1+\frac{\beta}{\theta}\right) + \alpha\beta\gamma(1-\eta) + \eta\left\{\beta(1+\gamma) + \frac{\theta}{1-\theta}(1+\beta(1+\gamma))\right\}\right].$$

■

B Supplementary Materials

B.1 Derivation of V_t^M and V_t^o

To derive V_t^M in (3), recall that the utility of the middle-aged is given by $V_t^M = \ln c_t + \beta(\ln d_{t+1} + \gamma \ln w_{t+1}h_{t+1})$. Given the consumption and savings functions and human capital formation function in Section 2, this utility function is rewritten as

$$V_t^M = \ln \frac{1}{1+\beta}(1-\tau_t)w_t h_t + \beta \ln \frac{\beta\left(1-\tau_{t+1}^k\right)R_{t+1}}{1+\beta}(1-\tau_t)w_t h_t + \beta\gamma \ln w_{t+1} D\left(x_t\right)^\eta \left(h_t\right)^{1-\eta},$$

or,

$$V_t^M = (1+\beta)\ln(1-\tau_t)w_t h_t + \beta \ln\left(1-\tau_{t+1}^k\right)R_{t+1} + \beta\gamma \ln w_{t+1} + \beta\gamma\eta \ln x_t + \tilde{\phi}\left(h_t\right), \quad (35)$$

where

$$\tilde{\phi}\left(h_t\right) \equiv \beta\gamma \ln D\left(h_t\right)^{1-\eta} + \ln \frac{1}{1+\beta} + \beta \ln \frac{\beta}{1+\beta}.$$

$(1-\tau_t)w_t h_t$, in (35), is rewritten as follows:

$$\begin{aligned}(1-\tau_t)w_t h_t &= (1-\alpha)A\left(k_t\right)^\alpha h_t - \left((1+n)x_t - \frac{\tau_t^k R_t s_{t-1}}{1+n}\right)\\ &= (1-\alpha)A\left(k_t\right)^\alpha h_t - (1+n)x_t + \tau_t^k \alpha A\left(k_t\right)^{\alpha-1} k_t h_t, \end{aligned} \quad (36)$$

where the first equality comes from the first-order conditions for profit maximization with respect to H_t, $w_t = (1-\alpha)A\left(k_t\right)^\alpha$, and the government budget constraint in (1), and the second equality comes from the first-order conditions for profit maximization with respect to K_t, $\rho_t = \alpha A\left(k_t\right)^{\alpha-1}$, and the capital market-clearing condition, $(1+n)\cdot k_t h_t = s_{t-1}$.

The term $\beta \ln R_{t+1} + \beta\gamma \ln w_{t+1}$ is reformulated as follows:

$$\begin{aligned}\beta \ln R_{t+1} + \beta\gamma \ln w_{t+1} &= \beta \ln \alpha A\left(k_{t+1}\right)^{\alpha-1} + \beta\gamma \ln(1-\alpha)A\left(k_{t+1}\right)^\alpha\\ &= \beta\left(\alpha - 1 + \gamma\alpha\right)\ln k_{t+1} + \beta\left(\ln \alpha A + \gamma \ln(1-\alpha)A\right), \end{aligned} \quad (37)$$

where the equality on the first line comes from the first-order conditions for profit maximization with respect to K_{t+1} and H_{t+1}. The term k_{t+1} in (37) is reformulated by using the capital market-clearing condition as follows:

$$\begin{aligned}k_{t+1} &= \frac{s_t}{(1+n)h_{t+1}}\\ &= \frac{1}{(1+n)D\left(x_t\right)^\eta\left(h_t\right)^{1-\eta}} \times \frac{\beta}{1+\beta}(1-\tau_t)w_t h_t\\ &= \frac{1}{(1+n)D\left(x_t\right)^\eta\left(h_t\right)^{1-\eta}} \times \frac{\beta}{1+\beta}\left[(1-\alpha)A\left(k_t\right)^\alpha h_t - (1+n)x_t + \tau_t^k \alpha A\left(k_t\right)^{\alpha-1} k_t h_t\right], \end{aligned} \quad (38)$$

where the first line comes from the capital market-clearing condition, second line comes from the savings function, and third line comes from the government budget constraint.

By using (36)–(38) and rearranging the terms, we can reformulate V_t^M in (35) as follows:

$$\begin{aligned} V_t^M &= (1+\alpha\beta(1+\gamma))\ln\left[\left((1-\alpha)+\tau_t^k\alpha\right)A(k_t)^\alpha h_t-(1+n)x_t\right] \\ &\quad+\beta\eta(1-\alpha)(1+\gamma)\ln x_t+\beta\ln\left(1-\tau_{t+1}^k\right)+\beta(1-\alpha)(1+\gamma)\ln D(h_t)^{1-\eta}+C, \end{aligned}$$

where

$$C\equiv\beta(\alpha-1+\alpha\gamma)\ln\frac{\beta}{(1+n)(1+\beta)}+\beta(\ln\alpha A+\gamma\ln(1-\alpha)A)+\ln\frac{1}{1+\beta}+\beta\ln\frac{\beta}{1+\beta}.$$

We next derive V_t^o in (4). Recall that V_t^o is defined as $V_t^o=\ln d_t+\gamma\ln w_t h_t$. This is rewritten as follows:

$$\begin{aligned} V_t^o &= \ln R_t\left(1-\tau_t^k\right)s_{t-1}+\gamma\ln(1-\alpha)A(k_t)^\alpha h_t \\ &= \ln\left(1-\tau_t^k\right)+\{\ln\alpha A(k_t)^\alpha h_t(1+n)+\gamma\ln(1-\alpha)A(k_t)^\alpha h_t\}. \end{aligned}$$

■

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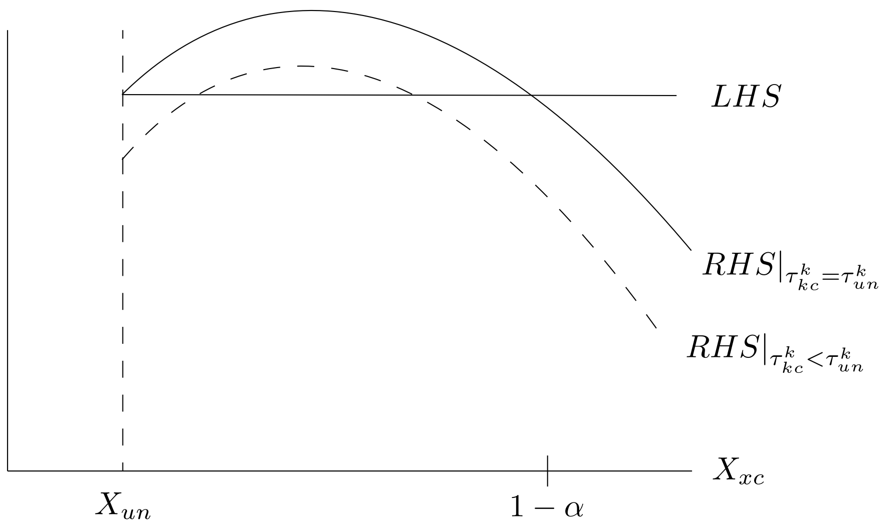


Figure 1: LHS and RHS of Eq. (14).

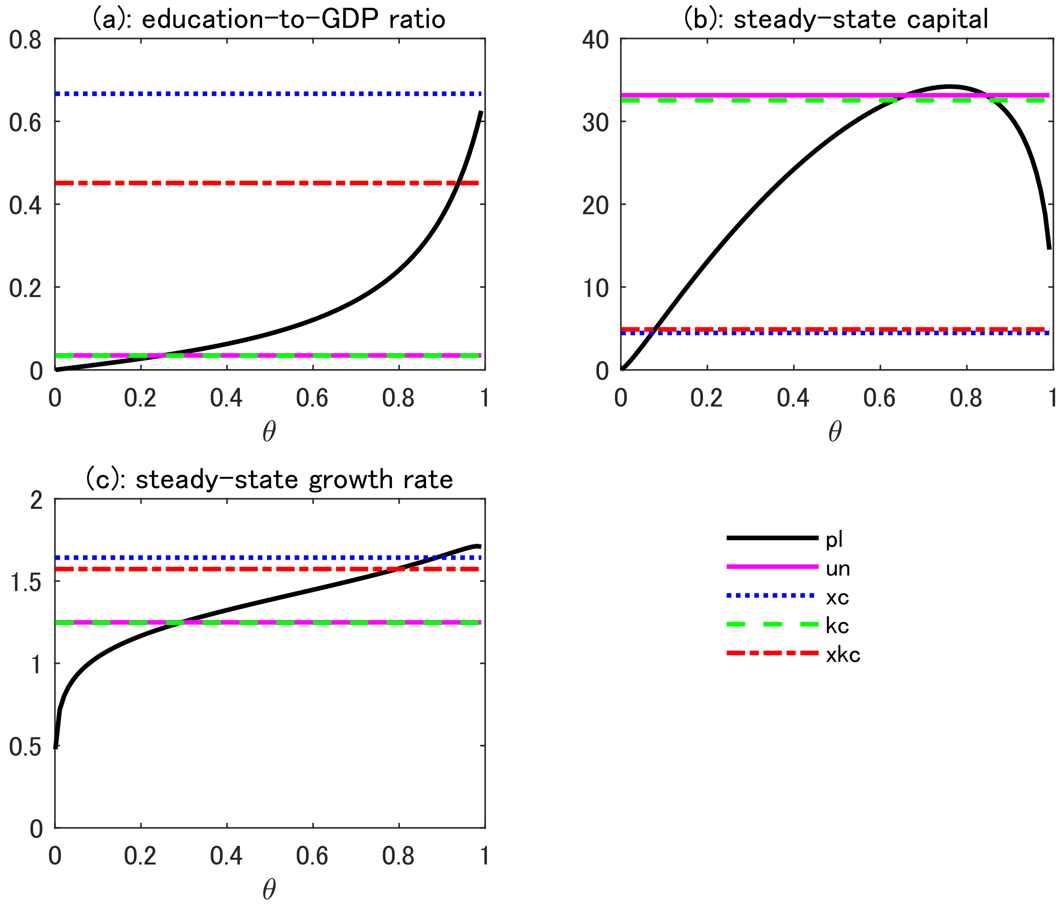
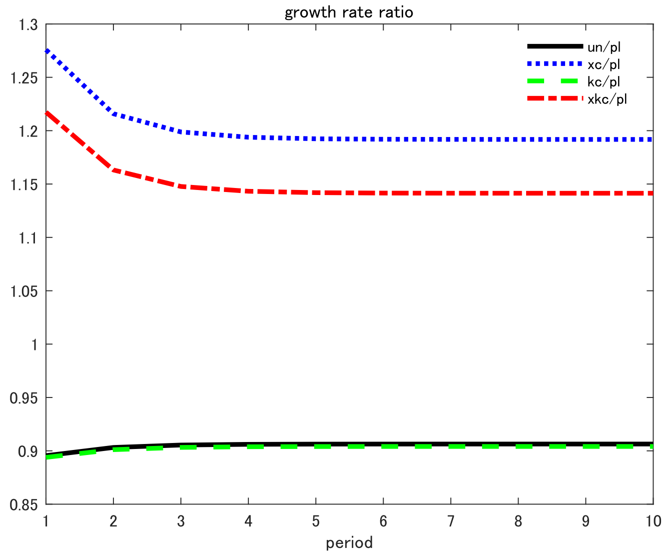


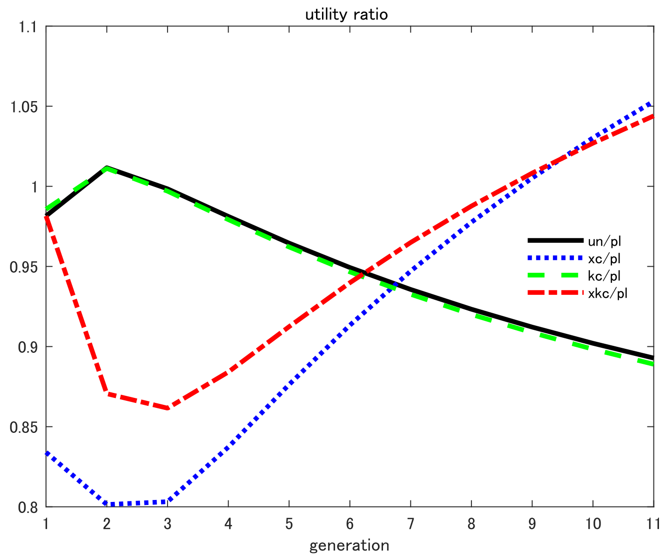
Figure 2: Ratio of public education expenditure to GDP (Panel (a)), steady-state capital (Panel (b)), and steady-state growth rates (Panel (c)). The symbols pl, un, xc, kc, and xkc denote the corresponding values in the planner's allocation (pl), political equilibrium in the absence of any constraint (un), political equilibrium with the minimum constraint on public education expenditure (xc), political equilibrium with the upper limit constraint on capital income tax (kc), and political equilibrium with the combination of the two constraints (xkc), respectively.

	Labor income tax rate	Capital income tax rate	Ratio of public education expenditure to GDP
Absence of constraints	0.0374	0.0299	0.0349
Minimum constraint on public education expenditure	0.6675	0.6649	0.6667*
Upper limit constraint on capital income tax rate	0.0516	0*	0.0344
Combination of the two constraints	0.6617	0.0299*	0.451*

Table 1: Numbers marked with asterisks denote the values of the constraints and numbers without asterisks are the resulting fiscal variables.



(a)



(b)

Figure 3: Panel (a): Evolution of growth rates across periods; Panel (b); Distribution of utility across generations.