Factor Returns and Circular Causality

Haiwen Zhou

5 May 2018
Factor Returns and Circular Causality
Haiwen Zhou

Abstract
The presence of circular causality in a region through factor returns is studied in a general equilibrium model in which firms producing final products engage in oligopolistic competition. The intermediate input is produced by capital and labor with a constant returns to scale technology. If the degree of increasing returns in the production of final products is sufficiently high, the return to a factor can increase with the amount of this factor. Thus a higher amount of a factor in a region leads to a higher return to this factor and attracts additional amount of this factor to move in. Capital movement and labor movement can be reinforcing. This type of circular causality means that unbalanced regional development can persist over time.

Keywords: Factor return, circular causation, increasing returns, oligopolistic competition, intermediate input

JEL Classification Numbers: O10, R10

1. Introduction
With constant returns to scale in production and perfect competition, the return to a factor of production is determined by its marginal productivity. When the amount of a factor increases, other things equal, the marginal productivity of this factor decreases and the return to this factor decreases. Since developed countries and cities have higher ratios of capital to labor than developing countries and rural areas, we may expect that capital will move from developed countries to developing countries and from cities to rural areas. This expectation is valid in some cases. For example, before World War I, with its abundant supply of capital and high ratio of capital to labor, Britain invested heavily in other countries (Williamson, 2006). Also, some developing countries with low ratio of capital to labor such as China receive billions of dollars of capital inflow in recent years (Huang, 2005). However, capital does not necessarily flow from developed countries to developing countries (Lucas, 1990). Also, capital does not always flow from cities to rural areas (Jacobs, 1985). Instead, developed countries and cities attract capital and labor. In fact, if diminishing marginal returns were always valid, with factor mobility economic activities would be relatively uniformly distributed over space: cities may disappear and the huge income differences between developed countries and developing countries may disappear. Thus at least under some conditions, returns to a factor of production can increase

---

1 I thank Leo Michelis, Laura Razzolini, and two anonymous referees for their detailed and insightful comments. I am solely responsible for all the remaining errors.
with the amounts of this factor.\textsuperscript{2} Regional concentration of economic activities as a cumulative process is discussed in Myrdal (1957, 1968). Myrdal argues that market forces may lead to production to be concentrated in given locations rather than dispersed uniformly across spaces.\textsuperscript{3} In his discussion, capital mobility may contribute to the concentration of industries in given regions. This may increase the return to capital and attract additional capital to move into those regions.

In this paper, we study the conditions for the presence of circular causality through factor returns in a region in a general equilibrium model. There is a continuum of final products. Final products are produced by an intermediate input and the intermediate input is produced by both labor and capital with a constant returns to scale technology.\textsuperscript{4} The production of a final product requires a fixed cost.\textsuperscript{5} The existence of fixed cost is the source of increasing returns in the production of final products.\textsuperscript{6} The existence of fixed costs in the production of a final product also leads to imperfect competition in the sector producing final products. More specifically, similar to Lahiri and Ono (1988, 1995, 2004) and Neary (2003), firms producing the same final product are assumed to engage in oligopolistic competition.

We show that the return to a factor of production can increase with the amount of this factor in a region. The reason is as follows. In addition to the effect from a diminishing marginal product in the production of the intermediate input, there is one additional effect affecting the return to a factor: increasing returns to scale in the production of final products. While the diminishing marginal product tends to decrease the return to a factor, increasing returns in the production of final products tends to increase the return to a factor. If the effect from increasing returns dominates the effect from the diminishing marginal product, the return to a factor increases with the amount of this factor in a region. Thus a circular causality results: a higher amount of capital in a region increases the return to capital and attracts additional capital to move in; this increases the return to capital. A higher amount of capital also increases the

\textsuperscript{2} Bai, Hsieh, and Qian (2006) show that even with high levels of investment over years in China, the return to capital in China is not low.

\textsuperscript{3} Porter (1990) has a detailed discussion of various factors leading to the concentration of industries in various developed countries.

\textsuperscript{4} Baldwin et al. (2003, chapter 8) provide a synthesis of the literature on economic geography based on the usage of intermediate inputs and vertical linkages.

\textsuperscript{5} The importance of fixed costs in modern production is discussed in Chandler (1990).

\textsuperscript{6} In this model, increasing returns are internal. Holod and Reed (2009) study regional external economies. Zhou (2007a) shows that the choice of technology can be a link between internal and external increasing returns.
return to labor. If factor mobility is possible, capital mobility and labor mobility can be reinforcing. This type of circular causality can lead to the concentration of economic activities in given regions. As the return to a factor in a region with a lower ratio of this factor may not necessarily be higher, an undeveloped region can remain undeveloped for a long period of time.

In the literature, Nurkse (1953) and Myrdal (1957, 1968) have illustrated the implications of circular causality on economic development. Matsuyama (1995) provides a survey of models of cumulative processes based on monopolistic competition. Baldwin et al. (2003) provide a discussion of various mechanisms of circular causation in which firms engage in monopolistic competition. In a formal model, Krugman (1991) studies the location of economic activities in which firms producing manufactured products engage in monopolistic competition. In Krugman (1991), labor is the only factor of production and firms are more interested in locating at a region with a larger market size. As the number of varieties produced in a region increases, a lower price index means that the real income of a consumer is higher and this attracts workers to move into this region. This leads to a process of circular causation. In this model, the number of varieties of final products is fixed and firms producing final products engage in oligopolistic competition. With oligopolistic competition, a firm’s scale of production increases with the size of the market and thus average cost decreases. This change of a firm’s scale of production is the source of the benefit of locating at a region with larger amounts of factors of production. By incorporating capital as a factor of production, we show that labor mobility and capital mobility can be reinforcing. Rather than relying on computer simulations frequently used in the literature, results in this model are derived analytically.

The paper is organized as follows. Section 2 specifies the model and establishes the equilibrium conditions. Section 3 explores the properties of the equilibrium. Section 4 discusses some possible generalizations and extensions of the model and concludes.

2. The model

In this section, we specify the model. To make the intuition as clear as possible, we focus on a closed economy. First, we study a representative consumer’s utility maximization. Second,

---

we study firms’ profit maximization, including profit maximization for a firm producing the intermediate input and the profit maximization for a firm producing a final product. Finally, we establish the market clearing conditions, including factor markets and product markets clearing conditions.

There is a continuum of final products indexed by a number $\sigma \in [0,1]$. All final products are assumed to have the same costs of production and enter a consumer’s utility function in the same way. A representative consumer’s consumption of the product $\sigma$ is $c(\sigma)$ and her utility function is specified as $U = \int_0^1 \ln c(\sigma)d\sigma$. The wage rate is $w$. The return to capital is $r$ and the amount of capital in this economy is $K$. It is assumed that capital is equally owned by all the $L$ residents in this economy. A consumer’s income is the sum of her wage income and her income as a capital owner. Thus a consumer’s total income is $w + (rK/L)$. The price of the final product $\sigma$ is $p(\sigma)$. A consumer’s budget constraint is $\int_0^1 p(\sigma)c(\sigma)d\sigma = w + (rK/L)$. A consumer takes the wage rate, the return to capital, and the prices of final products as given and chooses quantities of consumption of final products to maximize her utility.

The intermediate input is produced by capital and labor. Total output of the intermediate input produced by all firms is $Q$. For $\theta$ denoting a constant between zero and one, output of the intermediate input is specified as $Q = L^\theta K^{1-\theta}$. Thus there are constant returns to scale in the production of the intermediate input. Firms producing the intermediate input are assumed to engage in perfect competition. The price of the intermediate input is $p_i$, $I$ for intermediate. For a firm producing the intermediate input, it takes the wage rate and the return to capital as given and chooses the amounts of labor and capital to maximize its profit $p_i L^\theta K^{1-\theta} - rK - wL$. To produce the intermediate input, the optimal choice of the amount of labor requires that

---

8 As discussed in Neary (2003), the main purpose of having a continuum of final products rather than one final product is to eliminate the market power of a firm producing a final product in the market for the intermediate product. Otherwise, it can be viewed that there is only one final product. When there is only one final product and there are only few firms producing it, a firm producing the final product is one of the small number of firms purchasing the intermediate input and will have market power in the market for the intermediate input. With a continuum of final products, even though each final product is still produced by a small number of firms, a firm is only one of the infinite number of buyers of the intermediate input and does not have market power in the market for the intermediate input.

9 With homothetic preferences, the distribution of ownership of capital does not affect the production pattern in this model.
The optimal choice of the amount of capital requires that

\[ (1 - \theta) p_1 L^\theta K^{-\theta} - r = 0. \]

Final products are produced by using the intermediate input only. For the product \( \sigma \), there are \( m(\sigma) \) identical firms producing it. Firms producing the same final product are assumed to engage in Cournot competition. For each final product, the fixed cost is \( f \) units of the intermediate input and the marginal cost is \( \beta \) units of the intermediate input. For a firm producing a final product, if this firm’s output is \( x \), its revenue is \( px \) and its cost of purchasing the intermediate input is \((f + \beta x)p_1\). Thus its profit is \( px - (f + \beta x)p_1 \). A firm producing a final product takes the price of the intermediate input as given and chooses its output to maximize its profit. This firm’s optimal choice of output requires that

\[ p + x \frac{\partial p}{\partial x} - \beta p_1 = 0. \]

As the number of firms producing a final product is a real number rather than restricted to be an integer number, free entry into the production of final products leads to zero profits for a firm producing a final product:\(^{10}\)

\[ px - (f + \beta x)p_1 = 0. \]

Total income in this economy is the sum of the labor income \( wL \) and the capital income \( rK \). Thus total income is \( wL + rK \). Total value of a final product is \( pmx \). The clearance of the market for a final product requires that demand equals supply:

\[ wL + rK = pmx. \]

For a final product, each of the \( m \) firms produces \( x \) units of output and the total quantity of production is \( mx \). Each of the \( L \) consumers consumes \( c_m \) units of output and the total quantity of consumption is \( Lc_m \). In equilibrium \( mx = Lc_m \). In a Cournot equilibrium, when a firm chooses its level of output, it views the output of other firms as given. With this in mind, by using the result that the absolute value of a consumer’s elasticity of demand for a final product is one, partial differentiation of \( mx = Lc_m \) with respect to \( p \) leads to \( \partial x / \partial p = -m \). Plugging this

\(^{10}\) Examples of oligopolistic competition with free entry include Mankiw and Whinston (1986), Brander (1995), Lahiri and Ono (1995), and Zhang (2007).
expression into a firm’s optimal output choice condition \( p + x \hat{c} p / \hat{c} x - \beta p_I = 0 \) yields the familiar condition that a firm’s price is a markup over its marginal cost of production:

\[
p \left(1 - \frac{1}{m}\right) = \beta p_I. \tag{5}\]

For each final product, each of the \( m \) firms demands \( f + \beta x \) units of the intermediate input. For this economy, the total demand for the intermediate input is \( \int_0^1 m(\sigma)[f + \beta x(\sigma)]d\sigma \). Total supply of the intermediate input is \( L^{(1-\theta)} K^{\theta} \). The clearance of the market for the intermediate input requires that demand equals supply:

\[
\int_0^1 m(\sigma)[f + \beta x(\sigma)]d\sigma = L^{(1-\theta)} K^{\theta}. \tag{6}\]

We focus on a symmetric equilibrium in which the number of firms, the price, the output, and the consumption of all final products are the same. In a symmetric equilibrium, Equations (1)-(6) form a system of six equations defining six variables \( p, p_I, m, x, r, \) and \( w \). An equilibrium is a tuple \((p, p_I, m, x, r, w)\) satisfying Equations (1)-(6). For the rest of the paper, the price of a final product is used as the numeraire: \( p \equiv 1 \). With this normalization, returns to capital and labor are “real” as they are measured in terms of the price of a final product.

3. Properties of the Equilibrium

In this section, we explore the properties of the equilibrium. Interestingly, there is a closed form solution for Equations (1)-(6).\(^\text{11}\) First, we conduct comparative static analysis to establish some properties of the model without solving Equations (1)-(6) explicitly. This type of exercise is useful if no explicit solution is available. Also, this approach shows the structure of the model in a clear way. Second, we provide a closed form solution for the variables to establish additional properties of the equilibrium.

\(^{11}\) The reason that a closed form solution is available is because preferences are homothetic and the production function for the intermediate input is Cobb-Douglas. With homothetic preference, a fixed percentage of income is spent on each final product. With Cobb-Douglas production function, a fixed percentage of expenditure is spent on each factor of production. As a result, the equilibrium is easier to solve.
To conduct comparative statics, we need to reduce the system of six equations to a manageable number of equations. We arrive at the following system of three equations defining three variables \( w, r, \) and \( x \) as functions of exogenous parameters:

\[
V_1 \equiv (1-\theta)wL - \theta rK = 0, 
\tag{7a}
\]

\[
V_2 \equiv fL^\theta K^{1-\theta} - (f + \beta x)^2 = 0, 
\tag{7b}
\]

\[
V_3 \equiv \partial x (f + \beta x) - fwL = 0. 
\tag{7c}
\]

The derivation of \( V_1 - V_3 \) is as follows. Dividing Equation (1) by Equation (2) leads to \( V_1 \). From Equation (6), the number of firms producing the same final product is given by \( m = Q/(f + \beta x) \). Plugging this value of \( m \) into Equation (5) and using Equation (3) to eliminate \( p \), yield

\[
(f + \beta x)^2 - fQ = 0. 
\tag{8}
\]

Replacing \( Q \) in Equation (8) with \( L^\theta K^{1-\theta} \) yields \( V_2 \). Plugging \( m = Q/(f + \beta x) \) into Equation (4) yields

\[
\frac{Qx}{f + \beta x} = wL + rK. 
\tag{9}
\]

Plugging the value of \( Q \) from Equation (8) into Equation (9) yields

\[
x(f + \beta x) = f(wL + rK). 
\tag{10}
\]

From Equation (7a), \( rK = (1-\theta)wL/\theta \). Plugging this value of \( rK \) into Equation (10) yields \( V_3 \).

The following proposition studies the impact of capital endowment on this economy’s level of output, the wage rate, and the return to capital.

Proposition 1: The output of a firm producing a final product always increases with the capital endowment. The wage rate always increases with the capital endowment. The return to capital increases with the amount of capital in this economy if and only if

\[
L^\theta K^{1-\theta} < f \left[ \frac{(1-\theta) + 2\theta}{2\theta} \right]^2. 
\tag{11}
\]

Proof: Differentiation of \( V_1, V_2, \) and \( V_3 \) with respect to \( w, r, x, \) and \( K \) yields
\[
\begin{pmatrix}
\frac{\partial V_1}{\partial w} & \frac{\partial V_1}{\partial r} & 0 \\
0 & 0 & \frac{\partial V_2}{\partial x} \\
\frac{\partial V_3}{\partial w} & 0 & \frac{\partial V_3}{\partial x}
\end{pmatrix}\begin{pmatrix} dw \\ dr \\ dx \end{pmatrix} = -\begin{pmatrix} \frac{\partial V_1}{\partial K} \\ \frac{\partial V_2}{\partial K} \\ 0 \end{pmatrix} dK.
\]

Since \( \frac{\partial V_1}{\partial r} < 0 \), \( \frac{\partial V_2}{\partial x} < 0 \), and \( \frac{\partial V_3}{\partial w} < 0 \), for \( \Delta \) denoting the determinant of the coefficient matrix, it is clear that \( \Delta = \frac{\partial V_1}{\partial r} \frac{\partial V_2}{\partial x} \frac{\partial V_3}{\partial w} < 0 \). An application of Cramer’s rule leads to

\[
\frac{dx}{dK} = \frac{\frac{\partial V_1}{\partial r} \frac{\partial V_2}{\partial x} \frac{\partial V_3}{\partial w}}{\Delta} > 0,
\]

\[
\frac{dw}{dK} = \frac{\frac{\partial V_1}{\partial r} \frac{\partial V_2}{\partial K} \frac{\partial V_3}{\partial x}}{\Delta} > 0,
\]

\[
\frac{dr}{dK} = \frac{-\left(\frac{\partial V_1}{\partial K} \frac{\partial V_2}{\partial x} \frac{\partial V_3}{\partial w} + \frac{\partial V_1}{\partial x} \frac{\partial V_2}{\partial K} \frac{\partial V_3}{\partial w}\right)}{\Delta}.
\]

From Equations (7a) and (7c), the return to capital can be expressed as

\[
r = (1 - \theta)x(f + \beta x)/(fK).
\]

Using this value of \( r \) to replace \( r \) in \( \partial V_1 / \partial K \) yields

\[
\frac{\partial V_1}{\partial w} \frac{\partial V_2}{\partial K} \frac{\partial V_3}{\partial x} + \frac{\partial V_1}{\partial x} \frac{\partial V_2}{\partial K} \frac{\partial V_3}{\partial w}
\]

\[
= \frac{\theta(1-\theta)L(f + \beta x)^2}{K}[(1-\theta)(f + 2\beta x) - 2\beta x].
\]

Thus \( dr/dK > 0 \) if and only if \( (1 - \theta)(f + 2\beta x) - 2\beta x < 0 \), or \( x < (1 - \theta)f/(2\beta \theta) \). Plugging the value of \( x \) from Equation (7b) into the inequality \( x < (1 - \theta)f/(2\beta \theta) \), we need

\[
L^0 K^{1-\theta} < f\left(\frac{(1-\theta) + 2\theta}{2\theta}\right)^2.
\]

The intuition behind Proposition 1 is as follows. As capital is complementary to labor in the production of the intermediate input, a higher amount of capital increases the return to labor. When the amount of capital in this economy increases, the production of the intermediate input increases. The higher level of the intermediate input could not be absorbed solely by an increase of the number of firms in the production of each final product. The reason is that a firm
producing a final product will make a negative profit if its output does not increase while a higher number of competing firms leads to a lower price of a final product. To absorb the additional amount of the intermediate input, the level of output for a firm producing a final product will be higher. When the amount of capital increases, there are two effects working in opposite directions on the return to capital. First, other things equal, a higher amount of capital decreases the marginal product of capital. This decreases the return to capital. Second, there are increasing returns in the production of final products. As the output for a firm producing a final product increases, the average cost is lower and this increases the return to capital. Whether the return to capital increases or not depends on which of the two effects is stronger. From the specification of the production function of the intermediate input, a lower value of \( \theta \) increases the marginal product of capital. When the value of \( \theta \) decreases, the first effect decreases and it can be dominated by the second effect.\(^{12}\) As a result, the return to capital increases with the amount of capital in this region.

The reason that the return to capital may increase with the amount of capital is the existence of increasing returns in the production of final products, or the existence of fixed costs in the production of final products. From inequality (11), if the fixed cost is zero (no increasing returns to scale), the return to capital always decreases with the amount of capital in a region. For inequality (11), the left-hand side \( L^\theta K^{1-\theta} \) is the output of the intermediate input. Thus inequality (11) can be interpreted as that the level of increasing returns in the production of final products \( (f) \) is relatively high compared with the constant returns industry (the level of output of the intermediate input), the return to capital will increase with the amount of capital.

The following proposition studies the impact of the labor endowment on the output, the wage rate, and the return to capital.

Proposition 2: The output of a firm producing a final product always increases with the labor endowment. The return to capital always increases with the labor endowment. The return to labor increase with the labor endowment in this region if and only if

\(^{12}\) The right-hand side of inequality (11) goes to infinity while the left hand side is bounded if \( \theta \) is equal to zero. Thus inequality (11) will be valid if \( \theta \) is sufficiently small. In the extreme case that \( \theta \) is equal to zero, there are constant returns rather than decreasing returns result for the usage of capital in the production of the intermediate input.
\[ L^\theta K^{1-\theta} < f \left( \frac{\theta+2(1-\theta)}{2(1-\theta)} \right)^2. \] (12)

Proof: From Equations (7a)-(7c), we can establish the following system of three equations:

\[ \Omega_1 \equiv (1-\theta)wL - \theta rK = 0, \]
\[ \Omega_2 \equiv f L^\theta K^{1-\theta} - (f + \beta x)^2 = 0, \]
\[ \Omega_3 \equiv (1-\theta)x(f + \beta x) - f rK = 0. \]

For the above system, \( \Omega_1 \) is the same as Equation (7a), \( \Omega_2 \) is the same as Equation (7b), and \( \Omega_3 \) follows from Equations (7a) and (7c). Differentiation of \( \Omega_1, \Omega_2, \) and \( \Omega_3 \) with respect to \( w, r, x, \) and \( L \) yields

\[
\begin{pmatrix}
\frac{\partial \Omega_1}{\partial w} & \frac{\partial \Omega_1}{\partial r} & 0 \\
0 & 0 & \frac{\partial \Omega_2}{\partial x} \\
0 & \frac{\partial \Omega_3}{\partial r} & \frac{\partial \Omega_3}{\partial x}
\end{pmatrix}
\begin{pmatrix}
dw \\
dr \\
dx
\end{pmatrix}
=
\begin{pmatrix}
\frac{\partial \Omega_1}{\partial L} \\
\frac{\partial \Omega_2}{\partial L} \\
0
\end{pmatrix}
\begin{pmatrix}
dL
\end{pmatrix}.
\]

Since \( \frac{\partial \Omega_1}{\partial w} > 0, \frac{\partial \Omega_2}{\partial x} < 0, \) and \( \frac{\partial \Omega_3}{\partial r} < 0, \) it is clear that the determinant of the above coefficient matrix \( \Delta_\Omega \) is negative: \( \Delta_\Omega = -\frac{\partial \Omega_1}{\partial w} \frac{\partial \Omega_2}{\partial x} \frac{\partial \Omega_3}{\partial r} < 0. \) An application of Cramer’s rule leads to

\[
\frac{dx}{dL} = \left( \frac{\partial \Omega_1}{\partial w} \frac{\partial \Omega_2}{\partial L} \frac{\partial \Omega_3}{\partial r} \right) / \Delta_\Omega > 0,
\]
\[
\frac{dr}{dL} = \left( \frac{\partial \Omega_1}{\partial w} \frac{\partial \Omega_2}{\partial L} \frac{\partial \Omega_3}{\partial x} \right) / \Delta_\Omega > 0,
\]
\[
\frac{dw}{dL} = \left( \frac{\partial \Omega_1}{\partial L} \frac{\partial \Omega_2}{\partial x} \frac{\partial \Omega_3}{\partial r} + \frac{\partial \Omega_1}{\partial r} \frac{\partial \Omega_2}{\partial L} \frac{\partial \Omega_3}{\partial x} \right) / \Delta_\Omega.
\]

From Equation (7c), the wage rate can be expressed as \( w = \frac{\theta x(f + \beta x)}{f L}. \) Using this value of wage rate to replace \( w \) in \( \frac{\partial \Omega_1}{\partial L} \) yields
\[
\frac{\partial \Omega_1}{\partial L} \frac{\partial \Omega_2}{\partial x} \frac{\partial \Omega_3}{\partial r} + \frac{\partial \Omega_1}{\partial r} \frac{\partial \Omega_2}{\partial L} \frac{\partial \Omega_3}{\partial x} = (1-\theta)\theta K (f + \beta x)(2\beta x - \theta f - 2\beta \theta x)/L.
\]

Thus \(dw/dL > 0\) if and only if \(x < \theta f/[2(1-\theta)\beta]\). Plugging the value of \(x\) from Equation (7b) into \(x < \theta f/[2(1-\theta)\beta]\), we need

\[
L^\theta K^{1-\theta} < f \left( \frac{\theta + 2(1-\theta)}{2(1-\theta)} \right)^2.
\]

Because the right-hand side of inequality (12) goes to infinity while the left-hand side of (12) is finite when \(\theta\) goes to one, a higher value of \(\theta\) makes it more likely that the return to labor increases with the amount of labor in this economy. As the right-hand side of inequality (11) decreases with \(\theta\), a lower value of \(\theta\) makes it more likely that the return to capital increases with capital endowment in a region. Because the left-hand sides of inequalities (11) and (12) are the same, with an intermediate value of \(\theta\), it is possible that both the return to capital increases with the amount of capital and the return to labor increases with the amount of labor.\(^{13}\) In this case, capital mobility and labor mobility are reinforcing. As capital moves into a region, a higher amount of capital in this region increases the return to labor in this region and makes labor movement into this region more desirable.

The presence of circular causality is limited by the size of the fixed cost of production in the production of final products. As the amounts of labor and capital in a region increase, if \(f\) does not change, inequality (11) will eventually be violated and the return to capital will eventually decrease. However, the process of industrialization is associated with the continuous adoption of machines in production. As illustrated vividly in Ford (1922) and discussed systematically in Chandler (1990), modern production is associated with the usage of machines, which are fixed costs. The adoption of new technologies can lead to high levels of fixed costs. As fixed cost increases over time, this phenomenon of circular causality can persist over time. This result is consistent with the observation that city size can increase over time if technologies with higher levels of fixed costs are frequently introduced.

\(^{13}\) For \(\theta = 1/2\), the right-hand sides of inequalities (11) and (12) coincide. Thus, the set of \(\theta\) such that inequalities (11) and (12) both hold is nonempty.
From the above analysis, capital does not necessarily move to a region with a lower capital-labor ratio because the return to capital will not necessarily be higher in a region with a lower amount of capital if regions have the same amount of labor. This provides an explanation to the Lucas puzzle (1990) that capital does not always move from developed countries to developing countries.

From Equation (7b), the value of $x$ can be solved. Then other variables can be solved. By solving the system of Equations (1)-(6), it can be shown that

$$m = \frac{1}{f} \left( \sqrt{L^\theta K^{1-\theta}} \right),$$

$$x = \frac{1}{\beta} \left( \sqrt{fL^\theta K^{1-\theta} - f} \right).$$

From the above equations, first, the number of firms producing the same final product increases with the amount of capital and the amount of labor in a region and decreases with the level of fixed cost of producing a final product. Second, a firm’s output decreases with the marginal cost in the production of a final product. With a lower marginal cost, for a firm producing a final product, the level of output needed to break even is smaller.

4. Conclusion

In this paper, we have studied the presence of circular causality in a region through factor returns in a general equilibrium model. The intermediate input is produced by both capital and labor. Firms producing final products engage in oligopolistic competition and there are increasing returns in the production of final products. We show that the return to a factor can increase with the amount of this factor in a region and capital mobility and labor mobility can be reinforcing.

In this paper, we have explored some channels of circular causations of regional development. There are some interesting generalizations and extensions of this model. Here we briefly discuss some of them. First, in this model, the marginal cost of a final product is constant. Variable marginal cost in the production of a final product can be incorporated into the model. With increasing marginal cost, the tendency for economic activities to concentrate in a given region will be smaller.
Second, in this model, there is only one type of final product. If we name final products in this model as manufactured products, an agricultural product can be incorporated into the model. With two types of final products, a price index based on the prices of the agricultural product and manufactured products can be defined (see Baldwin et al., 2003 for examples). The return to a factor can be defined in terms of the price index. The agricultural sector can be a constraint for the concentration of production in a region if the production of the agricultural product has decreasing or constant returns to scale. With a higher amount of capital or labor in a region, the return to a factor in terms of the agricultural product can decrease. However, the return to a factor in terms of the price index can still increase with a higher amount of a factor in a region. This increase can be achieved in at least two ways. First, if the increase of the return to a factor in terms of manufactured products dominates the decrease of the return to a factor in terms of the agricultural product, the return to a factor in terms of the price index will increase. Second, in a dynamic model, suppose that the level of output in the agricultural sector is affected by the level of agricultural technology and there are spillovers from the manufacturing sector to the agricultural sector in the sense that the level of agricultural technology increases with the level of technology in the manufacturing sector (Zhou, 2009). If the degree of spillovers to the agricultural sector is strong enough, the production of the agricultural product can keep pace with the amount of factors in a region and the return to a factor in terms of the agricultural product will not decrease. Thus the incorporation of the agricultural sector may not necessarily eliminate the tendency for factors of production to concentrate in given regions.

Third, with the incorporation of one additional type of final product such as the agricultural product, the model can be extended to study trade between different economies. With the explicit modeling of trade, the impact of factor mobility on factor returns can be addressed. The existence of transportation costs among regions can lead to different returns to factors in different regions. The direction of capital mobility can be affected by whether a capital owner moves together with her capital or not.

Fourth, in this model, firms producing final products use the same technology regardless of the levels of output. A firm’s choice of technology can be incorporated into this model. Suppose that there are technologies with different levels of fixed and marginal costs of production (Zhou, 2004). A more specialized technology has a higher fixed but a lower marginal cost of production. It can be shown that that a firm producing a final product in a region with a
larger market size will choose a more specialized technology. A more specialized technology leads to a lower average cost and thus a lower price level and thus a higher real wage. This will create another channel of the circular causation process because workers will be more interested in moving to the region with a larger market size.

Finally, as pointed out in Neary (2001) and Baldwin et al. (2003, chapter 19), while the spatial distribution of economic activities is interesting to policy makers, compared with theoretical research, policy implications of economic geography models have not been sufficiently explored. Various policy issues such as tax competition among regions are interesting avenues for future research.

References


