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1 Title: Cost-share induced technological change and Kaldor's stylized facts

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3

4 **Abstract**

5 This paper presents a theory of induced technological change in which firms pursue a random,
6 local, and bounded search for productivity-enhancing innovations. Firms implement profitable
7 innovations at fixed prices, which then spread through the economy. After diffusion, all firms
8 adjust prices and wages. The model is consistent with a variety of price-setting behaviors, which
9 determine equilibrium positions characterized by constant cost shares and productivity growth
10 rates. Target-return pricing yields Harrod-neutral technological change with a fixed wage share as
11 a stable equilibrium, consistent with Kaldor's stylized facts, while allowing for deviations from
12 equilibrium, as observed in the longer historical record.

13

14 *Keywords:* classical; post-Keynesian; evolutionary; induced technological change; Harrod-neutral
15 technological change

16 JEL: E12, E14, O33

17

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21

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1 **1 Introduction**

2 In a well-known paper, Kaldor (1961, pp. 178–179) introduced six “stylized facts”, of
3 which the first four are: rising labor productivity and output; rising capital per worker; a
4 steady rate of profit; and steady capital-output ratios. The last two imply a steady profit
5 share, and thus a steady wage share. However, shortly after the publication of Kaldor’s
6 paper, the wage share rose briefly in many high-income countries and then began to fall,
7 a trend that continues today. By compiling long historical data series, Maddison (1994, p.
8 10) demonstrated that capital productivity wanders over a wide range, without exhibiting
9 a long-run trend, suggesting that over many decades technological change appears to
10 have a weak Harrod-neutral tendency, but with substantial variations. Observations such
11 as these put Kaldor’s stylized facts into question and renewed interest in a major concern
12 of the classical economists – the intersection between growth, distribution and
13 technological change (Acemoglu, 2002; Brugger & Gehrke, 2016; Tavani & Zamparelli,
14 2017).

15 The present paper is a contribution to this literature. It presents a theory of cost-share
16 induced technological change, an idea first proposed by Hicks (1932, p. 124 ff.). We
17 show that when combined with a price and wage-setting regime that leaves the profit and
18 wage shares fixed, the theory is consistent with Marx-biased technological change
19 (although other outcomes are possible). A regime of target-return pricing generates a
20 stable dynamic with Harrod-neutral technological change as the equilibrium position,
21 while allowing for substantial variation.

22 We build on a theory proposed by Duménil and Lévy (1995), who sought, as we do, to
23 motivate a theory of cost-share induced technological change in order to explain the
24 observed behavior of labor productivity and capital-output ratios. Their theory is
25 evolutionary, in the sense of Nelson and Winter (1982), in that innovation is stochastic,
26 locally bounded, and shaped by past decisions. They also follow Nelson and Winter

1 (1982, pp. 168–169) by separating innovation into a series of distinct processes. With
2 Duménil and Lévy (1995), we view firms as seeking innovations in a bounded region
3 close to their existing technologies. They only accept innovations that increase their
4 profit rate under prevailing prices, which induces a bias in the direction of technological
5 change. Productivity change is random, and constrained by the potential for discovery.

6 In the theory presented in this paper, innovation is a microeconomic phenomenon,
7 located within the firm. It translates into macroeconomic impacts as innovations are taken
8 up by other firms (Nelson & Winter, 1982, p. 202). Diffusion of innovations weakens the
9 temporary monopoly earned by the innovating firm, and all firms adjust their prices and
10 the wages they pay in response to the new cost structure, resulting in a new functional
11 income distribution. The innovating firm thus faces unintended consequences from its
12 innovation, as proposed by Marx. The functional income distribution which results from
13 their and other firms' innovation, followed by diffusion and price and wage adjustment,
14 induces subsequent rounds of innovation and price setting, thereby setting up a repeating
15 dynamic.

16 The price- and wage-setting step avoids the implication of Okishio's theorem (Okishio,
17 1961; Bowles, 1981), which states that with fixed wage rates and the establishment of a
18 new set of Sraffian production prices, the rate of profit must rise. When wages and prices
19 are allowed to adjust dynamically after innovation and diffusion, the profit rate can rise,
20 fall, or stay the same, depending on the price- and wage-setting regime.

21 While we begin with Duménil and Lévy's (1995) model, we go significantly beyond it.
22 First, we relax their restrictive assumptions about technological discovery. Second, we
23 expand their model from two inputs to production – labor and capital – to an arbitrary
24 number. Third, we follow Kaldor (1961) by tying the rate of technological change to the
25 pace of capital accumulation. We illustrate the theory by applying it to a classically-

1 inspired model along lines proposed by Foley and Michl (1999),² showing that the
2 direction of technological change depends on firm pricing strategy: a fixed wage share
3 leads to Marx-biased technological change, whereas target-return pricing leads to
4 Kaldor's stylized fact of trendless capital productivity as an equilibrium position. We
5 generate scenarios with the model that exhibit changes in productivity and cost shares
6 similar to those experienced by the US in the latter half of the 20th century.

7 **2 Theories of induced technological change**

8 The existing body of theory on induced technological change offers competing
9 representations of technological potential and processes of wage and price determination.
10 The classical economists,³ writing at a time when European countries had large pools of
11 agricultural workers, developed theories with subsistence wages. They saw that labor
12 productivity could be improved both through embodied technological change, which
13 arises from the capabilities of new machines, and disembodied technological change,
14 which arises from learning-by-doing and procedural improvements. Technological
15 progress was reflected in increasing output for a given level of total labor, which was
16 taken to include both the contemporary labor directly used in producing goods and the
17 historical labor embodied in machines. For Ricardo, a rising money wage rate motivated
18 technological innovation, thereby driving down the cost of subsistence. Marx, observing
19 a growing array of machine-built goods, saw more clearly than Ricardo the importance of
20 cumulative technological change and the increasingly intertwined system of industrial
21 production. Famously, Marx concluded that an endogenous process drives a falling rate
22 of profit. Competition for profits drives a substitution of labor for machinery, but the gain
23 is short-lived as the innovation changes the demand for, and cost of, labor. Marx's

² The author is applying the theory in a more elaborate post-Keynesian Kaleckian-Harrodian model (Kemp-Benedict, 2017). That model exhibits Kondratieff-type long waves driven in part by cost-induced technological change and in part by wage and price dynamics that operate with lags.

³ This discussion draws mainly on Kurz's (2010) paper on the views of Smith, Ricardo and Marx on technological progress and income distribution.

1 general story of rational action at a microeconomic level being frustrated by
2 macroeconomic processes (Kurz, 2016, p. 44) finds an echo in the theory presented in
3 this paper.

4 The neoclassical theory of technological change was initiated by Hicks (1932), who
5 assumed, on marginalist grounds, that wages and prices are determined endogenously by
6 cost-minimizing firms given a production function. Technical progress was captured in
7 changes to the production function. Hicks' theory was criticized by both Keynesian and
8 neoclassical theorists. Kaldor (1961) argued that neoclassical theory could not account
9 for the stylized facts he had documented, and proposed a "technical progress function"
10 that relates capital accumulation and output growth in a Keynesian theory in which firms
11 have a minimum profit margin and money wages are exogenously specified. Salter (1966
12 [1960], pp. 43-44) argued that Hicks' entrepreneur, focused on total cost savings, should
13 be indifferent to whether those savings come from capital or labor costs. Kennedy⁴
14 (1964) responded to Salter by arguing that the entrepreneur should seek to minimize unit
15 costs within an "innovation-possibility frontier", which he acknowledged is a "disguised"
16 form of Kaldor's technical progress function. Given rather broad conditions for the shape
17 of the frontier, but arguing that it should not depend on the cost shares themselves,
18 Kennedy showed that his assumption implies technological change biased in the direction
19 of the factor with the highest cost share. Samuelson (1965), while disagreeing with much
20 of Kennedy's argument, nevertheless took it as a starting point for a thorough
21 development of induced technological change with a neoclassical production function,
22 something Kennedy had sought to avoid with the introduction of the innovation-
23 possibility frontier (Kennedy, 1966). This burst of activity faded because of internal

⁴ Kennedy credits an unpublished manuscript by Ahmad for inspiration, while Samuelson (1965) credits an unpublished manuscript by Weizäcker with proposing ideas similar to Kennedy's.

1 problems with the model (Acemoglu, 2002, p. 785; Brugger & Gehrke, 2016; Nordhaus,
2 1973).

3 Research within the neoclassical tradition resumed with the emergence of endogenous
4 growth theories (Romer, 1994). This literature follows convention by assuming a
5 production function, but abandons perfect competition. Instead, firms engage in
6 monopolistic competition, gaining monopoly rents for their inventions. Endogenous
7 growth theories thus change the price and wage-setting assumptions of conventional
8 neoclassical theory. Acemoglu (2002) built on this body of work to develop a theory of
9 “directed technological change” in which technological developments are guided by the
10 interaction of a “price effect”, under which relative prices drive substitution, and a
11 “market size effect”, under which increasing availability of a factor lowers its price and
12 raises demand for goods produced with that factor.

13 Nelson and Winter (1982) sharply criticized the neoclassical theory of technological
14 change, arguing that it is disconnected from discoveries about the microeconomic
15 processes of innovation. They proposed an evolutionary theory in which firms engage in
16 a search for new techniques, starting from their existing technology, knowledge and
17 practices; technological potential is represented by the possible and probable outcomes of
18 that search. Nelson and Winter offered a rather complex simulation model featuring
19 multiple interacting processes of innovation and imitation. Duménil and Lévy (1995)
20 took their ideas and built a formal model of cost-share induced technological change that
21 can be more easily incorporated into pen-and-paper models of growth and distribution.

22 The theories of Kennedy (1964) and Duménil and Lévy (1995) have been applied to
23 classical and post-Keynesian models. Julius (2005) argued that to some extent Duménil
24 and Lévy’s theory is formally equivalent to that of Kennedy and he applied their common
25 core to different distributional regimes. Among other exercises, he introduced
26 endogenous technological change into a Goodwin (1967) model, as did Shah and Desai

1 (1981). Both papers found that the Goodwin cycles disappeared when coupled with cost-
2 shared induced technological change. Zamparelli (2015) applied a modified form of
3 Kennedy's theory to a classical model, one in which the innovation-possibility frontier
4 can be altered through R&D expenditure. Julius (2005, p. 110) noted incidentally that
5 Duménil and Lévy's theory places few restrictions on the relationship between the cost
6 share and technological progress; one of the novel contributions of this paper is to
7 discover such restrictions.

8 Foley (2003, p. 42 ff.) approached biased technological change from a classical
9 perspective. He noted that labor is different from other inputs, and developed a theory
10 based on that proposition. In particular, he made the plausible suggestion that labor-
11 saving innovation is easier to generalize from one process to another, whereas capital-
12 saving innovation is more likely to differ from one process to another. Foley considered,
13 but dismissed, Duménil and Lévy's (1995) theory out of concern that their findings are
14 dictated by the particulars of their model. In this paper we do not address Foley's own
15 theory, which offers a plausible alternative, but propose a model along Duménil and
16 Lévy's lines that addresses his critique.

17 Post-Keynesian theorists typically endogenize technological change by assuming the
18 Kaldor-Verdoorn law, in which labor productivity growth increases with the pace of
19 capital accumulation. The Kaldor-Verdoorn law has undergone extensive empirical
20 testing, and appears to be robust (Lavoie, 2014, pp. 428–430). However, it applies solely
21 to labor productivity, while, following Kaldor, the capital-output ratio is held fixed. We
22 derive a generalization of the Kaldor-Verdoorn law by drawing a distinction between
23 embodied and disembodied technological change.

24 The model presented in this paper is more consistent with evolutionary, classical, or post-
25 Keynesian theory than it is with neoclassical theory. Unlike in neoclassical theory, we do
26 not assume a smooth production function, and firms need not know the production

1 possibilities frontier. Along evolutionary lines, firms make a local search relative to their
2 current technology, and implement discoveries that increase their return on capital in the
3 short run. We consider this to be a plausible conception of technological change. The
4 history of technology is one of incremental improvements on existing technologies, rather
5 than a series of leaps to close the gaps opened up by new advances (Basalla, 1988).
6 Consistent with our non-neoclassical orientation, we assume that prices are set by firms
7 engaged in oligopolistic competition (Coutts & Norman, 2013), while wages are
8 influenced by social processes. Thus, both wage and price setting involves choice and
9 conflict, which adds a degree of freedom – and realism – lacking in neoclassical models.

10 **3 The model of Duménil and Lévy**

11 We begin by presenting a version of Duménil and Lévy's (1995) evolutionary model,
12 which has one sector and two factors of production – labor and capital. Firms are
13 continually searching for innovations, which may or may not be biased towards saving on
14 one input or another. A bias will arise in any case because, as argued by Okishio (1961),
15 firms only adopt those innovations that increase their return on capital at fixed prices and
16 wages.

17 Duménil and Lévy's theory incorporates a version of Marxian unintended consequences.
18 Firms take the individually rational decision to adopt technologies that increase their rate
19 of profit at constant prices and wages. This gives them a temporary monopoly and the
20 excess profits that come with it. In the model, the advantage is erased by the following
21 period, thereby abstracting from the details of diffusion, imitation, and adaptation. Under
22 oligopolistic competition and guided by institutional norms for wage setting, firms then
23 adjust prices and wages to reflect their new cost structure. Thus, pricing and wage-setting
24 behaviors translate the microeconomic behavior proposed by Okishio into
25 macroeconomic outcomes. Duménil and Lévy assumed a fixed mark-up, but as shown by

1 Julius (2005), their theory is consistent with a variety of pricing and wage-setting
2 strategies.

3 Firms have a capital productivity κ and labor productivity λ . They have considerable
4 flexibility to set both wages and prices, employing labor at a wage w and setting a price
5 P . The profit rate r is then

$$6 \quad r = \kappa\pi = \kappa \left(1 - \frac{w}{P\lambda} \right), \quad (1)$$

7 where π is the profit share. If a firm makes a discovery that would, if implemented,
8 change productivities by amounts $\Delta\kappa$, $\Delta\lambda$, the firm then asks whether implementing it will
9 raise profitability in the short run while keeping prices fixed. Using a hat to denote a
10 growth rate, $\hat{x} = \Delta x/x$, we have

$$11 \quad \hat{r} = \hat{\kappa} + \frac{\omega}{\pi} \hat{\lambda}, \quad (2)$$

12 where $\omega = 1 - \pi$ is the wage share. Firms adopt an innovation if it raises the profit rate at
13 constant costs, giving the viability condition

$$14 \quad \pi\hat{\kappa} + \omega\hat{\lambda} > 0. \quad (3)$$

15 We illustrate this condition for the average viable technology in . In the figure we
16 suppose, with Duménil and Lévy (1995), that innovation is entirely neutral. Innovations
17 move productivities in a random direction, with a probability distribution that is
18 circularly symmetric and also symmetric around the $\hat{\kappa} = \hat{\lambda}$ line. With these assumptions,
19 the firm is just as likely to make a labor-saving or a capital-saving innovation. However,
20 viable technologies must lie on the positive side of the vector perpendicular to a line that
21 passes through the origin and the point (π, ω) ; this introduces a bias.

22 Two such lines are shown, one with $\pi = 0.3$ (so $\omega = 0.7$) and one with $\pi = 0.4$ (so $\omega =$
23 0.6). The average viable technology lies along a vector perpendicular to the lines, and in a
24 positive direction away from them, shown in the figure by the vectors labeled $(\hat{\kappa}_1, \hat{\lambda}_1)$ and

1 $(\hat{\kappa}_2, \hat{\lambda}_2)$. As shown, in both cases capital productivity is falling, because both $\hat{\kappa}_1$ and $\hat{\kappa}_2$
 2 are negative. As the profit share rises, the vector rotates clockwise, so when $\pi = 0.4$,
 3 capital productivity is falling more slowly, and labor productivity rising more slowly,
 4 than when $\pi = 0.3$.

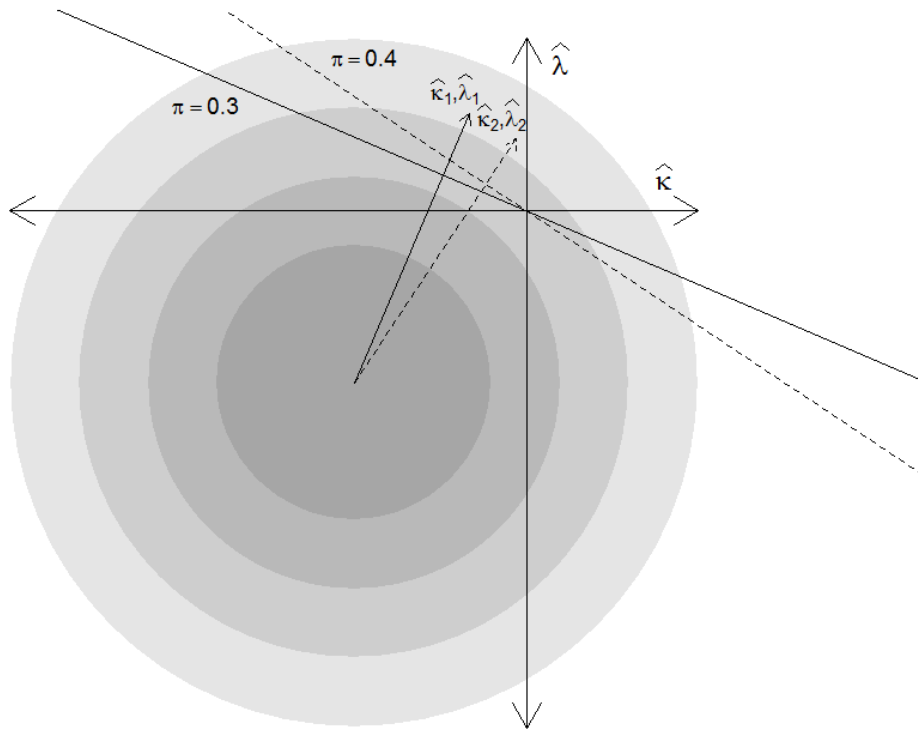


Figure 1: Biased technological change in the model of Duménil and Lévy

5 As successful discoveries are implemented through innovation and emulation, firms
 6 subsequently adjust their prices. They engage in explicit or implicit bargaining over
 7 wages and may reduce prices to expand their market or discourage entry by rival firms.
 8 Duménil and Lévy (1995, p. 237) argued that labor productivity growth depends on the
 9 wage share such that the system approaches an equilibrium with constant labor
 10 productivity growth and a fixed wage share. In the example shown in , this drives a
 11 steady improvement in labor productivity and a steadily falling rate of profit. However,
 12 the result depends on the probability distribution of technological innovation. As the
 13 origin of the probability distribution shifts, and as its boundary expands and contracts, it
 14 is reasonable to suppose that labor productivity growth continues to be biased in a

1 positive direction, but capital productivity can either rise or fall, as the average rate of
2 capital productivity growth becomes positive or negative. In this way Duménil and Lévy
3 (1995) explained why labor productivity could grow, while the capital-labor ratio
4 wanders across a broad range.

5 **4 A general model of biased technological change**

6 Duménil and Lévy's model is compelling, but it has some limitations. As noted by Foley
7 (2003), their model requires a specific form for the probability distribution of
8 technological discoveries. Furthermore, neither Marx-biased nor Harrod-neutral change is
9 guaranteed, because capital productivity growth is determined by exogenous changes in
10 the probability distribution of technological discovery, and can be positive, negative, or
11 zero. In this section we generalize their model and show how the capital productivity
12 growth rate can emerge endogenously.

13 First, we allow for an arbitrary number of inputs to production and replace the symmetric
14 probability distribution of technological potential shown in with an arbitrary probability
15 distribution. Next, we separate embodied and disembodied technological change and
16 show how the modified theory implies the Kaldor-Verdoorn law. Finally, we show, in
17 general terms, how cost share induced technological change, when combined with
18 specific price and wage-setting dynamics, sets up equilibrating forces with an equilibrium
19 characterized by constant productivity growth rates and cost shares.

20 *4.1 A multi-factor viability condition*

21 We continue to denote capital productivity by κ and the price level by P , but we now
22 allow for an arbitrary number of inputs, N , indexed by $i = 1, \dots, N$, with costs per unit input
23 q_i . Those inputs, including labor, are used in production with productivity v_i , so the profit
24 rate is

$$25 \quad r = \kappa \left(1 - \frac{1}{P} \sum_{i=1}^N \frac{q_i}{v_i} \right). \quad (4)$$

1 Suppose that a firm makes a discovery that would, if implemented, change productivities
 2 at rates \hat{v}_i . The change in the profit rate at constant prices is then

$$3 \quad \hat{r} = \hat{\kappa} + \left(1 - \frac{1}{P} \sum_{i=1}^N \frac{q_i}{v_i}\right)^{-1} \frac{1}{P} \sum_{i=1}^N \frac{q_i}{v_i} \hat{v}_i. \quad (5)$$

4 The expression in parentheses is the profit share,

$$5 \quad \pi = \left(1 - \frac{1}{P} \sum_{i=1}^N \frac{q_i}{v_i}\right), \quad (6)$$

6 while the cost share for each factor is

$$7 \quad \sigma_i = \frac{1}{P} \frac{q_i}{v_i}. \quad (7)$$

8 Using this notation, we can write equation (5) as

$$9 \quad \hat{r} = \hat{\kappa} + \frac{1}{\pi} \sum_{i=1}^N \sigma_i \hat{v}_i. \quad (8)$$

10 This equation, a generalization of equation (2), is the change in the profit rate that would
 11 obtain if a firm were to introduce the innovation. An innovation is viable if it increases
 12 the profit rate, and should be rejected if it does not. That is, a viable technology should
 13 satisfy $\hat{r} > 0$, or

$$14 \quad \pi \hat{\kappa} + \sum_{i=1}^N \sigma_i \hat{v}_i > 0. \quad (9)$$

15 To simplify the notation, we extend the set of factors to include capital, defining a cost
 16 share and productivity for index $i = 0$,

$$17 \quad \sigma_0 \equiv \pi, \quad v_0 \equiv \kappa. \quad (10)$$

18 We can then write equation (9) compactly as

$$19 \quad \boldsymbol{\sigma} \cdot \hat{\mathbf{v}} > 0. \quad (11)$$

20 This is the viability condition for new discoveries – when it holds, discoveries tend to be
 21 accepted, and when it fails, discoveries are certainly rejected.

1 In reality the decision will be less clear-cut. First, innovation is a messy process; in its
 2 transit from the R&D department's bench to the plant floor, a new invention or proposed
 3 innovation will undergo numerous changes that ultimately affect productivity. Second,
 4 while firms do track costs, it is not always straightforward to assign those costs to the
 5 specific product line or process where technological innovation takes place, so firms may
 6 not be confident in their ability to estimate the cost implications of a particular
 7 innovation. Third, a firm may decide to take a strategic loss in order to gain experience
 8 with a new technology or method. These reflections suggest that the crisp cutoff in
 9 expression (9) be replaced with a fuzzy boundary.

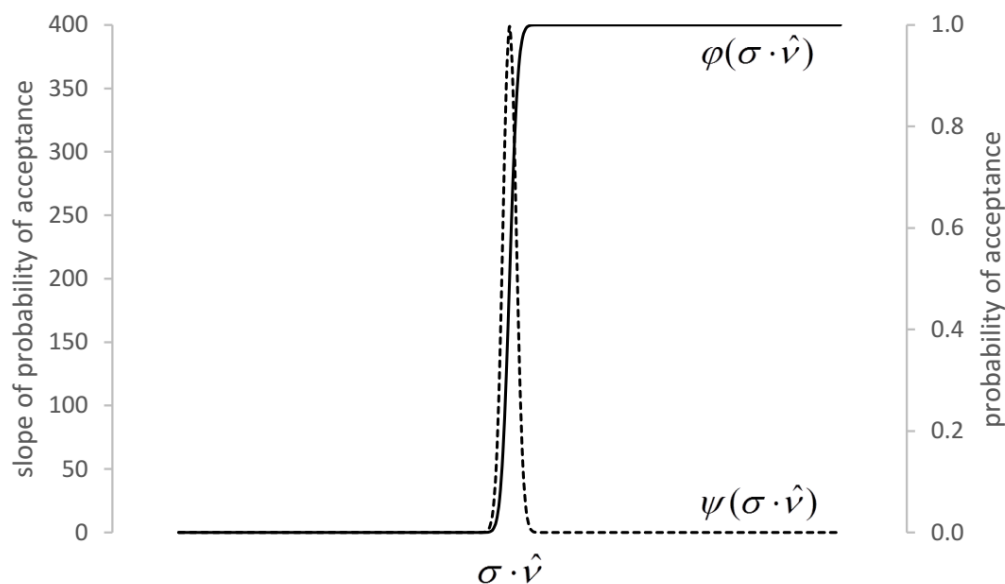


Figure 2: Probability that a firm will accept a candidate innovation

10 The probability of acceptance $\varphi(\sigma \cdot \hat{v})$ of a candidate innovation is shown schematically
 11 in Figure 2. It drops quickly to zero when the estimated change in profit rate is negative
 12 and rises quickly to one when the estimated change is positive. The slope of the curve
 13 $\psi(\sigma \cdot \hat{v})$ is very high near $\sigma \cdot \hat{v} = 0$, dropping rapidly towards zero on either side. In the
 14 ideal case, in which the viability condition in equation (11) holds exactly, $\varphi(\sigma \cdot \hat{v})$

1 becomes the Heaviside (or step) function, while $\psi(\boldsymbol{\sigma} \cdot \hat{\mathbf{v}})$ becomes the Dirac delta
 2 function.

3 *4.2 Induced bias in a random innovation process*

4 We follow Hicks (1932, p. 125), Nelson and Winter (1982), and Duménil and Lévy
 5 (1995) in supposing that innovation is a random process. Specifically, we label
 6 innovations by their effect on productivities, and assume that any individual innovation is
 7 drawn from a probability distribution $f(\hat{\mathbf{v}})$. The average productivity improvement, $\langle \hat{\mathbf{v}} \rangle$,
 8 is then given by integrating over all possible values of $\hat{\mathbf{v}}$ that satisfy the viability
 9 condition (11). We enforce the viability condition in a probabilistic form using the
 10 probability of acceptance function $\varphi(\boldsymbol{\sigma} \cdot \hat{\mathbf{v}})$ shown in Figure 2, which is equal to one
 11 when its argument becomes slightly positive and zero when its argument becomes
 12 slightly negative,

$$13 \quad \langle \hat{\mathbf{v}} \rangle = \int d\hat{\mathbf{v}} \varphi(\boldsymbol{\sigma} \cdot \hat{\mathbf{v}}) \hat{\mathbf{v}} f(\hat{\mathbf{v}}). \quad (12)$$

14 The cost share $\boldsymbol{\sigma}$ only appears within the acceptance probability function in equation (12),
 15 so we can take the derivative of the i th element of $\langle \hat{\mathbf{v}} \rangle$, $\langle \hat{v}_i \rangle$, with respect to σ_j to find

$$16 \quad \frac{\partial \langle \hat{v}_i \rangle}{\partial \sigma_j} = \int d\hat{\mathbf{v}} \psi(\boldsymbol{\sigma} \cdot \hat{\mathbf{v}}) \hat{v}_j \hat{v}_i f(\hat{\mathbf{v}}). \quad (13)$$

17 In this expression we have introduced the slope (or derivative) of the acceptance
 18 probability function, $\psi(\boldsymbol{\sigma} \cdot \hat{\mathbf{v}})$. As shown in Figure 2, the acceptance probability has zero
 19 slope everywhere except very close to zero, where its slope becomes extremely large.

20 Equation (13) is an essential result. It holds for any innovation probability distribution
 21 $f(\hat{\mathbf{v}})$, so it is a more general and flexible expression than that found by Duménil and
 22 Lévy (1995), who assumed a specific form for the probability distribution. By allowing
 23 for an arbitrary innovation probability distribution, we address Foley's (2003) objection
 24 that Duménil and Lévy's result might depend crucially on their assumption of a
 25 symmetric distribution.

1 We can express the right-hand side of equation (13) as the entries M_{ij} in a matrix \mathbf{M} (the
 2 Jacobian matrix),

$$3 \quad M_{ij} = \frac{\partial \langle \hat{v}_i \rangle}{\partial \sigma_j} = \int d\hat{\mathbf{v}} \psi(\boldsymbol{\sigma} \cdot \hat{\mathbf{v}}) \hat{v}_j \hat{v}_i f(\hat{\mathbf{v}}). \quad (14)$$

4 As shown in the mathematical appendix, \mathbf{M} is, to a good approximation, an
 5 $(N + 1) \times (N + 1)$ dimensional positive semi-definite matrix of rank N that satisfies⁶

$$6 \quad \boldsymbol{\sigma} \cdot \mathbf{M} = \mathbf{M} \cdot \boldsymbol{\sigma} \approx 0. \quad (15)$$

7 The properties of the \mathbf{M} matrix characterize acceptable models of cost-share induced
 8 technological change consistent with the assumptions that led to equation (12): aggregate
 9 productivity change is an average over the randomly-distributed efforts of many firms,
 10 with each firm accepting only those innovations that increase its profit rate at prevailing
 11 prices and wages. From these properties, we get the immediate result that the own-
 12 response of the productivity growth rate to a change in the cost share is positive,

$$13 \quad \frac{\partial \langle \hat{v}_i \rangle}{\partial \sigma_i} = M_{ii} > 0. \quad (16)$$

14 By itself, this is not a statement of cost share induced technological change, which
 15 obtains when the *total* change in the productivity growth rate depends positively on the
 16 change in the cost share,

$$17 \quad \frac{\Delta \langle \hat{v}_i \rangle}{\Delta \sigma_i} > 0. \quad (17)$$

18 As explained in the mathematical appendix, this inequality can be violated for
 19 complementary inputs – for example, capital and energy.⁷ However, the resulting
 20 dynamics are unstable, and so unlikely to persist. We expect (17) to hold except for
 21 possible transient conditions for complementary factors.

⁶ This relationship holds if the cost shares are treated as independent of each other. In fact, they must sum to one, but if that condition is imposed at this stage, then constraints on the resulting Jacobian are not as easy to state. In applying the theory it is best to impose the condition at a later stage in the analysis.

⁷ The author treats this case in a separate paper (Kemp-Benedict, 2018).

1 It is convenient, when building explicit models, to assume that the approximate equality
 2 in equation (15) is exactly true. With two factors, capital and wages, and associated profit
 3 and wage shares π and ω , it becomes

$$4 \quad \begin{pmatrix} M_{00} & M_{01} \\ M_{10} & M_{11} \end{pmatrix} \begin{pmatrix} \pi \\ \omega \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \quad (18)$$

5 Because \mathbf{M} is symmetric, $M_{01} = M_{10}$, and we find

$$6 \quad M_{01} = M_{10} = -\frac{\pi}{\omega} M_{00} = -\frac{\omega}{\pi} M_{11}. \quad (19)$$

7 All entries can be expressed in terms of a single entry, for example M_{00} , so conditions on
 8 \mathbf{M} restrict the relationship between cost shares and productivity growth rates. This
 9 addresses the critique of Julius (2005) that Duménil and Lévy's theory placed few
 10 constraints on the functional form of the cost share-productivity relationship.

11 *4.3 Capital accumulation and the Kaldor-Verdoorn law*

12 Up to now, we have treated technological innovation as though it were independent of
 13 capital accumulation. For disembodied innovation this is the case: changing operating
 14 procedures, retrofitting, and learning-by-doing can lead to higher capital and labor
 15 productivity. However, the resource-saving qualities of some innovations are embodied
 16 in new capital. In this section we extend the theory to take capital accumulation into
 17 account.

18 We start with a generic input X that is associated with a given capital stock, K . For
 19 example, X could be the amount of labor, energy or materials required to operate a given
 20 stock of capital. We use a lower-case x to denote the average amount of X per unit of
 21 capital stock,

$$22 \quad x = \frac{X}{K}. \quad (20)$$

1 New capital has a marginal X -to-capital ratio x_m . If capital of all vintages depreciates at a
 2 uniform rate δ (the perpetual inventory model), the time rate of change of X (indicated by
 3 a dot) is given by

$$4 \quad \dot{X} = -x\delta K + x_m I. \quad (21)$$

5 Dividing both sides by $X = xK$, the growth rate of x is found to be

$$6 \quad \hat{x} = -\hat{K} - \delta + \frac{x_m}{x} g_K, \quad (22)$$

7 where $g_K = I/K$ is the gross investment rate. In a perpetual inventory model, the growth
 8 rate of the capital stock is

$$9 \quad \hat{K} = -\delta + g_K. \quad (23)$$

10 Substituting this expression into equation (22) gives

$$11 \quad \hat{x} = g_K \left(\frac{x_m}{x} - 1 \right). \quad (24)$$

12 The quantity in parentheses on the right-hand side of this equation is the relative
 13 difference between the marginal and average values of the X -to-capital ratio, which we
 14 denote by r_x . The rate of change of the output-to- X ratio – the X -productivity v_x – is then
 15 given by

$$16 \quad \hat{v}_x = \hat{\kappa} - \hat{x} = \hat{\kappa} - g_K r_x. \quad (25)$$

17 Using the same expressions, but setting X equal to total output, Y , equation (24) gives an
 18 expression for capital productivity growth,

$$19 \quad \hat{\kappa} = g_K \left(\frac{\kappa_m}{\kappa} - 1 \right) = g_K r_Y. \quad (26)$$

20 Substituting into equation (25) then gives

$$21 \quad \hat{v}_x = g_K (r_Y - r_x). \quad (27)$$

22 This is the expression for X -productivity growth due to embodied technological change.

1 Regardless of the source of productivity change, whether embodied or disembodied, the
 2 choice of technology is governed by the viability condition (9). In the case of embodied
 3 technological change, the equations above allow us to write the viability condition as

$$4 \quad \pi r_Y + \sum_{i=1}^N \sigma_i (r_Y - r_i) > 0. \quad (28)$$

5 By defining a vector with elements

$$6 \quad z_0 = r_Y, z_i = r_Y - r_i, \quad (29)$$

7 we find an expression equivalent to equation (11),

$$8 \quad \boldsymbol{\sigma} \cdot \mathbf{z} > 0. \quad (30)$$

9 Proceeding by analogy with the derivation for disembodied technical change, we have

$$10 \quad \langle \hat{\nu} \rangle = g_K \int d\mathbf{z} \varphi_z(\boldsymbol{\sigma} \cdot \mathbf{z}) \mathbf{z} f_x(\mathbf{z}), \quad (31)$$

11 where φ_z and f_z are the equivalents, for the \mathbf{z} variables, to the functions φ and f . This
 12 expression is of precisely the same form as equation (12), and it has similar properties.

13 Firms may discover, in any time step, a number of possible innovations. Some will
 14 involve embodied technological change and others disembodied technological change.
 15 The first type are governed by equation (31), which we summarize by g_K multiplied by a
 16 function $\Psi_i(\boldsymbol{\sigma})$. The second are characterized by equation (12), which we summarize by a
 17 function $\Phi_i(\boldsymbol{\sigma})$. Combining the two sources of change, the average rate of productivity
 18 change is given by

$$19 \quad \langle \hat{\nu}_i \rangle = g_K \Psi_i(\boldsymbol{\sigma}) + \Phi_i(\boldsymbol{\sigma}). \quad (32)$$

20 This is a generalization of the Kaldor-Verdoorn law. In the Kaldor-Verdoorn law, $\Psi_i(\boldsymbol{\sigma})$
 21 and $\Phi_i(\boldsymbol{\sigma})$ are constants, but here they are functions of the cost shares. The matrices of
 22 their partial derivatives,

$$23 \quad M_{ij} \equiv \frac{\partial \Psi_i}{\partial \sigma_j}, \quad N_{ij} \equiv \frac{\partial \Phi_i}{\partial \sigma_j}, \quad (33)$$

1 are positive $(N + 1) \times (N + 1)$ dimensional matrices that are approximately semi-definite.
 2 The gross investment rate g_K may also depend on cost shares, so, through equation (32),
 3 productivity growth rates can have a complex dependence on the cost share.

4 Separating embodied and disembodied technological change thus allows for richer
 5 dynamics, and the theory developed in this paper offers a generalized form of the Kaldor-
 6 Verdoorn law. However, in order to simplify the examples, we do not explore it further in
 7 this paper.

8 *4.4 An equilibrating dynamic*

9 The theory of cost share induced technological change present above suggests that aside
 10 from a transient condition for some complementary factors, we expect the following to
 11 hold:

$$12 \quad \frac{\Delta \langle \hat{v}_i \rangle}{\Delta \sigma_i} > 0. \quad (34)$$

13 Thus, a rise (fall) in the cost share of a factor stimulates faster (slower) productivity
 14 growth in that factor. We have provided a justification for the assumption using a general
 15 probabilistic model of innovation, along with constraints on the relationship between cost
 16 shares and productivity growth rates.

17 After innovation and diffusion, all firms respond to productivity changes by setting prices
 18 (following their pricing policies) and wages (through tacit or explicit negotiation with
 19 their employees). From the expression for the cost shares in equation (7), the subsequent
 20 growth in cost shares is, using the average productivity growth rate,

$$21 \quad \hat{\sigma} = (\hat{\mathbf{q}} - \hat{P}) - \langle \hat{\mathbf{v}} \rangle. \quad (35)$$

22 The term in parentheses is a vector of growth rates of real unit costs. These, in turn, can
 23 be expected to respond to changes in productivity. From equation (34), an increase in the
 24 cost share associated with input i leads to a rise in its productivity growth rate.

25 Subsequently, from equation (35), the cost share is driven downward, so this is an

1 equilibrating process. The equilibrium is reached when cost shares and productivity
2 growth rates are not changing: $\Delta \hat{\mathbf{v}} = \Delta \boldsymbol{\sigma} = 0$.

3 An equilibrium, once established, can be disturbed by changes in pricing strategy – e.g.,
4 from a fixed mark-up to target-return pricing – or by changes in technological potential as
5 captured in the probability distribution $f(\hat{\mathbf{v}})$. We explore this possibility in the next
6 section.

7 **5 Applying the theory to a classically-inspired growth model**

8 In this section we apply the theory elaborated above to a simple classically-inspired
9 growth model with two inputs to production – labor and capital. In constructing the
10 model we largely follow Foley and Michl (1999).

11 Different model closures lead to quite different outcomes. We introduce two closures,
12 which we express in terms of the gross profit rate r . Because it is a product, $r = \pi\kappa$, the
13 growth rate of the gross profit rate is the sum of the growth rates of the profit share and
14 capital productivity,

$$15 \hat{r} = \hat{\pi} + \hat{\kappa}. \tag{36}$$

16 We first close the model by fixing the profit and wage shares, which corresponds to Foley
17 and Michl's (1999, p. 118) model of Marx-biased technological change and to the
18 standard post-Keynesian assumption of mark-up pricing. In this case,

$$19 \text{Fixed profit share: } \hat{r} = \hat{\kappa}. \tag{37}$$

20 A fixed profit (or wage) share is a realistic assumption when labor has a strong
21 bargaining position and is able to defend a constant real wage. When labor is not strong,
22 firms are freer to set prices so as to target a desired rate of return. While target-return
23 pricing was not considered by Foley and Michl, it is a recognized pricing procedure (Lee,
24 1999) used particularly by large firms with sufficiently sophisticated accounting systems

1 (Lavoie, 1995, p. 793, 2016, p. 174). Assuming a steady depreciation rate, target-return
 2 pricing implies a fixed profit rate. In that case,

$$3 \quad \text{Fixed profit rate: } \hat{\pi} = -\hat{\kappa}. \quad (38)$$

4 The conditions expressed by equations (37) and (38) distinguish different pricing regimes
 5 that depend on firms' accounting practices and the ability of workers to influence their
 6 wages.

7 *5.1 Capital and labor productivity*

8 For purposes of presentation it is helpful to have a specific model for capital and labor
 9 productivity. We construct one by setting $M_{10} = M_{01} = -a/\pi$ in equation (19).⁹ That
 10 determines the other two coefficients, which allows us to write

$$11 \quad \frac{\partial \hat{\kappa}}{\partial \pi} = M_{00} = a \frac{\omega}{\pi^2}, \quad \frac{\partial \hat{\kappa}}{\partial \omega} = M_{01} = -a \frac{1}{\pi}, \quad (39)$$

$$12 \quad \frac{\partial \hat{\lambda}}{\partial \omega} = M_{11} = a \frac{1}{\omega}, \quad \frac{\partial \hat{\lambda}}{\partial \pi} = M_{10} = -a \frac{1}{\pi}. \quad (40)$$

13 These partial derivatives correspond to the following two functions,

$$14 \quad \hat{\kappa} = k - a \frac{\omega}{\pi}, \quad (41)$$

$$15 \quad \hat{\lambda} = \ell + a \ln \frac{\omega}{\pi}. \quad (42)$$

16 The expression for capital productivity growth will prove to be particularly convenient
 17 when considering target-return pricing.

18 *5.2 Marx-biased change with a fixed profit share*

19 If the profit share (and therefore the wage share) is fixed at a value $\bar{\pi}$, then capital and
 20 labor productivity growth rates are given by

⁹ The condition $\sigma \cdot \mathbf{M} \approx 0$ implies that $\hat{\kappa}$ and $\hat{\lambda}$ are (approximately) homogeneous functions of σ of order zero. Their first derivatives with respect to the profit share are then homogeneous of order negative one. So, \mathbf{M} could scale like π^{-1} , ω^{-1} , $(\pi^2 + \omega^2)^{-1/2}$, or any other function of order negative one in the cost shares. This particular choice leads to a convenient expression for the capital productivity growth rate.

1
$$\hat{\kappa} = k - a \frac{1 - \bar{\pi}}{\bar{\pi}}, \quad (43)$$

2
$$\hat{\lambda} = \ell + a \ln \frac{1 - \bar{\pi}}{\bar{\pi}}. \quad (44)$$

3 These expressions will not change unless the parameters k , ℓ , or a change. Suppose that,
4 initially,

5
$$\frac{k}{a} = \frac{1 - \bar{\pi}}{\bar{\pi}} \Rightarrow \bar{\pi} = \frac{a}{a + k}. \quad (45)$$

6 With this assumption, capital productivity is constant (by design), while labor
7 productivity is growing at a rate

8
$$\hat{\lambda} = \ell + a \ln \frac{k}{a}. \quad (46)$$

9 This situation might have characterized high-income countries from the 1960s through
10 the early 1970s, with a stable wage share and steady profit rates. In the early 1970s that
11 pattern changed.

12 Gordon (1999, 2012, 2016) has suggested that the pattern changed because the potential
13 for innovation was declining at the end of a wave of technological progress that started in
14 the late 19th century¹⁰. In the theory presented in this paper, Gordon's suggestion
15 translates into a contraction of the probability distribution $f(\hat{\nu})$ of new discoveries. We
16 can implement Gordon's hypothesis by proposing that k changes to a new value $k' < k$.
17 After the change, capital productivity growth becomes

18
$$\hat{\kappa} \rightarrow k' - a \frac{1 - \bar{\pi}}{\bar{\pi}} = a(k' - k) < 0. \quad (47)$$

¹⁰ We do not advocate either for or against Gordon's thesis. The validity of the cost-share induced theory of technological change does not depend on it. Oil crises, war, social conflict, the decline of industrial production, and gradually weakening unions could all have plausibly influenced cost shares, wage determination, and pricing strategies. For the purposes of this paper, the virtue of Gordon's mechanism is that it can be implemented in a straightforward manner by changing one or two model parameters.

1 Thus, under mark-up pricing, after the probability distribution of new discoveries
 2 contracts, capital productivity begins to shrink, and continues to shrink indefinitely.
 3 Labor productivity continues to grow at the same pace.

4 The combination of a constant profit share and falling capital productivity corresponds to
 5 the conditions in high-income countries that prevailed through the 1970s to the early
 6 1980s. From equation (37), with a fixed profit share, if the capital productivity is falling
 7 then the profit rate is also falling. We therefore find Marx-biased technological change as
 8 a special – but important – case of a fixed profit share.

9 *5.3 Harrod-neutral technological change under target-return pricing*

10 During the 1980s, the position of labor was considerably weakened, effectively vanishing
 11 by the 1990s, a few years into what Goldstein (2012) has called the “shareholder value
 12 era”. Whether labor’s decline was driven by falling profitability or other causes is a moot
 13 point, but the increasing strength of firms and their investors vis-à-vis labor allowed firms
 14 to set prices so as to achieve desired returns.

15 Under target-return pricing, firms adjust their profit margins to maintain a fixed value for
 16 the gross profit rate, r . From equation (38), this implies that the growth rate of the profit
 17 share is the negative of the growth rate of capital productivity. The time rate of change of
 18 the profit share, indicated by a dot, is then given by

$$19 \quad \dot{\pi} = -\pi\hat{k}. \quad (48)$$

20 Substituting from equation (41), we can write this as

$$21 \quad \dot{\pi} = -\pi \left(k - a \frac{1-\pi}{\pi} \right) = -(a+k)\pi + a. \quad (49)$$

22 The solution is a trajectory that asymptotically approaches a new equilibrium,

$$23 \quad \pi = e^{-(a+k)t} \pi_0 + \frac{a}{a+k} (1 - e^{-(a+k)t}), \quad (50)$$

1 where π_0 is the value for the profit share at time $t = 0$. Suppose that initially the profit
 2 share is $\pi_0 = \bar{\pi}$, where $\bar{\pi}$ satisfies equation (45). Then the exponential terms in equation
 3 (50) cancel and we find (as we should) that the profit share is constant,

$$4 \quad \pi = \frac{a}{a+k} = \bar{\pi}. \quad (51)$$

5 If k then changes to $k' < k$, as we assumed above, while $\pi_0 = \bar{\pi}$, then we find

$$6 \quad \pi = \frac{a}{a+k'} - a \frac{k-k'}{(a+k)(a+k')} e^{-(a+k')t}. \quad (52)$$

7 Over time, the profit share approaches its new steady-state value of $a/(a+k')$. Because
 8 k' is less than k , the new profit share is higher than the original. Thus, under target-return
 9 pricing we expect a constant profit rate but a rising profit share, until it has reached its
 10 new steady-state value. During that time, capital productivity continues to fall. From
 11 equation (42), labor productivity growth slows as the profit share increases at the expense
 12 of the wage share.

13 The combination of steady profit rate, falling capital productivity, slowing labor
 14 productivity growth, and rising profit share, characterized the 1990s in high-income
 15 countries. The 1980s were a transitional period.

16 *5.4 Growth and distribution*

17 We now consider the two cases described above – fixed profit share and fixed profit
 18 rate – in a simple classically-inspired growth model, by adding capital accumulation.
 19 Following Foley and Michl (1999, p. 98) capitalists' net income is the net profit rate (the
 20 gross profit rate, r , less the depreciation rate δ) multiplied by the previous-period capital
 21 stock, K_{-1} . That income is then divided between net investment (the difference between
 22 current and previous-period capital stocks) and capitalists' consumption C_c ,

$$23 \quad C_c + (K - K_{-1}) = (r - \delta) K_{-1}. \quad (53)$$

24 If the capitalists' marginal rate of saving out of end-of-period wealth is β , then

1
$$C_c = (1 - \beta)(1 + r - \delta)K_{-1}. \quad (54)$$

2 Substituting into equation (53) gives a growth rate of the capital stock g_K that depends on
3 the rate of profit,

4
$$g_K \equiv \frac{K}{K_{-1}} - 1 = \beta(1 + r - \delta) - 1. \quad (55)$$

5 Output, Y , is given by the product of capital productivity and the capital stock,

6
$$Y = \kappa K, \quad (56)$$

7 while the real wage per worker depends on both the profit share and labor productivity,

8
$$w = (1 - \pi)\lambda. \quad (57)$$

9 Employment, L , is the ratio of GDP to labor productivity,

10
$$L = \frac{Y}{\lambda}. \quad (58)$$

11 In Figure 3 we show indices for κ , r , π , w , Y , and L , as generated by this model. We
12 assume firms to set a target profit rate starting in year 20. The profit rate ramps up to its
13 final value, which is achieved in year 30.

14 To generate the curves in Figure 3, we set a to 1.75%/year and k to 3.25%/year,
15 corresponding to a profit share of 35%. Labor productivity growth is determined by
16 setting ℓ to 0.40%/year. Capital productivity, κ , starts at 0.3/year. In the first decade,
17 steady capital accumulation and a constant capital productivity lead to steadily rising
18 GDP. The real wage rises at the same as the growth rate of labor productivity, so the
19 profit share remains at 35%. Employment expands modestly because GDP is growing
20 faster than labor productivity.

21 In year 10, we assume that k falls to 80% of its initial value, causing capital productivity
22 to begin to fall, while ℓ declines to -0.04%/year, causing labor productivity growth to
23 slow. As the wage continues to grow at historical rates, the profit share falls, and as the
24 GDP growth rate falls below the labor productivity growth rate, employment begins to

1 fall as well. Falling capital productivity and declining profit share combine to drive the
 2 profit rate down.

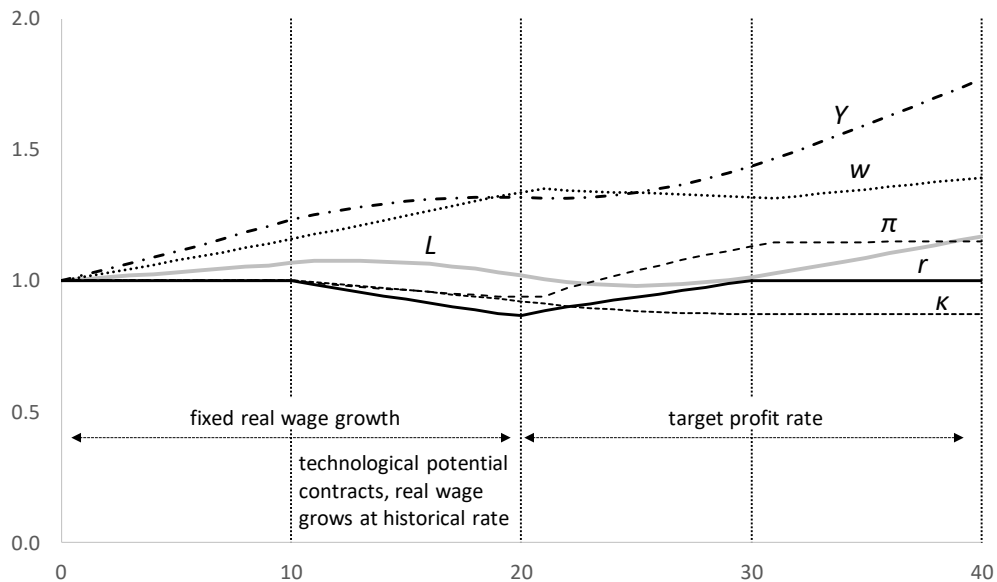


Figure 3: Growth and distribution in a classically-inspired model

3 In year 20, firms and investors react to falling profitability by targeting returns. The target
 4 level rises between years 20 and 30, eventually returning to its initial level. Capital
 5 productivity continues to fall, stabilizing around year 30, while the profit share rises.
 6 GDP growth outstrips labor productivity and wage growth, leading to rising employment
 7 with stagnant wages.

8 The event that drives the scenario is a contraction in the technological frontier starting in
 9 year 10. This corresponds to Gordon's (1999, 2012, 2016) proposed mechanism. The
 10 divergent trajectory after that event in Figure 3 follows from changes in price- and wage-
 11 setting behaviors. Initially, firm wage- and price-setting strategies are aligned, in which a
 12 steady increase in the wage rate is consistent with a fixed mark-up. However, the
 13 alignment depends on a steady labor and capital productivity growth rates. Once they
 14 begin to slow in year 10, the contradictions become apparent. Firms shift to target-return

1 pricing. The result is rising employment with a sharply rising profit share. This is similar
2 in broad outline to trends in the US from the 1960s through the 1990s.

3 **6 Discussion**

4 The classical preoccupation with the link between growth and distribution has returned to
5 the research agenda. Hicks' (1932) theory of cost share induced technological change has
6 proven both fruitful and frustrating in this effort. Above, we built on a promising but
7 incomplete theory of cost share induced technological change proposed by Duménil and
8 Lévy (1995), who sought to construct an evolutionary theory along the lines laid down by
9 Nelson and Winter (1982). This paper generalized Duménil and Lévy's theory in
10 important ways. We allowed for an arbitrary probability distribution of productivity-
11 enhancing innovations and expanded from two to an arbitrary number of inputs to
12 production. The generalized theory suggests that aside from an edge case involving
13 complementary inputs, cost shares are related positively to the associated productivity
14 growth rate. That is, the theory implies cost share induced technological change. When
15 embodied and disembodied technological change are disentangled, it implies a
16 generalized form of the Kaldor-Verdoorn law.

17 The expressions that relate cost shares to productivity growth rates are not arbitrary. To a
18 good approximation, the Jacobian matrix \mathbf{M} is symmetric and positive semi-definite. It is
19 also of order negative one in the cost shares. This finding addresses Julius' (2005)
20 concern that Duménil and Lévy's theory did not constrain the relationships. The
21 constraints are sufficient to ensure cost share induced technological change, with the
22 possible exception of complementary factors. For example, if energy is cheap then
23 energy-intensive capital can readily substitute for labor without regard to the energy costs
24 of operating the capital. However, this exceptional case cannot persist because it
25 generates an unstable dynamic that pushes the cost share of one of the factors upward

1 without limit. At some point the cost will be high enough that we expect it to spur a shift
2 to a different technological regime; for example, toward more energy-efficient capital.

3 The constraints on the matrix \mathbf{M} can potentially be tested empirically, but tests using
4 macroeconomic data will be difficult. The reason is that \mathbf{M} characterizes the change
5 within a firm, before the technology spreads, triggering changes in prices and wages.
6 Macroeconomic data reflect the full process of firm-level innovation followed by
7 diffusion and wage and price adjustment. Tests of the constraints on \mathbf{M} must be carried
8 out using microeconomic data at the level of the appropriate decision unit.

9 Alternatively, the underlying assumptions of the model can also be explored using
10 surveys. If firms evaluate potential innovations in a manner consistent with Okishio's
11 (1961) viability criterion, then it increases confidence in Duménil and Lévy's theory and
12 models derived from it, including the one in this paper. Surveys might also ask about
13 R&D processes. We have assumed that the probability of improving productivities with a
14 particular innovation is independent of the cost share, at least in the short run. But if
15 R&D research programs adapt rapidly to a changing cost structure, then that assumption
16 may be violated. We further assumed that innovations made by any firm can be rapidly
17 taken up by other firms. However, diffusion takes time, while, as argued by Foley (2003),
18 labor-saving innovations may diffuse more readily than capital-saving innovations. An
19 extended theory could take such additional biases and time lags into account.

20 **7 Conclusion**

21 Kaldor's stylized facts faced an almost immediate challenge, as the patterns he observed
22 were violated soon after his paper was published. This drove considerable work on
23 technological change, resurrecting the classical research question of the links between
24 growth and distribution. Building on the work of Duménil and Lévy (1995), in this paper
25 we developed a quite general model of cost-share induced technological change. The
26 theory makes specific predictions that can potentially be tested empirically, although the

1 tests are likely to be challenging. When combined with a particular price- and wage-
2 setting regime, the model generates different distributional and growth outcomes. We
3 illustrate this point with a classically-inspired growth model.

4 **8 References**

- 5 Acemoglu, D. (2002). Directed technical change. *The Review of Economic Studies*, 69(4), 781–
6 809. <https://doi.org/10.1111/1467-937X.00226>
- 7 Basalla, G. (1988). *The Evolution of Technology*. Cambridge University Press.
- 8 Bowles, S. (1981). Technical change and the profit rate: a simple proof of the Okishio theorem.
9 *Cambridge Journal of Economics*, 5(2), 183–186.
- 10 Brugger, F., & Gehrke, C. (2016). The neoclassical approach to induced technical change: From
11 Hicks to Acemoglu. *Metroeconomica*. <https://doi.org/10.1111/meca.12141>
- 12 Coutts, K., & Norman, N. (2013). Post-Keynesian approaches to industrial pricing. In G. C.
13 Harcourt & P. Kriesler (Eds.), *The Oxford Handbook of Post-Keynesian Economics*,
14 *Volume 1: Theory and Origins*.
- 15 Duménil, G., & Lévy, D. (1995). A stochastic model of technical change: An application to the
16 US economy (1869–1989). *Metroeconomica*, 46(3), 213–245.
17 <https://doi.org/10.1111/j.1467-999X.1995.tb00380.x>
- 18 Foley, D. K. (2003). *Unholy Trinity: Labor, Capital, and Land in the New Economy*. London;
19 New York: Routledge.
- 20 Foley, D. K., & Michl, T. R. (1999). *Growth and distribution*. Cambridge, Mass: Harvard
21 University Press.
- 22 Goldstein, A. (2012). Revenge of the managers: Labor cost-cutting and the paradoxical
23 resurgence of managerialism in the shareholder value era, 1984 to 2001. *American*
24 *Sociological Review*, 77(2), 268–294. <https://doi.org/10.1177/0003122412440093>

- 1 Goodwin, R. M. (1967). A growth cycle. In C. H. Feinstein (Ed.), *Socialism, Capitalism, and*
2 *Economic Growth: Essays Presented to Maurice Dobb* (pp. 54–58). Cambridge, UK:
3 Cambridge University Press.
- 4 Gordon, R. J. (1999). U.S. Economic Growth since 1870: One Big Wave? *The American*
5 *Economic Review*, 89(2), 123–128. <https://doi.org/10.2307/117092>
- 6 Gordon, R. J. (2012). *Is U.S. Economic Growth Over? Faltering Innovation Confronts the Six*
7 *Headwinds* (Working Paper No. 18315). National Bureau of Economic Research.
8 Retrieved from <http://www.nber.org/papers/w18315>
- 9 Gordon, R. J. (2016). *The Rise and Fall of American Growth: The U.S. Standard of Living since*
10 *the Civil War*. Princeton: Princeton University Press.
- 11 Hicks, J. R. (1932). *The Theory of Wages*. London: MacMillan and Company Limited.
- 12 Julius, A. J. (2005). Steady-state growth and distribution with an endogenous direction of
13 technical change. *Metroeconomica*, 56(1), 101–125. [https://doi.org/10.1111/j.1467-](https://doi.org/10.1111/j.1467-999X.2005.00209.x)
14 [999X.2005.00209.x](https://doi.org/10.1111/j.1467-999X.2005.00209.x)
- 15 Kaldor, N. (1961). Capital accumulation and economic growth. In F. A. Lutz & D. Hague (Eds.),
16 *The Theory of Capital* (pp. 177–222). Macmillan and Co. Ltd.
- 17 Kemp-Benedict, E. (2017). *A multi-sector Kaleckian-Harrodian model for long-run analysis*
18 (Working Paper No. 1702). Leeds, UK: Post Keynesian Economics Study Group.
19 Retrieved from <https://www.postkeynesian.net/working-papers/1702/>
- 20 Kemp-Benedict, E. (2018). Dematerialization, decoupling, and productivity change. *Ecological*
21 *Economics*, 150, 204–216. <https://doi.org/10.1016/j.ecolecon.2018.04.020>
- 22 Kennedy, C. (1964). Induced bias in innovation and the theory of distribution. *The Economic*
23 *Journal*, 74(295), 541–547. <https://doi.org/10.2307/2228295>
- 24 Kennedy, C. (1966). Samuelson on induced innovation. *The Review of Economics and Statistics*,
25 48(4), 442–444. <https://doi.org/10.2307/1924623>
- 26 Kurz, H. D. (2010). Technical progress, capital accumulation and income distribution in Classical
27 economics: Adam Smith, David Ricardo and Karl Marx. *The European Journal of the*

- 1 *History of Economic Thought*, 17(5), 1183–1222.
2 <https://doi.org/10.1080/09672567.2010.522242>
- 3 Kurz, H. D. (2016). *Economic Thought: A Brief History*. (J. Riemer, Trans.). Columbia
4 University Press.
- 5 Lavoie, M. (1995). The Kaleckian model of growth and distribution and its neo-Ricardian and
6 neo-Marxian critiques. *Cambridge Journal of Economics*, 19(6), 789–818.
- 7 Lavoie, M. (2014). *Post-Keynesian Economics: New Foundations*. Cheltenham UK: Edward
8 Elgar Publishing Limited.
- 9 Lavoie, M. (2016). Frederic Lee and post-Keynesian pricing theory. *Review of Political*
10 *Economy*, 28(2), 169–186. <https://doi.org/10.1080/09538259.2016.1149375>
- 11 Lee, F. S. (1999). *Post Keynesian Price Theory*. Cambridge University Press.
- 12 Maddison, A. (1994). *Standardised Estimates of Fixed Capital Stock: A Six Country Comparison*
13 (GGDC Research Memorandum No. GGDC Research Memorandum 199409).
14 Groningen, Netherlands: Groningen Growth and Development Centre, University of
15 Groningen. Retrieved from <http://ideas.repec.org/p/dgr/rugggd/199409.html>
- 16 Nelson, R. R., & Winter, S. G. (1982). *An evolutionary theory of economic change*. Cambridge,
17 Mass.: Belknap Press of Harvard University Press.
- 18 Nordhaus, W. D. (1973). Some skeptical thoughts on the theory of induced innovation. *The*
19 *Quarterly Journal of Economics*, 87(2), 208–219. <https://doi.org/10.2307/1882184>
- 20 Okishio, N. (1961). Technical changes and the rate of profit. *Kobe University Economic Review*,
21 7, 86–99.
- 22 Romer, P. M. (1994). The origins of endogenous growth. *The Journal of Economic Perspectives*,
23 8(1), 3–22. Retrieved from <http://www.jstor.org/stable/2138148>
- 24 Salter, W. E. G. (1966). *Productivity and Technical Change* (2nd ed.). London, UK: Cambridge
25 University Press.
- 26 Samuelson, P. A. (1965). A theory of induced innovation along Kennedy-Weisäcker lines. *The*
27 *Review of Economics and Statistics*, 47(4), 343–356. <https://doi.org/10.2307/1927763>

1 Shah, A., & Desai, M. (1981). Growth cycles with induced technical change. *The Economic*
2 *Journal*, 91(364), 1006–1010. <https://doi.org/10.2307/2232506>

3 Tavani, D., & Zamparelli, L. (2017). *Endogenous technical change in alternative theories of*
4 *growth and distribution* (Working Paper No. 1/2017). Rome, Italy: Dipartimento di
5 Scienze Sociali ed Economiche, Sapienza Università di Roma.

6 Zamparelli, L. (2015). Induced innovation, endogenous technical change and income distribution
7 in a labor-constrained model of classical growth. *Metroeconomica*, 66(2), 243–262.
8 <https://doi.org/10.1111/meca.12068>

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10

1 **9 Mathematical Appendix**

2 We start with equation (14), which we repeat here for convenience,

$$3 \quad M_{ij} = \frac{\partial \langle \hat{v}_i \rangle}{\partial \sigma_j} = \int d\hat{\mathbf{v}} \psi(\boldsymbol{\sigma} \cdot \hat{\mathbf{v}}) \hat{v}_j \hat{v}_i f(\hat{\mathbf{v}}). \quad (\text{A1})$$

4 Note that if we exchange the i and j indices under the integral the expression is
5 unchanged, because

$$6 \quad \hat{v}_i \hat{v}_j = \hat{v}_j \hat{v}_i. \quad (\text{A2})$$

7 This means that the off-diagonal elements of \mathbf{M} are symmetric, $M_{ij} = M_{ji}$. The diagonal
8 elements are positive, because after setting $i = j$, all factors under the integral are positive,

$$9 \quad M_{ii} = \int d\hat{\mathbf{v}} \psi(\boldsymbol{\sigma} \cdot \hat{\mathbf{v}}) \hat{v}_i^2 f(\hat{\mathbf{v}}) > 0. \quad (\text{A3})$$

10 For a similar reason, for an arbitrary vector \mathbf{x} , a quadratic form constructed with \mathbf{M} is
11 non-negative,

$$12 \quad \mathbf{x}^T \cdot \mathbf{M} \cdot \mathbf{x} = \int d\hat{\mathbf{v}} \psi(\boldsymbol{\sigma} \cdot \hat{\mathbf{v}}) \left(\sum_{i=0}^N x_i \hat{v}_i \right)^2 f(\hat{\mathbf{v}}) \geq 0. \quad (\text{A4})$$

13 We can derive a further useful property. Compute

$$14 \quad \mathbf{M} \cdot \boldsymbol{\sigma} = \int d\hat{\mathbf{v}} \boldsymbol{\sigma} \cdot \hat{\mathbf{v}} \psi(\boldsymbol{\sigma} \cdot \hat{\mathbf{v}}) \hat{v}_i f(\hat{\mathbf{v}}). \quad (\text{A5})$$

15 Because $\psi(\boldsymbol{\sigma} \cdot \hat{\mathbf{v}})$ is sharply peaked near zero (see Figure 2), and is zero everywhere else,
16 we have

$$17 \quad \boldsymbol{\sigma} \cdot \hat{\mathbf{v}} \psi(\boldsymbol{\sigma} \cdot \hat{\mathbf{v}}) \approx 0, \quad (\text{A6})$$

18 to a very good approximation. We conclude, from equation , that

$$19 \quad \mathbf{M} \cdot \boldsymbol{\sigma} = \boldsymbol{\sigma} \cdot \mathbf{M} \approx 0, \quad (\text{A7})$$

20 again to a very good approximation. The quadratic form in equation is then seen to be
21 (approximately) zero only for vectors \mathbf{x} that are proportional to $\boldsymbol{\sigma}$.

22 From the approximate relationship in equation (A7) we also find that each productivity
23 growth rate $\langle \hat{v}_i \rangle$ is (approximately) homogeneous of order zero in the cost shares,

1
$$\mathbf{M} \cdot \boldsymbol{\sigma} = \sum_{j=1}^N \sigma_j \frac{\partial \langle \hat{v}_i \rangle}{\partial \sigma_j} \approx 0. \quad (\text{A8})$$

2 If the probability distribution $f(\hat{\mathbf{v}})$ does not change, the total change in $\langle \hat{v}_i \rangle$ is given by

3
$$\Delta \langle \hat{v}_i \rangle = \sum_{j=0}^N \Delta \sigma_j \frac{\partial \langle \hat{v}_i \rangle}{\partial \sigma_j} = \sum_{j=0}^N M_{ij} \Delta \sigma_j. \quad (\text{A9})$$

4 Because $\boldsymbol{\sigma}$ is a vector of shares that must sum to one, the change $\Delta \boldsymbol{\sigma}$ has to sum to zero,
5 so it cannot be proportional to $\boldsymbol{\sigma}$. From this fact, and the positive semi-definiteness of \mathbf{M} ,
6 we find that

7
$$\Delta \boldsymbol{\sigma}^T \cdot \Delta \langle \hat{\mathbf{v}} \rangle = \Delta \boldsymbol{\sigma}^T \cdot \mathbf{M} \cdot \Delta \boldsymbol{\sigma} > 0. \quad (\text{A10})$$

8 When the approximate inequality in (A8) holds exactly, it says that the productivity
9 growth rate for each input is a homogeneous function of the cost shares of order zero.

10 That means that the matrix of derivatives, \mathbf{M} , must be of order negative one in the cost
11 shares.

12 *9.1 Complementary factors as a special case*

13 The positivity of the own-response in (16) does not guarantee that the total response of
14 the productivity growth rate of a particular factor to a rise in the cost share will generally
15 be positive. A possible exception can arise when two factors are complementary. We say
16 that two factors i and j are complements if M_{ij} is positive, and substitutes if negative.

17 The distinction between complements and substitutes is important when determining the
18 total change in productivity growth rates. Cost shares must sum to one, so any increase in
19 the cost share of one factor must be compensated by a net decrease in the cost shares of
20 other factors. The most problematic case arises when price and wage setting dynamics are
21 such that an increase z in the cost share for factor i is compensated entirely by a fall in the
22 cost share for a complementary factor j ,

23
$$\Delta \sigma_i = z, \quad \Delta \sigma_j = -z, \quad \Delta \sigma_k = 0 \text{ for } k \neq i, j. \quad (\text{A11})$$

24 Then the total change in $\langle \hat{v}_i \rangle$ is equal to

$$1 \quad \Delta \langle \hat{v}_i \rangle = z \left(\frac{\partial \langle \hat{v}_i \rangle}{\partial \sigma_i} - \frac{\partial \langle \hat{v}_i \rangle}{\partial \sigma_j} \right) = z (M_{ii} - M_{ij}). \quad (\text{A12})$$

2 If factors i and j are substitutes, this is certainly positive, because in that case M_{ij} is
 3 negative. However, if they are complements then this expression could, in principle, be
 4 negative. To constrain the possibilities, we use a property of positive semi-definite
 5 matrices, that the absolute value of the off-diagonal elements is bounded by the average
 6 of the corresponding diagonal entries,¹¹

$$7 \quad |M_{ij}| \leq \frac{1}{2} (M_{ii} + M_{jj}). \quad (\text{A13})$$

8 If i and j are complements, then $|M_{ij}| = M_{ij}$, and we have

$$9 \quad \Delta \langle \hat{v}_i \rangle \geq \frac{z}{2} (M_{ii} - M_{jj}) = \frac{z}{2} \left(\frac{\partial \langle \hat{v}_i \rangle}{\partial \sigma_i} - \frac{\partial \langle \hat{v}_j \rangle}{\partial \sigma_j} \right). \quad (\text{A14})$$

10 As we have assumed z to be positive, the total change in $\langle \hat{v}_i \rangle$ is certainly positive when
 11 factor i 's own-response exceeds that for factor j . However, using the same argument, a
 12 rise in the cost share of the other factor, j , could be negative, which can introduce a
 13 positive feedback. Under the assumed conditions, when the j th factor's cost share rises
 14 and the i th factor's cost share falls by the same amount, processes that use more of factor
 15 i become profitable even if they drive unit inputs of the j th factor upward. If they are
 16 adopted, we expect the cost share σ_j to increase, driving a further decline in v_j .

17 This positive feedback, which drives the cost share of factor j steadily upward and that of
 18 factor i downward, clearly cannot continue indefinitely; at some point, factor j figures so
 19 heavily in costs that firms can no longer accept increases in its use even if accompanied
 20 by declines in the cost of factor i . We therefore argue, on economic grounds, that pairs of
 21 factors should normally satisfy

¹¹ This can be shown by defining \mathbf{x}^\pm such that $x_i^\pm = 1$, $x_j^\pm = \pm 1$, $x_k = 0$ for $k \neq i, j$, and constructing quadratic forms with \mathbf{M} .

1
$$\frac{\partial \langle \hat{v}_j \rangle}{\partial \sigma_i} = \frac{\partial \langle \hat{v}_i \rangle}{\partial \sigma_j} \leq \min \left(\frac{\partial \langle \hat{v}_i \rangle}{\partial \sigma_i}, \frac{\partial \langle \hat{v}_j \rangle}{\partial \sigma_j} \right). \quad (\text{A15})$$

2 For complementary factors, this inequality may be temporarily violated, accompanied by
3 an unstable dynamic that halts when cost shares have shifted sufficiently that the
4 inequality is re-established.

5