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2008

Online at http://mpra.ub.uni-muenchen.de/8662/
MPRA Paper No. 8662, posted 8. May 2008 18:42 UTC
Indirect Network Effects and
the Impact of Trade Liberalization: A Note

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April 10, 2008

Abstract

Indirect network effects exist when the utility of consumers is increasing in the variety of complementary software products available for use with an electronic hardware device. In this note, we examine how trade liberalization affects production structure in the presence of indirect network effects. For these purposes we construct a simple

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‡We are grateful to Jota Ishikawa, Eiji Ogawa, Takatoshi Tabuchi, Makoto Tawada, Shigemi Yabuuchi, Dao-Zhi Zeng and Laixun Zhao for their helpful comments.
two-country model of trade with two incompatible hardware technologies. It is shown that, given that both types of hardware exist before trade liberalization, liberalization may *reduce* the variety of hardware technology via intensified network effects. It is also shown that, contrary to the findings of previous studies, some consumers may become worse off as the result of trade. In other words, trade liberalization, which forms the basis for a greater variety of software products, may work as a catalyst for Pareto inferior outcomes.

Key Words: Indirect network effects; hardware/software systems; trade liberalization; Pareto inferior trade

JEL Classification: D43, F12
1 Introduction

The proliferation of trade liberalization through both economic integration (e.g., the European Union) and preferential trade agreements (e.g., NAFTA) has spawned a vast literature on the implications of trade liberalization. Since liberalization often provides an opportunity to acquire varieties of products not available from domestic producers, welfare gains via increased product diversification are emphasized in the literature.\(^1\) As yet, however, the cases of “hardware/software” systems (i.e., hardware devices and the varieties of complementary software products) are downplayed in the trade literature. In other words, little attention has been paid to the implications of trade liberalization in the presence of products with indirect (or virtual) network effects.

Indirect network effects exist when the utility of consumers is increasing in the variety of complementary products available for an electronic hardware device. Examples of such devices include personal computers, video cassette recorders, and consumer electronics products. In systems that pair hardware with software, an indirect network effect arises because increases in

\(^1\)One of the seminal contributions on the gains from variety is Krugman (1979). In a recent empirical study, Broda and Weinstein (2006) find that the value to U.S. consumers of global variety growth in the 1972-2001 period was about 3 percent of GDP.
the number of users of hardware increase the demand for compatible software and hence the supply of software varieties. Since larger and more integrated markets often provide greater product variation, these characteristics affect the degree to which indirect network effects exist.

Despite the fact that many industries have indirect network effects that are supported by trade liberalization, the literature on indirect network effects is almost exclusively focused on a closed economy. Because the role of indirect network effects is amplified in the globalized world, it seems important to explore the impact of trade liberalization in the presence of products with indirect network effects.

As our primary contribution, we examine how trade liberalization affects production structure in the presence of indirect network effects. For these


3Gandal and Shy (2001, p. 364) note that, in 1992, it was estimated that seventy-two percent of all personal computers throughout the world were IBM-compatibles. That is, they ran the MS-DOS operating system and were compatible with applications software written for the MS-DOS operating system.
purposes we construct a simple, two-country model of trade with two incompatible hardware technologies which is an extension of Church and Gandal’s (1992) closed economy model.\(^4\) We modify their approach to include aspects of trading economies such as two-way trade flow of complementary software products and gains/losses from trade. Also, to provide a better understanding of Church and Gandal’s (1992) model, we present a new graphical exposition for equilibrium configuration. It is shown that, given that two incompatible hardware devices exist before trade liberalization, trade liberalization may reduce the variety of hardware devices. It is also shown that, if the variety of hardware devices is reduced by trade liberalization, some consumers are made worse off by trade. In other words, trade liberalization, which forms the basis for a greater variety of software products (i.e., intensified indirect network effects), may work as a catalyst for Pareto inferior outcomes.

The rest of this note is organized as follows. Section 2 describes both consumer preferences and technologies. Section 3 describes the basic model and derives an autarky equilibrium. Section 4 considers the impact of trade liberalization. Section 5 contains concluding remarks.

\(^4\)In order to analyze the possibility of coalition formation among suppliers of retail services, Henkel et al. (2000) adapt the work of Church and Gandal (1992) to a spatial economy setting.
2 Consumer Preferences and Technology

Suppose that there are two countries, Home and Foreign, and that they are identical in regard to tastes, size, and technology. In each country there are three types of goods: hardware, a large variety of software products, and the outside good. We assume that there are two hardware technologies in both countries: Hardware 0 and Hardware 1. We also assume that the hardware technologies are incompatible: software written for one hardware will not work with the other’s. The characterization of the two hardware technologies is exogenous: each is located at the end point of the unit line: let Hardware 0’s technology be at the left end point and Hardware 1’s technology at the right end point. We denote the marginal cost of each hardware production by c. We further assume that the hardware technologies are non-proprietary and that they will be offered at marginal cost. In this and next sections, we consider the Home autarky situation.

Following Church and Gandal (1992), consumer preferences over the combination of hardware and software are modelled as a Dixit-Stiglitz (1977) CES utility function. We assume that the distribution of the tastes of Home (Foreign) consumers is uniform along a line of unit length $t \in [0, 1]$. We normalize the total number of consumers in each country to 1.

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5In this way, we rule out Ricardian comparative advantage.
The preferences of a consumer of type $t$ for system $h$ are:

$$U(t) = \left[ \sum_{i=1}^{n_h} (x^h_i) \right]^{(1/\theta)} + \phi - k|t-h|, \quad 1/2 < \theta < 1,$$

where $n_h$ is the number of software products written for Hardware $h$ ($h = 0, 1$), $x^h_i$ is the level of consumption of software product $i$ written for Hardware $h$, $\sigma \equiv 1/(1-\theta) > 2$ is the elasticity of substitution between every pair of software products, and we assume that $\phi > k$. $k$ is a measure of the degree of product differentiation between the hardware technologies: the greater $k$, the greater the degree of differentiation.

The representative consumer who purchases Hardware $h$ will maximize (1) subject to the following budget constraint:

$$\sum_{i}^{n_h} p^h_i x^h_i = e - c,$$

where $p^h_i$ is the price of software variety $i$ for Hardware $h$, $e$ is the total expenditure allocated to hardware and software, and $c$ is the price (i.e., cost) of a unit of Hardware $h$.

The solution to this problem consists of the following demand functions:

$$x^h_i = (e - c)(P^h)^{\sigma-1}/(p^h_i)^\sigma,$$

where

$$P^h = \left[ \sum_{j=1}^{n_h} (p^h_j)^{1-\sigma} \right]^{1/(1-\sigma)}.$$
The indirect utility of a type-$t$ consumer who purchases a system $h$ is

$$V(t) = n_h^{1/(\sigma-1)}(e - c)/p^h + \phi - k|t - h|. \quad (5)$$

The indirect utility function is concave in $n_h$: the marginal benefit of another software variety is decreasing.

Now, turn to the cost structure of software provision. The technology for the production of software is characterized by increasing returns to scale, since software creation typically involves fixed costs. We denote the constant marginal cost of software production for every product by $b$, and the software development cost by $f$.

We assume that software firms are monopolistic competitors, and thus, each product is priced at a markup over marginal cost $b$:\(^6\)

$$p = b\sigma/(\sigma - 1). \quad (6)$$

## 3 The Model

In this section, we specify a simple game in which the strategy of each software firm in a decision to provide software for either hardware, 0 or 1. The timing of the game is as follows:\(^7\) In the first stage software firms enter the

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\(^6\)Hereafter, we drop the superscript $h$.

\(^7\)This is taken from Church and Gandal’s (1992) closed economy model.
industry. There is free entry into the software industry and software firms have rational expectations. Although there may be more than one equilibrium software configuration, we show that the free-entry number of software firms, \( N = n_0 + n_1 \), is unique, where \( n_h \) is the number of firms providing software for Hardware \( h \). In the second stage, software firms simultaneously choose which platform to provide software for. In the final stage, each consumer purchases either a Hardware 0 or a Hardware 1 system and some of the compatible software. We solve this problem backward. To obtain a better understanding of the model, we extensively use a new graphical exposition for equilibrium configuration.

3.1 Final Stage

Since we assume the marginal costs (prices) of hardware and software are equal for both systems, consumers determine which hardware to purchase considering only their tastes and the amount of software available for each system. From (5), a consumer located at \( t \) purchases Hardware 0 if the following inequality holds:

\[
n_0^{1/(\sigma-1)}(e - c)/p + \phi - kt > (N - n_0)^{1/(\sigma-1)}(e - c)/p + \phi - k(1 - t), \tag{7}
\]

where use has been made of the equation \( n_0 + n_1 = N \). Therefore, the location of the marginal consumer who purchase Hardware 0 is given by a
function of \( n_0 \), that is,

\[
t(n_0) = \left[ n_0^{1/(\sigma-1)} - (N - n_0)^{1/(\sigma-1)} \right] (e - c)(\sigma - 1)/2kb\sigma + 1/2. \tag{8}
\]

And the first derivative of \( t(n_0) \) is positive:

\[
t'(n_0) \equiv \frac{dt(n_0)}{dn_0} = \left[ n_0^{(2-\sigma)/(\sigma-1)} + (N - n_0)^{(2-\sigma)/(\sigma-1)} \right] (e - c) \frac{1}{2kb\sigma} > 0. \tag{9}
\]

This means that the share of Hardware 0 is increasing in the amount of software for it. It can also be shown that

\[
t(0) \geq 0 \quad \text{and} \quad t(N) \leq 1 \iff N^{1/(\sigma-1)} \leq k\sigma/[(e - c)(\sigma - 1)] \tag{10}
\]

and

\[
t'(N/2) \geq 1/N \iff N^{1/(\sigma-1)} \geq 2^{1/(\sigma-1)} k\sigma/2(e - c). \tag{11}
\]

Based on the above, we can draw the function \( t(n_0) \) as shown in Figure 1,\(^8\) where curves \( A \), \( B \), and \( C \) correspond to the graph of \( t(n_0) \) under each of the following three cases: in case \( A \), \( N^{1/(\sigma-1)} \leq k\sigma/[(e - c)(\sigma - 1)] \); in case \( B \), \( k\sigma/[(e - c)(\sigma - 1)] < N^{1/(\sigma-1)} < 2^{1/(\sigma-1)} k\sigma/2(e - c) \); and in case \( C \), \( N^{1/(\sigma-1)} \geq 2^{1/(\sigma-1)} k\sigma/2(e - c). \(^9\)

\(^8\)The second derivative of \( t(n_0) \) is negative (positive) if \( n_0 \) is smaller (greater) than \( N/2 \), since

\[
\frac{d^2t(n_0)}{dn_0^2} = -\frac{\left( n_0^{(3-2\sigma)/(\sigma-1)} - (N - n_0)^{(3-2\sigma)/(\sigma-1)} \right) (e - c)}{2k\sigma(\sigma - 1)},
\]

where \( \sigma > 2 \) from the assumption \( \theta > 1/2 \).

\(^9\)The importance of discrimination between case \( B \) and \( C \) will appear in the following.
Note that in cases B and C, \( t(n_0) \) can reach 0 or 1, even if there are still two types of software. Since the market is of unit length, that is, \( 0 \leq t \leq 1 \), there exists a critical number of software firms for each type of hardware such that if the number of software firms for one technology exceeds the critical number, then all consumers purchase the dominant hardware. On the other hand, in case A, there are two types of consumers unless one hardware is standardized; no software for the other hardware exists.\(^{10}\)

### 3.2 Second Stage

In the second stage, software firms simultaneously select the network for which to supply software are. Given the marginal consumer, \( t \), and the number of competing software firms \( (n_0 \text{ or } n_1) \), the profit of a software firm writing software for Hardware 0 is

\[
\pi^0(t, n_0) = t(p - b)x^0 - f = t(e - c)/n_0\sigma - f,
\]

and that for Hardware 1 is

\[
\pi^1(t, n_1) = (1 - t)(p - b)x^1 - f = (1 - t)(e - c)/n_1\sigma - f,
\]

\(^{10}\)Since we assume that hardware only facilitates the consumption of software and provides no stand-alone benefits, in case A, the marginal consumer, \( t \), changes discontinuously to 0 or 1 when \( n_0 \) is equal to 0 or \( N \).
where $x^1 = (e - c)/n_1 p$. From these equations, it is easily derived that

$$\pi^0(t, n_0) \begin{cases} \geq \pi^1(t, n_1) & \iff t \geq \frac{n_0}{N} \end{cases}$$

(14)

Based on the latter inequality, each firm considers whether $t(n_0)$ is greater than $n_0/N$ or not, and then chooses the network to supply.

### 3.3 First Stage

At any equilibrium where two networks coexist, $\pi^0(t, n_0) = \pi^1(t, n_1)$ must be satisfied. Therefore, $t = n_0/N$ holds at the equilibrium and

$$\pi^0 = \pi^1 = (e - c)/N\sigma - f.$$  

(15)

On the other hand, if all software firms provide software for one network at equilibrium, then $(t, n_0) = (1, N)$ or $(t, n_1) = (0, N)$ hold and

$$\pi^0 = (e - c)/N\sigma - f \quad \text{or} \quad \pi^1 = (e - c)/N\sigma - f.$$  

(16)

Thus, the profit of each firm is independent of equilibrium software configurations, and the free-entry number of firms, $N$, is uniquely given by $N = (e - c)/f\sigma$ from the zero-profit condition.

Based on the foregoing argument, we can conclude that $\pi^0 = \pi^1 = 0$ holds for any pair $(t, n_0)$ on the dotted line in Figure 1, $\pi^0 = 0$ at $(1, N)$, and $\pi^1 = 0$ at $(0, 0)$, while $\pi^0$ ($\pi^1$) is positive (negative) at any pair above the line and vice versa.
3.4 Nash Equilibrium Configurations

Based on the foregoing argument, we obtain the Nash equilibrium configurations as follows: In order for a configuration to be a Nash equilibrium, it must be impossible for a software firm to switch networks and increase its profit.

In case A, the graph of \( t(n_0) \) is drawn as curve \( A \) in Figure 1. So, there are three equilibrium candidates; \((n_0 = n_1 = N/2)\), \((n_0 = N, n_1 = 0)\), and \((n_0 = 0, n_1 = N)\). Since

\[
\begin{align*}
t(n_0) & \begin{cases} > n_0/N \quad & \text{if} \quad n_0 < N/2, \\
< n_0/N \quad & \text{if} \quad n_0 > N/2,
\end{cases} \\
\end{align*}
\] (17)

we can conclude that only symmetric equilibrium \((n_0 = n_1 = N/2)\) is stable in the sense of a Nash equilibrium.

On the other hand, in case C, the graph is drawn as curve \( C \) and

\[
\begin{align*}
t(n_0) & \begin{cases} < n_0/N \quad & \text{if} \quad n_0 < N/2, \\
> n_0/N \quad & \text{if} \quad n_0 > N/2.
\end{cases} \\
\end{align*}
\] (18)

Therefore, only two equilibria, \((n_0 = N, n_1 = 0)\) and \((n_0 = 0, n_1 = N)\), are stable.\(^{11}\)

Finally, in case B, the graph of \( t(n) \) is drawn as curve \( B \) and it is apparent from the discussion above that all three of the equilibria, \((n_0 = n_1 = N/2)\),

\(^{11}\)In the interval of \( n \) where \( t(n_0) \) is greater than 1 (smaller than 0), the actual marginal consumer, \( t \), is equal to 1 (0) and is still above (below) the line \( t = n_0/N \).
Lemma: Depending on the parameter values, the following three cases emerge:

Case A: If \( N^{1/(\sigma - 1)} \leq k\sigma/[(e - c)(\sigma - 1)] \), a unique symmetric equilibrium exists, \((n_0 = n_1 = N/2)\).

Case B: If \( k\sigma/[(e - c)(\sigma - 1)] < N^{1/(\sigma - 1)} < 2^{1/(\sigma - 1)}k\sigma/2(e - c) \), three equilibria, \((n_0 = n_1 = N/2), (n_0 = N, n_1 = 0), and (n_0 = 0, n_1 = N),\) exist.

Case C: If \( N^{1/(\sigma - 1)} \geq 2^{1/(\sigma - 1)}k\sigma/2(e - c) \), only two equilibria, \((n_0 = N, n_1 = 0)\) and \((n_0 = 0, n_1 = N),\) exist.

Although the present result is the same as that stated in Church and Gandal’s (1992) closed economy model, we believe that our graphical exposition provides better understanding of the equilibrium configuration. Also, one major advantage of our graphical exposition is that it makes it easier to extend the analysis to the case of unequal preference distribution.

4 The Impact of Trade Liberalization

Now let us turn to the impact of trade liberalization. Trade liberalization between two identical countries implies one basic change: the total number of
consumers becomes 2. This implies that the integrated market can support a larger number of software products: the total number of complementary software products changes from $N$ to $2N$. Since we have assumed away Ricardian comparative advantage, there is no incentive for inter-industry trade. Still, since each software firm specializes in different range of products, an incentive for two-way trade of software products remains. Since consumers prefer to consume a wide variety of software products, trade liberalization might result in gains from product diversification. However, we have to check the changes in the variety of hardware. Depending on parameter values, several possible cases emerge. In order to highlight the interaction between indirect network effects and trade liberalization, let us examine the following two representative cases (these cases are summarized in Figure 2).

4.1 The Case of Hardware Differentiation

First, let us assume that the following condition is satisfied:

$$\left(2N\right)^{1/(\sigma-1)} \leq k\sigma/[((e-c)(\sigma-1)].$$

Note that this condition holds when the degree of hardware differentiation ($k$) is relatively large (or the degree to which indirect network effects exist is relatively low). In this case, two types of hardware exist both before and after trade liberalization. Thus, no consumer changes his or her hardware
and trade liberalization induces twice as many software varieties for each type of hardware: $n_0$ becomes $2n_0$ and $n_1$ becomes $2n_1$. From (5), this clearly increases every consumer's utility.

**Proposition 1:** Given that condition (19) holds, both types of hardware remain in the equilibrium and both countries gain from trade liberalization.

Note that these gains correspond to those obtained from the “love-of-variety” approach to trade gains (e.g., Helpman and Krugman, 1985). Through trade liberalization, consumers in each country can obtain a wider variety of products, which results in mutual gains.

### 4.2 The Case of Hardware Standardization

Next, let us assume that the following condition is satisfied:\[12\]

\[kb\sigma/[2(e - c)] \leq N^{1/(\sigma-1)} \leq kb\sigma/[(e - c)(\sigma - 1)]. \tag{20}\]

In this case, while both types of hardware exist before trade liberalization, only one type of hardware remains after liberalization. In other words, intensified indirect network effects result in a *reduced* number of hardware varieties (2 rather than 1). For simplicity, let us suppose that only Hardware 1 remains after trade liberalization. In this case, some consumers have to switch

\[12\text{Note that } \sigma \leq 3 \text{ is required for this condition.}\]
from Hardware 0 to Hardware 1. While there are gains from the increased
diversity of software available, there are losses from switching to the other
network. The change in the indirect utility of a type-$t$ consumer who switches
to the other network is:

$$
\Delta V(t) = [(4^{1/(\sigma-1)} - 1)(N/2)^{1/(\sigma-1)}(e - c)(\sigma - 1)]/(b\sigma) - k(1 - 2t). \tag{21}
$$

Note that the first term on the RHS represents the gains from software diver-
sification while the second term on the RHS represents costs from increased
disutility. Let us define a type-$\bar{t}$ consumer who is indifferent to switching
hardware as follows:

$$
\bar{t} = (1/2) - [(4^{1/(\sigma-1)} - 1)(N/2)^{1/(\sigma-1)}(e - c)(\sigma - 1)]/2k\sigma. \tag{22}
$$

Let us define the solution of $2^{1/(\sigma-1)} - 4^{1/(\sigma-1)} + 1 = 0$ as $\tilde{\sigma}$. Then we can
show that $\bar{t} > 0$ holds when $\sigma > \tilde{\sigma}$:

$$
\bar{t} \geq (1/2) - (4^{1/(\sigma-1)} - 1)/2^{1+1/(\sigma-1)}
= (2^{1/(\sigma-1)} - 4^{1/(\sigma-1)} + 1)/2^{\sigma/(\sigma-1)} \begin{cases} < 0 & \text{if } 2 < \sigma < \tilde{\sigma} \\ > 0 & \text{if } \sigma > \tilde{\sigma} \end{cases}
$$

Now we can state the possibility of losses from trade.

---

13Note that, in the case of hardware standardization, the number of software varieties
for Hardware 1 increases from $n_1$ to $4n_1$ (or from $N/2$ to $2N$).
Proposition 2: If condition (20) and $\bar{\sigma} \leq \sigma \leq 3$ are satisfied and Hardware 1 (resp. 0) dominates the integrated market, both countries’ consumers who located at $t \in [0, \bar{t}]$ (resp. $t \in [1 - \bar{t}, 1]$) are made worse off by trade liberalization.

This implies that trade liberalization leads some consumers to “switch” to an other-dominated brand, thereby increasing disutility. Note that this case is highly contrasted with the cases of universal gains from trade, which are emphasized in the literature.\(^{14}\) We would like to emphasize that trade liberalization, which forms a basis for a greater variety of software products (i.e., intensified indirect network effects), may work as a catalyst for Pareto inferior outcome.

5 Conclusions

Indirect network effects exist when the utility of consumers is increasing in the variety of complementary software products available for a hardware device. In this note, we examine how trade liberalization affects production structure in the presence of indirect network effects. For these purposes we \(^{14}\)See, for example, Helpman and Krugman (1985). Related to the present result, Chou and Shy (1991) considered the case where the variety of non-traded domestic products is reduced by trade liberalization.
construct a simple, two-country model of trade with incompatible hardware technologies. It is shown that, given that both hardware devices remain after liberalization, every consumer gains from trade (Proposition 1). It is also shown that, if the number of hardware varieties is reduced by trade liberalization, some consumers may be made worse off by trade (Proposition 2). It must be noted that competition in the integrated market is likely to lead to standardization on a single hardware/software system.

The present analysis must be regarded as tentative. Hopefully it provides a useful paradigm for considering how indirect network effects (or hardware/software systems) affect both the structure of production and the gains or losses from trade.
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versity Press.
Figure 1

Figure 2