Probabilistic Approaches to Cleaning the Ganges in Varanasi to Attract Tourists

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Probabilistic Approaches to Cleaning the Ganges in Varanasi to Attract Tourists[1]

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1

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2

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Abstract

We study two probabilistic approaches to cleaning the Ganges river when the underlying goal is to use the cleanup to sustain tourism in Varanasi, India. The first approach models the idea that because resources are scarce and cleanup is costly, *not all* pollutants in the Ganges can be removed. Therefore, a cleaning agency first establishes a benefit-cost ratio rule and then it uses this rule to remove from the Ganges only those pollutants whose removal satisfies the ratio rule. In contrast, the second approach focuses on removing *all* pollutants from the Ganges but the emphasis now is on the frequency of cleanup given that pollutants accumulate temporally and hence water quality deteriorates over time. Finally, we compare and contrast these two approaches and discuss the connections between the two approaches and the sustainability of tourism in Varanasi.

Recommendations for Resource Managers

1. When cleanup resources are scarce, strategic management of Ganges water pollution calls for removing only those pollutants whose removal satisfies a benefit-cost criterion.
2. When cleanup resources are adequate, holistic management of Ganges water pollution calls for attaining the highest possible water quality by removing all pollutants from the river.
3. Strategic management is more realistic and also likely to cost less than holistic management.

Keywords

Cleanup, Ganges River, Tourism, Uncertainty, Varanasi

JEL Codes: Q53, L83

1. Introduction

The Ganges (Ganga in Hindi) river is not only the longest river in India but it also occupies a central place in the Hindu religion. Hindus generally consider the Ganges to be sacred and therefore millions of them routinely visit the holy city of Varanasi in the state of Uttar Pradesh to perform a purification ritual that involves, among other things, bathing in the river. The city of Varanasi is important not only for what Rinschede (1992) calls “religious tourism,” but also because it is one of the oldest inhabited cities in the world. Alley (1992) and Chitravanshi (2014) point out that in contemporary times, in addition to being a major center for both domestic and foreign tourists, Varanasi is also well known for its art, culture, and music.

Regrettably, pollutants of all types are now routinely deposited into the Ganges. In addition, in Varanasi, one can find animal carcasses, partially cremated corpses, and the material offerings of Hindu devotees in the river. In this regard, Dhillon (2014) contends that 32,000 bodies are cremated every year in Varanasi and that this process results in 300 tons of ash and 200 tons of half burnt human flesh being deposited into the Ganges. Given this extremely insalubrious state of the river, questions are now frequently being asked about the sustainability of the tourism industry in Varanasi.

The cleanup of the Ganges has been discussed many times in the past but this discussion has led to very little change in the extremely polluted status of the river. However, the Ganges now appears to have a champion in Mr. Narendra Modi who is not only a devout Hindu but also the current Prime Minister of India. Bhandari (2015) and Parth (2017) point out that Mr. Modi contested the 2014 election from Varanasi and that he has promised to convert Varanasi into a vibrant city for tourists by launching a major campaign to clean the Ganges.

Despite the salience of this campaign from both environmental and touristic perspectives, to the best of our knowledge, there is only *one brief* paper by Batabyal and Beladi (2017) in the literature that sheds some *theoretical* light on how to study the stochastic nature of the Ganges cleanup problem and its connection to the sustainability of tourism.[4] Given this lacuna in the extant literature, we substantially generalize the discussion in Batabyal and Beladi (2017) and analyze two probabilistic approaches to cleaning the river Ganges when the underlying objective of the cleanup is to contribute to the sustainability of the tourism industry in Varanasi, India.

The rest of this paper is organized as follows. Section 2 provides a detailed discussion of the first approach in which we explicitly model the idea that because resources are scarce and cleanup is costly, *not all* pollutants in the Ganges can be removed. Therefore, in this approach, a cleaning agency first establishes a benefit-cost ratio rule and then it uses this rule to remove from the Ganges only those pollutants whose removal satisfies the above mentioned ratio rule. Section 3 discusses the second approach which, in contrast to the first approach, focuses on removing *all* pollutants from the Ganges but the emphasis now is on the frequency of cleanup given that pollutants accumulate temporally and therefore water quality deteriorates over time. The concluding section 4 first discusses the key points about the two probabilistic approaches and the sustainability of tourism in Varanasi and then suggests two ways in which the research delineated in this paper might be extended.

4

We are also aware of two studies that have addressed the cleanup of the river Ganges. Markandya and Murty (2000) provide a detailed analysis of a particular program designed to clean up the polluted Ganges, namely, the Ganga Action Plan (GAP). Markandya and Murty (2004) discuss environmental and development issues in the context of river cleanup programs and then provide estimates of the social benefits from cleaning the Ganges. It should be noted that neither of these two studies undertakes any *stochastic* modeling of the Ganges cleanup problem.

2. Approach in which all pollutants are not removed

2.1. Preliminaries

Consider a designated portion of the Ganges river in the city of Varanasi. The objective of an appropriate authority in this city (CA) is to cleanup this portion of the Ganges. This CA classifies the different possible pollutants in the relevant portion of the Ganges such as animal carcasses, partly cremated corpses, and ash (see section 1) into i possible types where $i = 1,2,\ldots,n$. A decision by the CA to remove a type i pollutant from the Ganges takes this CA τ_i units of time. At the same time, because this removal of a type i pollutant improves the water quality of the Ganges, this act also gives rise to an environmental benefit which we assume can be measured in dollars and is given by b_i. Note that the CA's pollution control problem is non-point in nature because the different pollutants that it confronts in the designated portion of the Ganges are deposited into the Ganges at multiple points and not at a single point.

At the CA's designated inspection point, possibly at a particular "ghat"[5] in Varanasi, the various possible pollutants arrive in accordance with independent and stationary Poisson processes[6] with rates $\lambda_1, \lambda_2, \ldots, \lambda_n$. Two points are now worth emphasizing. First, our CA is operating in an environment of *uncertainty*. Second, since this CA does not possess the resources to remove every possible pollutant from the designated portion of the Ganges, the CA first establishes a benefit-cost ratio rule to help it determine which pollutants to remove from the Ganges. To this end, we suppose that the CA cleans up the Ganges in the sense that it removes all type i pollutants for which the benefit-cost ratio $E[b_i]/E[\tau_i]$ is at least B dollars per unit time

5

A "ghat" refers to a series of steps that descend to the river Ganges.

6

See Ross (2003, pp. 288-348) or Tijms (2003, pp. 1-32) for textbook descriptions of the Poisson process.

for a carefully selected value of B.[7] Given this ratio rule, our CA only accepts cleanup or pollutant removal tasks of type i where $i = 1, 2, \ldots, n_0$.

The pollutant removal tasks in the relevant portion of the Ganges in Varanasi are onerous. This means that our CA can take on a cleanup task only if it is not already occupied with an alternate cleanup task. Finally, we suppose that cleanup tasks that are not taken up by our CA are dealt with by some other waste management entity whose activities we do not study here. With these preliminary details out of the way, our next task is to first define an appropriate regenerative process and to then identify the time points or epochs at which the pertinent regenerations occur.

2.2. The regenerative process

We begin with a brief description of the renewal-reward theorem that will form the centerpiece for a large part of our subsequent analysis in this paper.[8] A stochastic process $\{Q(t): t \geq 0\}$ is said to be a counting process if $Q(t)$ denotes the total number of events that have occurred by time t. Now let X_1 denote the time or epoch at which the first event occurs. Further, for $q \geq 1$, let X_q denote the time between the $(q-1)th$ and the qth event. These $X_q, q \geq 1$, are known as the interarrival times. A counting process for which these interarrival times have an arbitrary distribution is called a renewal process.

Consider a renewal process $\{Q(t): t \geq 0\}$ with interarrival times $X_q, q \geq 1$, that have cumulative distribution function $F(\cdot)$. Further, suppose that a monetary reward R_q is earned when the qth renewal is completed. Let $R(t)$, the total reward by time t, be given by $\sum_{q=1}^{Q(t)} R_q$.

7

The threshold B is given to the CA exogenously and we do not model how B is selected in this paper. That said, B would typically be determined by information provided to the CA by toxicologists. For example, suppose the specific pollutant of interest is the fecal coliform count in the Ganges water. Then, as noted by Ramachandran (2014), when this count exceeds 500 per 100 ml of water, this water is unsafe for bathing. Therefore, with regard to this particular use of the Ganges water, the threshold B would be 500.

8

This discussion is taken from Batabyal (2000).

In addition, let $E[R_q] = E[R]$ and let $E[X_q] = E[X]$, where $E[\cdot]$ is the expectation operator. The renewal-reward theorem tells us that if $E[R]$ and $E[X]$ are finite, then with probability one,

$$\lim_{t\to\infty} \frac{E[R(t)]}{t} = \frac{E[R]}{E[X]}. \quad (1)$$

In other words, if we think of a cycle being completed every time a renewal occurs, then the long run expected reward is simply the expected reward in a cycle divided by the expected amount of time it takes to complete that cycle. Two points now ought to be noted by the reader. First, the renewal-reward theorem holds for both positive rewards such as revenues and for negative rewards such as costs. Second, a renewal process is also a so called regenerative process and, in the remainder of this section, we shall be working with certain specific properties of an appropriately defined regenerative process.[9]

To this end, let the stochastic process $Y(t) = 1$ if the CA in Varanasi is at work on a particular cleanup task at time t and let $Y(t) = 0$ otherwise. Then, the preceding discussion in this section and some thought together tell us that the continuous-time stochastic process $\{Y(t): t \geq 0\}$ is a regenerative process. We now need to identify the regeneration epochs for this process. This identification is easily accomplished by thinking of the time points or epochs at which the CA completes a particular cleanup task and hence becomes idle as the regeneration epochs. Having defined the relevant regenerative process and having identified the regeneration epochs, we would now like to compute the long run expected benefit per unit time to the CA as a result of its cleanup or pollutant removal activities.

9

See Ross (2003, pp. 425-434) or Tijms (2003, pp. 39-50) for textbook accounts of regenerative processes.

2.3. The expected benefit function

We know from the discussion in section 2.1 that the cleanup or pollutant removal tasks confronted by the CA are of types $1, 2, \ldots, n_0$ and that they arrive at this CA's designated inspection point in accordance with a stationary Poisson process with rate $\lambda = \lambda_1 + \lambda_2 + \cdots + \lambda_{n_0}$. Now, given that a cleanup task belonging to one of the above types arrives, this particular task is of type i with probability λ_i/λ for $i = 1,2, \ldots, n_0$. Because the stationary Poisson process possesses the memoryless property,[10] we deduce that the expected amount of time that our CA is idle in one regeneration cycle is equal to $1/\lambda$.

We now want to use the renewal-reward theorem stated in equation (1) to compute the expected benefit per unit time to the CA. To do so, we will first need to compute the two expectations in the right-hand-side (RHS) of equation (1). The expected reward (environmental benefit) to our CA from removing a type i pollutant in a cycle is given by weighting the expected environmental benefit $E[b_i]$ with the probability λ_i/λ that this cleanup task is of type i, and then summing across all possible cleanup tasks. In symbols, we get

$$E[benefit\ obtained\ in\ one\ cycle] = \sum_{i=1}^{n_0} \frac{\lambda_i}{\lambda} E[b_i]. \tag{2}$$

The expected length of one cycle can be computed by recognizing that this cycle is composed of time during which our CA is either working on a specific cleanup task or is idle. To account for the time during which our CA is working, we use reasoning similar to that employed in the previous paragraph. Specifically, we infer that the expectation of this time is given by

10

See Ross (2003, pp. 272-273) or Tijms (2003, pp. 2-3) for additional details on the memoryless property.

weighting the expected time it takes the CA to remove a type i pollutant from the Ganges or $E[\tau_i]$ with the probability λ_i/λ that this clean up task is of type i, and then summing across all possible cleanup tasks. In symbols, we get $\sum_{i=1}^{n_0}(\lambda_i/\lambda)\,E[\tau_i]$. To this expression, we have to add the expected length of time during which our CA is idle. From the discussion in the first paragraph of this section, we know that this expectation is given by $1/\lambda$. Adding these last two expressions, we get the expected length of one cycle. In symbols, we get

$$E[length\ of\ one\ cycle] = \sum_{i=1}^{n_0}\frac{\lambda_i}{\lambda}E[\tau_i] + \frac{1}{\lambda}. \tag{3}$$

Dividing the RHSs and the left-hand-sides (LHSs) of equation (2) by equation (3) and then simplifying gives us a closed-form expression for the CA's long run expected benefit per unit time that we seek. The specific expression of interest is

$$CA's\ Long\ Run\ Expected\ Benefit = \frac{\sum_{i=1}^{n_0}\lambda_i E[b_i]}{\sum_{i=1}^{n_0}\lambda_i E[\tau_i]+1}. \tag{4}$$

It is reasonable to suppose that our CA will want to remove pollutants from and thereby improve the water quality of the Ganges in Varanasi to maximize the expression in the RHS of equation (4). In this regard, there are two potential choice variables to consider. For any type i pollutant, these are the environmental benefit or b_i and the cleanup time or τ_i variables. Inspection of equation (4) tells us that the CA's long run expected benefit function is an increasing (decreasing) function of b_i (τ_i). Therefore, since the expectation operator $E[\cdot]$ is a linear operator, to maximize the long run expected benefit from cleaning up the Ganges, for any type i pollutant, our CA will want to either optimally raise the environmental benefit from removing this pollutant from the Ganges or optimally lower the amount of time it takes to remove this same pollutant.

To see the connection between this cleanup problem that we have been discussing thus far in this section and the sustainability of the tourism industry in Varanasi, let us focus, for instance, on the CA's cleanup time τ_i for any pollutant i. It is reasonable to suppose that the sustainability of the tourism industry (S) is an increasing function $F_1\{\cdot\}$ of the cleanliness of the Ganges and some sites along the Ganges (C). In symbols, we have $S = F_1\{C\}$. Now, cleanliness (C) is an increasing function $F_2(\cdot)$ of the CA's cleanup time τ_i and hence we get $C = F_2(\tau_i)$. Combining the preceding two functional relationships, we see that $S = F_1\{F_2(\tau_i)\}$. This tells us that the sustainability of the tourism industry is itself an increasing function of the CA's cleanup time.

We know that in the cleanup approach that we are studying in this section, the CA does *not* remove every possible pollutant from the Ganges because it is prohibitively costly to do so. Therefore, it is of considerable interest to determine the long run fraction of time the CA spends removing pollutants from the Ganges and the long run fraction of the cleanup tasks $1,2,\ldots,n_0$ that the CA is unable to take on. We now proceed to ascertain these two long run fractions.

2.4. Two long run fractions

The long run fraction of time that our CA spends removing pollutants from the Ganges can be easily determined by applying the renewal-reward theorem delineated in equation (1). The expected length of one cycle (or the denominator of the ratio expression we seek) remains unchanged from what we have already obtained in equation (3) above. What is different now is the numerator of the pertinent ratio expression. To see this, observe that in any regenerative cycle, the

CA is either removing pollutants from the Ganges or is idle. However, to compute the long run fraction of time spent cleaning up the Ganges, we do *not* need to account for the expected amount of time during which the CA is idle. Therefore, we deduce that the long run fraction of time spent removing pollutants from the Ganges is given by

$$Long\ Run\ Fraction\ of\ Time\ Spent\ Cleaning\ Up = \frac{\sum_{i=1}^{n_0} \lambda_i E[\tau_i]}{\sum_{i=1}^{n_0} \lambda_i E[\tau_i]+1}, \tag{5}$$

with probability one.

To compute the long run fraction of all cleanup tasks that our CA is unable to take on, note the following two points. First, the fraction we seek equals the expected number of cleanup tasks that are declined in one cycle divided by the expected number of cleanup tasks arriving during one cycle. Second, the expected number of arrivals of a pollutant cleanup task of one of the types $1, 2, \dots, n_0$ in the time period τ_i is equal to $\lambda E[\tau_i]$. Using these two points, we modify the numerator and the denominator in equation (5). This tells us that the long run fraction of all the declined cleanup tasks is given by the ratio $\sum_{i=1}^{n_0}(\lambda_i/\lambda)\lambda E[\tau_i] / \{\sum_{i=1}^{n_0}(\lambda_i/\lambda)\lambda E[\tau_i] + 1\}$. This ratio can be simplified and this simplification gives

$$Long\ Run\ Fraction\ of\ Declined\ Tasks = \frac{\sum_{i=1}^{n_0} \lambda_i E[\tau_i]}{\sum_{i=1}^{n_0} \lambda_i E[\tau_i]+1}. \tag{6}$$

Inspecting equations (5) and (6), we see that the long run fraction of time the CA spends removing pollutants from the Ganges is *equal* to the long run fraction of the cleanup tasks

$1, 2, \ldots, n_0$ that the CA is unable to take on. This equality result arises because pollutants or cleanup tasks arrive at our CA's designated inspection point in accordance with independent and stationary Poisson processes. When this happens, the Poisson arrivals see time averages or, put differently, the prominent PASTA property holds.[11] We now move on to analyze the second probabilistic approach to cleaning the Ganges in Varanasi. In this second approach, the CA concentrates on removing *all* pollutants from the Ganges but the emphasis now is on the temporal frequency with which cleanups are carried out.

3. Approach in which all pollutants are removed

3.1. Preliminaries

As in section 2.1, once again consider a designated portion of the Ganges in the city of Varanasi. As a result of the discharge of a variety of pollutants into the Ganges along the lines discussed in section 1, water quality of this river deteriorates over time. At the designated inspection point, our CA inspects the quality of the water at fixed times denoted by $\tau = 0, 1, \ldots$ In each time period between two successive inspections, the quality of the water declines by a *random* amount. In addition, these declines in water quality accumulate over time. We suppose that the amounts of the declines in water quality in successive time periods can be described by independent random variables that have a common exponential distribution with mean $1/\beta$.

Suppose that an inspection reveals the composite level of pollutants to be higher than some critical threshold denoted by T. When this happens, our CA launches what we shall call a *mandatory* cleanup operation whose objective is to remove all the deleterious pollutants from the designated portion of the Ganges.[12] A mandatory cleanup operation involves a cost of $c_{\mathrm{T}} > 0$. In

11

See Ross (2003, p. 480) or Tijms (2003, pp. 53-58) for additional details on the PASTA property.
12

contrast with a mandatory cleanup, the CA can also undertake a *discretionary* cleanup which involves a lower cost of $c_{\Delta} > 0$ with $c_{\mathrm{T}} > c_{\Delta} > 0$. Such a discretionary cleanup is undertaken when water quality inspection by the CA reveals that the composite level of pollutants is either at or below the critical threshold T.

The CA uses the following decision rule to determine when it ought to commence a cleanup operation. Specifically, this CA begins a cleanup operation (removes all pollutants) at each inspection that shows the accumulated composite level of pollutants to be larger than some cleanup limit l where $0 \leq l < \mathrm{T}$. Now, to keep the subsequent mathematical analysis tractable, we shall abstract away from the amount of time it takes to clean up the polluted Ganges and, because all pollutants are being removed, we shall suppose that after a cleanup operation has been completed, the relevant portion of the Ganges in Varanasi is essentially unpolluted. As in section 2.1, our next task is to first define an apposite regenerative process and to then identify the time points or epochs at which the relevant regenerations occur.

3.2. The regenerative process

Recall that in the model of this section, in each time period between two successive inspections, the quality of the water in the Ganges declines by a *random* amount. Also, these declines in water quality accumulate over time. Keeping these two points in mind, consider the continuous-time stochastic process which delineates the cumulative amount of the declines in water quality. Given the description of the cleanup problem in section 3.1, this stochastic process is a regenerative process. Further, the time points or epochs at which our CA undertakes a cleanup operation can be thought of as the regeneration epochs. The section 2 approach to the pollution cleanup problem focused on the computation of the long run expected *benefit* per unit time to the

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Note that instead of casting the problem in terms of the composite level of pollutants being *higher* than some critical threshold, we could also think of the problem in terms of water quality being *lower* than some similar critical threshold.

CA as a result of its cleanup activities. Now, to illustrate a different approach to the cleanup problem, we concentrate on the CA's long run expected *cost* per unit time from its cleanup activities.

3.3. The expected cost function

Once again, we shall use the renewal-reward theorem given in equation (1) above to determine a closed-form expression for the expected cost function. However, before we can use this theorem, we will first need to define two additional concepts from renewal theory. The first concept is the renewal function.[13] This function, often denoted by $M(t)$, tells us the mean number of renewals that have occurred by time t. The second concept is the excess variable.[14] The excess variable $\gamma(t)$ tells us the random amount of time that has elapsed from epoch t until the next renewal after epoch t.

Let us now compute the expected length of a cycle. Note that the length of a cycle is the number of time periods that are needed for the cumulative water quality decline amounts to exceed the limit l. To this end, let the sequence $W_1, W_2, W_3, \ldots$ denote the water quality decline amounts that arise in the successive time periods 1, 2, 3,… From the description of the problem in section 3.1, we know that the W_i are exponentially distributed with mean $1/\beta$. Then, from either Ross (2003, p. 405) or Tijms (2003, p. 36), we can tell that the renewal function associated with the renewal process that we have just described is given by

$$M(t) = \beta t, \tag{7}$$

and, given the exponential structure of the problem, the excess variable $\gamma(t)$ is exponentially

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13

The renewal function is sometimes also known as the mean value function. See Ross (2003, pp. 403-404) or Tijms (2003, pp. 35-37) for additional details on the renewal function.

14

See Ross (2003, pp. 414-415) or Tijms (2003, pp. 37-39) for more on the excess variable.

distributed with mean $1/\beta$ for each time t. With these two pieces of information in hand, we reason that the expected length of a cycle is now given by

$$E[length\ of\ one\ cycle] = 1 + M(l) = 1 + \beta l. \tag{8}$$

Let us now focus on the costs incurred by our CA in one cycle. We know that the two possible costs are either c_{T} or c_{Δ}. These two cost terms have to be weighted by the probabilities that they will, in fact, be incurred. Using the above described excess variable, the two probabilities of interest are $Pr\{\gamma(l) > \mathrm{T} - l\}$ and $Pr\{\gamma(l) \leq \mathrm{T} - l\}$. Putting these pieces of information together, we get an expression for the costs incurred by our CA in one cycle. That expression is

$$E\{costs\ incurred\ in\ one\ cycle] = c_{\mathrm{T}} \Pr\{\gamma(l) > \mathrm{T} - l\} + c_{\Delta} \Pr\{\gamma(l) \leq \mathrm{T} - l\}. \tag{9}$$

Because of the exponential structure of our problem, the two probabilities on the RHS of equation (9) can be simplified. This gives us

$$E[costs\ incurred\ in\ one\ cycle] = (c_{\mathrm{T}} - c_{\Delta})e^{-\beta(\mathrm{T}-l)} + c_{\Delta}. \tag{10}$$

Now, using the renewal-reward theorem stated in equation (1), we divide equation (10) by equation (8). This gives us a closed-form expression for our CA's long run expected cost per unit time from its cleanup activities. We get

$$CA's\ long\ run\ expected\ cost = \frac{(c_{\mathrm{T}} - c_{\Delta})e^{-\beta(\mathrm{T}-l)} + c_{\Delta}}{1+\beta l}, \tag{11}$$

with probability one. Our final task in this paper is to determine the value of the cleanup limit l for which the CA's long run expected cost in equation (11) is minimal.

3.4. The cost minimization problem

Our CA solves

$$min_{\{l>0\}}\left[\frac{(c_{\mathrm{T}}-c_{\Delta})e^{-\beta(\mathrm{T}-l)}+c_{\Delta}}{1+\beta l}\right]. \qquad (12)$$

Differentiating the minimand in equation (12) with respect to $l > 0$ and then setting the resulting derivative equal to zero gives us the first order necessary condition for an optimum. After several steps of algebra, we see that the CA's long run expected cost of cleanup is minimal for the unique value of the cleanup limit l that solves the equation

$$\beta l e^{-\beta(\mathrm{T}-l)} = \frac{c_{\Delta}}{c_{\mathrm{T}}-c_{\Delta}}. \qquad (13)$$

This completes our discussion of the second probabilistic approach to the cleanup of the Ganges in which the CA attempts to remove all the pollutants from the designated portion of the river.[15]

As in section 2, we can once again demonstrate the nexus between the cleanup problem being studied in this section and the sustainability of the tourism industry in Varanasi. To this end, let us concentrate on the cleanup limit l. We posit that the sustainability of the tourism industry (S) is an increasing function $G_1\{\cdot\}$ of the cleanliness of the Ganges and some sites along the Ganges (C). In symbols, we have $S = G_1\{C\}$. What is different now is that cleanliness (C) is a decreasing function $F_2(\cdot)$ of the cleanup limit l and hence we get $C = G_2(l)$. Combining the preceding two functional relationships, we see that $S = G_1\{G_2(l)\}$. This tells us that as l rises, the cleanliness of the Ganges *falls* and hence so does the sustainability of the tourism industry in Varanasi. We now generally discuss the connections between the two approaches to the cleanup problem studied here and the sustainability of the tourism industry in Varanasi and then suggest

15

The second order sufficiency condition is satisfied because $\left(1/e^{\beta(T-l)}\right)(1+\beta l) > 0$.

two ways in which the research delineated in this paper might be extended.

4. Conclusions

There are three main points to recognize about the two probabilistic approaches to the cleanup of the Ganges that we have studied in this paper. The first (section 2) approach is *strategic* in the sense that the CA recognizes that it does not have the resources to remove every pollutant in the designated portion of the Ganges and hence this CA prioritizes the removal of pollutants with a specific control rule. In contrast, the second (section 3) approach is not strategic but *holistic* in the sense that by following this approach, the CA seeks to attain the highest possible water quality by ridding the designated portion of the Ganges of all possible pollutants. Given these observations, it seems fair to say that the first approach is the more realistic approach and, in addition, that it may also cost less to implement than the second approach.

Next, observe that even though the CA focuses on the long run in both approaches, the specific objective functions in the two approaches are different. In particular, in the first approach, the CA concentrates on the expected *benefit* from removing pollutants from the Ganges but in the second approach, this same CA focus on the expected *cost* of removing pollutants. As such, the first approach is more general than the second approach because expected benefit is a broader criterion than is expected cost.

Finally, a key rationale for cleaning the Ganges is to ensure the sustainability of the tourism industry in the ancient city of Varanasi. In this regard, we know from the discussion in section 1 that the number of present and future tourists in Varanasi depends significantly on the cleanliness of the Ganges. In turn, this cleanliness depends on the enactment of a cleanup program along the lines discussed in this paper. It is not possible to unambiguously say which of the two approaches studied in this paper is likely to make the designated portion of the Ganges cleaner. However, what we can say is that if properly implemented then both approaches will positively impact Varanasi's long run ability to attract tourists and hence the sustainability of the tourism industry.

Here are two suggestions for extending the research described in this paper. First, it would be interesting to use studies such as the one in Tai *et al.* (2016) and generalize the analysis conducted here by studying the maximization of social welfare in Varansi when this welfare depends not only on the actions of a CA but also on the actions of city residents and tourists. Second, it is often the case that those who benefit by polluting the Ganges are distinct from those who bear the actual cost of this pollution. Therefore, it would be useful to analyze the role that regulations on the discharge of pollutants in the Ganges along with temporal guidelines on alternate touristic activities together have on the ability of Varanasi to utilize the services provided by the Ganges in a sustainable manner. Studies of the cleanup of the Ganges with a view to promoting tourism that incorporate these aspects of the problem into the analysis will provide further insights into river water management questions that have both theoretical and practical implications.

References

1. Alley, K.D. 1992. On the banks of the Ganga, *Annals of Tourism Research*, 19, 125-127.
2. Batabyal, A.A. 2000. A renewal theoretic approach to environmental standard setting, *Applied Mathematics Letters*, 13, 115-119.
3. Batabyal, A.A., and Beladi, H. 2017. Cleaning the Ganges in Varanasi to attract tourists, *Atlantic Economic Journal*, 45, 511-513.
4. Bhandari, A. 2015. Varanasi: Development, branding, and marketing of Varanasi as a smart heritage city, TOURISMFIRST.org, May 13, http://www.tourismfirst.org/varanasi%E2%80%8B-development-branding-marketing-of-varanasi-as-a-smart-heritage-city/ Accessed on April 10, 2018
5. Chitravanshi, R. 2014. Tourist guides to hold annual meeting in Varanasi, devise clean Ganga strategy, *Business Standard*, June 28, http://www.business-standard.com/article/current-affairs/tourist-guides-to-hold-annual-meeting-in-varanasi-devise-clean-ganga-strategy-114062500741_1.html Accessed on April 10, 2018.
6. Dhillon, A. 2014. Ganga management, Post Magazine, *South China Morning Post*, http://www.scmp.com/magazines/post-magazine/article/1589301/ganga-management Accessed on April 10, 2018.
7. Markandya, A., and Murty, M.N. 2000. *Cleaning-up the Ganges*. Oxford University Press, New Delhi, India.
8. Markandya, A., and Murty, M.N. 2004. Cost-benefit analysis of cleaning the Ganges: Some emerging environment and development issues, *Environment and Development*

Economics, 9, 61-81.

9. Parth, M.N. 2017. India tries to clean up holy city of Varanasi, but buffaloes and humans stand in the way, *Los Angeles Times*, April 8, http://www.latimes.com/world/asia/la-fg-india-varanasi-2017-story.html Accessed on April 10, 2018.
10. 10. Ramachandran, S. 2014. Cleaning up the Ganges, *The Diplomat*, September 15, https://thediplomat.com/2014/09/cleaning-up-the-ganges/ Accessed on April 10, 2018.
11. Rinschede, G. 1992. Forms of religious tourism, *Annals of Tourism Research*, 19, 51-67.
12. Ross, S.M. 2003. *Introduction to Probability Models*, 8th edition. Academic Press, San Diego, CA.
13. Tai, M., Chao, C.C., Lu, L., Hu, S, and Wang, V. 2016. Land conservation, growth, and welfare, *North American Journal of Economics and Finance*, 38, 102-110.
14. Tijms, H.C. 2003. *A First Course in Stochastic Models*. Wiley, Chichester, UK.