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Status, fertility, growth and the great transition*

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Abstract

We develop an overlapping generation model to examine how the relationship between status concerns, fertility and education affect growth performances. Results are threefold. First, we show that stronger status motives heighten the desire of parents to have fewer but better educated children, which may foster economic development. Second, government should sometimes postpone the introduction of an economic policy in order to maintain the process of economic development, although such a policy aims to implement the social optimum. Third, status can alter the dynamic path of the economy and help to explain the facts about fertility during the “great transition”.

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1 Introduction

The idea that individuals derive utility not only from their level of consumption but also from their relative position in society is by now well established. Also nicknamed “keeping up with the Joneses”, this concept has been used to explain the equity premium puzzle in finance (Abel, 1990), to raise issues about taxation (Ljungqvist and Uhlig, 2000) and competition (Dixon, 2000). In growth models, it has been used to analyze the impact of social relations on growth performance (Corneo and Jeanne, 1997; Fershtman, Murphy and Weiss, 1996; Futagami and Shibata, 1998; Pham, 2005; Tournemaine and Tsoukis, 2008).

In this paper, we investigate how such status considerations can alter individuals’ decisions to bring up and educate children. This issue is important for two reasons. First, there seems to be a trade-off between the expectation to achieve a position in society and the desire to bring up children (Tournemaine, 2008). Recent empirical studies have shown a parallel increase in women’s level of education and participation into the labor force at the same time as a decrease in fertility rates (see Black and Juhn, 2000; Dolado, Felgueroso and Jimeno, 2001 and 2002; Sheran, 2007). In this paper, we then attempt to give a possible explanation to the upstream factor inducing such behavior. We argue that the decision to have children can be seen as a threat to achieving any career and obtain one’s place in society. This notion parallels that of Becker (1991) who pointed out that bringing up children into adulthood is costly, especially in terms of the mother’s time because women are the primary providers of child care.

Second, the link between quality (education) and quantity of children has been recognized for many years in the literature. It is now well admitted that economic development goes along with a general decline in fertility rates and an increase in investment in education (Galor and Weil, 1999; Doepke, 2004). The existence of a trade-off regarding the parents’ decisions between the quality and quantity of children is often considered as a factor which has contributed to the transition of economies from a stage of stagnation (poverty trap) to perpetual growth (Becker, Murphy and Tamura, 1990; hereafter BMT). The bottom line of this is that if obtaining a higher social status in society affects the number of children parents decide to bring up (see above), this is likely to alter the
trade-off between quality and quantity of children. More precisely, if seeking greater social status induces parents to increase their investment in the education of their children, social status is a factor which can perhaps help to explain why countries have left the poverty trap in which they seemed to be stuck and initiated a “great transition” to a state of development.¹

Galor and Weil (1999) argue that the process of economic development can be divided in three main stages: a Malthusian regime, characterized by stagnation and underdevelopment where fertility and mortality are high; then, a Post Malthusian Regime where there is an acceleration of technological progress and an increase in per capita income accompanied by a decline first in mortality, and a rise, then a fall in the fertility rate; and finally, a modern regime where incomes per capita is high and fertility and mortality rates are low. Thus, in a plot of net fertility (i.e., the population growth rate) against growth, or the stage of development, the data appears to show an inverted U-shaped curve (see Galor, 2005; and Section 4 for the case of some Asian countries). Accounting for this evidence seems a challenge for BMT, as their analysis implies only a drop of fertility across the two regimes. However, reconsidering the BMT framework enhanced to account for social status, we show that the transitional dynamics can replicate such empirical evidence.

Moreover, we show that introducing social aspirations in the basic BMT framework has important policy implications. In order to eliminate the distortion arising from the consumption externality (“keeping up with the Joneses”) and implement the optimum, a major conclusion of the relevant literature is that government intervention by means of a tax on consumption is desirable. In contrast, the interesting result here is that the policy-maker should sometimes postpone the introduction of such a policy in order to maintain a long-run self-sustaining economic growth. The idea that an economic policy is

¹While the broad outlines of the Industrial Revolution and the "great transition" are known, by no means are all the details clear or agreed upon; a rapidly expanding literature attempts to engage with the specifics. E.g., the Crafts and Mills (2007) findings suggest that the "Malthusian" regime may have ended in Britain earlier than generally thought, possibly during the 17th century. If so, an intermediate "Smithian (pre-industrial) growth" period characterised the late 17th and most of the 18th centuries, whereby some growth occurred by the improvement of agriculture, better organisation of markets, etc. (see Mokyr, 2000). However, engaging with the literature on the "great transition" in a thorough way, apart from analysing the fertility-growth relationship, is beyond the scope of this.
not desirable and might be postponed is well known in other fields of economics such as trade and finance.\(^2\) However, to our knowledge, this notion has not been raised in a theoretical growth model, although some authors have investigated the relationship between trade liberalisation and growth. For instance, empirical findings of Greenaway, Morgan and Wright (2002) suggest that trade liberalization has resulted in both an increase and decrease in the growth rate depending on country circumstances. In this paper, the argument in favour of (or against) the postponement of the consumption-tax policy is that this tax reduces the return to investment in children’s education. That is, it gives incentives to parents to replace quality of children with more quantity; as a result, parents could stop investing in education. Thus, although the policy aims to maximize welfare, it could also break the process of economic development and the country could end up in a poverty trap. In order to avoid this situation, the government should implement the social optimum only when the return to investment in education is sufficiently high to maintain incentives to educate children even after the introduction of the tax.

Thus, our contribution is summarised as follows: We introduce status considerations in the BMT framework, and investigate how they alter individuals’ decisions to bring up and educate children, ceteris paribus we examine how they affect growth and development. It is worthwhile to note here that in comparison with the basic literature focusing on growth and social status, we give an alternative approach to the growth process by placing both human capital and fertility at the centre of the analysis. We also derive policy implications, particularly the desirability of postponing the otherwise optimal policy of offsetting consumption externalities by taxation. We finally develop the transitional dynamics of the model, hitherto ignored, and show that our framework can replicate the empirical observation that there is an inverted U-shaped relationship between fertility and development.

The remainder of the paper is organized as follows. In Section 2, we present the model.

\(^2\)For example, since the early 90’s, the “Washington Consensus” has largely influenced the policy reforms like trade and financial liberalization in emerging countries. Some authors such as Stiglitz (1998) and Williamson (2000) criticised these reforms arguing that the “Washington Consensus” did not account for the difference in the countries’ state of development. As a result, the policy reforms have not been as successful as expected in many countries of Latin America, Africa and South East Asia.
In Section 3, we examine its key properties regarding the status motives of individuals, fertility, education and growth. The transitional dynamics is developed in Section 4. We conclude in Section 5.

2 Model

The main building block of the model is taken from BMT. Time, denoted by $t$, is discrete and goes from 0 to $\infty$. The economy is populated by overlapping generations of people who live for two periods: childhood and adulthood. All decisions are made in the adult period of life. Each adult individual is endowed with one unit of labor that she supplies inelastically between the production of a consumption good and raising children to adulthood. Parents and children are linked through altruism, i.e. parents care about their own welfare but also that of each of their children. Following Rauscher (1997), Fisher and Hof (2000), Tsoukis (2007), the social status of adult individuals is measured by the ratio of their level of consumption, $c_t$, to the average level of consumption of all other individuals, $\bar{c}_t$, which is taken as given. Formally, preferences of an adult individual at time $t$ are given by

$$V_t = [(c_t/\bar{c}_t)^\gamma]^{1-\sigma}(1-\sigma)/(1-\sigma) + \alpha (n_t)^{1-\varepsilon} V_{t+1},$$

(1)

where $\gamma > 0$, $0 < \sigma < 1$, $0 < \alpha < 1$, $0 < \varepsilon < 1$, $n_t$ is the number of children that a parent has, $V_{t+1}$ is the level of utility that a child will attain as an adult and $\Psi(c_t/\bar{c}_t)$, where $\Psi(\cdot)$ is strictly increasing, represents the preference regarding social status. The parameters $\varepsilon$ and $\alpha$ are respectively the elasticity of altruism with respect to the number of children and the degree of altruism of parents toward children.

Observe that the standard preferences (e.g. Barro and Becker 1988, 1989) correspond to the case of $\gamma = 0$ whereby individuals do not derive utility from their social status. When $\gamma > 0$, the utility of an adult individual exhibits a consumption externality: the “keeping up with the Joneses”. That is, the average level of consumption of people has a negative effect on the level of utility of an individual. It will be useful to keep this feature in mind when we will study the properties of the model.

A second remark is that the restriction on the value of $\sigma$ to the range $(0, 1)$ is not directly comparable to its value in more standard growth models where we should as-
sume values higher than 1 because this is the case which is empirically supported (see Hall, 1988). As explained by Ehrlich and Lui (1997) this is “because in the generational frameworks $\sigma$ denotes the inverse of the elasticity of substitution in consumption across consecutive generations, rather than years. Thus, if a generation spans 25 calendar years, a value of $\sigma$ less than 1 could be equivalent to a value substantially above 1 in these other models” (see footnote 8, pp. 229).

Finally let us mention that there exists other definitions of social status. For instance, Corneo and Jeanne (1997), Futagami and Shibata (1998), Long and Shimomura (2004), Pham (2005), define it as the level of wealth relative to that of others. This would suggest introducing physical capital accumulation in the model. Adding this variable would complicate, but not “wash away”, the effects we are discussing here. Similarly, like Fershtman, Murphy and Weiss (1996), we could define social status as the relative level of human capital of individuals. Such a specification would not affect the qualitative results of the paper either.

Raising a child is costly. We assume that it takes a fixed amount of consumption good, $f$, and a fixed amount of time $v$ to bring up one child to adulthood, where $f > 0$ and $v > 0$. Moreover, parents can use an extra amount of their time, $e_t > 0$, to teach each child. The level of human capital of children depends on the level of human capital of their teachers-parents and on the amount of time their parents spend to teach. The technology of human capital is

$$H_{t+1} = \phi e_t H_t + H_0,$$

(2)

where $\phi > 0$ is a productivity parameter and $H_0$ is the innate level of skills of an individual: if individuals do not invest in education ($e_t = 0$), the level of skills of individuals is given by $H_0$.

Assuming a linear technology for the production of the consumption good, the resource constraint is given by

$$l_t H_t = c_t + fn_t,$$

(3)

$^3$We have changed the specification of the human capital technology slightly from that employed by BMT. Our equation here ensures that human capital is positive even with no investment in education and is internally consistent.
where \( l_t \) is the time spent by an adult to the production of the consumption good and \( fn_t \) represents the total cost in terms of the consumption good of raising \( n_t \) children. Finally, since people have a fixed time endowment equal to unity, the time constraint is

\[
1 = l_t + n_t (v + \varepsilon_t),
\]

(4)

where \( n_t (v + \varepsilon_t) \) represents the total cost in time to raise \( n_t \) children.

3 Equilibrium

3.1 The representative individual’s problem

In this Section, we compute the equilibrium conditions of a representative individual’s maximization problem. For simplicity, we focus on a symmetric equilibrium, whereby individuals are identical. The problem of an adult individual consists of choosing \( l_t, \varepsilon_t, n_t, H_{t+1} \) that maximize (1) subject to (2), (3), (4). After substitution, the problem can be written as

\[
V_t(H_t) = \max_{n_t, H_{t+1}} \frac{1}{1 - \sigma} \left\{ H_t \left[ 1 - n_t \left( v + \frac{H_{t+1} - H_0}{\phi H_t} \right) \right] - fn_t \right\}^{(1-\sigma)} \times
\Psi \left\{ \left\{ H_t \left[ 1 - n_t \left( v + \frac{H_{t+1} - H_0}{\phi H_t} \right) \right] - fn_t \right\}^{\gamma(1-\sigma)} \right. \\
+ \alpha (n_t)^{1-\varepsilon} V_{t+1}(H_{t+1}).
\]

Manipulation of the first order condition with respect to \( n_t \) and using the property of symmetry (e.g. \( c_t = \bar{c} \)) yields

\[
\alpha (1 - \varepsilon) (n_t)^{-\varepsilon} V_{t+1}(H_{t+1}) = [H_t (v + \varepsilon_t) + f] [1 + \gamma \Delta] (c_t)^{-\sigma} [\Psi(1)]^{\gamma(1-\sigma)},
\]

(5)

where \( \Delta \equiv \Psi'(c_t/\bar{c}_t)c_t/\bar{c}_t/ \Psi(c_t/\bar{c}_t)\bar{c}_t = \Psi'(1)/\Psi(1) \) is the elasticity related to the status effect: \( \Delta \) measures how much individuals care about their status. Futagami and Shibata (1998) explain that \( \Delta \) is the strength of status preference: the greater the value of \( \Delta \), the stronger are the status motives of individuals. The left hand side of (5) represents the marginal benefit of an additional child and the right hand side is the marginal cost in terms of utility. Note that the cost of an additional child increases in \( \Delta \). This is because raising
children takes time that could alternatively be used to increase the production of the consumption good which in turn would increase the relative position of the adult-parent in the society.

Manipulation of the first order condition with respect to $H_{t+1}$ yields

$$
(1 + \gamma \Delta) (c_t)^{-\sigma} [\Psi(1)]^{\gamma(1-\sigma)} \geq \alpha \phi (n_t)^{-\sigma} \frac{dV_{t+1}}{dH_{t+1}},
$$

where equality holds if $e_t > 0$. Moreover, the envelope condition implies

$$
\frac{dV_{t+1}}{dH_{t+1}} = (1 - v n_{t+1}) (1 + \gamma \Delta) (c_{t+1})^{-\sigma} [\Psi(1)]^{\gamma(1-\sigma)}.
$$

Combining (6) and (7) yields

$$
\left( \frac{c_{t+1}}{c_t} \right)^{\sigma} \geq \alpha \phi (n_t)^{-\sigma} (1 - v n_{t+1}),
$$

where the left hand side is the marginal rate of substitution between consumption of parents and children and the right hand side represents the return of investments in human capital.

### 3.2 Basic properties

In this sub-section, we analyse how social status affects the choice of fertility and education of individuals and its welfare implication. For simplicity, we restrict our attention to steady-state equilibria, i.e. we focus on the long-run. The short-run effects of social status are examined in Section 3.3 in which we characterise the transitional dynamics.

#### 3.2.1 Steady-state equilibria

The basic structure of the model combined with the fact that we focus on the symmetric individuals case implies that three steady-state equilibria can be shown to exist (see BMT for more details): a stable Malthusian poverty trap, a stable state of persistent and self-sustaining growth and an unstable intermediate state of development. Proposition 1 summarizes these results, where the symbols “$u^*$”, “$*$” are used to denote respectively the value of variables in the Malthusian and self-sustained growth steady-states. A hat, “$\hat{\cdot}$”, on a variable denotes its value in the unstable intermediate steady-state.
For convenience, the following parameter restrictions are assumed throughout the paper:

**Assumption 1:**

\[
\frac{[1 - v(\alpha/\varepsilon)^{1/(\varepsilon-1)}] H_0 - f(\alpha/\varepsilon)^{1/(\varepsilon-1)}}{(H_0v + f)(1 + \gamma \Delta)} < \frac{(1 - \sigma) [((\alpha/\varepsilon)^{\varepsilon/(\varepsilon-1)} - \alpha(\alpha/\varepsilon)^{1/(\varepsilon-1)}]}{\alpha (1 - \varepsilon)}.
\]

**Assumption 2:**

\[1 - \varepsilon - (1 - \sigma)(1 + \gamma \Delta) > 0.\]

Assumption 1 guarantees the uniqueness of the solution for the equilibrium fertility rate in the Malthusian steady-state. Assumption 2 guarantees the existence of a growing steady-state where the amount of time allocated to teaching activities is strictly positive.

**Proposition 1** *In the Malthusian steady-state, parents do not invest in education of their children (e.g. \( e_u = 0 \)) which leads to zero economic growth. Under Assumption 1, the number of children they bring up, \( n_u \), is unique. It is the solution of*

\[
\frac{(1 - vn_u)H_0 - fn_u}{(H_0v + f)(1 + \gamma \Delta)} = \frac{(1 - \sigma) [(n_u)^{\varepsilon} - \alpha n_u]}{\alpha (1 - \varepsilon)},
\]

*and verifies*

\[(n_u)^{\varepsilon} > \alpha \phi [1 - vn_u].\]

*In the growing steady-state, individuals choose to bring up a number of children, \( n^* \), which is the solution of*

\[
\left[\frac{v \phi (1 - \sigma)(1 + \gamma \Delta)}{1 - \varepsilon - (1 - \sigma)(1 + \gamma \Delta)}\right]^{\sigma} = \alpha \phi (n^*)^{-\varepsilon} (1 - vn^*).
\]

*The amount of time they allocate to teaching activities, \( e^* \), is given by*

\[e^* = \frac{v (1 - \sigma)(1 + \gamma \Delta)}{1 - \varepsilon - (1 - \sigma)(1 + \gamma \Delta)}.\]

*The common growth rate of consumption and human capital, \( g^* \), is given by*

\[g^* = \phi e^* - 1.\]

*In the intermediate state of development, the fertility rate is the solution of*

\[(\hat{n})^{\varepsilon} = \alpha \phi (1 - v\hat{n}).\]
The human capital of an individual, $\hat{H}$, is constant over time which implies an economic growth rate equal to zero. It is given by
\[
\left[1 - \hat{n} (v + \tilde{e})\right] \frac{\hat{H} - f \hat{n}}{\hat{H} + (v + \tilde{e}) + f} = \frac{(1 - \sigma) \left[\left(\hat{n}\right)^{\varepsilon} - \alpha \hat{n}\right]}{\alpha (1 - \varepsilon)}.
\]

where investment in education, $\tilde{e}$, is given by
\[
\tilde{e} = \frac{\hat{H} - H_0}{\phi \hat{H}}.
\]

**Proof.** See Appendix.

A Malthusian steady state is a corner solution: $e = 0$. It arises because the rate of return to investment in the quality of children (human capital) is too low relative to the return to investments in the quantity of children. The return from investing in children is given by the left hand side of (9). If such a steady-state occurs, it is stable. The reason is that the condition (10) holds with a strict inequality. Thus, even if the level of human capital becomes strictly positive, for sufficiently low values of $H_t$ this does not reverse the inequality. Consequently, the economy will return to the steady-state where $H_t = H_0$ in all periods. When the level of human capital is sufficiently high, however, the returns to investment in human capital are large enough to break the corner solution (see equation (8)): the quantity-quality trade-off turns out towards children quality instead of quantity. This is the force that puts the economy on a convergent path to the steady state with positive growth.

Between the Malthusian and self sustained growth steady-states, an intermediate state of development can emerge. This steady-state is however unstable. If the initial level of human capital is higher than the threshold level $\hat{H}$, individuals find it profitable to invest in education and the economy will keep on growing. On the other hand, if the initial level of human capital is lower than the threshold level, $\hat{H}$, the economy will converge towards the Malthusian steady-state because, in this case, the income effect dominates the substitution one. Figure 1 gives a graphical representation of the above results.

Insert Figure 1 here
3.2.2 Effects of social aspirations

Proposition 1 allows us to analyze the effects of social aspirations on the choice of fertility and education in each of the three steady-states in which the economy can end up. From equation (9), the choice of fertility of individuals is negatively correlated with the strength of social aspirations, $\Delta$. The reason is that higher social aspirations increase the cost of bringing up additional children (see equation (5)). Thus, adult individuals reduce the number of children and allocate more time to the production of the consumption good because they expect to improve their relative position in the society. The outcome is that condition (10) becomes less restrictive. Thus, as social aspirations increase, a Malthusian steady state is less likely to occur.

Examination of equations (14), (15), (16) gives a confirmation of this. We can see that stronger status motives reduce the threshold level of human capital required to switch to the self sustained growth steady-state. Therefore, social status heightens the parents’ decision to substitute quality for quantity of children.

Once the economy is on the self-sustained growth steady-state, stronger social aspirations induce a reduction of the fertility rate, $n^*$, and an increase of investments in education, $e^*$. Thus, in turn, stronger status motives foster economic growth, $g^*$, (see equations (11), (12), (13)). The reason is that parents take into account that, for their children, education is the means to increase the production of the consumption good, thereby their relative position in the society: as generations are linked through altruism, the success of children in terms of social achievement affects the welfare of parents who prefer to reduce the number of children so as to improve their quality.

Figure 2 gives a graphical representation of the effects of an increase in the strength of social status, $\Delta$, on the three possible steady-states. The plain line, called $C_1$, represents the solution of the model. The dotted line, called $C_2$, represents the polar case in which individuals do not derive utility from social status (e.g. we set $\gamma = 0$). Note that $C_2$ represents the socially optimal, benchmark solution of BMT in which social aspirations do not affect preferences of individuals. In Figure 2, we denote by $\left(\hat{H}\right)^0$ the threshold level of human capital of the unstable intermediate state of development in this benchmark case.
It is important to mention that in the perpetual growth regime and in the unstable intermediate state of development, investments in education are excessive relative to the situation which is socially optimal (for which $\gamma = 0$). This feature finds empirical support in the literature (see for instance Kodde and Ritzen, 1984; Oosterbeek and Webbink, 1995). Hence, from the model, we can then argue that preferences for social status are a possible factor causing these excessive investments.

3.2.3 Social optimum and economic policy

As explained in the previous sub-section, the solution given in Proposition 1 is not socially optimal because of the presence of the external effect caused by social aspirations. It is thus natural to think about an economic policy which can eliminate this market failure in order to implement the optimum. To this end, we assume the government’s intervention by means of a tax $\tau_t$ charged on the level of consumption of an adult individual. By increasing the price of consumption, the policy-maker increases the cost at which individuals can improve their relative position in society. Then, by choosing an appropriate level of tax, she can obtain an optimal allocation of resources.

For simplicity, we assume that this policy is funded through a lump sum transfer $T_t$ from individuals and that the budget constraint of the government is balanced at each moment: $T_t = \tau_t c_t$, at all times. Computations lead to the following Proposition:

**Proposition 2** If government authorities choose a tax rate, $\tau^o$, such that

$$\tau^o = \gamma \Delta \text{ at all times},$$

the equilibrium is optimal.

**Proof.** See Appendix. ■

The relevance of this tax is illustrated in Figure 2. The introduction of the economic policy will induce a downward shift of the curve $C_1$ for which $\gamma > 0$ to make it coincide with the optimal solution represented by the curve $C_2$ for which $\gamma = 0$. The question we
raise at this stage is the following: should the government introduce the economic policy tool as proposed in Proposition 2? The answer to this question depends on the level of human capital of individuals when the policy is introduced. From Figure 2, if the level of human capital of individuals is lower than $\hat{H}$ or greater than $(\hat{H})^o$, the government can introduce the economic policy tool in order to reach the optimum; in this case, the introduction of the tax does not alter the kind of steady-state the economy will end up in (i.e. Malthusian or self sustained growth). If the human capital of individuals belongs to the set $[\hat{H},(\hat{H})^o]$, however, the tax will tilt the economy which was developing towards the Malthusian steady-state poverty trap. Thus, one should wonder whether it is optimal for a country to choose to become poor. If it is not, government authorities should postpone the introduction of their economic policy until individuals attain a sufficiently high level of human capital (e.g. slightly greater than $(\hat{H})^o$) in order to maintain the incentives to invest in education. In this case, a long-run self-sustaining growth can be maintained and the optimum can be reached later.

4 Transitional dynamics

In this Section, we study the transitional dynamics of the model; to our knowledge this has not been done before. As mentioned, this issue is interesting because it allows us to confront the model to a larger extent to empirical evidence. In particular, as shown by Galor (2005), there is an inverted U-shaped relationship between net fertility (i.e. population growth) and income during the transition from the Malthusian steady-state to the state of perpetual development. We may view the 18th century, with its prolonged period of stability and processes of "Smithian growth" as mentioned above, as a "shock" that gave an impetus to fertility and growth beyond the threshold derived in the previous section. Such empirical feature can also be observed for some Asian countries which displayed good growth performances since the 1960s. Indeed, using annual data from the IMF (International Financial) over the period 1951-2005, Figure 3 shows the evolution of the population growth rate in Hong-Kong (HK), Singapore (SI), Malaysia (MA), Thailand (THA), South Korea (KO), Indonesia (IN) and Philippines (PHI).
We can observe that population growth rate increase before falling down in these
countries: so, countries that exhibited a transition from Malthusian to endogenous growth
regimes showed an inverted U-shaped relationship between population growth and income.
In such context, the aim of this Section is to show that an increase in preferences for
social status can help to explain the above empirical facts. As mortality here is fixed (all
individuals live for two periods), our fertility rate \( n_t \) is equivalent to population growth.

To characterise the transitional dynamics we develop a \( 2 \times 2 \) linearized system in the
amount of time devoted to schooling and human capital around the asymptotic steady-
state of perpetual development. We give a more intuitive exposition here, and relegate
the more formal details to the Appendix. A reminder that \( H_t, e_t \) and \( n_t \) denote the actual
values of human capital, time devoted to schooling and fertility while \( H_t^*, e^*, n^* \) denote
the paths of the variables that would result if the economy were on its steady-state growth
path, growing at a rate \( g^* \) (see Proposition 1). In this Section, deviations, indicated by a
tilde, are the percentage deviations from the path of perpetual development: we denote
\( \tilde{h}_t = (H_t - H_t^*)/H_t^*, \tilde{e}_t = e_t - e^* \) and \( \tilde{n}_t = n_t - n^* \) these deviations. After tedious
computations described in the Appendix, one gets the following system:

\[
\begin{bmatrix}
\tilde{h}_{t+1} \\
\tilde{e}_{t+1}
\end{bmatrix} = \begin{bmatrix}
1 & \phi/(1 + g^*) \\
0 & \chi
\end{bmatrix} \begin{bmatrix}
\tilde{h}_t \\
\tilde{e}_t
\end{bmatrix}.
\]  
(17)

Obviously, there exist a unit root and a stable root \( \chi \), where
\[
\chi \equiv \frac{\sigma M - \varepsilon N/n^* - \sigma \phi/(1 + g^*)}{N(v(1 - vn^*))^{-1} + \sigma M}.
\]  
(18)

It will also be useful below to note during the transition, deviations of the fertility rate
are related to education effort by:

\[
\tilde{n}_t = N\tilde{e}_t,
\]  
(19)

with
\[
N \equiv -\frac{(1 - \varepsilon) e^*/(v + e^*)^3 + [\varepsilon + (1 - \varepsilon) n^*e^*]/(v + e^*)^2}{1 + \varepsilon [1 - n^*(v + e^*)]/[n^*(v + e^*)]} < 0.
\]

Thus, fertility and educational effort move in opposite directions during transition.
We assume that the parameters of the model are such that $0 < \chi < 1$ to ensure saddle-point stability and avoid oscillatory trajectories. To require saddle-point stability makes sense as $e_t$ is a “jump variable” (closely linked to $n_t$, the number of children per parent). Human capital, however, is a state variable and likely to change more slowly: it is a “predetermined” variable.

From (17), we can plot the phase diagram (see Figure 4). Standard computations show that the slope of the stable arm is given by $\phi/[\chi(1 + g^*)] > 0$. In terms of deviations, the origin of the graph $(0,0)$ represents both the Malthusian and steady-state growth equilibria. Point B represents the situation at $T$, i.e. after a shock to the system (17) has occurred, corresponding here to an increase in the preferences for social status, $\Delta$, that has moved the system from the Malthusian to the endogenous growth regime.

While educational effort $e_t$ is a variable that is free to jump, human capital in terms of deviations also jumps, but not freely. Its instantaneous jump is determined as follows. Let us assume a positive shock such as a perpetual increase in the preferences for social status occurring at time $t = T$ so that $e_t = 0$ for all $t < T$ (i.e. the economy is initially in the Malthusian regime). At that time, actual human capital is $H_T = \phi e_{T-1}H_0 + H_0 = H_0$ because in the Malthusian steady-state we have $e_{T-1} = 0$. By definition, reference human capital is the one that grows at rate $g^*$. Hence, the instantaneous jump at $t = T$ is $\tilde{h}_T = -(e^*\phi - 1)/e^*\phi < 0$. This means that the jump is downward on the upward-sloping stable arm. Since social status affects positively the amount of time parents allocate to teaching activities, we can argue that the size of the jump of human capital at the time of the shock is positively correlated with the size of the shock (formalised here by an

\[ \tilde{h}_T = (H_T - H^*_T)/H^*_T = (H_0 - (1 + g^*)H_0)/(1 + e^*\phi)H_0 \] from $e_{T-1} = 0$, with $1 + g^* = e^*\phi$. It is implicitly assumed that the shock is large enough to induce perpetual investments in human capital, i.e. $H_{T+1} > \tilde{H}$. The reason is that this is the relevant case. Indeed, if $H_{T+1} = \tilde{H}$, there are no transitional dynamics because the economy has jumped to the (unstable) intermediate state of development; and if $H_{T+1} < \tilde{H}$, this implies that individuals will stop investing in education; the economy will then stay in the Malthusian regime.

\[ 4 \] The shock is calculated as $\tilde{h}_T = (H_T - H^*_T)/H^*_T = (H_0 - (1 + g^*)H_0)/(1 + e^*\phi)H_0$ from $e_{T-1} = 0$, with $1 + g^* = e^*\phi$. It is implicitly assumed that the shock is large enough to induce perpetual investments in human capital, i.e. $H_{T+1} > \tilde{H}$. The reason is that this is the relevant case. Indeed, if $H_{T+1} = \tilde{H}$, there are no transitional dynamics because the economy has jumped to the (unstable) intermediate state of development; and if $H_{T+1} < \tilde{H}$, this implies that individuals will stop investing in education; the economy will then stay in the Malthusian regime.
increase of preferences for social status). Graphically, at the time of the shock \((t = T)\), both \(\tilde{h}_T\) and \(\tilde{e}_T\) jump from the origin to point \(B\) (see Figure 4). Then, both variables start sluggishly approaching the origin again along the upward-sloping thick stable arm. In economic terms, this means that \(c_t\) approaches its (asymptotic) steady-state value from below: along the transition, individuals invest an increasing amount of time to education (see Figure 5a). While doing so, human capital grows at a rate which is lower than the asymptotic growth rate \(g^*\) and approaches its steady-state value from below too (see Figure 5b).

Regarding the fertility rate, we have that \(n_t\) approaches its steady-state value from above (see equation (19)). The interesting result here is that we can obtain an inverted U-shaped relationship between fertility and income as shown by empirical observations.

To prove this result, we convert the above system in terms of deviations of the growth rate from its endogenous growth equilibrium. Using (17) and (19) yields

\[
\tilde{h}_{t+1} - \tilde{h}_t = \phi \tilde{n}_t / [N(1+g^*)].
\]

Taking a Taylor approximation of (2), we can approximate \(g_t \equiv g_t - g^* \approx (1 + g^*)(\tilde{h}_{t+1} - \tilde{h}_t)\). Simple computations yield:

\[
\tilde{n}_t = \frac{N \tilde{g}_t}{\phi},
\]

which represents the saddle path of fertility and income; it is the equivalent of (19) in \((n, g)\) space. As \(N < 0\), the saddle path is downward sloping (see equation (20)).

The empirical evidence and the predictions of the model regarding fertility and growth are summarised in Figure 5. \(M\) and \(EG\) represent the steady-state Malthusian and endogenous growth equilibria, respectively. The continuous hump-shaped line represents the real-world evidence on the movement of fertility and growth in countries along the ”great transition” (Galor, 2005). The dotted line \((M' - EG)\) represents the negatively-sloped saddle path (20); section \((M - M')\) in Figure 6 (the broken line) is the instantaneous jump at \(T\).

Insert Figures 5a, 5b, 6
To check that the model indeed fits the data, we have to ensure that fertility jumps temporarily (at the time of the shock) before settling on the lower endogenous growth steady state level, $n^*$. From Figure 5, we have that the model produces such an inverted U-shaped relation between fertility and growth if the absolute value of the slope of the saddle arm ($M' - EG$) is higher than the slope of the locus ($M - EG$) represented by the dotted line in the graph. Formally, we must have,

$$\left| \frac{n_u - n^*}{0 - g^*} \right| < \left| \frac{\tilde{n}_t}{g_t} \right| = -\frac{N}{\phi},$$

where the first expression is the slope of ($M - EG$) and the last that of ($M' - EG$) in Figure 5. Rearranging this expression, we can state that at the time of the shock, the economy moves from the Malthusian equilibrium $M$ onto the saddle path, ($M' - EG$) if

$$n_u < n^* + \frac{(1 - \varepsilon) e^*/(v + e^*)^3 + [\varepsilon + (1 - \varepsilon) n^* e^*]/(v + e^*)^2}{1 + \varepsilon [1 - n^*(v + e^*)]/[n^*(v + e^*)]} \left[ \frac{g^*}{\phi} \right].$$

(21)

Under the assumption that (21) is satisfied, the dynamics of the fertility rate is represented in Figure 7.

Insert Figure 7

If (21) holds, ($M - M'$) in Figure 6 is upward sloping, giving rise an upward jump of fertility $n_t$ onto the stable arm; after that, $n_t$ will gradually decline towards its long-run value in the endogenous growth regime, $n^*$. Thus, if this condition holds, a movement can be predicted that can match the inverted U-shaped graph of fertility versus growth found in the data. We assume that this condition holds without loss of generality, as we have some degrees of freedom among the parameters. For example, from the results of Proposition 1, we can see that the inverted relationship between fertility and income is likely to happen if the innate level of skills of individuals, $H_0$, and the cost in terms of consumption good of raising children, $f$, are high. This is because higher values of these parameters have a negative effect on the Malthusian steady-state number of children $n_u$, while they have no effect on the determination of the number of children in the endogenous growth steady-state, $n^*$. 

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The reason why fertility can increase at the time of the shock is that an increase in the level of human capital induces both an income and a substitution effect. While the income effect is positive because children are a normal good, the substitution effect is negative because the time allocated to bring up children can be used for the production of the consumption good. When the level of human capital is low (as it is the case at the time of the shock), the income effect dominates, leading individuals to bring up more children. However, as income (or human capital) rises, the costs of child quality increases as well. For sufficiently high levels of human capital, the substitution effect will dominate leading the individuals to reduce the number of children they bring up. This is the standard quality-quantity trade-off on children, represented by the move \((M' - EG)\) in Figure 6.

From the analysis above, we can note that the effects of status on the slope of the stable arm, (given by \(N/\phi\)) is ambiguous. However, in the case where status makes the stable arm steeper, the jump of fertility at \(T\) will be greater. Thus, the theoretical prediction will be more pronounced, having the potential of matching the data more closely. In that case, status can both decrease the basin of attraction around the Malthusian steady state (see Sections 3.2.2-3.2.3), and change the dynamic path of the economy to match more closely the facts about the “great transition”.

5 Conclusion

This paper has investigated the implication and importance of social aspirations for the great transition in the BMT model. We have shown that social aspirations of people are a possible factor that can contribute to a country’s switch to development. This is because higher status motives increase the return to investment in children’s education relative to the return to investment in the quantity of children. Moreover, we have argued that there exist cases in which government authorities should postpone the introduction of an economic policy, although such a policy aims to implement the social optimum. The reason is that the policy could induce parents to substitute quantity for quality of children, and thus it could break the process of economic development. Finally, the transitional dynamics analysis of the framework has revealed that changes in status concerns affect the dynamic path of the economy and could perhaps help to explain facts about the “great
transition”.

Our framework can be extended in many ways. For instance, the analysis can be extended to frameworks in which there is some social status accruing to children. On the empirical side, an interesting issue would be to test our hypothesis regarding social status, choice of fertility and education in order to measure its impact on economic growth.

6 Appendix

The Malthusian steady-state

As explained in the text, a Malthusian steady state is a corner solution. Adult individuals choose not to invest in education of their children. One has \( c_t = 0 \) in all periods which implies that economic growth is zero: \( H_{t+1} = H_t = H_0 \) in all periods. Then, the levels of consumption and utility of any individual are respectively given by \( c_t = c_{t+1} \) and \( V_t = V_{t+1} \) in all periods. Equation (10) follows directly from equation (8) where \( c_t = c_{t+1} \). This condition must hold with strict inequality because we have a corner solution. To compute (9), we use (1), (5) and the fact that \( c_t = c_{t+1} \) and \( V_t = V_{t+1} \) in all periods.

The growing steady-state

When growth is strictly positive, the level of innate human capital of an individual, \( H_0 \), and the fixed cost of raising a child in terms of the consumption good, \( f \), become negligible in the long-run (as ratios over \( H_t \)). Thus, we can skip these variables in the computation of the steady-state. From (3) and (2), one has \( 1 + g^* = c_{t+1}/c_t = H_{t+1}/H_t = \phi e^* \) at the steady state. Using the fact that equation (6) reads with equality with strictly positive investments in education and combining this equation with (5) one gets after some manipulation

\[
\frac{H_{t+1}}{H_t} = \phi e^* = \frac{\phi(v + e^*)}{(1 - \varepsilon)} \frac{dV_{t+1}}{dH_{t+1}} \frac{H_{t+1}}{V_{t+1}}.
\]

Since the growth rate of human capital and consumption are the same at steady-state, one has

\[
\frac{dV_{t+1}}{dH_{t+1}} \frac{H_{t+1}}{V_{t+1}} = \frac{dV_{t+1}/V_{t+1}}{dH_{t+1}/H_{t+1}} = \frac{dV_{t+1}/V_{t+1}}{dc_{t+1}/c_{t+1}} = \frac{dV_{t+1}/c_{t+1}}{dc_{t+1}/V_{t+1}}.
\]
Assuming that the economy has reached the steady-state, recursive substitution leads to

\[ V_{t+1} = \left( \frac{(c_{t+1}) \Psi (c_{t+1}/t_{t+1})^\gamma}{1-\sigma} \right)^{1-\sigma} \sum \left[ \alpha (n^*)^{1-\varepsilon} (1 + g^*)^{1-\sigma} \right] i. \]

(24)

Thus, from the above equation we have that

\[ \frac{dV_{t+1}}{dc_{t+1}} = (1 - \sigma)(1 + \gamma \Delta). \]

(25)

Plugging the above result in (22), one gets \( e^* \). Then, using (8) which holds with equality, one deduces \( n^* \).

The intermediate steady-state

Using (8) with the fact that \( c_{t+1} = c_t \) in all periods, one gets (14). Using (5) with the fact that growth is zero, one gets (15). Since the level of human capital is such that \( H_{t+1} = H_t = \hat{H} \) in all periods, one gets (16) by using (2).

Optimal taxation

To compute the optimal tax, we first characterize the optimum. This is given by the individuals’s maximization problem stated in section 3.1 in which we set \( \gamma \) to zero. This is because at optimum, the social planner takes into account that individuals are identical: \( c_t = \pi_t \). Her problem is to choose \( l_t, e_t, n_t, H_{t+1} \) that maximize \( V_t = \left[ \frac{(c_t) \Psi (1)^{1-\sigma}/(1 - \sigma) + \alpha (n_t)^{1-\varepsilon} V_{t+1}}{1-\sigma} \right] \), subject to (2), (3), (4). This problem is equivalent to maximize (1) with \( \gamma = 0 \), subject to the same constraints.

Second, to find the optimal tax rate, we must take into account that the resource constraint (3) becomes \( T_t + l_t (H_t + H_0) = (1 + \tau_t) c_t + f n_t \). Following the same steps as in Section 3.1 and comparing the set of first order conditions with the optimum conditions, one gets Proposition 2.

Transitional dynamics

Combining (5) and (6) (with equality in (6) as we are considering the endogenous growth case of \( e_t > 0 \)), we get

\[ \frac{(1 - \varepsilon) V_{t+1}}{[H_t (v + e_t) + f] \phi} = \frac{dV_{t+1}}{dH_{t+1}} \]

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which is a differential equation in \( V_{t+1} \) and \( dV_{t+1}/dH_{t+1} \). The solution is:

\[
V_{t+1} = \Upsilon_t \exp \left[ \frac{(1 - \varepsilon) H_{t+1}/\phi}{H_t(v + e_t) + f} \right],
\]

where \( \Upsilon_t \) is a variable independent of \( H_{t+1} \) that must grow at the same rate as \((c_t)^{1-\sigma}\). Indeed, plugging back the above equation in (5), we get

\[
\alpha (1 - \varepsilon) (n_t)^{-\varepsilon} \Upsilon_t \exp \left[ \frac{(1 - \varepsilon) H_{t+1}/\phi}{H_t(v + e_t) + f} \right] = [H_t(v + e_t) + f] [1 + \gamma \Delta] (c_t)^{-\sigma} [\Psi(1)]^{\gamma(1-\sigma)},
\]

where \( \Upsilon_t \) is a variable independent of \( H_{t+1} \). For simplicity, we specify \( \Upsilon_t = (c_t)^{1-\sigma} \) so that both sides grow at the same rate. Taking a Taylor expansion of this equation and dividing by the reference values (asymptotic steady-state of perpetual development), we get

\[
-\varepsilon \tilde{n}_t/n^* + \frac{(1 - \varepsilon)}{(v + e^*)^2} \tilde{c}_t = \tilde{h}_t + \frac{\tilde{e}_t}{(v + e^*)^2} - \tilde{c}_t,
\]

where \( \tilde{c}_t = (c_t - c^*_t)/c^*_t \) and \( \tilde{n}_t = (n_t - n^*)/n^* \).

Then, combining (3) and (4), and proceeding in the same way as before, we get

\[
\tilde{h}_t = \tilde{c}_t + \frac{(v + e^*) \tilde{n}_t + n^* \tilde{e}_t}{1 - n^*(v + e^*)}.
\]

Finally, linearisation of (2) yields

\[
\tilde{h}_{t+1} = \frac{\phi \tilde{c}_t}{1 + g^*} + \tilde{h}_t,
\]

and linearisation of (8) yields

\[
\sigma (\tilde{c}_{t+1} - \tilde{c}_t) = -\frac{\varepsilon \tilde{n}_t}{n^*} - \frac{v}{1 - vn^*} \tilde{n}_{t+1}.
\]

Equations (26), (27), (28) and (29) is a 4 \times 4 system in deviation from the reference paths for fertility, time allocated to schooling, consumption and human capital. Now, the strategy is to reduce this system to a 2 \times 2 system in deviation from the reference paths for time allocated to school and human capital. To do it, we use (26) and (27) to eliminate \( \tilde{c}_t \) and \( \tilde{n}_t \), and re-write the system in terms of \( \tilde{h}_t \) and \( \tilde{c}_t \). From (27), we can write \( \tilde{n}_t \) as a function of the other variables. Plugging the result in (26) yields the deviations of consumption:
\[ \tilde{c}_t = \tilde{h}_t + M \tilde{e}_t, \]  

(30)

with

\[ M \equiv \frac{(1 - \varepsilon) e^*/(v + e^*)^2}{1 + \varepsilon [1 - n^*(v + e^*)]/[n^*(v + e^*)]} > 0. \]

Combining (30) with (27) yields (19) given in the text. Straightforward manipulations of (28), (29), (30) and (19) yields the system (17) given in the main text.
7 Reference


Growth, The Hague, Kluwer; also available from the authors website, Northwestern University.


Figure 1: Steady state equilibria
Figure 2: Effect of social aspirations on the steady state equilibria
Figure 3: Population Growth rates over the period 1951-2005
Source IMF/IFS
Figure 4: Phase diagram for the system (17)
Figure 5a: Dynamics of the amount of time devoted to education
Figure 5b: Dynamics of the growth rate

Variation of the growth rate one period after the shock occurred at $t=T$. 

$g^* \rightarrow g=0$ from $t=T$ to $t=T+1$. 

$g=0$ 

$t=T+1$ 

Time, $t$
Figure 6: Phase diagram for the fertility rate and income growth
Fertility rate

Variation of the fertility at the time of the shock, $t=T$.

$n_u$

$n^*$

$t=T$

Time, $t$

Figure 7: Dynamics of the fertility rate